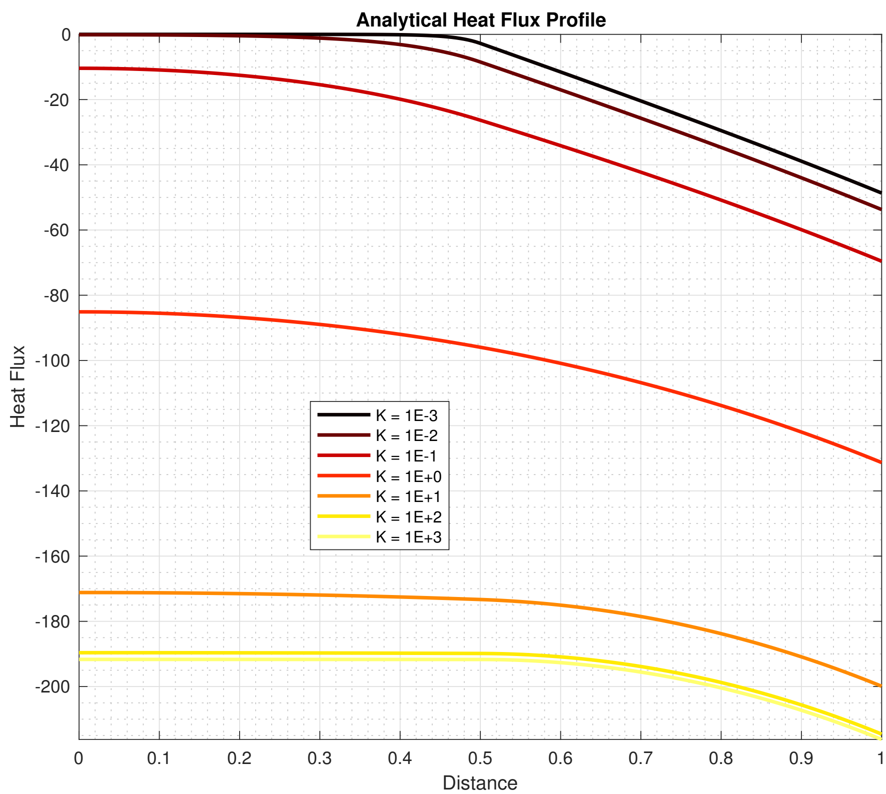
AERO 430 – Numerical Solution

Homework 1 – Solution



Ross Alexander
2.11.18

Analytical Solution

For the given problem, the governing equation is:

$$-u'' + \frac{b}{k}u = 0 \qquad x \in [0, 1] \tag{1}$$

Since the differential equation is split into two regions with different coefficients, the governing equations become:

$$-u'' + \alpha_1^2 u = 0 \qquad x \in [0, \tfrac{1}{2}] \tag{2}$$

$$-u'' + \alpha_2^2 u = 0 \qquad x \in [\tfrac{1}{2}, 1] \tag{3}$$

with $\alpha_i^2 = \frac{b}{k_i}$. The boundary conditions are temperatures at the left and right ends of the domain, 0 and 100, respectively, and the continuity conditions for temperature and heat flux ($\dot{Q} = -kAu'$) at the midpoint of the domain:

$$\begin{aligned} u(0) &= 0 & u(1) &= 100 \\ u(\tfrac{1}{2}) &= u(\tfrac{1}{2}) & -k_1 A u'(\tfrac{1}{2}) &= -k_2 A u'(\tfrac{1}{2}) \end{aligned} \tag{4}$$

The general solution for the second-order inhomogeneous partial differential equation of similar form is:

$$u(x) = A\cosh(\alpha x) + B\sinh(\alpha x) + C \tag{5}$$

In the case of this problem, we have:

$$u_1(x) = A\cosh(\alpha_1 x) + B\sinh(\alpha_1 x) + T_a \qquad x \in [0, \tfrac{1}{2}] \tag{6}$$

$$u_2(x) = C\cosh(\alpha_2 x) + D\sinh(\alpha_2 x) + T_a \qquad x \in [\tfrac{1}{2}, 1] \tag{7}$$

A straightforward application of the boundary conditions and the use of a numerical solver yields the coefficients A, B, C, and D. For conditions $b = 1$, $k_1 = K * k_2$, $k_2 = 1$, $T_a = 0$, $A = 1$, and for $K \in \{10^{-3}, 10^{-2}, 10^{-1}, 10^{+0}, 10^{+1}, 10^{+2}, 10^{+3}\}$, the analytical temperature and heat flux profiles are calculated below.

