O6. Measurement of The Speed of Light Using Foucault Method

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Group 4

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I. Introduction

The speed of light, denoted by the symbol c, is a fundamental physical constant that plays a crucial role in many fields of physics, such as electromagnetism, relativity, and cosmology [1]. It is the underlying constant for many physics rules and equations, any variations in the value of the speed of light will completely change how the reality that we are living in. Thus, the precise measurements of the speed of light play an important role in validating scientific theories and models throughout history.

One of the earliest successful attempts to measure the speed of light was made by the Danish astronomer Ole Rømer in 1676. He used observations of the eclipses of Jupiter's moons to estimate the finite speed of light. Before then, humanities believed that light travels instantaneously, and many people at that time still doubted the result of the experiment. It was not until the 19th century that more precise experiments and solid physics models were developed. Many famous experiments such as the Fizeau experiment.

However, there was a problem with many attempts to measure the speed of light. Most of the experiment relies on the kinematic method, which measures the speed of light with displacement and time-lapse. This is very hard to achieve as the scale of light speed is very big. The Fizeau experiment prevented this problem by using a rotating gear, but the error is still relatively large and requires a very big experiment space [2].

The Foucault method is a clever and interesting approach to measuring the speed of light. This is done through a rotating mirror with respect to a fixed mirror which reflects two separate beams with angular separation representing the time taken for a given distance traversed by light beam to reach the fixed mirror. This method can achieve a relatively high accuracy within a small space [3].

II. Theoretical Background

Foucault's method measures the speed of light mechanically. It uses a fast-rotating mirror to change the direction of the reflected beam, and one could measure the small displacement between the original beam position and the final beam position to estimate the speed of light. Based on this theory, there exist two experimental parameters that require careful measurement, one is the rotation speed, and the other is the position shift [4].

To derive the equation for calculating the speed of light, one can look at the theoretical experimental setup shown in Fig.1. From the figure, the light coming from S first passes through L, then is reflected by RM. The light then hits M and bounces back to RM. However, since RM has already rotated for a few radii, the beam's trajectory is changed, thus eventually landing at S`. Note that V is the original imaginary image position of the beam, while V` is the position where the beam eventually lands after the RM has rotated. With the rotation speed set to 0, the distance between V & V` and S & S` will as well be 0.

To calculate the time taken as the light travels between RM and M, we take $\Delta t = 2d_3/c$. During this moment, the RM is rotated by an angle of $\omega\Delta t$. Using the changed in angle of RM, one could calculate the distance between V and V` as $\Delta S` = 2\omega\Delta t * d_3$. Take the imaginary position of the beam spot as a reference, we can deduce the ΔS at the other end of the lens using the lens' magnification M:

$$\Delta S = M * \Delta S` = \frac{d_1}{d_2 + d_3} * \Delta S` = 2\omega\Delta t * d_3 \frac{d_1}{d_2 + d_3} = 2\omega(\frac{2d_3}{c}) * d_3 \frac{d_1}{d_2 + d_3}$$

Note that during the experiment, we do not measure the rotation speed directly, thus ω should be converted to frequency $f = \omega/2\pi$:

$$\Delta S = \frac{8\pi d_1 {d_3}^2}{c*(d_2+d_3)} f \quad (1)$$

Using this equation, one can derive the speed of light by finding ΔS and f under controlled environments, where $d_{1,2,3}$ can be measured directly. This equation also hints that ΔS will be incredibly small if f is not large enough given a limited environment of $d_3 + d_2 \approx 5m$.

III. Experimental Setup

Our experimental setup is shown at in Fig. 2. We used a HeNe laser which emits a 632.8nm wavelength laser. The light beam is then guided to two lenses, each with a focal length of 48mm (L1) and 252mm (L2). Note that the separation between L1 and L2 is 315mm, with 15mm longer than the focal length. The lens is used to guide the laser beam so that when the light is reflected to the observing panel, it can be perfectly focused on a small spot. The beam is also capable to focus to a small spot-on M, thus helping the alignment process to be conducted. A beam splitter is placed in front of the focal point of L2 to guide the reflected beam to the observing panel. d_1 is 252mm, d_2 is 486mm, and d_3 is 4000mm. This value is chosen as from equation (1), one should minimize d_2 and maximize d_3, but any distance further than 4m will make the alignment very difficult to perform, as one could not see whether the reflected light is hitting the rotating mirror or not during the alignment. Note that due to the large distance and the lack of equipment, it is hard to get a precise measurement of distance, thus the error can be as large as 5%.

