The *Covariance* kernel was analyzed as shown in the following:

- Memory Traffic β: Inside the loop, for each iteration $n = 1, 2, \ldots, N$, there are 2 reads from memory: $x[i]$ and $y[i]$. a, b and c are scalars so we assume they fit in registers. Hence the total number of reads and writes are $(2N)$ reads $= 2N$. Because we are using double precision numbers, the total memory traffic is $(2N) \times 8\text{Byte} = 16N$ Byte.
- FLOPs π: Inside the loop, for each iteration $n = 1, 2, \ldots, N$, there are 3 addition operations for a and $x[i]$, b and $y[i]$, c and the sum of $x[i]$ and $y[i]$, 1 multiplication operations for x and y. Outside the loop, there are 3 divisions, 1 multiplication and 1 subtraction which amount to a total of 5 operations. If N is large, this part becomes negligible.
- Operational Intensity: Given the above computations, we find the operational intensity of this kernel to be

$$I = \frac{\pi}{\beta} = \frac{4N}{16N} = \frac{1}{4} = 0.25 \text{ Flops/Byte}$$

- Performance: We then measured the runtime of both sequential *Covariance()* and ISPC *Covariance()* and computed the performance for each kernel. For a data size of 16000, we have $T_s = 27.032094$ microseconds, ISPC $T_p = 7.227561$ microseconds. Take the sequential calculation as example we have performance $=16000 \times 4\, flops \div (27.032094 \times 10^{-6}\, seconds) \div (10^9) = 2.37\, Gflops$. Hence the performances of the sequential, the ISPC optimized Covariance kernels are 2.37 Gflops/s and 8.85 Gflops/s , respectively.
- Theoretical peak: We find that Intel Xeon E5-2683 has a double precision peak of 537.6Gflops/s and memory bound kernels up to 7 flops/s. Since we are in the memory bound region, the peak attainable performance for 0.25 flop/Byte is given by $P = beta \times Operational\, Intensity = 76.8GB/s \times 0.25 = 19.2Gflops/s$.