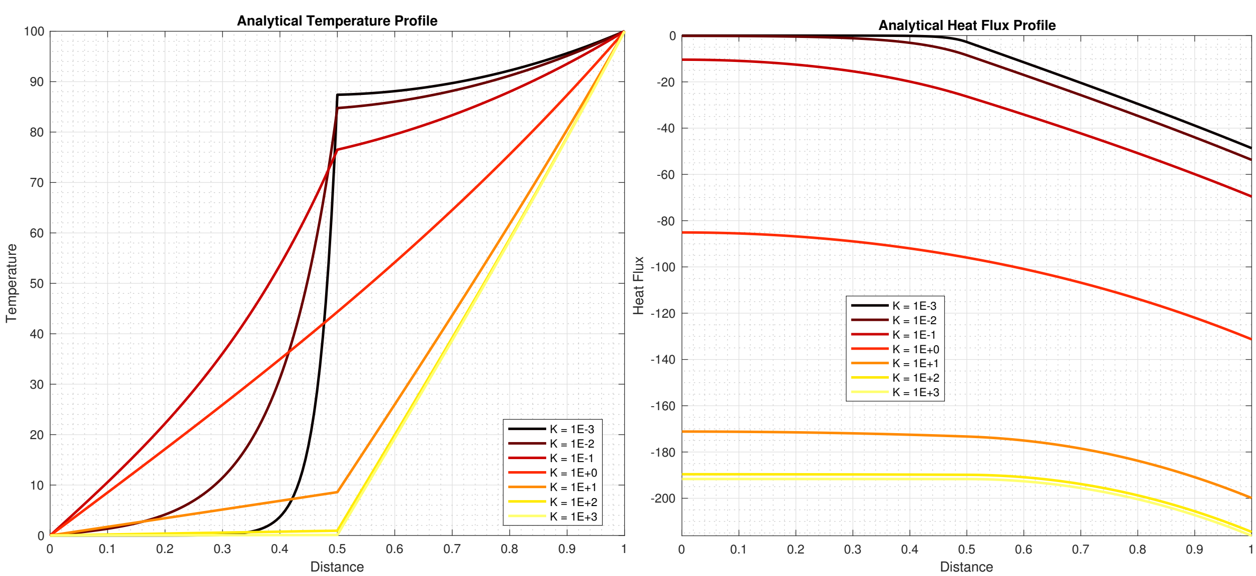


Figure 1 – Analytical Temperature and Heat Flux Profiles

Finite Difference Method

2nd-Order Central Difference Scheme

The second-order discretization scheme is achieved using the heat flux gradient form of the governing equation, where $Q = -ku'$ and $Q' = -ku''$:

$$Q' + bu = 0 \tag{8}$$

Or in discrete form:

$$Q_i' + b_i u_i = 0 \tag{9}$$

First the heat flux gradient is discretized using the midpoints, rearranged, and then exchanged for the temperature gradients:

$$\frac{Q_{i+\frac{1}{2}} - Q_{i-\frac{1}{2}}}{\Delta x} + b_i u_i = 0 \tag{10}$$

$$Q_{i+\frac{1}{2}} - Q_{i-\frac{1}{2}} + b_i u_i \Delta x = 0 \tag{11}$$

$$-k_{i+\frac{1}{2}} u'_{i+\frac{1}{2}} + k_{i-\frac{1}{2}} u'_{i-\frac{1}{2}} + b_i u_i \Delta x = 0 \tag{12}$$

Then, the temperature gradients are discretized using the nodes and rearranged:

$$-k_{i+\frac{1}{2}} \frac{u_{i+1} - u_i}{\Delta x} + k_{i-\frac{1}{2}} \frac{u_i - u_{i-1}}{\Delta x} + b_i u_i \Delta x = 0 \tag{13}$$

$$-k_{i+\frac{1}{2}} (u_{i+1} - u_i) + k_{i-\frac{1}{2}} (u_i - u_{i-1}) + b_i u_i \Delta x^2 = 0 \tag{14}$$

Simplifying yields the three-point second-order stencil:

$$\mathbf{\left(-k_{i-\frac{1}{2}}\right) u_{i-1} + \left(b_i \Delta x^2 + k_{i-\frac{1}{2}} + k_{i+\frac{1}{2}}\right) u_i + \left(-k_{i+\frac{1}{2}}\right) u_{i+1}} \tag{15}$$

2nd-Order Extraction Scheme

Since the quanties of interest are $u(\frac{1}{2})$ and $Q(\frac{1}{2})$, formulas must be developed to extract them from the numerical results. However, this is trivial for $u(\frac{1}{2})$ since it is obtained directly from the numerical results. But, for the midpoint heat flux $Q(\frac{1}{2})$, a second-order extraction formula must be developed.

The two choices are to evaluate the heat flux using a forward or backward difference. Both formulas are given below:

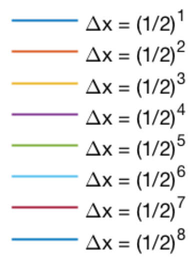
$$\mathbf{Q(\tfrac{1}{2})^- = -k_{i-\frac{1}{2}} \frac{u_i - u_{i-1}}{\Delta x} - \frac{b \Delta x}{2} u_i} \tag{16}$$

$$\mathbf{Q(\tfrac{1}{2})^+ = -k_{i+\frac{1}{2}} \frac{u_{i+1} - u_i}{\Delta x} + \frac{b \Delta x}{2} u_i} \tag{17}$$

Results

Finite Difference Method

A legend is provided for the figures below:



