CALCULATING INFORMATION LEAKAGE USING MODEL CHECKING TOOLS

A Thesis presented to the Faculty of the Graduate School at the University of Missouri

 $\label{eq:continuous} \mbox{In Partial Fulfillment}$ of the Requirements for the Degree $\mbox{Master of Science}$

by

JIA CHEN

Dr. Rohit Chadha, Thesis Supervisor JUL 2014 The undersigned, appointed by the Dean of the Graduate School, have examined the thesis entitled:

CALCULATING INFORMATION LEAKAGE USING MODEL CHECKING TOOLS

presented by Ji	a Chen,							
a candidate for	the degree of	Master of	Science and	hereby	certify	that,	in	their
opinion, it is we	orthy of acceptai	nce.						
-		Dr. Rohi	t Chadha					
_		Dr. Prasa	ad Calyam					

Dr. Michela Becchi

ACKNOWLEDGMENTS

I would like to express my sincere gratitude to my advisor Dr.Rohit Chadha, for guiding me through the research process when I was working on the previous thesis subject of browser cross-site scripting defence, and furthermore for providing me the current subject when I was struggling with the previous one. He helped me all the way during research and writing of this thesis, with his deep knowledge on information security.

I also sincerely give my appreciation to Dr.Prasad Calyam and Dr.Michela Becchi, for their time and work serving as my thesis committee members.

I'd like to thank Umang Mathur for providing MOPED source code and building instructions during the tests. His email saved us plenty of time.

Last but not the least, I would like to thank my parents and friends, for encouraging me to fight on.

TABLE OF CONTENTS

A	CKN	OWLEDGMENTS	ii
LI	ST (OF TABLES	\mathbf{v}
Ll	ST (OF FIGURES	vi
\mathbf{A}	BST	RACT	vii
\mathbf{C}	HAP	PTER	
1	Intr	roduction	1
2	The	eory background	2
	2.1	Min-entropy	2
	2.2	Information leak	2
	2.3	Two intuitive solutions	2
		2.3.1 Double loop	3
		2.3.2 Single loop and array	4
3	Mo	notonic programs	7
	3.1	Discovery by mistake	7
	3.2	Improvements	9
4	Exp	periment with Getafix and jMoped	12
	4.1	Getafix	13
		4.1.1 The converter	13
	4.2	iMoped	17

	4.3	Tests a	and results	18
		4.3.1	Sanity check	19
		4.3.2	Implict flow	19
		4.3.3	Mix and duplicate	20
		4.3.4	Masked copy	21
		4.3.5	Binary search	21
		4.3.6	Electronic purse	22
		4.3.7	Sum query	23
5	Sun	nmary	and concluding remarks	24
\mathbf{A}	PPE	NDIX		
\mathbf{A}	\mathbf{Titl}	e of fir	st appendix	25
	A.1	Section	n title	25
		A.1.1	Subsection title	25
ΒI	BLI	OGRA	PHY	26
1 71	TT A			27

LIST OF TABLES

Table		Page
2.1	Execution time of an empty double loop with different bit length	5
2.2		6
4.1	Examples of input and output of the parser, with bit length of 4	15
4.2	Timing results for sanity check	19
4.3	Timing results for implicit flow	20
4.4	Timing results for mix and duplicate	21
4.5	Timing results for masked copy	21
4.6	Timing results for binary search	22
4.7	Timing results for electronic purse	23
4.8	Timing results for sum query	23

LIST OF FIGURES

Figure	I	Page
4.1	Workflow of calculating information leakage with Getafix	13

ABSTRACT

A confidential program should not allow any information about its secret inputs to be inferred from its public outputs. As such confidentiality is difficult to achieve in practice, it has been proposed in literature to evaluate security of programs by computing the amount of information it leaks. In this thesis, we consider the problem of computing information leaked by a deterministic program and use the information-theoretic measure of min-entropy to quantify the amount of information.

