Image Geometry

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Interpolation of Data

- affine transformations

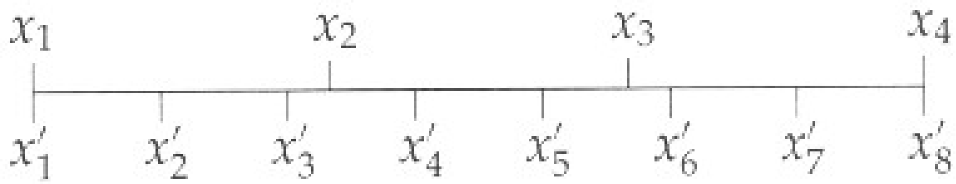


FIGURE 6.1 *Replacing four points with eight.*

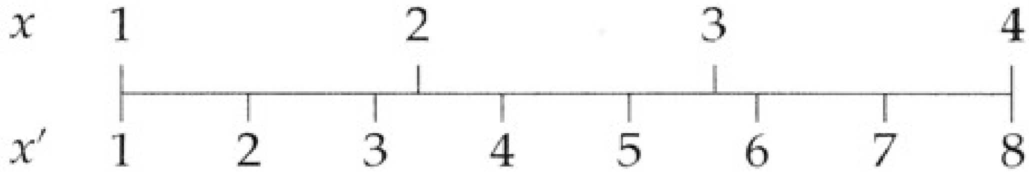


FIGURE 6.2 *Figure 6.1 slightly redrawn.*

Interpolation of Data

- The estimation of function values based on surrounding values is called **interpolation**.