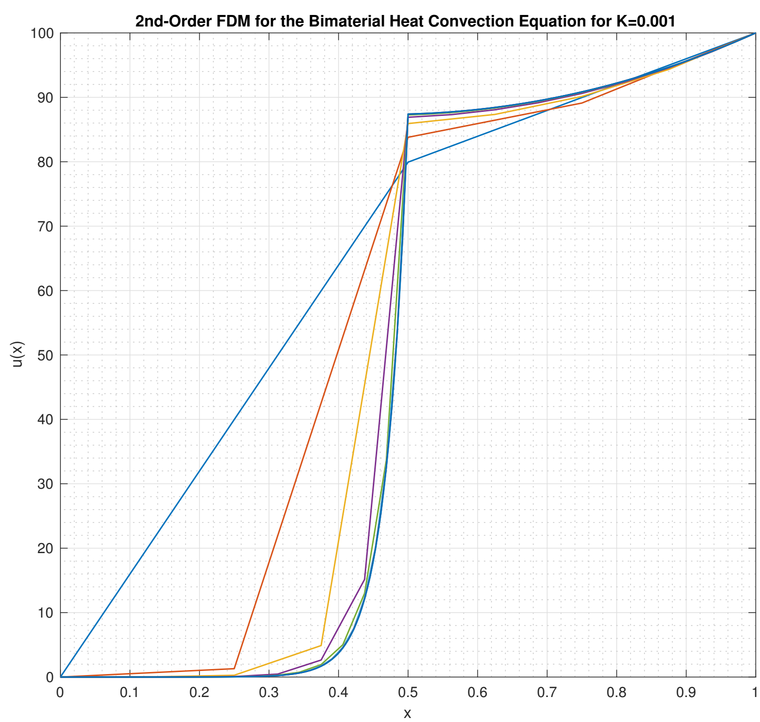


Figure 2 – FDM Solution for the Bimaterial Heat Convection Equation for K = 1E-3

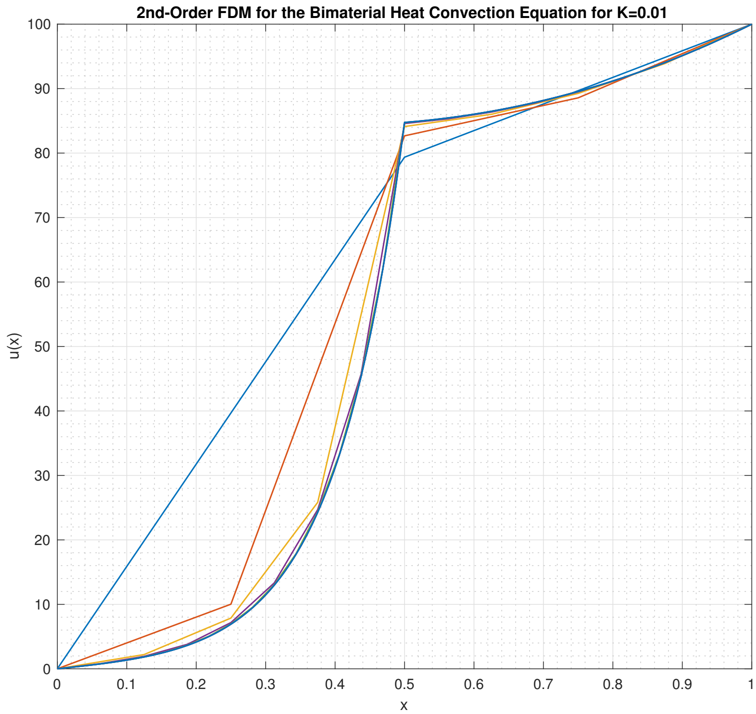


Figure 3 – FDM Solution for the Bimaterial Heat Convection Equation for K = 1E-2

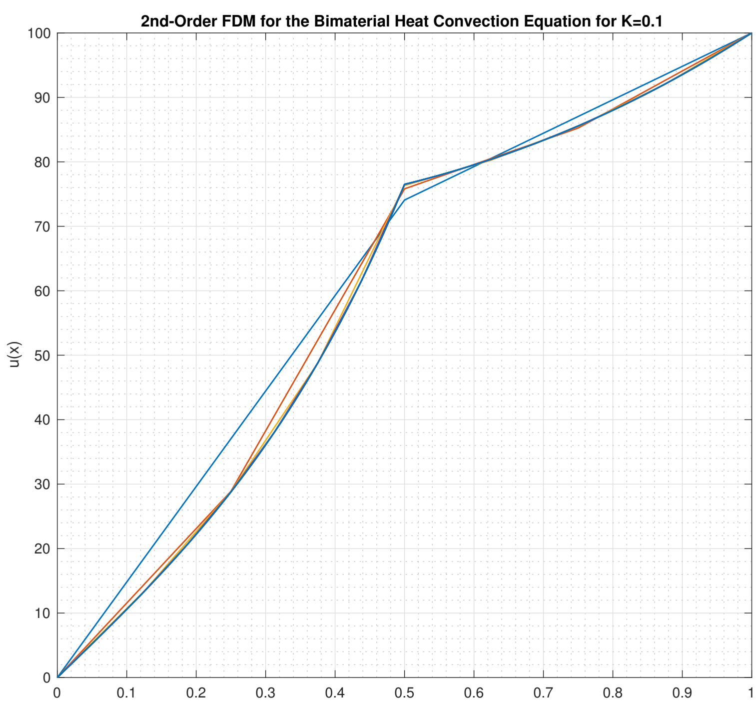


Figure 4 – FDM Solution for the Bimaterial Heat Convection Equation for K = 1E-1

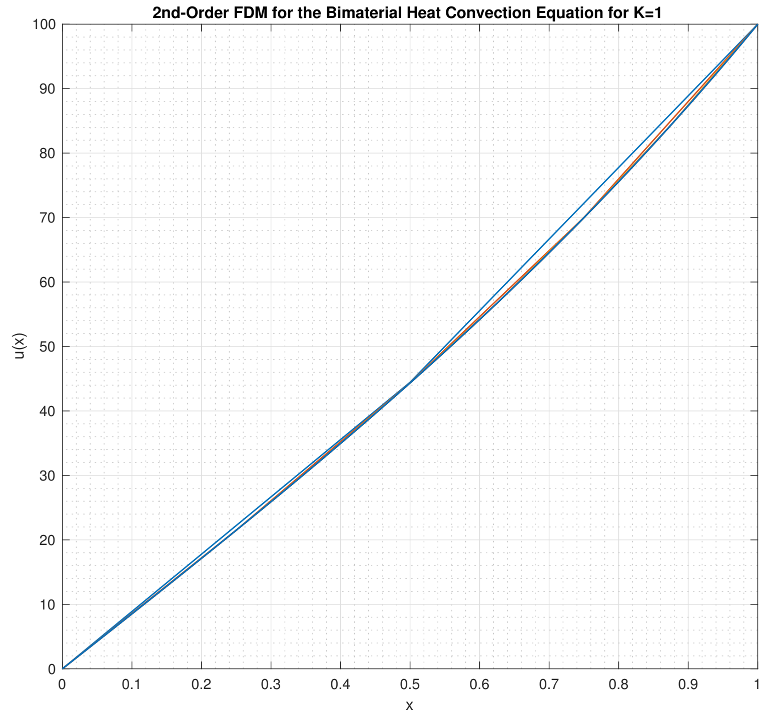


Figure 5 – FDM Solution for the Bimaterial Heat Convection Equation for K = 1E+0

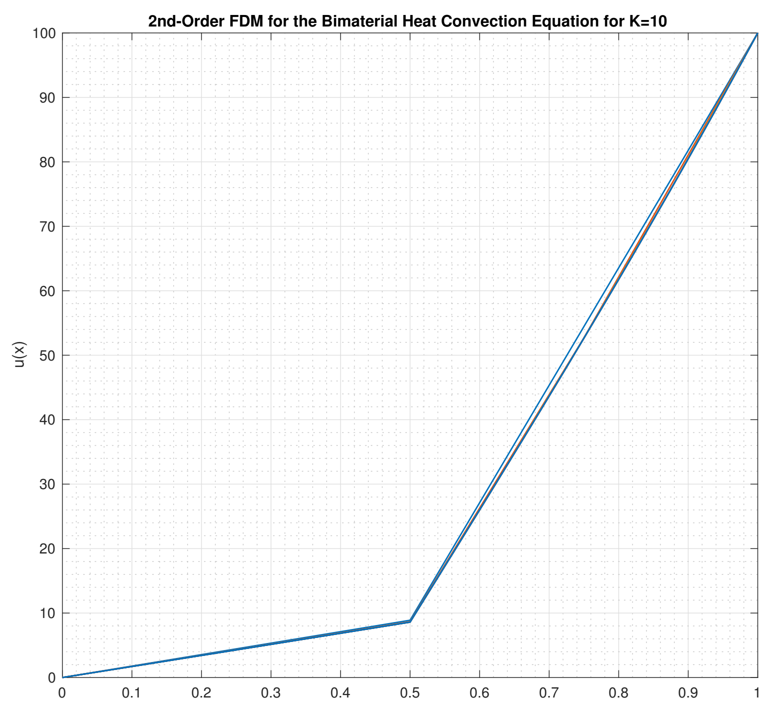


Figure 6 – FDM Solution for the Bimaterial Heat Convection Equation for K = 1E+1