The main challenge in computing information leakage by a program using minentropy is that one has to count the number of distinct outputs by that program. We find a polynomial-time reduction from the problem of counting outputs to the problem of checking safety in programs. Thus we propose a hypothesis that we can estimate leakage using model checking tools which are originally developed for checking safety.

We test the above hypothesis using two popular model checking tools, JMoped and Getafix. Our tests indicate that they do not scale as the number of bits in the input increases. However, we find that if the program enjoys the additional property of monotonicity then we can use a different reduction to the problem of checking safety. We observe a dramatic improvement in performance with this new reduction.

Chapter 1

Introduction

Introduce the reader to the current problem that you wish to solve, and why anyone should care about it.

Chapter 2

Theory background

2.1 Min-entropy

The introduction counts as chapter 1. This page shows how the bulk of your thesis will be organized: through chapters and sections. Here is a citation.[?]

2.2 Information leak

2.3 Two intuitive solutions

To count the number of outputs of a program, we come up with two approaches:

1. Put the program in a double loop and count the number of outputs. The outer loop is for possible outputs and the inner loop is for inputs. When the program

in the inner loop produces an output which matches the outer loop, counter increases.

2. Let the program iterate through all input values and record the output hit results in a bit array. Counter increases when a bit flips.

The first approach is time-consuming, while the second one is memory-consuming. We will discuss these two approaches in detail in the subsections.

2.3.1 Double loop

In Algorithm 1, for each possible output value, we iterate through the input range to see if an input can result in this output. If we hit this output, OCounter increases and the code breaks out of the inner loop to continue testing the next possible output value. After the double loop finishes, the value of OCounter is the number of outputs of program P.

In this approach, we declare seven variables, and all of them require bitLength bits except for OCounter which needs to be bitLength + 1 bits. The total memory usage for variables is $7 \times bitLength + 1$ at O(bitLength). As with execution time, we assume program P takes t(P) seconds to execute, and the total execution time for the double loop when break is never reached is $2^{bitLength} \times 2^{bitLength} \times t(P)$. Thus the time complexity is $(2^{O(bitLength)}) \times t(P)$.

In order to get an estimation of how much time the double loop will take to execute, we implemented a piece of C code with an empty while loop which loops 2^{32} times. On our experiment PC, this loop takes on average 10.30 seconds to complete. Were we to run a double loop in bit length of 32, the execution time would be $2^{32} \times 10.30$

```
Algorithm 1: Calculate the number of outputs using double loop.
    S \leftarrow 0
    O \leftarrow 0
    SIn \leftarrow 0
    OOut \leftarrow 0
    OCounter \leftarrow 0
    SMax \leftarrow 1 << bitLength - 1
    OMax \leftarrow 1 << bitLength - 1
    for O = 0 to OMax do
       for S = 0 to SMax do
          SIn \leftarrow S
          OOut \leftarrow P(SIn) // \text{ the program } P \text{ takes } SIn \text{ as input}
          if OOut = O then
            OCounter \leftarrow OCounter + 1
            break
          end if
       end for
    end for
```

seconds, which is around 1403 years. Running the double loop at 32 bits would be infeasible.

Starting with bit length n, the time requirement for a full double loop is $2^{2\times n} \times t(P)$. Increase the bit length by one and the time becomes $4 \times 2^{2\times n} \times t(P)$, four times the previous time. Table 2.1 shows the actual execution time of an empty double loop under different bit length, and the time increase follows the theoretical analysis. At bit length of 23, the execution time would exceed a day, and executing at higher bit length is impractical.

2.3.2 Single loop and array

In Algorithm 2, we create a bit array with size equal to the maximum number of possible outputs (1 << bitLength), or $2^{bitLength}$ and initialize it with zeros. While we

Bit length	Time(s)	Multiplier
14	0.708	
15	2.797	3.951
16	11.196	4.003
17	44.515	3.976
18	178.970	4.020

Table 2.1: Execution time of an empty double loop with different bit length.

