SAOT: An Enhanced Locality-Aware Spectral Transformer for Solving PDEs Supplementary Material

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Details of Benchmarks

Here we provide detailed descriptions of benchmarks.

Darcy We utilize the Darcy dataset proposed in (Li et al. 2021), where the 2-D Darcy flow equation models the flow of fluid through a porous medium within a unit square domain:

$$-\nabla \cdot (a(x)\nabla u(x)) = f(x), \tag{1}$$

with a Dirichlet boundary $u(x) = 0$, where $a \in \mathbb{R}^+$ is the diffusion coefficient function and f means the forcing function. This dataset aims to learn the operator mapping the diffusion a to the solution u.

Navier-Stokes (NS) We use the 2-D Navier-Stokes dataset from (Li et al. 2021), which models a viscous and incompressible flow on the unit torus:

$$\begin{aligned} \partial_t w(x,t) + u(x,t) \cdot \nabla w(x,t) &= \nu \nabla^2 w(x,t) + f(x), \\ \nabla \cdot u(x,t) &= 0, \\ w(x,0) &= w_0(x) \end{aligned} \tag{2}$$

where $x \in (0,1)^2, t \in [0,T]$, u is the velocity field, $w = \nabla \times u$ is the vorticity, w_0 is the initial vorticity, ν is the viscosity coefficient and set as 10^{-5} in this dataset. The task is to predict the future 10 frames based on the past 10 frames.

Airfoil The Airfoil dataset (Li et al. 2023) models the transonic flow over an airfoil, which is governed by the Euler equations, expressed as:

$$\begin{aligned} \frac{\partial \rho^f}{\partial t} + \nabla \cdot (\rho^f \boldsymbol{v}) &= 0, \\ \frac{\partial \rho^f \boldsymbol{v}}{\partial t} + \nabla \cdot (\rho^f \boldsymbol{v} \otimes \boldsymbol{v} + p\mathbb{I}) &= 0, \\ \frac{\partial E}{\partial t} + \nabla \cdot \Big((E+p)\boldsymbol{v}\Big) &= 0 \end{aligned} \tag{3}$$

where ρ^f is the fluid density, $\boldsymbol{v}$ is the velocity vector, p is the pressure, and E is the total energy. The shape of the airfoil is discretized into structured meshes. This task takes the mesh point locations as inputs and predicts the Mach number on these mesh points.

Pipe This dataset (Li et al. 2023) models the incompressible flow in a pipe, where the governing equation can be formulated as follows,

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\nabla)\boldsymbol{v} = -\nabla p + \nu \nabla^2 \boldsymbol{v}, \tag{4}$$

$$\nabla \cdot \boldsymbol{v} = 0 \tag{5}$$

where $\boldsymbol{v}$ is the velocity vector, p is the pressure, and $\nu = 0.005$ is the viscosity. The input is the mesh structure, and the output is the velocity of the horizontal fluid within the pipe.

Plasticity This dataset (Li et al. 2023) focuses on the plastic forging problem, where a plastic material is clamped on the bottom edge and struck from above by a die of any shape. The input is the shape of the die, and the task is to predict the deformation of the input mesh points in the future 20 timesteps.

Elasticity We used the Elasticity dataset in (Li et al. 2023). This elasticity problem has an unstructured input format of a set of point clouds. We want to estimate the inner stress of the elasticity material based on the material structure. The input is the point clouds describing the structure of the material. The task is to predict the stress of each point.

Evaluation metrics

We use the mean relative L^2 error as the evaluation metric in the experiments. Given a finite set of function observations $\{a_i, u_i\}_{i=1}^N$ and the well-trained neural operator Φ_θ, the mean relative L^2 error ε can be computed as follows:

$$\varepsilon = \frac{1}{N}\sum_{i=1}^{N} \frac{||\Phi_\theta(a_i) - u_i||_2}{||u_i||_2}. \tag{6}$$

Visualization of Results

In this section, we compare the prediction results of Transolver (Wu et al. 2024) and our SAOT model and show some examples in Figure 1. We can see that SAOT achieves lower prediction errors than Transolver across diverse datasets with different geometries.