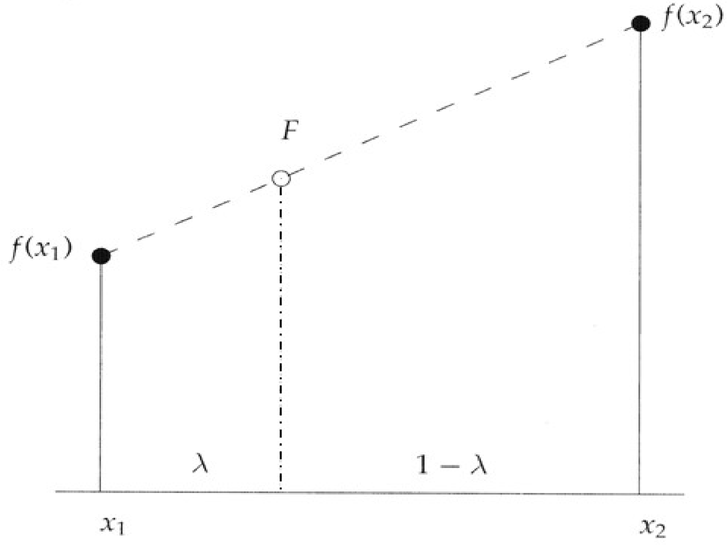


FIGURE 6.5 *Calculating linearly interpolated values.*

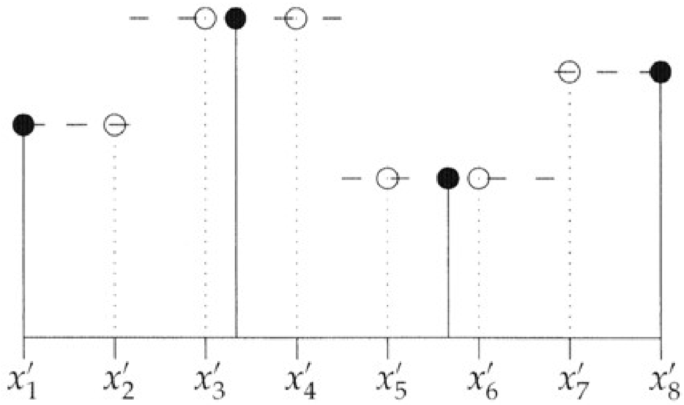


FIGURE 6.3 *Nearest-neighbor interpolation.*

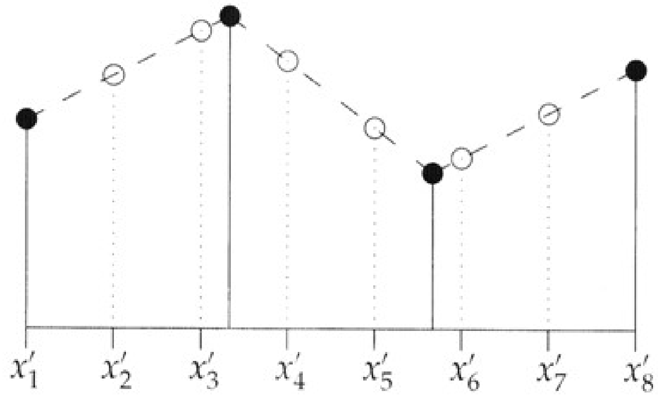


FIGURE 6.4 *Linear interpolation.*

Image Interpolation

- bilinear interpolation

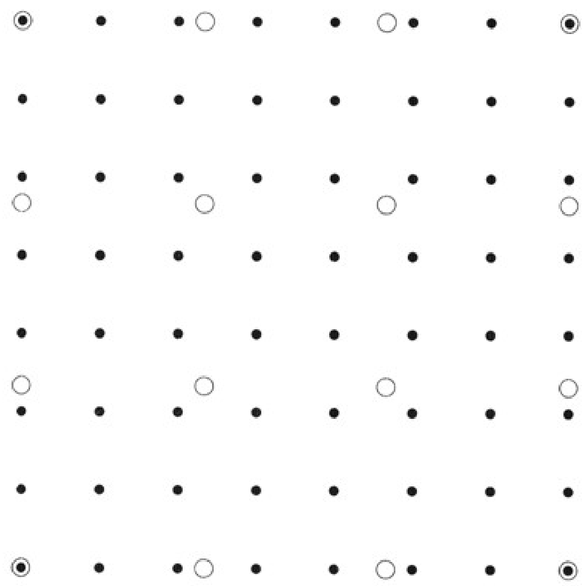


FIGURE 6.6 *Interpolation on an image.*

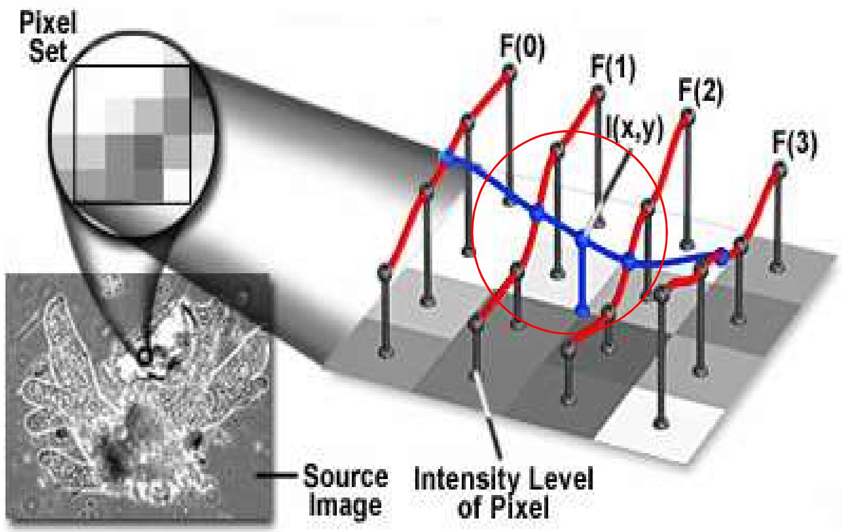


Image Interpolation

- bilinear interpolation

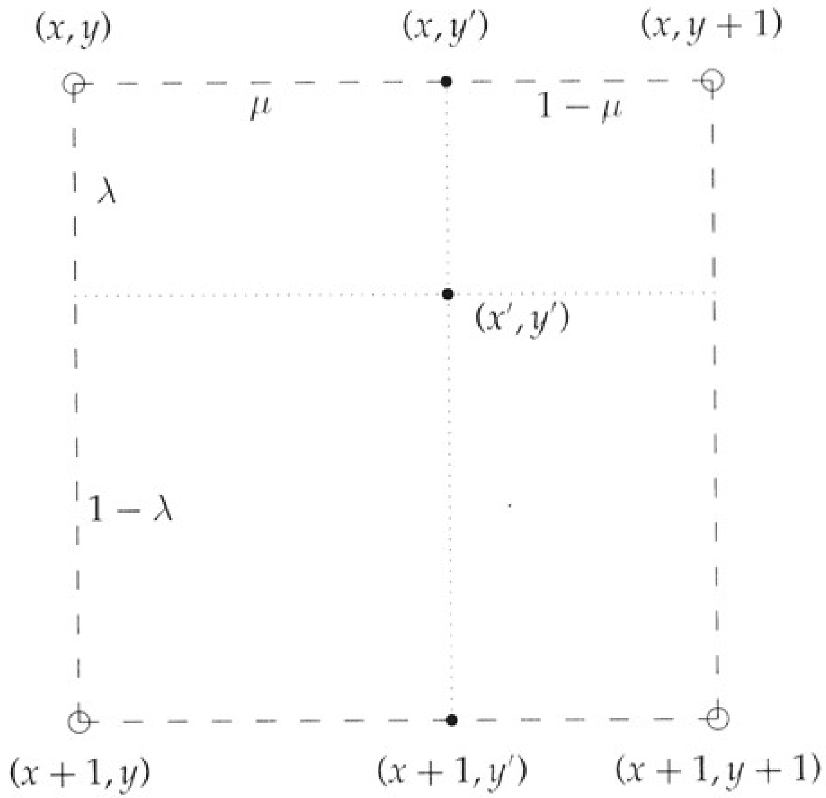


FIGURE 6.7 *Interpolation between four image points.*