To measure the tiny movement of the beam displacement, we used the webcam Aoni A20. Since the displacement of the light spot is relatively small, we removed the lens on the webcam and used the CMOS for the measurement. For a resolution of 1920x1080 and a dimension of 3.5mm * 6.2mm, one can calculate the measured pixel width to be 3.24μm. An image of the webcam CMOS is shown in Fig. 3. In the original proposal, we planned to use a microscope to observe the light displacement, but we found that a CMOS is better for this job as data can be recorded digitally and it is much cheaper to do so.

As the experiment requires an extremely fast spinning mirror, we choose a polynomial printer mirror that is taken down from a Fuji Xerox 4110 printer. The side of the mirror can be used for the experiment. The number of mirrors does not affect the experiment outcome as the displacement of light only depends on rotation speed. We originally planned to use a spinning cooling fan for the experiment, but it turned out to be too slow for any proper measurement. This printer mirror was able to achieve incredible spinning speed while maintaining stability and low noise. An image of the printer mirror is shown in Fig. 4. Since there is no manual on how to operate this device, a series of experiments were conducted, and we were able to find how to operate this device. A constant current power supply is used to deliver power to the R, with an oscilloscope to observe any potential output signal from the RM. Pin 1 is power input; it accepts DC voltage input, and the voltage should be higher than 9V for the mirror to operate. The mirror is found to be controlled by the applied voltage at Pin 1, however, any voltage above 31.7V is not tested, we worried that any voltage higher than this will burn the circuit. Pin 2 and Pin 3 should be grounded, we suspect that one of them should be GND, and the other one should be the enabling switch. Pin 5 should be connected to a square wave signal of 5V, with a frequency higher than 2Hz. We were not able to determine the use of Pin 4, and many questions on how this mirror operates were unable to be answered during the lab sections. For example, the mirror will always stop after 15 second when we turn on the power. But we eventually managed to make it spin at a very fast speed.

To determine the turning speed of the rotating mirror, we used a light source that alters at a controllable frequency and drew a line on the rotating mirror. By controlling the altering frequency of the light source and the turning speed of the mirror, we can measure the turning frequency of the rotating mirror when the line on the mirror is stable. However, the light source can only achieve a frequency of around 250Hz, extrapolation is required to estimate the turning frequency above this. Therefore, we proposed to use extrapolation to estimate the turning frequency of the RM.

IV. Data Analysis

Since turning speed is related to the applied voltage to the RM, we proposed that there exists a linear relationship between the applied voltage and the frequency. Thus, a series of measurements is conducted, and the result is shown in Fig. 5. From the graph, there exists a linear relationship between the applied voltage and the frequency, and the accuracy of the result is extremely high.

To further calculate the displacement of the light spot, we need to analyze the pictures taken by the Webcam. However, there exists one big issue, that is, the resulting beam spot shifting does not match our expectations. A picture of the spot guided by the experiment setup is shown in Fig. 6. There is a very wide light spot, which contradicts our original assumption of a small dot. To increase the accuracy of the measurement, a total of 12 sets of images were taken at voltages of 11.0, 13.8, 15.6, 18.1, 19.0, 21.1, 23.1, 25.9, 26.2, 28.0, 28.9 and 31.7V with error of 0.1V. Using the fitted equation, we can then convert the voltage to frequency. The calculation is shown in Python notebook Part 2 [5].

As the wide light spot is very hard to find its centroid, a K-Mean clustering method is proposed using the SkLearn library in Python to perform the search, and the result is shown in Fig. 7. The calculation process is shown in Python notebook Part 1 [5]. We choose a grayscale light intensity with a threshold of 200 to preprocess the image before training to achieve a better result. From Fig. 7, a trace of displacement can be seen. This should be the displacement that we are looking for.

To further analyze the data, a PCA dimension reduction method is proposed, and we managed to map every centroid point to a 1D line, where 0 indicates the position of the data with the smallest turning frequency of 185Hz. This method suited our situation very well, as the displacement of the light spot is in diagonal direction. Note that each data is then subtracted from the first data. This is because we choose not use the initial position for the calculation of the light speed due to the large width of the following data. The calculation process is shown in Python notebook Part 3 [5], and the resulting ΔS versus Δf is shown in Fig. 8. Using the slope of ΔS versus Δf, the slope is 0.039 pixel/Hz and the y-intercept is 0.89 pixel.

