(a)

$$\frac{\partial f}{\partial x} \doteq f(x+1,y) - f(x,y) \doteq f(x,y) - f(x-1,y)$$

$$\begin{aligned}
\frac{\partial^2 f}{\partial x^2} &= \frac{\partial}{\partial x}\left[\frac{\partial f}{\partial x}\right] \\
&\doteq \frac{\partial}{\partial x}\left[f(x+1,y) - f(x,y)\right] \\
&= \left[f(x+1,y) - f(x,y)\right] - \left[f(x,y) - f(x-1,y)\right] \\
&= \left[f(x+1,y) - 2f(x,y) + f(x-1,y)\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 f}{\partial y^2} &= \frac{\partial}{\partial y}\left[\frac{\partial f}{\partial y}\right] \\
&\doteq \frac{\partial}{\partial x}\left[f(x,y+1) - f(x,y)\right] \\
&= \left[f(x,y+1) - f(x,y)\right] - \left[f(x,y) - f(x,y-1)\right] \\
&= \left[f(x,y+1) - 2f(x,y) + f(x,y-1)\right]
\end{aligned}$$

$$\begin{aligned}
\nabla^2 f &\doteq \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \\
&= f(x+1,y) - 2f(x,y) + f(x-1,y) + f(x,y+1) - 2f(x,y) + f(x,y-1) \\
&= f(x+1,y) - 4f(x,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) \\
&= f(x+1,y) - 4f(x,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) \\
&\quad + 0f(x-1,y-1) + 0f(x+1,y+1) + 0f(x+1,y-1) + 0f(x-1,y+1) \\
&= \begin{bmatrix} f(x-1,y-1) & f(x,y-1) & f(x+1,y-1) \\ f(x-1,y) & f(x,y) & f(x+1,y) \\ f(x-1,y+1) & f(x,y+1) & f(x+1,y+1) \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}
\end{aligned}$$

(b)

$$\frac{\partial f}{\partial x} \doteq f(x+1,y) - f(x,y) \doteq f(x,y) - f(x-1,y)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left[\frac{\partial f}{\partial x}\right]$$

$$\doteqdot \frac{\partial}{\partial x}[f(x,y) - f(x+1,y)]$$
$$= [f(x,y) - f(x-1,y)] - [f(x+1,y) - f(x,y)]$$
$$= [-f(x+1,y) + 2f(x,y) - f(x-1,y)]$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left[\frac{\partial f}{\partial y}\right]$$
$$\doteqdot \frac{\partial}{\partial x}[f(x,y) - f(x,y+1)]$$
$$= [f(x,y) - f(x,y-1)] - [f(x,y+1) - f(x,y)]$$
$$= [-f(x,y-1) + 2f(x,y) - f(x,y-1)]$$

$$\nabla^2 f \doteqdot \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$
$$= -f(x+1,y) + 2f(x,y) - f(x-1,y) - f(x,y+1) + 2f(x,y) - f(x,y-1)$$
$$= -f(x+1,y) + 4f(x,y) - f(x-1,y) - f(x,y+1) - f(x,y-1)$$
$$= -f(x+1,y) + 4f(x,y) - f(x-1,y) - f(x,y+1) - f(x,y-1)$$
$$+ 0f(x-1,y-1) + 0f(x+1,y+1) + 0f(x+1,y-1) + 0f(x-1,y+1)$$
$$= \begin{bmatrix} f(x-1,y-1) & f(x,y-1) & f(x+1,y-1) \\ f(x-1,y) & f(x,y) & f(x+1,y) \\ f(x-1,y+1) & f(x,y+1) & f(x+1,y+1) \end{bmatrix} \otimes \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$