$$f(x+1, y') = \mu f(x+1, y+1) + (1-\mu) f(x+1, y).$$

$$f(x', y') = \lambda f(x+1, y') + (1-\lambda) f(x, y'),$$

$$\begin{aligned} f(x', y') &= \lambda(\mu f(x+1, y+1) + (1-\mu) f(x+1, y)) \\ &\quad + (1-\lambda)(\mu f(x, y+1) + (1-\mu) f(x, y)) \\ &= \lambda\mu f(x+1, y+1) + \lambda(1-\mu) f(x+1, y) + (1-\lambda)\mu f(x, y+1) \\ &\quad + (1-\lambda)(1-\mu) f(x, y). \end{aligned}$$

Image Interpolation

General Interpolation

- A general interpolation function

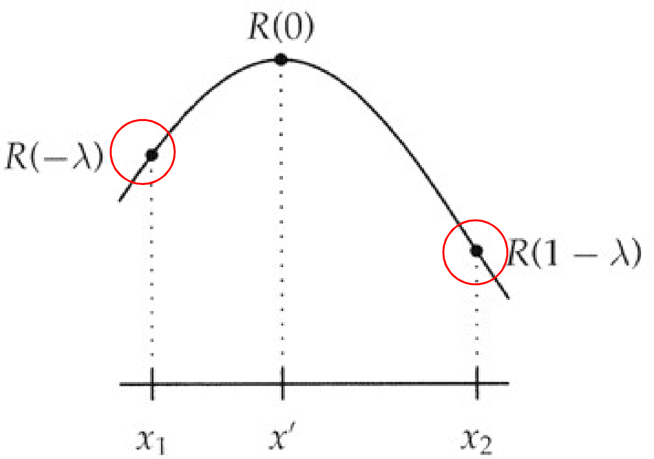
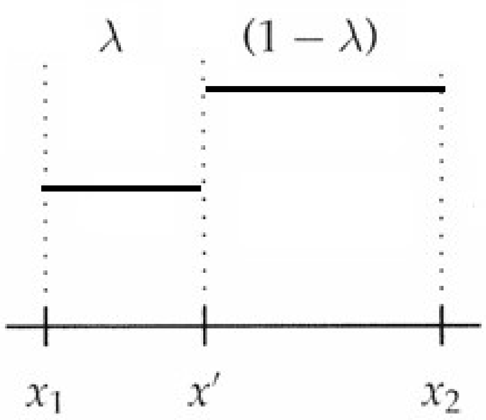