With the founded slope, one can calculate the light speed using equation (1). Our result of the measured speed of light is $1.78 * 10^8$m/s, and it has a 40.3% percentage error compared to the expected value of $2.98 * 10^8$m/s. The result has the same degree as the expected value, but it contains a very large error. Thus, it can be concluded that the experiment was proven to be able to find the speed of light, but it requires more time and effort to improve the overall setup to achieve a higher accuracy.

V. Further Discussion

There exist many points that requires improvement for this experiment. I would like to divide them into 3 parts.

First, the experiment requires very high accuracy, but the setup is very hard to minimize its error. There exist many measurements or alignment for this experiment that require very large efforts, for example, the large distance of d_3= 4000mm resulting in an extremely hard alignment problem. Since the reflected beam must hit the rotating mirror, and the distance is very large, any tiny adjustment to the mirror's position or direction will result in a very large shift in the final position of the beam spot. Thus, the distance of d_3 contains a very big error. The entire setup is also not perfectly horizontal, thus resulting in the displacement of the light spot not horizontally, but diagonally. The webcam should be also placed perfectly perpendicular to the incline light ray, any angle will result in a change of the measured light speed [4], but without proper equipment or stand, we were not able to achieve this. Some measurements could be achieved with higher accuracy if one can get a mirror with a higher turning speed, however, our power supply is limited to 30V, and we are unsure what the maximum operating voltage of the RM is. Therefore, the experiment can increase its accuracy with better experiment equipment such as a faster rotating mirror, better environment setup for accurate alignment, and more time for performing the alignment.

Second, the experiment was not well prepared, and much time was wasted before the experiment could proceed. We were stuck on finding a mirror that could achieve a 500Hz turning frequency, and it took a long time for us to find this printer mirror that suited our experiment. We were also not prepared for the experiment procedure, as our groupmates were unable to understand what the next step of the experiment should be, and the purpose of some of the experiment's measurements. This can be improved by having more conversations and communication between the teammate and the technician.

Third, some experiment finding was not expected. The resulting beam spot was originally expected to be a simple dot, but in our measurement, it is shown to be a very wide horizontal fringe. The fringe also shifted diagonally instead of horizontally. We were confused by why these happened, and the wide fringe induced many errors and was very hard for computation. Many machine learning techniques were proposed to minimize this problem, as well as the diagonal shifting issue. We suspect that this is caused by the experiment setup not being perfectly horizontal. The size of the reflecting mirror could also be the reason why this happens. Due to the width of the reflecting mirror, the light that is reflected will be a line instead of a dot. This can be solved by using a reflecting mirror with a smaller width or covering the side of the mirror to limit its width. To improve this, more time should be placed to examine whether different setups can affect the result of the experiment.

VI. Personal Evaluation

I was the one that suggested this experiment. Since I was always interested in how physics constants are found, I think this experiment was the way to go. However, I was not able to prepare and understand the difficulty of this experiment before the lab section is conducted. I tried my best to understand the materials and get prepared for the experiment, and we were at least able to complete the entire experiment. The same degree of light speed was calculated, proving that this method of measurement does work, and with enough time and better equipment, results with higher accuracy can be achieved. I was responsible for purchasing the equipment for the experiments, such as the rotating mirror and the wire connecting it to the breadboard. I was also in charge of wiring the Python code to calculate the result of the measurement, and I have applied many data analysis techniques such as PCA dimension reduction and K-Mean clustering. I have also communicated multiple times with technician Peter, and he gave our team a lot of assistance. I am very enthusiastic about the experiment process, and I am always willing to contribute more to the teamwork. However, as a leader, sometimes I feel too overwhelmed by the workload and the stress of this experiment. I tried to divide my work among our teammates, but they seemed to need more time to understand the materials than me. The fear of not being able to complete the experiment surrounded me for 4 weeks, and I tried my best to find solutions to all the difficulties that we faced.

