Convergence Analysis of 2nd & 4th Order Finite Difference Methods for Axisymmetric Heat Conduction Problems [FIRST PROJECT]

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October 27th, 2015

Aggie Code of Honor

My signature below serves as confirmation of my adherence to the Aggie Code of Honor –
"An Aggie does not lie, cheat or steal or tolerate those who do."

Sepehr Seyedi

AERO 430 – 500

Texas A&M University – College Station

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I. INTRODUCTION

In this report, we utilize 2nd and 4th order finite difference methods (FDM) to determine their effectiveness in approximating the solutions to axisymmetric heat conduction problems. The heat conduction problems in question are taken from the required textbook for this course[1], namely problems 115, 116, and 117 at the end of chapter 8. The primary objective of this pursuit is to determine the derivative of the temperature at the outer boundary of the cylindrical pipes of interest for these problems. In order to complete this objective, we first solve for the analytical solutions of the temperature profiles for each problem in section II. In section III, we introduce 2nd and 4th order finite difference methods for numerically computing the temperature profiles, as well as the derivatives of the temperature profiles. We discuss methods for extrapolating these values to an approximation of the exact solution as well. In section IV, we present graphically the results of applying the difference methods to each problem, for several different values of the spacing of the numerical grids. Finally, in section V we analyze the convergence of the methods employed to the exact solution, as well as to the extrapolated solutions.

II. FORMULATION OF THE CONTINUOUS MODEL & SOLUTIONS

The three problems of interest all concern the temperature distribution in a radially symmetric pipe. The heat flow, Q, through a pipe can be represented as the heat flux, $q(x)$, multiplied by the total cross sectional area, A, of the pipe. Using Fourier's law of heat conduction we obtain

$$Q(x) = Aq(x) = -AkT'(x)$$

For simplicity, we assume a unit pipe length, such that $A = 2\pi x$. We now set the derivative of the heat equal to a function $f(x)$ representing internal heating sources,

$$Q'(x) = -2\pi k(xT')' = -2\pi k(xT'' + T') = Af(x)$$

Rearranging terms, we obtain the differential equation,

$$T'' + \frac{1}{x}T' = -\frac{1}{k}f(x)$$

In general, the temperature distribution in an axisymmetric pipe satisfies the differential equation

$$\frac{d^2T}{dx^2} + \frac{1}{x}\frac{dT}{dx} = f(x) \tag{1}$$

where T is the temperature and x is the radial distance from the axis of the pipe. The problem statements for each of these problems are shown in Figure 1.

[1] Hoffman, Joe. *Numerical Methods for Engineers and Scientists.* Boca Raton: CRC Press, 2001. p.498. Print.

Problem 115

$$\frac{d^2T}{dx^2} + \frac{1}{x}\frac{dT}{dx} = 0 \qquad 1 < x < 2 \qquad T(1) = 100 \text{ and } T(2) = 0 \tag{I}$$

Problem 116

$$\dot{q}_{cond} = Fourier's\ Law = -kA\frac{dT}{dx} = \dot{q}_{conv} = Robin = hA(T - T_a) \tag{II}$$

$1 < x < 2$ $\qquad T_a = 0.0$ C, h = $500\frac{J}{s-m^2-K}$, k = $100\frac{J}{s-m-K}$, $T(1) = 100,\ T'(2) = -5T(2)$

Problem 117

$$\frac{d^2T}{dx^2} + \frac{1}{x}\frac{dT}{dx} = A\left[1 + \left(\frac{r}{R}\right)^2\right] \tag{III}$$

$0 < x < 1$ $\qquad T'(0) = 0$ and $T(R) = 0$ C $\quad R = 1.0$, $A = -100$

Figure 1: Problem 115, 116, and 117 from Feibus "Conduction and Convection in a 1-D Pipe", 2014, p.2.

Our objective in this section is to determine the analytical solution for the temperature distribution for all three problems.

II.A. Solution of 115

To solve (1) for problem 115, we can rewrite (1) in the form,

$$\frac{d}{dx}\left(x\frac{dT}{dx}\right) = 0 \tag{2.1}$$

Integrating (2),

$$x\frac{dT}{dx} = C_1 \tag{2.2}$$

$$T(x) = C_1 + C_2\ln(x) \tag{2.3}$$

Using the boundary conditions for 115 we obtain,

$$T(x) = 100\left(1 - \frac{\ln(x)}{\ln(2)}\right) \tag{2.4}$$

$$T'(x) = -\frac{100}{x*\ln(2)} \tag{2.5}$$

II.B. Solution of 116

The solution to the problem of 116 has the same general form as (2.3), and the boundary condition $T(1) = 100$ *C* can be used to find that $C_1 = 100$. The heat conduction equation in Figure 1 can be used to obtain C_2. Rearranging (II),

$$\frac{dT}{dx}(2) = -\frac{h}{k}T(2) = -5T(2) \tag{3.1}$$

where we have used the condition of $T_a = 0.0\ C$. We can differentiate (2.3) and substitute it into (3.1) at $x = 2$ to obtain C_2, using the derivative boundary condition,

$$\frac{dT}{dx}(2) = \frac{C_2}{2} = -5[100 + C_2\ln(2)] \tag{3.2}$$

Solving for C_2 we obtain the analytical solution for the temperature distribution,

$$T(x) = 100\left[\frac{1+\ln(1024)-10\ln(x)}{1+\ln(1024)}\right] \tag{3.3}$$

$$T'(x) = -\frac{1}{x}\left[\frac{1000}{1+\ln(1024)}\right] \tag{3.4}$$

II.C. Solution of 117

The solution of problem 117 is the sum of the homogenous solution (2.3) and the inhomogeneous solution for the differential equation in (III) in Figure 1. Equation (III) can be rewritten using $R = 1.0$ and $A = -100$, and integrated twice to obtain the inhomogeneous solution for the temperature distribution T,

$$\frac{d}{dx}\left(x\frac{dT}{dx}\right) = -100x(1 + x^2) \tag{4.1}$$

$$x\frac{dT}{dx} = -100\left(\frac{x^2}{2} + \frac{x^4}{4}\right) \tag{4.2}$$

$$T_{inh}(x) = -100\left(\frac{x^2}{4} + \frac{x^4}{16}\right) \tag{4.3}$$

Combining (2.3) and (4.3) we obtain the full solution for the temperature distribution,

$$T(x) = C_1 + C_2\ln(x) - 100\left(\frac{x^2}{4} + \frac{x^4}{16}\right) \tag{4.4}$$

$$T'(x) = \frac{C_2}{x} - 100\left(\frac{x}{2} + \frac{x^3}{4}\right) \tag{4.5}$$

Applying the boundary condition $T(1) = 1$ to (4.4), we obtain $C_1 = 125/4$. Applying the boundary condition $T'(0) = 0$ to (4.5), we find that C_2 must be zero, which results in the solution,

$$T(x) = \frac{125}{4} - 100\left(\frac{x^2}{4} + \frac{x^4}{16}\right) \tag{4.6}$$

$$T'(x) = -25x(2 + x^2) \tag{4.7}$$

III. FORMULATION OF DIFFERENCE EQUATIONS

In this section we use finite difference formulas in place of the derivatives in the relevant differential equation. This gives us an approximation of the temperature distribution at discrete points, where the acquired values of the temperature will depend on the finite difference formulas used. We will first use 2nd order difference formulas, for which the truncation error will be of the order of the square of the spacing Δx, and then 4th order difference formulas, for which the truncation error will be of the order of the fourth power of the spacing Δx.

***III.a.* 2^nd Order FDM**

For the 2nd order method, we replace the derivative terms in the expanded form of Equation (1) with finite difference formulas that have a truncation error of the order of Δx^2. For the first derivative, this will be the two-point central difference, and for the second derivative the three-point central difference,

$$T'(x_i) = \frac{T(x_{i+1}) - T(x_{i-1})}{2\Delta x} + H.O.T. \tag{5.1}$$

$$T''(x_i) = \frac{T(x_{i+1}) - 2T(x_i) + T(x_{i-1})}{\Delta x^2} + H.O.T. \tag{5.2}$$

Neglecting the higher order terms and substituting these formulas into (1) we obtain the 2nd order finite difference,

$$x_i \frac{T(x_{i+1}) - 2T(x_i) + T(x_{i-1})}{\Delta x^2} + \frac{T(x_{i+1}) - T(x_{i-1})}{2\Delta x} = x_i f(x_i) \tag{5.3}$$

Rearranging (5.3), we obtain

$$\left(x_i - \frac{\Delta x}{2}\right) T(x_{i-1}) - 2x_i T(x_i) + \left(x_i + \frac{\Delta x}{2}\right) T(x_{i+1}) = x_i \Delta x^2 f(x_i) \tag{5.4}$$

***III.b.* 4^th Order FDM**

For the 4th order method, we use the same difference formulas as in the 2nd order method, except we keep terms of the order of higher order terms, leading to the approximations,

$$T'(x_i) = \frac{T(x_{i+1}) - T(x_{i-1})}{2\Delta x} - \frac{\Delta x^2}{6} T'''(x_i) + \ H.O.T. \tag{5.5}$$

$$T''(x_i) = \frac{T(x_{i+1}) - 2T(x_i) + T(x_{i-1})}{\Delta x^2} - \frac{\Delta x^2}{12} T^{iv}(x_i) + H.O.T. \tag{5.6}$$

In order to use these formulas, we have to derive expressions for T''' and T^{iv} from Equation (1),

$$T'''(x) = -\frac{2}{x} T''(x) + f'(x) + \frac{f(x)}{x} \tag{5.7}$$

$$T^{iv}(x) = -\frac{3}{x} T'''(x) + \frac{2}{x} f'(x) + f''(x) = -\frac{3}{x}\left[-\frac{2}{x} T''(x) + f'(x) + \frac{f(x)}{x}\right] + \frac{2}{x} f'(x) + f''(x) \tag{5.8}$$

Equation (5.8) can be substituted into (5.6) to derive a formula for T'', which can then substituted into (5.7) to determine T''', which can then be substituted into (5.5) to obtain T'. The results of this procedure are,

$$T''(x_i) = \frac{T(x_{i+1}) - 2T(x_i) + T(x_{i-1})}{\alpha \Delta x^2} - \frac{\Delta x^2}{12\alpha}\left(f''(x_i) - 3\frac{f(x_i)}{x_i^2} - \frac{f'(x_i)}{x_i}\right) \tag{5.9}$$

$$T'(x_i) = \frac{T(x_{i+1}) - T(x_{i-1})}{2\Delta x} - \frac{\Delta x^2}{6}\left[-\frac{2}{x_i} T''(x_i) + f'(x_i) + \frac{f(x_i)}{x_i}\right] \tag{5.10}$$

where $\alpha = 1 + \frac{\Delta x^2}{2x_i^2}$. We can now substitute these equations into Equation (1) to obtain a system of equations,

$$\left(\frac{1}{\alpha\Delta x^2} + \frac{1}{3\alpha x_i{}^2} - \frac{1}{2x_i\Delta \mathrm{x}}\right)T_{i-1} - \left(\frac{2}{\alpha\Delta \mathrm{x}^2} + \frac{2}{3\alpha \mathrm{x}_i{}^2}\right)T_i + \left(\frac{1}{\alpha\Delta \mathrm{x}^2} + \frac{1}{3\alpha \mathrm{x}_i{}^2} + \frac{1}{2x_i\Delta \mathrm{x}}\right)T_{i+1}$$
$$= \left(1 + \frac{\Delta \mathrm{x}^2}{6x_i{}^2}\right)f_i + \left(\frac{\Delta \mathrm{x}^2}{6x_i}\right)f_i{}' + \frac{\Delta \mathrm{x}^2}{12\alpha}\left(1 + \frac{\Delta \mathrm{x}^2}{3x_i{}^2}\right)\left(f_i{}'' - \frac{1}{x_i}f_i{}' - \frac{3}{x_i{}^2}f_i\right) \quad (5.11)$$

where f_i, f'_i, and f''_i, are $f(x_i)$, $f'(x_i)$, and $f''(x_i)$, respectively, and T_{i-1}, T_i, and T_{i+1} are $T(x_{i-1})$, $T(x_i)$, and $T(x_{i+1})$, respectively.

III.A. Formulation for 115

III.A.a. ***2nd Order FDM***

For problem 115, $f(x) = 0$, and Equation (5.4) therefore reduces to

$$\left(x_i - \frac{\Delta x}{2}\right)T(x_{i-1}) - 2x_iT(x_i) + \left(x_i + \frac{\Delta x}{2}\right)T(x_{i+1}) = 0 \quad (5.12)$$

This represents a system of equations for the 2nd order finite difference approximation to the temperature distribution. The boundary conditions for this problem give us the temperatures at the boundaries, which enables us to solve this system of equations in matrix form without any additional equations.

III.A.b. ***4th Order FDM***

For the 4th order approximation, we simply set the r.h.s. of (5.11) equal to zero to obtain a tridiagonal system of equations for temperatures between the boundaries. We obtain a matrix equation that enables us to solve for the interior temperatures.

III.B. Formulation for 116

Since we do not have the boundary condition at the outer boundary, we have to introduce an extra equation for the outer boundary using Fourier's Law.

III.B.a. ***2nd Order FDM***

For the 2nd order method we use Equation (5.4) for the interior points, and the temperature value at the inner boundary is given. To derive a relation for the outer boundary, we substitute the 2-point central difference formula (5.1) into (3.1) to obtain a system of equations,

$$T(x_{i-1}) - 10\Delta x T(x_i) - T(x_{i+1}) = 0 \quad (6.1)$$

We then substitute this equation into Equation (5.4) for $T(x_{i+1})$ to obtain the equation for the outer boundary,

$$x_NT(x_{N-1}) - \left[x_N + 5\Delta x\left(x_N + \frac{\Delta x}{2}\right)\right]T(x_N) = 0 \quad (6.2)$$

Equation (5.6) can be combined with the system of equations generated by Equation (5.4) to find the temperature values of the interior points as well as that of the outer boundary.

III.B.b. ***4th Order FDM***

For the 4th order finite difference method, the system of equations for the interior points is defined by Equation (5.11) with the r.h.s. set to zero. To obtain the equation at the outer boundary we use the 4th order difference equation for *T'*, which is (5.10), and substitute into (3.1) using $f(x) = 0$ to obtain,

$$\left(\frac{1}{3\alpha x_N} - \frac{1}{2\Delta x}\right) T(x_{N-1}) + \left(5 - \frac{2}{3\alpha x_N}\right) T(x_N) + \left(\frac{1}{2\Delta x} + \frac{1}{3\alpha x_N}\right) T(x_{N+1}) = 0 \tag{6.3}$$

We then solve this equation for $T(x_{i+1})$ and substitute back into Equation (5.11) to obtain,

$$\left[\frac{1}{\alpha \Delta x^2} + \frac{1}{3\alpha {x_N}^2} - \frac{1}{2x_N \Delta \mathrm{x}} + \frac{1}{\gamma}\left(\frac{1}{2\Delta x} - \frac{1}{3\alpha x_N}\right)\left(\frac{1}{\alpha \Delta \mathrm{x}^2} + \frac{1}{3\alpha \mathrm{x}_N{}^2} + \frac{1}{2x_N \Delta \mathrm{x}}\right)\right] T(x_{N-1}) -$$
$$\left[\frac{2}{\alpha \Delta \mathrm{x}^2} + \frac{2}{3\alpha \mathrm{x}_N{}^2} + \frac{1}{\gamma}\left(5 - \frac{2}{3\alpha x_N}\right)\left(\frac{1}{\alpha \Delta \mathrm{x}^2} + \frac{1}{3\alpha \mathrm{x}_N{}^2} + \frac{1}{2x_N \Delta \mathrm{x}}\right)\right] T(x_N) = 0 \tag{6.4}$$

where $\gamma = \left(\frac{1}{2\Delta x} + \frac{1}{3\alpha x_N}\right)$.

III.C. Formulation for 117

III.C.a. ***2nd Order FDM***

For the 2nd order method, Equation (5.4) is used to find the values of the temperature at the interior points of the selected grid, with the function *f(x)* being,

$$f(x_i) = -100(1 + x_i^2) \tag{7.1}$$

We already have the value of the temperature at the exterior boundary, but we do not have a value for the interior boundary. For the 2nd order method, we use the boundary condition at the interior, $T'(0) = 0$, and equate it with a first order approximation,

$$T'(0) = \frac{T(x_1) - T(0)}{\Delta x} = 0 \quad \xrightarrow{yields} \quad T(0) = T(x_1) \tag{7.2}$$

Therefore, we will equate the temperature at the interior boundary with the temperature value of the adjacent grid point.

III.C.b. ***4th Order FDM***

For the 4th order method, Equation (5.11) is used to find the temperature values at the interior grid points using Equation (7.1) for *f(x)*. For the value of the temperature at the interior boundary, we use the Taylor series expansion at the adjacent grid point,

$$T(x_1) = T(0) + \Delta x T'(0) + \frac{\Delta x^2}{2} T''(0) + \frac{\Delta x^3}{6} T'''(0) + O(\Delta x^4) \tag{7.3}$$

From the boundary conditions we know the second term on the r.h.s. of Equation (7.3) is zero. In order to derive values for the higher order derivatives, we use Equation (1) and differentiate to obtain,

$$xT'''(x) + 2T''(x) = xf'(x) + f(x) \tag{7.4}$$

At the interior boundary, $x = 0$, Equation (7.4) yields the result,

$$T''(0) = \frac{f(0)}{2} \tag{7.5}$$

Differentiating Equation (7.4), we obtain,

$$xT'^{v}(x) + 3T'''(x) = xf''(x) + 2f'(x) \tag{7.6}$$

At $x = 0$, this equation yields the result,

$$T'''(0) = \frac{2f'(0)}{3} = 0 \tag{7.7}$$

where we have used Equation (7.1) to determine the value of $f'(0)$. Substituting the results of Equations (7.1), (7.5) and (7.7) into Equation (7.3), we find,

$$T(0) - T(x_1) = -\frac{\Delta x^2}{4} f(0) = 25\Delta x^2 \tag{7.8}$$

Equation (7.8) can be used in combination with the tridiagonal matrix formed by applying Equation (5.11) for a specific value of the grid spacing Δx to find values for the interior points as well as for the inner boundary.

III.D. Difference Equations for the Derivative at the Outer Boundary

The value of the derivative at the boundary requires a different formulation than for the interior points, as we can only use two-points to determine the derivative.

III.D.a. *4th Order FDM*

We begin with the Taylor series expansion for the temperature at the point adjacent to the boundary,

$$T(x_{N-1}) = T(x_N) - \Delta x T'(x_N) + \frac{\Delta x^2}{2} T''(x_N) - \frac{\Delta x^3}{6} T'''(x_N) + \frac{\Delta x^4}{24} T^{iv}(x_N) + O(\Delta x^5) \tag{8.1}$$

where the last term on the r.h.s. is the truncation error. In order to derive an expression for $T'(x_N)$, we use Equation (1) to substitute for the higher order derivatives,

$$T''(x_N) = f(x_N) - \frac{1}{x_N} T'(x_N) \tag{8.2}$$

$$T'''(x_N) = f'(x_N) + \frac{f(x_N)}{x_N} - \frac{2}{x_N} T''(x_N) = f'(x_N) - \frac{f(x_N)}{x_N} + \frac{2}{x_N^2} T'(x_N) \tag{8.3}$$

$$T^{iv}(x_N) = f''(x_N) + \frac{f'(x_N)}{x_N} - \frac{3}{x_N} T'''(x_N) = f''(x_N) - \frac{f'(x_N)}{x_N} + \frac{3f(x_N)}{x_N^2} - \frac{6}{x_N^3} T'(x_N) \tag{8.4}$$

We substitute these expressions into Equation (8.1), and rearrange the equation to solve for $T'(x_N)$, obtaining,

$$T'(x_N) = \frac{1}{\eta}\left[\frac{T(x_N)-T(x_{N-1})}{\Delta x} + f(x_N)\left(\frac{\Delta x}{2} + \frac{\Delta x^2}{6x_N} + \frac{\Delta x^3}{8x_N^2}\right) - f'(x_N)\left(\frac{\Delta x^2}{6} + \frac{\Delta x^3}{24x_N}\right) + f''(x_N)\frac{\Delta x^3}{24}\right] \quad (8.5)$$

$$\eta = 1 + \frac{\Delta x}{2x_N} + \frac{\Delta x^2}{3x_N^2} + \frac{\Delta x^3}{4x_N^3}$$

where we have omitted the truncation error, which is of the order of Δx^4. Therefore, this approximation for the derivative of the temperature at the boundary is a 4th order approximation.

III.D.b. ***2nd Order FDM***

This approximation can be used to derive a 2nd order approximation, where we ignore all derivatives of higher order than 2. The process is identical, and the result is,

$$T'(x_N) = \frac{1}{1+(\Delta x/2x_N)}\left[\frac{T(x_N)-T(x_{N-1})}{\Delta x} + f(x_N)\frac{\Delta x}{2}\right] + O(\Delta x^2) \quad (8.6)$$

where the last term on the r.h.s. is the truncation error that is of the desired order.

III.E. Extrapolation Methods

In this subsection we assume the analytical solution to the three problems are not known, and proceed to analyze the convergence of the finite difference methods outlined in subsections III.A. and III.B. To this end, we utilize two commonly used extrapolation methods.

III.E.a. ***Richardson's Extrapolation***

For this method, three distinct values for the quantity of interest, Q, are required, with the spacing sizes for these values being defined as,

$$\Delta x_a = \Delta x \quad ; \quad \Delta x_b = \frac{\Delta x}{2} \quad ; \quad \Delta x_c = \frac{\Delta x}{4}$$

For each of these spacing sizes, the extrapolated value, Q_{EX}, will be equal to the calculated value, Q_i, with a truncation error of the order of Δx^{β}, where β is the convergence rate of the approximation method (e.g. 2nd order finite difference $\beta = 2$). Therefore we can write three equations for the extrapolated value of Q_{EX},

$$Q_{EX} = Q_{\Delta x} + C\Delta x^{\beta} \quad (9.1)$$

$$Q_{EX} = Q_{\Delta x/2} + C\frac{\Delta x^{\beta}}{2^{\beta}} \quad (9.2)$$

$$Q_{EX} = Q_{\Delta x/4} + C\frac{\Delta x^{\beta}}{4^{\beta}} \quad (9.3)$$

where C is a constant. We can move the approximated values to the l.h.s. and divide Equation (9.1) by Equation (9.2) to obtain 2^{β}. We can then do the same division with Equation (9.2) and Equation (9.3) to obtain 2^{β} as well,

$$\frac{Q_{EX}-Q_{\Delta x}}{Q_{EX}-Q_{\Delta x/2}} = 2^{\beta} = \frac{Q_{EX}-Q_{\Delta x/2}}{Q_{EX}-Q_{\Delta x/4}} \tag{9.4}$$

We can now solve for QEX in terms of the three approximated values,

$$Q_{EX} = \frac{Q_{\Delta x/2}^2 - Q_{\Delta x}Q_{\Delta x/4}}{2Q_{\Delta x/2}-Q_{\Delta x}-Q_{\Delta x/4}} \tag{9.5}$$

Note that Equation (9.5) does not depend on the convergence rate explicitly. Also of note is that the quantity of values that are extrapolated is limited by the largest grid size, Δx.

III.E.b. ***Standard Extrapolation***

In the standard approach to extrapolation, we only use two approximated values with the ratio of their spacing being two, as with Richardson's extrapolation. Therefore, Equations (9.1) and (9.2) are applicable for this approach. We multiply Equation (9.2) by 2^{β}, and subtract Equation (9.1) from this new equation to eliminate the Δx terms. The result of this procedure is the following result for the extrapolated quantity, Equation (9.6),

$$Q_{EX} = \frac{2^{\beta}Q_{\Delta x/2}-Q_{\Delta x}}{2^{\beta}-1} \tag{9.6}$$

Equation (9.6) can be used for both the 2nd and 4th order finite difference methods, for which the convergence rate, β, will be two and four, respectively. The explicit equations for the 2nd and 4th order methods are Equation (9.7) and Equation (9.8), respectively,

$$Q_{EX} = \frac{4Q_{\Delta x/2}-Q_{\Delta x}}{3} \tag{9.7}$$

$$Q_{EX} = \frac{16Q_{\Delta x/2}-Q_{\Delta x}}{15} \tag{9.8}$$

IV. SOLUTIONS TO THE DIFFERENCE EQUATIONS

In this section we determine solutions to Equation (5.4) and Equation (5.11) for the cases outlined in problems 115, 116 and 117. All computations and plotting were carried out in the MATLAB® software environment. The symbol *h* is used to designate the spacing size Δx in plots where it is used on the horizontal axis label, and "log" refers to the natural logarithm whereas "log10" refers to the base 10 logarithm.

Derivatives for all interior points were calculated using a two-point central difference for the 2nd order method, Equation (5.5), and the compact three-point difference for the 4th order method, Equations (5.9) and (5.10).

Spacing sizes of $\Delta x = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{1}{11}, \frac{1}{12}, \frac{1}{13}, \frac{1}{14}, \frac{1}{15}, \frac{1}{16}$ correspond to interval quantities N = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, respectively.

IV.A. FDM Solutions to 115

The temperature distribution calculated using Equation (5.12) for P115 is shown in Figure 2. MATLAB code files 'P115.m' and 'Order2FDM.m' were used to generate the plot.

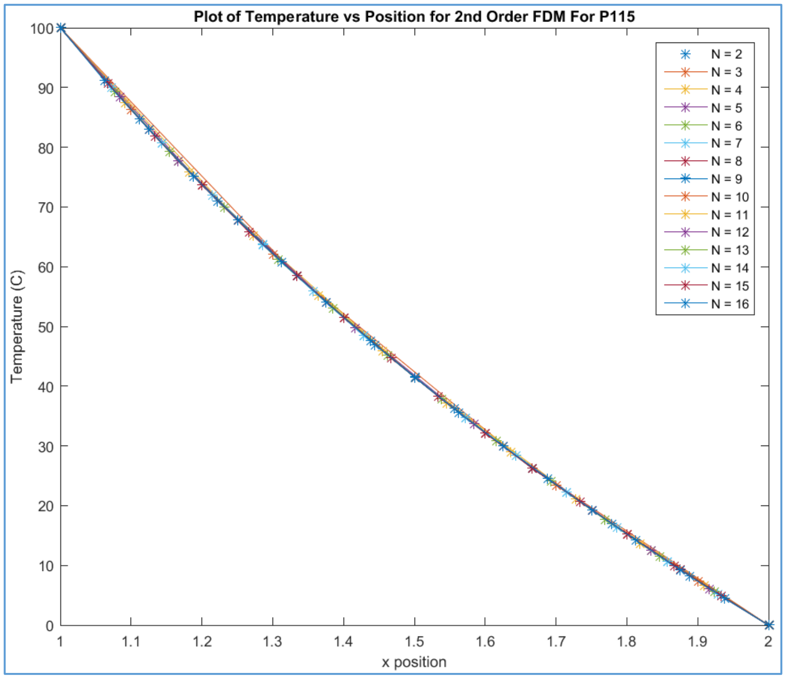

Figure 2: Plot of Temperature for P115 using 2nd Order FDM.

Figure 3 displays a plot of the first derivative of the temperature, generated using the same files.

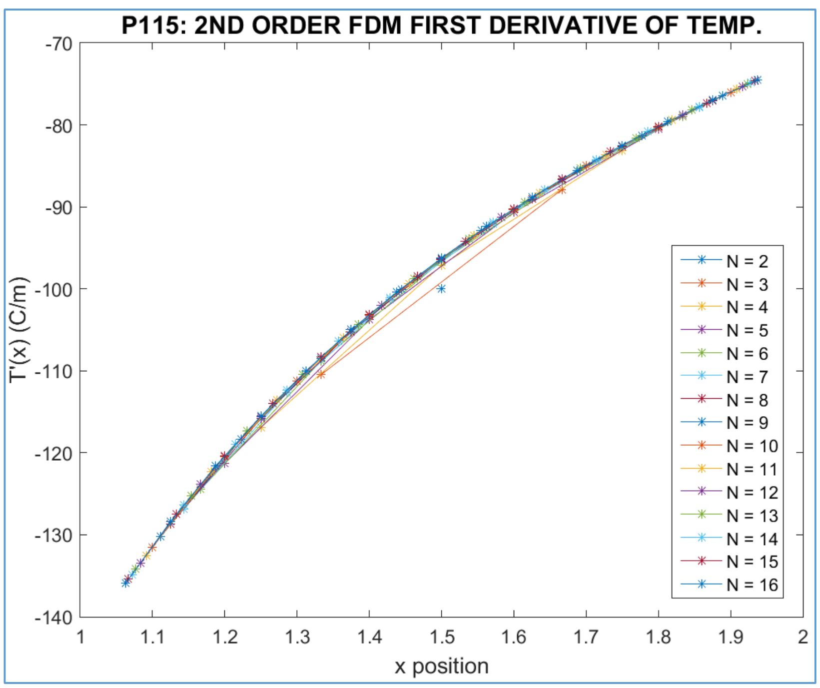

Figure 3: Plot of Derivative of Temperature for P115 using 2nd Order FDM.

Figure 4 shows the relative error values of the first derivative of the temperature for each spacing size, in log-log format, for the 2nd order FDM using the same code files. The error clearly decreases with decreasing spacing size.

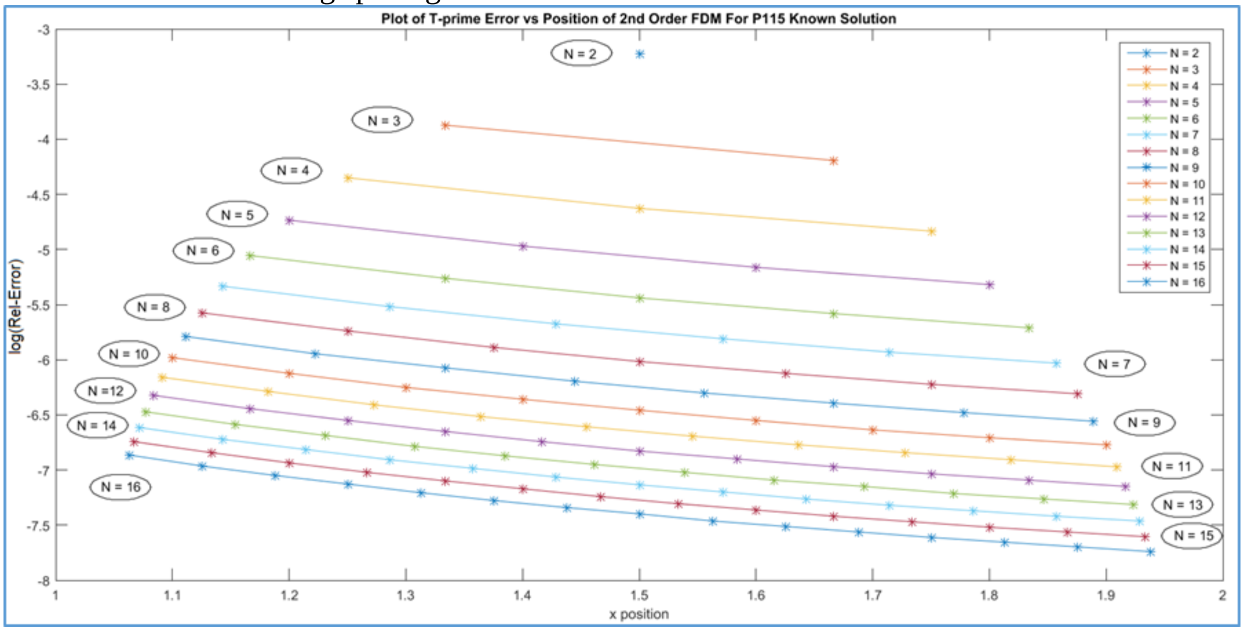

Figure 4: Plot of Relative Error for the First Derivative of the Temperature for P115 - 2nd Order FDM.

