```
# -*- coding: utf-8 -*-
"""a1.ipynb

Automatically generated by Colaboratory.

Original file is located at
    https://colab.research.google.com/drive/1Ez0ICGL6wIW22OxU_UBJeJ_8h427R_Bo

# CSC413 Assignment 1: Word Embeddings

**Deadline**: February 5, 2021 by 10pm

**Submission**: Submit a PDF report containing your code, outputs,
and your written solutions.
You may export the completed notebook, but if you do so
**it is your responsibly to make sure that your code and answers do not get cut off**.

**Late Submission**: Please see the syllabus for the late submission criteria.

**Working with a partner**: You may work with a partner for this assignment.
If you decide to work with a partner, please create your group on Markus by
February 5, 10pm, even if you intend to use grace tokens. Markus does not allow
you to create groups past the deadline, even if you have grace tokens remaining.

Based on an assignment by George Dahl, Jing Yao Li, and Roger Grosse

In this assignment, we will make a neural network that can predict the next word
in a sentence given the previous three. We will apply an idea called *weight sharing*
to go beyond multi-layer perceptrons with only fully-connected layers.

We will also solve this problem problem twice: once in numpy, and once
using PyTorch. When using numpy, you'll implement the backpropagation
computation manually.

The prediction task is not very interesting on its own, but in learning to predict
subsequent words given the previous three, our neural networks will learn
about how to *represent* words. In the last part of the assignment, we'll explore
the *vector representations* of words that our model produces, and analyze these
representations.

You may modify the starter code, including changing the signatures of helper
functions and adding/removing helper functions. However, please make sure that your
TA can understand what you are doing and why.
"""
```

```
import pandas
import numpy as np
import matplotlib.pyplot as plt

import torch
import torch.nn as nn
import torch.optim as optim

"""## Question 1. Data

With any machine learning problem, the first thing that we would want to do
is to get an intuitive understanding of what our data looks like.
Download the file `raw_sentences.txt` from Quercus.

If you're using Google Colab, upload the file to Google Drive.
Then, mount Google Drive from your Google Colab notebook:
"""

from google.colab import drive
drive.mount('/content/gdrive')

"""Find the path to `raw_sentences.txt`:"""

file_path = '/content/gdrive/My Drive/CSC413/A/A1/raw_sentences.txt' # TODO - UPDATE ME!

"""You might find it helpful to know that you can run shell commands (like `ls`) by
using `!` in Google Colab, like this:
"""

# !ls /content/gdrive/My\ Drive/
# !mkdir /content/gdrive/My\ Drive/CSC413

"""The following code reads the sentences in our file, split each sentence into
its individual words, and stores the sentences (list of words) in the
variable `sentences`.
"""

sentences = []
for line in open(file_path):
    words = line.split()
    sentence = [word.lower() for word in words]
    sentences.append(sentence)
```

```
"""There are 97,162 sentences in total, and
these sentences are composed of 250 distinct words.
"""

vocab = set([w for s in sentences for w in s])
print(len(sentences)) # 97162
print(len(vocab)) # 250

"""We'll separate our data into training, validation, and test.
We'll use 10,000 sentences for test, 10,000 for validation, and
the rest for training.
"""

test, valid, train = sentences[:10000], sentences[10000:20000], sentences[20000:]

"""### Part (a) -- 2 pts

Display 10 sentences in the training set.
Explain how punctuations are treated in our word representation, and how words
with apostrophes are represented.

(Note that for questions like this, you'll need to supply both your code **and**
the output of your code to earn full credit.)
"""

# Your code goes here
for i in range(10):
    print(train[i])

"""Punctuations are treated as a single words in our word representation exceopt for apostrophes, it will combine with the rest of the word/letters after the apostrophes and it will be inside a double quotation mark in the display.

### Part (b) -- 2 pts

What are the 10 most common words in the vocabulary? How often does each of these
words appear in the training sentences? Express the second quantity a percentage
(i.e. number of occurrences of the   word / total number of words in the training set).

These are good quantities to compute, because one of the first things that most
machine learning model will learn is to predict the **most common** class.
Getting a sense of the distribution of our data will help you understand our
model's behaviour.
```

```
You might find Python's `collections.Counter` class helpful.
"""

# Your code goes here
from collections import Counter
count = Counter([item for sublist in train for item in sublist])
common_word = sorted(count, key=count.get,reverse=True)
times = sorted(count.values(),reverse=True)
total = sum(count.values())
print("10 most common words in the vocabulary : ", common_word[:10])
print("total time each of these words appear in the training sentences : ",times[:10])
print("Precentage of these words appear in the training sentences : ", [x / total for x in times[:10]])

"""### Part (c) -- 4 pts

Complete the helper functions `convert_words_to_indices` and
`generate_4grams`, so that the function `process_data` will take a
list of sentences (i.e. list of list of words), and generate an
$N \times 4$ numpy matrix containing indices of 4 words that appear
next to each other. You can use the constants `vocab`, `vocab_itos`,
and `vocab_stoi` in your code.
"""

# A list of all the words in the data set. We will assign a unique
# identifier for each of these words.
vocab = sorted(list(set([w for s in train for w in s])))
# A mapping of j => word (string)
vocab_itos = dict(enumerate(vocab))
# A mapping of word => its j
vocab_stoi = {word:j for j, word in vocab_itos.items()}

def convert_words_to_indices(sents):
    """
    This function takes a list of sentences (list of list of words)
    and returns a new list with the same structure, but where each word
    is replaced by its j in `vocab_stoi`.

    Example:
    >>> convert_words_to_indices([['one', 'in', 'five', 'are', 'over', 'here'],
                                  ['other', 'one', 'since', 'yesterday'],
                                  ['you']])
    [[148, 98, 70, 23, 154, 89], [151, 148, 181, 246], [248]]
```

```
        """

        # Write your code here
        ans = []
        curr = 0
        for i in sents:
            ans.append([])
            for j in range(len(i)):
                ans[curr].append(vocab_stoi[i[j]])
            curr += 1
        return ans

def generate_4grams(seqs):
        """
        This function takes a list of sentences (list of lists) and returns
        a new list containing the 4-grams (four consequentively occuring words)
        that appear in the sentences. Note that a unique 4-gram can appear multiple
        times, one per each time that the 4-gram appears in the data parameter `seqs`.

        Example:

        >>> generate_4grams([[148, 98, 70, 23, 154, 89], [151, 148, 181, 246], [248]])
        [[148, 98, 70, 23], [98, 70, 23, 154], [70, 23, 154, 89], [151, 148, 181, 246]]
        >>> generate_4grams([[1, 1, 1, 1, 1]])
        [[1, 1, 1, 1], [1, 1, 1, 1]]
        """

        # Write your code here
        ans = []
        for i in seqs:
            for j in range(len(i)):
                if j <= len(i) - 4 :
                    ans.append([i[j], i[j+1], i[j+2], i[j+3]])
        return ans



def process_data(sents):
        """
        This function takes a list of sentences (list of lists), and generates an
        numpy matrix with shape [N, 4] containing indices of words in 4-grams.
        """
        indices = convert_words_to_indices(sents)
        fourgrams = generate_4grams(indices)
        return np.array(fourgrams)

train4grams = process_data(train)
valid4grams = process_data(valid)
test4grams = process_data(test)
```

"""## Question 2. MLP Math

Suppose we were to use a 2-layer multilayer perceptron to solve this prediction problem. Our model will look like this:

<img src="https://www.cs.toronto.edu/~lczhang/321/hw/p2_model1.png" />

```
\begin{align*}
\bf{x} &= \text{concatenation of the one-hot vector for words 1, 2 and 3} \\
\bf{m} &= \bf{W^{(1)}} \bf{x} + \bf{b^{(1)}} \\
\bf{h} &= \textrm{ReLU}(\bf{m}) \\
\bf{z} &= \bf{W^{(2)}} \bf{h} + \bf{b^{(2)}} \\
\bf{y} &= \textrm{softmax}(\bf{z}) \\
\end{align*}
```

### Part (a) -- 2 pts

What is the shape of the input vector $\bf{x}$?
What is the shape of the output vector $\bf{y}$?
Let $k$ represent the size of the hidden layer. What are the
dimension of $W^{(1)}$ and $W^{(2)}$?

### Part (b) -- 2 pts

Draw a computation graph for $\bf{y}$. Your graph should include
the quantities $\bf{W^{(1)}}$, $\bf{W^{(2)}}$, $\bf{b^{(1)}}$, $\bf{b^{(2)}}$,
$\bf{x}$, $\bf{m}$, $\bf{h}$, $\bf{z}$ and $\bf{y}$.

