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Calculation of Moments for Valuing Basket Options

Consider the problem of calculating the first two moments of the value of a basket of assets at a future time, T, in a risk-neutral world. The price of each asset in the basket is assumed to be lognormal. Define

n : The number of assets
S_i : The value of the ith asset at time T[1]
F_i : The forward price of the ith asset for a contract maturing at time T.
σ_i : The volatility of the ith asset between time zero and time T
ρ_{ij} : Correlation between returns from the ith and jth asset
P : Value of basket at time T
M_1 : First moment of P in a risk-neutral world
M_2 : Second moment of P in a risk-neutral world

Because $P = \sum_{i=1}^{n} S_i$, $\hat{E}(S_i) = F_i$, $M_1 = \hat{E}(P)$ and $M_2 = \hat{E}(P^2)$ where $\hat{E}$ denotes expected value in a risk-neutral world, it follows that

$$M_1 = \sum_{i=1}^{n} F_i$$

Also,

$$P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} S_i S_j$$

From the properties of lognormal distributions

$$\hat{E}(S_i S_j) = F_i F_j e^{\rho_{ij}\sigma_i\sigma_j T}$$

Hence

$$M_2 = \sum_{i=1}^{n} \sum_{j=1}^{n} F_i F_j e^{\rho_{ij}\sigma_i\sigma_j T}$$

[1] If the ith asset is a certain stock and there are, say, 200 shares of the stock in the basket, then the ith "asset" is defined as 200 shares of the stock and S_i is the value of 200 shares of the stock.