Figures 5, 6, and 7 were generated using files ‘P115.m’ and ‘Order4FDM.m’.

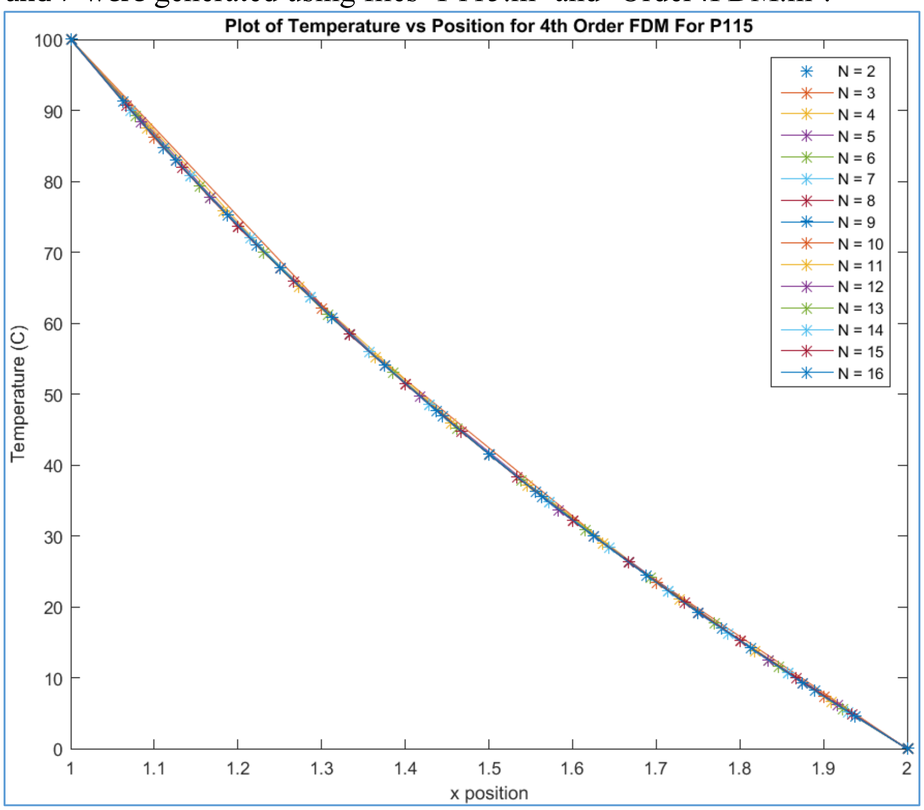

Figure 5: Plot of Temperature for P115 using 4th Order FDM.

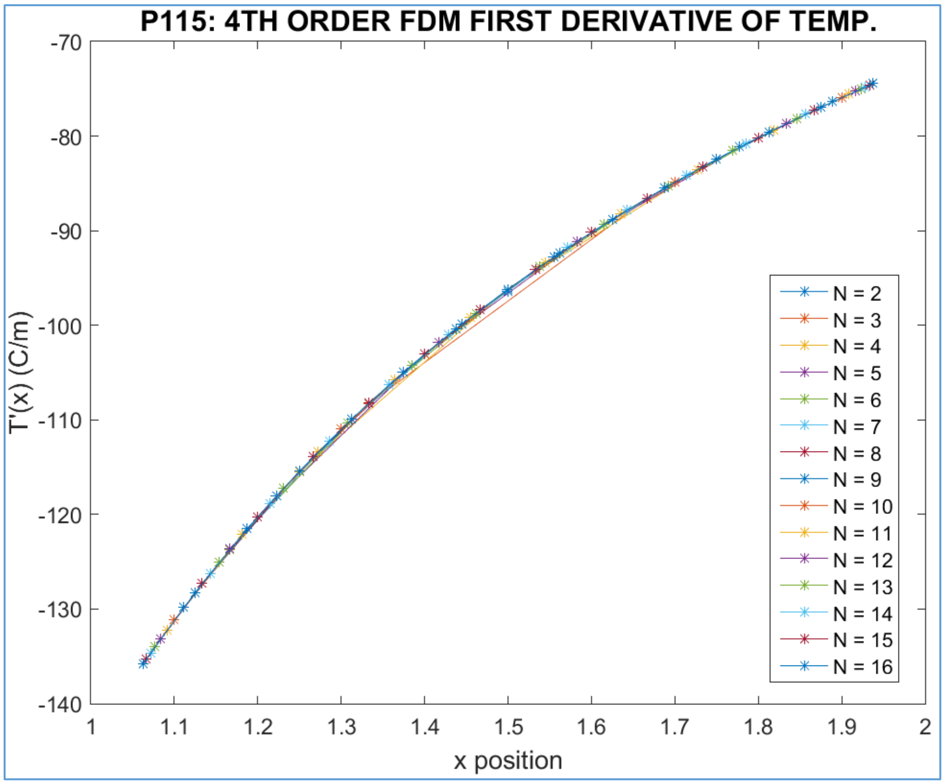

Figure 6: Plot of First Derivative of Temperature for P115 using 4th Order FDM.

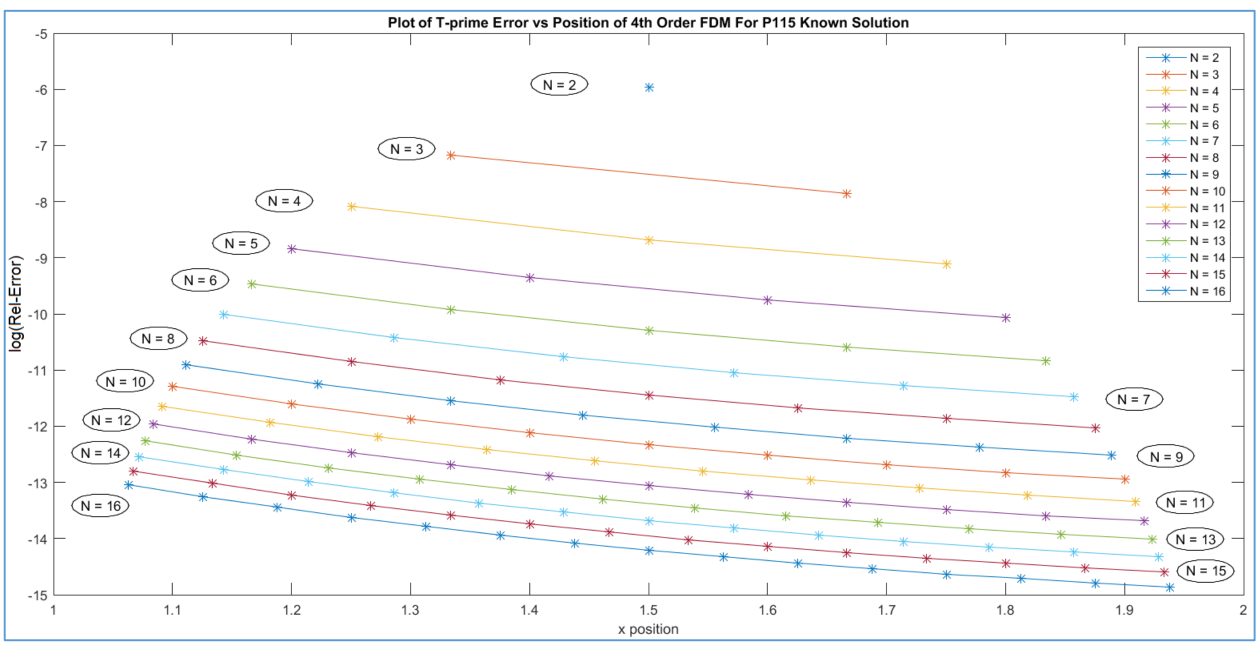

Figure 7: Plot of Relative Error for the First Derivative of the Temperature for P115 – 4th Order FDM.

IV.B. FDM Solutions to 116

Figures 8, 9, and 10 were generated using code files ‘P116.m’ and ‘Order2FDM116.m’.

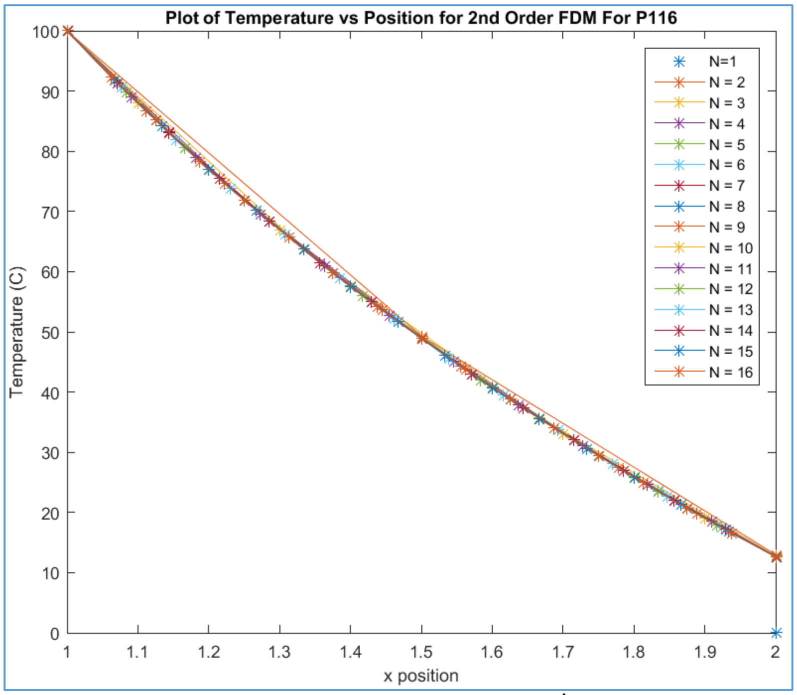

Figure 8: Plot of Temperature for P116 using 2nd Order FDM.

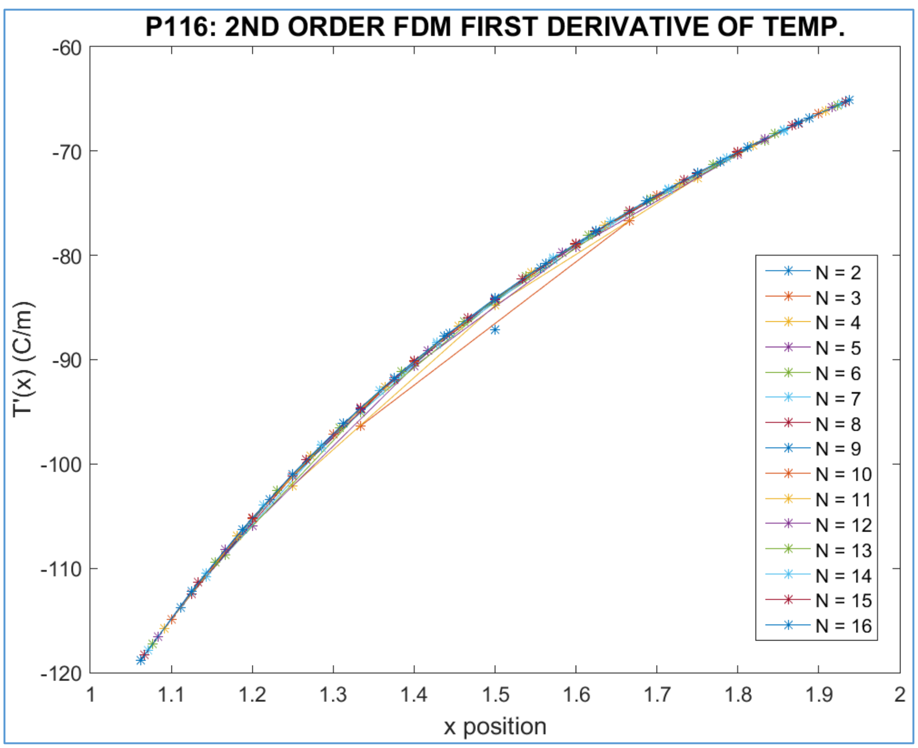

Figure 9: Plot of First Derivative of Temperature for P116 using 2nd Order FDM.

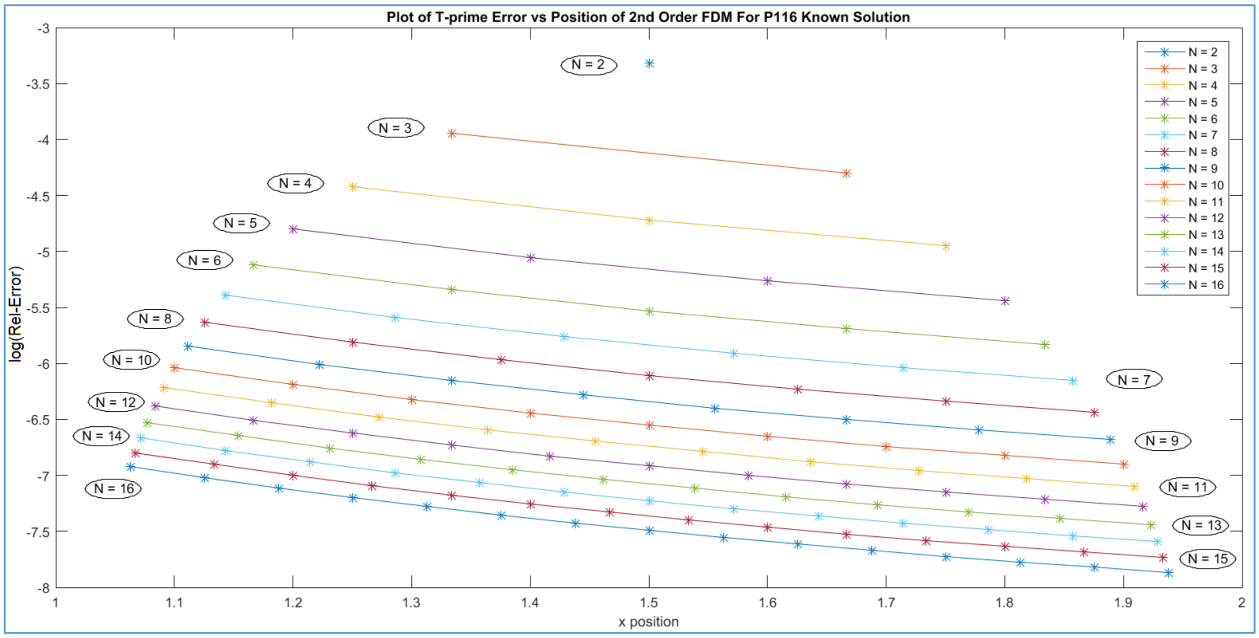

Figure 10: Plot of Relative Error for the First Derivative of the Temperature for P116 – 2nd Order FDM.

Figures 11, 12, and 13 were generated using code files 'P116.m' and 'Order4FDM116.m'.

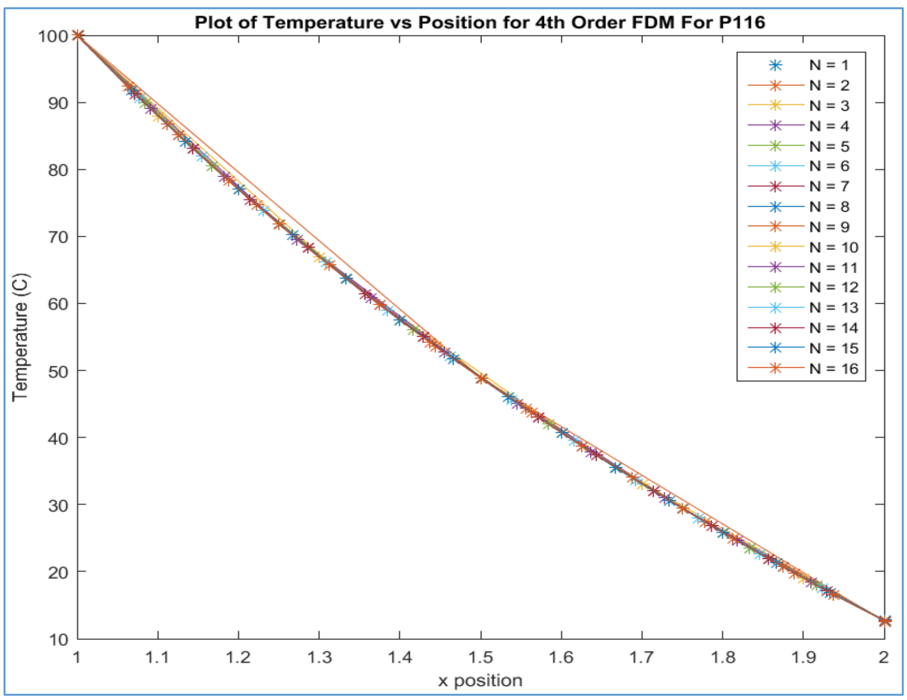

Figure 11: Plot of Temperature for P116 using 4th Order FDM.

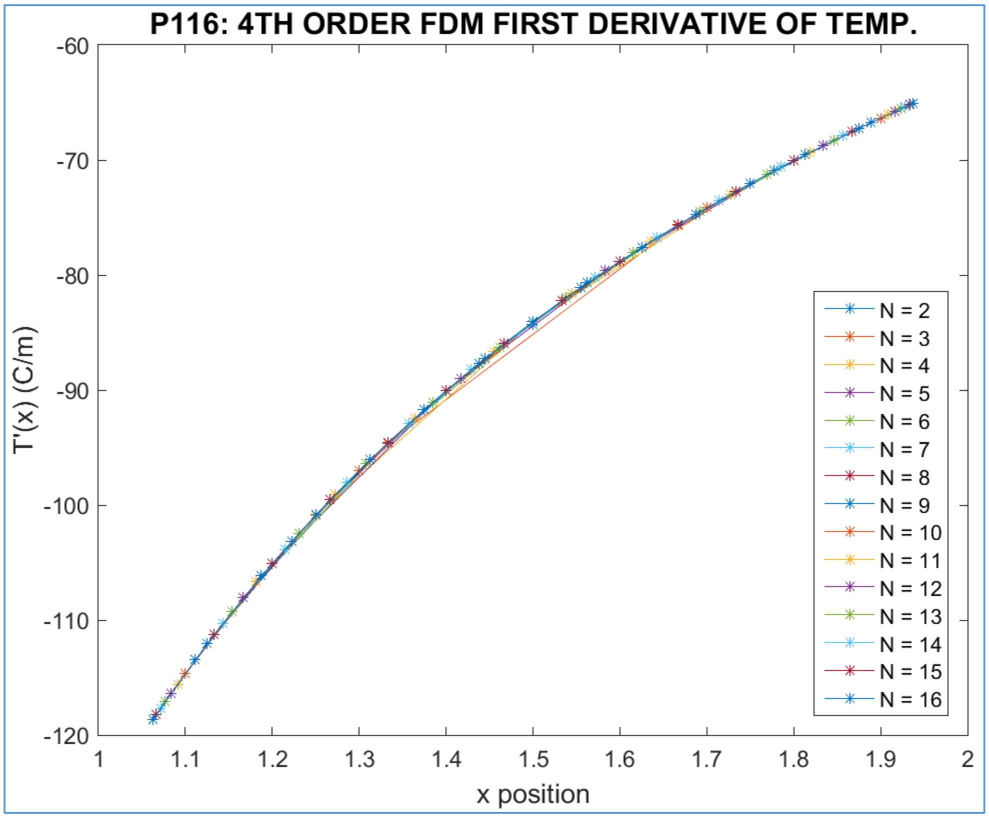

Figure 12: Plot of First Derivative of Temperature for P116 using 4th Order FDM.

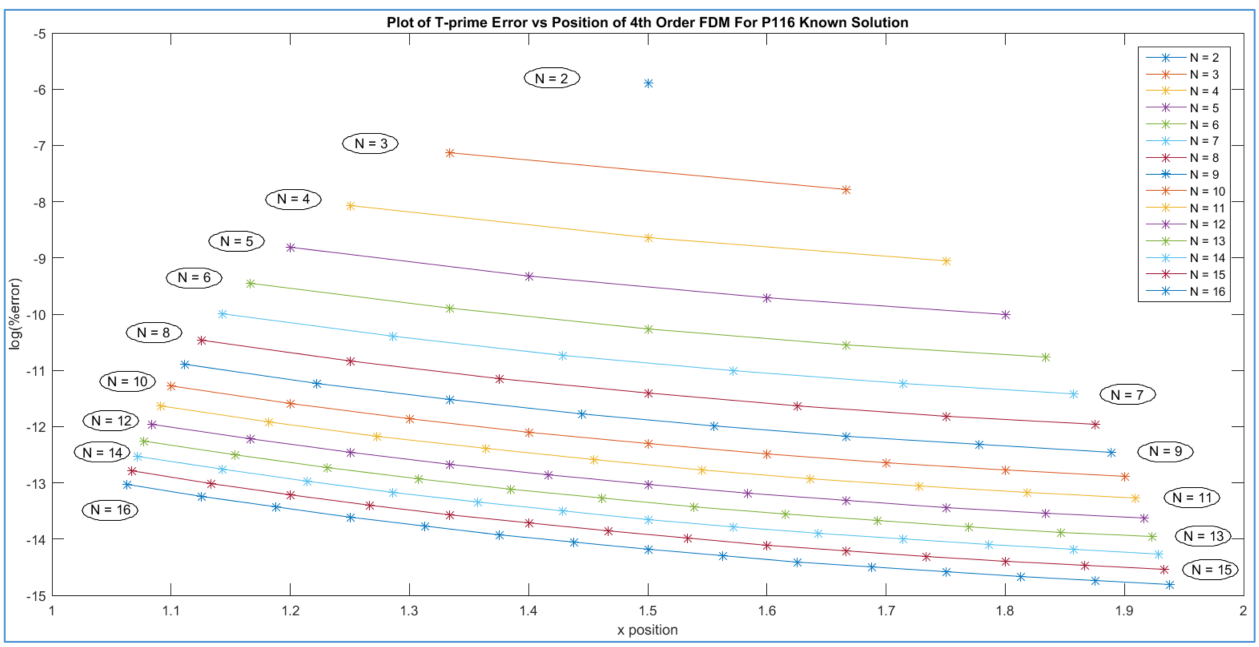

Figure 13: Plot of Relative Error for the First Derivative of the Temperature for P116 – 4th Order FDM.

IV.C. FDM Solutions to 117

Figures 14, 15, and 16 were generated using code files 'P117.m' and 'Order2FDM117.m'.

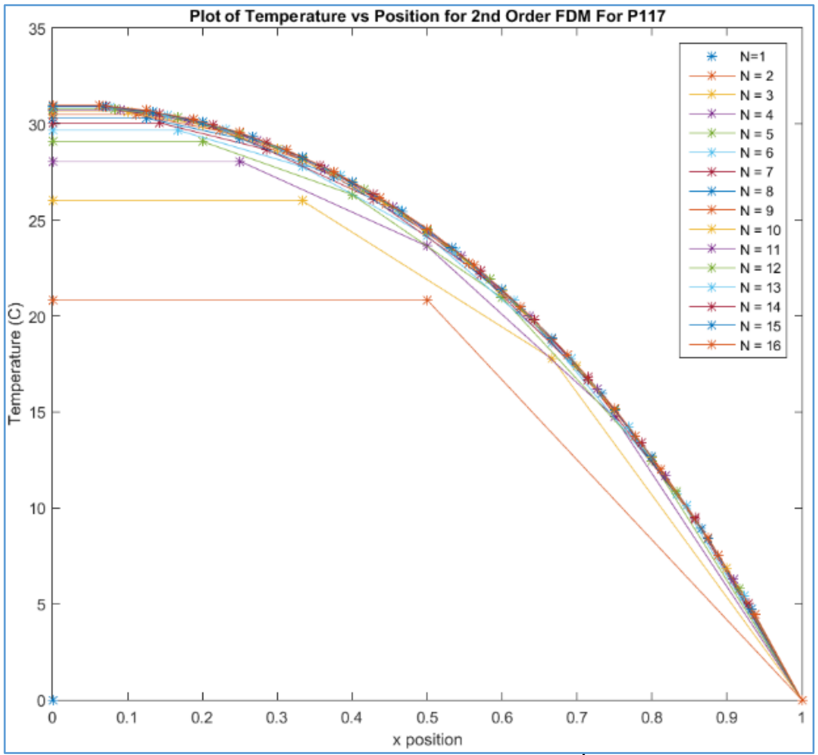

Figure 14: Plot of Temperature for P117 using 2nd Order FDM.

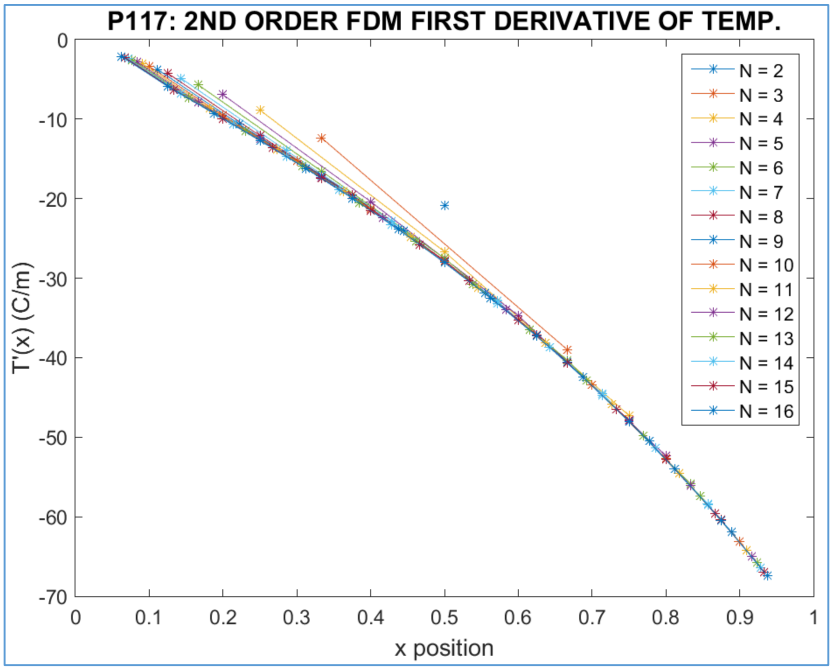

Figure 15: Plot of First Derivative of Temperature for P117 using 2nd Order FDM.

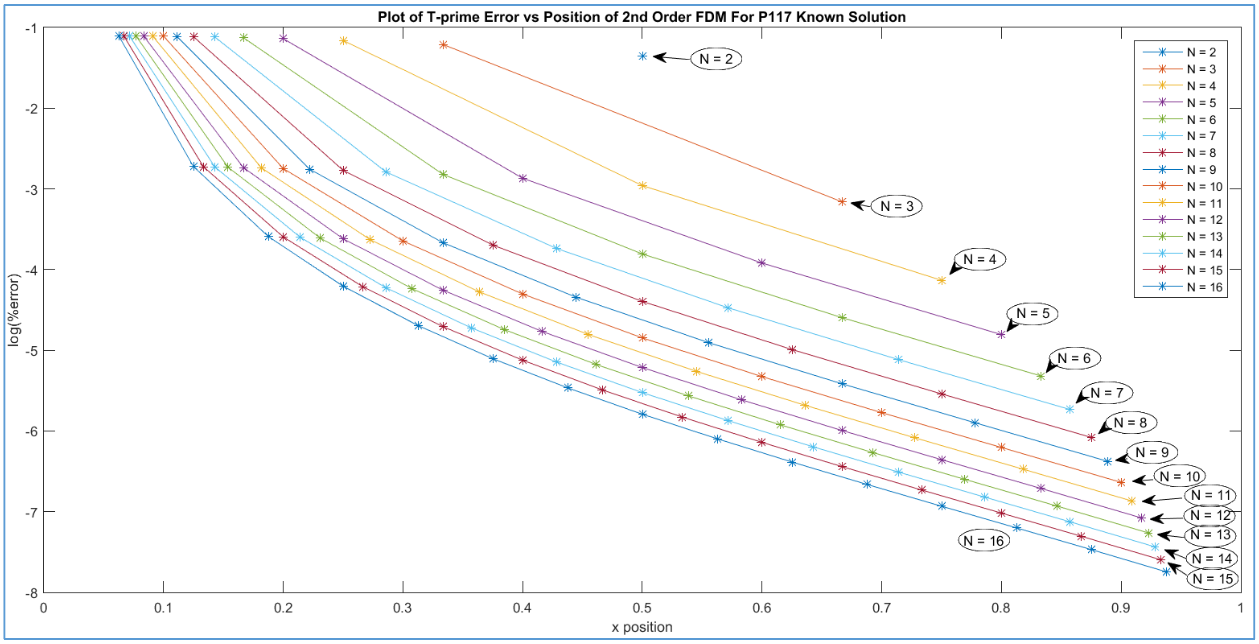

Figure 16: Plot of Relative Error for the First Derivative of the Temperature for P117 – 2nd Order FDM.

Figures 17, 18, and 19 were generated using code files 'P117.m' and 'Order4FDM117.m'.

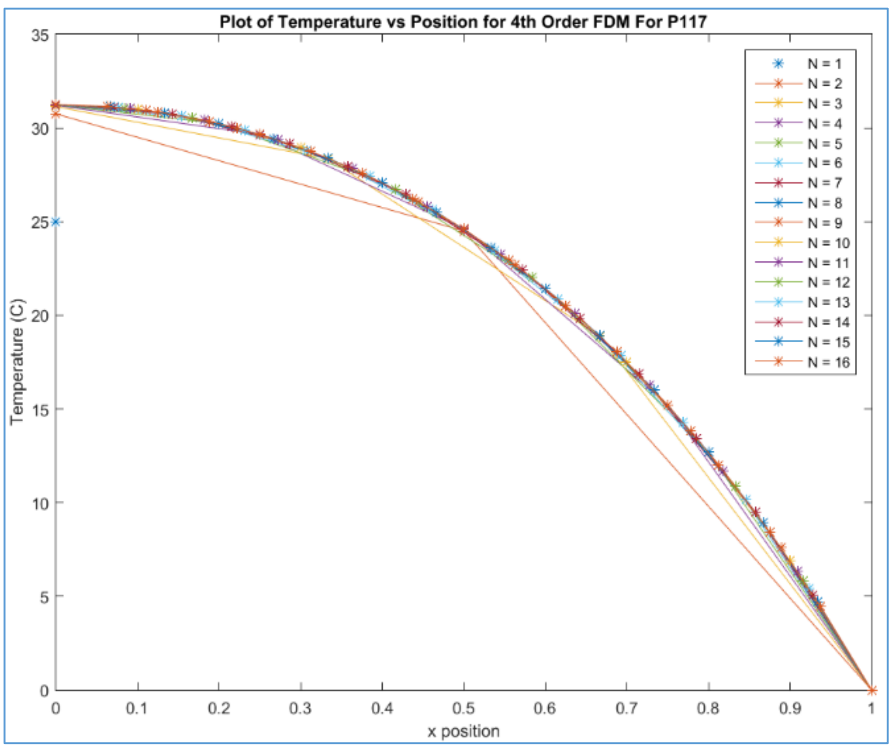

Figure 17: Plot of Temperature for P117 using 4th Order FDM.