### Part (c) -- 3 pts

Derive the gradient descent update rule for ${\bf W}^{(2)}$.
You should begin by deriving the update rule for $W^{(2)}_{ij}$,
and then vectorize your answer. Assume that we will use the softmax
activation and cross-entropy loss.

Note: if you use the derivative of the softamx activation and
the cross-entropy loss, you **must** derive them.

### Part (d) -- 4 pts

What would be the update rule for $W^{(2)}_{ij}$, if we use the square loss
$\mathcal{L}_{SE}(\bf{y}, \bf{t}) = \frac{1}{2}(\bf{y} - \bf{t})^2$?

Show that we will not get good gradient signal
to update $W^{(2)}_{ij}$ if we use this square loss.

### Part (e) -- 4 pts

In this question, we'll show a similar issue with using
the sigmoid activation. Let's assume we have a deep neural network
as follows:

\begin{align*}
h_1 &= \sigma(w_1 x + b_1) \\
h_2 &= \sigma(w_2 h_1 + b_2) \\
\dots
\end{align*}

where, for simplicity, $x$, $w_1$, $b_1$, $h_1$, $w_2$, $b_2$, $h_2$, etc., are all scalars. Show that

\begin{align*}
|\frac{\partial h_1}{\partial x}| \le \frac{1}{4} |w_1|
\end{align*}

In order to do so, you will need to first
show that $\sigma'(z) = \sigma(z) (1 - \sigma(z))$ (worth 1 point).
Include a plot (or sketch) of the function $\sigma'(z)$ (worth 1 point).

### Part (f) -- 2 pts

Continue from the previous question, show that for a deeper neural network.

\begin{align*}
|\frac{\partial h_N}{\partial x}| \le \frac{1}{4^N} |w_1| |w_2| \cdots |w_N|
\end{align*}

What would be a problem with this result?

### Part (g) -- 1 pts

Would we have the same issue as in part(f) we we replaced the sigmoid activation with ReLU activations? Why or why not?

## Question 3. Weight Sharing - Math

From this point onward, we will change our architecture to introduce weight sharing. In particular, the input $\bf{x}$ consists of three one-hot vectors concatenated together. We can think of $\bf{h}$ as a representation of those three words (all together). However, $\bf{W^{(1)}}$ needs to learn about the first word separately from the second and third word, when some of the information could be shared. Consider the following architecture:

<img src="https://www.cs.toronto.edu/~lczhang/321/hw/p2_model2.png" />

Here, we add an extra *embedding* layer to the neural network, where we compute the representation of **each** word before concatenating them together! We use the same weight $\bf{W}^{(word)}$ for each of the three words:

```
\begin{align*}
\bf{x_a} &= \textrm{the one-hot vector for word 1} \\
\bf{x_b} &= \textrm{the one-hot vector for word 2} \\
\bf{x_c} &= \textrm{the one-hot vector for word 3} \\
\bf{v_a} &= \bf{W}^{(word)} \bf{x_a} \\
\bf{v_b} &= \bf{W}^{(word)} \bf{x_b} \\
\bf{v_c} &= \bf{W}^{(word)} \bf{x_c} \\
\bf{v} &= \textrm{concatenation of } \bf{v_a}, \bf{v_b}, \bf{v_c} \\
\bf{m} &= \bf{W^{(1)}} \bf{v} + \bf{b^{(1)}} \\
\bf{h} &= \textrm{ReLU}(\bf{m}) \\
\bf{z} &= \bf{W^{(2)}} \bf{h} + \bf{b^{(2)}} \\
\bf{y} &= \textrm{softmax}(\bf{z}) \\
\end{align*}
```

Note that there are no biases in the embedding layer.

### Part (a) -- 4 pts

Draw a computation graph for $\bf{y}$. Your graph should include
the quantities $\bf{W}^{(word)}$, $\bf{W^{(1)}}$, $\bf{W^{(2)}}$,
$\bf{b^{(1)}}$, $\bf{b^{(2)}}$,
$\bf{x_a}$,$\bf{x_b}$, $\bf{x_c}$,
$\bf{v_a}$,$\bf{v_b}$, $\bf{v_c}$, $\bf{v}$,
$\bf{m}$, $\bf{h}$, $\bf{z}$ and $\bf{y}$.

### Part (b) -- 2 pts

Using the computation graph from part (e), use the chain rule to
write the quantity $\frac{\partial{\bf y}}{\partial{\bf W}^{(word)}}$
in terms of derivatives along the edges in the computation graph
(e.g. $\frac{\partial \bf{y}}{\partial {\bf z}} \frac{\partial \bf{z}}{\partial {\bf \cdot}} \dots$)

You don't need to compute the actual derivatives along the edges
for this question. However, you will need to in Q4(a).

## Question 4. NumPy

In this question, we will implement the model from Question 3
using NumPy. Start by reviewing these helper functions,
which are given to you:

```
"""


def make_onehot(indicies, total=250):
    """
    Convert indicies into one-hot vectors by
        1. Creating an identity matrix of shape [total, total]
        2. Indexing the appropriate columns of that identity matrix
    """
    I = np.eye(total)
    return I[indicies]

def softmax(x):
    """
    Compute the softmax of vector x, or row-wise for a matrix x.
    We subtract x.max(axis=0) from each row for numerical stability.
    """
    x = x.T
    exps = np.exp(x - x.max(axis=0))
    probs = exps / np.sum(exps, axis=0)
    return probs.T

def get_batch(data, range_min, range_max, onehot=True):
    """
    Convert one batch of data in the form of 4-grams into input and output
    data and return the training data (xs, ts) where:
      - `xs` is an numpy array of one-hot vectors of shape [batch_size, 3, 250]
      - `ts` is either
             - a numpy array of shape [batch_size, 250] if onehot is True,
             - a numpy array of shape [batch_size] containing indicies otherwise

    Preconditions:
      - `data` is a numpy array of shape [N, 4] produced by a call
          to `process_data`
      - range_max > range_min
    """
    xs = data[range_min:range_max, :3]
    xs = make_onehot(xs)
    ts = data[range_min:range_max, 3]
    if onehot:
        ts = make_onehot(ts).reshape(-1, 250)
    return xs, ts

def estimate_accuracy(model, data, batch_size=5000, max_N=100000):
    """
    Estimate the accuracy of the model on the data. To reduce
    computation time, use at most `max_N` elements of `data` to
    produce the estimate.
    """
    correct = 0
    N = 0
    for i in range(0, data.shape[0], batch_size):
        xs, ts = get_batch(data, i, i + batch_size, onehot=False)
        z = model(xs)
        pred = np.argmax(z, axis=1)
        correct += np.sum(ts == pred)
        N += ts.shape[0]

        if N > max_N:
            break
    return correct / N
```

"""### Part (a) -- 8 point

Your first task is to implement the model from Question 3 in NumPy.
We will represent the model as a Python class. We set up the
class methods and APIs to be similar to that of PyTorch, so that you
have some intuition about what PyTorch is doing under the hood.
Here's what you need to do:

1. in the `__init__` method, initialize the weights and biases to have the correct shapes. You may want to look back at your answers in the previous question. (0 points)
2. complete the `forward` method to compute the predictions given a **batch** of inputs. This function will also store the intermediate values obtained in the computation; we will need these values for gradient descent. (3 points)
3. complete the `backward` method to compute the gradients of the loss with respect to the

```
weights and biases. (4 points)
4. complete the `update` method that uses the stored gradients to update the weights and biases. (1 point)
"""

class NumpyWordEmbModel(object):
    def __init__(self, vocab_size=250, emb_size=100, num_hidden=100):
        """
        Initialize the weights and biases to zero. Update this method
        so that weights and baises have the correct shape.
        """
        self.vocab_size = vocab_size
        self.emb_size = emb_size
        self.num_hidden = num_hidden
        self.emb_weights = np.zeros((self.emb_size, self.vocab_size)) # W^{(word)}
        self.weights1 = np.zeros((self.num_hidden, 3 * self.emb_size))       # W^{(1)}
        self.bias1 = np.zeros(self.num_hidden)                        # b^{(1)}
        self.weights2 = np.zeros((self.vocab_size, self.num_hidden))       # W^{(2)}
        self.bias2 = np.zeros(self.vocab_size)                        # b^{(2)}
        self.cleanup()

    def initializeParams(self):
        """
        Randomly initialize the weights and biases of this two-layer MLP.
        The randomization is necessary so that each weight is updated to
        a different value.

        You do not need to change this method.
        """
        self.emb_weights = np.random.normal(0, 2/self.emb_size, self.emb_weights.shape)
        self.weights1 = np.random.normal(0, 2/self.emb_size, self.weights1.shape)
        self.bias1 = np.random.normal(0, 2/self.emb_size, self.bias1.shape)
        self.weights2 = np.random.normal(0, 2/self.num_hidden, self.weights2.shape)
        self.bias2 = np.random.normal(0, 2/self.num_hidden, self.bias2.shape)

    def forward(self, inputs):
        """
        Compute the forward pass prediction for inputs.

        Note that for vectorization, `inputs` will be a rank-3 numpy array
        with shape [N, 3, vocab_size], where N is the batch size.
        The returned value will contain the predictions for the N
        data points in the batch, so the return value shape should be
        [N, something].
        You should refer to the mathematical expressions we provided in Q3
        when completing this method. However, because we are computing
        forward pass for a batch of data at a time, you may need to rearrange
        some computation (e.g. some matrix-vector multiplication will become
        matrix-matrix multiplications, and you'll need to be careful about
        arranging the dimensions of your matrices.)

        For numerical stability reasons, we will return the **logit z**
        instead of the **probability y**. The loss function assumes that
        we return the logits from this function.

        After writing this function, you might want to check that your code
        runs before continuing, e.g. try

            xs, ts = get_batch(train4grams, 0, 8, onehot=True)
            m = NumpyWordEmbModel()
            m.forward(xs)
        """
        self.N = inputs.shape[0]
        ipt = np.split(inputs,3,axis=1)
        self.xa = ipt[0].reshape((self.N,inputs.shape[2])) # todo
        self.xb = ipt[1].reshape((self.N,inputs.shape[2])) # todo
        self.xc = ipt[2].reshape((self.N,inputs.shape[2])) # todo
        self.va = self.xa @ (self.emb_weights).T # todo
        self.vb = self.xb @ (self.emb_weights).T # todo
        self.vc = self.xc @ (self.emb_weights).T # todo
        self.v = np.concatenate([self.va,self.vb,self.vc],axis=1) # todo
        self.m = self.v @ (self.weights1).T + self.bias1 # todo
        self.h = np.maximum(self.m, 0) # todo
        self.z = self.h @ (self.weights2).T + self.bias2 # todo
        self.y = softmax(self.z)
        return self.z



    def __call__(self, inputs):
        """
        This function is here so that if you call the object like a function,
        the `backward` method will get called. For example, if we have
            m = NumpyWordEmbModel()
        Calling `m(foo)` is equivalent to calling `m.forward(foo)`.

        You do not need to change this method.
        """
        return self.forward(inputs)

    def backward(self, ts):
        """
        Compute the backward pass, given the ground-truth, one-hot targets.
        Note that `ts` needs to be a numpy array with shape [N, vocab_size].
        Complete this method. You might want to refer to your answers to Q2
        and Q3. But be careful: we are computing the backward pass for an
        entire batch of data at a time! Carefully track the dimensions of your
        quantities!

        You may assume that the forward() method has already been called, so
        you can access values like self.N, self.y, etc..

        This function needs to be called before calling the update() method.
        """
        z_bar = (self.y - ts) / self.N
        self.w2_bar = z_bar.T @ self.h # todo, compute gradient for W^{(2)}
        self.b2_bar = z_bar.T @ np.ones(self.N) # todo, compute gradient for b^{(2)}
        h_bar = z_bar @ self.weights2 # todo
        m_bar = (self.m > 0) * h_bar # todo
        self.w1_bar = m_bar.T @ self.v # todo
        self.b1_bar = m_bar.T @ np.ones(self.N) # todo
        # ...
        v_bar = m_bar @ self.weights1
        v_bar0 = (v_bar[:,:self.num_hidden]).T @ self.xa
        v_bar1 = (v_bar[:,self.num_hidden:self.num_hidden*2]).T @ self.xb
        v_bar2 = (v_bar[:,self.num_hidden*2:]).T @ self.xc
        self.emb_bar = v_bar0 + v_bar1 + v_bar2# todo, compute gradient for W^{(word)}

    def update(self, alpha):
        """
        Compute the gradient descent update for the parameters.
        Complete this method. Use `alpha` as the learning rate.

        You can assume that the forward() and backward() methods have already
        been called, so you can access values like self.w1_bar.
        """
        self.weights1 = self.weights1 - alpha * self.w1_bar
        # todo... update the other weights/biases
        self.bias1 = self.bias1 - alpha * self.b1_bar
        self.weights2 = self.weights2 - alpha * self.w2_bar
        self.bias2 = self.bias2 - alpha * self.b2_bar
```

```
        self.emb_weights = self.emb_weights - alpha * self.emb_bar

    def cleanup(self):
        """
        Erase the values of the variables that we use in our computation.

        You do not need to change this method.
        """
        self.N = None
        self.xa = None
        self.xb = None
        self.xc = None
        self.va = None
        self.vb = None
        self.vc = None
        self.v = None
        self.m = None
        self.h = None
        self.z = None
        self.y = None
        self.z_bar = None
        self.w2_bar = None
        self.b2_bar = None
        self.w1_bar = None
        self.b1_bar = None
        self.emb_bar = None

"""### Part (b) -- 2 points

Complete the `run_gradient_descent` function. Train your numpy model
to obtain a training accuracy of at least 25%. You do not need to train
this model to convergence, but you do need to clearly show
that your model reached at least 25% training accuracy.
"""

def run_gradient_descent(model,
                         train_data=train4grams,
                         validation_data=valid4grams,
                         batch_size=100,
                         learning_rate=0.1,
                         max_iters=5000):
    """
    Use gradient descent to train the numpy model on the dataset train4grams.
    """
    n = 0
    while n < max_iters:
        # shuffle the training data, and break early if we don't have
        # enough data to remaining in the batch
        np.random.shuffle(train_data)
        for i in range(0, train_data.shape[0], batch_size):
            if (i + batch_size) > train_data.shape[0]:
                break

            # get the input and targets of a minibatch
            xs, ts = get_batch(train_data, i, i + batch_size, onehot=True)

            # erase any accumulated gradients
            model.cleanup()

            # forward pass: compute prediction

            # TODO: add your code here
            y = softmax(model.forward(xs))
            # backward pass: compute error
            model.backward(ts)
            model.update(learning_rate)
            # increment the iteration count
            n += 1

            # compute and plot the *validation* loss and accuracy
            if (n % 100 == 0):
                train_cost = -np.sum(ts * np.log(y)) / batch_size
                train_acc = estimate_accuracy(model, train_data)
                val_acc = estimate_accuracy(model, validation_data)
                model.cleanup()
                print("Iter %d. [Val Acc %.0f%%] [Train Acc %.0f%%, Loss %f]" % (
                      n, val_acc * 100, train_acc * 100, train_cost))

        if n >= max_iters:
            return


numpy_model= NumpyWordEmbModel()
numpy_model.initializeParams()
run_gradient_descent(numpy_model)

"""### Part (c) -- 2 pts
```

If we do not call `numpy_model.initializeParams()`, your model weights will not change. Clearly explain (mathematically) why this is the case.

### Part (d) -- 2 pts

The `estimate_accuracy` function takes the continuous predictions `z` and turns it into a discrete prediction `pred`. Show that for a given data point, `pred` is equal to 1 only if the predictive probability `y` is at least 0.5.

## Question 5. PyTorch

Now, we will build the same model in PyTorch.

### Part (a) -- 2 pts

Since PyTorch uses
automatic differentiation, we only need to write the *forward pass* of our model. Complete the `__init__` and `forward` methods below.