FIGURE 6.10 *Using a general interpolation function.*



$$f(x') = R(-\lambda)f(x_1) + R(1-\lambda)f(x_2).$$

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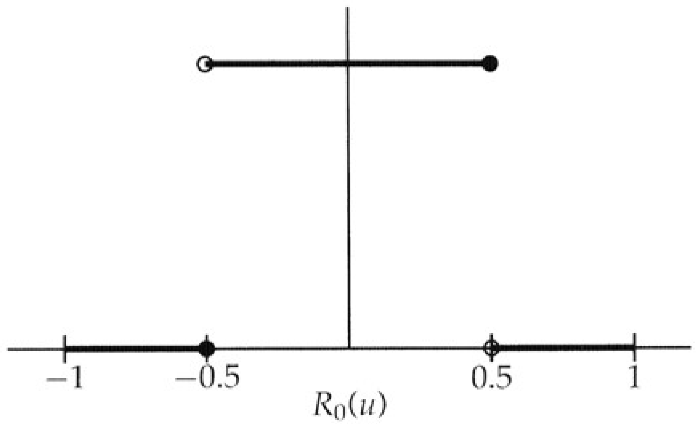
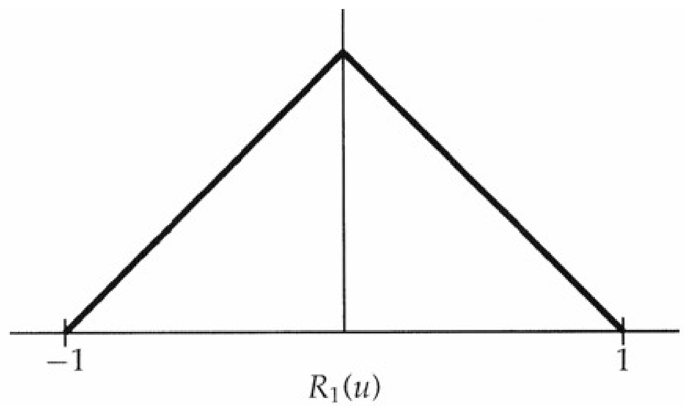
General linear Interpolation

- two interpolation functions

$$R_0(u) = \begin{cases} 0 & \text{if } u \le -0.5 \\ 1 & \text{if } -0.5 < u \le 0.5 \\ 0 & \text{if } u > 0.5 \end{cases}$$



$$R_1(u) = \begin{cases} 1 + u & \text{if } u \le 0 \\ 1 - u & \text{if } u \ge 0 \end{cases}.$$



General linear Interpolation

- Cubic interpolation

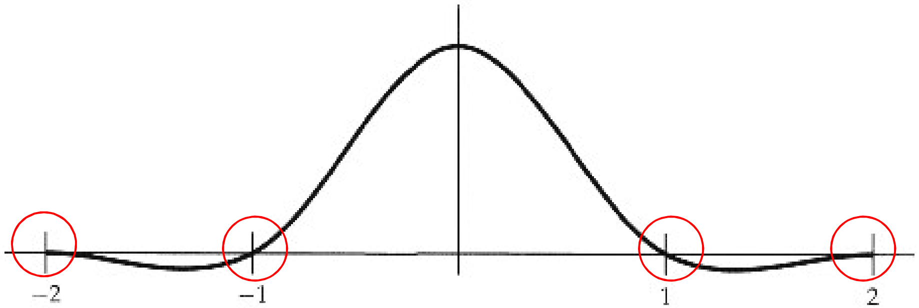


FIGURE 6.12 *The cubic interpolation function $R_3(u)$.*

$$R_3(u) = \begin{cases} 1.5|u|^3 - 2.5|u|^2 + 1 & \text{if } |u| \le 1, \\ -0.5|u|^3 + 2.5|u|^2 - 4|u| + 2 & \text{if } 1 < |u| \le 2. \end{cases}$$

$$f(x') = R_3(-1-\lambda)f(x_1) + R_3(-\lambda)f(x_2) + R_3(1-\lambda)f(x_3) + R_4(2-\lambda)f(x_4)$$