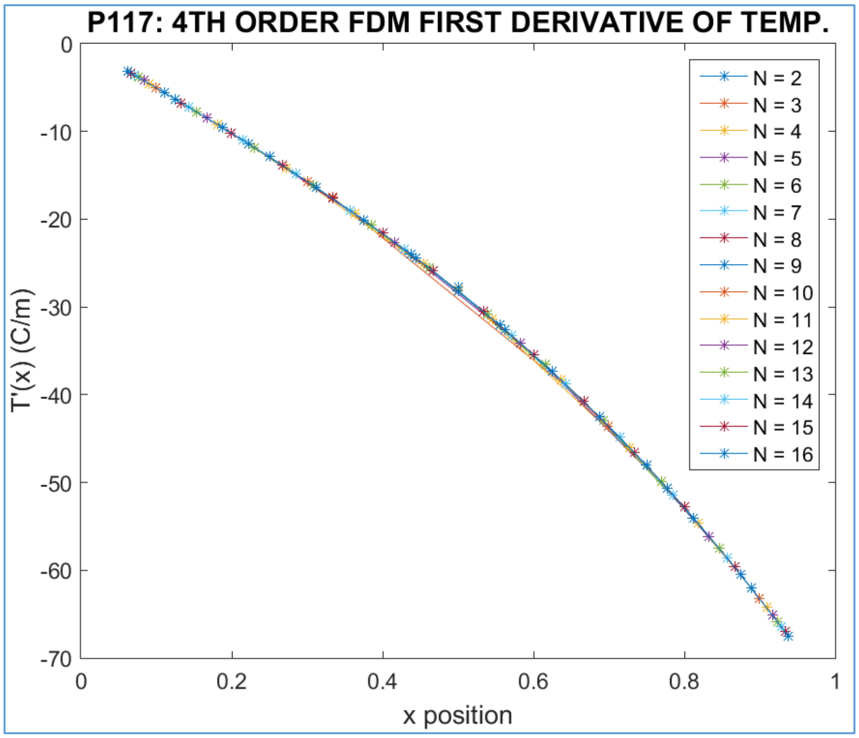

Figure 18: Plot of First Derivative of Temperature for P117 using 4th Order FDM.

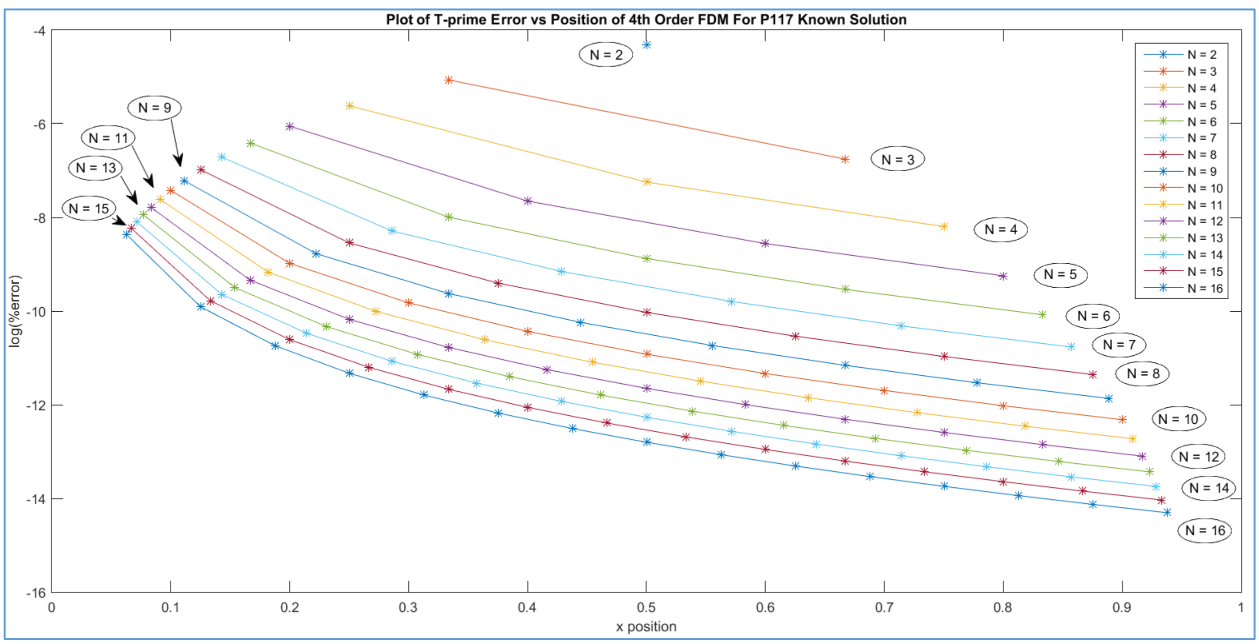

Figure 19: Plot of Relative Error for the First Derivative of the Temperature for P117 – 4th Order FDM.

V. CONVERGENCE ANALYSIS

From the solutions in part IV, it is clear that the 4th order FDM approximations converge to a solution faster than the 2nd order FDM approximations. In fact, as shown in section III.E, the solutions found using the 2nd and 4th order methods should converge to the exact solution with the 2nd and 4th power of the spacing size, respectively. If we take the logarithm of these functions, it is apparent that the 2nd and 4th order approximations should converge to the exact solutions as a function of decreasing spacing size, Δx. Specifically, on a log-log plot, the convergence will be linear, with the slope of the line being equal to the order of the approximation method. This means the slope of the lines on the log-log plot will be 2 for the 2nd order method, and 4 for the second order method. This reasoning lays the foundation for our analysis of the convergence of the approximations to the exact or extrapolated solutions as a function of the spacing size.

As a foreword, unless otherwise mentioned, the approximate solutions shown on figures displaying convergence are taken at the midpoint of the range of x-values. For problems 115 and 116, this will be $x = 1.5$, and for problem 117 this will be $x = 0.5$.

Regarding the convergence plots using an extrapolation method, the standard extrapolation (quadratic or quartic) was used in all cases due to the error being lower than the error for the Richardson's extrapolation values. This can be seen in figures 22, 23, 30, 31, 36, and 37, where the standard extrapolation error is lower than the Richardson's extrapolation error in all cases. In addition, the standard extrapolation methods provide us with seven extrapolated values with the grid spacing sizes we used, whereas with Richardson's extrapolation we can only obtain four extrapolated values.

In the graphs displaying the error for the extrapolation methods, the terms "QuadEx" refer to values derived using quadratic extrapolation, "QuartEx" refer to values derived using quartic extrapolation, and "RichEx" refer to values derived using Richardson's extrapolation.

In all convergence plots, the linear fits are within 0.5 of the nominal slope values of two and four for the corresponding approximation methods, as predicted.

V.A. Convergence Analysis for 115

Figures 20 and 21 show the convergence of the 2nd and 4th order approximations to the derivative of the temperature at the outer boundary, respectively. The slopes are near the nominal values, and would most likely be closer to the nominal values with finer grid spacing sizes.

Figures 22 and 23 show the error of the first derivative values derived using extrapolation methods with respect to the exact solutions found in part II.A. Figure 24 shows the convergence of the FDM solutions to the exact and extrapolated solutions for the Temperature at $x = 1.5$. Figure 25 shows the convergence of the FDM solutions to the exact and extrapolated solutions for the first derivative of the temperature at $x = 1.5$. Once again, the slope values for the linear fits to the FDM values are close to the expected 2 and 4 for the 2nd and 4th order methods, respectively.

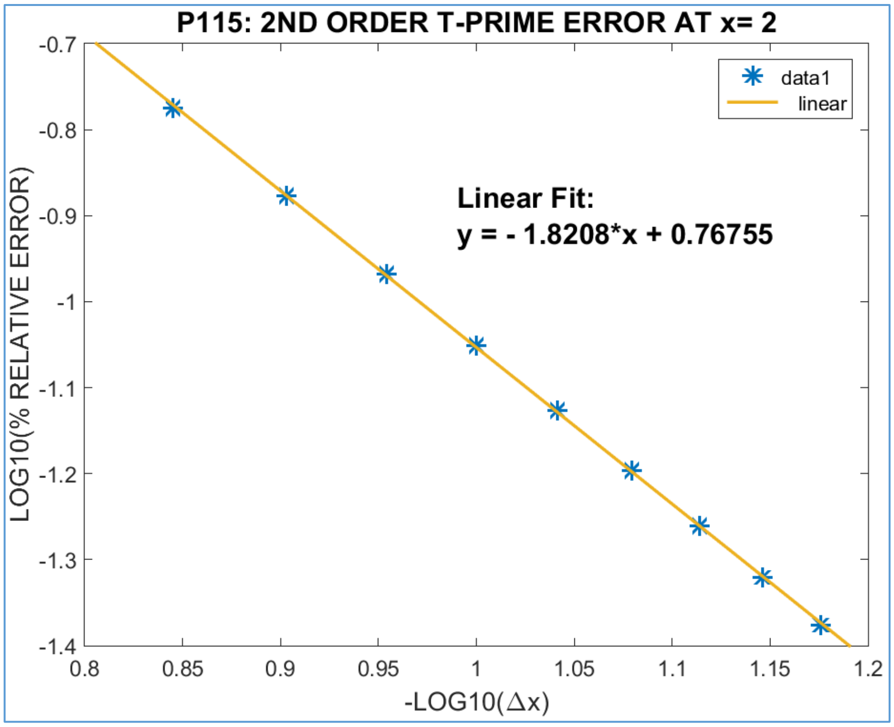

Figure 20: Convergence of the Derivative of the Temperature at x = 2 using 2nd order FDM.

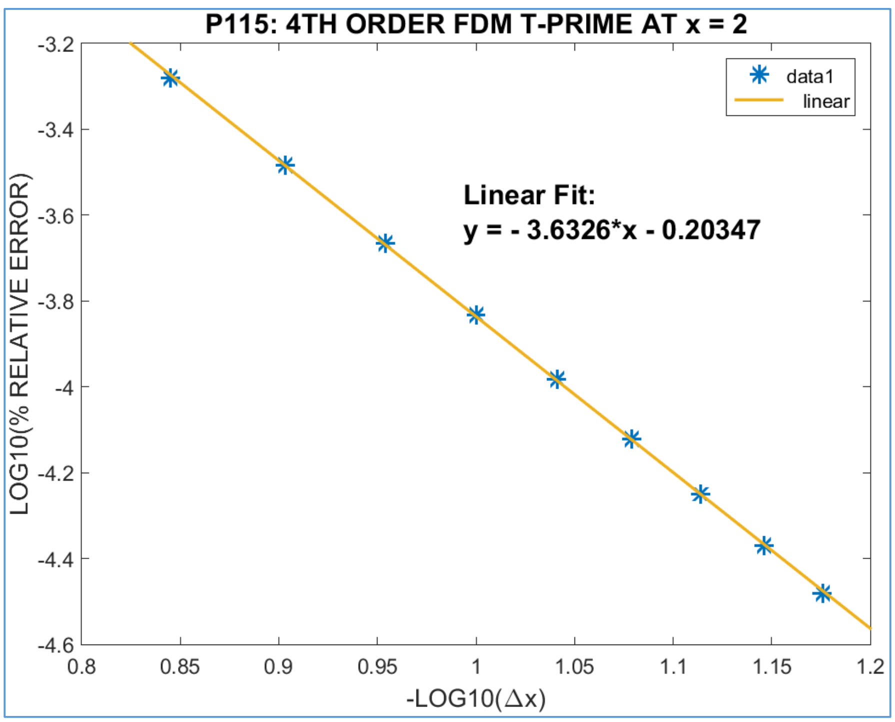

Figure 21: Convergence of the Derivative of the Temperature at x = 2 using 4th order FDM.

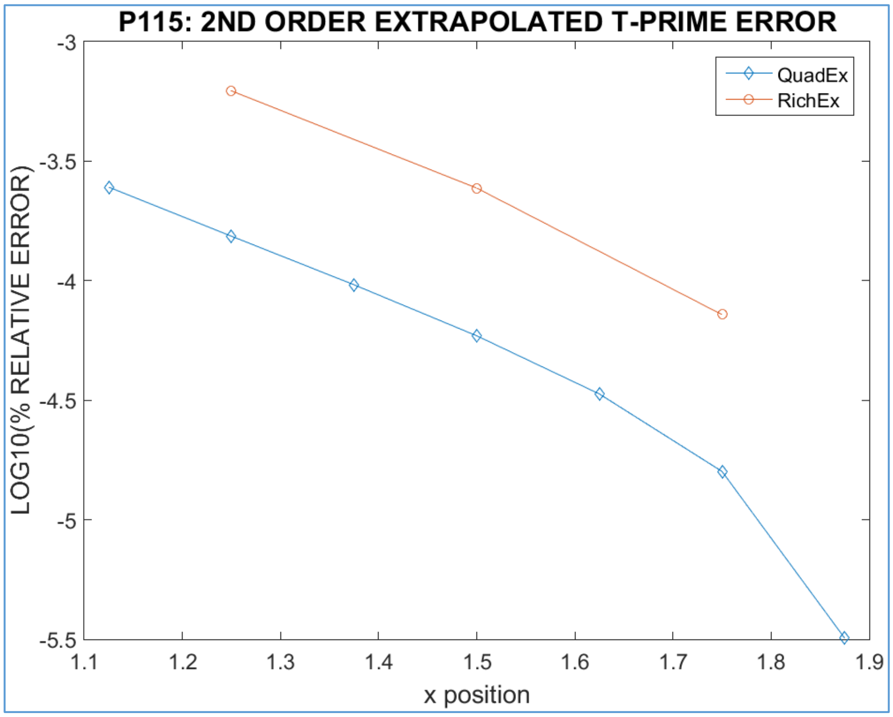

Figure 22: Error of the Extrapolated value of the Derivative of Temperature using 2nd order FDM.

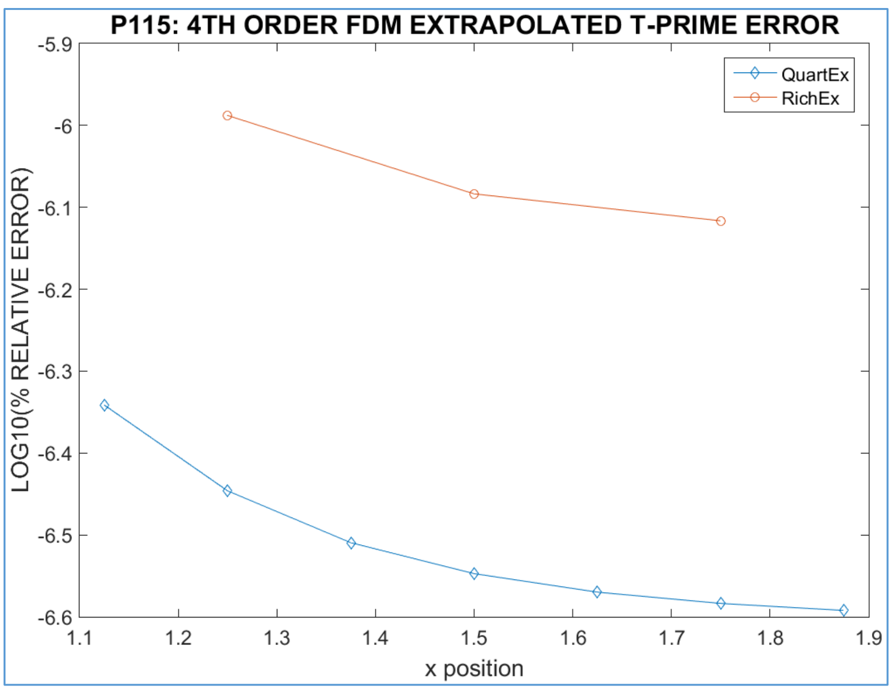

Figure 23: Error of the Extrapolated value of the Derivative of Temperature using 4th order FDM.

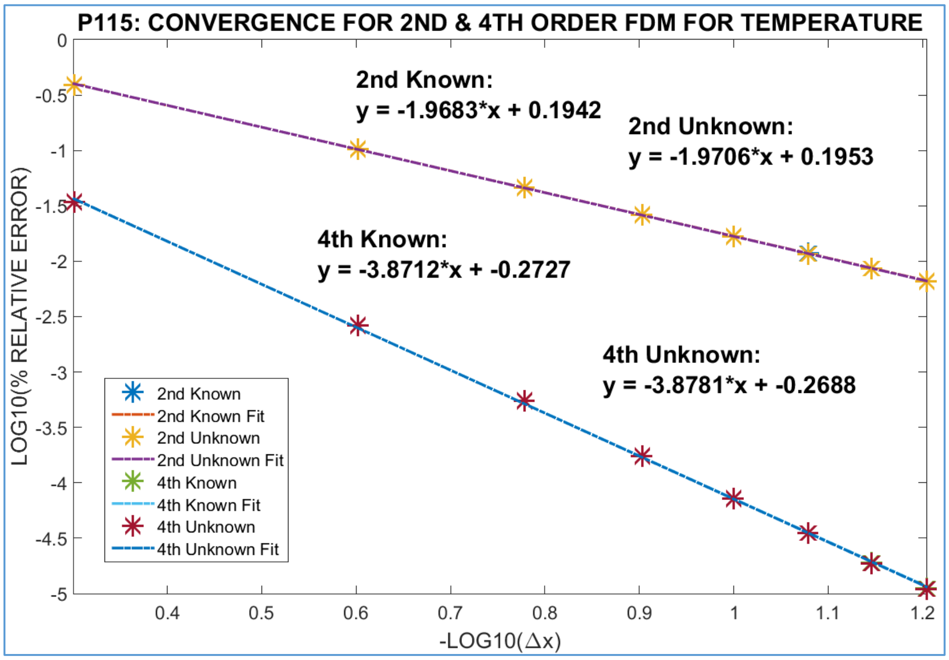

Figure 24: Convergence of the Temperature using exact (known) and extrapolated (unknown) values.

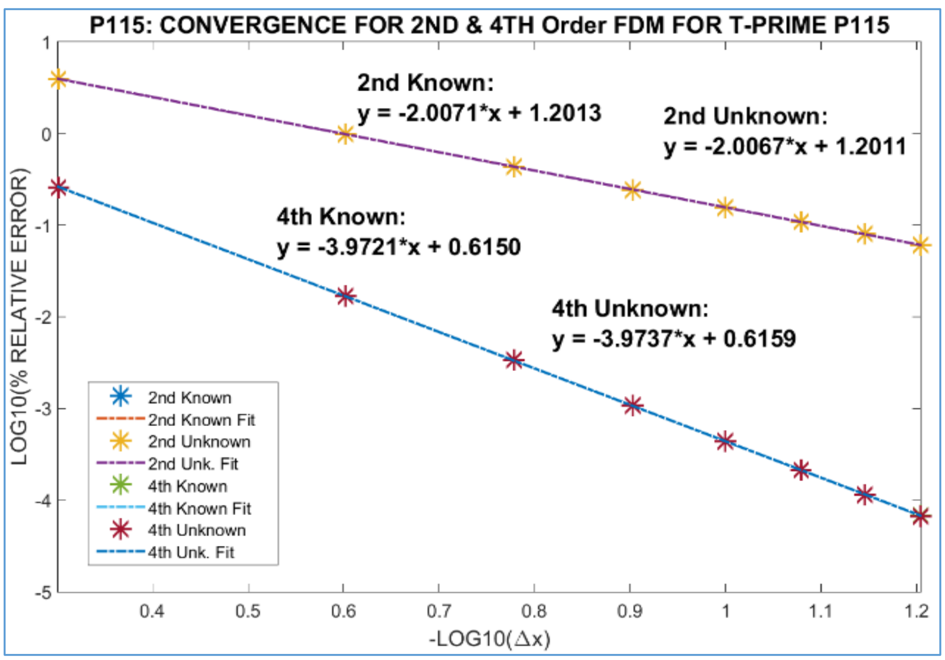

Figure 25: Convergence of the Derivative using exact (known) and extrapolated (unknown) values.

V.B. Convergence Analysis for 116

Figures 26 and 28 show the convergence of the 2nd and 4th order approximations to the temperature at the outer boundary, respectively. Figures 27 and 29 show the convergence of the 2nd and 4th order approximations to the derivative of the temperature at the outer boundary, respectively. Figures 30 and 31 show the error of the first derivative values derived using extrapolation methods with respect to the exact solutions found in part II.B. Figure 32 shows the convergence of the FDM solutions to the exact and extrapolated solutions for the Temperature at x = 1.5. Figure 33 shows the convergence of the FDM solutions to the exact and extrapolated solutions for the first derivative of the temperature at x = 1.5. Once again, the slope values for the linear fits to the FDM values are close to the expected 2 and 4 for the 2nd and 4th order methods, respectively.

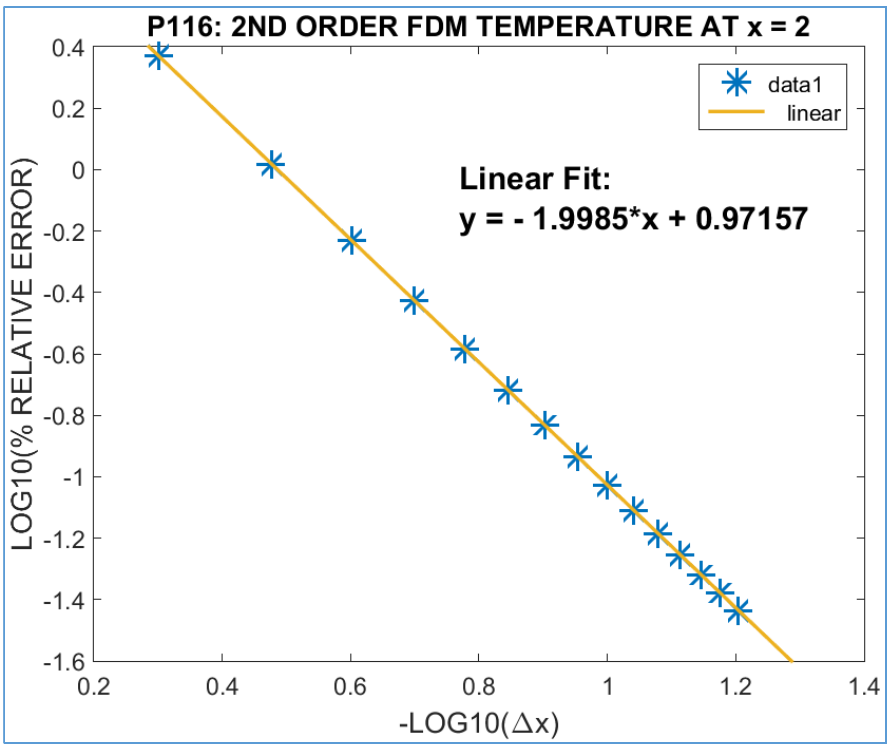

Figure 26: Convergence of the Temperature at x = 2 using 2nd order FDM.

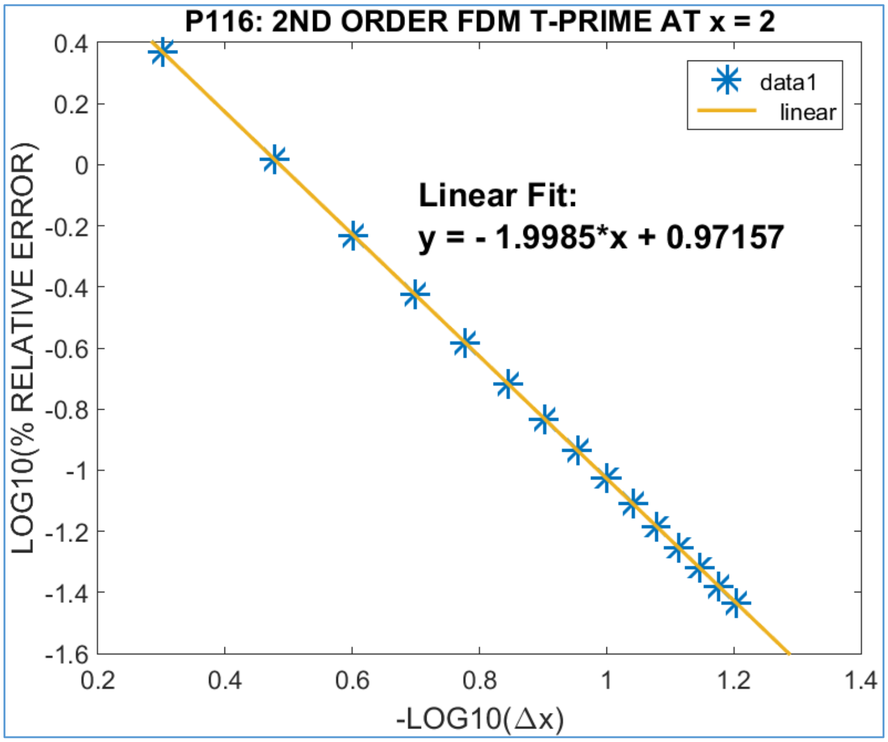

Figure 27: Convergence of the Derivative of the Temperature at x = 2 using 2nd order FDM.

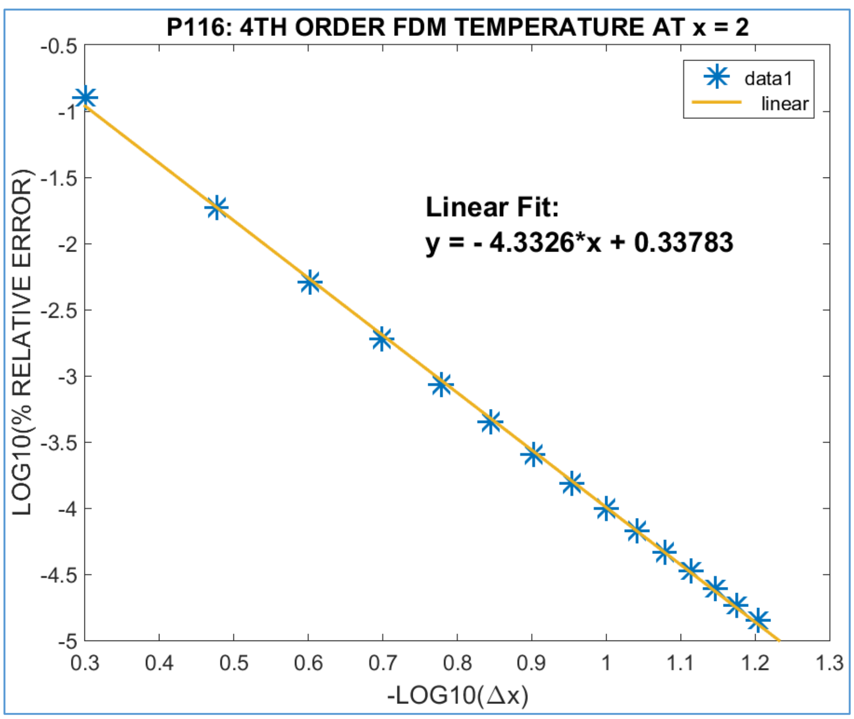

Figure 28: Convergence of the Temperature at x = 2 using 4th order FDM.

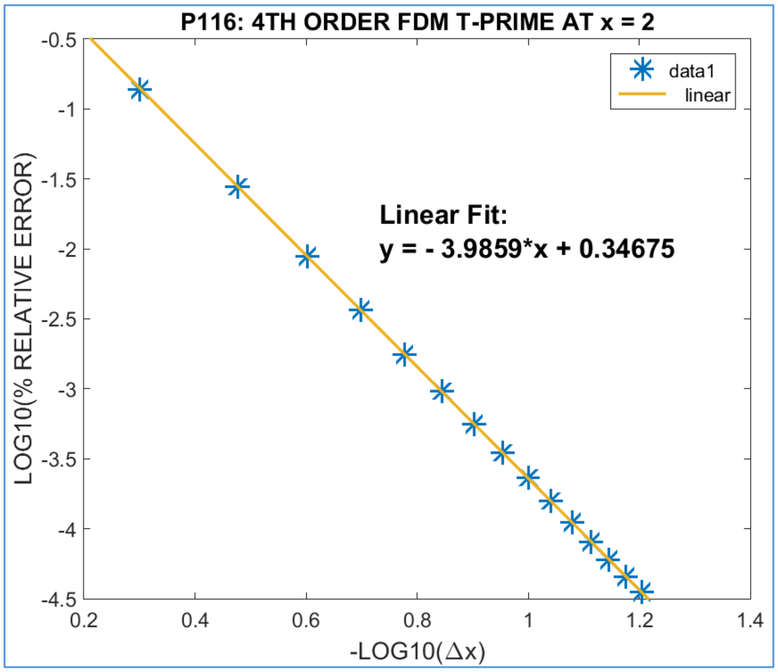

Figure 29: Convergence of the Derivative of the Temperature at x = 2 using 4th order FDM.

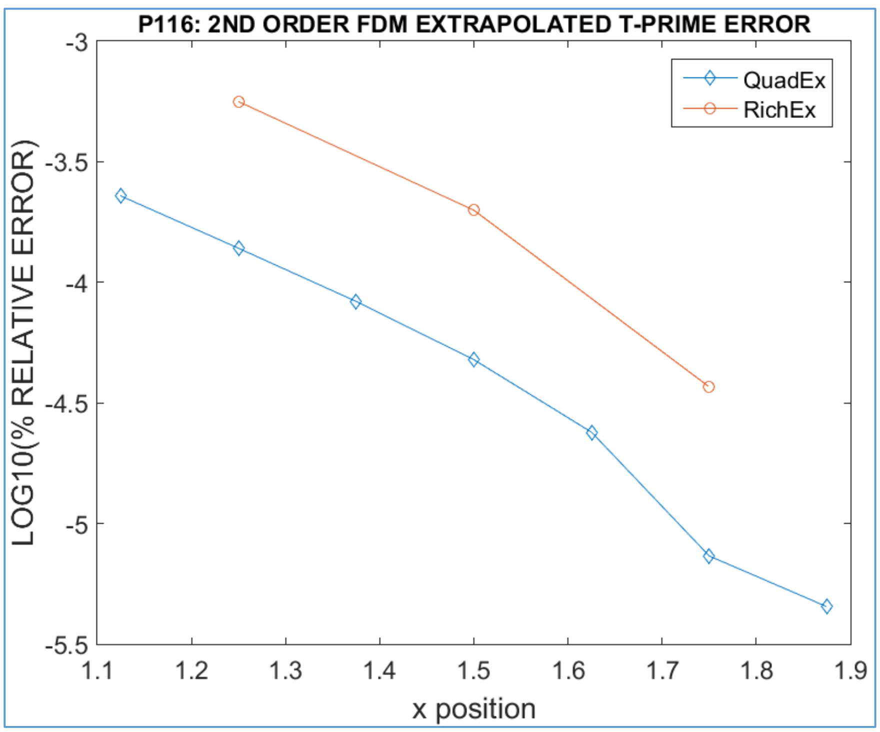

Figure 30: Error of the Extrapolated value of the Derivative of Temperature using 2nd order FDM.

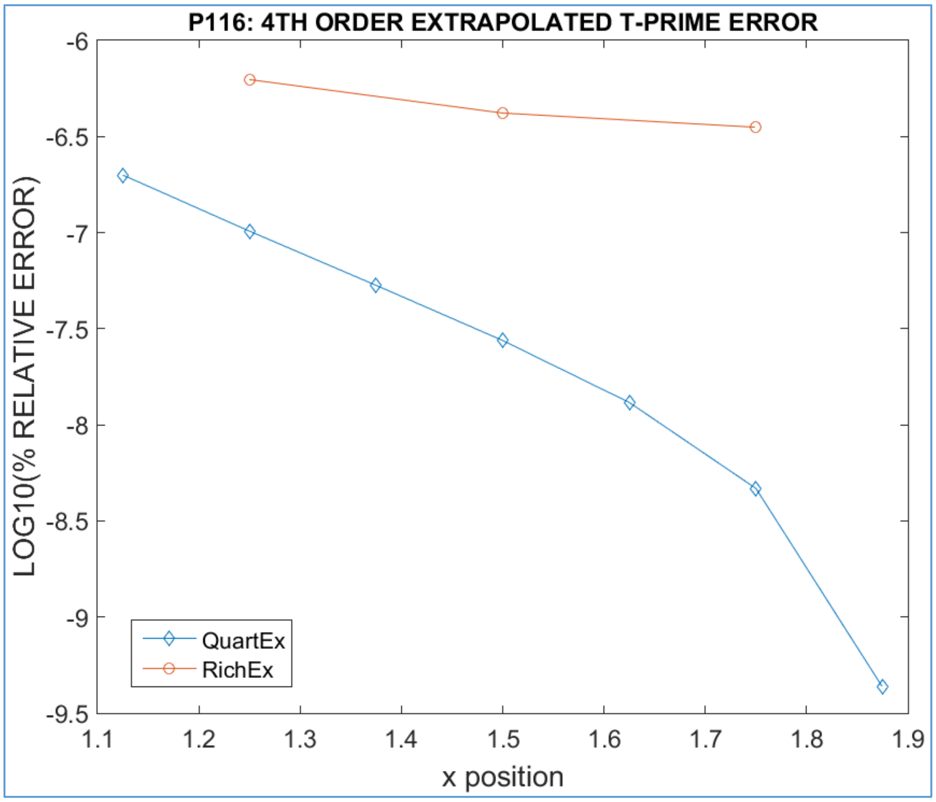

Figure 31: Error of the Extrapolated value of the Derivative of Temperature using 4th order FDM.