Hint: You might want to look up the `reshape` method in PyTorch.
"""

```
class PyTorchWordEmb(nn.Module):
    def __init__(self, emb_size=100, num_hidden=300, vocab_size=250):
        super(PyTorchWordEmb, self).__init__()
        self.word_emb_layer = nn.Linear(vocab_size,       # num input W^(word)
                        emb_size,       # num output W^(word)
                                        bias=False)
        self.fc_layer1 = nn.Linear(3 * emb_size, # num input W^(1)
                                   num_hidden) # num output W^(1)
        self.fc_layer2 = nn.Linear(num_hidden, # num input W^(2)
                                   vocab_size) # num output W^(2)
        self.num_hidden = num_hidden
        self.emb_size = emb_size

    def forward(self, inp):
        vs = self.word_emb_layer(inp)
        v = vs.reshape((-1,3*self.emb_size)) # TODO: what do you need to do here?
        m = self.fc_layer1(v)
        h = torch.relu(m)
        z = self.fc_layer2(h) # TODO: what do you need to do here?
        return z
```

```
"""### Part (b) -- 2 pts

The function `run_pytorch_gradient_descent` is given to you. It is similar
to the code that you wrote fro the PyTorch model, with a few differences:

1. We will use a slightly fancier optimizer called **Adam**. For this optimizer,
   a smaller learning rate usually works better, so the default learning
   rate is set to 0.001.
2. Since we get weight decay for free, you are welcome to use weight decay.


Use this function and train your PyTorch model to obtain a training
accuracy of at least 37%.  Plot the learning curve using the `plot_learning_curve`
function provided to you, and include your plot in your PDF submission.
"""

def estimate_accuracy_torch(model, data, batch_size=5000, max_N=100000):
    """
    Estimate the accuracy of the model on the data. To reduce
    computation time, use at most `max_N` elements of `data` to
    produce the estimate.
    """
    correct = 0
    N = 0
    for i in range(0, data.shape[0], batch_size):
        # get a batch of data
        xs, ts = get_batch(data, i, i + batch_size, onehot=False)

        # forward pass prediction
        z = model(torch.Tensor(xs))
        z = z.detach().numpy() # convert the PyTorch tensor => numpy array
        pred = np.argmax(z, axis=1)
        correct += np.sum(pred == ts)
        N += ts.shape[0]

        if N > max_N:
            break
    return correct / N

def run_pytorch_gradient_descent(model,
                                 train_data=train4grams,
                                 validation_data=valid4grams,
                                 batch_size=100,
                                 learning_rate=0.001,
                                    weight_decay=0,
                                    max_iters=1000,
                                    checkpoint_path=None):
    """
    Train the PyTorch model on the dataset `train_data`, reporting
    the validation accuracy on `validation_data`, for `max_iters`
    iteration.

    If you want to **checkpoint** your model weights (i.e. save the
    model weights to Google Drive), then the parameter
    `checkpoint_path` should be a string path with `{}` to be replaced
    by the iteration count:

    For example, calling

    >>> run_pytorch_gradient_descent(model, ...,
            checkpoint_path = '/content/gdrive/My Drive/CSC413/mlp/ckpt-{}.pk')

    will save the model parameters in Google Drive every 500 iterations.
    You will have to make sure that the path exists (i.e. you'll need to create
    the folder CSC413, mlp, etc...). Your Google Drive will be populated with files:

    - /content/gdrive/My Drive/CSC413/mlp/ckpt-500.pk
    - /content/gdrive/My Drive/CSC413/mlp/ckpt-1000.pk
    - ...

    To load the weights at a later time, you can run:

    >>> model.load_state_dict(torch.load('/content/gdrive/My Drive/CSC413/mlp/ckpt-500.pk'))

    This function returns the training loss, and the training/validation accuracy,
    which we can use to plot the learning curve.
    """
    criterion = nn.CrossEntropyLoss()
    optimizer = optim.Adam(model.parameters(),
                           lr=learning_rate,
                           weight_decay=weight_decay)

    iters, losses = [], []
    iters_sub, train_accs, val_accs  = [], [] ,[]

    n = 0 # the number of iterations
    while True:
```

```
    for i in range(0, train_data.shape[0], batch_size):
        if (i + batch_size) > train_data.shape[0]:
            break

        # get the input and targets of a minibatch
        xs, ts = get_batch(train_data, i, i + batch_size, onehot=False)

        # convert from numpy arrays to PyTorch tensors
        xs = torch.Tensor(xs)
        ts = torch.Tensor(ts).long()

        zs = model(xs)
        loss = criterion(zs, ts) # compute the total loss
        loss.backward()                  # compute updates for each parameter
        optimizer.step()                # make the updates for each parameter
        optimizer.zero_grad()       # a clean up step for PyTorch

        # save the current training information
        iters.append(n)
        losses.append(float(loss)/batch_size)    # compute *average* loss

        if n % 500 == 0:
            iters_sub.append(n)
            train_cost = float(loss.detach().numpy())
            train_acc = estimate_accuracy_torch(model, train_data)
            train_accs.append(train_acc)
            val_acc = estimate_accuracy_torch(model, validation_data)
            val_accs.append(val_acc)
            print("Iter %d. [Val Acc %.0f%%] [Train Acc %.0f%%, Loss %f]" % (
                  n, val_acc * 100, train_acc * 100, train_cost))

            if (checkpoint_path is not None) and n > 0:
                torch.save(model.state_dict(), checkpoint_path.format(n))

        # increment the iteration number
        n += 1

        if n > max_iters:
            return iters, losses, iters_sub, train_accs, val_accs


def plot_learning_curve(iters, losses, iters_sub, train_accs, val_accs):
    """
    Plot the learning curve.
    """
    plt.title("Learning Curve: Loss per Iteration")
    plt.plot(iters, losses, label="Train")
    plt.xlabel("Iterations")
    plt.ylabel("Loss")
    plt.show()

    plt.title("Learning Curve: Accuracy per Iteration")
    plt.plot(iters_sub, train_accs, label="Train")
    plt.plot(iters_sub, val_accs, label="Validation")
    plt.xlabel("Iterations")
    plt.ylabel("Accuracy")
    plt.legend(loc='best')
    plt.show()

pytorch_model = PyTorchWordEmb()
learning_curve_info                                                        =
run_pytorch_gradient_descent(pytorch_model,max_iters=8000,checkpoint_path=
'/content/gdrive/My Drive/CSC413/A/A1/Q5/ckpt-{}.pk')

# you might want to save the `learning_curve_info` somewhere, so that you can plot
# the learning curve prior to exporting your PDF file

plot_learning_curve(*learning_curve_info)

"""### Part (c) -- 3 points

Write a function `make_prediction` that takes as parameters
a PyTorchWordEmb model and sentence (a list of words), and produces
a prediction for the next word in the sentence.

Start by thinking about what you need to do, step by step, taking
care of the difference between a numpy array and a PyTorch Tensor.
"""

def make_prediction_torch(model, sentence):
    """
    Use the model to make a prediction for the next word in the
    sentence using the last 3 words (sentence[:-3]). You may assume
    that len(sentence) >= 3 and that `model` is an instance of
    PyTorchWordEmb. You might find the function torch.argmax helpful.

    This function should return the next word, represented as a string.
    Example call:
    >>> make_prediction_torch(pytorch_model, ['you', 'are', 'a'])
    """
    global vocab_stoi, vocab_itos

    #   Write your code here
    sentence = [word.lower() for word in sentence]
    last = [sentence[-3:]]
    index = convert_words_to_indices(last)
    one_hot = make_onehot(index)
    pred = torch.Tensor(one_hot)
    y = model.forward(pred)
    word_idx = torch.argmax(y)
    return vocab_itos[word_idx.item()]

"""### Part (d) -- 4 points

Use your code to predict what the next word should be in each
of the following sentences:

- "You are a"
- "few companies show"
- "There are no"
- "yesterday i was"
- "the game had"
- "yesterday the federal"

Do your predictions make sense? (If all of your predictions are the same,
train your model for more iterations, or change the hyper parameters in your
model. You may need to do this even if your training accuracy is >=37%)

One concern you might have is that our model may be "memorizing" information
from the training set.   Check if each of 3-grams (the 3 words appearing next
to each other) appear in the training set. If so, what word occurs immediately
following those three words?
"""

# Write your code and answers here
print(make_prediction_torch(pytorch_model, ['You', 'are', 'a']))
print(make_prediction_torch(pytorch_model, ['few', 'companies', 'show']))
print(make_prediction_torch(pytorch_model, ['There', 'are', 'no']))
print(make_prediction_torch(pytorch_model, ['yesterday', 'i', 'was']))
print(make_prediction_torch(pytorch_model, ['the', 'game', 'had']))
print(make_prediction_torch(pytorch_model, ['yesterday', 'the', 'federal']))
```

```
"""### Part (3) -- 1 points

Report the test accuracy of your model. The test accuracy is the percentage
of correct predictions across your test set.
"""

# Write your code here
print("The test accuracy is", estimate_accuracy_torch(pytorch_model,test4grams))

"""## Question 6. Visualizing Word Embeddings

While training the `PyTorchWordEmb`, we trained the `word_emb_layer`, which takes a one-hot
representation of a word in our vocabulary, and returns a low-dimensional vector
representation of that word. In this question, we will explore these word embeddings.

### Part (a) -- 1 pts

The code below extracts the **weights** of the word embedding layer,
and converts the PyTorch tensor into an numpy array.
Explain why each *row* of `word_emb` contains the vector representing
of a word. For example `word_emb[vocab_stoi["any"],:]` contains the
vector representation of the word "any".
"""

word_emb_weights = list(pytorch_model.word_emb_layer.parameters())[0]
word_emb = word_emb_weights.detach().numpy().T

# Write your explanation here

"""### Part (b) -- 1 pts

Once interesting thing about these word embeddings is that distances
in these vector representations of words make some sense! To show this,
we have provided code below that computes the cosine similarity of
every pair of words in our vocabulary.
"""

norms = np.linalg.norm(word_emb, axis=1)
word_emb_norm = (word_emb.T / norms).T
similarities = np.matmul(word_emb_norm, word_emb_norm.T)

# Some example distances. The first one should be larger than the second
```

```
print(similarities[vocab_stoi['any'], vocab_stoi['many']])
print(similarities[vocab_stoi['any'], vocab_stoi['government']])

"""Compute the 5 closest words to the following words:

- "four"
- "go"
- "what"
- "should"
- "school"
- "your"
- "yesterday"
- "not"
"""

# Write your code here
def closest_words(w):
    similar = similarities[vocab_stoi[w]]
    index = (np.argsort(similar))[::-1]
    print("5 closest words for", w,":")
    for i in range(5):
        print(vocab_itos[index[i]])
    return

following_words = ["four", "go","what","should","school","your","yesterday","not"]
for i in following_words:
    closest_words(i)

"""### Part (c) -- 2 pts

We can visualize the word embeddings by reducing the dimensionality of
the word vectors to 2D. There are many dimensionality reduction techniques
that we could use, and we will use an algorithm called t-SNE.
(You don’t need to know what this is for the assignment,
but we may cover it later in the course.)
Nearby points in this 2-D space are meant to correspond to nearby points
in the original, high-dimensional space.

The following code runs the t-SNE algorithm and plots the result.
Look at the plot and find two clusters of related words.
What do the words in each cluster have in common?

Note that there is randomness in the initialization of the t-SNE
algorithm. If you re-run this code, you may get a different image.
```

```
Please make sure to submit your image in the PDF file for your TA to see.
"""

import sklearn.manifold
tsne = sklearn.manifold.TSNE()
Y = tsne.fit_transform(word_emb)

plt.figure(figsize=(10, 10))
plt.xlim(Y[:,0].min(), Y[:, 0].max())
plt.ylim(Y[:,1].min(), Y[:, 1].max())
for i, w in enumerate(vocab):
    plt.text(Y[i, 0], Y[i, 1], w)
plt.show()

"""## Question 7. Work Allocation -- 2 pts

This question is to make sure that if you are working with a partner, that
you and your partner contributed equally to the assignment.

Please have each team member write down the times that you worked on the
assignment, and your contribution to the assignment.
"""

# Example answer:
# I worked on the assignment on Jan 20 afternoon, Jan 26th 12pm-2pm,
# and then Feb 4th in the evening. My partner and I had a meeting on
# Jan 20th to read the entire assignment, and we did Question 1 together
# while screensharing. I worked out the math for Q2, and checked my
# partner's implementation in Q3. I also wrote the Q3 helper functions,
# and Q4(b).

"""This assignment is finished individually by Hongyu Chen"""
```

