

E2 15-Puzzle Problem (IDA*)

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1 IDA* Algorithm

1.1 Description

Iterative deepening A* (IDA*) was first described by Richard Korf in 1985, which is a graph traversal and path search algorithm that can find the shortest path between a designated start node and any member of a set of goal nodes in a weighted graph.

It is a variant of **iterative deepening depth-first search** that borrows the idea to use a heuristic function to evaluate the remaining cost to get to the goal from the **A* search algorithm**.

Since it is a depth-first search algorithm, its memory usage is lower than in A*, but unlike ordinary iterative deepening search, it concentrates on exploring the most promising nodes and thus does not go to the same depth everywhere in the search tree.

Iterative-deepening-A* works as follows: at each iteration, perform a depth-first search, cutting off a branch when its total cost $f(n) = g(n) + h(n)$ exceeds a given threshold. This threshold starts at the estimate of the cost at the initial state, and increases for each iteration of the algorithm. At each iteration, the threshold used for the next iteration is the minimum cost of all values that exceeded the current threshold.

1.2 Pseudocode





```
path          current search path (acts like a stack)
node          current node (last node in current path)
g            the cost to reach current node
f            estimated cost of the cheapest path (root..node..goal)
h(node)      estimated cost of the cheapest path (node..goal)
cost(node, succ) step cost function
is_goal(node) goal test
successors(node) node expanding function, expand nodes ordered by g + h(node)
ida_star(root) return either NOT_FOUND or a pair with the best path and its cost

procedure ida_star(root)
  bound := h(root)
  path := [root]
  loop
    t := search(path, 0, bound)
    if t = FOUND then return (path, bound)
    if t = ∞ then return NOT_FOUND
    bound := t
  end loop
end procedure

function search(path, g, bound)
  node := path.last
  f := g + h(node)
  if f > bound then return f
  if is_goal(node) then return FOUND
  min := ∞
  for succ in successors(node) do
    if succ not in path then
      path.push(succ)
      t := search(path, g + cost(node, succ), bound)
      if t = FOUND then return FOUND
      if t < min then min := t
      path.pop()
    end if
  end for
  return min
end function
```

2 Tasks

- Please solve 15-Puzzle problem by using IDA* (Python or C++). You can use one of the two commonly used heuristic functions: h_1 = the number of misplaced tiles. h_2 = the sum of the distances of the tiles from their goal positions.
- Here are 4 test cases for you to verify your algorithm correctness. You can also play this game (15puzzle.exe) for more information.

	<p>TextOut of Result</p> <pre> 11 3 1 7 4 6 8 2 15 9 10 13 14 12 5 0 LowerBound 38 moves A optimal solution 56 moves Used time 3 sec 13 10 8 6 9 12 5 13 10 8 12 15 14 5 13 12 15 14 5 13 14 9 4 11 3 1 6 4 11 3 1 6 4 2 8 10 12 15 10 8 7 4 2 11 3 5 9 10 11 3 6 2 3 7 8 12 </pre>
	<p>TextOut of Result</p> <pre> 0 5 15 14 7 9 6 13 1 2 12 10 8 11 4 3 LowerBound 44 moves A optimal solution 62 moves Used time 4 sec 7 9 2 1 9 2 5 7 2 5 1 11 8 9 5 1 6 12 10 3 4 8 11 10 12 13 3 4 8 12 13 15 14 3 4 8 12 13 15 14 7 2 1 5 10 11 13 15 14 7 3 4 8 12 15 14 11 10 9 13 14 15 </pre>
	<p>TextOut of Result</p> <pre> 14 10 6 0 4 9 1 8 2 3 5 11 12 13 7 15 LowerBound 37 moves A optimal solution 49 moves Used time 0 sec 6 10 9 4 14 9 4 1 10 4 1 3 2 14 9 1 3 2 5 11 8 6 4 3 2 5 13 12 14 13 12 7 11 12 7 14 13 9 5 10 6 8 12 7 10 6 7 11 15 </pre>
	<p>TextOut of Result</p> <pre> 6 10 3 15 14 8 7 11 5 1 0 2 13 12 9 4 LowerBound 32 moves A optimal solution 48 moves Used time 0 sec 9 12 13 5 1 9 7 11 2 4 12 13 9 7 11 2 15 3 2 15 4 11 15 8 14 1 5 9 13 15 7 14 10 6 1 5 9 13 14 10 6 2 3 4 8 7 11 12 </pre>

- We encourage you to use better heuristic functions and compare efficiency of different functions, but that's not necessary.
- Please send E02_YourNumber.pdf to ai_course2021@163.com, you can certainly use E02_15puzzle.tex as the L^AT_EX template.

3 Methods

In this experiment, we use IDA* algorithm to solve 15 digital problems. In the selection of H function, we select the sum of the distances of the tiles from their goal positions.

```

def h2(a):
    """
    @description :the sum of the distances of the tiles from their goal positions.
    -----
    @param :
    a:Matrix
    -----
    @Returns :
    return h
    -----
    """
    # aa = deepcopy(a)
    ans = 0
    for i in range(4):
        for j in range(4):

```

```

    ans = ans+abs(i-(a[i][j]-1) // 4)+abs(j-(a[i][j]-1) % 4)
pass
return ans

```

4 Results

Take matrix[[5, 1, 3, 4],[9, 2, 7, 8],[13, 6, 10, 11],[14, 16, 15, 12]] as an example. The result is below:

	16 01 03 04	
05 01 03 04	05 02 07 08	
09 02 07 08	09 06 10 11	
13 06 10 11	13 14 15 12	
14 16 15 12		
===== step 0 =====	===== step 4 =====	
05 01 03 04	01 16 03 04	01 02 03 04
09 02 07 08	05 02 07 08	05 06 07 08
13 06 10 11	09 06 10 11	09 10 16 11
16 14 15 12	13 14 15 12	13 14 15 12
===== step 1 =====	===== step 5 =====	===== step 8 =====
05 01 03 04	01 02 03 04	01 02 03 04
09 02 07 08	05 16 07 08	05 06 07 08
16 06 10 11	09 06 10 11	09 10 11 16
13 14 15 12	13 14 15 12	13 14 15 12
===== step 2 =====	===== step 6 =====	===== step 9 =====
05 01 03 04	01 02 03 04	01 02 03 04
16 02 07 08	05 06 07 08	05 06 07 08
09 06 10 11	09 16 10 11	09 10 11 12
13 14 15 12	13 14 15 12	13 14 15 16
===== step 3 =====	===== step 7 =====	===== step 10 =====

More results of examples put in the file test1_h2_2.txt, test1_h2_3.txt, test1_h2_4.txt.

5 Source Code

```
def search(path, cost, bound, heuristic):
    global step

    arr = path[-1]
    f = cost + heuristic(arr)

    if f > bound:
        return f, False
    if is_goal(arr):
        return f, True

    (bi, bj) = blank(arr)
    min = 0x3f3f3f3f

    for d in ['U', 'L', 'D', 'R']:

        newArr = tryy(arr, bi, bj, d)
        if newArr == None:
            continue
        else:
            if newArr not in path:
                path.append(newArr)
                t, flag = search(path, cost + 1, bound, heuristic)
                if flag == True:
                    return t, True
                if bound < min:
                    min = t
                path.pop()
            pass

    return min, False

def ida():
    global step
    bound = h2(matr)
    path = [matr]
    while True:
        bound, flag = search(path, 0, bound, h2)
        if flag == True:
            break
        if bound == 0x3f3f3f3f:
            print("impossible!")
            return
        visial_path(path)

ida()
```