CALCULATING BOUNDS ON INFORMATION LEAKAGE USING MODEL-CKECINGG TOOLS

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by

JIA CHEN

Dr. Rohit Chadha, Thesis Supervisor

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The undersigned, appointed by the Dean of the Graduate School, have examined the thesis entitled:

CALCULATING BOUNDS ON INFORMATION LEAKAGE
USING MODEL-CKECINGG TOOLS

presented by Jia Chen,
a candidate for the degree of Master of Science and hereby certify that, in their opinion, it is worthy of acceptance.

Dr. Rohit Chadha

Dr. Prasad Calyam

Dr. Michela Becchi

ACKNOWLEDGMENTS

This page is where you would acknowledge all those who helped you with your academic research. This is not necessarily where you would recognize loved ones who supported you during your studies. That would be more appropriately done in an optional Dedication page. I would like to thank Professor Smith Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Vestibulum eu tellus. Nullam et odio eget sapien porttitor interdum. Donec vel ante. Maecenas in sem a nunc viverra hendrerit. Quisque ut massa quis pede blandit pharetra.

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ABSTRACT

A *confidential* program should not allow *any* information about its secret inputs to be inferred from its public outputs. Such confidentiality is, however, difficult to achieve in practice. Therefore, it has been proposed in literature to evaluate security of programs by computing the *amount* of information leaked (a low amount of information leakage is desirable). We consider the problem of *computing* information leaked by a deterministic program, when the information-theoretic measure of *min-entropy* is used to quantify the amount of information.

In order to measure the information leaked by a program P using min-entropy, one has to count the number of different possible outputs that may be achieved when the program is run with different inputs. Now, if the input to the program P consists of n-bits, this quantity can be computed by running the program on each of the 2^n different inputs and remembering the outputs observed on each of the different inputs. Since different inputs can lead to different outputs, this naive algorithm can take $O(2^n)$-additional space. The same computation can be actually carried out in polynomial additional space, but in this case the program has to be run 2^{2n} times. The latter observation lies at the heart of recent results which show that the problem of checking whether information leaked by a program is equal to a given number has the same complexity of checking safety in programs.

Chapter 1

Introduction

Introduce the reader to the current problem that you wish to solve, and why anyone should care about it.

Chapter 2

Theory background

2.1 Min-entropy

The introduction counts as chapter 1. This page shows how the bulk of your thesis will be organized: through chapters and sections. Here is a citation.[**?**]

2.2 Information leak

2.3 Two intuitive solutions

To count the number of outputs of a program, we come up with two approaches: Put the program in a double loop and count the number of outputs, or let the program iterate through all input values and record the outputs in an array. The first approach is time-consuming, while the second one is space-consuming.

2.3.1 Double loop and counter

In algorithm 1, for each possible output value, we iterate through the input range to see if an input can result in this output. If we hit such an input, counter increases and the code breaks out of the inner loop to continue testing the next possible output value. After the double loop finishes, the value of $OCounter$ is the number of outputs of program P.

Algorithm 1: Calculate the number of outputs using double loop.

```
S ← 0
O ← 0
SIn ← 0
OOut ← 0
OCounter ← 0
SMax ← 1 << bitLength − 1
OMax ← 1 << bitLength − 1
for O = 0 to OMax do
  for S = 0 to SMax do
    SIn ← S
    OOut ← P(SIn) // the program P takes SIn as input
    if OOut = O then
      OCounter ← OCounter + 1
      break
    end if
  end for
end for
```

In algorithm 1, we declared seven variables, and all of them require $bitLength$ bits except for $OCounter$ which is $bitLength$+1 bits. The total memory usage for variables is $7 \times bitLength + 1$ at $O(bitLength)$. As with execution time, we assume program P takes time $t(P)$ to execute, and the total execution time for the double loop when break is never reached is $2^{bitLength} \times 2^{bitLength} \times t(P)$. Thus the time complexity is $(2^{O(bitLength)}) \times t(P)$.

