Homework 3 PHYS 585

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Problem 1:

a)

$$\tau_m \frac{dV}{dt} = a_L(V-V_L)(V-V_C)+R_m I_e V = \alpha x+V_0, I_e = \gamma I+I_0 \frac{dx}{dt} = \frac{a_L}{\tau_m \alpha}[(\alpha x+V_0-V_L)(\alpha x+V_0-V_C)]+\frac{R_m I_e}{\tau_m \alpha}\text{Let } V_0 = \frac{V_C}{}$$

b) $\frac{dx}{dt} = 0 = x^2 + I$ Suppose $I = 0$, then $x = 0$ Suppose $I < 0$, then $x = \sqrt{-I}, x = -\sqrt{-I}$ Suppose $I > 0$, then there are no solutions

c) For $x = 0, I = 0$ it is a saddle point since dx/dt is of higher value on either side of x For $x = \sqrt{-I}, I < 0$, it is an unstable point since $(\epsilon+\sqrt{-I})^2+I > 0, (-\epsilon+\sqrt{-I})^2+I < 0$ for some small ϵ For $x = -\sqrt{-I}, I < 0$, it is a stable point since $(\epsilon - \sqrt{-I})^2 + I < 0, (-\epsilon - \sqrt{-I})^2 + I > 0$

d) It would be the saddle point $x = 0, I = 0$ and the unstable point $x = \sqrt{-I}, I < 0$. It also goes to infinity whenever $I > 0$ since dx/dt would always be positive.

e) $\frac{dx}{dt}(\frac{1}{x^2+I}) = 1$ $x(t) = tan(t\sqrt{I}+c)\sqrt{I}, x(0) = -\infty$ Therefore, $c = 3\pi/2$ since $\sin(3\pi/2) = -1, \cos(3\pi/2) = 0$ $x(t) = tan(t\sqrt{I} + 3\pi/2)\sqrt{I}$ And spikes would occur when the tangent is ∞ which means that $\sin(t\sqrt{I} + 3\pi/2) = 1, cos(t\sqrt{I} + 3\pi/2) = 0$ $t\sqrt{I} + 3\pi/2 = \frac{(5+4k)\pi}{2}$ for some non-negative integer k $t\sqrt{I} = (2k + 1)\pi$ If you fix t, then the values of I that cause a spike are the ones which $I = (\frac{(2k+1)\pi}{t})^2$ Likewise, fixing I, the time at which the spike occurs would be $t = \frac{(2k+1)\pi}{\sqrt{I}}$

f) Having $t\sqrt{I} = (2k + 1)\pi$, we can see that $t\sqrt{\frac{I_e-I_0}{\gamma}} = (2k + 1)\pi$ So the condition on I_e to spike is that $I_e = (\frac{(2k+1)\pi}{t})^2\gamma + I_0$ Likewise, we can see that it fires every time $t\sqrt{\frac{I_e-I_0}{\gamma}}$ increases by 2π. So the firing rate is such that it fires once every time t changes by $\frac{2\pi}{\sqrt{\frac{I_e-I_0}{\gamma}}}$. So the overall firing rate is $1/t = \sqrt{\frac{I_e-I_0}{\gamma}}/(2\pi)$