School of Computer Science, Sun Yat-sen University

P02 CSP

Haoran Chen 19335016

2021 年 11 月 2 日

目录

1 Futoshiki (GAC, C++/Python)

1.1 Description

Futoshiki is a board-based puzzle game, also known under the name Unequal. It is playable on a square board having a given fixed size (4 × 4 for example), please see Figure 1.

The purpose of the game is to discover the digits hidden inside the board's cells; each cell is filled with a digit between 1 and the board's size. On each row and column each digit appears exactly once; therefore, when revealed, the digits of the board form a so-called Latin square.

At the beginning of the game some digits might be revealed. The board might also contain some inequalities between the board cells; these inequalities must be respected and can be used as clues in order to discover the remaining hidden digits.

Each puzzle is guaranteed to have a solution and only one.

You can play this game online: `http://www.futoshiki.org/`.

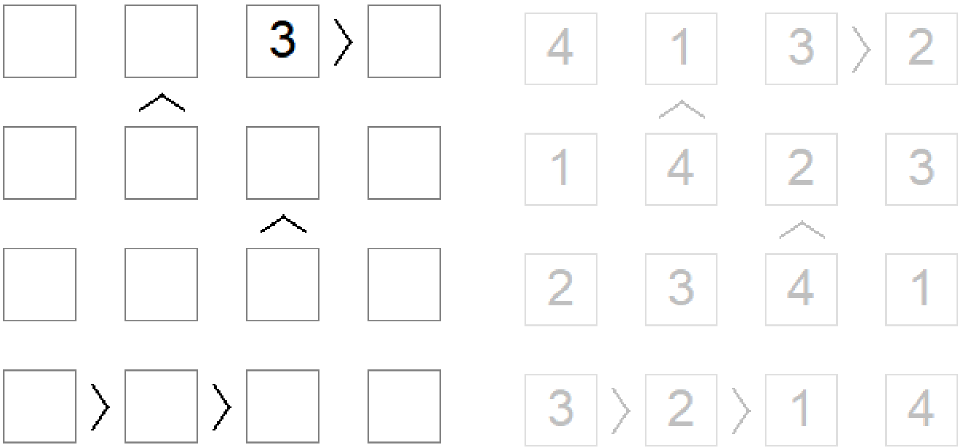


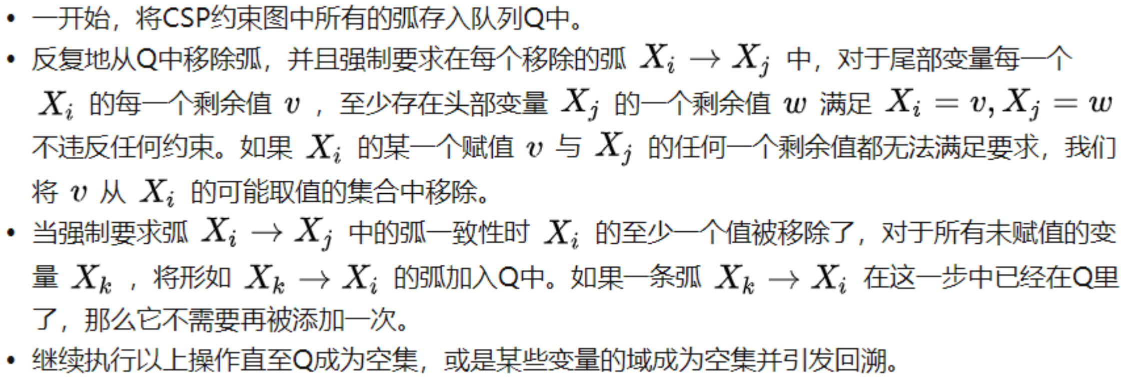
图 1: Futoshiki Puzzles

1.2 Tasks

1. Describe with sentences the main ideas of the GAC algorithm and the main differences between the GAC and the forward checking (FC) algorithm. (10 points)

   Answer: GAC 算法的主要内容是检测每个变量的可扩展性以满足约束。即，如果变量的每个值都可以以满足约束的方式扩展到约束的其他变量，则变量与约束一致。至于他的优化问题,

   GAC 算法主要步骤如下:



