

計量  $\hat{\theta}$  為不偏估計量 (unbiased estimator)。若估計量的期望值與母體參數相同  $E(\hat{\theta}) = \theta$ ，此估計量為不偏估計量 (unbiased estimator)。

$$E(X_i) = \mu$$

$$V(X_i) = \sigma^2 = E(X_i^2) - \mu^2$$

$$\therefore E(\bar{X}) = \mu, V(\bar{X}) = \frac{\sigma^2}{n} = E(\bar{X}^2) - \mu^2$$

$$E(\hat{\theta}_1) = E\left(\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n}\right)$$

$$= \frac{1}{n} E\left(\sum_{i=1}^n X_i^2 - n\bar{X}^2\right)$$

$$= \frac{1}{n} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2)$$

$$= \frac{n-1}{n} \sigma^2$$

$$E(\hat{\theta}_2) = E\left(\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}\right)$$

$$= \frac{1}{n-1} E\left(\sum_{i=1}^n X_i^2 - n\bar{X}^2\right)$$

$$= \frac{1}{n-1} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2)$$

$$= \sigma^2$$

$\hat{\theta}_2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$  是  $\sigma^2$  的偏估計量  
 $\hat{\theta}_1 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n}$  是母體變異數  $\sigma^2$  的偏估計量