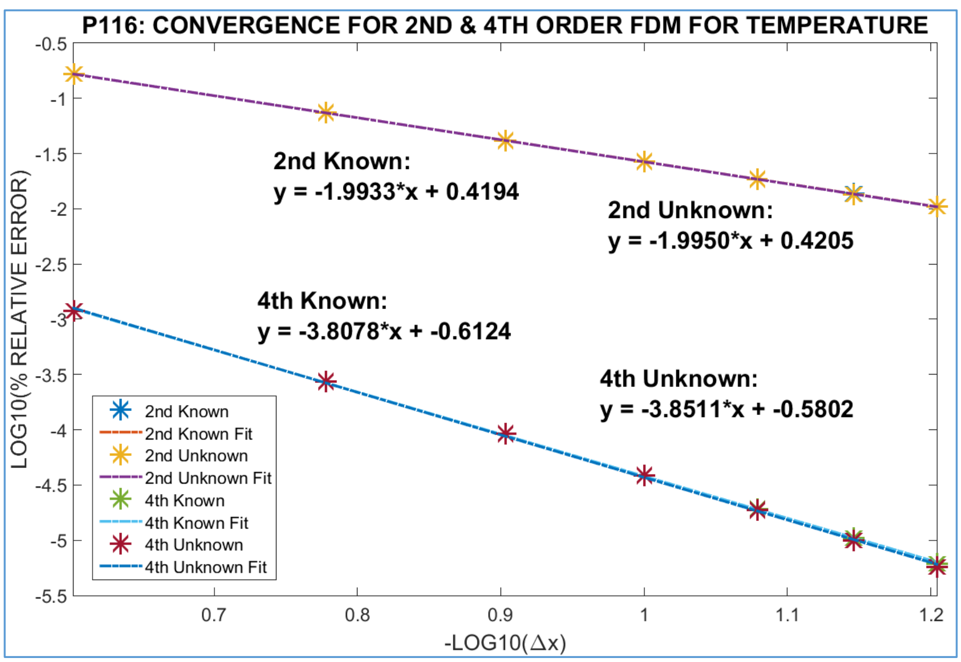

Figure 32: Convergence of the Temperature using exact (known) and extrapolated (unknown) values.

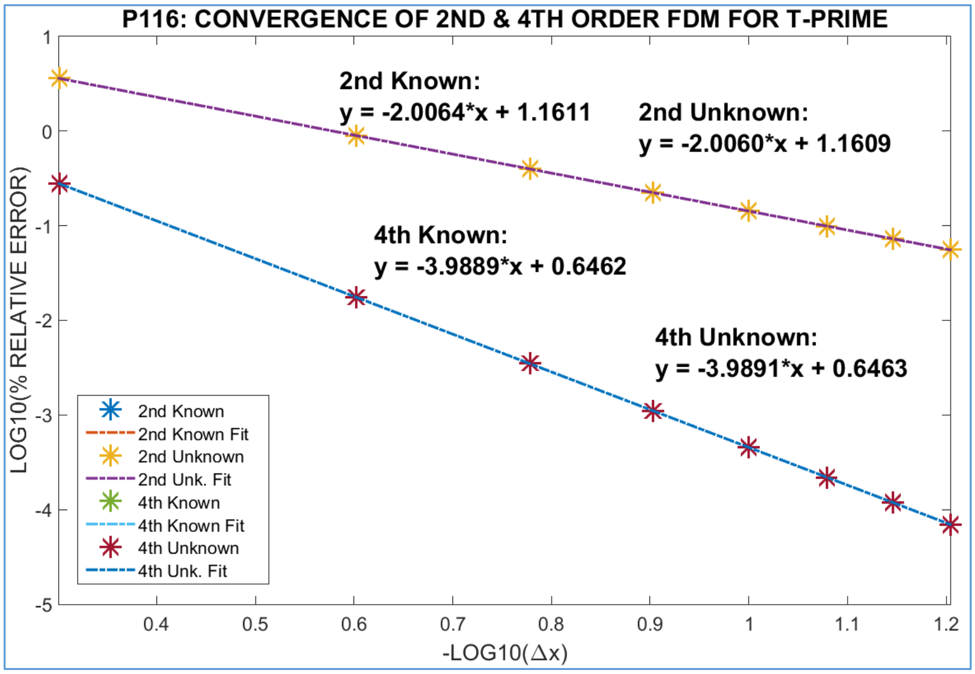

Figure 33: Convergence of the Derivative using exact (known) and extrapolated (unknown) values.

V.C. Convergence Analysis for 117

Figures 34 and 35 show the convergence of the 2nd and 4th order approximations to the derivative of the temperature at the outer boundary, respectively. Figures 36 and 37 show the error of the first derivative values derived using extrapolation methods with respect to the exact solutions found in part II.B. Figure 38 shows the convergence of the FDM solutions to the exact and extrapolated solutions for the Temperature at x = 0.5. Figure 39 shows the convergence of the FDM solutions to the exact and extrapolated solutions for the first derivative of the temperature at x = 0.5. Once again, the slope values for the linear fits to the FDM values are close to the expected 2 and 4 for the 2nd and 4th order methods, respectively.

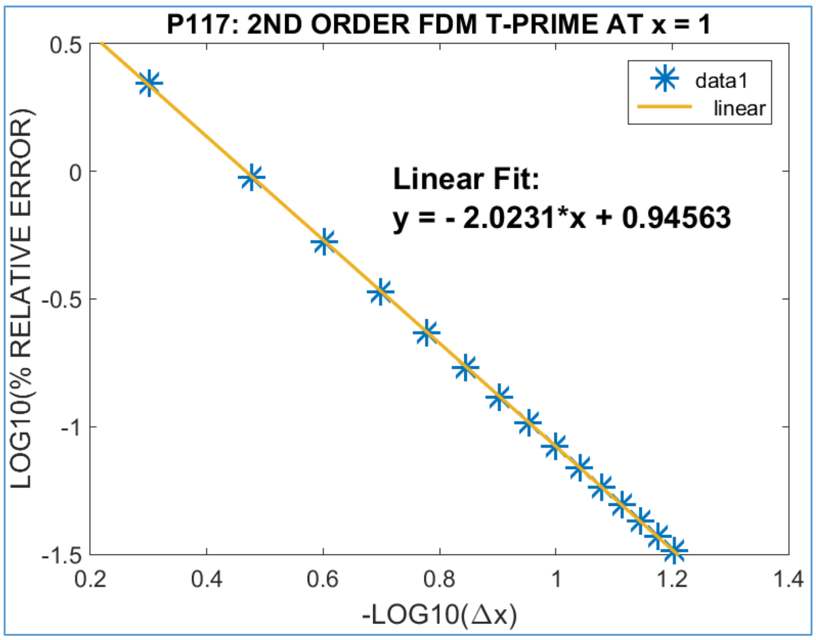

Figure 34: Convergence of the Derivative of the Temperature at x = 1 using 2nd order FDM.

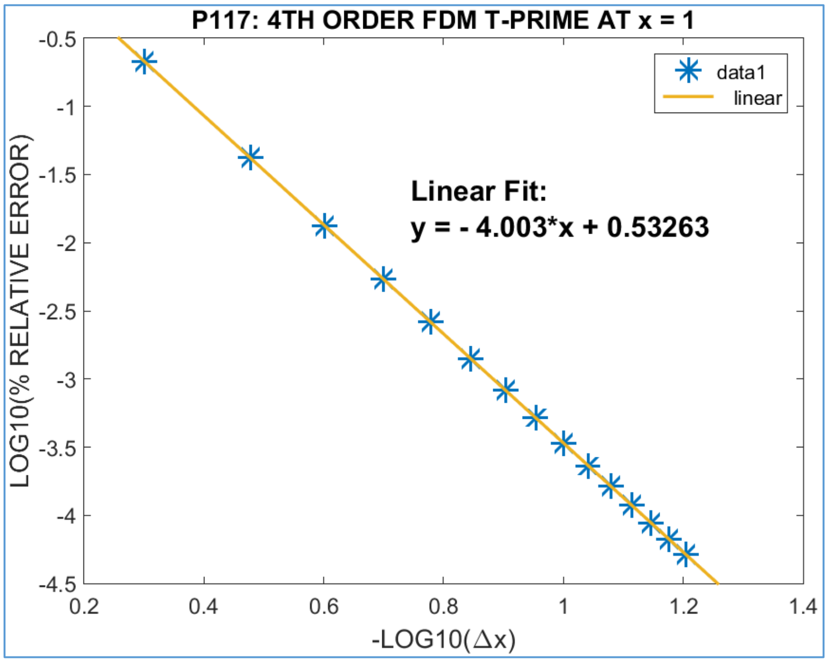

Figure 35: Convergence of the Derivative of the Temperature at x = 1 using 4th order FDM.

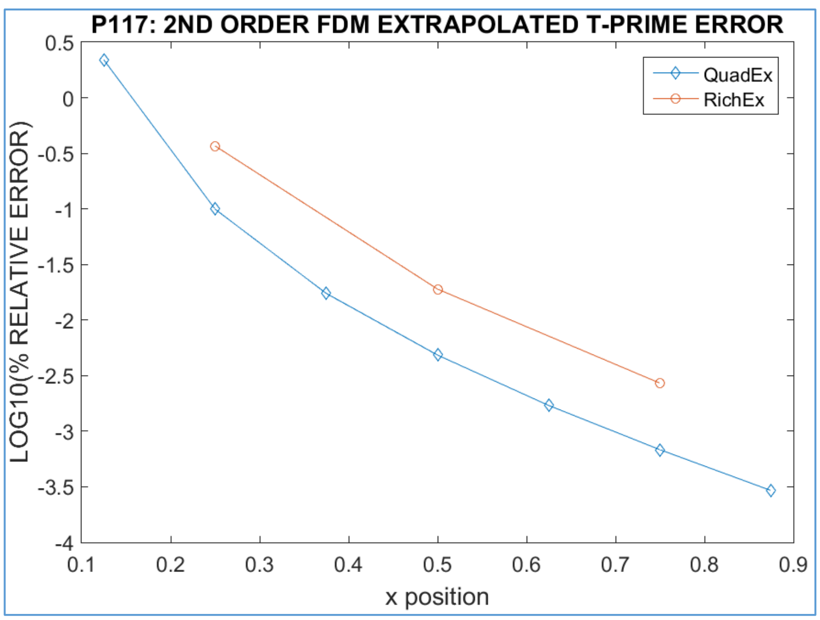

Figure 36: Error of the Extrapolated value of the Derivative of Temperature using 2nd order FDM.

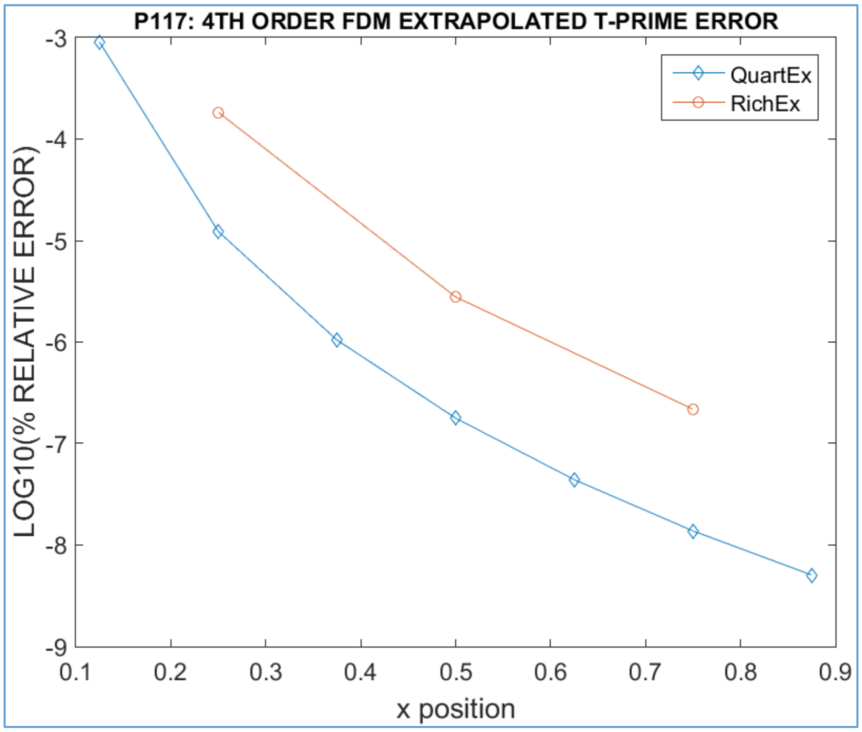

Figure 37: Error of the Extrapolated value of the Derivative of Temperature using 4th order FDM.

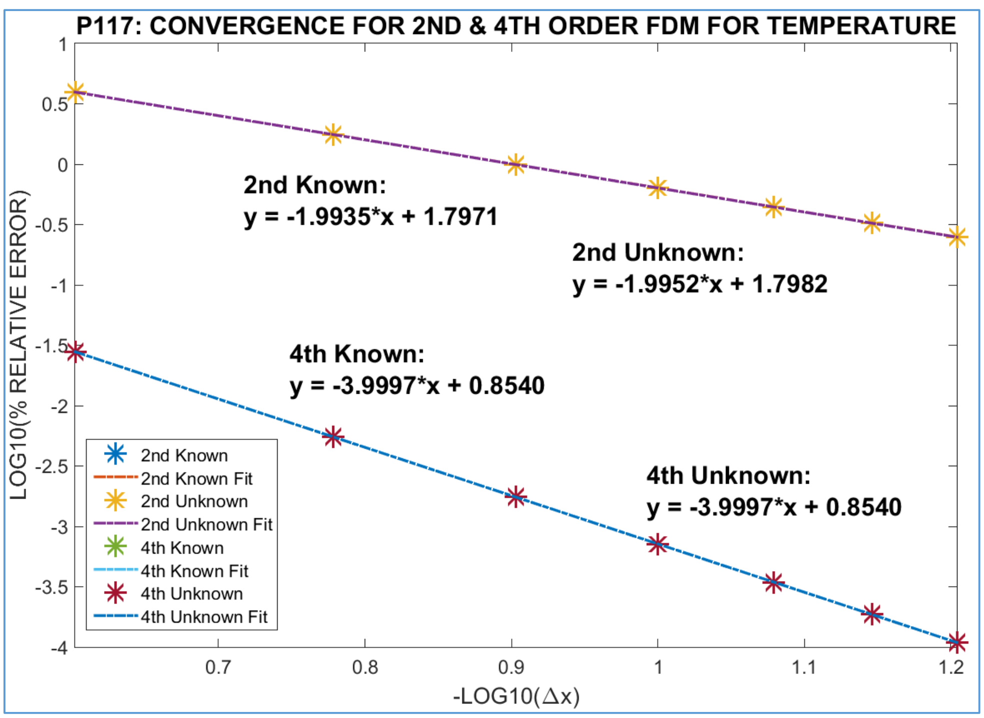

Figure 38: Convergence of the Temperature using exact (known) and extrapolated (unknown) values.

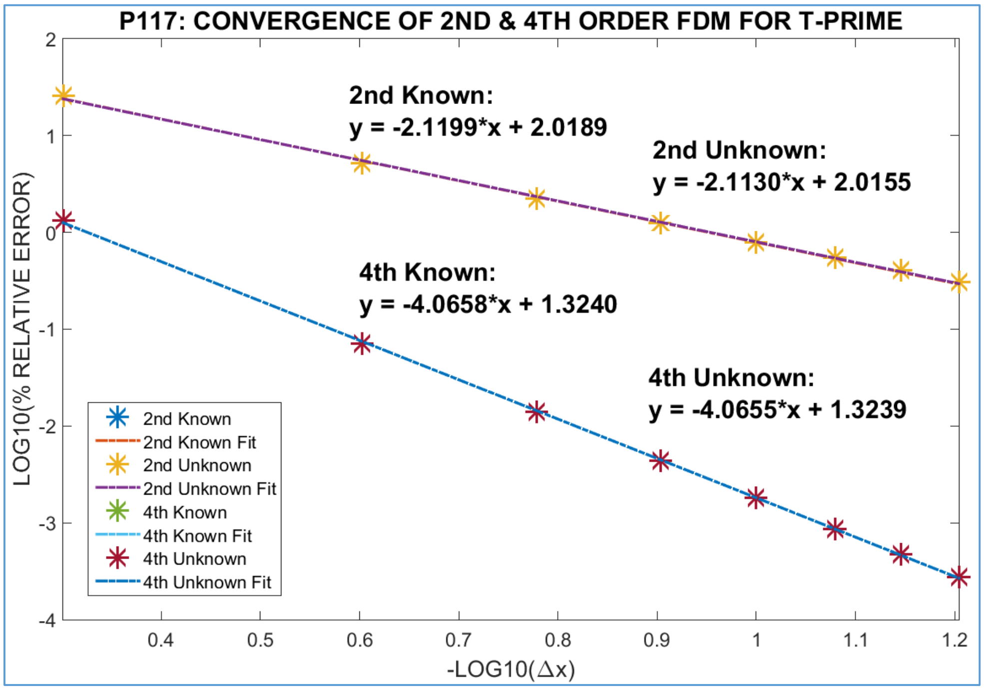

Figure 39: Convergence of the Derivative using exact (known) and extrapolated (unknown) values.

APPENDIX A. DATA TABLES

In this appendix, we list all relevant data acquired in sections IV and V. In the tables that follow, the symbol "T" refers to the value of the temperature calculated using a numerical method, whereas the symbol "Ť" refers to the analytically derived exact value of the temperature. Similarly, the symbol "T' " refers to the value of the derivative of the temperature calculated using a numerical method, whereas the symbol "Ť' " refers to the analytically derived exact value of the derivative of the temperature. The symbol "e" refers to the difference between the calculated and exact values of the relevant quantities in each table.

Table 1: P115 2nd Order FDM Temp. values, △x = 1/2

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.5	41.666667	41.503750	0.162917
2	0.000000	0.000000	0.000000

Table 2: P115 2nd Order FDM Temp. values, △x = 1/3

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.3333	58.577406	58.499857	0.077549
1.6667	26.359833	26.300555	0.059278
2	0.000000	0.000000	0.000000

Table 3: P115 2nd Order FDM Temp. values, △x = 1/4

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.25	67.850719	67.807191	0.043528
1.5	41.546763	41.503750	0.043013
1.75	19.289568	19.264508	0.025060
2	0.000000	0.000000	0.000000

Table 4: P115 2nd Order FDM Temp. values, △x = 1/5

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.2	73.722198	73.696559	0.025639
1.4	51.487135	51.457317	0.029818
1.6	32.216748	32.192809	0.023939
1.8	15.213464	15.200309	0.013155
2	0.000000	0.000000	0.000000

Table 5: P115 2nd Order FDM Temp. values, △x = 1/6

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1667	77.777028	77.756636	0.020392
1.3333	58.517118	58.499857	0.017261
1.5	41.523080	41.503750	0.019330
1.6667	26.317889	26.300555	0.017334
1.8333	12.560811	12.555711	0.005100
2	0.000000	0.000000	0.000000

Table 6: P115 2nd Order FDM Temp. values, △x = 1/7

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1429	80.746429	80.730082	0.016347
1.2857	63.757985	63.744595	0.013390
1.4286	48.557797	48.539797	0.018000
1.5714	34.805247	34.794953	0.010294
1.7143	22.248570	22.238040	0.010530
1.8571	10.696428	10.694850	0.001578
2	0.000000	0.000000	0.000000

Table 7: P115 2nd Order FDM Temp values, △x = 1/8

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.125	83.015193	83.007500	0.007693
1.25	67.818260	67.807191	0.011069
1.375	54.068655	54.056838	0.011817
1.5	41.514667	41.503750	0.010917
1.625	29.964998	29.956028	0.008970
1.75	19.270860	19.264508	0.006352
1.875	9.314249	9.310940	0.003309
2	0.000000	0.000000	0.000000

Table 8: P115 2nd Order FDM Temp. values, △x = 1/9

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1111	84.805302	84.801133	0.004169
1.2222	71.057718	71.051961	0.005757
1.3333	58.505576	58.499857	0.005719
1.4444	46.957605	46.952967	0.004638
1.5556	36.265040	36.252886	0.012154
1.6667	26.309893	26.300555	0.009338
1.7778	16.997013	16.990697	0.006316
1.8889	8.248550	8.245367	0.003183
2	0.000000	0.000000	0.000000

Table 9: P115 2nd Order FDM Temp. values, △x = 1/10

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1	86.253863	86.249648	0.004215
1.2	73.703043	73.696559	0.006484
1.3	62.156288	62.148838	0.007450
1.4	51.464849	51.457317	0.007532
1.5	41.510750	41.503750	0.007000
1.6	32.198851	32.192809	0.006042
1.7	23.451309	23.446525	0.004784
1.8	15.203627	15.200309	0.003318
1.9	7.401766	7.400058	0.001708
2	0.000000	0.000000	0.000000

Table 10: P115 2nd Order FDM Temp. values, △x = 1/11

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0909	87.450158	87.448114	0.002044
1.1818	75.904304	75.901410	0.002894
1.2727	65.213698	65.210761	0.002937
1.3636	55.260375	55.257950	0.002425
1.4545	45.949202	45.947670	0.001532
1.5455	37.202342	37.192635	0.009707
1.6364	28.955303	28.947456	0.007847
1.7273	21.154050	21.148133	0.005917
1.8182	13.752861	13.748910	0.003951
1.9091	6.712706	6.710733	0.001973
2	0.000000	0.000000	0.000000

Table 11: P115 2nd Order FDM Temp. values, △x = 1/12

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0833	88.454831	88.456717	0.001886
1.1667	77.764859	77.756636	0.008223
1.25	67.812126	67.807191	0.004935
1.3333	58.501506	58.499857	0.001649
1.4167	49.755165	49.746571	0.008594
1.5	41.508616	41.503750	0.004866
1.5833	33.707826	33.706536	0.001290
1.6667	26.307076	26.300555	0.006521
1.75	19.267338	19.264508	0.002830
1.8333	12.555031	12.555711	0.000680
1.9167	6.141048	6.137545	0.003503
2	0.000000	0.000000	0.000000

Table 12: P115 2nd Order FDM Temp. values, △x = 1/13

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0769	89.310522	89.311571	0.001049
1.1538	79.358250	79.360683	0.002433
1.2308	70.048059	70.040365	0.007694
1.3077	61.302123	61.296839	0.005284
1.3846	53.055954	53.053075	0.002879
1.4615	45.255524	45.255017	0.000507
1.5385	37.855116	37.847556	0.007560
1.6154	30.815704	30.810856	0.004848
1.6923	24.103706	24.101466	0.002240
1.7692	17.690020	17.690285	0.000265
1.8462	11.549256	11.544115	0.005141
1.9231	5.659135	5.656622	0.002513
2	0.000000	0.000000	0.000000

Table 13: P115 2nd Order FDM Temp. values, △x = 1/14

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0714	90.048092	90.050280	0.002188
1.1429	80.738243	80.730082	0.008161
1.2143	71.992628	71.987511	0.005117
1.2857	63.746761	63.744595	0.002166
1.3571	55.946618	55.947297	0.000679
1.4286	48.546481	48.539797	0.006684
1.5	41.507327	41.503750	0.003577
1.5714	34.795575	34.794953	0.000622
1.6429	28.382124	28.375533	0.006591
1.7143	22.241585	22.238040	0.003545
1.7857	16.351681	16.351027	0.000654
1.8571	10.692753	10.694850	0.002097
1.9286	5.247369	5.244605	0.002764
2	0.000000	0.000000	0.000000

Table 14: P115 2nd Order FDM Temp. values, △x = 1/15

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0667	90.690426	90.684551	0.005875
1.1333	81.945070	81.947019	0.001949
1.2	73.699447	73.696559	0.002888
1.2667	65.899534	65.892512	0.007022
1.3333	58.499617	58.499857	0.000240
1.4	51.460671	51.457317	0.003354
1.4667	44.749118	44.742619	0.006499
1.5333	38.335856	38.336000	0.000144
1.6	32.195499	32.192809	0.002690
1.6667	26.305769	26.300555	0.005214
1.7333	20.647009	20.647862	0.000853
1.8	15.201786	15.200309	0.001477
1.8667	9.954572	9.950991	0.003581
1.9333	4.891471	4.893447	0.001976
2	0.000000	0.000000	0.000000

Table 15: P115 2nd Order FDM Temp. values, △x = 1/16

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0625	91.254855	91.253716	0.001139
1.125	83.009432	83.007500	0.001932
1.1875	75.209708	75.207249	0.002459
1.25	67.809970	67.807191	0.002779
1.3125	60.771195	60.768258	0.002937
1.375	54.059804	54.056838	0.002966
1.4375	47.646698	47.643804	0.002894
1.5	41.506490	41.503750	0.002740
1.5625	35.616902	35.614381	0.002521
1.625	29.958279	29.956028	0.002251
1.6875	24.513189	24.511250	0.001939
1.75	19.266102	19.264508	0.001594
1.8125	14.203123	14.201900	0.001223
1.875	9.311770	9.310940	0.000830
1.9375	4.580790	4.580369	0.000421
2	0.000000	0.000000	0.000000

Table 16: P115 2nd Order FDM T-prime values, △x = 1/2

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.5	-100.000000	-96.179669	3.820331

Table 17: P115 2nd Order FDM T-prime values, △x = 1/3

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.3333	-110.460251	-108.204833	2.255418
1.6667	-87.866109	-86.559971	1.306138

Table 18: P115 2nd Order FDM T-prime values, △x = 1/4

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.25	-116.906475	-115.415603	1.490872
1.5	-97.122302	-96.179669	0.942633
1.75	-83.093525	-82.439717	0.653808

Table 19: P115 2nd Order FDM T-prime values, △x = 1/5

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.2	-121.282161	-120.224587	1.057574
1.4	-103.763627	-103.049646	0.713981
1.6	-90.684178	-90.168440	0.515738
1.8	-80.541869	-80.149724	0.392145

Table 20: P115 2nd Order FDM T-prime values, △x = 1/6

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1667	-124.448646	-123.656042	0.792604
1.3333	-108.761842	-108.204833	0.557009
1.5	-96.597688	-96.179669	0.418019
1.6667	-86.886810	-86.559971	0.326839
1.8333	-78.953666	-78.693888	0.259778

Table 21: P115 2nd Order FDM T-prime values, △x = 1/7

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1429	-126.847053	-126.231082	0.615971
1.2857	-112.660212	-112.210861	0.449351
1.4286	-101.334582	-100.986633	0.347949
1.5714	-92.082294	-91.809536	0.272758
1.7143	-84.380866	-84.156509	0.224357
1.8571	-77.869997	-77.685372	0.184625

Table 22: P115 2nd Order FDM T-prime values, △x = 1/8

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.125	-128.726959	-128.239559	0.487400
1.25	-115.786154	-115.415603	0.370551
1.375	-105.214374	-104.923276	0.291098
1.5	-96.414627	-96.179669	0.234958
1.625	-88.975226	-88.781233	0.193993
1.75	-82.602995	-82.439717	0.163278
1.875	-77.083440	-76.943736	0.139704

Table 23: P115 2nd Order FDM T-prime values, △x = 1/9

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1111	-130.240270	-129.843852	0.396418
1.2222	-118.348767	-118.040831	0.307936
1.3333	-108.450507	-108.204833	0.245674
1.4444	-100.082412	-99.881961	0.200451
1.5556	-92.914706	-92.742031	0.172675
1.6667	-86.706120	-86.559971	0.146149
1.7778	-81.276040	-81.150582	0.125458
1.8889	-76.486559	-76.377523	0.109036

Table 24: P115 2nd Order FDM T-prime values, △x = 1/10

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1	-131.484785	-131.154095	0.330690
1.2	-120.487876	-120.224587	0.263289
1.3	-111.190972	-110.976542	0.214430
1.4	-103.227693	-103.049646	0.178047
1.5	-96.329990	-96.179669	0.150321
1.6	-90.297202	-90.168440	0.128762
1.7	-84.976117	-84.864414	0.111703
1.8	-80.247717	-80.149724	0.097993
1.9	-76.018136	-75.931318	0.086818

Table 25: P115 2nd Order FDM T-prime values, △x = 1/11

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0909	-132.526330	-132.248147	0.278183
1.1818	-122.300533	-122.076074	0.224459
1.2727	-113.541609	-113.357039	0.184570
1.3636	-105.954727	-105.800458	0.154269
1.4545	-99.319179	-99.188384	0.130795
1.5455	-93.466441	-93.348110	0.118331
1.6364	-88.265606	-88.162738	0.102868
1.7273	-83.613430	-83.523131	0.090299
1.8182	-79.427392	-79.347434	0.079958
1.9091	-75.640737	-75.569380	0.071357

Table 26: P115 2nd Order FDM T-prime values, △x = 1/12

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0833	-133.410847	-133.175948	0.234899
1.1667	-123.856224	-123.656042	0.200182
1.25	-115.580117	-115.415603	0.164514
1.3333	-108.341766	-108.204833	0.136933
1.4167	-101.957341	-101.834901	0.122440
1.5	-96.284039	-96.179669	0.104370
1.5833	-91.209239	-91.119500	0.089739
1.6667	-86.642923	-86.559971	0.082952
1.75	-82.512272	-82.439717	0.072555
1.8333	-78.757745	-78.693888	0.063857
1.9167	-75.330184	-75.269737	0.060447

Table 27: P115 2nd Order FDM T-prime values, $\Delta x = 1/13$

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0769	-134.171378	-133.967410	0.203968
1.1538	-125.206009	-125.038572	0.167437
1.2308	-117.364825	-117.216042	0.148783
1.3077	-110.448683	-110.323089	0.125594
1.3846	-104.302890	-104.195800	0.107090
1.4615	-98.805444	-98.713311	0.092133
1.5385	-93.858830	-93.772833	0.085997
1.6154	-89.384165	-89.308842	0.075323
1.6923	-85.316949	-85.250549	0.066400
1.7692	-81.603929	-81.545051	0.058878
1.8462	-78.200748	-78.144028	0.056720
1.9231	-75.070163	-75.019242	0.050921

Table 28: P115 2nd Order FDM T-prime values, $\Delta x = 1/14$

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0714	-134.832296	-134.655128	0.177168
1.1429	-126.388253	-126.231082	0.157171
1.2143	-118.940374	-118.808782	0.131592
1.2857	-112.322070	-112.210861	0.111209
1.3571	-106.401961	-106.307202	0.094759
1.4286	-101.075034	-100.986633	0.088401
1.5	-96.256340	-96.179669	0.076671
1.5714	-91.876422	-91.809536	0.066886
1.6429	-87.877932	-87.813929	0.064003
1.7143	-84.213102	-84.156509	0.056593
1.7857	-80.841826	-80.791569	0.050257
1.8571	-77.730179	-77.685372	0.044807
1.9286	-74.849270	-74.805301	0.043969

Table 29: P115 2nd Order FDM T-prime values, $\Delta x = 1/15$

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0667	-135.411979	-135.248434	0.163545
1.1333	-127.432344	-127.300365	0.131979
1.2	-120.341514	-120.224587	0.116927
1.2667	-113.998728	-113.893980	0.104748
1.3333	-108.291474	-108.204833	0.086641
1.4	-103.128741	-103.049646	0.079095
1.4667	-98.436111	-98.363335	0.072776
1.5333	-94.152141	-94.090852	0.061289
1.6	-90.225654	-90.168440	0.057214
1.6667	-86.613679	-86.559971	0.053708
1.7333	-83.279870	-83.234007	0.045863
1.8	-80.193273	-80.149724	0.043549
1.8667	-77.327367	-77.285854	0.041513
1.9333	-74.659292	-74.623444	0.035848