- 一开始，将CSP约束图中所有的弧存入队列Q中。
- 反复地从Q中移除弧，并且强制要求在每个移除的弧 $X_i \rightarrow X_j$ 中，对于尾部变量每一个 X_i 的每一个剩余值 v ，至少存在头部变量 X_j 的一个剩余值 w 满足 $X_i = v, X_j = w$ 不违反任何约束。如果 X_i 的某一个赋值 v 与 X_j 的任何一个剩余值都无法满足要求，我们将 v 从 X_i 的可能取值的集合中移除。
- 当强制要求弧 $X_i \rightarrow X_j$ 中的弧一致性时 X_i 的至少一个值被移除了，对于所有未赋值的变量 X_k ，将形如 $X_k \rightarrow X_i$ 的弧加入Q中。如果一条弧 $X_k \rightarrow X_i$ 在这一步中已经在Q里了，那么它不需要再被添加一次。
- 继续执行以上操作直至Q成为空集，或是某些变量的域成为空集并引发回溯。

图 2: GAC 算法主要步骤

至于 GAC 与 FC 的区别, 广义弧一致性 (GAC, or 弧相容, arc consistency) 是前向检测的扩展, FC 算法在运行过程中, 每当一个新的变量被赋值，我们就能进行前向检测，并删去约束图中与刚赋值的变量相邻的未赋值变量的域；而 GAC 算法在运行过程中, 不仅仅是删去相邻的未赋值变量的域, 而是通过比较全部约束条件不断进行比较直到满足所有条件或者存储约束条件的队列变成空集, 效率较 FC 算法有较大提升.

2. The GAC_Enforce procedure from class acts as follows: when removing d from CurDom[V], push all constraints C' such that $V \in scope(C')$ and $C' \notin$ GACQueue onto GACQueue. What's the reason behind this operation? Can it be improved and how? (20 points)

   Answer: 在进行操作 removing d from CurDom[V] 之后, CurDom[V] 发生变化, 而与域 V 有关的约束 C' 受到操作影响也可能会导致与约束 C' 有关的其他变量域取值范围发生变化, 因此需要将与域 V 相关联且不在 GACQueue 的约束 C' 放入 GACQueue 中, 这样能简化约束满足问题并且减少域的取值范围进而缩短搜索时间. 这显然是可以优化的.一个简单的想法是, 在 GACQueue 中取出一个约束条件后, 对其进行分类讨论, 如果是不等式约束, 将显然不符合不等关系的域中的值删除并标记是否删除, 如果删除后域中没有值了, 这显然是无解的; 如果是行列关系, 在判断在域范围大小是否为 1 后删除不符合约束条件的值并进行空域判断.