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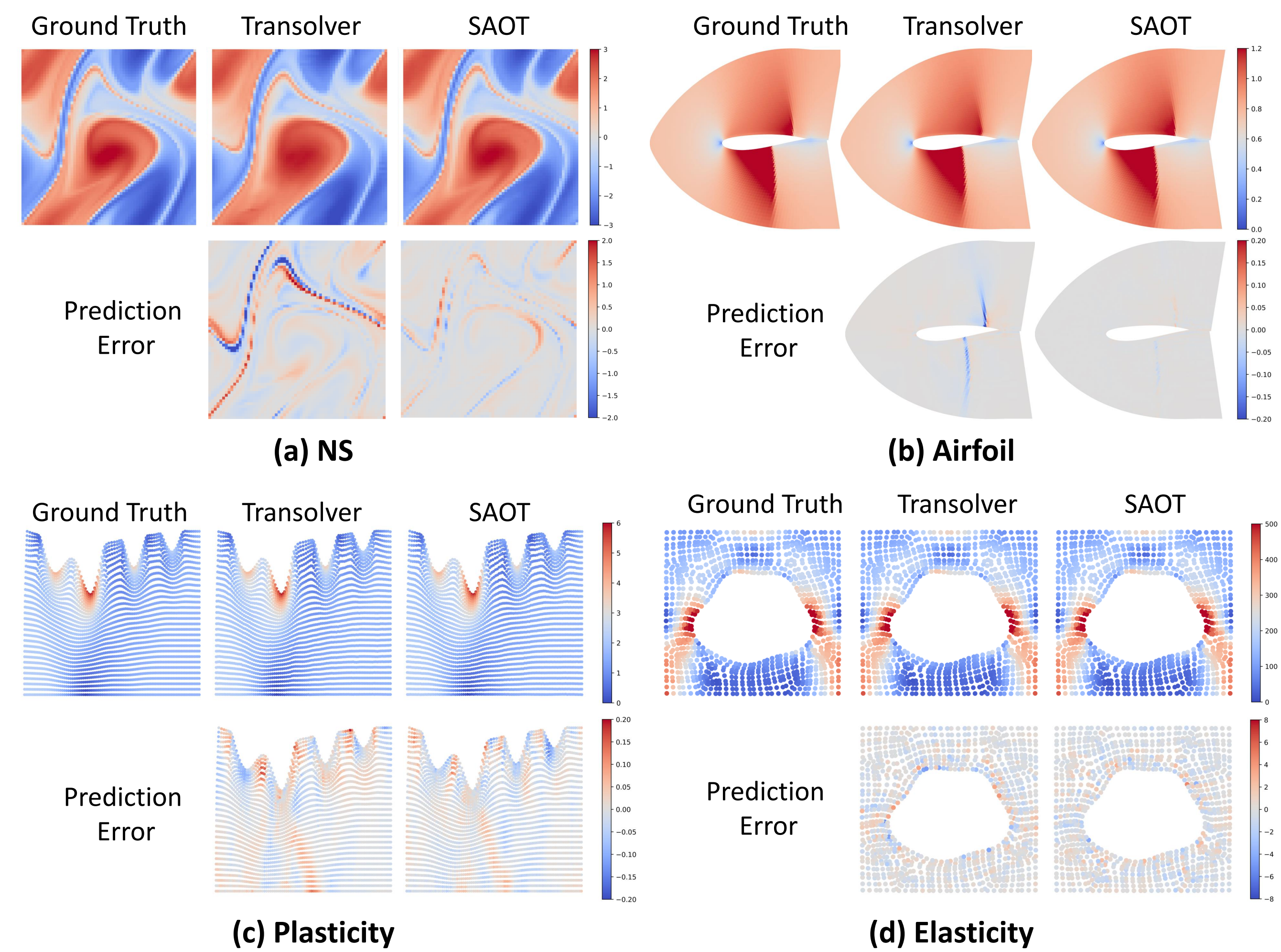

Figure 1: Ground truth, predictions, and pointwise errors of Transolver and our SAOT model.

References

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