In order to get an estimation of how much time the double loop will take to execute, we implemented a piece of C code with an empty while loop which loops 2^{32} times. On out experiment PC, this loop takes on average 10.30 seconds to complete. Were we to run a double loop in bit length of 32, the execution time would be $2^{32} \times 10.30$ seconds, which is around 1403 years. Running the double loop directly would be infeasible.

2.3.2 Single loop and array

In algorithm 2, we create an array with size equal to the maximum number of possible outputs($1 << bitLength$, or $2^{bitLength}$) and initialize it with zeros. While we iterate through the range of S, we set each $OHit[P(SIn)]$ to 1. When a 0 turns to 1, we increase $OCounter$. After the loop, the value of $OCounter$ is the number of outputs by program P.

In algorithm 2 except for the array we have 7 variables using $7 \times bitLength + 1$ memory. The array $OHit[]$ is of size $2^{bitLength} \times bitLength$ making a total of $2^{bitLength} \times bitLength + 7 \times bitLength + 1$ at $2^{O(bitLength}$). As with execution time, we assume program P takes time $t(P)$ to execute, and the execution time for the single loop is $2^{bitLength} \times t(P)$. Thus the time complexity is $(2^{O(bitLength)}) \times t(P)$.

We set the bit length to 32. In array $OHit[]$, each element is 32 bits and the number of elements is 2^{32}. The total memory usage for this array is $2^{32} \times 32$ bits, which is 16 gigabytes. To our knowledge, it is neither difficult nor expensive to build a PC with more than 16 gigabytes of memory, and we can get such a PC off-the-shelf from top gaming PC brands like Alienware. However, as this memory requirement grows exponentially, adding two to three bits to the bit length and the requirement

if this is not hard we need reasons not doing this..

Algorithm 2: Calculate the number of outputs using single loop and a table.

$S \leftarrow 0$
$O \leftarrow 0$
$SIn \leftarrow 0$
$OOut \leftarrow 0$
$OCounter \leftarrow 0$
$SMax \leftarrow 1 << bitLength - 1$
$OMax \leftarrow 1 << bitLength - 1$
$OHit[OMax + 1] \leftarrow [0]$
for $S = 0$ to $SMax$ **do**
  $SIn \leftarrow S$
  $OOut \leftarrow P(SIn)$ // the program P takes SIn as input
  if $OHit[OOut] = 0$ **then**
    $OCounter \leftarrow OCounter + 1$
    $OHit[OOut] \leftarrow 1$
  end if
end for

will exceed the capacity of current PCs.

Chapter 3

Experiment with Getafix

We want to use model-checking tools to see if we can reduce the time requirement of algorithm **??**. Specifically, we choose the reachability property and append algorithm 3 to the end of algorithm **??**. The statement within the if statement has a label. Although the exact statement following that label is irrelevant, reaching this line means *OCounter* satisfies the constrains in the condition block.

(why and what other property are there)

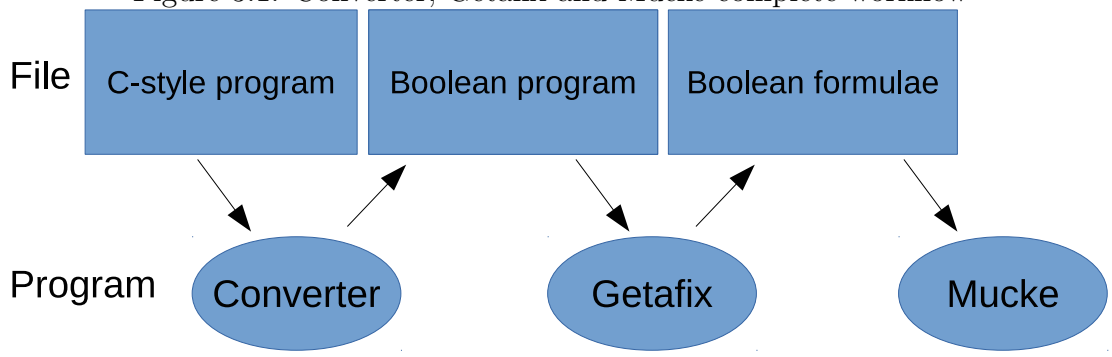
We experiment on several model-checking tools, including Interproc from [1], Berkeley Lazy Abstraction Software Verification Tool (Blast) from [2] and Getafix from [3]. We can not get correct reachability results from Interproc and Blast, so we shift our focus onto Getafix.