3. Use the GAC algorithm to implement a Futoshiki solver by **C++** or **Python**. (20 points)

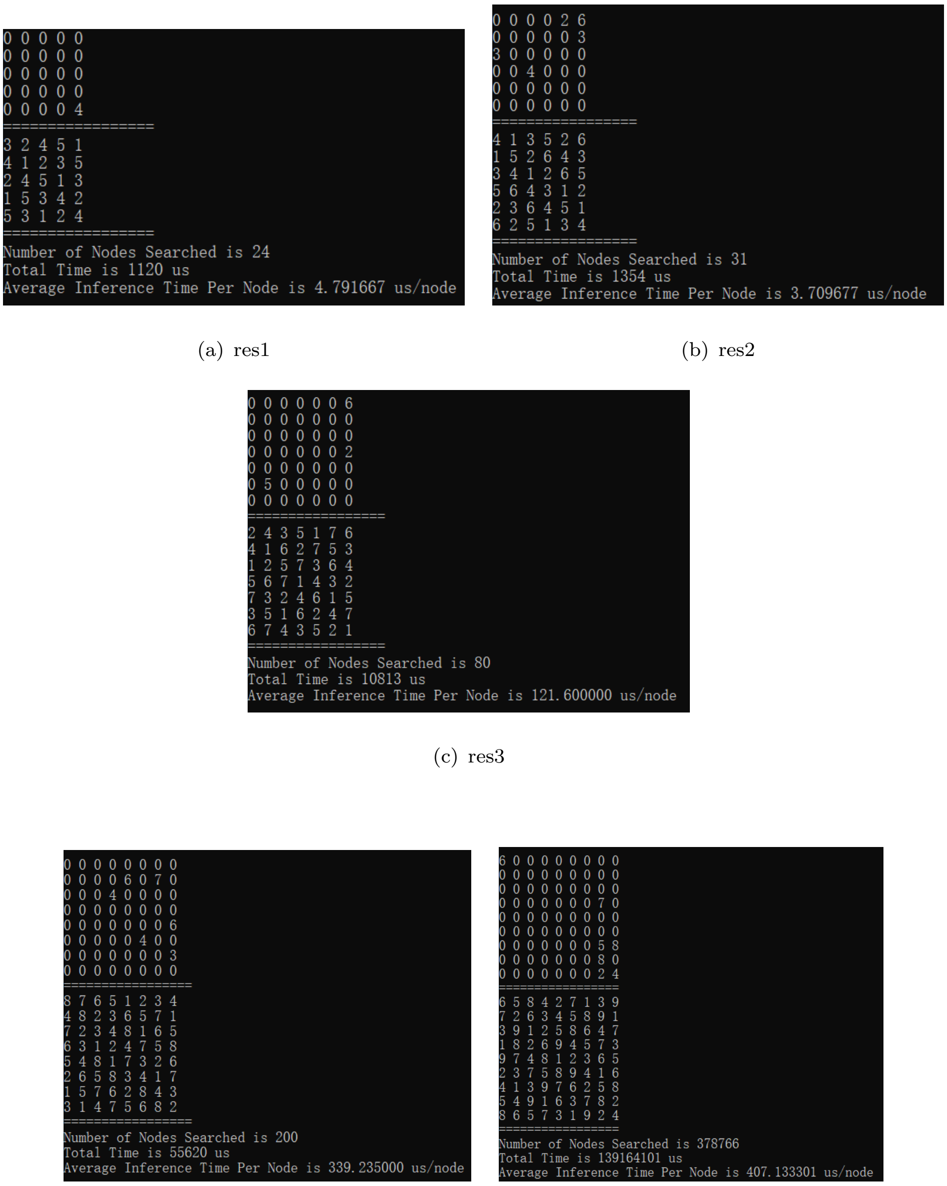
   Answer: Please see the subsecion GAC Source Code in 2.1.

4. Explain any ideas you use to speed up the implementation. (10 points)

   Answer: 见第二问2

5. Run the following 5 test cases to verify your solver's **correctness**. We also provide test file “datai.txt” for every test case i. Refer to the “readme.txt” for more details. (20 points)

   Answer: 运行结果如下:(图8与 `testcase5.txt` 不一致)



```
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 4
=================
3 2 4 5 1
4 1 2 3 5
2 4 5 1 3
1 5 3 4 2
5 3 1 2 4
=================
Number of Nodes Searched is 24
Total Time is 1120 us
Average Inference Time Per Node is 4.791667 us/node
```

(a) res1

```
0 0 0 0 2 6
0 0 0 0 0 3
3 0 0 0 0 0
0 0 4 0 0 0
0 0 0 0 0 0
0 0 0 0 0 0
=================
4 1 3 5 2 6
1 5 2 6 4 3
3 4 1 2 6 5
5 6 4 3 1 2
2 3 6 4 5 1
6 2 5 1 3 4
=================
Number of Nodes Searched is 31
Total Time is 1354 us
Average Inference Time Per Node is 3.709677 us/node
```

(b) res2

```
0 0 0 0 0 0 6
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 2
0 0 0 0 0 0 0
0 5 0 0 0 0 0
0 0 0 0 0 0 0
=================
2 4 3 5 1 7 6
4 1 6 2 7 5 3
1 2 5 7 3 6 4
5 6 7 1 4 3 2
7 3 2 4 6 1 5
3 5 1 6 2 4 7
6 7 4 3 5 2 1
=================
Number of Nodes Searched is 80
Total Time is 10813 us
Average Inference Time Per Node is 121.600000 us/node
```

(c) res3

```
0 0 0 0 0 0 0 0
0 0 0 0 6 0 7 0
0 0 0 4 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 6
0 0 0 0 0 4 0 0
0 0 0 0 0 0 0 3
0 0 0 0 0 0 0 0
=================
8 7 6 5 1 2 3 4
4 8 2 3 6 5 7 1
7 2 3 4 8 1 6 5
6 3 1 2 4 7 5 8
5 4 8 1 7 3 2 6
2 6 5 8 3 4 1 7
1 5 7 6 2 8 4 3
3 1 4 7 5 6 8 2
=================
Number of Nodes Searched is 200
Total Time is 55620 us
Average Inference Time Per Node is 339.235000 us/node
```