iterate through the range of S, we set each OHit[P(SIn)] to 1. When a 0 turns to 1, we increase OCounter. After the loop, the value of OCounter is the number of outputs by program P.

```
Algorithm 2: Calculate the number of outputs using single loop and a table.
    S \leftarrow 0
    O \leftarrow 0
    SIn \leftarrow 0
    OOut \leftarrow 0
    OCounter \leftarrow 0
    SMax \leftarrow 1 << bitLength - 1
    OMax \leftarrow 1 << bitLength - 1
    OHit[OMax + 1] \leftarrow [0]
    for S = 0 to SMax do
       SIn \leftarrow S
       OOut \leftarrow P(SIn) // \text{ the program } P \text{ takes } SIn \text{ as input}
       if OHit[OOut] = 0 then
          OCounter \leftarrow OCounter + 1
          OHit[OOut] \leftarrow 1
       end if
    end for
```

In Algorithm 2 except for the array we have 7 variables using $7 \times bitLength + 1$ bits memory. The array OHit[] is of size $2^{bitLength} \times 1$ bits making a total of $2^{bitLength} + 7 \times bitLength + 1$ bits at $2^{O(bitLength)}$). As with execution time, we assume program P takes t(P) seconds to execute, and the execution time for the single loop is $2^{bitLength} \times t(P)$.

Thus the time complexity is $(2^{O(bitLength)}) \times t(P)$.

We set the bit length to 32. In array OHit[], each element is 1 bit and the number of elements is 2^{32} . The total memory usage for this array is $2^{32} \times 1$ bits, which is 0.5 gigabytes. To our knowledge, it is neither difficult nor expensive to build a PC with more than 16 gigabytes of memory, and we can get such a PC off-the-shelf from top gaming PC brands like Alienware. However, as this memory requirement grows exponentially, adding five or six bits to the bit length and the requirement will exceed the capacity of current PCs.

if this is
not hard
we need
reasons
not doing
this

	Execution time	Growth	Memory requirement	Growth
Double loop		*4		
Single loop & array				

Table 2.2:

Chapter 3

Monotonic programs

In this chapter we describe our discovery that if the test program P meets a certain property, then we can execute the program in a loop and get a direct output count in a reasonable time.

Need to insert somewhere else.

Definition 1. Let $bitLength \in \{0, 1, 2, ..., 32\}$ and let P be a function whose input $S \in \{0, 1, 2, ..., 1 \ll bitLength - 1\}$ and output $O \in \{0, 1, 2, ..., 1 \ll bitLength - 1\}$.

3.1 Discovery by mistake

During the initial tests with the sanity check program of bit length 32, the program terminates in less than a minute and gives the correct output count. Later we time the execution of an empty loop from 0 to $2^{32} - 1$ and the result is 10.30 seconds, so theoretically the double loop would take a much longer time to terminate, longer than our test result with sanity check. Listing 3.1 shows the C code we used.

```
uint32_t S = 0;
uint32_t O = 0;
uint32_t SMax = S - 1;
uint32_t OMax = O - 1;
uint32_t OCounter = 0;
uint32_t OTemp = 0;
uint32_t base = 0;
for (; O < OMax; O++){
        for (; S \leq SMax; S++)
                //program start here
                if (S < 16)
                        OTemp = base + S;
                else
                        OTemp = base;
                //program end here
                OCounter ++;
                        break;
        }
                }
```

Listing 3.1: Initial implementation of the double loop with sanity check in C.

Due to the large difference in actual execution time and the theoretical time, we decided to inspect the code for errors. Our code differs from Algorithm 1 at two places: First, S and O can not reach SMax and OMax due to the use of <, thus

the upper bound is not tested. Second, in the inner loop, we did not initialize S to 0 except for its first iteration. The first point does not affect the output count of the sanity check example as every S greater than 16 will lead to a P(S) value of base which is 0. The second point is the cause of the large time difference, because essentially P only executes once for each S, reaching a total of 2^{32} times, much smaller than the full double loop in which P executes $2^{2\times 32}$ times.

We found that removing the initialization of S to 0 can be a way to speed up some test cases, but not all. Specifically, program P has to obey certain properties:

- 1. Function P is monotonically increasing within range [0, n].
- 2. The output of function P with S in range [0, n] has to start with zero and is continuous.
- 3. Each output of function P with S in range $[n+1,1 \ll bitLength-1]$ can be produced by at least one S in range [0,n].