correct citations of web page

Algorithm 3: Determine if *OCounter* meets certain constrains.

```
if value of OCounter meets certain constrains then
    reach: OCounter // a label followed by a statement
end if
```

Figure 3.1: Converter, Getafix and Mucke complete workflow



3.1 Getafix

Getafix is a symbolic model checker for Boolean programs implemented in [3]. Getafix only supports reachability check. It translates sequential and concurrent Boolean programs into Boolean formulae and uses the model-checker Mucke to solve the reachability problem symbolically using Boolean Decision Diagrams [4].

Mostly copied from getafix website. Need rewrite

3.2 The converter

Input for Getafix are boolean programs, meaning it only supports binary variables which can be either 0 or 1. We represent our problem in c-style code, thus we need to translate it into boolean form. We implemented a converter to automate this process, and figure 3.1 shows the complete workflow including the converter. The converter has three components, a parser, a built-in function generator and a piece of script which calls the first two components and assemble the output file. The converter has these properties:

1. The input to the converter is a c-style code file and a positive integer which

represent the bit length. The converter supports 32 bits and less. Also note that the converter represents a number with bit length of $bitLength$ using $bitLength + 1$ bits. We do this so that writing a loop which iterate from 0 to $1 << bitLength - 1$ is easier.

2. The output of the converter is a boolean program which follows the syntax of Getafix input file.

3. In the language that we defined for the input file, we support only one variable type: non-negative decimal integer. Again we support the length up to 32 bits.

4. Our converter does not support function definitions. The input file has two parts, variable declarations block and statements block. The parser will print these blocks into the main function of the output boolean program. Also we require all variable declarations to appear before statements in the input code. Getafix input syntax has this requirement, and we decide to keep it in our converter.

5. We implement support for the following symbolic operators in the language: plus +, minus -, and &, or |, xor ^, greater than >, equal ==, less than <, not equal !=, greater than or equal >=, less than or equal <=, left shift <<and right shift >>.

6. We implement support for three control statements: if...else, while loop, goto and statements with labels. We currently do not support for loop, do...while loop, switch statements, break and continue, but these statements can be easily expressed using the supported ones.

Input to the parser is a c-style code file and a desired bit length. Output of the parser is its corresponding boolean program which follows the syntax of Getafix input file. First we define the syntax of the input code and second we create the parser using flex and bison. The parser scans the input code and builds a syntax tree. Then the parser prints the syntax tree as a boolean program. The parser has three points worth noting:

Table 3.1: Examples of input and output of the parser, with bit length of 4

Input	Output
var SMax = 16;	decl SMax4,SMax3,SMax2,SMax1,SMax0; Smax4, SMax3, SMax2, SMax1, SMax1 := 1,0,0,0,0;
STemp = STemp - 5;	STemp4,STemp3,STemp2,STemp1,STemp0 := minus(STemp4,STemp3,STemp2,STemp1,STemp0,0,0,1,0,1);

1. When printing the output code, the parser "stretches" each variable and literal into its binary form. Assume the desired bit length is *bitLength*. We split each variable into *bitLength* variables by copying the name of the variable *bitLength* times and append a counter value to each one. Also we convert a literal to its corresponding binary value and prepend it with zeros to reach the desired length. Table 3.1 contains an example of how the parser deals with a variable declaration.