(d) res4

```
6 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 7 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 5 8
0 0 0 0 0 0 0 8 0
0 0 0 0 0 0 0 2 4
=================
6 5 8 4 2 7 1 3 9
7 2 6 3 4 5 8 9 1
3 9 1 2 5 8 6 4 7
1 8 2 6 9 4 5 7 3
9 7 4 8 1 2 3 6 5
2 3 7 5 8 9 4 1 6
4 1 3 9 7 6 2 5 8
5 4 9 1 6 3 7 8 2
8 6 5 7 3 1 9 2 4
=================
Number of Nodes Searched is 378766
Total Time is 139164101 us
Average Inference Time Per Node is 407.133301 us/node
```

(e) res5

图 3: 运行结果

6. Run the FC algorithm you implemented in E04 and the GAC algorithm you implemented in Task 3 on the 5 test cases, and fill in the following table. In the table, “Total Time” means the total time the algorithm uses to solve the test case, “Number of Nodes Searched” means the total number of nodes traversed by the algorithm, and “Average Inference Time Per Node” means the average time for constraint propagation (inference) used in each node (note that this time is not equal to the total time divided by the number of nodes searched). Analyse the reasons behind the experimental results, and write them in your report. (20 points)

Test Case	Algorithm	Total Time	Number of Nodes Searched	Average Inference Time Per Node
1	FC	0.002363 s	65	33.708 us/node
	GAC	0.001120 s	24	4.792 us/node
2	FC	0.002703 s	92	47.054 us/node
	GAC	0.001354 s	31	3.710 us/node
3	FC	0.052703 s	1328	69.760 us/node
	GAC	0.010813 s	80	121.600 us/node
4	FC	0.091053 s	1386	165.013 us/node
	GAC	0.055620 s	200	339.235 us/node
5	FC	1702.582241 s	26742542	81.598 us/node
	GAC	139.164101 s	378766	407.133 us/node

可以看到, 不论在什么情况下, GAC 算法搜索的节点数总是要小于 FC 算法搜索的节点数, 在 puzzle 大小比较小且者约束条件比较多时, GAC 算法与 FC 算法程序运行时间相差无几, 但是但问题规模比较大时, 性能差距就显现出来了 (样例 5 的运行时间与搜索节点数差距十分明显); 至于每个节点的平均推断时间, 可以看到, 在问题规模小的时候, GAC 的消耗时间是比 FC 小的, 甚至可以忽略不计, 但是但问题规模变大的时候, GAC 的消耗时间逐渐增大, 超过了 FC 的平均推断时间. 究其原因, 是因为 GAC 算法在实现弧一致性期间, 需要不断判断, 删除, 插入约束条件直到 GacQueue 为空, 在问题规模较大的时候, 这种不断地判断、插入与删除是比较消耗时间的，而且 FC 算法判断的条件相对来说比较少, 这就导致了在节点平均推断时间方面 FC 算法比较稳定, 而 GAC 在问题规模变大的情况下推断时间也随之变大.