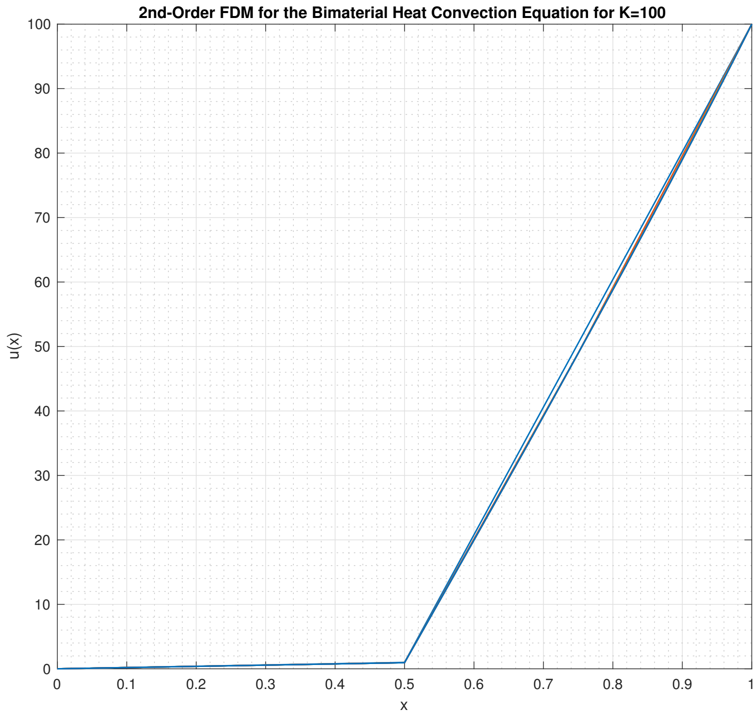


Figure 7 – FDM Solution for the Bimaterial Heat Convection Equation for K = 1E+2

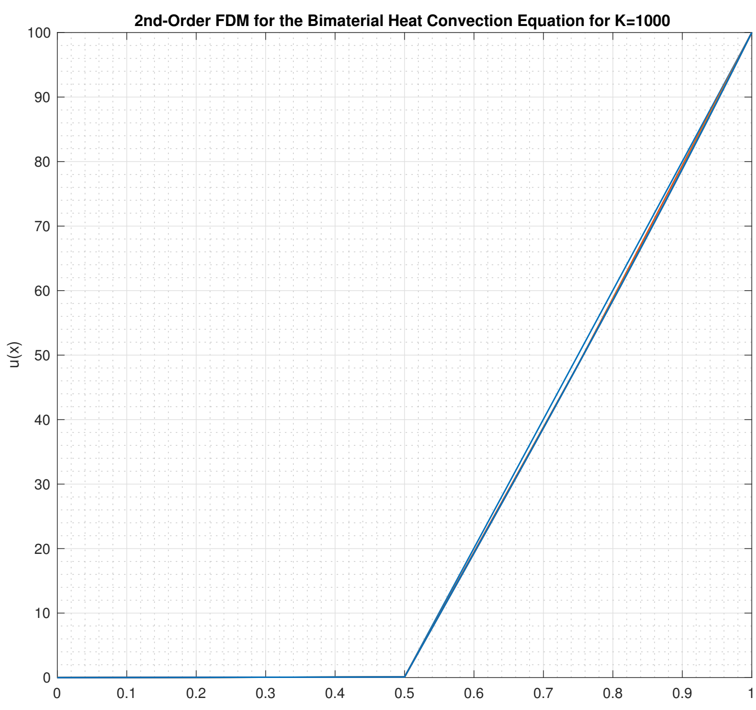


Figure 8 – FDM Solution for the Bimaterial Heat Convection Equation for K = 1E+3

Rate of Convergence

Error plots and rate of convergence values are depicted in Figures 9 and 10 and tabulated below for the midpoint temperature and the midpoint heat flux. Rate of convergence (β) values are calculated using a first-order forward difference approximation of the first-derivative of the plot.