Table 30: P115 2nd Order FDM T-prime values, $\Delta x = 1/16$

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0625	-135.924541	-135.783063	0.141478
1.125	-128.361172	-128.239559	0.121613
1.1875	-121.595697	-121.490109	0.105588
1.25	-115.508108	-115.415603	0.092505
1.3125	-110.001326	-109.919622	0.081704
1.375	-104.995975	-104.923276	0.072699
1.4375	-100.426517	-100.361394	0.065123
1.5	-96.238366	-96.179669	0.058697
1.5625	-92.385687	-92.332483	0.053204
1.625	-88.829709	-88.781233	0.048476
1.6875	-85.537419	-85.493039	0.044380
1.75	-82.480526	-82.439717	0.040809
1.8125	-79.634649	-79.596968	0.037681
1.875	-76.978660	-76.943736	0.034924
1.9375	-74.494163	-74.461680	0.032483

Table 31: P115 2nd Order FDM Quadratic Extrapolation using $\Delta x = 1/8$ and $\Delta x = 1/16$

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.125	-128.239243	-128.239559	0.000316
1.25	-115.415426	-115.415603	0.000177
1.375	-104.923175	-104.923276	0.000101
1.5	-96.179613	-96.179669	0.000056
1.625	-88.781203	-88.781233	0.000030
1.75	-82.439703	-82.439717	0.000014
1.875	-76.943733	-76.943736	0.000003

Table 32: P115 2nd Order FDM Richardson's Extrapolation using Δx = 1/4, Δx = 1/8 and Δx = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.25	-115.416322	-115.415603	0.000719
1.5	-96.179904	-96.179669	0.000235
1.75	-82.439776	-82.439717	0.000059

Table 33: P115 2nd Order FDM T-prime at x = 2

△x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.5	-74.074074	-72.134752	1.939322
0.3333	-72.996460	-72.134752	0.861708
0.25	-72.619551	-72.134752	0.484799
0.2	-72.445067	-72.134752	0.310315
0.1667	-72.350269	-72.134752	0.215517
0.1429	-72.293100	-72.134752	0.158348
0.125	-72.255993	-72.134752	0.121241
0.1111	-72.230550	-72.134752	0.095798
0.1	-72.212350	-72.134752	0.077598
0.0909	-72.198884	-72.134752	0.064132
0.0833	-72.188641	-72.134752	0.053889
0.0769	-72.180670	-72.134752	0.045918
0.0714	-72.174345	-72.134752	0.039593
0.0667	-72.169242	-72.134752	0.034490
0.0625	-72.165066	-72.134752	0.030314

Table 34: P115 4th Order FDM Temp. values, △x = 1/2

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.5	41.517857	41.503750	0.014107
2	0.000000	0.000000	0.000000

Table 35: P115 4th Order FDM Temp. values, △x = 1/3

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.3333	58.499986	58.499857	0.000129
1.6667	26.305750	26.300555	0.005195
2	0.000000	0.000000	0.000000

Table 36: P115 4th Order FDM Temp. values, △x = 1/4

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.25	67.808422	67.807191	0.001231
1.5	41.504843	41.503750	0.001093
1.75	19.265098	19.264508	0.000590
2	0.000000	0.000000	0.000000

Table 37: P115 4th Order FDM Temp. values, △x = 1/5

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.2	73.697048	73.696559	0.000489
1.4	51.457833	51.457317	0.000516
1.6	32.193195	32.192809	0.000386
1.8	15.200509	15.200309	0.000200
2	0.000000	0.000000	0.000000

Table 38: P115 4th Order FDM Temp. values, △x = 1/6

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1667	77.760981	77.756636	0.004345
1.3333	58.496512	58.499857	0.003345
1.5	41.503976	41.503750	0.000226
1.6667	26.303601	26.300555	0.003046
1.8333	12.553170	12.555711	0.002541
2	0.000000	0.000000	0.000000

Table 39: P115 4th Order FDM Temp. values, △x = 1/7

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1429	80.735605	80.730082	0.005523
1.2857	63.743135	63.744595	0.001460
1.4286	48.542818	48.539797	0.003021
1.5714	34.792440	34.794953	0.002513
1.7143	22.239318	22.238040	0.001278
1.8571	10.691559	10.694850	0.003291
2	0.000000	0.000000	0.000000

Table 40: P115 4th Order FDM Temp. values, △x = 1/8

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.125	83.007562	83.007500	0.000062
1.25	67.807273	67.807191	0.000082
1.375	54.056921	54.056838	0.000083
1.5	41.503823	41.503750	0.000073
1.625	29.956086	29.956028	0.000058
1.75	19.264547	19.264508	0.000039
1.875	9.310960	9.310940	0.000020
2	0.000000	0.000000	0.000000

Table 41: P115 4th Order FDM Temp. values, Δx = 1/9

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1111	84.799727	84.801133	0.001406
1.2222	71.049389	71.051961	0.002572
1.3333	58.496303	58.499857	0.003554
1.4444	46.948577	46.952967	0.004390
1.5556	36.257050	36.252886	0.004164
1.6667	26.303473	26.300555	0.002918
1.7778	16.992522	16.990697	0.001825
1.8889	8.246227	8.245367	0.000860
2	0.000000	0.000000	0.000000

Table 42: P115 4th Order FDM Temp. values, Δx = 1/10

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1	86.249670	86.249648	0.000022
1.2	73.696591	73.696559	0.000032
1.3	62.148873	62.148838	0.000035
1.4	51.457351	51.457317	0.000034
1.5	41.503780	41.503750	0.000030
1.6	32.192835	32.192809	0.000026
1.7	23.446545	23.446525	0.000020
1.8	15.200322	15.200309	0.000013
1.9	7.400065	7.400058	0.000007
2	0.000000	0.000000	0.000000

Table 43: P115 4th Order FDM Temp. values, Δx = 1/11

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0909	87.446926	87.448114	0.001188
1.1818	75.899211	75.901410	0.002199
1.2727	65.207693	65.210761	0.003068
1.3636	55.254126	55.257950	0.003824
1.4545	45.943184	45.947670	0.004486
1.5455	37.196897	37.192635	0.004262
1.6364	28.950677	28.947456	0.003221
1.7273	21.150423	21.148133	0.002290
1.8182	13.750360	13.748910	0.001450
1.9091	6.711424	6.710733	0.000691
2	0.000000	0.000000	0.000000

Table 44: P115 4th Order FDM Temp. values, Δx = 1/12

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0833	88.452288	88.456717	0.004429
1.1667	77.760772	77.756636	0.004136
1.25	67.807207	67.807191	0.000016
1.3333	58.496267	58.499857	0.003590
1.4167	49.749982	49.746571	0.003411
1.5	41.503765	41.503750	0.000015
1.5833	33.703511	33.706536	0.003025
1.6667	26.303451	26.300555	0.002896
1.75	19.264516	19.264508	0.000008
1.8333	12.553093	12.555711	0.002618
1.9167	6.140057	6.137545	0.002512
2	0.000000	0.000000	0.000000

Table 45: P115 4th Order FDM Temp. values, Δx = 1/13

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0769	89.308486	89.311571	0.003085
1.1538	79.354922	79.360683	0.005761
1.2308	70.043984	70.040365	0.003619
1.3077	61.297700	61.296839	0.000861
1.3846	53.051484	53.053075	0.001591
1.4615	45.251232	45.255017	0.003785
1.5385	37.851172	37.847556	0.003616
1.6154	30.812238	30.810856	0.001382
1.6923	24.100817	24.101466	0.000649
1.7692	17.687781	17.690285	0.002504
1.8462	11.547725	11.544115	0.003610
1.9231	5.658355	5.656622	0.001733
2	0.000000	0.000000	0.000000

Table 46: P115 4th Order FDM Temp. values, ⊿x = 1/14

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0714	90.046437	90.050280	0.003843
1.1429	80.735499	80.730082	0.005417
1.2143	71.989217	71.987511	0.001706
1.2857	63.743001	63.744595	0.001594
1.3571	55.942750	55.947297	0.004547
1.4286	48.542691	48.539797	0.002894
1.5	41.503758	41.503750	0.000008
1.5714	34.792337	34.794953	0.002616
1.6429	28.379303	28.375533	0.003770
1.7143	22.239247	22.238040	0.001207
1.7857	16.349877	16.351027	0.001150
1.8571	10.691523	10.694850	0.003327
1.9286	5.246743	5.244605	0.002138
2	0.000000	0.000000	0.000000

Table 47: P115 4th Order FDM Temp. values, ⊿x = 1/15

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0667	90.689063	90.684551	0.004512
1.1333	81.942781	81.947019	0.004238
1.2	73.696566	73.696559	0.000007
1.2667	65.896315	65.892512	0.003803
1.3333	58.496257	58.499857	0.003600
1.4	51.457324	51.457317	0.000007
1.4667	44.745904	44.742619	0.003285
1.5333	38.332870	38.336000	0.003130
1.6	32.192814	32.192809	0.000005
1.6667	26.303445	26.300555	0.002890
1.7333	20.645091	20.647862	0.002771
1.8	15.200312	15.200309	0.000003
1.8667	9.953569	9.950991	0.002578
1.9333	4.890961	4.893447	0.002486
2	0.000000	0.000000	0.000000

Table 48: P115 4th Order FDM Temp. values, ⊿x = 1/16

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0625	91.253718	91.253716	0.000002
1.125	83.007504	83.007500	0.000004
1.1875	75.207253	75.207249	0.000004
1.25	67.807196	67.807191	0.000005
1.3125	60.768263	60.768258	0.000005
1.375	54.056843	54.056838	0.000005
1.4375	47.643809	47.643804	0.000005
1.5	41.503755	41.503750	0.000005
1.5625	35.614385	35.614381	0.000004
1.625	29.956032	29.956028	0.000004
1.6875	24.511253	24.511250	0.000003
1.75	19.264510	19.264508	0.000002
1.8125	14.201902	14.201900	0.000002
1.875	9.310942	9.310940	0.000002
1.9375	4.580370	4.580369	0.000001
2	0.000000	0.000000	0.000000

Table 49: P115 4th Order FDM T-prime values, ⊿x = 1/2

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.5	-96.428571	-96.179669	0.248902

Table 50: P115 4th Order FDM T-prime values, ⊿x = 1/3

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.3333	-108.285428	-108.204833	0.080595
1.6667	-86.595373	-86.559971	0.035402

Table 51: P115 4th Order FDM T-prime values, ⊿x = 1/4

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.25	-115.450967	-115.415603	0.035364
1.5	-96.195945	-96.179669	0.016276
1.75	-82.448810	-82.439717	0.009093

Table 51: P115 4th Order FDM T-prime values, ⊿x = 1/5

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.2	-120.242064	-120.224587	0.017477
1.4	-103.058556	-103.049646	0.008910
1.6	-90.173655	-90.168440	0.005215
1.8	-80.153139	-80.149724	0.003415

Table 52: P115 4th Order FDM T-prime values, △x = 1/6

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1667	-123.669177	-123.656042	0.013135
1.3333	-108.207433	-108.204833	0.002600
1.5	-96.182919	-96.179669	0.003250
1.6667	-86.563872	-86.559971	0.003901
1.8333	-78.694014	-78.693888	0.000126

Table 53: P115 4th Order FDM T-prime values, △x = 1/7

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1429	-126.241521	-126.231082	0.010439
1.2857	-112.212974	-112.210861	0.002113
1.4286	-100.990797	-100.986633	0.004164
1.5714	-91.809331	-91.809536	0.000205
1.7143	-84.158272	-84.156509	0.001763
1.8571	-77.684388	-77.685372	0.000984

Table 54: P115 4th Order FDM T-prime values, △x = 1/8

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.125	-128.243158	-128.239559	0.003599
1.25	-115.417836	-115.415603	0.002233
1.375	-104.924752	-104.923276	0.001476
1.5	-96.180702	-96.179669	0.001033
1.625	-88.781991	-88.781233	0.000758
1.75	-82.440297	-82.439717	0.000580
1.875	-76.944197	-76.943736	0.000461

Table 55: P115 4th Order FDM T-prime values, △x = 1/9

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1111	-129.844935	-129.843852	0.001083
1.2222	-118.040227	-118.040831	0.000604
1.3333	-108.203180	-108.204833	0.001653
1.4444	-99.879639	-99.881961	0.002322
1.5556	-92.745240	-92.742031	0.003209
1.6667	-86.562134	-86.559971	0.002163
1.7778	-81.151940	-81.150582	0.001358
1.8889	-76.378254	-76.377523	0.000731

Table 56: P115 4th Order FDM T-prime values, △x = 1/10

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1	-131.155732	-131.154095	0.001637
1.2	-120.225686	-120.224587	0.001099
1.3	-110.977312	-110.976542	0.000770
1.4	-103.050207	-103.049646	0.000561
1.5	-96.180093	-96.179669	0.000424
1.6	-90.168770	-90.168440	0.000330
1.7	-84.864678	-84.864414	0.000264
1.8	-80.149941	-80.149724	0.000217
1.9	-75.931500	-75.931318	0.000182

Table 57: P115 4th Order FDM T-prime values, △x = 1/11

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0909	-132.248208	-132.248147	0.000061
1.1818	-122.075001	-122.076074	0.001073
1.2727	-113.355188	-113.357039	0.001851
1.3636	-105.798065	-105.800458	0.002393
1.4545	-99.185612	-99.188384	0.002772
1.5455	-93.351113	-93.348110	0.003003
1.6364	-88.164904	-88.162738	0.002166
1.7273	-83.524620	-83.523131	0.001489
1.8182	-79.348370	-79.347434	0.000936
1.9091	-75.569863	-75.569380	0.000483

Table 58: P115 4th Order FDM T-prime values, △x = 1/12

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0833	-133.172698	-133.175948	0.003250
1.1667	-123.660178	-123.656042	0.004136
1.25	-115.416045	-115.415603	0.000442
1.3333	-108.202461	-108.204833	0.002372
1.4167	-101.837555	-101.834901	0.002654
1.5	-96.179874	-96.179669	0.000205
1.5833	-91.117747	-91.119500	0.001753
1.6667	-86.561839	-86.559971	0.001868
1.75	-82.439832	-82.439717	0.000115
1.8333	-78.692555	-78.693888	0.001333
1.9167	-75.271131	-75.269737	0.001394

Table 59: P115 4th Order FDM T-prime values, Δx = 1/13

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0769	-133.965173	-133.967410	0.002237
1.1538	-125.034031	-125.038572	0.004541
1.2308	-117.219316	-117.216042	0.003274
1.3077	-110.324002	-110.323089	0.000913
1.3846	-104.194848	-104.195800	0.000952
1.4615	-98.710878	-98.713311	0.002433
1.5385	-93.775312	-93.772833	0.002479
1.6154	-89.309805	-89.308842	0.000963
1.6923	-85.250256	-85.250549	0.000293
1.7692	-81.543713	-81.545051	0.001338
1.8462	-78.146051	-78.144028	0.002023
1.9231	-75.020204	-75.019242	0.000962

Table 60: P115 4th Order FDM T-prime values, Δx = 1/14

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0714	-134.652019	-134.655128	0.003109
1.1429	-126.236174	-126.231082	0.005092
1.2143	-118.810451	-118.808782	0.001669
1.2857	-112.209825	-112.210861	0.001036
1.3571	-106.304012	-106.307202	0.003190
1.4286	-100.988787	-100.986633	0.002154
1.5	-96.179780	-96.179669	0.000111
1.5714	-91.807958	-91.809536	0.001578
1.6429	-87.816298	-87.813929	0.002369
1.7143	-84.157278	-84.156509	0.000769
1.7857	-80.790980	-80.791569	0.000589
1.8571	-77.683630	-77.685372	0.001742
1.9286	-74.806455	-74.805301	0.001154

Table 61: P115 4th Order FDM T-prime values, Δx = 1/15

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0667	-135.253034	-135.248434	0.004600
1.1333	-127.296903	-127.300365	0.003462
1.2	-120.224804	-120.224587	0.000217
1.2667	-113.897148	-113.893980	0.003168
1.3333	-108.202265	-108.204833	0.002568
1.4	-103.049757	-103.049646	0.000111
1.4667	-98.365663	-98.363335	0.002328
1.5333	-94.088884	-94.090852	0.001968
1.6	-90.168505	-90.168440	0.000065
1.6667	-86.561759	-86.559971	0.001788
1.7333	-83.232455	-83.234007	0.001552
1.8	-80.149767	-80.149724	0.000043
1.8667	-77.287272	-77.285854	0.001418
1.9333	-74.622191	-74.623444	0.001253

Table 62: P115 4th Order FDM T-prime values, Δx = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0625	-135.783357	-135.783063	0.000294
1.125	-128.239785	-128.239559	0.000226
1.1875	-121.490285	-121.490109	0.000176
1.25	-115.415743	-115.415603	0.000140
1.3125	-109.919735	-109.919622	0.000113
1.375	-104.923368	-104.923276	0.000092
1.4375	-100.361471	-100.361394	0.000077
1.5	-96.179734	-96.179669	0.000065
1.5625	-92.332538	-92.332483	0.000055
1.625	-88.781281	-88.781233	0.000048
1.6875	-85.493081	-85.493039	0.000042
1.75	-82.439753	-82.439717	0.000036
1.8125	-79.597000	-79.596968	0.000032
1.875	-76.943765	-76.943736	0.000029
1.9375	-74.461706	-74.461680	0.000026

Table 63: P115 4th Order FDM Quartic Extrapolation using Δx = 1/8 and Δx = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.125	-128.23955977	-128.239559	0.00000058
1.25	-115.41560368	-115.415603	0.00000041
1.375	-104.92327603	-104.923276	0.00000032
1.5	-96.17966967	-96.179669	0.00000027
1.625	-88.78123352	-88.781233	0.00000024
1.75	-82.43971684	-82.439717	0.00000021
1.875	-76.94373571	-76.943736	0.00000020

Table 64: P115 4th Order FDM Richardson's Extrapolation using △x = 1/4, △x = 1/8 and △x = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.25	-115.41560208	-115.415603	119E-6
1.5	-96.17966860	-96.179669	7.9E-7
1.75	-82.43971599	-82.439717	6.3E-7

Table 65: P115 4th Order FDM T-prime at x = 2

△x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.5	-72.221323	-72.134752	0.086571
0.3333	-72.152914	-72.134752	0.018162
0.25	-72.140644	-72.134752	0.005892
0.2	-72.137196	-72.134752	0.002444
0.1667	-72.135939	-72.134752	0.001187
0.1429	-72.135396	-72.134752	0.000643
0.125	-72.135130	-72.134752	0.000378
0.1111	-72.134989	-72.134752	0.000237
0.1	-72.134908	-72.134752	0.000155
0.0909	-72.134858	-72.134752	0.000106
0.0833	-72.134827	-72.134752	0.000075
0.0769	-72.134807	-72.134752	0.000055
0.0714	-72.134793	-72.134752	0.000041
0.0667	-72.134783	-72.134752	0.000031
0.0625	-72.134776	-72.134752	0.000024

Table 66: P116 2nd Order FDM Temp. values, △x = 1

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
2	0.137931	12.608000	12.470069

Table 67: P116 2nd Order FDM Temp. values, △x = 1/2

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.5	49.193548	48.878957	0.314591
2	12.903226	12.608000	0.295226

Table 68: P116 2nd Order FDM Temp. values, △x = 1/3

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.3333	63.854407	63.732195	0.122212
1.6667	35.741167	35.592582	0.148585
2	12.739426	12.608000	0.131426

Table 69: P116 2nd Order FDM Temp. values, △x = 1/4

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.25	71.927888	71.866060	0.061828
1.5	48.959796	48.878957	0.080839
1.75	29.525257	29.443639	0.081618
2	12.681990	12.608000	0.073990

Table 70: P116 2nd Order FDM Temp. values, △x = 1/5

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.2	77.047753	77.012897	0.034856
1.4	57.626620	57.577579	0.049041
1.6	40.794972	40.741940	0.053032
1.8	25.943518	25.891855	0.051663
2	12.655375	12.608000	0.047375

Table 71: P116 2nd Order FDM Temp. values, △x = 1/6

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1667	80.586213	80.561079	0.025134
1.3333	63.760931	63.732195	0.028736
1.5	48.915094	48.878957	0.036137
1.6667	35.631976	35.592582	0.039394
1.8333	23.613918	23.580688	0.033230
2	12.640908	12.608000	0.032908

Table 72: P116 2nd Order FDM Temp. values, △x = 1/7

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1429	83.178575	83.159634	0.018941
1.2857	68.336142	68.315677	0.020465
1.4286	55.056070	55.027900	0.028170
1.5714	43.040766	43.016006	0.024760
1.7143	32.070272	32.042268	0.028004
1.8571	21.977417	21.954443	0.022974
2	12.632181	12.608000	0.024181

Table 73: P116 2nd Order FDM Temp. values, △x = 1/8

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.125	85.159782	85.149914	0.009868
1.25	71.881693	71.866060	0.015633
1.375	59.868183	59.849352	0.018831
1.5	48.899326	48.878957	0.020369
1.625	38.807978	38.787172	0.020806
1.75	29.464138	29.443639	0.020499
1.875	20.764700	20.745017	0.019683
2	12.626516	12.608000	0.018516

Table 74: P116 2nd Order FDM Temp. values, △x = 1/9

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1111	86.723272	86.717407	0.005865
1.2222	74.710995	74.701730	0.009265
1.3333	63.743264	63.732195	0.011069
1.4444	53.652951	53.641138	0.011813
1.5556	44.310068	44.290122	0.019946
1.6667	35.611523	35.592582	0.018941
1.7778	27.474174	27.456510	0.017664
1.8889	19.829997	19.813792	0.016205
2	12.622631	12.608000	0.014631

Table 75: P116 2nd Order FDM Temp. values, △x = 1/10

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1	87.988605	87.983292	0.005313
1.2	77.021680	77.012897	0.008783
1.3	66.932109	66.921112	0.010997
1.4	57.589913	57.577579	0.012334
1.5	48.892006	48.878957	0.013049
1.6	40.755255	40.741940	0.013315
1.7	33.111640	33.098388	0.013252
1.8	25.904804	25.891855	0.012949
1.9	19.087526	19.075059	0.012467
2	12.619852	12.608000	0.011852

Table 76: P116 2nd Order FDM Temp. values, △x = 1/11

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0909	89.033671	89.030656	0.003015
1.1818	78.944649	78.939760	0.004889
1.2727	69.602962	69.596989	0.005973
1.3636	60.905529	60.899027	0.006502
1.4545	52.769221	52.762588	0.006633
1.5455	45.126022	45.111387	0.014635
1.6364	37.919578	37.905761	0.013817
1.7273	31.102671	31.089776	0.012895
1.8182	24.635349	24.623448	0.011901
1.9091	18.483506	18.472644	0.010862
2	12.617795	12.608000	0.009795

Table 77: P116 2nd Order FDM Temp. values, △x = 1/12

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0833	89.911396	89.912095	0.000699
1.1667	80.570096	80.561079	0.009017
1.25	71.873023	71.866060	0.006963
1.3333	63.737052	63.732195	0.004857
1.4167	56.094170	56.082524	0.011646
1.5	48.888024	48.878957	0.009067
1.5833	42.071400	42.064816	0.006584
1.6667	35.604346	35.592582	0.011764
1.75	29.452758	29.443639	0.009119
1.8333	23.587290	23.580688	0.006602
1.9167	17.982510	17.971724	0.010786
2	12.616231	12.608000	0.008231

Table 78: P116 2nd Order FDM Temp. values, △x = 1/13

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0769	90.659001	90.659168	0.000167
1.1538	81.962209	81.962888	0.000679
1.2308	73.826501	73.817676	0.008825
1.3077	66.183865	66.176534	0.007331
1.3846	58.977952	58.972143	0.005809
1.4615	52.161547	52.157265	0.004282
1.5385	45.694702	45.683736	0.010966
1.6154	39.543313	39.534223	0.009090
1.6923	33.678034	33.670753	0.007281
1.7692	28.073435	28.067894	0.005541
1.8462	22.707329	22.696633	0.010696
1.9231	17.560248	17.551435	0.008813
2	12.615014	12.608000	0.007014

Table 79: P116 2nd Order FDM Temp. values, △x = 1/14

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0714	91.303431	91.304741	0.001310
1.1429	83.167931	83.159634	0.008297
1.2143	75.525491	75.519326	0.006165
1.2857	68.319762	68.315677	0.004085
1.3571	61.503532	61.501462	0.002070
1.4286	55.036853	55.027900	0.008953
1.5	48.885621	48.878957	0.006664
1.5714	43.020493	43.016006	0.004487
1.6429	37.416037	37.405946	0.010091
1.7143	32.050069	32.042268	0.007801
1.7857	26.903120	26.897490	0.005630
1.8571	21.958012	21.954443	0.003569
1.9286	17.199511	17.191365	0.008146
2	12.614048	12.608000	0.006048

Table 80: P116 2nd Order FDM Temp. values, △x = 1/15

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0667	91.864668	91.859043	0.005625
1.1333	84.222386	84.223139	0.000753
1.2	77.016807	77.012897	0.003910
1.2667	70.200718	70.192784	0.007934
1.3333	63.734172	63.732195	0.001977
1.4	57.583067	57.577579	0.005488
1.4667	51.718060	51.709470	0.008590
1.5333	46.113720	46.110598	0.003122
1.6	40.747863	40.741940	0.005923
1.6667	35.601020	35.592582	0.008438
1.7333	30.656014	30.652580	0.003434
1.8	25.897613	25.891855	0.005758
1.8667	21.312244	21.304371	0.007873
1.9333	16.887765	16.884482	0.003283
2	12.613268	12.608000	0.005268

Table 81: P116 2nd Order FDM Temp. values, △x = 1/16

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0625	92.357848	92.356447	0.001401
1.125	85.152390	85.149914	0.002476
1.1875	78.336416	78.333119	0.003297
1.25	71.869980	71.866060	0.003920
1.3125	65.718979	65.714596	0.004383
1.375	59.854072	59.849352	0.004720
1.4375	54.249827	54.244874	0.004953
1.5	48.884060	48.878957	0.005103
1.5625	43.737304	43.732120	0.005184
1.625	38.792382	38.787172	0.005210
1.6875	34.034061	34.028872	0.005189
1.75	29.448770	29.443639	0.005131
1.8125	25.024366	25.019325	0.005041
1.875	20.749942	20.745017	0.004925
1.9375	16.615663	16.610876	0.004787
2	12.612630	12.608000	0.004630

Table 82: P116 2nd Order FDM T-prime values, △x = 1/2

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.5	-87.096774	-84.053336	3.043438

Table 83: P116 2nd Order FDM T-prime values, △x = 1/3

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.3333	-96.388249	-94.562367	1.825882
1.6667	-76.672471	-75.646490	1.025981

Table 84: P116 2nd Order FDM T-prime values, △x = 1/4

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.25	-102.080408	-100.864004	1.216404
1.5	-84.805262	-84.053336	0.751926
1.75	-72.555613	-72.045717	0.509896

Table 85: P116 2nd Order FDM T-prime values, △x = 1/5

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.2	-105.933450	-105.066670	0.866780
1.4	-90.631951	-90.057146	0.574805
1.6	-79.207756	-78.800003	0.407753
1.8	-70.348994	-70.044447	0.304547

Table 87: P116 2nd Order FDM T-prime values, △x = 1/6

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1667	-108.717207	-108.065488	0.651719
1.3333	-95.013358	-94.562367	0.450991
1.5	-84.386864	-84.053336	0.333528
1.6667	-75.903528	-75.646490	0.257038
1.8333	-68.973206	-68.772162	0.201044

Table 88: P116 2nd Order FDM T-prime values, △x = 1/7

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1429	-110.823504	-110.315867	0.507637
1.2857	-98.428770	-98.063315	0.365455
1.4286	-88.533814	-88.254238	0.279576
1.5714	-80.450292	-80.234189	0.216103
1.7143	-73.721722	-73.546056	0.175666
1.8571	-68.033318	-67.890800	0.142518

Table 89: P116 2nd Order FDM T-prime values, △x = 1/8

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.125	-112.473229	-112.071115	0.402114
1.25	-101.166397	-100.864004	0.302393
1.375	-91.929465	-91.694549	0.234916
1.5	-84.240819	-84.053336	0.187483
1.625	-77.740756	-77.587695	0.153061
1.75	-72.173115	-72.045717	0.127398
1.875	-67.350488	-67.242669	0.107819

Table 90: P116 2nd Order FDM T-prime values, △x = 1/9

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1111	-113.800522	-113.473139	0.327383
1.2222	-103.410039	-103.158243	0.251796
1.3333	-94.761200	-94.562367	0.198833
1.4444	-87.449379	-87.288843	0.160536
1.5556	-81.186426	-81.049116	0.137310
1.6667	-75.761526	-75.646490	0.115036
1.7778	-71.016865	-70.919116	0.097749
1.8889	-66.831943	-66.747845	0.084098

Table 91: P116 2nd Order FDM T-prime values, △x = 1/10

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1	-114.891600	-114.618186	0.273414
1.2	-105.282485	-105.066670	0.215815
1.3	-97.158836	-96.984619	0.174217
1.4	-90.200511	-90.057146	0.143365
1.5	-84.173288	-84.053336	0.119952
1.6	-78.901829	-78.800003	0.101826
1.7	-74.252257	-74.164708	0.087549
1.8	-70.120574	-70.044447	0.076127
1.9	-66.424760	-66.357897	0.066863

Table 92: P116 2nd Order FDM T-prime values, △x = 1/11

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0909	-115.804429	-115.574300	0.230129
1.1818	-106.868902	-106.684722	0.184180
1.2727	-99.215161	-99.064983	0.150178
1.3636	-92.585577	-92.461136	0.124441
1.4545	-86.787288	-86.682712	0.104576
1.5455	-81.673037	-81.578780	0.094257
1.6364	-77.128433	-77.047179	0.081254
1.7273	-73.063259	-72.992534	0.070725
1.8182	-69.405406	-69.343309	0.062097
1.9091	-66.096544	-66.041593	0.054951

Table 93: P116 2nd Order FDM T-prime values, △x = 1/12

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0833	-116.579427	-116.385124	0.194303
1.1667	-108.230237	-108.065488	0.164749
1.25	-100.998262	-100.864004	0.134258
1.3333	-94.673119	-94.562367	0.110752
1.4167	-89.094167	-88.995556	0.098611
1.5	-84.136622	-84.053336	0.083286
1.5833	-79.702071	-79.631153	0.070918
1.6667	-75.711852	-75.646490	0.065362
1.75	-72.102333	-72.045717	0.056616
1.8333	-68.821486	-68.772162	0.049324
1.9167	-65.826354	-65.779728	0.046626

Table 94: P116 2nd Order FDM T-prime values, △x = 1/13

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0769	-117.245640	-117.076799	0.168841
1.1538	-109.411254	-109.273708	0.137546
1.2308	-102.559236	-102.437443	0.121793
1.3077	-96.515567	-96.413554	0.102013
1.3846	-91.145066	-91.058793	0.086273
1.4615	-86.341124	-86.267536	0.073588
1.5385	-82.018526	-81.949954	0.068572
1.6154	-78.108340	-78.048783	0.059557
1.6923	-74.554204	-74.502159	0.052045
1.7692	-71.309582	-71.263851	0.045731
1.8462	-68.335713	-68.291628	0.044085
1.9231	-65.600052	-65.560816	0.039236

Table 95: P116 2nd Order FDM T-prime values, △x = 1/14

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0714	-117.824486	-117.677809	0.146677
1.1429	-110.445579	-110.315867	0.129712
1.2143	-103.937179	-103.829370	0.107809
1.2857	-98.153711	-98.063315	0.090396
1.3571	-92.980367	-92.903990	0.076377
1.4286	-88.325381	-88.254238	0.071143
1.5	-84.114520	-84.053336	0.061184
1.5714	-80.287086	-80.234189	0.052897
1.6429	-76.792968	-76.742349	0.050619
1.7143	-73.590422	-73.546056	0.044366
1.7857	-70.644400	-70.605367	0.039033
1.8571	-67.925257	-67.890800	0.034457
1.9286	-65.407747	-65.373849	0.033898