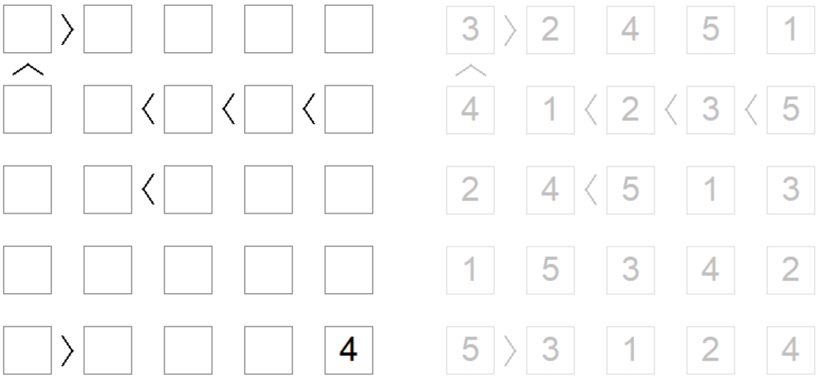


图 4: Futoshiki Test Case 1

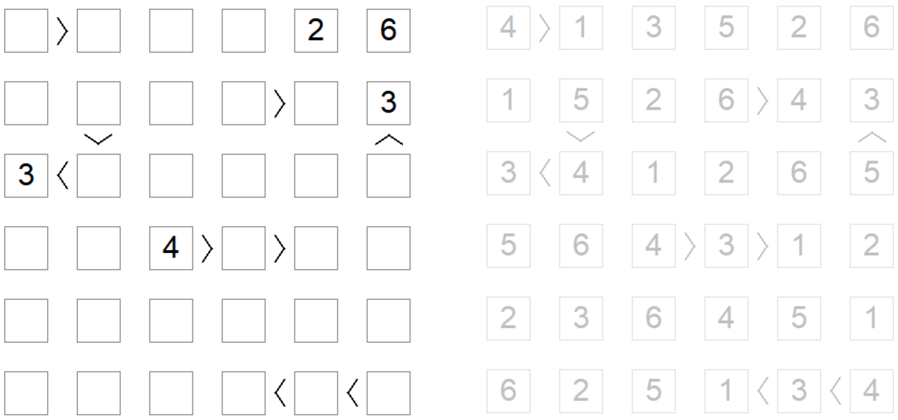


图 5: Futoshiki Test Case 2

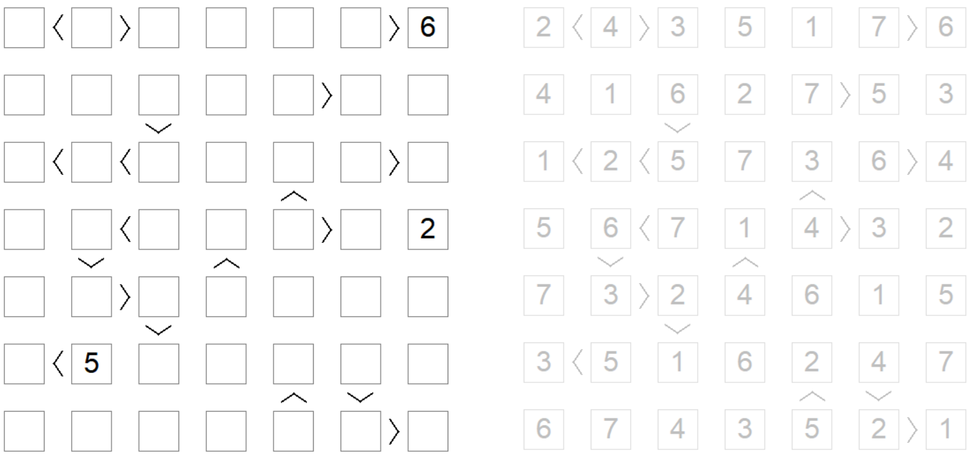


图 6: Futoshiki Test Case 3

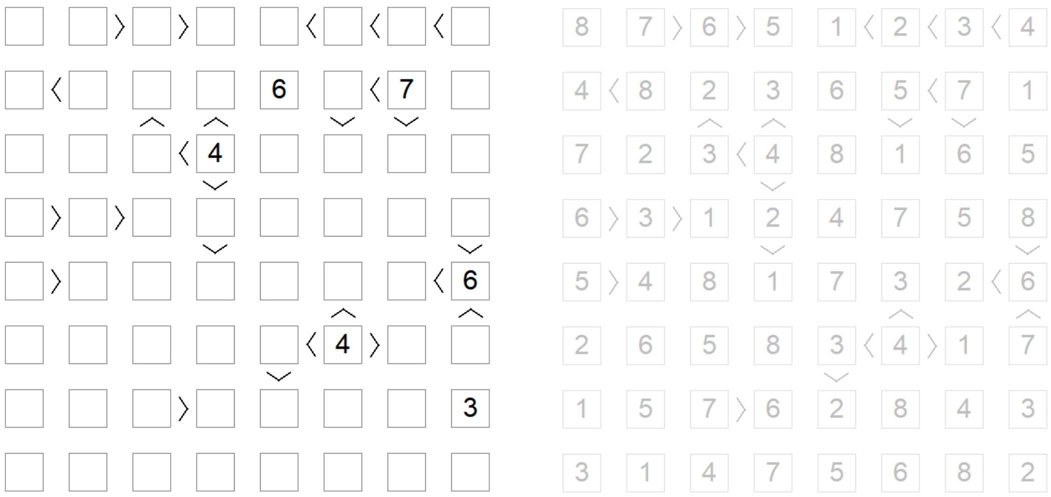


图 7: Futoshiki Test Case 4

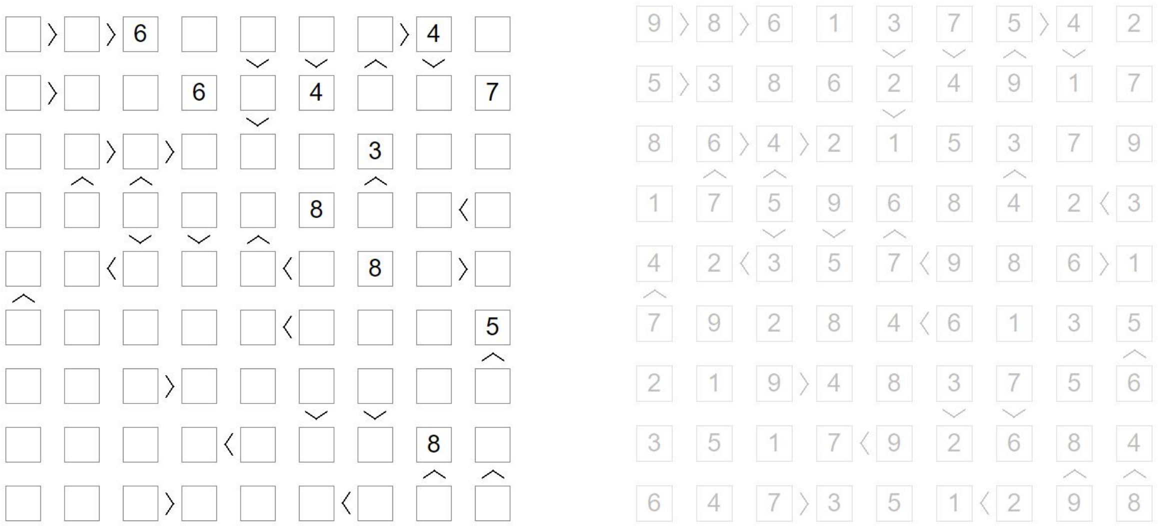


图 8: Futoshiki Test Case 5

2 To be announced

2.1 GAC Source Code

```
#include <iostream>
#include <fstream>
#include <sstream>
#include <vector>
#include <string>
#include <set>
#include <algorithm>
#include <list>
#include <ctime>
#include <windows.h>
using namespace std;

#define POINT pair<int, int>
#define PUZZLE vector<vector<int> >
#define DOMAINS vector<vector<set<int> > >

long long node_find = 0;

