ERRATUM

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Here I collect my personal erratum to the book "Algebraic Varieties: Minimal Models and Finite Generation" of Kawamata, translated by myself.

(1) P136, L-3: here $N^1(Y)_{\mathbf{R}}$ might not be generated by those divisors (think of product of elliptic curves). The correction is as the following: Since C_0, T_0 and exceptional divisors of μ are linearly independent in $N^1(Y)_{\mathbf{R}}$, there exist integers c and $e_{i,j}$ such that

$$h^*H \equiv cC_0 + q(H \cdot C)T_0 - \sum_{i,j} e_{i,j}E_{i,j} + E.$$

Here E is a divisor satisfying $(C_0 \cdot E) = (T_0 \cdot E) = (E_{i,j} \cdot E) = 0$. This implies that $E^2 \leq 0$ by the Hodge index theorem as $C_0 + T_0$ is nef and big. (Thanks Jia Jia for pointing out)

(2) P146, L-10: here I made a very wrong claim that the support of D_m does not intersect C (the original Japanese version is correct, so this is totally on me). This can not be true as $(D_m \cdot C) > 0$. The correct statement is as the following: there is an effective $\mathbb{R}$-divisor $D_m \equiv mD + tH$ such that the support of D_m does not contain C and (X, D_m) is KLT in a neighborhood of C (this is because we can make the multiplicity of D_m at every point of C smaller than 1).

(3) P151, L7: here (Y', C'') might not be a minimal model in our sense: it might not be DLT, but it is only $\overline{\mathrm{KLT}}$. So we might consider to weaken the condition on singluarties of minimal models to allow this situation.

(4) P183 & P204: in the proof of the nonvanishing theorem (Theorem 3.5.1), there is no need to assume the existence of PL flips, because the existence of minimal models for $K_X + B$ relatively big implies the existence of flips already.

(5) P189: in the beginning of Proof of Theorem 3.2.1, I should be careful when claiming that all pairs have the same minimal model by the finiteness of minimal models. Here we should say that there are finitely many minimal models of $(Y, B_{Y,m})$, and all of them have natural morphisms to the canonical model of (Y, B_Y) if m is sufficiently large. This is sufficient for taking a common higher model Y' and taking limit to get P.

(6) P197–P202: this one might be subtle. In this section our aim is to prove the existence of minimal models. In the last part of Step 4, we need to show that P does not contain any LC center, but the argument is not rigorous. So we correct this by strengthen the statement of this section as the following: we will show the existence

Date: October 20, 2025.

of **strong** minimal models in Theorem 3.4.1. Here by a **strong** minimal model, I mean a minimal model $\alpha : (X, B) \dashrightarrow (Y, C)$ such that α is isomorphic over the generic point of any LC center of (Y, C). Note that any minimal model from the MMP is strong, and any minimal model for KLT pairs is strong. We can prove this strengthen statement by the same argument, and in the last part of Step 4, P does not contain any LC center as P is contracted. Also the minimal model we construct from weak minimal model by Lemma 3.4.5 is also strong, as no LC center is affected.

(7) P206, L7: here the correct equation should be $N_t = (1+t)N_\sigma(g^*(K_X + B)) + E_t$.

(8) P207, L3: here as we assumed the existence and finiteness of minimal models for $K_X + B$ relatively big, we may assume that the minimal model $Y_{t,v}$ is obtained by a $(K_X + B_{t,v})$-MMP over S with scaling of $v + A$. This will be used in the last part of Step 4 (derive a contradiction by special termination).

(9) P208, L7: here it is claimed that $\alpha_Z : (Z, B_Z) \dashrightarrow (W, C_W)$ is a weak minimal model except that W might not be $\mathbb{Q}$-factorial. This means that $K_W + C_W$ is nef and $p^*(K_Z + B_Z) \geq q^*(K_W + C_W)$ for a common resolution $p : U \to Z, q : U \to W$. But there is a small issue that α_Z might not be surjective in codimension 1 (so that C_W may not be equal to $\alpha_{Z*}B_Z$), and this will affect the definition of N_W. One possible solution is the following: in the beginning of Step 3, we may take further resolution to assume that (Z, B_Z) is already a very log resolution to itself, in this case, $p^*(K_Z + B_Z) \geq q^*(K_W + C_W)$ garentees that α_Z is surjective in codimension 1.

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