There exist many points that I should improve before any assignments like this in the future. I should have better time management and get prepared before the experiment. I should also have better communication skills so that if any of my teammates have difficulty understanding the materials that I assigned them to do, I can help them understand it. I should also assess the capabilities of our team and the difficulty of the experiment before conducting it. This experiment is rather too hard for our team, and I should choose an experiment with lower difficulty. Being a team leader, it is a shame for me to not be able to lead the team well and get a better accuracy of the speed of light. However, if there are more assignments like this in the future, I am still willing to take the challenge, as I found doing experiments to be a very fun subject.

VII. Conclusion

Overall, the experiment result satisfied our expectations. The Foucault method provided an important early measurement of the speed of light. By using a rapidly rotating mirror to reflect light over a known distance, we were able to calculate the time taken for the round trip and compare it to the displacement to obtain the speed of light value. Our observation has proved that this method does work, and the theory behind is correct. However, many unexpected findings have hindered the final result of the experiment, such as the wide fringe induced and diagonal shifting. We managed to obtain a speed of light value of $1.78 * 10^8$m/s, a value with the same degree as the expected value. A more precise measurement should be conducted to obtain a value with higher accuracy and less error.

VIII. References

[1] Grant R. Fowles Introduction to Modern Optics. Second Edition, Dover Publications, 1989, p.4

[2] A modern Fizeau experiment for education and outreach purposes https://arxiv.org/pdf/1011.1770.pdf

[3] Pasco Speed of Light Apparatus https://cdn.pasco.com/product_document/Basic-Speed-of-Light-Apparatus-Manual-OS-9262.pdf

[4] Foucault's Method For Measuring The Speed of light with modern apparatus https://ph208.edu.physics.uoc.gr/refs/Exp10/Voros_EJP3_1_2015.pdf

[5] Python program code https://github.com/chenhh17/PHYS3153-O6-Program/blob/main/PHYS3153_O6%20Group4.ipynb

IX. Tables and Figures

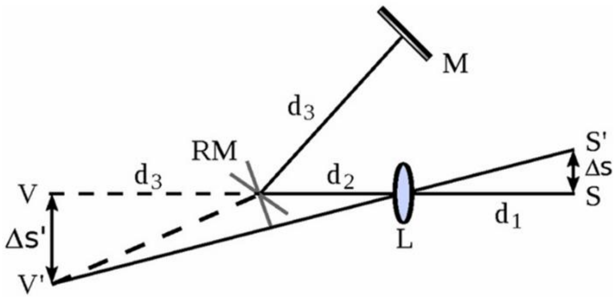

Figure 1: A demonstration of the theoretical experimental setup, where RM is the rotating mirror; M is the reflecting mirror; L is the convex lens used for converging the light beam. Sourced from [4].

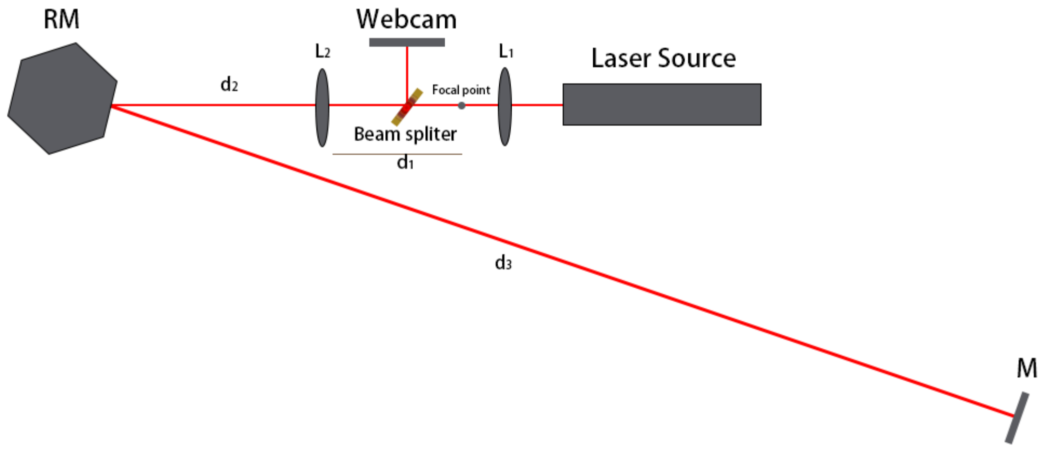

Figure 2: A demonstration of the actual setup during the experiment process. The symbols have a similar meaning as indicated in Fig. 1.