1a)

```
['last', 'night', ',', 'he', 'said', ',', 'did', 'it', 'for', 'me', '.']
['on', 'what', 'can', 'i', 'do', '?']
['now', 'where', 'does', 'it', 'go', '?']
['what', 'did', 'the', 'court', 'do', '?']
['but', 'at', 'the', 'same', 'time', ',', 'we', 'have', 'a', 'long', 'way', 'to', 'go', '.']
['that', 'was', 'the', 'only', 'way', '.']
['this', 'team', 'will', 'be', 'back', '.']
['so', 'that', 'is', 'what', 'i', 'do', '.']
['we', 'have', 'a', 'right', 'to', 'know', '.']
['now', 'they', 'are', 'three', '.']
```

1b)

10 most common words in the vocabulary :   ['.', 'it', ',', 'i', 'do', 'to', 'nt', '?', 'the', "'s"]

total time each of these words appear in the training sentences :   [64297, 23118, 19537, 17684, 16181, 15490, 13009, 12881, 12583, 12552]

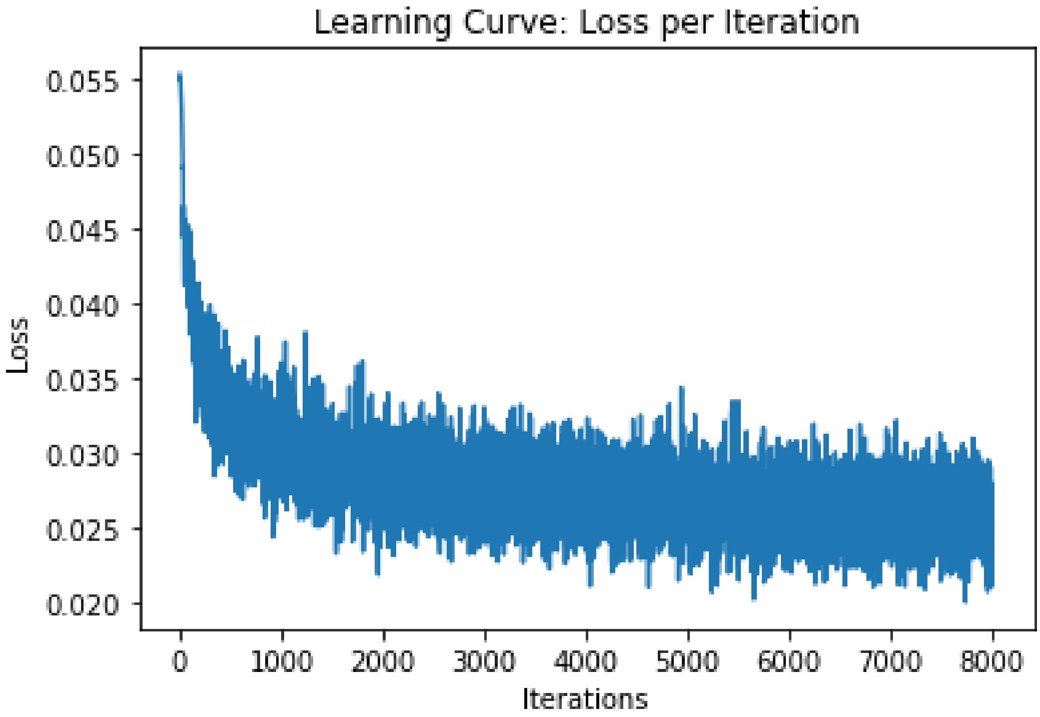
Precentage of these words appear in the training sentences :   [0.10695720015237538, 0.038456484021379134, 0.032499538382458865, 0.029417097648328946, 0.026916877236349845, 0.02576740797176065, 0.021640297631028683, 0.021427371341784955, 0.020931652324639397, 0.020880084238963183]