Bicubic Interpolation

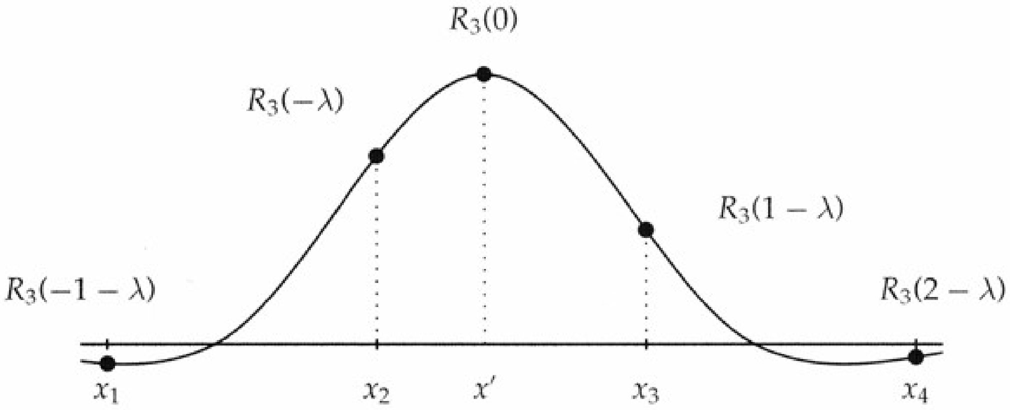


FIGURE 6.13 *Using $R_3(u)$ for interpolation.*

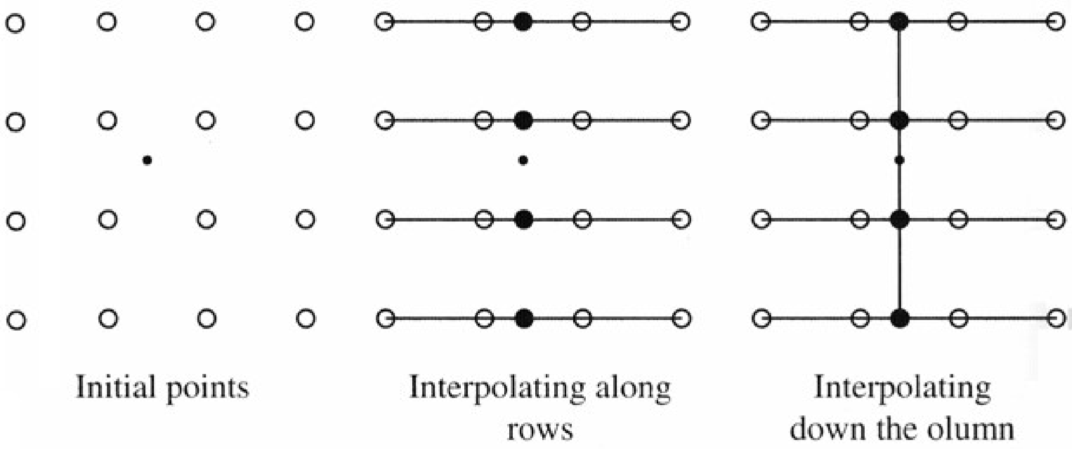
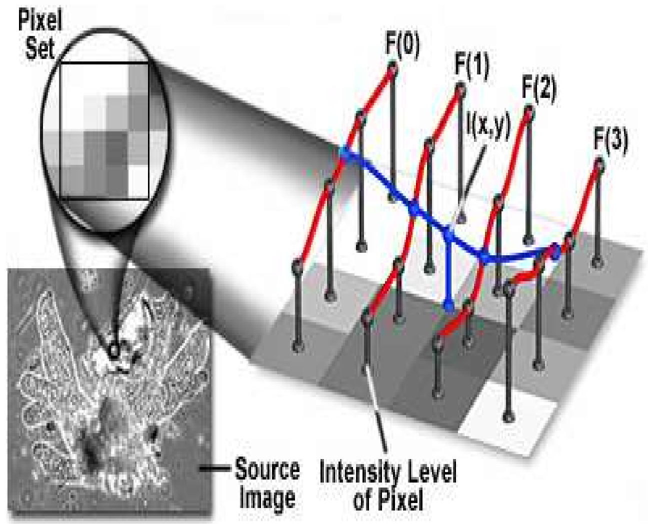