Table 96: P116 2nd Order FDM T-prime values, △x = 1/15

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0667	-118.332102	-118.196310	0.135792
1.1333	-111.358961	-111.250335	0.108626
1.2	-105.162516	-105.066670	0.095846
1.2667	-99.619762	-99.534226	0.085536
1.3333	-94.632380	-94.562367	0.070013
1.4	-90.120836	-90.057146	0.063690
1.4667	-86.020100	-85.961686	0.058414
1.5333	-82.276479	-82.227877	0.048602
1.6	-78.845250	-78.800003	0.045247
1.6667	-75.688864	-75.646490	0.042374
1.7333	-72.775556	-72.739863	0.035693
1.8	-70.078280	-70.044447	0.033833
1.8667	-67.573858	-67.541653	0.032205
1.9333	-65.242315	-65.214920	0.027395

Table 97: P116 2nd Order FDM T-prime values, △x = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0625	-118.780881	-118.663534	0.117347
1.125	-112.171452	-112.071115	0.100337
1.1875	-106.259281	-106.172635	0.086646
1.25	-100.939497	-100.864004	0.075493
1.3125	-96.127265	-96.060956	0.066309
1.375	-91.753220	-91.694549	0.058671
1.4375	-87.760092	-87.707829	0.052263
1.5	-84.100177	-84.053336	0.046841
1.5625	-80.733421	-80.691203	0.042218
1.625	-77.625946	-77.587695	0.038251
1.6875	-74.748901	-74.714077	0.034824
1.75	-72.077562	-72.045717	0.031845
1.8125	-69.590625	-69.561382	0.029243
1.875	-67.269626	-67.242669	0.026957
1.9375	-65.098490	-65.073551	0.024939

Table 98: P116 2nd Order FDM Quadratic Extrapolation using △x = 1/8 and △x = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.125	-112.070860	-112.071115	0.000255
1.25	-100.863864	-100.864004	0.000140
1.375	-91.694472	-91.694549	0.000077
1.5	-84.053296	-84.053336	0.000040
1.625	-77.587676	-77.587695	0.000019
1.75	-72.045712	-72.045717	0.000005
1.875	-67.242672	-67.242669	0.000003

Table 99: P116 2nd Order FDM Richardson's Extrapolation using △x = 1/4, △x = 1/8 and △x = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.25	-100.864570	-100.864004	0.000566
1.5	-84.053504	-84.053336	0.000168
1.75	-72.045743	-72.045717	0.000026

Table 100: P116 2nd Order FDM T-prime at x = 2

△x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1	-79.889655	-63.040002	16.849653
0.5	-64.516129	-63.040002	1.476127
0.3333	-63.697130	-63.040002	0.657128
0.25	-63.409948	-63.040002	0.369946
0.2	-63.276873	-63.040002	0.236871
0.1667	-63.204538	-63.040002	0.164536
0.1429	-63.160905	-63.040002	0.120903
0.125	-63.132578	-63.040002	0.092576
0.1111	-63.113154	-63.040002	0.073152
0.1	-63.099259	-63.040002	0.059257
0.0909	-63.088976	-63.040002	0.048974
0.0833	-63.081155	-63.040002	0.041153
0.0769	-63.075068	-63.040002	0.035066
0.0714	-63.070238	-63.040002	0.030236
0.0667	-63.066342	-63.040002	0.026340
0.0625	-63.063152	-63.040002	0.023150

Table 102: P116 4th Order FDM Temp. values, △x = 1

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
2	12.782956	12.608000	0.174956

Table 103: P116 4th Order FDM Temp. values, △x = 1/2

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.5	48.882014	48.878957	0.003057
2	12.592147	12.608000	0.015853

Table 104: P116 4th Order FDM Temp. values, △x = 1/3

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.3333	63.731330	63.732195	0.000865
1.6667	35.595386	35.592582	0.002804
2	12.605645	12.608000	0.002355

Table 105: P116 4th Order FDM Temp. values, △x = 1/4

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.25	71.866930	71.866060	0.000870
1.5	48.879538	48.878957	0.000581
1.75	29.443637	29.443639	2.25E-06
2	12.607359	12.608000	0.000641

Table 106: P116 4th Order FDM Temp. values, △x = 1/5

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.2	77.013262	77.012897	0.000365
1.4	57.577914	57.577579	0.000335
1.6	40.742114	40.741940	0.000174
1.8	25.891826	25.891855	0.000029
2	12.607760	12.608000	0.000240

Table 107: P116 4th Order FDM Temp. values, △x = 1/6

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1667	80.564852	80.561079	0.003773
1.3333	63.729227	63.732195	0.002968
1.5	48.879092	48.878957	0.000135
1.6667	35.595163	35.592582	0.002581
1.8333	23.578372	23.580688	0.002316
2	12.607892	12.608000	0.000108

Table 108: P116 4th Order FDM Temp. values, △x = 1/7

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1429	83.164449	83.159634	0.004815
1.2857	68.314380	68.315677	0.001297
1.4286	55.030510	55.027900	0.002610
1.5714	43.013772	43.016006	0.002234
1.7143	32.043341	32.042268	0.001073
1.8571	21.951517	21.954443	0.002926
2	12.607944	12.608000	0.000056

Table 109: P116 4th Order FDM Temp. values, △x = 1/8

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.125	85.149963	85.149914	0.000049
1.25	71.866122	71.866060	0.000062
1.375	59.849410	59.849352	0.000058
1.5	48.879002	48.878957	0.000045
1.625	38.787200	38.787172	0.000028
1.75	29.443647	29.443639	7.99E-06
1.875	20.745006	20.745017	0.000011
2	12.607968	12.608000	0.000032

Table 110: P116 4th Order FDM Temp. values, △x = 1/9

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1111	86.716174	86.717407	0.001233
1.2222	74.699476	74.701730	0.002254
1.3333	63.729081	63.732195	0.003114
1.4444	53.637291	53.641138	0.003847
1.5556	44.293749	44.290122	0.003627
1.6667	35.595117	35.592582	0.002535
1.7778	27.458089	27.456510	0.001579
1.8889	19.814525	19.813792	0.000733
2	12.607981	12.608000	0.000019

Table 111: P116 4th Order FDM Temp. values, △x = 1/10

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.1	87.983310	87.983292	0.000018
1.2	77.012922	77.012897	0.000025
1.3	66.921138	66.921112	0.000026
1.4	57.577602	57.577579	0.000023
1.5	48.878976	48.878957	0.000019
1.6	40.741954	40.741940	0.000014
1.7	33.098395	33.098388	7.22E-06
1.8	25.891855	25.891855	7.62E-07
1.9	19.075053	19.075059	6.22E-06
2	12.607988	12.608000	0.000012

Table 112: P116 4th Order FDM Temp. values, △x = 1/11

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0909	89.029617	89.030656	0.001039
1.1818	78.937837	78.939760	0.001923
1.2727	69.594305	69.596989	0.002684
1.3636	60.895682	60.899027	0.003345
1.4545	52.758663	52.762588	0.003925
1.5455	45.115107	45.111387	0.003720
1.6364	37.908570	37.905761	0.002809
1.7273	31.091771	31.089776	0.001995
1.8182	24.624708	24.623448	0.001260
1.9091	18.473240	18.472644	0.000596
2	12.607992	12.608000	0.000008

Table 113: P116 4th Order FDM Temp. values, △x = 1/12

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0833	89.908223	89.912095	0.003872
1.1667	80.564693	80.561079	0.003614
1.25	71.866073	71.866060	0.000013
1.3333	63.729055	63.732195	0.003140
1.4167	56.085502	56.082524	0.002978
1.5	48.878967	48.878957	0.000010
1.5833	42.062169	42.064816	0.002647
1.6667	35.595108	35.592582	0.002526
1.75	29.443641	29.443639	0.000002
1.8333	23.578395	23.580688	0.002293
1.9167	17.973914	17.971724	0.002190
2	12.607995	12.608000	0.000005

Table 114: P116 4th Order FDM Temp. values, △x = 1/13

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0769	90.656472	90.659168	0.002696
1.1538	81.957853	81.962888	0.005035
1.2308	73.820837	73.817676	0.003161
1.3077	66.177285	66.176534	0.000751
1.3846	58.970751	58.972143	0.001392
1.4615	52.153954	52.157265	0.003311
1.5385	45.686894	45.683736	0.003158
1.6154	39.535428	39.534223	0.001205
1.6923	33.670183	33.670753	0.000570
1.7692	28.065703	28.067894	0.002191
1.8462	22.699785	22.696633	0.003152
1.9231	17.552946	17.551435	0.001511
2	12.607996	12.608000	0.000004

Table 115: P116 4th Order FDM Temp. values, △x = 1/14

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0714	91.301382	91.304741	0.003359
1.1429	83.164367	83.159634	0.004733
1.2143	75.520816	75.519326	0.001490
1.2857	68.314283	68.315677	0.001394
1.3571	61.497487	61.501462	0.003975
1.4286	55.030427	55.027900	0.002527
1.5	48.878962	48.878957	4.61E-06
1.5714	43.013718	43.016006	0.002288
1.6429	37.409238	37.405946	0.003292
1.7143	32.043321	32.042268	0.001053
1.7857	26.896482	26.897490	0.001008
1.8571	21.951533	21.954443	0.002910
1.9286	17.193231	17.191365	0.001866
2	12.607997	12.608000	0.000003

Table 116: P116 4th Order FDM Temp. values, △x = 1/15

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0667	91.862986	91.859043	0.003943
1.1333	84.219435	84.223139	0.003704
1.2	77.012902	77.012897	4.68E-06
1.2667	70.196107	70.192784	0.003323
1.3333	63.729048	63.732195	0.003147
1.4	57.577584	57.577579	5.07E-06
1.4667	51.712339	51.709470	0.002869
1.5333	46.107860	46.110598	0.002738
1.6	40.741943	40.741940	2.63E-06
1.6667	35.595105	35.592582	0.002523
1.7333	30.650157	30.652580	0.002423
1.8	25.891855	25.891855	2.88E-07
1.8667	21.306621	21.304371	0.002250
1.9333	16.882307	16.884482	0.002175
2	12.607998	12.608000	0.000002

Table 117: P116 4th Order FDM Temp. values, △x = 1/16

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
1	100.000000	100.000000	0.000000
1.0625	92.356449	92.356447	1.97E-06
1.125	85.149917	85.149914	3.14E-06
1.1875	78.333123	78.333119	3.77E-06
1.25	71.866064	71.866060	4.02E-06
1.3125	65.714600	65.714596	4.00E-06
1.375	59.849356	59.849352	3.79E-06
1.4375	54.244877	54.244874	3.45E-06
1.5	48.878960	48.878957	3.00E-06
1.5625	43.732123	43.732120	2.49E-06
1.625	38.787174	38.787172	1.94E-06
1.6875	34.028873	34.028872	1.35E-06
1.75	29.443640	29.443639	7.33E-07
1.8125	25.019325	25.019325	1.09E-07
1.875	20.745017	20.745017	5.22E-07
1.9375	16.610875	16.610876	1.16E-06
2	12.607999	12.608000	1.79E-06

Table 118: P116 4th Order FDM T-prime values, △x = 1/2

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.5	-84.286144	-84.053336	0.232808

Table 119: P116 4th Order FDM T-prime values, Δx = 1/3

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.3333	-94.635351	-94.562367	0.072984
1.6667	-75.679468	-75.646490	0.032978

Table 120: P116 4th Order FDM T-prime values, Δx = 1/4

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.25	-100.895649	-100.864004	0.031645
1.5	-84.068177	-84.053336	0.014841
1.75	-72.054193	-72.045717	0.008476

Table 121: P116 4th Order FDM T-prime values, Δx = 1/5

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.2	-105.082233	-105.066670	0.015563
1.4	-90.065180	-90.057146	0.008034
1.6	-78.804777	-78.800003	0.004774
1.8	-70.047623	-70.044447	0.003176

Table 122: P116 4th Order FDM T-prime values, Δx = 1/6

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1667	-108.077102	-108.065488	0.011614
1.3333	-94.564757	-94.562367	0.002390
1.5	-84.056281	-84.053336	0.002945
1.6667	-75.649993	-75.646490	0.003503
1.8333	-68.772359	-68.772162	0.000197

Table 123: P116 4th Order FDM T-prime values, Δx = 1/7

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1429	-110.325060	-110.315867	0.009193
1.2857	-98.065225	-98.063315	0.001910
1.4286	-88.257934	-88.254238	0.003696
1.5714	-80.234062	-80.234189	0.000127
1.7143	-73.547644	-73.546056	0.001588
1.8571	-67.889984	-67.890800	0.000816

Table 124: P116 4th Order FDM T-prime values, Δx = 1/8

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.125	-112.074301	-112.071115	0.003186
1.25	-100.865992	-100.864004	0.001988
1.375	-91.695873	-91.694549	0.001324
1.5	-84.054269	-84.053336	0.000933
1.625	-77.588385	-77.587695	0.000690
1.75	-72.046250	-72.045717	0.000533
1.875	-67.243097	-67.242669	0.000428

Table 125: P116 4th Order FDM T-prime values, Δx = 1/9

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1111	-113.474110	-113.473139	0.000971
1.2222	-103.157738	-103.158243	0.000505
1.3333	-94.560944	-94.562367	0.001423
1.4444	-87.286833	-87.288843	0.002010
1.5556	-81.051938	-81.049116	0.002822
1.6667	-75.648396	-75.646490	0.001906
1.7778	-70.920319	-70.919116	0.001203
1.8889	-66.748499	-66.747845	0.000654

Table 126: P116 4th Order FDM T-prime values, Δx = 1/10

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.1	-114.619633	-114.618186	0.001447
1.2	-105.067646	-105.066670	0.000976
1.3	-96.985306	-96.984619	0.000687
1.4	-90.057650	-90.057146	0.000504
1.5	-84.053718	-84.053336	0.000382
1.6	-78.800302	-78.800003	0.000299
1.7	-74.164950	-74.164708	0.000242
1.8	-70.044646	-70.044447	0.000199
1.9	-66.358065	-66.357897	0.000168

Table 127: P116 4th Order FDM T-prime values, Δx = 1/11

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0909	-115.574365	-115.574300	0.000065
1.1818	-106.683794	-106.684722	0.000928
1.2727	-99.063375	-99.064983	0.001608
1.3636	-92.459053	-92.461136	0.002083
1.4545	-86.680298	-86.682712	0.002414
1.5455	-81.581412	-81.578780	0.002632
1.6364	-77.049080	-77.047179	0.001901
1.7273	-72.993843	-72.992534	0.001309
1.8182	-69.344134	-69.343309	0.000825
1.9091	-66.042020	-66.041593	0.000427

Table 128: P116 4^{th} Order FDM T-prime values, $\Delta x = 1/12$

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0833	-116.382292	-116.385124	0.002832
1.1667	-108.069109	-108.065488	0.003621
1.25	-100.864397	-100.864004	0.000393
1.3333	-94.560301	-94.562367	0.002066
1.4167	-88.997882	-88.995556	0.002326
1.5	-84.053521	-84.053336	0.000185
1.5833	-79.629627	-79.631153	0.001526
1.6667	-75.648127	-75.646490	0.001637
1.75	-72.045822	-72.045717	0.000105
1.8333	-68.771002	-68.772162	0.001160
1.9167	-65.780951	-65.779728	0.001223

Table 129: P116 4^{th} Order FDM T-prime values, $\Delta x = 1/13$

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0769	-117.074849	-117.076799	0.001950
1.1538	-109.269745	-109.273708	0.003963
1.2308	-102.440309	-102.437443	0.002866
1.3077	-96.414356	-96.413554	0.000802
1.3846	-91.057966	-91.058793	0.000827
1.4615	-86.265414	-86.267536	0.002122
1.5385	-81.952124	-81.949954	0.002170
1.6154	-78.049628	-78.048783	0.000845
1.6923	-74.501907	-74.502159	0.000252
1.7692	-71.262685	-71.263851	0.001166
1.8462	-68.293400	-68.291628	0.001772
1.9231	-65.561659	-65.560816	0.000843

Table 130: P116 4^{th} Order FDM T-prime values, $\Delta x = 1/14$

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0714	-117.675096	-117.677809	0.002713
1.1429	-110.320320	-110.315867	0.004453
1.2143	-103.830833	-103.829370	0.001463
1.2857	-98.062413	-98.063315	0.000902
1.3571	-92.901205	-92.903990	0.002785
1.4286	-88.256124	-88.254238	0.001886
1.5	-84.053436	-84.053336	0.000100
1.5714	-80.232813	-80.234189	0.001376
1.6429	-76.744421	-76.742349	0.002072
1.7143	-73.546730	-73.546056	0.000674
1.7857	-70.604856	-70.605367	0.000511
1.8571	-67.889280	-67.890800	0.001520
1.9286	-65.374859	-65.373849	0.001010

Table 131: P116 4^{th} Order FDM T-prime values, $\Delta x = 1/15$

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0667	-118.200334	-118.196310	0.004024
1.1333	-111.247312	-111.250335	0.003023
1.2	-105.066863	-105.066670	0.000193
1.2667	-99.536998	-99.534226	0.002772
1.3333	-94.560125	-94.562367	0.002242
1.4	-90.057245	-90.057146	0.000099
1.4667	-85.963722	-85.961686	0.002036
1.5333	-82.226159	-82.227877	0.001718
1.6	-78.800062	-78.800003	0.000059
1.6667	-75.648054	-75.646490	0.001564
1.7333	-72.738509	-72.739863	0.001354
1.8	-70.044486	-70.044447	0.000039
1.8667	-67.542895	-67.541653	0.001242
1.9333	-65.213827	-65.214920	0.001093

Table 132: P116 4^{th} Order FDM T-prime values, $\Delta x = 1/16$

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.0625	-118.663793	-118.663534	0.000259
1.125	-112.071314	-112.071115	0.000199
1.1875	-106.172791	-106.172635	0.000156
1.25	-100.864128	-100.864004	0.000124
1.3125	-96.061056	-96.060956	0.000100
1.375	-91.694631	-91.694549	0.000082
1.4375	-87.707898	-87.707829	0.000069
1.5	-84.053395	-84.053336	0.000059
1.5625	-80.691253	-80.691203	0.000050
1.625	-77.587738	-77.587695	0.000043
1.6875	-74.714114	-74.714077	0.000037
1.75	-72.045750	-72.045717	0.000033
1.8125	-69.561411	-69.561382	0.000029
1.875	-67.242696	-67.242669	0.000027
1.9375	-65.073575	-65.073551	0.000024

Table 133: P116 4^{th} Order FDM Quartic Extrapolation using $\Delta x = 1/8$ and $\Delta x = 1/16$

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.125	-112.071115	-112.071115	2.23E-07
1.25	-100.864004	-100.864004	1.03E-07
1.375	-91.694549	-91.694549	4.88E-08
1.5	-84.053336	-84.053336	2.31E-08
1.625	-77.587695	-77.587695	1.01E-08
1.75	-72.045717	-72.045717	3.40E-09
1.875	-67.242669	-67.242669	2.51E-10

Table 134: P116 4th Order FDM Richardson's Extrapolation using △x = 1/4, △x = 1/8 and △x = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1.25	-100.864003	-100.864004	6.31E-07
1.5	-84.053336	-84.053336	3.52E-07
1.75	-72.045717	-72.045717	2.55E-07

Table 135: P116 4th Order FDM T-prime at x = 2

△x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1	-63.914780	-63.040002	0.874778
0.5	-63.127108	-63.040002	0.087106
0.3333	-63.057574	-63.040002	0.017572
0.25	-63.045615	-63.040002	0.005612
0.2	-63.042311	-63.040002	0.002309
0.1667	-63.041118	-63.040002	0.001116
0.1429	-63.040605	-63.040002	0.000603
0.125	-63.040356	-63.040002	0.000354
0.1111	-63.040223	-63.040002	0.000221
0.1	-63.040147	-63.040002	0.000145
0.0909	-63.040101	-63.040002	0.000099
0.0833	-63.040072	-63.040002	0.000070
0.0769	-63.040053	-63.040002	0.000051
0.0714	-63.040040	-63.040002	0.000038
0.0667	-63.040031	-63.040002	0.000029
0.0625	-63.040024	-63.040002	0.000022

Table 136: P117 2nd Order FDM Temp. values, △x = 1

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	0.000000	31.250000	31.250000
1	0.000000	0.000000	0.000000

Table 137: P117 2nd Order FDM Temp. values, △x = 1/2

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	20.833333	31.250000	10.416667
0.5	20.833333	24.609375	3.776042
1	0.000000	0.000000	0.000000

Table 138: P117 2nd Order FDM Temp. values, △x = 1/3

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	26.008230	31.250000	5.241770
0.3333	26.008230	28.395648	2.387418
0.6667	17.777778	18.902963	1.125185
1	0.000000	0.000000	0.000000

Table 139: P117 2nd Order FDM Temp. values, △x = 1/4

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	28.065476	31.250000	3.184524
0.25	28.065476	29.663086	1.597610
0.5	23.638393	24.609375	0.970982
0.75	14.732143	15.209961	0.477818
1	0.000000	0.000000	0.000000

Table 140: P117 2nd Order FDM Temp. values, △x = 1/5

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	29.096635	31.250000	2.153365
0.2	29.096635	30.240000	1.143365
0.4	26.323302	27.090000	0.766698
0.6	20.947302	21.440000	0.492698
0.8	12.444444	12.690000	0.245556
1	0.000000	0.000000	0.000000

Table 141: P117 2nd Order FDM Temp. values, △x = 1/6

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	29.689799	31.250000	1.560201
0.1667	29.689799	30.550451	0.860652
0.3333	27.786507	28.395648	0.609141
0.5	24.175396	24.609375	0.433979
0.6667	18.619840	18.902963	0.283123
0.8333	10.732323	10.876678	0.144355
1	0.000000	0.000000	0.000000

Table 142: P117 2nd Order FDM Temp. values, △x = 1/7

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	30.063701	31.250000	1.186299
0.1429	30.063701	30.736884	0.673183
0.2857	28.675391	29.167747	0.492356
0.4286	26.076473	26.446645	0.370172
0.5714	22.149538	22.421295	0.271757
0.7143	16.688851	16.867331	0.178480
0.8571	9.419152	9.511570	0.092418
1	0.000000	0.000000	0.000000

Table 143: P117 2nd Order FDM Temp. values, $\Delta x = 1/8$

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	30.315292	31.250000	0.934708
0.125	30.315292	30.857849	0.542557
0.25	29.257350	29.663086	0.405736
0.375	27.294459	27.610779	0.316320
0.5	24.364772	24.609375	0.244603
0.625	20.350015	20.530701	0.180686
0.75	15.089894	15.209961	0.120067
0.875	8.385417	8.445740	0.060323
1	0.000000	0.000000	0.000000

Table 144: P117 2nd Order FDM Temp. values, $\Delta x = 1/9$

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	30.493082	31.250000	0.756918
0.1111	30.493082	30.940468	0.447386
0.2222	29.659876	30.000444	0.340568
0.3333	28.123525	28.395648	0.272123
0.4444	25.850352	26.068948	0.218596
0.5556	22.768166	22.937151	0.168985
0.6667	18.777644	18.902963	0.125319
0.7778	13.754957	13.838228	0.083271
0.8889	7.552651	7.594380	0.041729
1	0.000000	0.000000	0.000000

Table 145: P117 2nd Order FDM Temp. values, $\Delta x = 1/10$

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	30.623603	31.250000	0.626397
0.1	30.623603	30.999375	0.375772
0.2	29.950269	30.240000	0.289731
0.3	28.714269	28.949375	0.235106
0.4	26.897127	27.090000	0.192873
0.5	24.452682	24.609375	0.156693
0.6	21.316318	21.440000	0.123682
0.7	17.407088	17.499375	0.092287
0.8	12.628421	12.690000	0.061579
0.9	6.868421	6.899375	0.030954
1	0.000000	0.000000	0.000000

Table 146: P117 2nd Order FDM Temp. values, $\Delta x = 1/11$

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	30.722392	31.250000	0.527608
0.0909	30.722392	31.043003	0.320611
0.1818	30.166875	30.416892	0.250017
0.2727	29.150551	29.356304	0.205753
0.3636	27.663533	27.835638	0.172105
0.4545	25.675205	25.819048	0.143843
0.5455	23.141846	23.257318	0.115472
0.6364	20.008390	20.099695	0.091305
0.7273	16.209014	16.277091	0.068077
0.8182	11.667376	11.712688	0.045312
0.9091	6.296734	6.319425	0.022691
1	0.000000	0.000000	0.000000

Table 147: P117 2nd Order FDM Temp. values, $\Delta x = 1/12$

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	30.799059	31.250000	0.450941
0.0833	30.799059	31.076227	0.277168
0.1667	30.332881	30.550451	0.217570
0.25	29.482186	29.663086	0.180900
0.3333	28.242107	28.395648	0.153541
0.4167	26.591730	26.720587	0.128857
0.5	24.500505	24.609375	0.108870
0.5833	21.929725	22.020512	0.090787
0.6667	18.833017	18.902963	0.069946
0.75	15.156547	15.209961	0.053414
0.8333	10.839112	10.876678	0.037566
0.9167	5.812198	5.827969	0.015771
1	0.000000	0.000000	0.000000

Table 148: P117 2nd Order FDM Temp. values, △x = 1/13

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	30.859811	31.250000	0.390189
0.0769	30.859811	31.101941	0.242130
0.1538	30.462999	30.655142	0.192143
0.2308	29.740336	29.900549	0.160213
0.3077	28.689952	28.826992	0.137040
0.3846	27.297222	27.415324	0.118102
0.4615	25.540217	25.641935	0.101718
0.5385	23.390971	23.474883	0.083912
0.6154	20.815898	20.885654	0.069756
0.6923	17.775964	17.832339	0.056375
0.7692	14.226774	14.270335	0.043561
0.8462	10.118608	10.144052	0.025444
0.9231	5.396450	5.409050	0.012600
1	0.000000	0.000000	0.000000

Table 149: P117 2nd Order FDM Temp. values, △x = 1/14

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	30.908811	31.250000	0.341189
0.0714	30.908811	31.122389	0.213578
0.1429	30.566940	30.736884	0.169944
0.2143	29.945324	30.088706	0.143382
0.2857	29.043913	29.167747	0.123834
0.3571	27.852280	27.960356	0.108076
0.4286	26.354325	26.446645	0.092320
0.5	24.529364	24.609375	0.080011
0.5714	22.352493	22.421295	0.068802
0.6429	19.794736	19.849279	0.054543
0.7143	16.823112	16.867331	0.044219
0.7857	13.400679	13.435078	0.034399
0.8571	9.486549	9.511570	0.025021
0.9286	5.035903	5.045315	0.009412
1	0.000000	0.000000	0.000000

Table 150: P117 2nd Order FDM Temp. values, △x = 1/15

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	30.948936	31.250000	0.301064
0.0667	30.948936	31.138654	0.189718
0.1333	30.651323	30.803804	0.152481
0.2	30.110879	30.240000	0.129121
0.2667	29.328656	29.440157	0.111501
0.3333	28.297106	28.395648	0.098542
0.4	27.004177	27.090000	0.085823
0.4667	25.434262	25.508273	0.074011
0.5333	23.568518	23.634225	0.065707
0.6	21.384988	21.440000	0.055012
0.6667	18.858672	18.902963	0.044291
0.7333	15.961553	15.999576	0.038023
0.8	12.662616	12.690000	0.027384
0.8667	8.927860	8.944186	0.016326
0.9333	4.720307	4.731739	0.011432
1	0.000000	0.000000	0.000000

Table 151: P117 2nd Order FDM Temp. values, △x = 1/16

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	30.982229	31.250000	0.267771
0.0625	30.982229	31.152248	0.170019
0.125	30.720795	30.857849	0.137054
0.1875	30.246552	30.363369	0.116817
0.25	29.561214	29.663086	0.101872
0.3125	28.659250	28.748989	0.089739
0.375	27.531487	27.610779	0.079292
0.4375	26.165943	26.235867	0.069924
0.5	24.548104	24.609375	0.061271
0.5625	22.661041	22.714138	0.053097
0.625	20.485459	20.530701	0.045242
0.6875	17.999729	18.037319	0.037590
0.75	15.179905	15.209961	0.030056
0.8125	11.999729	12.022305	0.022576
0.875	8.430642	8.445740	0.015098
0.9375	4.441784	4.449368	0.007584
1	0.000000	0.000000	0.000000

Table 152: P117 2nd Order FDM T-prime values, △x = 1/2

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.5	-20.833333	-28.125000	7.291667

Table 153: P117 2nd Order FDM T-prime values, △x = 1/3

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.3333	-12.345679	-17.590648	5.244969
0.6667	-39.012346	-40.743519	1.731173

Table 154: P117 2nd Order FDM T-prime values, △x = 1/4

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.25	-8.854167	-12.890625	4.036458
0.5	-26.666667	-28.125000	1.458333
0.75	-47.276786	-48.046875	0.770089

Table 155: P117 2nd Order FDM T-prime values, △x = 1/5

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.2	-6.933333	-10.200000	3.266667
0.4	-20.373333	-21.600000	1.226667
0.6	-34.697143	-35.400000	0.702857
0.8	-52.368254	-52.800000	0.431746

Table 156: P117 2nd Order FDM T-prime values, △x = 1/6

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.1667	-5.709877	-8.450810	2.740933
0.3333	-16.543210	-17.590648	1.047438
0.5	-27.500000	-28.125000	0.625000
0.6667	-40.329218	-40.743519	0.414301
0.8333	-55.859521	-56.130857	0.271336

Table 157: P117 2nd Order FDM T-prime values, △x = 1/7

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.1429	-4.859086	-7.217952	2.358866
0.2857	-13.955296	-14.868003	0.912707
0.4286	-22.840483	-23.398324	0.557841
0.5714	-32.856680	-33.234023	0.377343
0.7143	-44.556351	-44.826334	0.269983
0.8571	-58.410977	-58.596079	0.185102

Table 158: P117 2nd Order FDM T-prime values, △x = 1/8

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.125	-4.231771	-6.298828	2.067057
0.25	-12.083333	-12.890625	0.807292
0.375	-19.570313	-20.068359	0.498046
0.5	-27.777778	-28.125000	0.347222
0.625	-37.099511	-37.353516	0.254005
0.75	-47.858392	-48.046875	0.188483
0.875	-60.359575	-60.498047	0.138472

Table 159: P117 2nd Order FDM T-prime values, △x = 1/9

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.1111	-3.749428	-5.589283	1.839855
0.2222	-10.663009	-11.384266	0.721257
0.3333	-17.142857	-17.590648	0.447791
0.4444	-24.099114	-24.414129	0.315015
0.5556	-31.827188	-32.067723	0.240535
0.6667	-40.559441	-40.743519	0.184078
0.7778	-50.512469	-50.653697	0.141228
0.8889	-61.897308	-62.003957	0.106649

Table 160: P117 2nd Order FDM T-prime values, △x = 1/10

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.1	-3.366667	-5.025000	1.658333
0.2	-9.546667	-10.200000	0.653333
0.3	-15.265714	-15.675000	0.409286
0.4	-21.307937	-21.600000	0.292063
0.5	-27.904040	-28.125000	0.220960
0.6	-35.227972	-35.400000	0.172028
0.7	-43.439487	-43.575000	0.135513
0.8	-52.693333	-52.800000	0.106667
0.9	-63.142105	-63.225000	0.082895