3.2 Improvements

The properties that P has to follow in order for the optimization to work is rather strict. For example in the sanity check test, the optimization only works when base is 0. A possible solution would be to start O at the lowest value P can output, in essence pulling the function down on y axis so that it starts with 0. However, this approach does not ease the restrictions.

In our second attempt to improve this optimization method as shown in Listing 3.2, we started with monotonicity and rearranged the architecture of the outer pro-

gram. The problem is to count the number of outputs of a monotonically increasing function, and our solution is to iterate through the entire range of S and keep track of the largest output value. If P generates a value OOut which is larger than the current largest value, then OOut replaces that value and OCounter increases. The properties that P has to follow in order to use this optimization is:

- 1. Set the range of n to $[0, 2^{bitLength} 2]$.
- 2. Calculate $P(S_n)$ to be O_n and store it in set O[].
- 3. Calculate $P(S_{n+1})$ to be O_{n+1} has to be either:
 - (a) Greater than O_n , in which case we store O_{n+1} in O[].
 - (b) Already in set O[].
- 4. Continue step 2 on the next n value until n reaches its upper bound.

This is wrong.

this.

1. Function P is monotonically increasing within range $[n_1, n_2]$.

Correct

2. Each output of function P with S in range $[n_2 + 1, 1 \ll bitLength - 1]$ can be produced by at least one S in range $[n_1, n_2]$.

So with the second attempt, we removed the requirement for P to be continuous and start with 0 from the first attempt.

```
unsigned int S = 0;
unsigned int SMax = 4294967295;
unsigned int base =4;
```

```
unsigned int STemp = S;
unsigned int OTemp = 0;
//program starts here
if (STemp< 16){OTemp = base+ STemp;}
else {OTemp = base;}
//program ends here
unsigned int OCounter = 1;
unsigned int OMax = OTemp;
S++;
while (S \le SMax)
        STemp = S;
        //program starts here
        if(STemp < 16){OTemp = base+ STemp;</pre>
                                                   }
        else{OTemp = base;}
        //program ends here
        if (OTemp > OMax)
                OMax = OTemp;
                 OCounter = OCounter + 1;
        }
        if(S < SMax) S=S+1;
        else break;
```

Listing 3.2: Implementation of the optimization with sanity check in C.

Chapter 4

Experiment with Getafix and jMoped

We want to use model-checking tools to see if we can reduce the time requirement of Algorithm 1. Specifically, we choose the reachability property and append Algorithm 3 to the end of Algorithm 1. The statement within the if statement has a label. Although the exact statement following that label is irrelevant, reaching this line means *OCounter* satisfies the constrains in the condition block.

(why and what other property are there)

We experiment on several model-checking tools, including Interproc from [1], Berkeley Lazy Abstraction Software Verification Tool (Blast) from [2], Getafix from [3] and jMoped from [?]. We can not get correct reachability results from Interproc and Blast, so we shift our focus onto Getafix and jMoped.

correct citations

Algorithm 3: Determine if *OCounter* meets certain constrains.

if value of *OCounter* meets certain constrains **then** reach: *OCounter* // a label followed by a statement **end if**

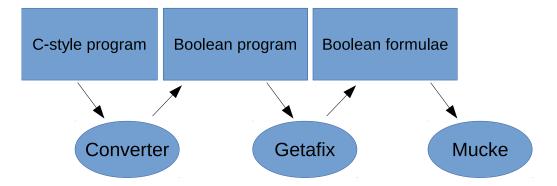


Figure 4.1: Workflow of calculating information leakage with Getafix.

4.1 Getafix

Getafix is a symbolic model checker for Boolean programs implemented in [3]. Getafix only supports reachability check. It translates sequential and concurrent Boolean programs into Boolean formulae and uses the model-checker Mucke to solve the reachability problem symbolically using Boolean Decision Diagrams [4].