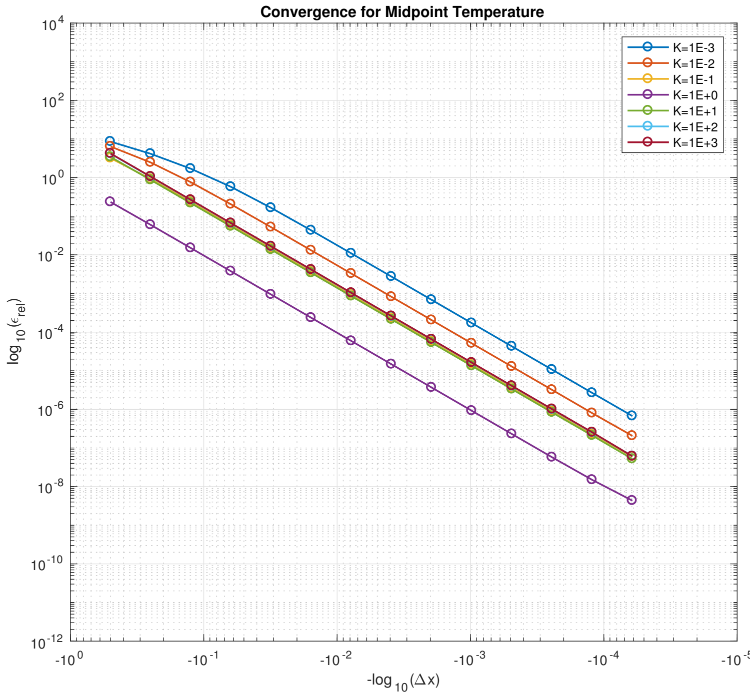


Figure 9 – FDM Convergence of the Midpoint Temperature

Δx	$\beta(K=0.001)$	$\beta(K=0.01)$	$\beta(K=0.1)$	$\beta(K=1)$	$\beta(K=10)$	$\beta(K=100)$	$\beta(K=1000)$
0.5000	1.0476	1.3708	1.7958	1.9670	1.9828	1.9906	1.9915
0.2500	1.2841	1.6997	1.9408	1.9916	1.9956	1.9976	1.9978
0.1250	1.5586	1.8965	1.9845	1.9979	1.9989	1.9994	1.9995
0.0625	1.8063	1.9711	1.9961	1.9995	1.9997	1.9998	1.9999
0.0312	1.9382	1.9925	1.9990	1.9999	1.9999	2.0000	2.0000
0.0156	1.9833	1.9981	1.9998	2.0000	2.0000	2.0000	2.0000
0.0078	1.9957	1.9995	1.9999	2.0000	2.0000	2.0000	2.0000
0.0039	1.9989	1.9999	2.0000	2.0000	2.0000	2.0000	2.0000
0.0020	1.9997	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
0.0010	1.9999	2.0000	2.0001	2.0000	2.0000	2.0000	2.0000
0.0005	2.0000	2.0001	1.9998	1.9980	1.9994	2.0003	1.9998
0.0002	2.0009	2.0016	1.9954	1.9438	1.9891	2.0001	1.9922
0.0001	1.9874	1.9525	1.9618	1.7850	2.0091	2.0208	2.0599
0.0001	1.9000	2.0463	0.1843	-2.8450	1.0282	1.8477	2.2216

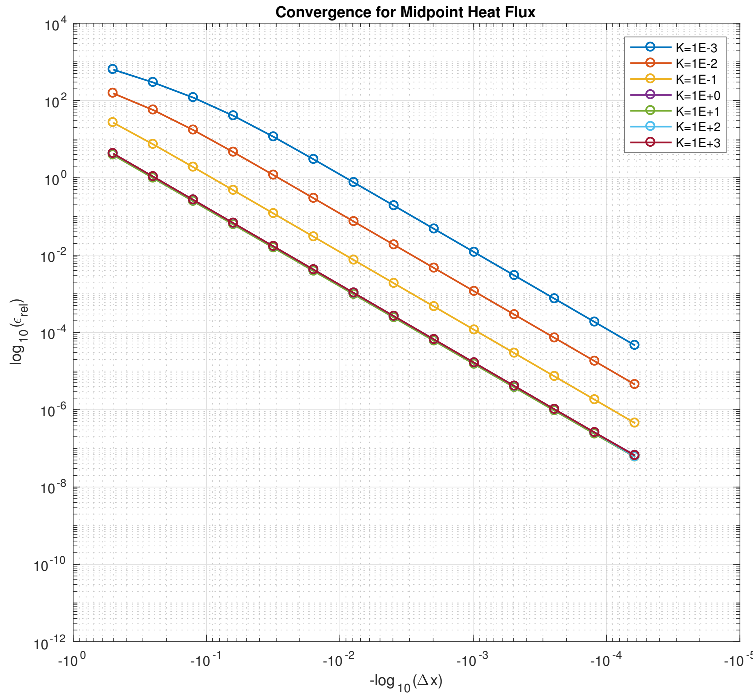


Figure 10 – FDM Convergence of the Midpoint Heat Flux

Δx	$\beta(K = 0.001)$	$\beta(K = 0.01)$	$\beta(K = 0.1)$	$\beta(K = 1)$	$\beta(K = 10)$	$\beta(K = 100)$	$\beta(K = 1000)$
0.5000	1.1122	1.4430	1.8705	1.9916	1.9885	1.9912	1.9916
0.2500	1.3048	1.7216	1.9608	1.9979	1.9971	1.9977	1.9978
0.1250	1.5655	1.9025	1.9896	1.9995	1.9993	1.9994	1.9995
0.0625	1.8084	1.9726	1.9974	1.9999	1.9998	1.9999	1.9999
0.0312	1.9388	1.9929	1.9993	2.0000	2.0000	2.0000	2.0000
0.0156	1.9835	1.9982	1.9998	2.0000	2.0000	2.0000	2.0000
0.0078	1.9958	1.9996	2.0000	2.0000	2.0000	2.0000	2.0000
0.0039	1.9989	1.9999	2.0000	2.0000	2.0000	2.0000	2.0000
0.0020	1.9997	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
0.0010	1.9999	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
0.0005	2.0000	2.0000	2.0000	2.0005	2.0000	2.0000	1.9999
0.0002	2.0000	1.9999	2.0005	1.9916	1.9955	2.0017	1.9937
0.0001	2.0001	2.0020	2.0061	1.9506	1.9732	2.0636	1.9811
0.0001	2.0006	1.9956	2.6006	2.5800	1.9122	1.9256	2.0921