2. In a boolean program, all operators operate on bit level, so we need to implement higher-level operators like plus, minus, greater than and left shift using operators that Getafix supports. In the parser, we print these high-level operators as function calls in the output boolean program, and the built-in function generator generates the body of the function. Table 3.1 contains an example of how the parser deals with a high-level operator.

3. In the boolean code syntax which Getafix defines, function call plus the semicolon is defined as a statement, and another rule allows the code to assign a function call to an identifier, but function call itself is not an expression. This means that a function call can not work as an expression as in many other languages, and it leads to two problems: First, the decider expression in if...else and while statements can not contain function calls. Second, parameters of a function call or operands to an operator can not be a function call. We automated a solution in the parser to the first problem, which assigns the decider expression to a temporary variable and use that variable as the decider, so we can use the c-style if...else and while in the input code. For the second problem, a possible solution would be to manage a set of internal temporary variables and assign each function call to a variable, but we did not implement it.

Input to the built-in function generator is a desired bit length. Output is a set of high-level operators like plus and left shift implemented as functions. We do not track the necessary functions in the parser, as experiments with Getafix indicate that the uncalled functions affect little on the execution time. Listing 3.1 shows a sample function by the generator.

Listing 3.1: Greater than operator as a function in boolean program with bit length of 2.

```
bool isGT(left2,left1,left0,right2,right1,right0)
begin
if (left2 != right2) then
        if (left2 = 1) then return 1; fi
else
```

```
        if (left1 != right1) then
                if (left1 = 1) then      return 1; fi
        else
                if (left0 != right0) then
                        if (left0 = 1) then      return 1; fi
fi      fi      fi
return 0;
end
```

Input to the third component, a piece of script, is a c-style code file and a desired bit length. Output of the script is a boolean program ready for Getafix to process. The script first passes the bit length to the built-in function generator and redirect its output to the output file. Then the script passes both the input to the parser and append its output to the output file. At this point the output is complete.

3.3 Tests and results

Before we have the converter, we coded a test case manually and Getafix gave us the correct answer. Now with the converter we can run more complicated tests and see if Getafix is a good solution to the problem of calculating information leakage. We decide to run the ten test cases from paper [**?**]. In each test, O represents the output of its test, and S is the input.

3.3.1 Sanity check

Listing 3.2 shows the code we use. In this test, O remains constant unless S is within a certain range. The program has 16 different outputs, ranging from 4 to 19.

Listing 3.2: Sanity check test program.

```
var base = 4;
if(S < 16){O = base+ S;}
else{O = base;}
```

Listing 3.3: Implict flow test program.

```
O = 0;
if(S == 0){O = 0;}
else{if(S == 1){O = 1;}
        else{if(S == 2){O = 2;}
                else{if(S == 3){O = 3;}
                        else{if(S == 4){O = 4;}
                                else{if(S == 5){O = 5;}
                                        else{if(S == 6){O = 6;}
        }       }       }       }       }       }
```

Chapter 4

Monotonic programs

In this chapter we describe our discovery that if the test program P meets a certain property, then we can execute the program in a loop and get a direct output count in a reasonable time.

Need to insert somewhere else.

Definition 1. *Let* $bitLength \in \{0, 1, 2, ..., 32\}$ *and let* P *be a function whose input* $S \in \{0, 1, 2, ..., 1 \ll bitLength - 1\}$ *and output* $O \in \{0, 1, 2, ..., 1 \ll bitLength - 1\}$.

4.1 Discovery by mistake

During the initial tests with the sanity check program of bit length 32, the program terminates in less than a minute and gives the correct output count. Later we time the execution of an empty loop from 0 to $2^{32} - 1$ and the result is 10.30 seconds, so theoretically the double loop would take a much longer time to terminate, longer than our test result with sanity check. Listing 4.1 shows the C code we used.