Mostly copied from getafix website. Need

rewrite

4.1.1 The converter

Input for Getafix are boolean programs, meaning it only supports binary variables which can be either 0 or 1. We represent our problem in c-style code, thus we need to translate it into boolean form. We implemented a converter to automate this process, and Figure 4.1 shows the workflow we used to calculate information leakage using Getafix. The converter has three components, a parser, a built-in function generator and a piece of script which calls the first two components and assembles the output file. The converter has these properties:

1. The input to the converter is a c-style code file and a positive integer which represents the bit length. The converter supports 32 bits and less. Also note

that the converter represents a number with bit length of bitLength using bitLength + 1 bits. We do this so that we do not need to deal with the upper bound explicitly when writing a loop iterating from 0 to 1 << bitLength - 1.

- 2. The output of the converter is a boolean program which follows the syntax of Getafix input file.
- 3. In the language that we defined for the input file, we support only one variable type: non-negative decimal integer. Again we support the length up to 32 bits.
- 4. Our converter does not support function definitions. The input file has two parts, variable declarations block and statements block. The parser will print these blocks into the main function of the output boolean program. Also we require all variable declarations to appear before statements in the input code. Getafix input syntax has this requirement, and we decide to keep it in our converter.
- 5. We implement support for the following symbolic operators in the language: plus +, minus -, and &, or |, xor ^, greater than >, equal ==, less than <, not equal !=, greater than or equal >=, less than or equal <=, left shift << and right shift >>.
- 6. We implement support for three control statements: if...else, while loop, goto and statements with labels. We currently do not support for loop, do...while loop, switch statements, break and continue, but these statements can be easily expressed using the supported ones.

Input to the parser is a c-style code file and a desired bit length. Output of the parser is its corresponding boolean program which follows the syntax of Getafix input file. First we define the syntax of the input code and second we create the parser using flex and bison. The parser scans the input code and builds a syntax tree. Then the parser prints the syntax tree as a boolean program. The parser has three points worth noting:

Table 4.1: Examples of input and output of the parser, with bit length of 4

Input	Output
von CMov. — 16.	decl SMax4,SMax3,SMax2,SMax1,SMax0;
var SMax = 16;	Smax4, SMax3, SMax2, SMax1, SMax1 := 1,0,0,0,0;
STown - STown 5.	STemp4,STemp3,STemp2,STemp1,STemp0 :=
STemp = STemp - 5;	minus(STemp4,STemp3,STemp2,STemp1,STemp0,0,0,1,0,1);

- 1. When printing the output code, the parser "stretches" each variable and literal into its binary form. Assume the desired bit length is bitLength. We split each variable into bitLength variables by copying the name of the variable bitLength times and appending a counter value to each one. Also we convert a literal to its corresponding binary value and prepend it with zeros to reach the desired length. Table 4.1 contains an example of how the parser deals with a variable declaration.
- 2. In a boolean program, all operators operate on bit level, so we need to implement higher-level operators like plus, minus, greater than and left shift using operators that Getafix supports. In the parser, we print these high-level operators as function calls in the output boolean program, and the built-in function generator generates the body of the function. Table 4.1 also contains an example of how the parser deals with a high-level operator.

3. In the boolean code syntax which Getafix defines, function call plus the semicolon is defined as a statement, and another rule allows the code to assign a
function call to an identifier, but function call itself is not an expression. This
means that a function call can not work as an expression as in many other languages, and it leads to two problems: First, the decider expressions in if...else
and while statements can not contain function calls. Second, parameters of a
function call or operands to an operator can not be a function call. We automated a solution in the parser to the first problem, which assigns the decider
expression to a temporary variable and use that variable as the decider, so we
can use the c-style if...else and while in the input code. For the second problem,
a possible solution would be to manage a set of internal temporary variables
and assign each function call to a variable, but we did not implement it.

Input to the built-in function generator is a desired bit length. Output is a set of high-level operators like plus and left shift implemented as functions. We do not track the necessary functions in the parser, as experiments with Getafix indicate that the uncalled functions affect little on the execution time. Listing 4.1 shows a sample function by the generator.