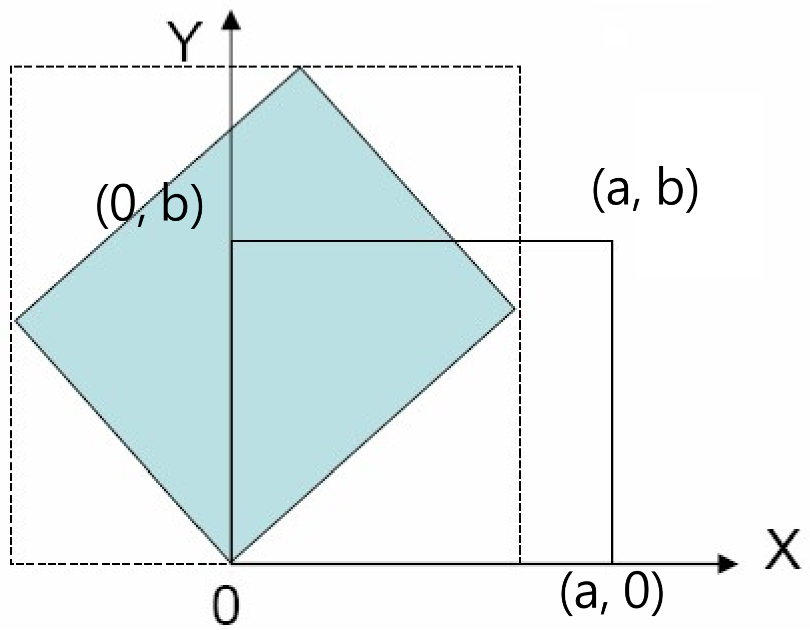
FIGURE 6.14 *How to apply bicubic interpolation.*



Rotation

$$\begin{pmatrix} X' \\ Y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\begin{pmatrix} X'' \\ Y'' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X' \\ Y' \end{pmatrix}$$



Rotation

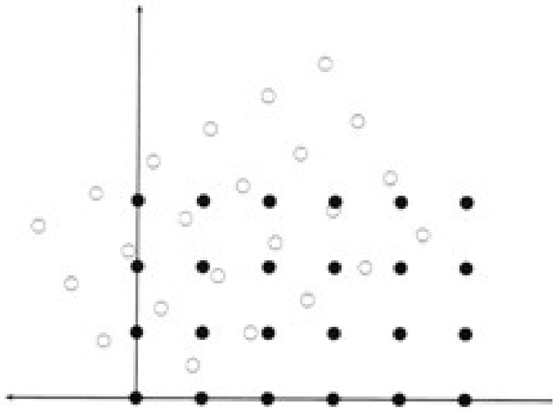


FIGURE 6.20 *Rotating a rectangle.*

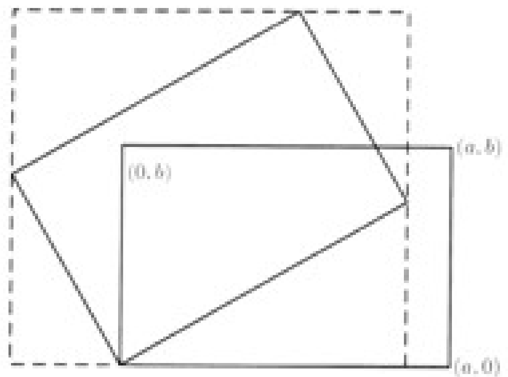


FIGURE 6.21 *A rectangle surrounding a rotated image.*

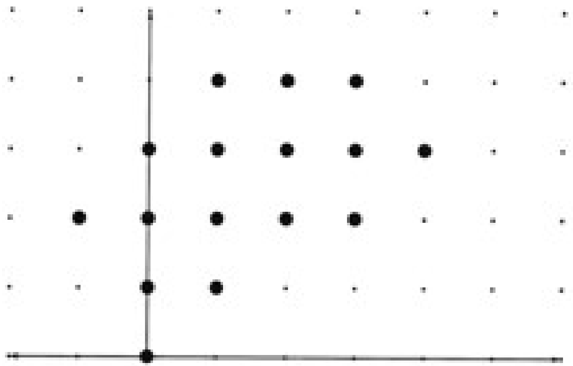


FIGURE 6.22 *The points on a grid after rotation.*

Transformations

$$\begin{array}{ccc} x & R & R_e \\ \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} & A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} & B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \text{point} & \text{rotation} & \text{reflection} \end{array}$$

$$\begin{array}{cc} D & T \\ C = \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & 1 \end{bmatrix} & E = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \\ \text{dilation/contraction} & \text{translation} \end{array}$$