```

```
LARGE_INTEGER node_start_time, node_end_time, frequency;
long long count_time = 0;

struct Constraint {
    //0为不等式约束，>0为行约束，<0为列约束；
    //行列数从1开始计数，且列值取负，否则与不等式约束重复冲突。
    int type;
    // 不等式约束，relation.first < relation.second
    pair<POINT, POINT> relation;
    Constraint(int t = 0, pair<POINT, POINT> r = {{-1, -1},{-1, -1}})
    : type(t), relation(r) {}
    //重载约束比较，不加这个会报错
    bool operator==(const Constraint& rhs) {
        return (type == rhs.type && relation == rhs.relation);
    }
};

class Solution{
    private:
    int size;     // puzzle尺寸，通过读取文件确定
    PUZZLE puzzle;
    vector<Constraint> conArray;
    public:
    Solution(const string filename){
        size = 0;
        ifstream fin(filename.c_str());
        string firstLine, tmp;
        int con_num = 0;
        //确定puzzle尺寸
        getline(fin, firstLine);
        stringstream ss;ss<<firstLine;
        while(ss>>tmp) size++;
        //重置文件指针
        fin.clear();fin.seekg(0);
        puzzle.assign(size, vector<int>(size, 0));

        for (int i = 0; i < size; i++)  for (int j = 0; j < size; j++)
        fin >> puzzle[i][j];

        //确定约束条件个数
        while(getline(fin, firstLine)) con_num++;
        fin.clear();fin.seekg(0);
        for(int _=1; _<=size; _++) getline(fin, firstLine);

        for (int i = 0; i < con_num; i++) {
            int x1, y1, x2, y2;
            fin >> x1 >> y1 >> x2 >> y2;
            conArray.push_back(Constraint(0, {{x1, y1}, {x2, y2}}));
        }
        fin.close();
    }
    bool isSolved(){
        for (int i = 0; i < size; i++) for (int j = 0; j < size; j++)
        if (puzzle[i][j] == 0)  return false;

        return true;
    }
    void printPuzzle(){
        for (int i = 0; i < size; i++) {
            for (int j = 0; j < size; j++) printf("%d ", puzzle[i][j]);
            puts("");
        }
        puts("================");
    }
    void FilePrintPuzzle(const string filename){
        ofstream fout (filename.c_str());
        for (int i = 0; i < size; i++) {
            for (int j = 0; j < size; j++) fout<<puzzle[i][j]<<" ";
            fout<<'\n';
        }
        fout<<"=================\n";
        fout.close();
    }

    // 初始化每个棋盘格子域
    DOMAINS makeDomains(list<Constraint>& gacQ){
        // 初始化
        DOMAINS domains(size, vector<set<int>>(size, set<int>()));
        for (int i = 0; i < size; i++) for (int j = 0; j < size; j++){
            if (puzzle[i][j] == 0) for (int k = 1; k <= size; k++)
                domains[i][j].insert(k);
            else domains[i][j].insert(puzzle[i][j]);
        }

        // 将所有行列约束加入gacQ
        for (int rc = 1; rc <= size; rc++) {
            gacQ.push_back(Constraint(rc));
            gacQ.push_back(Constraint(-rc));
        }
        for (int i = 0; i < conArray.size(); i++)
        gacQ.push_back(conArray[i]);

        return GacEnforce(domains, gacQ);  // 执行Gac_enforce
    }

    // MinimumPos, 与FC的MinimumPos不一样
    POINT MinimumPos(){
        for (int i = 0; i < size; i++) for (int j = 0; j < size; j++)
                if (puzzle[i][j] == 0) return {i, j};

        return POINT();
    }

    // 在具体实现中，队列使用C++ STL list
    // 把与变量pos相关的约束不重复地加入队列
    void AddConstraint2Q(POINT pos, list<Constraint>& gacQ){
        for (int i = 0; i < conArray.size(); i++) {
            if (conArray[i].type != 0) continue;// 不等式约束

            pair<POINT, POINT> relation = conArray[i].relation;
            if (pos == relation.first || pos == relation.second) {
                auto constr_position = find(gacQ.begin(),
                gacQ.end(), conArray[i]);
                if (constr_position == gacQ.end())   // 不重复加入，以下同理
                gacQ.push_back(conArray[i]);
            }
        }
        //行约束
        Constraint row_constraint(Constraint(pos.first + 1));
        auto row_position = find(gacQ.begin(), gacQ.end(), row_constraint);
        if (row_position == gacQ.end()) gacQ.push_back(row_constraint);

        //列约束
        Constraint col_constraint(Constraint(-pos.second - 1));
        auto col_position = find(gacQ.begin(), gacQ.end(), col_constraint);
        if (col_position == gacQ.end()) gacQ.push_back(col_constraint);
    }


    DOMAINS GacEnforce(DOMAINS domains, list<Constraint>& gacQ){
        while (!gacQ.empty()) {//类似于bfs
            Constraint constraint = gacQ.front(); gacQ.pop_front();
            // 已优化不等式约束
            if (constraint.type == 0) {
                POINT sp = constraint.relation.first;
                POINT lp = constraint.relation.second;
                int low = *domains[sp.first][sp.second].begin();
                for (int k = 1; k <= low; k++) {
                    bool flag = domains[lp.first][lp.second].erase(k);