Listing 4.1: Greater than operator as a function in boolean program with bit length of 2.

Input to the third component, a piece of script, is a c-style code file and a desired bit length. Output of the script is a boolean program ready for Getafix to process. The script first passes the bit length to the built-in function generator and redirects its output to the output file. Then the script passes both the input to the parser and appends its output to the output file. At this point the output is complete.

4.2 jMoped

jMoped is a model checker which checks for coverage in Java programs. The authors implemented it as a plug-in for eclipse, using its UI for parameters and output display. We rewrite our tests in Java so that we can use jMoped on them.

more explanation

4.3 Tests and results

Before we have the converter, we coded a few test cases manually and Getafix gave us

the correct answers. Now with the converter we can run more complicated tests and

see if Getafix is a good solution to the problem of calculating information leakage.

As with jMoped, we rewrite the tests in Java. We decide to run the eight test cases

from paper [?]. In each test, O represents the output of the test program, and S

is the input. Also in the tables, optimization refers to the final optimization in the

previous chapter, and an empty cell means that we did not do this experiment.

We did our tests on a laptop computer, and key hardware specifications are:

Model Lenovo ideapad Y580-IFI

CPU Intel Core i5-3210M @ 2.5GHz

Memory 4GBytes DDR3-1600 \times 2

And the software specifications are:

OS Ubuntu 14.04 LTS 32-bit

Getafix Version information not available. Source code retrieved on 2014/4/10

Mucke Version 0.4.4

Eclipse Eclipse juno sr2

jMoped Version 2.0.2

18

4.3.1 Sanity check

Listing 4.2 shows the code we use. In this test, O remains constant unless S is within a certain range. The program has 16 different outputs, ranging from 4 to 19. We can use the optimization in this test, and the timing results are in Table 4.2.

```
var base = 4;
if(S < 16){O = base+ S;}
else{O = base;}</pre>
```

Listing 4.2: Sanity check test program.

bit length(bit)	getafix(s)	getafix optimized(s)	jmoped(s)	jmoped optimized(s)
6	41.264		9.322	
7	235.64	3.3775	62.074	0.53
8	>600	6.448	326.122	0.875
9		13.491		1.69
10		27.479		3.045

Table 4.2: Timing results for sanity check.

4.3.2 Implict flow

Listing 4.3 shows the code. This test copies the value of S to O indirectly through the if statement when S is less than 7. For other S values, O is 0. The program has 7 different outputs, ranging from 0 to 6. We can use the optimization in this test, and the timing results are in Table 4.3.

```
O = 0;
if(S == 0)\{O = 0;\}
else\{if(S == 1)\{O = 1;\}
```

```
else{if(S == 2){O = 2;}

else{if(S == 3){O = 3;}

else{if(S == 4){O = 4;}

else{if(S == 5){O = 5;}

else{if(S == 6){O = 6;}

}

}
```

Listing 4.3: Impliet flow test program.

bit length(bit)	getafix(s)	getafix opt(s)	jmoped(s)	jmoped opt(s)
6	58.834		11.865	
7	289.919	8.2645	63.555	0.575
8	>600	22.6435	325.755	0.965
9		61.464		1.86
10		182.1295		3.57

Table 4.3: Timing results for implicit flow.

4.3.3 Mix and duplicate

Listing 4.4 shows the code. This test first calculates the XOR value of the two halves of S (mix) and second duplicates this XOR value twice in O (duplicate). The output count depends on the bit length, and at 8 bits, it has $2^4 = 16$ different outputs. We can use the optimization in this test, and the timing results are in Table 4.4.

$$O = ((S >> 4) \hat{S}) \& 15;$$

 $O = O | O << 4;$

Listing 4.4: Mix and duplicate test program at 8 bits.

bit length(bit)	getafix(s)	getafix opt(s)	jmoped(s)	jmoped opt(s)
4	2.7185	0.9825	1.14	0.375
6	44.728	3.719	32.94	0.63
8	>600	32.931	eclipse terminates 1	1.97
10		Memory allocation failed		9.765

Table 4.4: Timing results for mix and duplicate.