Table 161: P117 2nd Order FDM T-prime values, △x = 1/11

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.0909	-3.055347	-4.563777	1.508430
0.1818	-8.645129	-9.240218	0.595089
0.2727	-13.768380	-14.141985	0.373605
0.3636	-19.114403	-19.381743	0.267340
0.4545	-24.869279	-25.072154	0.202875
0.5455	-31.167480	-31.333114	0.165634
0.6364	-38.130575	-38.263629	0.133054
0.7273	-45.875576	-45.982911	0.107335
0.8182	-54.517542	-54.603625	0.086083
0.9091	-64.170569	-64.238434	0.067865

Table 162: P117 2nd Order FDM T-prime values, ∆x = 1/12

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.0833	-2.797068	-4.179450	1.382382
0.1667	-7.901235	-8.450810	0.549575
0.25	-12.544643	-12.890625	0.345982
0.3333	-17.342740	-17.590648	0.247908
0.4167	-22.449612	-22.643883	0.194271
0.5	-27.972028	-28.125000	0.152972
0.5833	-34.004926	-34.126534	0.121608
0.6667	-40.639070	-40.743519	0.104449
0.75	-47.963429	-48.046875	0.083446
0.8333	-56.066091	-56.130857	0.064766
0.9167	-65.034674	-65.093467	0.058793

Table 163: P117 2nd Order FDM T-prime values, ∆x = 1/13

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.0769	-2.579275	-3.856369	1.277094
0.1538	-7.276589	-7.780951	0.504362
0.2308	-11.524807	-11.847360	0.322553
0.3077	-15.880241	-16.113320	0.233079
0.3846	-20.473281	-20.652223	0.178942
0.4615	-25.390631	-25.532283	0.141652
0.5385	-30.708075	-30.828886	0.120811
0.6154	-36.497541	-36.596564	0.099023
0.6923	-42.829301	-42.910126	0.080825
0.7692	-49.772816	-49.837788	0.064972
0.8462	-57.397109	-57.458132	0.061023
0.9231	-65.770952	-65.819652	0.048700

Table 164: P117 2nd Order FDM T-prime values, ∆x = 1/14

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.0714	-2.393100	-3.579100	1.186000
0.1429	-6.744412	-7.217952	0.473540
0.2143	-10.661183	-10.961040	0.299857
0.2857	-14.651303	-14.868003	0.216700
0.3571	-18.827120	-18.993438	0.166318
0.4286	-23.260413	-23.398324	0.137911
0.5	-28.012821	-28.125000	0.112179
0.5714	-33.142400	-33.234023	0.091623
0.6429	-38.705670	-38.788092	0.082422
0.7143	-44.758397	-44.826334	0.067937
0.7857	-51.355942	-51.410796	0.054854
0.8571	-58.553429	-58.596079	0.042650
0.9286	-66.405842	-66.448247	0.042405

Table 165: P117 2nd Order FDM T-prime values, ∆x = 1/15

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.0667	-2.232099	-3.342419	1.110320
0.1333	-6.285432	-6.724215	0.438783
0.2	-9.920000	-10.200000	0.280000
0.2667	-13.603292	-13.809252	0.205960
0.3333	-17.433595	-17.590648	0.157053
0.4	-21.471329	-21.600000	0.128671
0.4667	-25.767445	-25.876285	0.108840
0.5333	-30.369557	-30.456882	0.087325
0.6	-35.323839	-35.400000	0.076161
0.6667	-40.675763	-40.743519	0.067756
0.7333	-46.470424	-46.522915	0.052491
0.8	-52.752696	-52.800000	0.047304
0.8667	-59.567320	-59.610952	0.043632
0.9333	-66.958952	-66.988748	0.029796

Table 166: P117 2nd Order FDM T-prime values, ∆x = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.0625	-2.091471	-3.131104	1.039633
0.125	-5.885417	-6.298828	0.413411
0.1875	-9.276646	-9.539795	0.263149
0.25	-12.698413	-12.890625	0.192212
0.3125	-16.237818	-16.387939	0.150121
0.375	-19.946460	-20.068359	0.121899
0.4375	-23.867062	-23.968506	0.101444
0.5	-28.039216	-28.125000	0.085784
0.5625	-32.501164	-32.574463	0.073299
0.625	-37.290492	-37.353516	0.063024
0.6875	-42.444429	-42.498779	0.054350
0.75	-48.000000	-48.046875	0.046875
0.8125	-53.994104	-54.034424	0.040320
0.875	-60.463562	-60.498047	0.034485
0.9375	-67.445138	-67.474365	0.029227

Table 167: P117 2nd Order FDM Quadratic Extrapolation using ∆x = 1/8 and ∆x = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.125	-6.436632	-6.298828	0.137804
0.25	-12.903439	-12.890625	0.012814
0.375	-20.071842	-20.068359	0.003483
0.5	-28.126362	-28.125000	0.001362
0.625	-37.354152	-37.353516	0.000636
0.75	-48.047203	-48.046875	0.000328
0.875	-60.498224	-60.498047	0.000177

Table 168: P117 2nd Order FDM Richardson's Extrapolation using △x = 1/4, △x = 1/8 and △x = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.25	-12.843137	-12.890625	0.047488
0.5	-28.119658	-28.125000	0.005342
0.75	-48.045575	-48.046875	0.001300

Table 168: P117 2nd Order FDM T-prime at x = 1

△x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1	-66.666667	-75.000000	8.333333
0.5	-73.333333	-75.000000	1.666667
0.3333	-74.285714	-75.000000	0.714286
0.25	-74.603175	-75.000000	0.396825
0.2	-74.747475	-75.000000	0.252525
0.1667	-74.825175	-75.000000	0.174825
0.1429	-74.871795	-75.000000	0.128205
0.125	-74.901961	-75.000000	0.098039
0.1111	-74.922601	-75.000000	0.077399
0.1	-74.937343	-75.000000	0.062657
0.0909	-74.948240	-75.000000	0.051760
0.0833	-74.956522	-75.000000	0.043478
0.0769	-74.962963	-75.000000	0.037037
0.0714	-74.968072	-75.000000	0.031928
0.0667	-74.972191	-75.000000	0.027809
0.0625	-74.975562	-75.000000	0.024438

Table 169: P117 4th Order FDM Temp. values, △x = 1

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	25.000000	31.250000	6.250000
1	0.000000	0.000000	0.000000

Table 170: P117 4th Order FDM Temp. values, △x = 1/2

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	30.750000	31.250000	0.500000
0.5	24.500000	24.609375	0.109375
1	0.000000	0.000000	0.000000

Table 171: P117 4th Order FDM Temp. values, △x = 1/3

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.138535	31.250000	0.111465
0.3333	28.360758	28.395648	0.034890
0.6667	18.891622	18.902963	0.011341
1	0.000000	0.000000	0.000000

Table 172: P117 4th Order FDM Temp. values, △x = 1/4

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.211879	31.250000	0.038121
0.25	29.649379	29.663086	0.013707
0.5	24.602504	24.609375	0.006871
0.75	15.207108	15.209961	0.002853
1	0.000000	0.000000	0.000000

Table 173: P117 4th Order FDM Temp. values, △x = 1/5

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.233479	31.250000	0.016521
0.2	30.233479	30.240000	0.006521
0.4	27.086279	27.090000	0.003721
0.6	21.437925	21.440000	0.002075
0.8	12.689094	12.690000	0.000906
1	0.000000	0.000000	0.000000

Table 174: P117 4th Order FDM Temp. values, △x = 1/6

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.241676	31.250000	0.008324
0.1667	30.547231	30.550451	0.003220
0.3333	28.392910	28.395648	0.002738
0.5	24.608017	24.609375	0.001358
0.6667	18.903527	18.902963	0.000564
0.8333	10.874450	10.876678	0.002228
1	0.000000	0.000000	0.000000

Table 175: P117 4th Order FDM Temp. values, △x = 1/7

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.245344	31.250000	0.004656
0.1429	30.735140	30.736884	0.001744
0.2857	29.166210	29.167747	0.001537
0.4286	26.446418	26.446645	0.000227
0.5714	22.419754	22.421295	0.001541
0.7143	16.867616	16.867331	0.000285
0.8571	9.508896	9.511570	0.002674
1	0.000000	0.000000	0.000000

Table 176: P117 4th Order FDM Temp. values, △x = 1/8

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.247188	31.250000	0.002812
0.125	30.856563	30.857849	0.001286
0.25	29.662227	29.663086	0.000859
0.375	27.610171	27.610779	0.000608
0.5	24.608945	24.609375	0.000430
0.625	20.530409	20.530701	0.000291
0.75	15.209783	15.209961	0.000178
0.875	8.445657	8.445740	0.000083
1	0.000000	0.000000	0.000000

Table 177: P117 4th Order FDM Temp. values, △x = 1/9

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.248199	31.250000	0.001801
0.1111	30.939557	30.940468	0.000911
0.2222	29.999609	30.000444	0.000835
0.3333	28.394637	28.395648	0.001011
0.4444	26.067549	26.068948	0.001399
0.5556	22.938349	22.937151	0.001198
0.6667	18.904164	18.902963	0.001201
0.7778	13.839257	13.838228	0.001028
0.8889	7.595024	7.594380	0.000643
1	0.000000	0.000000	0.000000

Table 178: P117 4th Order FDM Temp. values, △x = 1/10

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.248791	31.250000	0.001209
0.1	30.998791	30.999375	0.000584
0.2	30.239591	30.240000	0.000409
0.3	28.949069	28.949375	0.000306
0.4	27.089767	27.090000	0.000233
0.5	24.609199	24.609375	0.000176
0.6	21.439870	21.440000	0.000130
0.7	17.499284	17.499375	0.000091
0.8	12.689943	12.690000	0.000057
0.9	6.899348	6.899375	0.000027
1	0.000000	0.000000	0.000000

Table 179: P117 4th Order FDM Temp. values, △x = 1/11

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.249158	31.250000	0.000842
0.0909	31.042546	31.043003	0.000457
0.1818	30.416428	30.416892	0.000464
0.2727	29.355693	29.356304	0.000611
0.3636	27.834757	27.835638	0.000880
0.4545	25.817772	25.819048	0.001276
0.5455	23.258637	23.257318	0.001319
0.6364	20.101008	20.099695	0.001313
0.7273	16.278290	16.277091	0.001199
0.8182	11.713646	11.712688	0.000958
0.9091	6.319992	6.319425	0.000567
1	0.000000	0.000000	0.000000

Table 180: P117 4th Order FDM Temp. values, △x = 1/12

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.249395	31.250000	0.000605
0.0833	31.075784	31.076227	0.000443
0.1667	30.550514	30.550451	0.000062
0.25	29.662916	29.663086	0.000170
0.3333	28.394927	28.395648	0.000721
0.4167	26.721235	26.720587	0.000648
0.5	24.609290	24.609375	0.000085
0.5833	22.019309	22.020512	0.001204
0.6667	18.904271	18.902963	0.001308
0.75	15.209926	15.209961	0.000035
0.8333	10.874785	10.876678	0.001893
0.9167	5.830128	5.827969	0.002159
1	0.000000	0.000000	0.000000

Table 181: P117 4th Order FDM Temp. values, △x = 1/13

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.249554	31.250000	0.000446
0.0769	31.101625	31.101941	0.000317
0.1538	30.654616	30.655142	0.000526
0.2308	29.900783	29.900549	0.000234
0.3077	28.827011	28.826992	0.000019
0.3846	27.414922	27.415324	0.000403
0.4615	25.640884	25.641935	0.001051
0.5385	23.476014	23.474883	0.001131
0.6154	20.886174	20.885654	0.000520
0.6923	17.831976	17.832339	0.000363
0.7692	14.268779	14.270335	0.001557
0.8462	10.146689	10.144052	0.002637
0.9231	5.410561	5.409050	0.001512
1	0.000000	0.000000	0.000000

Table 182: P117 4th Order FDM Temp. values, △x = 1/14

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.249663	31.250000	0.000337
0.0714	31.122112	31.122389	0.000276
0.1429	30.737064	30.736884	0.000181
0.2143	30.088761	30.088706	0.000055
0.2857	29.167452	29.167747	0.000295
0.3571	27.959474	27.960356	0.000882
0.4286	26.447258	26.446645	0.000612
0.5	24.609329	24.609375	0.000046
0.5714	22.420309	22.421295	0.000987
0.6429	19.850912	19.849279	0.001633
0.7143	16.867949	16.867331	0.000618
0.7857	13.434328	13.435078	0.000750
0.8571	9.509049	9.511570	0.002522
0.9286	5.047209	5.045315	0.001894
1	0.000000	0.000000	0.000000

Table 183: P117 4th Order FDM Temp. values, △x = 1/15

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.249741	31.250000	0.000259
0.0667	31.138630	31.138654	0.000024
0.1333	30.803479	30.803804	0.000325
0.2	30.239919	30.240000	0.000081
0.2667	29.440551	29.440157	0.000394
0.3333	28.395007	28.395648	0.000641
0.4	27.089954	27.090000	0.000046
0.4667	25.509098	25.508273	0.000824
0.5333	23.633178	23.634225	0.001047
0.6	21.439974	21.440000	0.000026
0.6667	18.904301	18.902963	0.001338
0.7333	15.998009	15.999576	0.001566
0.8	12.689989	12.690000	0.000011
0.8667	8.946166	8.944186	0.001980
0.9333	4.729503	4.731739	0.002236
1	0.000000	0.000000	0.000000

Table 184: P117 4th Order FDM Temp. values, △x = 1/16

x (m)	T(x) (°C)	Ť(x) (°C)	e(T)
0	31.249797	31.250000	0.000203
0.0625	31.152141	31.152248	0.000107
0.125	30.857769	30.857849	0.000081
0.1875	30.363304	30.363369	0.000065
0.25	29.663032	29.663086	0.000054
0.3125	28.748944	28.748989	0.000045
0.375	27.610741	27.610779	0.000038
0.4375	26.235835	26.235867	0.000032
0.5	24.609348	24.609375	0.000027
0.5625	22.714116	22.714138	0.000022
0.625	20.530682	20.530701	0.000018
0.6875	18.037305	18.037319	0.000015
0.75	15.209950	15.209961	0.000011
0.8125	12.022296	12.022305	8.04E-06
0.875	8.445735	8.445740	5.17E-06
0.9375	4.449365	4.449368	2.50E-06
1	0.000000	0.000000	0.000000

Table 185: P117 4th Order FDM T-prime values, △x = 1/2

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.5	-27.750000	-28.125000	0.375000

Table 186: P117 4th Order FDM T-prime values, △x = 1/3

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.3333	-17.481481	-17.590648	0.109167
0.6667	-40.693243	-40.743519	0.050276

Table 187: P117 4th Order FDM T-prime values, △x = 1/4

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.25	-12.843750	-12.890625	0.046875
0.5	-28.104962	-28.125000	0.020038
0.75	-48.033625	-48.046875	0.013250

Table 188: P117 4th Order FDM T-prime values, △x = 1/5

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.2	-10.176000	-10.200000	0.024000
0.4	-21.589740	-21.600000	0.010260
0.6	-35.393216	-35.400000	0.006784
0.8	-52.794919	-52.800000	0.005081

Table 189: P117 4th Order FDM T-prime values, △x = 1/6

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.1667	-8.435185	-8.450810	0.015625
0.3333	-17.586655	-17.590648	0.003993
0.5	-28.121074	-28.125000	0.003926
0.6667	-40.737800	-40.743519	0.005719
0.8333	-56.131908	-56.130857	0.001051

Table 190: P117 4th Order FDM T-prime values, △x = 1/7

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.1429	-7.206997	-7.217952	0.010955
0.2857	-14.865066	-14.868003	0.002937
0.4286	-23.394029	-23.398324	0.004295
0.5714	-33.234300	-33.234023	0.000277
0.7143	-44.823592	-44.826334	0.002742
0.8571	-58.599349	-58.596079	0.003270

Table 191: P117 4th Order FDM T-prime values, △x = 1/8

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.125	-6.292969	-6.298828	0.005859
0.25	-12.888120	-12.890625	0.002505
0.375	-20.066703	-20.068359	0.001656
0.5	-28.123759	-28.125000	0.001241
0.625	-37.352524	-37.353516	0.000992
0.75	-48.046048	-48.046875	0.000827
0.875	-60.497338	-60.498047	0.000709

Table 192: P117 4th Order FDM T-prime values, △x = 1/9

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.1111	-5.585734	-5.589283	0.003549
0.2222	-11.383700	-11.384266	0.000566
0.3333	-17.591429	-17.590648	0.000781
0.4444	-24.416138	-24.414129	0.002009
0.5556	-32.063775	-32.067723	0.003948
0.6667	-40.740160	-40.743519	0.003359
0.7778	-50.651080	-50.653697	0.002617
0.8889	-62.002308	-62.003957	0.001649

Table 193: P117 4th Order FDM T-prime values, △x = 1/10

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.1	-5.022000	-5.025000	0.003000
0.2	-10.198718	-10.200000	0.001282
0.3	-15.674152	-15.675000	0.000848
0.4	-21.599365	-21.600000	0.000635
0.5	-28.124492	-28.125000	0.000508
0.6	-35.399577	-35.400000	0.000423
0.7	-43.574637	-43.575000	0.000363
0.8	-52.799683	-52.800000	0.000317
0.9	-63.224718	-63.225000	0.000282

Table 194: P117 4th Order FDM T-prime values, △x = 1/11

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.0909	-4.561983	-4.563777	0.001794
0.1818	-9.240209	-9.240218	0.000009
0.2727	-14.142864	-14.141985	0.000879
0.3636	-19.383445	-19.381743	0.001702
0.4545	-25.074750	-25.072154	0.002596
0.5455	-31.329509	-31.333114	0.003605
0.6364	-38.260434	-38.263629	0.003195
0.7273	-45.980227	-45.982911	0.002684
0.8182	-54.601591	-54.603625	0.002034
0.9091	-64.237225	-64.238434	0.001209

Table 195: P117 4th Order FDM T-prime values, △x = 1/12

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.0833	-4.179398	-4.179450	0.000052
0.1667	-8.448332	-8.450810	0.002478
0.25	-12.890134	-12.890625	0.000491
0.3333	-17.592225	-17.590648	0.001577
0.4167	-22.641488	-22.643883	0.002395
0.5	-28.124755	-28.125000	0.000245
0.5833	-34.128841	-34.126534	0.002307
0.6667	-40.740557	-40.743519	0.002962
0.75	-48.046712	-48.046875	0.000163
0.8333	-56.134112	-56.130857	0.003255
0.9167	-65.089566	-65.093467	0.003901

Table 196: P117 4th Order FDM T-prime values, △x = 1/13

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.0769	-3.856168	-3.856369	0.000201
0.1538	-7.782757	-7.780951	0.001806
0.2308	-11.845313	-11.847360	0.002047
0.3077	-16.112592	-16.113320	0.000728
0.3846	-20.652932	-20.652223	0.000709
0.4615	-25.534628	-25.532283	0.002345
0.5385	-30.825961	-30.828886	0.002925
0.6154	-36.595213	-36.596564	0.001351
0.6923	-42.910659	-42.910126	0.000533
0.7692	-49.840576	-49.837788	0.002788
0.8462	-57.453240	-57.458132	0.004892
0.9231	-65.816927	-65.819652	0.002725

Table 197: P117 4th Order FDM T-prime values, △x = 1/14

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.0714	-3.579446	-3.579100	0.000346
0.1429	-7.215276	-7.217952	0.002676
0.2143	-10.959968	-10.961040	0.001072
0.2857	-14.868573	-14.868003	0.000570
0.3571	-18.995806	-18.993438	0.002368
0.4286	-23.396347	-23.398324	0.001977
0.5	-28.124868	-28.125000	0.000132
0.5714	-33.236036	-33.234023	0.002013
0.6429	-38.784518	-38.788092	0.003574
0.7143	-44.824980	-44.826334	0.001354
0.7857	-51.412088	-51.410796	0.001292
0.8571	-58.600506	-58.596079	0.004427
0.9286	-66.444900	-66.448247	0.003347

Table 198: P117 4th Order FDM T-prime values, △x = 1/15

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.0667	-3.339852	-3.342419	0.002567
0.1333	-6.725546	-6.724215	0.001331
0.2	-10.199749	-10.200000	0.000251
0.2667	-13.807219	-13.809252	0.002033
0.3333	-17.592442	-17.590648	0.001794
0.4	-21.599875	-21.600000	0.000125
0.4667	-25.873967	-25.876285	0.002318
0.5333	-30.459165	-30.456882	0.002283
0.6	-35.399916	-35.400000	0.000084
0.6667	-40.740666	-40.743519	0.002853
0.7333	-46.525858	-46.522915	0.002943
0.8	-52.799937	-52.800000	0.000063
0.8667	-59.607350	-59.610952	0.003602
0.9333	-66.992539	-66.988748	0.003791

Table 199: P117 4th Order FDM T-prime values, △x = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.0625	-3.130371	-3.131104	0.000733
0.125	-6.298515	-6.298828	0.000313
0.1875	-9.539588	-9.539795	0.000207
0.25	-12.890470	-12.890625	0.000155
0.3125	-16.387815	-16.387939	0.000124
0.375	-20.068256	-20.068359	0.000103
0.4375	-23.968417	-23.968506	0.000089
0.5	-28.124923	-28.125000	0.000077
0.5625	-32.574394	-32.574463	0.000069
0.625	-37.353454	-37.353516	0.000062
0.6875	-42.498723	-42.498779	0.000056
0.75	-48.046823	-48.046875	0.000052
0.8125	-54.034376	-54.034424	0.000048
0.875	-60.498003	-60.498047	0.000044
0.9375	-67.474324	-67.474365	0.000041

Table 200: P117 4th Order FDM Quartic Extrapolation using △x = 1/8 and △x = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.125	-6.298885	-6.298828	0.000057
0.25	-12.890627	-12.890625	1.58E-06
0.375	-20.068360	-20.068359	2.10E-07
0.5	-28.125000	-28.125000	5.00E-08
0.625	-37.353516	-37.353516	1.64E-08
0.75	-48.046875	-48.046875	6.60E-09
0.875	-60.498047	-60.498047	3.10E-09

Table 201: P117 4th Order FDM Richardson's Extrapolation using △x = 1/4, △x = 1/8 and △x = 1/16

x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
0.25	-12.890601	-12.890625	0.000024
0.5	-28.124999	-28.125000	7.82E-07
0.75	-48.046875	-48.046875	1.05E-07

Table 202: P117 4th Order FDM T-prime at x = 1

△x (m)	T'(x) (°C)	Ť'(x) (°C)	e(T)
1	-72.000000	-75.000000	3.000000
0.5	-74.839695	-75.000000	0.160305
0.3333	-74.968592	-75.000000	0.031408
0.25	-74.990076	-75.000000	0.009924
0.2	-74.995937	-75.000000	0.004063
0.1667	-74.998041	-75.000000	0.001959
0.1429	-74.998942	-75.000000	0.001058
0.125	-74.999380	-75.000000	0.000620
0.1111	-74.999613	-75.000000	0.000387
0.1	-74.999746	-75.000000	0.000254
0.0909	-74.999827	-75.000000	0.000173
0.0833	-74.999878	-75.000000	0.000122
0.0769	-74.999911	-75.000000	0.000089
0.0714	-74.999934	-75.000000	0.000066
0.0667	-74.999950	-75.000000	0.000050
0.0625	-74.999961	-75.000000	0.000039