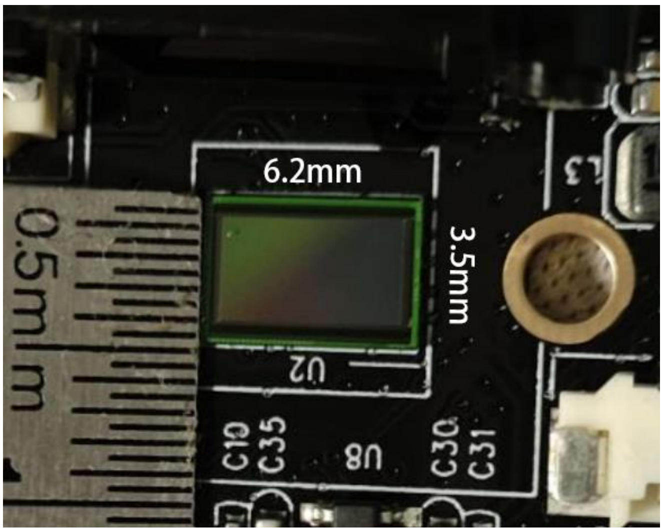

Figure 3: A close shot of the CMOS used for measuring the displacement of the light fringe. The dimension of the CMOS is shown in the picture.

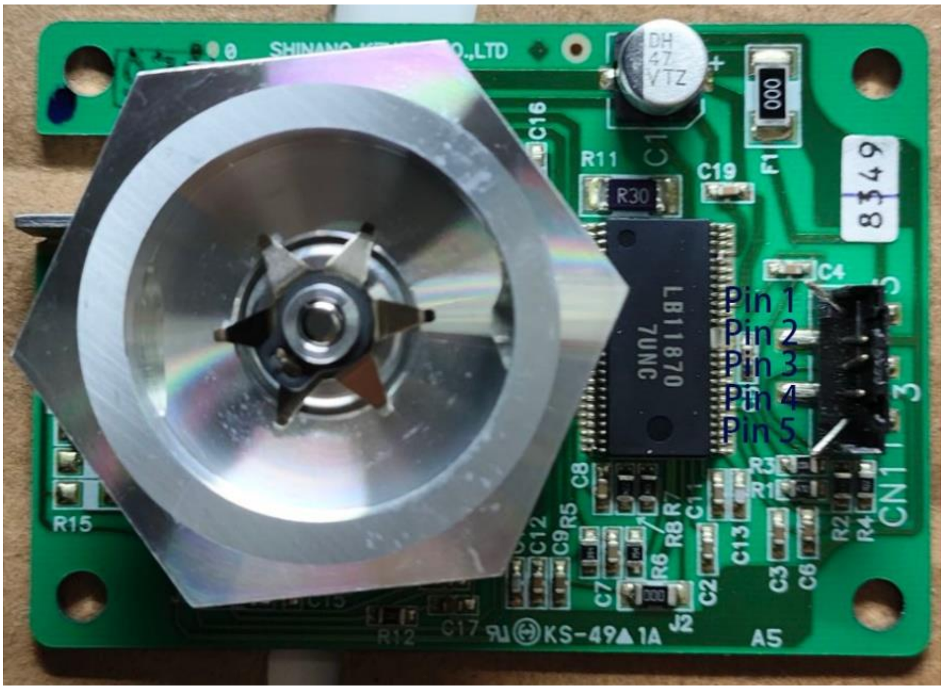

Figure 4: A close shot of the rotating laser printer mirror used during the experiment. The pin position is indicated in the picture. Note that pin 1 and 5 is bended for better wire connection.