Listing 4.1: Initial implementation of the double loop with sanity check in C.

```
uint32_t S = 0;
uint32_t O = 0;
uint32_t SMax = S - 1;
uint32_t OMax = O - 1;
uint32_t OCounter = 0;
uint32_t OTemp = 0;
uint32_t base = 0;
for (;O<OMax;O++){
        for (;S<SMax;S++){
                //program start here
                if (S < 16)
                        OTemp = base + S;
                else
                        OTemp = base;
                //program end here
                if (OTemp == O){
                        OCounter ++;
                        break;
}       }       }
```

Due to the large difference in actual execution time and the theoretical time, we decided to inspect the code for errors. Our code differs from Algorithm 1 at two places: First, S and O can not reach $SMax$ and $OMax$ due to the use of $<$, thus the upper bound is not tested. Second, in the inner loop, we did not initialize S

to 0 except for its first iteration. The first point does not affect the output count of the sanity check example as every S greater than 16 will lead to a $P(S)$ value of *base* which is 0. The second point is the cause of the large time difference, because essentially P only executes once for each S, reaching a total of 2^{32} times, much smaller than the full double loop in which P executes $2^{2\times32}$ times.

We found that removing the initialization of S to 0 can be a way to speed up some test cases, but not all. Specifically, program P has to obey certain properties:

1. Function P is monotonically increasing within range $[0, n]$.
2. The output of function P with S in range $[0, n]$ has to start with zero and is continuous.
3. Each output of function P with S in range $[n + 1, 1 \ll bitLength - 1]$ can be produced by at least one S in range $[0, n]$.

4.2 Improvements

The properties that P has to follow in order for the optimization to work is rather strict. For example in the sanity check test, the optimization only works when *base* is 0. A possible solution would be to start O at the lowest value P can output, in essence pulling the function down on y axis so that it starts with 0. However, this approach does not ease the restrictions.

In our second attempt to improve this optimization method, we started with monotonicity and rearranged the architecture of the outer program. The problem is to count the number of outputs of a monotonically increasing function,

Listing 4.2: Implementation of the optimization with sanity check in C.

```
unsigned int S = 0;
unsigned int SMax = 4294967295;
unsigned int base =4;
unsigned int STemp = S;
unsigned int OTemp = 0;
//program starts here
if (STemp< 16){OTemp = base+ STemp;}
else{OTemp = base;}
//program ends here
unsigned int OCounter = 1;
unsigned int OMax = OTemp;
S++;
while (S<= SMax){
        STemp = S;
        //program starts here
        if(STemp < 16){OTemp = base+ STemp;       }
        else{OTemp = base;}
        //program ends here
        if (OTemp > OMax ){
                OMax = OTemp;
                OCounter= OCounter+1;
        }
        if(S < SMax) S=S+1;
```

```
            else break;
        }
```

Chapter 5

Summary and concluding remarks

Congratulations on completing your dissertation.

Appendix A

Title of first appendix

A.1 Section title

Here is some additional information which would have detracted from the point being made in the main article.

A.1.1 Subsection title

This section even has subtitles

Bibliography

[1] Interproc, 2011.

[2] MTC (models and theory of computation): BLAST project, 2008.

[3] Salvatore La Torre, Madhusudan Parthasarathy, and Gennaro Parlato. Analyzing recursive programs using a fixed-point calculus. In *Proceedings of the 2009 ACM SIGPLAN Conference on Programming Language Design and Implementation*, PLDI '09, page 211222, New York, NY, USA, 2009. ACM.

[4] Getafix – boolean program checker, 2009.

VITA

This is a summary of your *professional* life, and should be written appropriately. This can be written in the following order: where your where born, what undergraduate university you graduated from, if you received a masters, and which institution you graduated from with your PhD (University of Missouri). You can describe when you began research with your current advisor.

In another paragraph, you could say if/when you were married, what the name of your kids are, and what your plans are for after graduation if you choose. Take a look at other vita's from other dissertations for examples.