Homework 3 PHYS 585

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Problem 1:

a.

$$\tau_m \frac{dV}{dt} = a_L(V - V_L)(V - V_C) + R_m I_e$$
$$V = \alpha x + V_0, I_e = \gamma I + I_0$$
$$\frac{dx}{dt} = \frac{a_L}{\tau_m \alpha}[(\alpha x + V_0 - V_L)(\alpha x + V_0 - V_C)] + \frac{R_m I_e}{\tau_m \alpha}$$
$$\text{Let } V_0 = \frac{V_C + V_L}{2}$$
$$\frac{dx}{dt} = \frac{a_L}{\tau_m \alpha}[(\alpha x - \frac{V_C - V_L}{2})(\alpha x + \frac{V_C - V_L}{2})] + \frac{R_m I_e}{\tau_m \alpha}$$
$$\frac{dx}{dt} = \frac{a_L}{\tau_m}\alpha x^2 - \frac{a_L}{\tau_m \alpha}(\frac{V_C - V_L}{2})^2 + \frac{R_m I_e}{\tau_m \alpha}$$
$$\text{Let } \alpha = \frac{\tau_m}{a_L}$$
$$\frac{dx}{dt} = x^2 - \alpha_L^2/\tau_m^2(\frac{V_C - V_L}{2})^2 + \frac{R_m I_e a_L}{\tau_m^2}$$
$$\frac{dx}{dt} = x^2 - \frac{a_L^2}{\tau_m^2}(\frac{V_C - V_L}{2})^2 + \frac{R_m a_L}{\tau_m^2}\gamma I + \frac{R_m a_L}{\tau_m^2} I_0$$
$$\text{Let } I_0 = \frac{a_L}{\tau_m R_m}(\frac{V_C - V_L}{2})^2, \gamma = \frac{\tau_m^2}{R_m a_L}$$
$$\text{Then } \frac{dx}{dt} = x^2 + I$$

b. $\frac{dx}{dt} = 0 = x^2 + I$ Suppose $I = 0$, then $x = 0$ Suppose $I < 0$, then $x = \sqrt{-I}, x = -\sqrt{-I}$ Suppose $I > 0$, then there are no solutions

c. For $x = 0, I = 0$ it is a saddle point since dx/dt is of higher value on either side of x For $x = \sqrt{-I}, I < 0$, it is an unstable point since $(\epsilon + \sqrt{-I})^2 + I > 0, (-\epsilon + \sqrt{-I})^2 + I < 0$ for some small ϵ For $x = -\sqrt{-I}, I < 0$, it is a stable point since $(\epsilon - \sqrt{-I})^2 + I < 0, (-\epsilon - \sqrt{-I})^2 + I > 0$

d. It would be the saddle point $x = 0, I = 0$ and the unstable point $x = \sqrt{-I}, I < 0$. It also goes to infinity whenever $I > 0$ since dx/dt would always be positive.

e. $\frac{dx}{dt}(\frac{1}{x^2+I}) = 1$ $x(t) = tan(t\sqrt{I} + c)\sqrt{I}, x(0) = -\infty$ Therefore, $c = 3\pi/2$ since $\sin(3\pi/2) = -1, \cos(3\pi/2) = 0$ $x(t) = tan(t\sqrt{I} + 3\pi/2)\sqrt{I}$ And spikes would occur when the tangent is ∞ which means that $\sin(t\sqrt{I} + 3\pi/2) = 1, cos(t\sqrt{I} + 3\pi/2) = 0$ $t\sqrt{I} + 3\pi/2 = \frac{(5+4k)\pi}{2}$ for some non-negative integer k $t\sqrt{I} = (2k+1)\pi$ If you fix t, then the values of I that cause a spike are the ones which $I = (\frac{(2k+1)\pi}{t})^2$ Likewise, fixing I, the time at which the spike occurs would be $t = \frac{(2k+1)\pi}{\sqrt{I}}$

f. Having $t\sqrt{I} = (2k+1)\pi$, we can see that $t\sqrt{\frac{I_e - I_0}{\gamma}} = (2k+1)\pi$ So the condition on I_e to spike is that $I_e = (\frac{(2k+1)\pi}{t})^2\gamma + I_0$ Likewise, we can see that it fires every time $t\sqrt{\frac{I_e - I_0}{\gamma}}$ increases by 2π. So the firing rate is such that it fires once every time t changes by $\frac{2\pi}{\sqrt{\frac{Ie - I_0}{\gamma}}}$. So the overall firing rate is $1/t = \sqrt{\frac{I_e - I_0}{\gamma}}/(2\pi)$