Conclusion

Using the second-order central difference scheme and a second-order extraction scheme for the midpoint heat flux, it was shown that the midpoint temperature and the midpoint heat flux demonstrate **quadratic convergence** ($\beta = 2$). In general, the stiffness variable K of the problem determines the overall conditioning of the interior matrix. That is, for values of K logarithmically far from unity, the value results in poor matrix conditioning which means that it takes a finer mesh to resolve the same amount of error. This is observed in Figure 9, where $K = 1$ has less relative error for similar mesh sizes compared to all other values of K.

aero_430_hw1_analytical.m

```
clear all; close all; clc

set(0,'DefaultFigureWindowStyle','docked')

figure
cmap = colormap(hot);
cIndex = 1;

for K = [1E-3 1E-2 1E-1 1E0 1E1 1E2 1E3]

b = 1;
k2 = 1;
k1 = K*k2;
Ta = 0;
A = 1;

a1 = sqrt(b/k1);
a2 = sqrt(b/k2);

C = [cosh(0) sinh(0) 0 0;
0 0 cosh(a2) sinh(a2);
cosh(a1/2) sinh(a1/2) -cosh(a2/2) -sinh(a2/2);
k1*a1*sinh(a1/2) k1*a1*cosh(a1/2) -k2*a2*sinh(a2/2) -k2*a2*cosh(a2/2)];

d = [-Ta
100-Ta;
0;
0];

aSolCoeff = C\d;


hold on;    box on;
grid on;    grid minor;
x1 = linspace(0.0, 0.5, 100);
x2 = linspace(0.5, 1.0, 100);
s1 = aSolCoeff(1)*cosh(a1*x1) + aSolCoeff(2)*sinh(a1*x1) + Ta;
s2 = aSolCoeff(3)*cosh(a2*x2) + aSolCoeff(4)*sinh(a2*x2) + Ta;
plot([x1 x2], [s1 s2],  'linewidth', 2, 'color', cmap(cIndex, :));
xlim([0 1]);    ylim([0 100]);
title('Analytical Temperature Profile')
xlabel('Distance');  ylabel('Temperature')

cIndex = cIndex + 9;

end

legend('K = 1E-3', 'K = 1E-2', 'K = 1E-1', 'K = 1E+0', 'K = 1E+1', 'K = 1E+2', 'K = 1E+3', ...
'location', 'best')
saveas(gcf, '430_hw1_analytical_temperature', 'epsc')

figure
cmap = colormap(hot);
cIndex = 1;

for K = [1E-3 1E-2 1E-1 1E0 1E1 1E2 1E3]

b = 1;
k2 = 1;
k1 = K;
Ta = 0;
A = 1;

a1 = sqrt(b/k1);
```

```
a2 = sqrt(b/k2);

C = [cosh(0) sinh(0) 0 0;
0 0 cosh(a2) sinh(a2);
cosh(a1/2) sinh(a1/2) -cosh(a2/2) -sinh(a2/2);
k1*a1*sinh(a1/2) k1*a1*cosh(a1/2) -k2*a2*sinh(a2/2) -k2*a2*cosh(a2/2)];

d = [-Ta
100-Ta;
0;
0];

aSolCoeff = C\d;

hold on;    box on;
grid on;    grid minor;
x1 = linspace(0.0, 0.5, 100);
x2 = linspace(0.5, 1.0, 100);
s1 = -k1*A*(aSolCoeff(1)*a1*sinh(a1*x1) + aSolCoeff(2)*a1*cosh(a1*x1));
s2 = -k2*A*(aSolCoeff(3)*a2*sinh(a2*x2) + aSolCoeff(4)*a2*cosh(a2*x2));
plot([x1 x2], [s1 s2],  'linewidth', 2, 'color', cmap(cIndex, :));
xlim([0 1]);    ylim([-inf 0]);
title('Analytical Heat Flux Profile')
xlabel('Distance');  ylabel('Heat Flux')

cIndex = cIndex + 9;

end

legend('K = 1E-3', 'K = 1E-2', 'K = 1E-1', 'K = 1E+0', 'K = 1E+1', 'K = 1E+2', 'K = 1E+3', ...
'location', 'best')
saveas(gcf, '430_hw1_analytical_heat_flux', 'epsc')
```