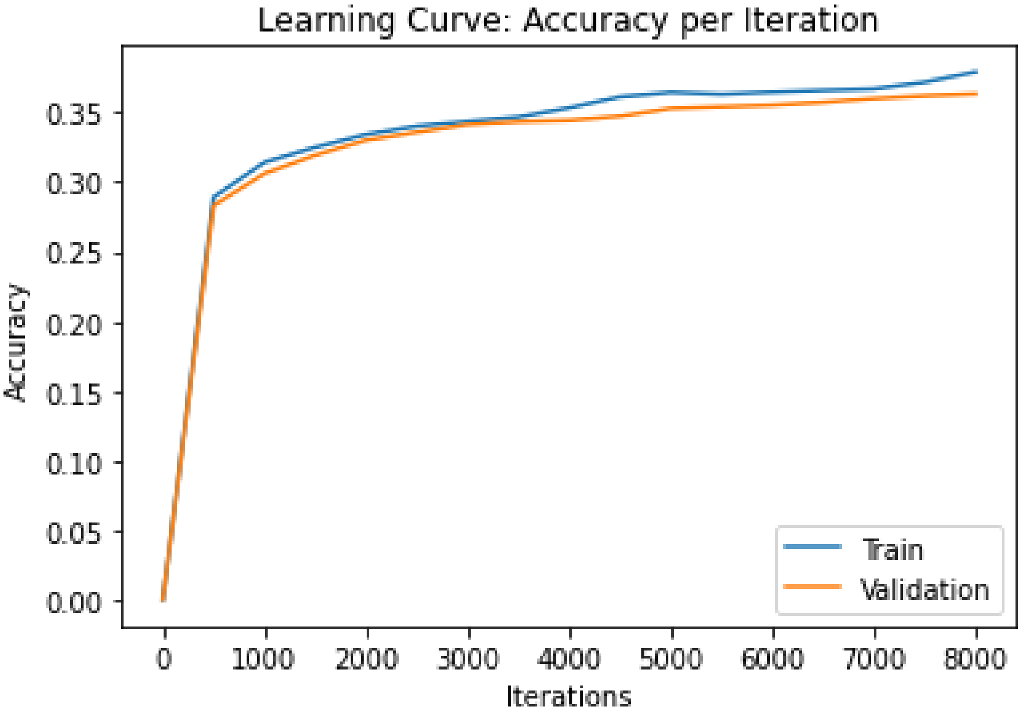
4b)

```
Iter 100. [Val Acc 17%] [Train Acc 17%, Loss 5.090020]
Iter 200. [Val Acc 17%] [Train Acc 17%, Loss 4.796250]
Iter 300. [Val Acc 17%] [Train Acc 17%, Loss 4.572593]
Iter 400. [Val Acc 17%] [Train Acc 17%, Loss 4.354664]
Iter 500. [Val Acc 17%] [Train Acc 17%, Loss 4.379154]
Iter 600. [Val Acc 17%] [Train Acc 17%, Loss 4.646561]
Iter 700. [Val Acc 17%] [Train Acc 17%, Loss 4.456584]
Iter 800. [Val Acc 17%] [Train Acc 17%, Loss 4.508409]
Iter 900. [Val Acc 17%] [Train Acc 17%, Loss 4.247037]
Iter 1000. [Val Acc 17%] [Train Acc 17%, Loss 4.555563]
Iter 1100. [Val Acc 17%] [Train Acc 17%, Loss 4.241536]
Iter 1200. [Val Acc 17%] [Train Acc 17%, Loss 4.439068]
Iter 1300. [Val Acc 17%] [Train Acc 17%, Loss 4.360120]
Iter 1400. [Val Acc 17%] [Train Acc 17%, Loss 4.341677]
Iter 1500. [Val Acc 17%] [Train Acc 17%, Loss 4.517631]
Iter 1600. [Val Acc 17%] [Train Acc 17%, Loss 4.507844]
Iter 1700. [Val Acc 17%] [Train Acc 17%, Loss 4.198820]
Iter 1800. [Val Acc 17%] [Train Acc 17%, Loss 4.433606]
Iter 1900. [Val Acc 18%] [Train Acc 18%, Loss 4.371648]
Iter 2000. [Val Acc 20%] [Train Acc 20%, Loss 4.145929]
Iter 2100. [Val Acc 21%] [Train Acc 21%, Loss 3.921186]
Iter 2200. [Val Acc 20%] [Train Acc 21%, Loss 3.862583]
Iter 2300. [Val Acc 21%] [Train Acc 21%, Loss 4.318796]
Iter 2400. [Val Acc 21%] [Train Acc 21%, Loss 3.898409]
Iter 2500. [Val Acc 21%] [Train Acc 21%, Loss 4.090193]
Iter 2600. [Val Acc 21%] [Train Acc 21%, Loss 4.117402]
Iter 2700. [Val Acc 21%] [Train Acc 21%, Loss 3.855835]
Iter 2800. [Val Acc 21%] [Train Acc 21%, Loss 3.803785]
Iter 2900. [Val Acc 21%] [Train Acc 21%, Loss 4.147912]
Iter 3000. [Val Acc 21%] [Train Acc 21%, Loss 3.660326]
Iter 3100. [Val Acc 21%] [Train Acc 21%, Loss 3.592342]
Iter 3200. [Val Acc 21%] [Train Acc 21%, Loss 3.954508]
Iter 3300. [Val Acc 21%] [Train Acc 21%, Loss 3.698666]
Iter 3400. [Val Acc 21%] [Train Acc 21%, Loss 3.844331]
Iter 3500. [Val Acc 21%] [Train Acc 21%, Loss 3.779545]
Iter 3600. [Val Acc 22%] [Train Acc 22%, Loss 3.729373]
Iter 3700. [Val Acc 21%] [Train Acc 22%, Loss 3.371281]
Iter 3800. [Val Acc 22%] [Train Acc 23%, Loss 3.893297]
Iter 3900. [Val Acc 22%] [Train Acc 23%, Loss 3.749753]
Iter 4000. [Val Acc 23%] [Train Acc 23%, Loss 3.609764]
Iter 4100. [Val Acc 23%] [Train Acc 24%, Loss 3.663236]
Iter 4200. [Val Acc 24%] [Train Acc 24%, Loss 3.491152]
Iter 4300. [Val Acc 24%] [Train Acc 24%, Loss 3.622248]
Iter 4400. [Val Acc 24%] [Train Acc 25%, Loss 3.669833]
Iter 4500. [Val Acc 25%] [Train Acc 25%, Loss 3.602762]
Iter 4600. [Val Acc 24%] [Train Acc 25%, Loss 3.615757]
Iter 4700. [Val Acc 25%] [Train Acc 25%, Loss 3.437219]
```

5b)

```
Iter 0. [Val Acc 0%] [Train Acc 0%, Loss 5.526132]
Iter 500. [Val Acc 28%] [Train Acc 29%, Loss 3.235706]
Iter 1000. [Val Acc 31%] [Train Acc 31%, Loss 2.894737]
Iter 1500. [Val Acc 32%] [Train Acc 32%, Loss 2.734710]
Iter 2000. [Val Acc 33%] [Train Acc 33%, Loss 2.784793]
Iter 2500. [Val Acc 34%] [Train Acc 34%, Loss 2.709608]
Iter 3000. [Val Acc 34%] [Train Acc 34%, Loss 2.727716]
Iter 3500. [Val Acc 34%] [Train Acc 35%, Loss 2.520512]
Iter 4000. [Val Acc 34%] [Train Acc 35%, Loss 2.556910]
Iter 4500. [Val Acc 35%] [Train Acc 36%, Loss 2.807450]
Iter 5000. [Val Acc 35%] [Train Acc 36%, Loss 2.643617]
Iter 5500. [Val Acc 35%] [Train Acc 36%, Loss 2.578403]
Iter 6000. [Val Acc 36%] [Train Acc 36%, Loss 2.730465]
Iter 6500. [Val Acc 36%] [Train Acc 37%, Loss 2.677843]
Iter 7000. [Val Acc 36%] [Train Acc 37%, Loss 2.825259]
Iter 7500. [Val Acc 36%] [Train Acc 37%, Loss 2.686172]
Iter 8000. [Val Acc 36%] [Train Acc 38%, Loss 2.393985]
```





5d)
good
.
other
nt
been
government

5 part3)