4.3.4 Masked copy

Listing 4.5 shows the code. In this test, O is S with its lower half set to 0, or "masked" out. The output count depends on the bit length, and at 8 bits, it has $2^4 = 16$ different outputs. We can use the optimization in this test, and the timing results are in Table 4.5.

```
O = S \& 240;
```

Listing 4.5: Masked copy test program at 8 bits.

bit length(bit)	getafix(s)	getafix opt(s)	jmoped(s)	jmoped opt(s)
4	1.728	0.7515	0.605	0.23
6	38.2435	1.3855	10.29	0.345
8	>600	5.254	320.87	0.81
10		39.1355		2.445

Table 4.5: Timing results for masked copy.

4.3.5 Binary search

Listing 4.6 shows the code. This test leaks the upper half of S to O through binary search. The output count depends on the bit length, and at 8 bits, it has $2^4 = 16$ different outputs. We can use the optimization in this test, and the timing results are in Table 4.6.

```
if (O + 128 \le S) \{O = O + 128;\}
if (O + 64 \le S) \{O = O + 64;\}
if (O + 32 \le S) \{O = O + 32;\}
if (O + 16 \le S) \{O = O + 16;\}
```

Listing 4.6: Binary search test program at 8 bits.

bit length(bit)	getafix(s)	getafix opt(s)	jmoped(s)	jmoped opt(s)
4	3.3590	1.094	0.905	0.330
6	136.0740	5.947	28.255	0.635
8	>600	65.235	eclipse error	2.315
10		>600		13.530

Table 4.6: Timing results for binary search.

4.3.6 Electronic purse

Listing 4.7 shows the code. Assume S is the account balance, we set the deduction to 5, and output O represents the number of times one can debit such an amount. In this test we set SMax to 19, and the program has 4 outputs, ranging from 0 to 3. We can use the optimization in this test, and the timing results are in Table 4.7.

```
O = 0;

while (S >= 5) {
S = S - 5;
O = O + 1;
}
```

Listing 4.7: Electronic purse test program.

bit length(bit)	getafix(s)	getafix opt(s)	jmoped(s)	jmoped opt(s)
6	10.411	2.7805	1.7	0.305

Table 4.7: Timing results for electronic purse.

4.3.7 Sum query

Listing 4.8 shows the code. This test leaks the sum of its three inputs to O. We set S1, S2 and S3 to be less than 10, so the program has 28 outputs ranging from 0 to 27. We can use the optimization in this test, and the timing results are in Table 4.8.

$$O = S1 + S2 + S3;$$

Listing 4.8: Sum query test program.

bit length(bit)	getafix(s)	getafix opt(s)	jmoped(s)	jmoped opt(s)
6	225.3995	33.488	21.115	2.595

Table 4.8: Timing results for sum query.

Chapter 5

Summary and concluding remarks

Congratulations on completing your dissertation.

Appendix A

Title of first appendix

A.1 Section title

Here is some additional information which would have detracted from the point being made in the main article.

A.1.1 Subsection title

This section even has subtitles

Bibliography

- [1] Interproc, 2011.
- [2] MTC (models and theory of computation): BLAST project, 2008.
- [3] Salvatore La Torre, Madhusudan Parthasarathy, and Gennaro Parlato. Analyzing recursive programs using a fixed-point calculus. In *Proceedings of the 2009 ACM SIGPLAN Conference on Programming Language Design and Implementation*, PLDI '09, page 211222, New York, NY, USA, 2009. ACM.
- [4] Getafix boolean program checker, 2009.

VITA

This is a summary of your *professional* life, and should be written appropriately. This can be written in the following order: where your where born, what undergraduate university you graduated from, if you received a masters, and which institution you graduated from with your PhD (University of Missouri). You can describe when you began research with your current advisor.

In another paragraph, you could say if/when you were married, what the name of your kids are, and what your plans are for after graduation if you choose. Take a look at other vita's from other dissertations for examples.