aero_430_hw1_fdm.m

```
% Ross Alexander
% 2.10.18

clc; close all; clear all;

meshOrder   = 1:15;
meshDx      = 0.5.^meshOrder;

rowID = 0;

plotGen         = true;
plotSave        = true;
tableSaveRoc    = true;

%% 2nd-Order FDM

for K = [1E-3 1E-2 1E-1 1E+0 1E+1 1E+2 1E+3]

% Enumerate constants
b = 1;
k2 = 1;
k1 = K*k2;

rowID = rowID + 1;
colID = 0;

if plotGen

xlabel('x');    ylabel('u(x)');
grid on;        grid minor;
box on;         hold on;
set(gcf, 'Position', [1 1 624 550])

titleString = strcat('2nd-Order FDM for the Bimaterial Heat Convection Equation for K=', ...
num2str(K));
title(titleString)

end

for dx = meshDx

% Develop mesh-specific values for FDM calculations
nxi = 1 / dx - 1;               % number of interior nodes
nxt = 1 / dx + 1;               % number of total nodes (includes boundary nodes)
xi = linspace(dx, 1-dx, nxi);   % x-values at interior nodes
xt = linspace(0, 1, nxt);       % x-values at total nodes
f = zeros(nxi, 1);              % interior load vector
colID = colID + 1;

nxiHalf = (nxi+1)/2;            % index for central interior node (x = 1/2) rel. to interior
nxtHalf = (nxt+1)/2;            % index for central interior node (x = 1/2) rel. to total

% Enumerate upper matrix and lower matrix constants
alpha1 = -k1;                       alpha2 = -k2;           % subdiagonal values
beta1  = b*dx^2+2*k1;               beta2  = b*dx^2+2*k2;   % diagonal values
gamma1 = -k1;                       gamma2 = -k2;           % superdiagonal values

if dx ~= 1/2

% Construct upper matrix and lower matrix
A1 = gallery('tridiag', nxiHalf, alpha1, beta1, gamma1);    % upper matrix for k1
A2 = gallery('tridiag', nxiHalf, alpha2, beta2, gamma2);    % lower matrix for k2

% Construct central vector for node at x = 1/2 for k1 and k2
```

```
a = zeros(1, nxi);
a(nxiHalf - 1) = -k1;
a(nxiHalf)     = b*dx^2+k1+k2;
a(nxiHalf + 1) = -k2;

% Assemble entire interior matrix by combining upper, central, and lower matrices
A = [A1(1:end-1, :) zeros(nxiHalf-1, nxiHalf-1);
a;
zeros(nxiHalf-1, nxiHalf-1) A2(2:end, :)];

% Enumerate load values at boundary nodes
f(1) = 0;       f(nxi) = 100*k2;

else

% If dx = 1/2 the solution is trivial
A = b*dx^2+k1+k2;
f = k2*100;

end

% Solve for u and append values at boundary nodes
ui = A\f;
ut = [0; ui; 100];

if plotGen && dx >= meshDx(8)
plot(xt, ut, 'linewidth', 1)
end

%% Quantities of Interest

% Develop analytical solution and corresponding coefficients
a1 = sqrt(b/k1);
a2 = sqrt(b/k2);

C = [cosh(0) sinh(0) 0 0;
0 0 cosh(a2) sinh(a2);
cosh(a1/2) sinh(a1/2) -cosh(a2/2) -sinh(a2/2);
k1*a1*sinh(a1/2) k1*a1*cosh(a1/2) -k2*a2*sinh(a2/2) -k2*a2*cosh(a2/2)];

d = [0
100;
0;
0];

aSolCoeff = C\d;

% Calculate midpoint temperature and heat flux from analytical and FDM solutions
u12.exact(rowID, colID) = aSolCoeff(1)*cosh(a1/2) + aSolCoeff(2)*sinh(a1/2);
u12.fdm(rowID, colID)   = ut(nxtHalf);
Q12.exact(rowID, colID) = -k1*(aSolCoeff(1)*a1*sinh(a1/2) + aSolCoeff(2)*a1*cosh(a1/2));
Q12.fdm(rowID, colID)   = -k1*(ut(nxtHalf)-ut(nxtHalf-1))/dx - b*dx/2*ut(nxtHalf);          % Q-
%         Q12.fdm(rowID, colID)   = -k2*(ut(nxtHalf+1)-ut(nxtHalf))/dx + b*dx/2*ut(nxtHalf); % Q+

end

if plotGen

%         legend('\Deltax = (1/2)^1', '\Deltax = (1/2)^2', '\Deltax = (1/2)^3', ...
%             '\Deltax = (1/2)^4', '\Deltax = (1/2)^5', '\Deltax = (1/2)^6', ...
%             '\Deltax = (1/2)^7', '\Deltax = (1/2)^8', ...
%             'location', 'eastoutside')

drawnow

if plotSave

figureString = strcat('solution_K_', num2str(rowID));
```

```
saveas(gcf, figureString, 'epsc')

close gcf

end

end

end

%% Convergence Plotting

% Calculate relative error
relErroru12 = abs(u12.exact-u12.fdm) ./ abs(u12.exact) * 100;
relErrorQ12 = abs(Q12.exact-Q12.fdm) ./ abs(Q12.exact) * 100;

if plotGen

%%% Midpoint Temperature

fig2 = figure(2);
xlabel('-log_{10}(\Deltax)');   ylabel('log_{10}(\epsilon_{rel})');
grid on;                        grid minor;
box on;                         hold on;
ylim([10^-12 10^4])
set(gcf, 'Position', [1 1 624 550])

% Plot log-log relative error in midpoint temperature v. -mesh size
for KID = 1:7
loglog(-meshDx, relErroru12(KID, :), '-o', 'linewidth', 1.25);
end

titleString = strcat('Convergence for Midpoint Temperature');
title(titleString)

legend('K=1E-3', 'K=1E-2', 'K=1E-1', 'K=1E+0', 'K=1E+1', 'K=1E+2', 'K=1E+3')

set(gca, 'XScale', 'log');  set(gca, 'YScale', 'log');
drawnow

%%% Midpoint Heat Flux

fig3 = figure(3);
xlabel('-log_{10}(\Deltax)');   ylabel('log_{10}(\epsilon_{rel})');
grid on;                        grid minor;
box on;                         hold on;
ylim([10^-12 10^4])
set(gcf, 'Position', [1 1 624 550])

% Plot log-log relative error in midpoint heat flux v. -mesh size
for KID = 1:7
loglog(-meshDx, relErrorQ12(KID, :), '-o', 'linewidth', 1.25);
end

titleString = strcat('Convergence for Midpoint Heat Flux');
title(titleString)

legend('K=1E-3', 'K=1E-2', 'K=1E-1', 'K=1E+0', 'K=1E+1', 'K=1E+2', 'K=1E+3')

set(gca, 'XScale', 'log');  set(gca, 'YScale', 'log');
drawnow

if plotSave

figure(2)
saveas(gcf, 'convergence_u12', 'epsc')

figure(3)
```

```
saveas(gcf, 'convergence_Q12', 'epsc')

end

end

%% Rate of Convergence Determination

logRelErroru12 = log10(relErroru12);

for KID = 1:7

for rocID = 1:length(logRelErroru12) - 1
rocu12(KID, rocID) = (logRelErroru12(KID, rocID+1) - logRelErroru12(KID, rocID)) / -log10(2);
end

end

if tableSaveRoc

colLabelsRoc = {'$\Delta x$', '$\beta(K=0.001)$', '$\beta(K=0.01)$', '$\beta(K=0.1)$', ...
'$\beta(K=1)$', '$\beta(K=10)$', '$\beta(K=100)$', '$\beta(K=1000)$'};
matrix2latex([meshDx(1:length(meshOrder)-1)' rocu12'], 'roc_u12.tex', ...
'columnLabels', colLabelsRoc, 'alignment', 'c', 'format', '%5.4f')

end

logRelErrorQ12 = log10(relErrorQ12);

for KID = 1:7

for rocID = 1:length(logRelErrorQ12) - 1
rocQ12(KID, rocID) = (logRelErrorQ12(KID, rocID+1) - logRelErrorQ12(KID, rocID)) / -log10(2);
end

end

if tableSaveRoc

colLabelsRoc = {'$\Delta x$', '$\beta(K=0.001)$', '$\beta(K=0.01)$', '$\beta(K=0.1)$', ...
'$\beta(K=1)$', '$\beta(K=10)$', '$\beta(K=100)$', '$\beta(K=1000)$'};
matrix2latex([meshDx(1:length(meshOrder)-1)' rocQ12'], 'roc_Q12.tex', ...
'columnLabels', colLabelsRoc, 'alignment', 'c', 'format', '%5.4f')

end
```