$(r > 0)$

Rotation

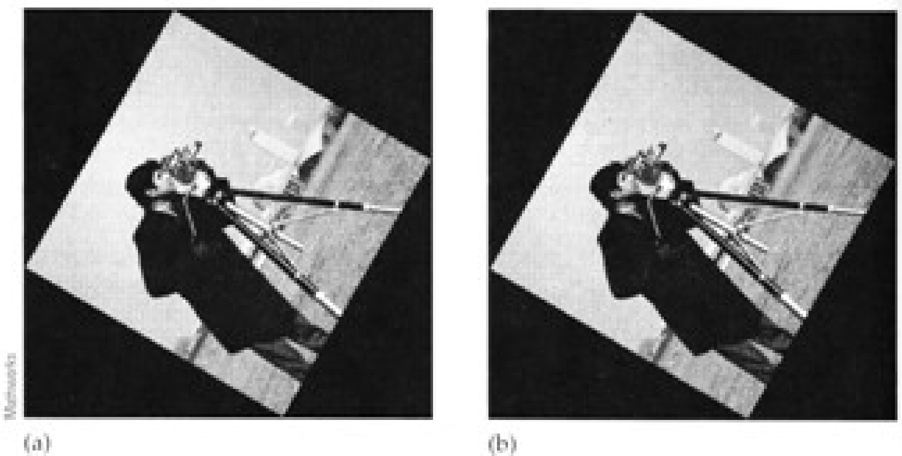


FIGURE 6.24 *Rotation with interpolation. (a) Nearest neighbor. (b) Bicubic interpolation.*

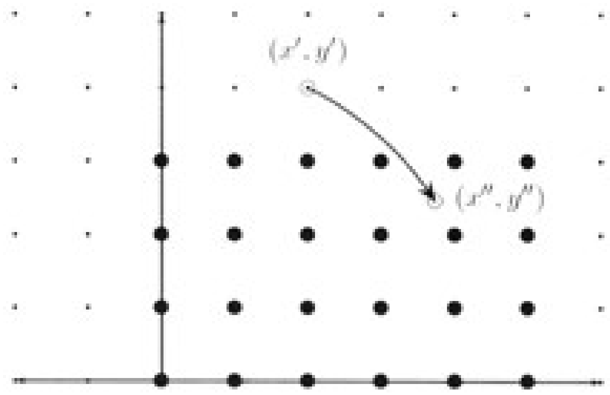
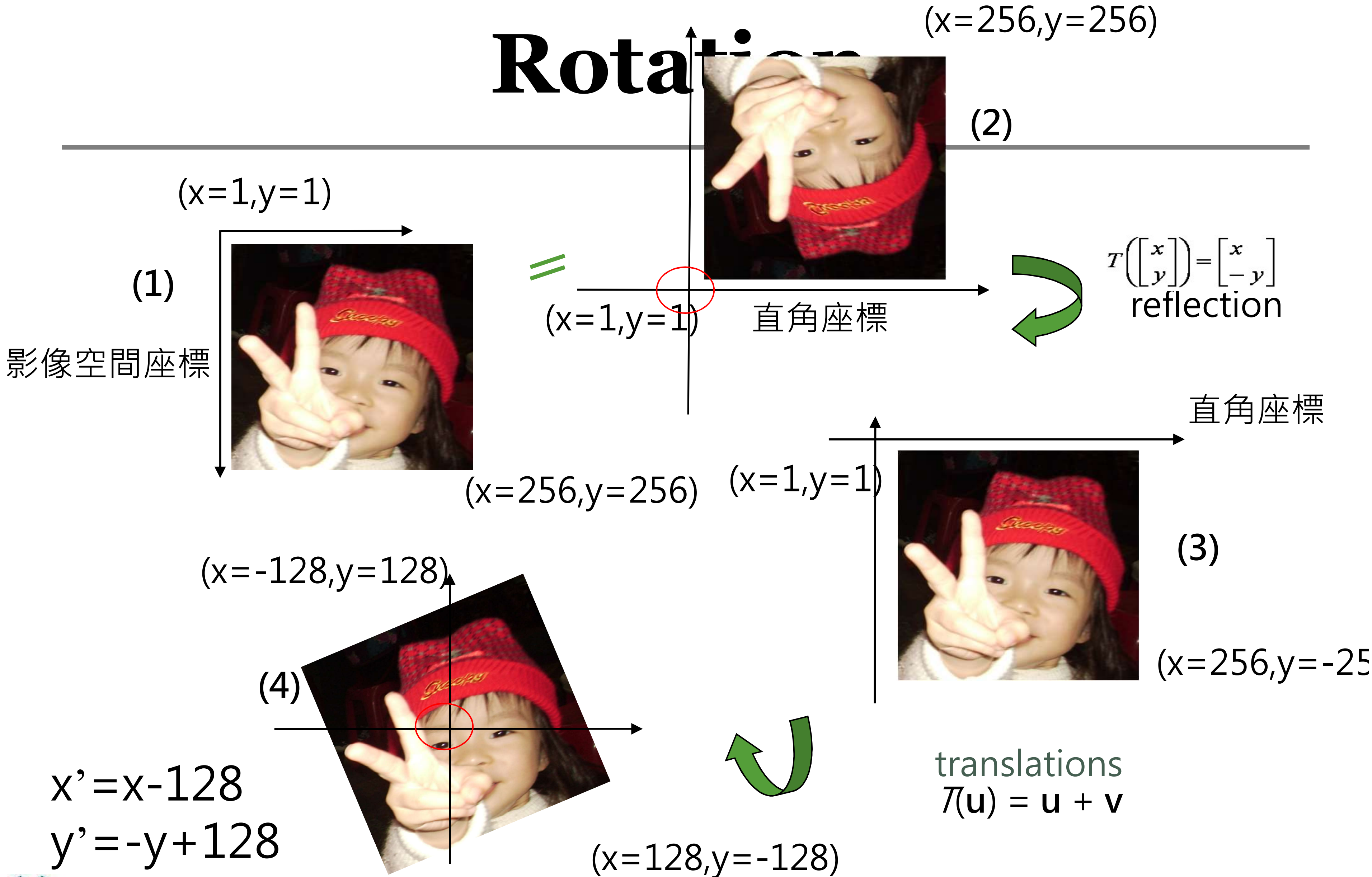


