## Exact Procedure for Valuing
## American Calls on Dividend-Paying Stocks

The Roll, Geske, and Whaley formula for the value of an American call option on a stock paying a single dividend $D_1$ at time $t_1$ is

$$C = (S_0 - D_1 e^{-rt_1}) N(b_1) + (S_0 - D_1 e^{-rt_1}) M \left( a_1, -b_1; -\sqrt{\frac{t_1}{T}} \right)$$

$$- K e^{-rT} M \left( a_2, -b_2; -\sqrt{\frac{t_1}{T}} \right) - (K - D_1) e^{-rt_1} N(b_2) \tag{1}$$

where

$$a_1 = \frac{\ln[(S_0 - D_1 e^{-rt_1})/K] + (r + \sigma^2/2)T}{\sigma \sqrt{T}}$$

$$a_2 = a_1 - \sigma \sqrt{T}$$

$$b_1 = \frac{\ln[(S_0 - D_1 e^{-rt_1})/S^*] + (r + \sigma^2/2)t_1}{\sigma \sqrt{t_1}}$$

$$b_2 = b_1 - \sigma \sqrt{t_1}$$

The variable $\sigma$ is the volatility of the stock price net of the present value of the dividend. The function, $M(a, b; \rho)$, is the cumulative probability, in a standardized bivariate normal distribution, that the first variable is less than $a$ and the second variable is less than $b$, when the coefficient of correlation between the variables is $\rho$. A procedure for calculating the $M$ function is given in Technical Note 5. The variable $S^*$ is the solution to

$$c(S^*) = S^* + D_1 - K$$

where $c(S^*)$ is the Black–Scholes–Merton option price when the stock price is $S^*$ and the time to maturity is $T - t_1$. When early exercise is never optimal, $S^* = \infty$. In this case $b_1 = b_2 = -\infty$ and equation (1) reduces to the Black–Scholes–Merton equation with $S_0$ replaced by $S_0 - D_1 e^{-rt_1}$. In other situations, $S^* < \infty$ and the option should be exercised at time $t_1$ when $S(t_1) > S^* + D_1$.

When several dividends are anticipated, early exercise is normally optimal only on the final ex-dividend date (see Section 15.12 of 9th edition). It follows that the Roll, Geske, and Whaley formula can be used with $S_0$ reduced by the present value of all dividends except the final one. The variable, $D_1$, should be set equal to the final dividend and $t_1$ should be set equal to the final ex-dividend date.