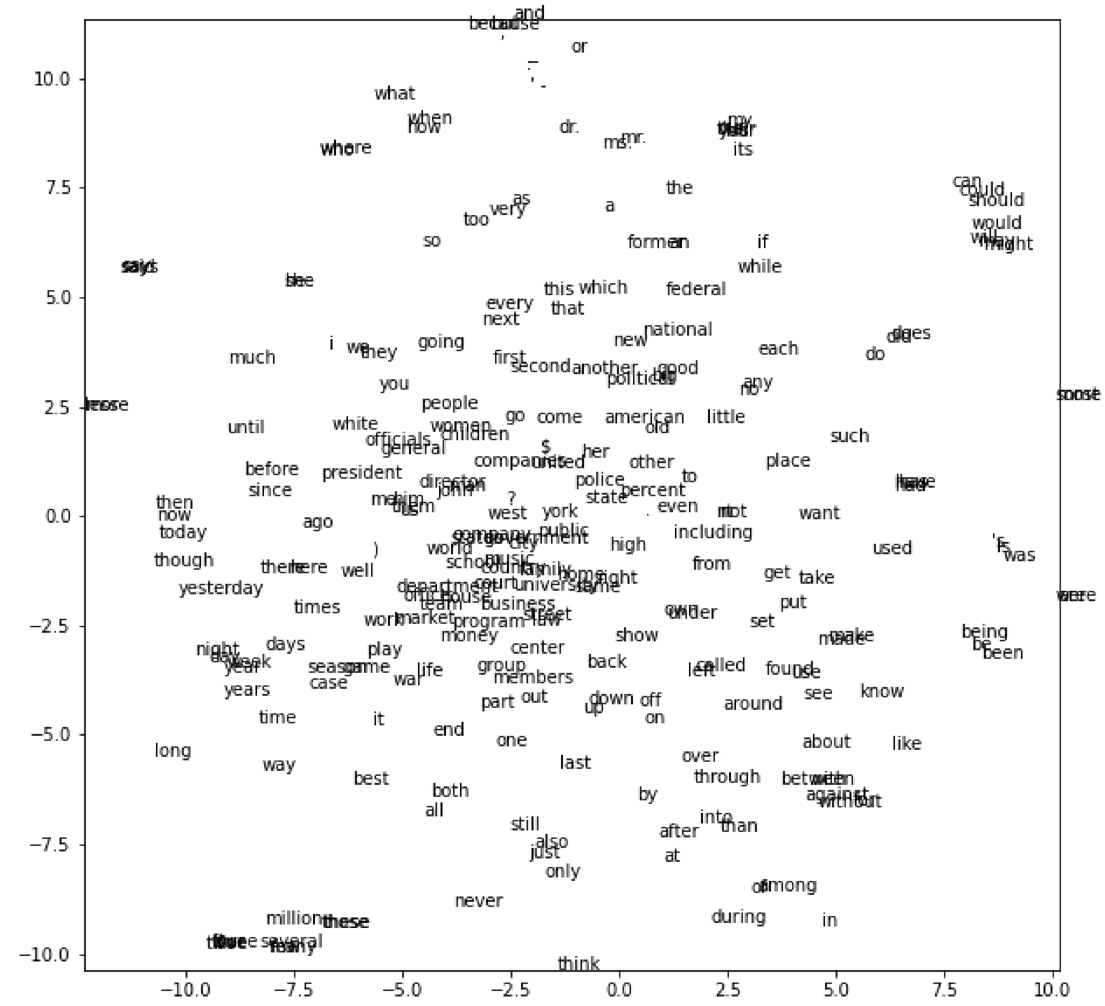
The test accuracy is 0.36428347771631353

6b)

```
5 closest words for four :
four
three
two
five
million
5 closest words for go :
go
back
come
going
get
5 closest words for what :
what
how
who
when
where
5 closest words for should :
should
could
would
might
may
5 closest words for school :
school
states
music
)
department
5 closest words for your :
your
their
our
my
his
5 closest words for yesterday :
yesterday
though
year
```

```
today
night
5 closest words for not :
not
nt
never
by
same
```

6c)



Assignment 1 non-prog

Question 2)

a)

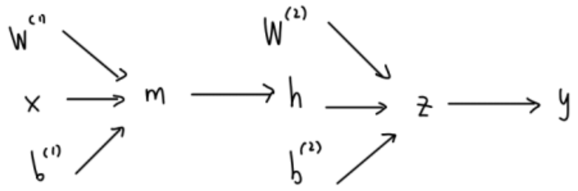
shape of the input vector x is: 750×1

shape of the output vector y is: 250×1

dimension of $W^{(1)}$ is : $K \times 750$

dimention of $W^{(2)}$ is : $250 \times K$

b)



c) $y = \text{softmax}\left(W^{(2)}h + b^{(2)}\right) = \text{Softmax}(z)$

$\mathcal{L}(y,t) = -t^T \log(y)$

$$\frac{\partial \mathcal{L}}{\partial z_i} = -\sum_{j=1}^{K} \frac{\partial t_j \log(y_j)}{\partial z_i} = -\sum_{j=1}^{K} t_j \frac{\partial \log(y_j)}{z_i}$$

$$= -\frac{t_i}{y_i}\frac{\partial y_i}{\partial z_i} - \sum_{j \neq i}^{K} \frac{t_j}{y_j}\frac{\partial y_j}{\partial z_i}$$

$$= -\frac{t_i}{y_i} y_i (1-y_i) - \sum_{j \neq i}^{K} \frac{t_j}{y_j}(-y_j y_i)$$

let $\Sigma_k = \sum_{a=1}^{k} e^{z_a}$ then we have $y_i = \frac{e^{z_i}}{\Sigma_k}$

if $i = j$: $\frac{\partial y_i}{\partial z_i} = \frac{\partial \frac{e^{z_i}}{\Sigma_k}}{\partial z_i} = \frac{e^{z_i}\Sigma_k - (e^{z_i})^2}{\Sigma_k^2} = \frac{e^{z_i}}{\Sigma_k}\frac{\Sigma_k - e^{z_i}}{\Sigma_k} = \frac{e^{z_i}}{\Sigma_k}\left(1 - \frac{e^{z_i}}{\Sigma_k}\right) = y_i(1-y_i)$

if $i \neq j$: $\frac{\partial y_i}{\partial z_j} = \frac{\partial \frac{e^{z_i}}{\Sigma_k}}{\partial z_j} = -\frac{e^{z_i}e^{z_j}}{\Sigma_k} = -y_i y_j$

$$= -t_i + t_i y_i + \sum_{j \neq i} t_j y_i = -t_i + y_i \sum_{j=1}^{K} t_j$$

$$= y_i - t_i$$

Gradient descent updates can be derived for each row of W

$$\frac{\partial \mathcal{L}}{\partial W_j^{(2)}} = \frac{\partial \mathcal{L}}{\partial z_j} \cdot \frac{\partial z_j}{\partial W_j^{(2)}} = (y_j - t_j) h$$

$$W_j^{(2)} \leftarrow W_j^{(2)} - \alpha \frac{1}{N} \sum_{i=1}^{N} (y_j^{(i)} - t_j^{(i)}) h^{(i)}$$

$$W^{(2)} \leftarrow W^{(2)} - \alpha \frac{1}{N} \sum_{i=1}^{N} (y^{(i)} - t^{(i)}) h$$

d) we have $\mathcal{L} = \frac{1}{2}(y-t)^2$

To find out the update rule, we will use chain rule.

$$\frac{\partial \mathcal{L}}{\partial W_{ij}^{(2)}} = \frac{\partial \mathcal{L}}{\partial y} \frac{\partial y}{\partial z_{ij}} \frac{\partial z_{ij}}{\partial W_{ij}^{(2)}}$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{\partial \left(\frac{1}{2}(y_i - t_i)^2\right)}{\partial y} = y_i - t_i$$

if $i = j$: $\frac{\partial y_i}{\partial z_i} = \frac{\partial \frac{e^{z_i}}{\Sigma_k}}{\partial z_i} = \frac{e^{z_i}\Sigma_k - (e^{z_i})^2}{\Sigma_k^2} = \frac{e^{z_i}}{\Sigma_k} \frac{\Sigma_k - e^{z_i}}{\Sigma_k} = \frac{e^{z_i}}{\Sigma_k}\left(1 - \frac{e^{z_i}}{\Sigma_k}\right) = y_i(1 - y_i)$

if $i \neq j$: $\frac{\partial y_i}{\partial z_j} = \frac{\partial \frac{e^{z_i}}{\Sigma_k}}{\partial z_j} = -\frac{e^{z_i} e^{z_j}}{\Sigma_k} = -y_i y_j$

$$\frac{\partial z_{ij}}{\partial W_{ij}^{(2)}} = \frac{\partial \left(W_{ij}^{(2)} \cdot h_{ij} + b_i^{(2)}\right)}{\partial W_{ij}^{(2)}} = h_{ij}$$

$$\frac{\partial \mathcal{L}}{\partial W_{ij}^{(2)}} = \begin{cases} (y_i - t_i) \cdot y_i (1 - y_i) h_{ij} & (i = j) \\ (y_i - t_i)(-y_i y_j) \cdot h_{ij} & (i \neq j) \end{cases}$$

Ideally, the gradient should give us strong signals regarding how to update w to do better.

But here $\frac{\partial \mathcal{L}}{\partial w_{ij}}$ is small.

Which means that update $W \leftarrow W - \alpha \frac{\partial \mathcal{L}}{\partial w}$ won't change W much.