FIGURE 6.23 *Rotating a point back into the original image.*

Rotation



(x=256,y=256)
(2)
(x=1,y=1)
(1)
影像空間座標
=
(x=1,y=1)
直角座標
$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} x \\ -y \end{bmatrix}$
reflection
直角座標
(x=256,y=256)
(x=1,y=1)
(3)
(x=256,y=-25
(x=-128,y=128)
(4)
translations
$T(\mathbf{u}) = \mathbf{u} + \mathbf{v}$
x'=x-128
y'=-y+128
(x=128,y=-128)

Example

Determine the matrix that defines a rotation of a plane through an angle θ about a point $P(h. k)$. Use this general result to find the matrix that defines a rotation of the plane through an angle of $\pi/2$ about the point (5, 4). Find the image of the triangle having the following vertices $A(1, 2)$, $B(2, 8)$, and $C(3, 2)$ under this rotation. See Figure 2.23.

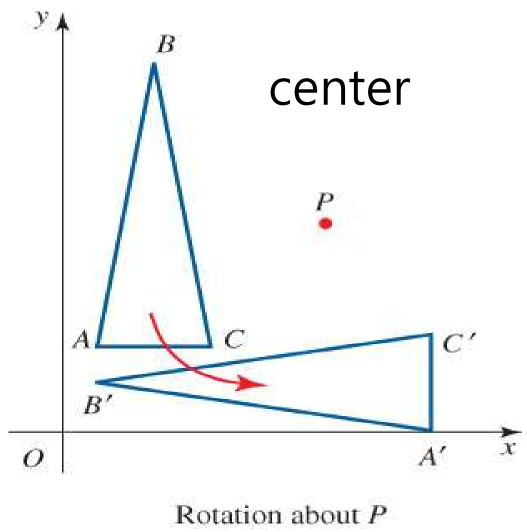


Figure 2.23

Solution

The rotation about P can accomplished by a sequence of three of the above transformations;

(a) A translation T_1 of the plane that takes P to origin O.

(b) A rotation R of the plane about the origin through an angle θ.

(c) A translation T_2 of the plane that takes O back to P.

The matrices that describe these transformations are as follows.

$$\overset{T_1}{\begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix}} \overset{R}{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{T_2}{\begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix}}$$

Solution

$$R_P\left(\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\right) = T_2 R T_1\left(\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & -h\cos\theta + k\sin\theta + h \\ \sin\theta & \cos\theta & -h\sin\theta - k\cos\theta + k \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Solution

To get the specific matrix that defined the rotation of the plane through an angle $\pi/2$ about the point $P(5, 4)$, for example, let $h = 5$, $k = 4$, and $\theta = \pi/2$. The rotation matrix is

$$M = \begin{bmatrix} 0 & -1 & 9 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$