Theoretical and Computational Neuroscience 2018

Problem Set 3

Due date: Thursday, February 15, 2018

1 Dynamics of quadratic neurons

In this question, you will analytically investigate an alternative to the linear leaky integrate-and-fire neuron.

One popular simple nonlinear integrate-and-fire neuron model is described by the equation

$$\tau_m \frac{dV}{dt} = a_L(V - V_L)(V - V_c) + R_m I_e \, , \tag{1}$$

where $a_L > 0$ and $V_c > V_L$.

This model has an instability for large-enough membrane potential that leads to a divergence in finite time. Suppose that whenever a divergence ($V = \infty$) is encountered, the potential is reset to $V = -\infty$.

(**a**) Many of the parameters in eq. (1) amount only to a choice of units and a choice of zero for the variables. This can obscure the dynamics, so let us rewrite it in a "nondimensionalized" form.

Find a rescaling and shift of the potential V and external current I_e that puts the differential equation (1) in the form

$$\frac{dx}{dt} = x^2 + I \, . \tag{2}$$

Hint: One way to do this is to plug $V = \alpha x + V_0$ and $I_e = \gamma I + I_0$ into eq. (1) and solve for α, γ, V_0, and I_0.

(**b**) Find the fixed points of the dynamical system from eq. (2)—these are the values of x for which $dx/dt = 0$. Your answer will depend on whether $I > 0$, $I = 0$, or $I < 0$.

(**c**) For any value of x, you can use eq. (2) to find the sign of the rate of change of x, dx/dt. This tells you what the system will do when started off at a particular x.

Consider now values of x close to one of the fixed points you found above, x_*. You will find that for some fixed points, $dx/dt < 0$ for $x > x_*$ and $dx/dt > 0$ for $x < x_*$. This means that the system relaxes towards x_* when started in its vicinity. This is called a *stable* fixed point. Conversely, if $dx/dt > 0$ for $x > x_*$ and $dx/dt < 0$ for $x < x_*$, any displacement from the fixed point gets amplified with time, resulting in an *unstable* fixed point. You may also find that for some points, $dx/dt < 0$ or $dx/dt > 0$ on both sides of the fixed point ($x < x_*$ and $x > x_*$). This is called a *saddle point*.

For part (**c**), determine whether the fixed points you found in part (**b**) are stable, unstable, or saddle points.

(**d**) For which values of I and the initial potential x_0 will the system generate a spike (*i.e.,* x will go to infinity)?

Hint: If the system is repelled by an unstable fixed point in a direction where there are no more fixed points, it will eventually go to $\pm\infty$.

(**e**) Assume that the potential starts off at $x_0 = -\infty$. For what values of I will the neuron spike? In these conditions, solve eq. (2) and sketch $x(t)$. Find the time T when the spike occurs (*i.e.,* $\lim_{t \to T} x(t) = \infty$). You should find that $T < \infty$.

(**f**) Using the results from above and the mapping you used in the first part of the problem, find the condition on I_e that allows the neuron to spike when started off with $V = V_0 = -\infty$. In this case, find the firing rate of the neuron and sketch your result.

Hint: Follow the approach from section 5.4 in Dayan and Abbott: the firing rate r is the reciprocal of the time between two spikes, T. This is exactly the time it takes for V to go to $+\infty$ when started at $-\infty$.

2 Tuning Curves

2.1 Introduction

Nerve cells in sensory systems can combine inputs to extract a certain aspect of the stimulus. We say that these cells are tuned to a certain stimulus parameter. For example *simple cells* in the primary visual cortex are tuned to recognize moving edges in certain orientations. Part of the function of the primary visual cortex is therefore to separate edges in close proximity according to directions. This can be important for later object recognition in other parts of the brain. The tuning curve for a simple cell tells us which direction it prefers and how good the discrimination is.

2.2 The Tuning Curve of a Simple Cell in the Cat Primary Visual Cortex

Read pages 14 - 16 in Dayan and Abbott covering tuning curves. Download `orientation_tuning_data.mat` from Canvas containing the response of a simple cell in the cat primary visual cortex to a drifting grating at different orientations. Being oriented at an angle θ means here that a series of bars is moving in the direction θ. The bars themselves are orthogonal to this direction. (The data is from experiments by Leif Vigeland, Contreras lab.) Load the file into MATLAB. Read the informative text given in the variable `information`. This text contains information in what format the data is stored. The sampling rate is 10 kHz.

Problem 2: (**a**) Plot the initial seconds of the recorded voltages `Vm` from the simple cell. You will see that, due to the intracellular recording, the action potentials are clearly visible.

(**b**) Create a list of spike times by scanning for action potentials in the data `Vm`. The simplest way to find the time point of an action potential is to look for crossings of a threshold voltage $V_{threshold}$ that can only be reached by the cell if it produces an action potential.

(**c**) Create the tuning curve (firing rate as a function of stimulus orientation) for the given cell. What are the preferred orientation angles? Why are there two angles which seem to be preferred? Are firing rates for these two preferred angles the same or different and what does this mean for the function of the cell?

(**d**) **(Extra Credit)** Find the preferred orientations by fitting a Gaussian tuning curve to the appropriate parts of the data similar to figure 1.5.

(**e**) Explain what the following YouTube video about a simple cell in the primary visual cortex is trying to show us: http://www.youtube.com/watch?v=n31XBMSSSpI&feature=related

Turning in the homework: Please upload the following files in response to all Part 2 (numerical simulation) questions to the Canvas Assignment Homework 2:

1) Your legible and commented code. If you have multiple files (for example, for different problems), please indicate which can be run to reproduce the output for each problem in file name. Please be sure that your code runs! If it does not, it partial credit may not be awarded for incorrect outputs.

2) A word document containing your answers to each question, with your MATLAB output figures embedded where necessary. Please include a caption accompanying each figure describing the relevant content. Be sure to label your axes, and make sure all relevant dynamics are clearly visible in the figures you make.

Furthermore, please turn in your written responses to all Part 1 questions in class on the day this assignment is due. If you have multiple pages, please make sure they are stapled together, and that your name appears on each page.