                    if (flag) {//确定是否删除了， 好进行下一步操作，以下同理
```

```
                        if (domains[lp.first][lp.second].size() == 0)
                        return DOMAINS();  // DWO
                        AddConstraint2Q(lp, gacQ);
                    }
                }
                int high = *domains[lp.first][lp.second].rbegin();
                for (int k = high; k <= size; k++) {
                    bool flag = domains[sp.first][sp.second].erase(k);
                    if (flag) {
                        if (domains[sp.first][sp.second].size() == 0)
                                return DOMAINS();  // DWO
                        AddConstraint2Q(sp, gacQ);
                    }
                }
            }
            // 已优化行约束
            else if (constraint.type > 0) {
                int row = constraint.type - 1;
                //判断唯一取值是否符合要求，以下同理
                for(int j = 0; j < size; j++) if(domains[row][j].size()==1){
                    int value = *domains[row][j].begin();
                    for (int jj = 0; jj < size; jj++) if (jj != j) {
                        bool flag = domains[row][jj].erase(value);
                        if (flag) {
                            if (domains[row][jj].size() == 0)
                                return DOMAINS();  // DWO
                            AddConstraint2Q({row, jj}, gacQ);
                        }
                    }
                }
            }
            // 已优化列约束
            else {
                int col = -constraint.type - 1;
                for(int i = 0; i < size; i++) if(domains[i][col].size()==1){
```

```
                int value = *domains[i][col].begin();
                for (int ii = 0; ii < size; ii++) if (ii != i) {
                    bool flag = domains[ii][col].erase(value);
                    if (flag) {
                        if (domains[ii][col].size() == 0)
                            return DOMAINS();  // DWO
                        AddConstraint2Q({ii, col}, gacQ);
                    }
                }
            }
        }
    }
    return domains;
}
PUZZLE Gac(const DOMAINS& domains, list<Constraint>& gacQ){
    if (domains.size() == 0) return PUZZLE();   //DWO
    if (isSolved()) return puzzle;
    POINT pos = MinimumPos();
    set<int> field = domains[pos.first][pos.second];
    for (auto pd = field.begin(); pd != field.end(); pd++) {
        QueryPerformanceCounter(&node_start_time);

        puzzle[pos.first][pos.second] = *pd;
        // 在domains副本上修改,否则原iteration失效
        auto temp_domains = domains;
        temp_domains[pos.first][pos.second].clear();
        temp_domains[pos.first][pos.second].insert(*pd);
        AddConstraint2Q(pos, gacQ);

        temp_domains = GacEnforce(temp_domains, gacQ);

        if (temp_domains.size() != 0) {  // not DWO
            node_find++;
            PUZZLE res = Gac(temp_domains, gacQ); //gac递归
            if (res.size() != 0) return res;
```

```cpp
            }
            QueryPerformanceCounter(&node_end_time);
            count_time+=(node_end_time.QuadPart-node_start_time.QuadPart)
                                        *1000000 / frequency.QuadPart;
        }
        puzzle[pos.first][pos.second] = 0;  //重新置0
        return PUZZLE();                    // DWO
    }
};

int main() {
    QueryPerformanceFrequency(&frequency);
    for(int t = 1; t <= 5; t++){
        node_find = 0;
        LARGE_INTEGER start_time, end_time;

        string testcase = "TestCase/data"+to_string(t)+".txt";
        Solution game(testcase);
        list<Constraint> gacQ;
        game.printPuzzle();
        // 初始化各变量域并执行GacEnforce
        auto domains = game.makeDomains(gacQ);

        QueryPerformanceCounter(&start_time);
        PUZZLE result = game.Gac(domains, gacQ);
        QueryPerformanceCounter(&end_time);

        long long dur_time = (end_time.QuadPart-start_time.QuadPart)*1000000
                                / frequency.QuadPart;

        game.printPuzzle();
        string filename = "TestCase/res_gac"+to_string(t)+".txt";
        game.FilePrintPuzzle(filename);
        ofstream fout (filename.c_str());
        fout<<"Number of Nodes Searched is "<<node_find<<"\n";
        fout<<"Total Time is "<<dur_time<<" us\n\n";
        fout<<"Average Inference Time Per Node is ";
        fout<<1.0*count_time/node_find<<" us/node\n\n";
        fout.close();
        printf("Number of Nodes Searched is %lld\n", node_find);
        printf("Total Time is %lld us\n", dur_time);
        printf("Average Inference Time Per Node is %lf us/node\n\n"
                , 1.0*count_time/node_find);
    }
    puts("end");
    return 0;
}
```