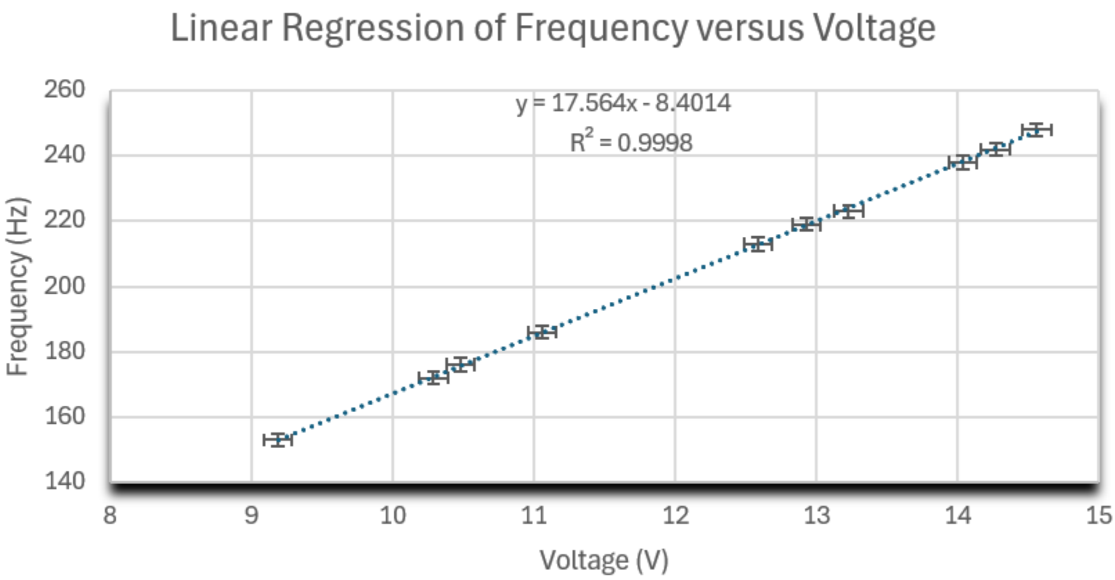

Figure 5: Linear regression line for estimating the voltage-frequency relationship of the rotating mirror.

Figure 6: The captured reflected beam under a rotating frequency of 185Hz.

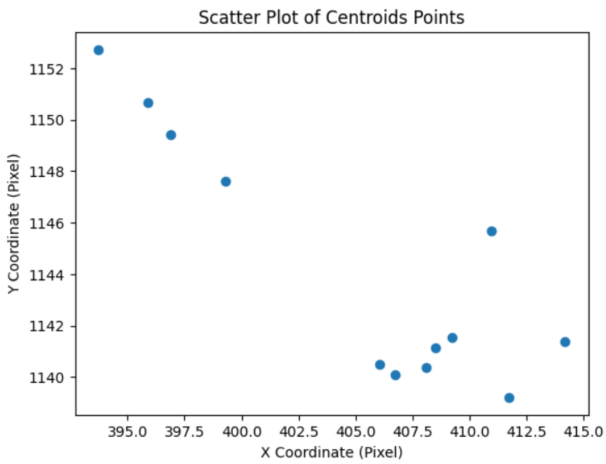

Figure 7: Scatter plot of the estimated centroid position using K-mean clustering.

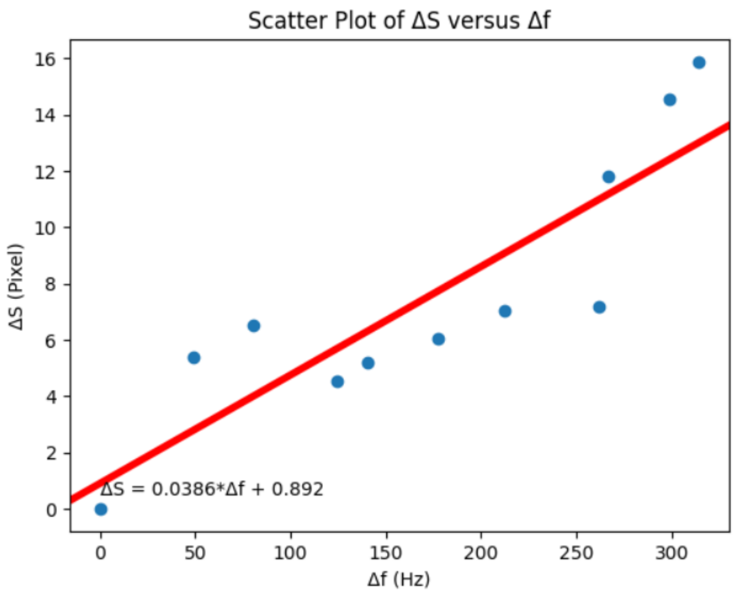

Figure 8: Graph of ΔS versus Δf. The slope of the fitted line can be then used for calculating the measured speed of light.