matrix2latex.m

```
function matrix2latex(matrix, filename, varargin)

% function: matrix2latex(...)
% Author:   M. Koehler
% Contact:  koehler@in.tum.de
% Version:  1.1
% Date:     May 09, 2004

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% foundation. For further reading see: http://www.gnu.org/licenses/licenses.html#GPL

% Usage:
% matrix2late(matrix, filename, varargs)
% where
%   - matrix is a 2 dimensional numerical or cell array
```

```
%   - filename is a valid filename, in which the resulting latex code will
%   be stored
%   - varargs is one ore more of the following (denominator, value) combinations
%      + 'rowLabels', array -> Can be used to label the rows of the
%      resulting latex table
%      + 'columnLabels', array -> Can be used to label the columns of the
%      resulting latex table
%      + 'alignment', 'value' -> Can be used to specify the alginment of
%      the table within the latex document. Valid arguments are: 'l', 'c',
%      and 'r' for left, center, and right, respectively
%      + 'format', 'value' -> Can be used to format the input data. 'value'
%      has to be a valid format string, similar to the ones used in
%      fprintf('format', value);
%      + 'size', 'value' -> One of latex' recognized font-sizes, e.g. tiny,
%      HUGE, Large, large, LARGE, etc.
%
% Example input:
%   matrix = [1.5 1.764; 3.523 0.2];
%   rowLabels = {'row 1', 'row 2'};
%   columnLabels = {'col 1', 'col 2'};
%   matrix2latex(matrix, 'out.tex', 'rowLabels', rowLabels, 'columnLabels', ...
%       columnLabels, 'alignment', 'c', 'format', '%-6.2f', 'size', 'tiny');
%
% The resulting latex file can be included into any latex document by:
% /input{out.tex}
%
% Enjoy life!!!

rowLabels = [];
colLabels = [];
alignment = 'l';
format = [];
textsize = [];
if (rem(nargin,2) == 1 || nargin < 2)
error('matrix2latex: ', 'Incorrect number of arguments to %s.', mfilename);
end

okargs = {'rowlabels','columnlabels', 'alignment', 'format', 'size'};
for j=1:2:(nargin-2)
pname = varargin{j};
pval = varargin{j+1};
k = strmatch(lower(pname), okargs);
if isempty(k)
error('matrix2latex: ', 'Unknown parameter name: %s.', pname);
elseif length(k)>1
error('matrix2latex: ', 'Ambiguous parameter name: %s.', pname);
else
switch(k)
case 1  % rowlabels
rowLabels = pval;
if isnumeric(rowLabels)
rowLabels = cellstr(num2str(rowLabels(:)));
end
case 2  % column labels
colLabels = pval;
if isnumeric(colLabels)
colLabels = cellstr(num2str(colLabels(:)));
end
case 3  % alignment
alignment = lower(pval);
if alignment == 'right'
alignment = 'r';
end
if alignment == 'left'
alignment = 'l';
end
if alignment == 'center'
alignment = 'c';
```

```
end
if alignment ~= 'l' && alignment ~= 'c' && alignment ~= 'r'
alignment = 'l';
warning('matrix2latex: ', 'Unkown alignment. (Set it to \''left\''.)');
end
case 4  % format
format = lower(pval);
case 5  % format
textsize = pval;
end
end
end

fid = fopen(filename, 'w');

width = size(matrix, 2);
height = size(matrix, 1);

if isnumeric(matrix)
matrix = num2cell(matrix);
for h=1:height
for w=1:width
if(~isempty(format))
matrix{h, w} = num2str(matrix{h, w}, format);
else
matrix{h, w} = num2str(matrix{h, w});
end
end
end
end

if(~isempty(textsize))
fprintf(fid, '\\begin{%s}', textsize);
end

fprintf(fid, '\\begin{tabular}{|');

if(~isempty(rowLabels))
fprintf(fid, 'l|');
end
for i=1:width
fprintf(fid, '%c|', alignment);
end
fprintf(fid, '}\r\n');

fprintf(fid, '\\hline\r\n');

if(~isempty(colLabels))
if(~isempty(rowLabels))
fprintf(fid, '&');
end
for w=1:width-1
fprintf(fid, '\\textbf{%s}&', colLabels{w});
end
fprintf(fid, '\\textbf{%s}\\\\\\hline\r\n', colLabels{width});
end

for h=1:height
if(~isempty(rowLabels))
fprintf(fid, '\\textbf{%s}&', rowLabels{h});
end
for w=1:width-1
fprintf(fid, '%s&', matrix{h, w});
end
fprintf(fid, '%s\\\\\\hline\r\n', matrix{h, width});
end

fprintf(fid, '\\end{tabular}\r\n');
```

```
if(~isempty(textsize))
fprintf(fid, '\\end{%s}', textsize);
end

fclose(fid);
```