So we will not get good gradient signal to update $W_{ij}^{(2)}$ if we use this square loss.

e) $\sigma(z) = \frac{1}{1+e^{-z}}$

$$\sigma'(z) = -\frac{\frac{d}{dz}(e^{-z}+1)}{(e^{-z}+1)^2} = \frac{e^{-z}}{(e^{-z}+1)^2}$$

$$\sigma(z)(1-\sigma(z)) = \frac{1}{1+e^{-z}} \cdot \left(\frac{e^{-z}}{1+e^{-z}}\right)$$

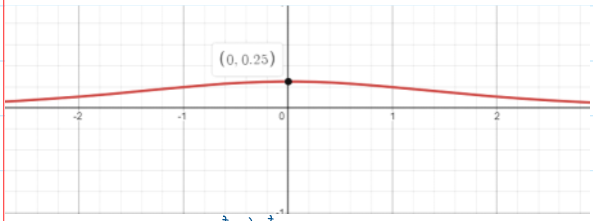
$$= \frac{e^{-z}}{}$$

$$\sigma(z)(1-\sigma(z)) = \frac{1}{1+e^{-z}} \cdot \left(\frac{e^{-z}}{1+e^{-z}}\right)$$

$$= \frac{e^{-z}}{(e^{-z}+1)^2}$$

So we showed that $\sigma'(z) = \sigma(z)(1-\sigma(z))$

plot of the function $\sigma'(z)$



Max : $\sigma''(z) = -\frac{(e^{z}-1)e^{z}}{(e^{z}+1)^3}$

$$-\frac{(e^{z}-1)e^{z}}{(e^{z}+1)^3} = 0$$

$$-(e^{z}-1)e^{z} = 0$$

$$e^{z}-1 = 0$$

$$e^{z} = 1$$

$$z = 0$$

When $z=0$ $\sigma'(0) = \frac{1}{4} = 0.25$

So the maximum is $(0, 0.25)$, and $\sigma'(z)$ must be positive since both e^{-z} and $(e^{-z}+1)$ are positive

let $z = W_1 x + b_1$

$$\frac{\partial h_1}{\partial x} = \frac{\partial h_1}{\partial z}\frac{\partial z}{\partial x} = \sigma'(z)\, W_1$$

$$= \sigma(z)(1-\sigma(z))\, W_1$$

Since for $\sigma'(z)$ it at most 0.25 which is $\frac{1}{4}$

And $\sigma'(z)$ is positive.

We have $\left|\frac{\partial h_1}{\partial x}\right| \leq \frac{1}{4}|W_1|$

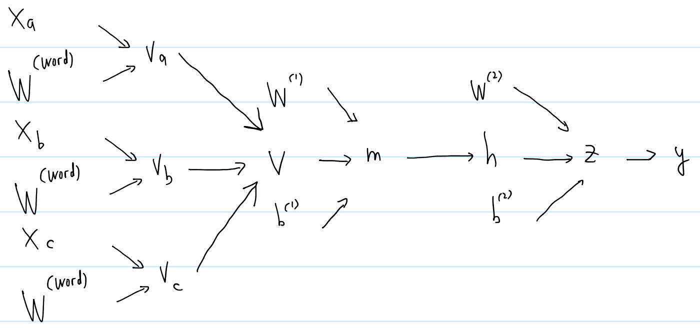
f) The problem for this result is when N is very large, $\left|\frac{\partial h_N}{\partial x}\right|$ will become extremely small. since it's smaller than $\frac{1}{4^N}|W_1||W_2|\cdots|W_N|$ and $\frac{1}{4^N}$ will surely be very small when N is big.

And this means that x's value will not effect hN a lot. The Nth layer will try to update its weight mostly based on its previous layer's updated weight. More closer to the last layer, the value of x will be less important. In this case, for many of the layers close to the Nth layer, it will only take few information from the input x. It shows that more layers will not help at all and it will even make the learning speed slower and cause problems. On the other hand, if we have many layers and we multiply these gradients together, it's possible that the product of many small values (less than one) will become zero very quickly (because "1/a large number" will be treated as 0 if the number is large enough) since the derivative of the sigmoid function is always smaller than 1. For deep learning, more layers will always improve the learning experience, but if we use sigmoid activation function, more layers means problems. That's why sigmoid activation function would be a problem with this result.

g) No, we will not have the same issue as in part f if we replaced the sigmoid activation with ReLu activation. ReLu activation function reduced likelihood of the gradient to vanish. For the gradient of sigmoid activation function, it will get smaller and smaller as the absolute value of the input x increases. And as we saw in the previous questions, the derivative of the sigmoid function is always smaller than 1/4. In Question e) we just proved that the maximum is (0,1/4). It means that the sigmoid activation function learns slow. But the constant gradient of ReLus will have a faster learning. The gradient of the ReLu activation function is either 0 (if <0) or 1 (if >0). This means the number of the layers will not cause any problems. The gradients will never vanish.

Q3)

a)



b) $$\frac{\partial y}{\partial W^{(word)}} = \frac{\partial y}{\partial z}\frac{\partial z}{\partial W^{(word)}} = \frac{\partial y}{\partial z}\frac{\partial z}{\partial h}\frac{\partial h}{\partial W^{(word)}} = \frac{\partial y}{\partial z}\frac{\partial z}{\partial h}\frac{\partial h}{\partial m}\frac{\partial m}{\partial W^{(word)}}$$

$$= \frac{\partial y}{\partial z}\frac{\partial z}{\partial h}\frac{\partial h}{\partial m}\frac{\partial m}{\partial V}\frac{\partial V}{\partial W^{(word)}}$$

$$= \frac{\partial y}{\partial z}\frac{\partial z}{\partial h}\frac{\partial h}{\partial m}\frac{\partial m}{\partial V}\left(\frac{\partial V}{\partial V_a}\frac{\partial V_a}{\partial W^{(word)}} + \frac{\partial V}{\partial V_b}\frac{\partial V_b}{\partial W^{(word)}} + \frac{\partial V}{\partial V_c}\frac{\partial V_c}{\partial W^{(word)}}\right)$$

4c)

If we do not call numpy_model.initializeParams(), then the **__init__** method in Numpy WordEmbModel class will initialize all weights and bias matrix with zeros. So no matter what the input is, every hidden unit will get zero. Since 0x+
0 = 0 (x is the input vector). It will cause all of them to have the same gradient. It means that if all the weights and bias are zero, the learning rate will only affects the scale of the weight v
ector, not the direction.

4d)

After applying softmax, each component will be in (0,1) range and they will have a total sum 1.

Then each column of Softmax (z) will be $\sigma(z)_j = \frac{e^{z_j}}{\sum_{k=1}^{K} e^{z_k}}$ for $j = 1 \ldots K$, K = the number of column.

Then if z_i is the maximum column in z. Since every column is divide by the same number $\sum_{k=1}^{K} e^{z_k}$, and e^{z_i} is positive, $\frac{e^{z_i}}{\sum_{k=1}^{K} e^{z_k}}$ will still be the maximum column in softmax(z)

The larger input components will correspond to larger probabilities. So the max column of z will still be the max column in y since it will have the largest probability.

5d)
Yes, these predictions do make sense since these are all part of some sentences.

For the 3-grams words, all of the 3-grams words are not appearing next to each other in the training set.

6a)
The shape of the word_emb_weights is (100,250). 100 here represents the emb size and 250 represents the vocab size. This embedding layer is to compute the representation of each word and we will get the result by multiply the word_emb_weights to the one-hot vector for each word (which has a shpe 250*1). For each word multiply by woed_emb_weights, we basically do a matrix multiplication of (100x250) x (250x1) and we will finally get a 100x1 vector and this vector is the representation vector of that word. If we multiply a (250x250)matrix which represent all of the one hot vectors for each word. We will finally get a (100x250) matrix which each column is a word representation vector. But since here we take the transpose, word_emb has a shape (250,100) which ith row is a 1x100 vector and this vector is the representation of the ith word in the vocab.In the example vocab_stoi["any"] will get the index of the word "any" and word_emb[vocab_stoi["any"],:] will take the corresponding row in word_emb matrix. Which is the vector representation of the word "any".

6c)
Cluster 1:
many, few,three,two,several

These word in cluster 1 are all able to describe numbers

Cluster 2: can,could,will,should,might

These words are all verb and they all can be followed with a personal pronoun

7)
This assignment is finished individually by Hongyu Chen