Technical Note No. 12*
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John Hull

The Calculation of the Cumulative Non-Central Chi Square Distribution

We present an algorithm proposed by Ding (1992).[1] Suppose that the non-centrality parameter is v and the number of degrees of freedom is k and we require the cumulative probability that the variable will be less than z. We define

$$t_0 = \frac{1}{\Gamma(k/2+1)}\left(\frac{z}{2}\right)^{k/2}\exp\left(-\frac{z}{2}\right)$$

$$t_i = t_{i-1}\frac{z}{k+2i}$$

We also define

$$w_0 = u_0 = \exp(-v/2)$$

$$u_i = \frac{u_{i-1}v}{2i}$$

$$w_i = w_{i-1} + u_i$$

The required probability that the variable with the non-central chi square distribution will be less than z is

$$\sum_{i=0}^{\infty} w_i t_i$$

By taking a sufficient number of terms in this series the required accuracy can be obtained.

The Gamma Function

In the above formulas $\Gamma(.)$ is the gamma function. It has the property that $\Gamma(n) = (n-1)!$ when n is an integer. In general $\Gamma(x+1) = x\Gamma(x)$. The computation of the gamma function is discussed in *Numerical Recipes*.[2].

$$\Gamma(x) = \left[\frac{\sqrt{2\pi}}{x}\left(p_0 + \sum_{n=1}^{6}\frac{p_n}{x+n}\right)\right](x+5.5)^{x+0.5}e^{-(x+5.5)}$$

where

$$p_0 = 1.000000000190015$$

[1] See C.G. Ding, "Algorithm AS275: Computing the non-central χ^2 distribution function," *Applied Statistics*, 41 (1992), 478–82.

[2] See W.H. Press, B.P. Flannery, S.A. Teukolsky, and W.T. Vetterling, *Numerical Recipes in C: The Art of Scientific Computing.* Cambridge University Press, Cambridge, 1988.

$$p_1 = 76.18009172947146$$
$$p_2 = -86.50532032941677$$
$$p_3 = 24.01409824083091$$
$$p_4 = -1.231739572450155$$
$$p_5 = 1.208650973866179 \times 10^{-3}$$
$$p_6 = -5.395239384953 \times 10^{-6}$$

To avoid overflow problems it is best to compute $\ln \Gamma(x)$ rather than $\Gamma(x)$.