MATLAB FILES

```
%% Project 1: Problem 115
% Seyed Sepehr Seyedi
%
% UIN 424006176
%
% AERO 430-500
%
% October 5th, 2015

%% Parameters - Boundary Conditions & Inhomogeneous Elements

Tf = 0;
T0 = 100;
xf = 2;
x0 = 1;
f = @(r) 0;
fd = @(r) 0;
fdd = @(r) 0;

% Define the exact solution function and its derivative.
Ta2 = @(r) 100*(1-((log(r))/(log(2))));
Td = @(r) -100/(r*log(2));

%% 2nd Order FDM

% Define variables as cell arrays to access data on the basis of grid spacing.
figure('name','Project 1: P115 2nd Order Temp')
T = cell(15,1);
x = cell(15,1);
xin = cell(15,1);
Er2 = cell(15,1);

% Evaluate T and x values using the interior equation, and plot temperature vs grid spacing.
T{1,1} = [T0 (1/(xf+x0))*(T0*1.25+Tf*1.75) Tf];
x{1,1} = [x0 1.5 xf];
xin{1,1} = 1.5;
Er2{1,1}(1) = log10(100*abs((Ta2(xin{1,1}(1))-T{1,1}(2))/(Ta2(xin{1,1}(1)))));
plot(x{1},T{1},'*');set(gca,'Fontsize',14);
hold on;
for i = 3:16
    [A,b] = Order2FDM(Tf,T0,xf,x0,i,f);
    T{i-1,1} = [T0 (A\b)' Tf];
    for j = 1:i+1
        x{i-1,1}(j) = x0 + (j-1)*((xf-x0)/i);
    end
    for u = 1:i-1
        xin{i-1,1}(u) = x0 + (u)*((xf-x0)/i);
    end
    plot(x{i-1,1},T{i-1,1},'-*');
    for e = 1:i-1
        Er2{i-1,1}(e) = log10(100*abs((Ta2(xin{i-1,1}(e))-T{i-1,1}(e+1))/(Ta2(xin{i-1,1}(e)))));
    end
end
xlabel('x position','Fontsize',16);
ylabel('Temperature (C)','Fontsize',16);
title('Plot of Temperature vs Position for 2nd Order FDM For P115','Fontsize',14);
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N =
12','N = 13','N = 14','N = 15','N = 16');

% Output temperature values for each grid size.
fprintf('TEMPERATURE VS RADIUS\n-----------------------------------');
for k = 1:15
    delta = 1/(k+1);
    fprintf('\ndeltaX = %6.4f\nx\t\tTemperature(C)\n',delta);
    for m = 1:k+2
        xi = x{k,1}(m);
        Ti = T{k,1}(m);
        fprintf('%6.4f\t\t%8.6f\n',xi,Ti);
```

```
    end
end


% Define derivative variables as cell arrays to access data on the basis of grid spacing.
Td2 = cell(15,1);
ErP = cell(15,1);
lx = cell(15,1);

% Evaluate and output derivative values using 2nd order difference, and plot error vs grid
spacing.
figure('name','Project 1: P115 2nd Order Error Plot')
fprintf('\nFirst Derivative vs Position');
for a = 1:15
    delta = 1/(a+1);
    fprintf('\ndeltaX = %6.4f\nx\t\tT prime(C)\n',delta);
    for b = 1:a
        Td2{a,1}(b) = (T{a,1}(b+2)-T{a,1}(b))/(2*delta);
        xi = x{a,1}(b+1);
        Ti = Td2{a,1}(b);
        fprintf('%6.4f\t\t%8.6f\n',xi,Ti);
        ErP{a,1}(b) = log10(100*abs((Ti-Td(x{a,1}(b+1)))/Td(x{a,1}(b+1))));
    end
    plot(xin{a,1},ErP{a,1},'-*');set(gca,'Fontsize',14);
    hold on;
end
xlabel('x position','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of T-prime Error vs Position of 2nd Order FDM For P115 Known
Solution','Fontsize',12);
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N =
12','N = 13','N = 14','N = 15','N = 16');

figure('name','P115: T-prime 2nd order')
for b = 1:15
    plot(xin{b,1},Td2{b,1},'-*');set(gca,'Fontsize',14);
    hold on;
end
xlabel('x position','Fontsize',16);
ylabel('T''(x)  (C/m)','Fontsize',16);
title('P115: 2ND ORDER FDM FIRST DERIVATIVE OF TEMP.','Fontsize',18);
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N =
12','N = 13','N = 14','N = 15','N = 16');
hold off;

% Convergence Analysis
N = zeros(1,8);
LEr2 = zeros(1,8);
LEr = zeros(1,8);
for w = 2:2:16
    temp = 0.5*w;
    N(temp) = log10(w);
    LEr2(temp) = Er2{w-1,1}(w/2);
    LEr(temp) = ErP{w-1,1}(w/2);
end

figure('name','Project 1: P115 2nd Order Convergence')
plot(N,LEr,'*');set(gca,'Fontsize',14);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Convergence of 2nd Order FDM For T-prime P115 - Known Solution','Fontsize',12);

% Extrapolation for Temperature Values.
figure('name','Project 1: P115 2nd Order Extrapolation');
xQ2 = [1.125:0.125:1.875];
for y = 1:7
    T2Q2(y) = (4*(T{15,1}(2*y+1))-(T{7,1}(y+1)))/3;
    ErQ2(y) = log10(100*abs((T2Q2(y)-Ta2(xQ2(y)))/(Ta2(xQ2(y)))));
end

% Convergence of Temperature - Unknown Solution using Quadratic
```

```matlab
% Extrapolation
LErT2 = zeros(1,8);
for w = 1:8
    LErT2(w) = log10(100*abs((T{2*w-1,1}(w+1)-T2Q2(4))/T2Q2(4)));
end


% Extrapolation for Derivative Values.
figure('name','Project 1: P115 2nd Order Extrapolation');
fprintf('\nQuadratic Extrapolation\nx\t\tT prime\n');
for y = 1:7
    TQ2(y) = (4*(Td2{15,1}(2*y))-(Td2{7,1}(y)))/3;
    fprintf('%6.4f\t\t%8.6f\n',xQ2(y),TQ2(y));
    ErPQ2(y) = log10(100*abs((TQ2(y)-Td(xQ2(y)))/(Td(xQ2(y)))));
end
plot(xQ2,ErPQ2,'-d');set(gca,'Fontsize',14);
hold on;

fprintf('\nRichardson''s Extrapolation\nx\t\tT prime\n');
xR2 = [1.25 1.5 1.75];
for z = 1:3
    TR2(z) = (((Td2{7,1}(2*z))^2)-(Td2{3,1}(z))*(Td2{15,1}(4*z)))/(2*(Td2{7,1}(2*z))-
(Td2{3,1}(z))-Td2{15,1}(4*z));
    fprintf('%6.4f\t\t%8.6f\n',xR2(z),TR2(z));
    ErPR2(z) = log10(100*abs((TR2(z)-Td(xR2(z)))/(Td(xR2(z)))));
end
plot(xR2,ErPR2,'-o');set(gca,'Fontsize',14);
xlabel('x position','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Extrapolated T-prime Error vs Position of 2nd Order FDM For P115','Fontsize',12);
legend('QuadEx','RichEx');

% Convergence of derivative - Unknown solution
LEr2U = zeros(1,8);
for w = 1:8
    Ti = Td2{2*w-1,1}(w);
    LEr2U(w) = log10(100*abs((Ti-TQ2(4))/TQ2(4)));
end

figure('name','Project 1: P115 2nd Order Convergence - Extrapolation')
plot(N,LEr2U,'*');set(gca,'Fontsize',14);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Convergence of 2nd Order FDM For T-prime P115 - Unknown Solution (Quadratic
Extrapolation)','Fontsize',12);

fprintf('\nT-prime at x = 2 (C)\ndelta-x\t\tT-prime\n');
for b = 1:15
    h = 1/(b+1);
    eta = 1+(h/4);
    TdB2(b) = (T{b,1}(b+2)-T{b,1}(b+1))/(h*eta);
    Ti = TdB2(b);
    fprintf('%6.4f\t\t%8.6f\n',h,Ti);
end

LErB2 = zeros(1,15);
h2 = zeros(1,15);
for w = 1:15
    h2(w) = log10(w);
    Ti = TdB2(w);
    LErB2(w) = log10(100*abs((Ti-Td(2))/Td(2)));
end

figure('name','Project 1: T-prime at Boundary');
plot(h2(7:end),LErB2(7:end),'*');set(gca,'Fontsize',14);
title('P115: T-prime at x = 2 Using 2nd Order Difference','Fontsize',16);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);


%% 4th Order FDM
```

```
figure('name','Project 1: P115 4th Order Temp')
T4 = cell(15,1);
Er4 = cell(15,1);
alpha = 1 + (0.25/(2*((x0 + 0.5)^2)));
T4one = T0*((1/(alpha*0.25))+(1/(3*alpha*2.25)) - (1/(1.5)))/((2/(alpha*0.25))+(2/(6.75*alpha)));
T4{1,1} = [T0 T4one Tf];
Er4{1,1}(1) = log10(100*abs((Ta2(xin{1,1}(1))-T4{1,1}(2))/(Ta2(xin{1,1}(1)))));
plot(x{1},T4{1},'*');set(gca,'Fontsize',14);
hold on;
for i = 3:16
    [A,b] = Order4FDM(Tf,T0,xf,x0,i,f,fd,fdd);
    T4{i-1,1} = [T0 (A\b)' Tf];
    plot(x{i-1,1},T4{i-1,1},'-*');
    for e = 1:i-1
        Er4{i-1,1}(e) = log10(100*abs((Ta2(xin{i-1,1}(e))-T4{i-1,1}(e+1))/(Ta2(xin{i-1,1}(e)))));
    end
end
xlabel('x position','Fontsize',16);
ylabel('Temperature (C)','Fontsize',16);
title('Plot of Temperature vs Position for 4th Order FDM For P115','Fontsize',14);
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N = 
12','N = 13','N = 14','N = 15','N = 16');

fprintf('\nTEMPERATURE VS RADIUS\n------------------------------------');
for k = 1:15
    delta = 1/(k+1);
    fprintf('\ndeltaX = %6.4f\nx\t\tTemperature(C)\n',delta);
    for m = 1:k+2
        xi = x{k,1}(m);
        Ti = T4{k,1}(m);
        fprintf('%6.4f\t\t%8.6f\n',xi,Ti);
    end
end

T4d2 = cell(15,1);
ErP4 = cell(15,1);

figure('name','Project 1: P115 4th Order Error Plot')
fprintf('\nFirst Derivative vs Position');
for a = 1:15
    delta = 1/(a+1);
    fprintf('\ndeltaX = %6.4f\nx\t\tT prime(C)\n',delta);
    for b = 1:a
        xi = x{a,1}(b+1);
        alpha2 = 1 + ((delta^2)/(2*(xi^2)));
        T4d2{a,1}(b) = (T4{a,1}(b+2)-T4{a,1}(b))/(2*delta) + ((delta^2)/6)*(2/xi)*((T4{a,1}(b+2)-
2*T4{a,1}(b+1)+T4{a,1}(b))/(alpha2*(delta^2)));
        Ti = T4d2{a,1}(b);
        fprintf('%6.4f\t\t%8.6f\n',xi,Ti);
        ErP4{a,1}(b) = log10(100*abs((Ti-Td(x{a,1}(b+1)))/Td(x{a,1}(b+1))));
    end
    plot(xin{a,1},ErP4{a,1},'-*');set(gca,'Fontsize',14);
    hold on;
end
xlabel('x position','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of T-prime Error vs Position of 4th Order FDM For P115 Known 
Solution','Fontsize',12);
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N = 
12','N = 13','N = 14','N = 15','N = 16');

figure('name','P115: T-prime 4th order')
for b = 1:15
    plot(xin{b,1},T4d2{b,1},'-*');set(gca,'Fontsize',14);
    hold on;
end
xlabel('x position','Fontsize',16);
ylabel('T''(x) (C/m)','Fontsize',16);
title('P115: 4TH ORDER FDM FIRST DERIVATIVE OF TEMP.','Fontsize',18);
```

```
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N =
12','N = 13','N = 14','N = 15','N = 16');
hold off;

% Convergence Analysis
LErT4 = zeros(1,8);
LEr4 = zeros(1,8);
for w = 2:2:16
    temp = 0.5*w;
    LErT4(temp) = Er4{w-1,1}(w/2);
    LEr4(temp) = ErP4{w-1,1}(w/2);
end

% Extrapolation for Temperature Values
figure('name','Project 1: P115 4th Order Extrapolation');
for y = 1:7
    T4Q2(y) = (16*(T4{15,1}(2*y+1))-(T4{7,1}(y+1)))/15;
    ErQ4(y) = log10(100*abs((T4Q2(y)-Ta2(xQ2(y)))/(Ta2(xQ2(y)))));
end


% Convergence of Temperature - Unknown Solution using Quadratic
% Extrapolation
LErT4U = zeros(1,8);
for w = 1:8
    LErT4U(w) = log10(100*abs((T4{2*w-1,1}(w+1)-T4Q2(4))/T4Q2(4)));
end


figure('name','Project 1: P115 4th Order Convergence')
plot(N,LEr4,'*');set(gca,'Fontsize',14);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Convergence of 4th Order FDM For T-prime P115 - Known Solution','Fontsize',12);

% Extrapolation for Derivative Values
figure('name','Project 1: P115 4th Order Extrapolation');
fprintf('\nQuartic Extrapolation\nx\t\tT prime\n');
for y = 1:7
    TQ4(y) = (16*(T4d2{15,1}(2*y))-(T4d2{7,1}(y)))/15;
    fprintf('%6.4f\t\t%12.10f\n',xQ2(y),TQ4(y));
    ErPQ4(y) = log10(100*abs((TQ4(y)-Td(xQ2(y)))/(Td(xQ2(y)))));
end
plot(xQ2,ErPQ4,'-d');set(gca,'Fontsize',14);
hold on;

fprintf('\nRichardson''s Extrapolation\nx\t\tT prime\n');
for z = 1:3
    TR4(z) = (((T4d2{7,1}(2*z))^2)-(T4d2{3,1}(z))*(T4d2{15,1}(4*z)))/(2*(T4d2{7,1}(2*z))-
(T4d2{3,1}(z))-T4d2{15,1}(4*z));
    fprintf('%6.4f\t\t%12.10f\n',xR2(z),TR4(z));
    ErPR4(z) = log10(100*abs((TR4(z)-Td(xR2(z)))/(Td(xR2(z)))));
end
plot(xR2,ErPR4,'-o');set(gca,'Fontsize',14);
xlabel('x position','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Extrapolated T-prime Error vs Position of 4th Order FDM For P115','Fontsize',12);
legend('QuartEx','RichEx');


% Convergence - Unknown solution for First Derivative

LEr4U = zeros(1,8);
for w = 1:8
    Ti = T4d2{2*w-1,1}(w);
    LEr4U(w) = log10(100*abs((Ti-TQ4(4))/TQ4(4)));
end

figure('name','Project 1: P115 4th Order Convergence - Extrapolation')
plot(N,LEr,'*');set(gca,'Fontsize',14);
z = [N(1) N(8)];
```

```
hold on;
P1 = polyfit(N,LEr,1); fit1 = @(r) P1(1)*r+P1(2); fplot(fit1,z,'-.'); s1 = sprintf('2nd Known:\ny
= %6.4f*x + %6.4f',P1(1),P1(2));text(0.5,0,s1);
plot(N,LEr2U,'*');P2 = polyfit(N,LEr2U,1);fit2 = @(r) P2(1)*r+P2(2); fplot(fit2,z,'-.'); s2 =
sprintf('2nd Unknown:\ny = %6.4f*x + %6.4f',P2(1),P2(2));text(1,0,s2);
plot(N,LEr4,'*');P3 = polyfit(N,LEr4,1);fit3 = @(r) P3(1)*r+P3(2); fplot(fit3,z,'-.'); s3 =
sprintf('4th Known:\ny = %6.4f*x + %6.4f',P3(1),P3(2));text(0.5,-3,s3);
plot(N,LEr4U,'*');P4 = polyfit(N,LEr4U,1);fit4 = @(r) P4(1)*r+P4(2); fplot(fit4,z,'-.'); s4 =
sprintf('4th Unknown:\ny = %6.4f*x + %6.4f',P4(1),P4(2));text(1,-3,s4);
legend('2nd Known','2nd K. Fit','2nd Unk.','2nd Unk. Fit','4th Known','4th K. Fit','4th
Unk.','4th Unk. Fit');
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('P115: CONVERGENCE FOR 2ND & 4TH ORDER FDM FOR T-PRIME ','Fontsize',18);

figure('name','Project 1: P115 4th Order Convergence - Extrapolation')
plot(N,LEr2,'*');set(gca,'Fontsize',14);
hold on;
P5 = polyfit(N,LEr2,1); fit5 = @(r) P5(1)*r+P5(2); fplot(fit5,z,'-.'); s5 = sprintf('2nd
Known:\ny = %6.4f*x + %6.4f',P5(1),P5(2));text(0.5,0,s5);
plot(N,LErT2,'*');P6 = polyfit(N,LErT2,1);fit6 = @(r) P6(1)*r+P6(2); fplot(fit6,z,'-.'); s6 =
sprintf('2nd Unknown:\ny = %6.4f*x + %6.4f',P6(1),P6(2));text(1,0,s6);
plot(N,LErT4,'*');P7 = polyfit(N,LErT4,1);fit7 = @(r) P7(1)*r+P7(2); fplot(fit7,z,'-.'); s7 =
sprintf('4th Known:\ny = %6.4f*x + %6.4f',P7(1),P7(2));text(0.5,-3,s7);
plot(N,LErT4U,'*');P8 = polyfit(N,LErT4U,1);fit8 = @(r) P8(1)*r+P8(2); fplot(fit8,z,'-.'); s8 =
sprintf('4th Unknown:\ny = %6.4f*x + %6.4f',P8(1),P8(2));text(1,-3,s8);
legend('2nd Known','2nd Known Fit','2nd Unknown','2nd Unknown Fit','4th Known','4th Known
Fit','4th Unknown','4th Unknown Fit');
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('P115: CONVERGENCE FOR 2ND & 4TH ORDER FDM FOR TEMPERATURE','Fontsize',18);


fprintf('\nT-prime at x = 2 (C)\ndelta-x\t\tT-prime\n');
for b = 1:15
    h = 1/(b+1);
    eta = 1+(h/4)+(h^2/12)+(h^3/32);
    TdB4(b) = (T4{b,1}(b+2)-T4{b,1}(b+1))/(h*eta);
    Ti = TdB4(b);
    fprintf('%6.4f\t\t%12.10f\n',h,Ti);
end

LErB4 = zeros(1,15);
h4 = zeros(1,15);

for w = 1:15
    h4(w) = log10(w);
    Ti = TdB4(w);
    LErB4(w) = log10(100*abs((Ti-Td(2))/Td(2)));
end

figure('name','Project 1: T-prime at Boundary');
plot(h4(7:end),LErB4(7:end),'*');set(gca,'Fontsize',14);
title('P115: T-prime at x = 2 Using 4th Order Difference','Fontsize',16);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);


%% Project 1: 2nd Order FDM P115
% Seyed Sepehr Seyedi
%
% UIN 424006176
%
% AERO 430-500
%
% October 5th, 2015

%% Function

function [A,b] = Order2FDM(Tf,T0,xf,x0,N,f)
```

```
h = (xf-x0)/N;
A = zeros(N-1,N-1);
A(1,1) = -2*(x0 + h);
A(1,2) = x0 + 1.5*h;
for i = 2:N-2
    A(i,i-1) = x0 + h*(i - 0.5);
    A(i,i) = -2*(x0 + h*i);
    A(i,i+1) = x0 + h*(i + 0.5);
end
A(N-1,N-2) = x0 + h*(N-1.5);
A(N-1,N-1) = -2*(x0 + h*(N-1));
b = zeros(N-1,1);
b(1) = -(x0 + 0.5*h)*T0 + (h^2)*(x0+h)*(f(x0+h));
b(N-1) = -(x0 + h*(N-0.5))*Tf + (h^2)*(xf-h)*(f(xf-h));
for j = 2:N-2
    b(j) = (h^2)*(x0+j*h)*(f(x0+j*h));
end


%% Project 1: 4th Order FDM P115
% Seyed Sepehr Seyedi
%
% UIN 424006176
%
% AERO 430-500
%
% October 5th, 2015

%% Function

function [A,b] = Order4FDM(Tf,T0,xf,x0,N,f,fd,fdd)

h = (xf-x0)/N;
A = zeros(N-1,N-1);
alpha = 1 + (h^2/(2*((x0 + h)^2)));
A(1,1) = -(2/(alpha*(h^2)))-(2/(3*alpha*((x0+h)^2)));
A(1,2) = (1/(alpha*(h^2)))+(1/(3*alpha*((x0+h)^2))) + (1/(2*h*(x0+h)));
for i = 2:N-2
    alpha2 = 1 + (h^2/(2*((x0 + i*h)^2)));
    A(i,i-1) = (1/(alpha2*(h^2)))+(1/(3*alpha2*((x0+i*h)^2))) - (1/(2*h*(x0+i*h)));
    A(i,i) = -(2/(alpha2*(h^2)))-(2/(3*alpha2*((x0+i*h)^2)));
    A(i,i+1) = (1/(alpha2*(h^2)))+(1/(3*alpha2*((x0+i*h)^2))) + (1/(2*h*(x0+i*h)));
end
alpha3 = 1 + (h^2/(2*((xf - h)^2)));
A(N-1,N-2) = (1/(alpha3*(h^2)))+(1/(3*alpha3*((xf-h)^2))) - (1/(2*h*(xf-h)));
A(N-1,N-1) = -(2/(alpha3*(h^2)))-(2/(3*alpha3*((xf-h)^2)));
b = zeros(N-1,1);
b(1) = f(x0+h)*(1+(h^2/(6*((x0+h)^2))))+fd(x0+h)*((h^2)/(6*(x0+h)))+(fdd(x0+h)-
(1/(x0+h))*fd(x0+h)-(3/((x0+h)^2))*f(x0+h))*(h^2/(12*alpha))*(1+((h^2)/(3*((x0+h)^2))))-
T0*((1/(alpha*(h^2)))+(1/(3*alpha*((x0+h)^2)))-(1/(2*h*(x0+h))));
b(N-1) = f(xf-h)*(1+(h^2/(6*((xf-h)^2))))+fd(xf-h)*((h^2)/(6*(xf-h)))+(fdd(xf-h)-(1/(xf-
h))*fd(xf-h)-(3/((xf-h)^2))*f(xf-h))*(h^2/(12*alpha3))*(1+((h^2)/(3*((xf-h)^2))))-
Tf*((1/(alpha*(h^2)))+(1/(3*alpha*((x0+h)^2)))+(1/(2*h*(x0+h))));
for j = 2:N-2
    alpha2 = 1 + (h^2/(2*((x0 + j*h)^2)));
    b(j) = f(x0+j*h)*(1+(h^2/(6*((x0+j*h)^2))))+fd(x0+j*h)*((h^2)/(6*(x0+j*h)))+(fdd(x0+j*h)-
(1/(x0+j*h))*fd(x0+j*h)-
(3/((x0+j*h)^2))*f(x0+j*h))*(h^2/(12*alpha2))*(1+((h^2)/(3*((x0+j*h)^2))));
end

%% Project 1: Problem 116
% Seyed Sepehr Seyedi
%
% UIN 424006176
%
% AERO 430-500
%
% October 5th, 2015

%% Parameters - Boundary Conditions & Inhomogeneous Elements
```

```
T0 = 100;
xf = 2;
x0 = 1;
f = @(r) 0;
fd = @(r) 0;
fdd = @(r) 0;

% Define the exact solution function and its derivative.
Ta2 = @(r) 100*((1+(log(1024))-10*(log(r)))/(1+(log(1024))));
Td = @(r) -(1/r)*(1000/(1+log(1024)));


%% 2nd Order FDM

% Define variables as cell arrays to access data on the basis of grid
% spacing.
figure('name','Project 1: P116 2nd Order Temp')
T = cell(16,1);
x = cell(16,1);
xin = cell(16,1);
Er2 = cell(16,1);

% Evaluate T and x values using the interior equation, and plot temperature vs grid spacing.
Tout = xf/(xf+5*(xf+0.5));
T{1,1} = [T0 Tout];
x{1,1} = [x0 xf];
Er2{1,1}(1) = log10(100*abs((Ta2(2)-T{1,1}(2))/(Ta2(2))));
plot(x{1},T{1},'*');set(gca,'Fontsize',14);
hold on;
for i = 2:16
    [A,b] = Order2FDM116(T0,xf,x0,i,f);
    T{i,1} = [T0 (A\b)'];
    for j = 1:i+1
        x{i,1}(j) = x0 + (j-1)*((xf-x0)/i);
    end
    for u = 1:i-1
        xin{i-1,1}(u) = x0 + (u)*((xf-x0)/i);
    end
    plot(x{i,1},T{i,1},'-*');set(gca,'Fontsize',14);
    for e = 1:i
        Er2{i,1}(e) = log10(100*abs((Ta2(x{i,1}(e+1))-T{i,1}(e+1))/(Ta2(x{i,1}(e+1)))));
    end
end
xlabel('x position','Fontsize',16);
ylabel('Temperature (C)','Fontsize',16);
title('Plot of Temperature vs Position for 2nd Order FDM For P116','Fontsize',14);
legend('N=1','N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N
= 12','N = 13','N = 14','N = 15','N = 16');

% Output temperature values for each grid size.
fprintf('TEMPERATURE VS RADIUS\n------------------------------------');
for k = 1:16
    delta = 1/(k);
    fprintf('\ndeltaX = %6.4f\nx\t\tTemperature(C)\n',delta);
    for m = 1:k+1
        xi = x{k,1}(m);
        Ti = T{k,1}(m);
        fprintf('%6.4f\t\t%8.6f\n',xi,Ti);
    end
end


% Define derivative variables as cell arrays to access data on the basis of grid spacing.
Td2 = cell(15,1);
Er = cell(15,1);
ErP = cell(15,1);
lx = cell(15,1);

% Evaluate and output derivative values using 2nd order difference, and plot error vs grid
spacing.
figure('name','Project 1: P116 2nd Order Error Plot')
```

```
fprintf('\nFirst Derivative vs Position');
for a = 2:16
    delta = 1/(a);
    fprintf('\ndeltaX = %6.4f\nx\t\tT prime(C)\n',delta);
    for b = 1:a-1
        Td2{a-1,1}(b) = (T{a,1}(b+2)-T{a,1}(b))/(2*delta);
        xi = x{a,1}(b+1);
        Ti = Td2{a-1,1}(b);
        fprintf('%6.4f\t\t%8.6f\n',xi,Ti);
        ErP{a-1,1}(b) = log10(100*abs((Ti-Td(x{a,1}(b+1)))/Td(x{a,1}(b+1))));
    end
    plot(xin{a-1,1},ErP{a-1,1},'-*');set(gca,'Fontsize',14);
    hold on;
end
xlabel('x position','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of T-prime Error vs Position of 2nd Order FDM For P116 Known
Solution','Fontsize',12);
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N =
12','N = 13','N = 14','N = 15','N = 16');

figure('name','P116: T-prime 2nd order')
for b = 1:15
    plot(xin{b,1},Td2{b,1},'-*');set(gca,'Fontsize',14);
    hold on;
end
xlabel('x position','Fontsize',16);
ylabel('T''(x)  (C/m)','Fontsize',16);
title('P116: 2ND ORDER FDM FIRST DERIVATIVE OF TEMP.','Fontsize',18);
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N =
12','N = 13','N = 14','N = 15','N = 16');
hold off;

% Convergence Analysis
N = zeros(1,8);
LEr2 = zeros(1,8);
LEr = zeros(1,8);
for w = 2:2:16
    temp = 0.5*w;
    N(temp) = log10(w);
    LEr2(temp) = Er2{w,1}(w/2);
    LEr(temp) = ErP{w-1,1}(w/2);
end

figure('name','Project 1: P116 Convergence')
plot(N,LEr,'*');set(gca,'Fontsize',14);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Convergence of 2nd Order FDM For T-prime P116 - Known Solution','Fontsize',12);

% Extrapolation for Temperature Values.
figure('name','Project 1: P116 2nd Order Extrapolation');
xQ2 = [1.125:0.125:1.875];
for y = 1:7
    T2Q2(y) = (4*(T{16,1}(2*y+1))-(T{8,1}(y+1)))/3;
    ErQ2(y) = log10(100*abs((T2Q2(y)-Ta2(xQ2(y)))/(Ta2(xQ2(y)))));
end

% Convergence of Temperature - Unknown Solution using Quadratic
% Extrapolation
LErT2 = zeros(1,8);
for w = 1:8
    LErT2(w) = log10(100*abs((T{2*w,1}(w+1)-T2Q2(4))/T2Q2(4)));
end


% Extrapolation for Derivative Values
figure('name','Project 1: P116 2nd Order Extrapolation');
fprintf('\nQuadratic Extrapolation\nx\t\tT prime\n');
xQ2 = [1.125:0.125:1.875];
for y = 1:7
```

```
    TQ2(y) = (4*(Td2{15,1}(2*y))-(Td2{7,1}(y)))/3;
    fprintf('%6.4f\t\t%8.6f\n',xQ2(y),TQ2(y));
    ErPQ2(y) = log10(100*abs((TQ2(y)-Td(xQ2(y)))/(Td(xQ2(y)))));
end
plot(xQ2,ErPQ2,'-d');set(gca,'Fontsize',14);
hold on;

fprintf('\nRichardson''s Extrapolation\nx\t\tT prime\n');
xR2 = [1.25 1.5 1.75];
for z = 1:3
    TR2(z) = (((Td2{7,1}(2*z))^2)-(Td2{3,1}(z))*(Td2{15,1}(4*z)))/(2*(Td2{7,1}(2*z))-
(Td2{3,1}(z))-Td2{15,1}(4*z));
    fprintf('%6.4f\t\t%8.6f\n',xR2(z),TR2(z));
    ErPR2(z) = log10(100*abs((TR2(z)-Td(xR2(z)))/(Td(xR2(z)))));
end
plot(xR2,ErPR2,'-o');set(gca,'Fontsize',14);
xlabel('x position','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Extrapolated T-prime Error vs Position of 2nd Order FDM For P116','Fontsize',13);
legend('QuadEx','RichEx');

% Convergence - Unknown solution

LEr2U = zeros(1,8);
for w = 1:8
    Ti = Td2{2*w-1,1}(w);
    LEr2U(w) = log10(100*abs((Ti-TQ2(4))/TQ2(4)));
end

figure('name','Project 1: P116 2nd Order Convergence - Extrapolation')
plot(N,LEr2U,'*');set(gca,'Fontsize',14);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Convergence of 2nd Order FDM For T-prime P116 - Unknown Solution (Quadratic
Extrapolation)','Fontsize',12);

fprintf('\nT-prime at x = 2 (C)\ndelta-x\t\tT-prime\n');
for b = 1:16
    h = 1/b;
    eta = 1+(h/4);
    TdB2(b) = (T{b,1}(b+1)-T{b,1}(b))/(h*eta);
    Ti = TdB2(b);
    fprintf('%6.4f\t\t%8.6f\n',h,Ti);
end

LErB2 = zeros(1,15);
LErBT2 = zeros(1,15);
h2 = zeros(1,15);
for w = 2:16
    h2(w-1) = log10(w);
    Ti = TdB2(w);
    LErB2(w-1) = log10(100*abs((Ti-Td(x{w,1}(w+1)))/Td(x{w,1}(w+1))));
    Tii = T{w,1}(w+1);
    LErBT2(w-1) = log10(100*abs((Tii-Ta2(2))/Ta2(2)));
end
figure('name','Project 1: Temperature at Boundary');
plot(h2,LErB2,'*');set(gca,'Fontsize',14);
title('P116: T-prime at x = 2 Using 2nd Order Difference','Fontsize',16);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);

figure('name','Project 1: Temperature at Boundary');
plot(h2,LErBT2,'*');set(gca,'Fontsize',14);
title('P116: Temperature at x = 2 Using 2nd Order Difference','Fontsize',16);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);


%% 4th Order FDM

figure('name','Project 1: P116 4th Order Temp')
```

```
T4 = cell(16,1);
Er4 = cell(15,1);
alpha1 = 1.125;
gamma1 = 0.5+(1/(6*alpha1));
Tout1 = T0*((1/alpha1)+(1/(12*alpha1))-0.25+(1/gamma1)*(0.5-
(1/(6*alpha1)))*((1/alpha1)+(1/(12*alpha1))+0.25))/((2/alpha1)+(1/(6*alpha1))+(1/gamma1)*(5-
(1/(3*alpha1)))*((1/alpha1)+(1/(12*alpha1))+0.25));
T4{1,1} = [T0 Tout1];
plot(x{1,1},T4{1,1},'*');set(gca,'Fontsize',14);
hold on;
for i = 2:16
    [A,b] = Order4FDM116(T0,xf,x0,i,f,fd,fdd);
    T4{i,1} = [T0 (A\b)'];
    plot(x{i,1},T4{i,1},'-*');set(gca,'Fontsize',14);
    for e = 1:i-1
        Er4{i-1,1}(e) = log10(100*abs((Ta2(x{i,1}(e+1))-T4{i,1}(e+1))/(Ta2(x{i,1}(e+1)))));
    end
end
xlabel('x position','Fontsize',16);
ylabel('Temperature (C)','Fontsize',16);
title('Plot of Temperature vs Position for 4th Order FDM For P116','Fontsize',14);
legend('N = 1','N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N =
11','N = 12','N = 13','N = 14','N = 15','N = 16');

fprintf('TEMPERATURE VS RADIUS\n------------------------------------');
for k = 1:16
    delta = 1/(k);
    fprintf('\ndeltaX = %6.4f\nx\t\tTemperature(C)\n',delta);
    for m = 1:k+1
        xi = x{k,1}(m);
        Ti = T4{k,1}(m);
        fprintf('%6.4f\t\t%12.10f\n',xi,Ti);
    end
end

T4d2 = cell(15,1);
ErP4 = cell(15,1);

figure('name','Project 1: P116 4th Order Error Plot')
fprintf('\nFirst Derivative vs Position');
for a = 2:16
    delta = 1/a;
    fprintf('\ndeltaX = %6.4f\nx\t\tT prime(C)\n',delta);
    for b = 1:a-1
        xi = x{a,1}(b+1);
        alpha2 = 1 + ((delta^2)/(2*(xi^2)));
        T4d2{a-1,1}(b) = (T4{a,1}(b+2)-T4{a,1}(b))/(2*delta) +
((delta^2)/6)*((2/xi)*(((T4{a,1}(b+2)-
2*T4{a,1}(b+1)+T4{a,1}(b))/(alpha2*(delta^2)))+(delta^2/(12*alpha2))*((3/(xi^2))*(f(xi))+(1/xi)*f
d(xi)-fdd(xi)))-fd(xi)-(1/xi)*f(xi));
        Ti = T4d2{a-1,1}(b);
        fprintf('%6.4f\t\t%8.6f\n',xi,Ti);
        ErP4{a-1,1}(b) = log10(100*abs((Ti-Td(x{a,1}(b+1)))/Td(x{a,1}(b+1))));
    end
    plot(xin{a-1,1},ErP4{a-1,1},'-*');set(gca,'Fontsize',14);
    hold on;
end
xlabel('x position','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of T-prime Error vs Position of 4th Order FDM For P116 Known
Solution','Fontsize',13);
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N =
12','N = 13','N = 14','N = 15','N = 16');

figure('name','P116: T-prime 4th order')
for b = 1:15
    plot(xin{b,1},T4d2{b,1},'-*');set(gca,'Fontsize',14);
    hold on;
end
xlabel('x position','Fontsize',16);
ylabel('T''(x) (C/m)','Fontsize',16);
```

```
title('P116: 4TH ORDER FDM FIRST DERIVATIVE OF TEMP.','Fontsize',18);
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N =
12','N = 13','N = 14','N = 15','N = 16');
hold off;

% Convergence Analysis
LErT4 = zeros(1,8);
LEr4 = zeros(1,8);
for w = 2:2:16
    temp = 0.5*w;
    LErT4(temp) = Er4{w-1,1}(w/2);
    LEr4(temp) = ErP4{w-1,1}(w/2);
end

% Extrapolation for Temperature Values
figure('name','Project 1: P115 4th Order Extrapolation');
for y = 1:7
    T4Q2(y) = (16*(T4{16,1}(2*y+1))-(T4{8,1}(y+1)))/15;
    ErQ4(y) = log10(100*abs((T4Q2(y)-Ta2(xQ2(y)))/(Ta2(xQ2(y)))));
end

% Convergence of Temperature - Unknown Solution using Quadratic
% Extrapolation
LErT4U = zeros(1,8);
for w = 1:8
    LErT4U(w) = log10(100*abs((T4{2*w,1}(w+1)-T4Q2(4))/T4Q2(4)));
end


figure('name','Project 1: P116 Convergence')
plot(N,LEr4,'*');set(gca,'Fontsize',14);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Convergence of 4th Order FDM For T-prime P116 - Known Solution','Fontsize',13);

% Extrapolation
figure('name','Project 1: P116 4th Order Extrapolation');
fprintf('\nQuartic Extrapolation\nx\t\tT prime\n');
for y = 1:7
    TQ4(y) = (16*(T4d2{15,1}(2*y))-(T4d2{7,1}(y)))/15;
    fprintf('%6.4f\t\t%12.10f\n',xQ2(y),TQ4(y));
    ErPQ4(y) = log10(100*abs((TQ4(y)-Td(xQ2(y)))/(Td(xQ2(y)))));
end
plot(xQ2,ErPQ4,'-d');set(gca,'Fontsize',14);
hold on;

fprintf('\nRichardson''s Extrapolation\nx\t\tT prime\n');
for z = 1:3
    TR4(z) = (((T4d2{7,1}(2*z))^2)-(T4d2{3,1}(z))*(T4d2{15,1}(4*z)))/(2*(T4d2{7,1}(2*z))-
(T4d2{3,1}(z))-T4d2{15,1}(4*z));
    fprintf('%6.4f\t\t%12.10f\n',xR2(z),TR4(z));
    ErPR4(z) = log10(100*abs((TR4(z)-Td(xR2(z)))/(Td(xR2(z)))));
end
plot(xR2,ErPR4,'-o');set(gca,'Fontsize',14);
xlabel('x position','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Extrapolated T-prime Error vs Position of 4th Order FDM For P116','Fontsize',14);
legend('QuartEx','RichEx');


% Convergence - Unknown solution

LEr4U = zeros(1,8);
for w = 1:8
    Ti = T4d2{2*w-1,1}(w);
    LEr4U(w) = log10(100*abs((Ti-TQ4(4))/TQ4(4)));
end

figure('name','Project 1: P116 4th Order Convergence - Extrapolation')
plot(N,LEr4U,'*');set(gca,'Fontsize',14);
z = [N(1) N(8)];
```

```
hold on;
P1 = polyfit(N,LEr,1); fit1 = @(r) P1(1)*r+P1(2); fplot(fit1,z,'-.'); s1 = sprintf('2nd Known:\ny
= %6.4f*x + %6.4f',P1(1),P1(2));text(0.5,0,s1);
plot(N,LEr2U,'*');P2 = polyfit(N,LEr2U,1);fit2 = @(r) P2(1)*r+P2(2); fplot(fit2,z,'-.'); s2 =
sprintf('2nd Unknown:\ny = %6.4f*x + %6.4f',P2(1),P2(2));text(1,0,s2);
plot(N,LEr4,'*');P3 = polyfit(N,LEr4,1);fit3 = @(r) P3(1)*r+P3(2); fplot(fit3,z,'-.'); s3 =
sprintf('4th Known:\ny = %6.4f*x + %6.4f',P3(1),P3(2));text(0.5,-3,s3);
plot(N,LEr4U,'*');P4 = polyfit(N,LEr4U,1);fit4 = @(r) P4(1)*r+P4(2); fplot(fit4,z,'-.'); s4 =
sprintf('4th Unknown:\ny = %6.4f*x + %6.4f',P4(1),P4(2));text(1,-3,s4);
legend('2nd Known','2nd K. Fit','2nd Unk.','2nd Unk. Fit','4th Known','4th K. Fit','4th
Unk.','4th Unk. Fit');
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('P116: CONVERGENCE OF 2ND & 4TH ORDER FDM FOR T-PRIME','Fontsize',18);

figure('name','Project 1: P116 4th Order Convergence - Extrapolation')
plot(N(2:end),LEr2(2:end),'*');set(gca,'Fontsize',14);
z = [N(2) N(8)];
hold on;
P5 = polyfit(N(2:end),LEr2(2:end),1); fit5 = @(r) P5(1)*r+P5(2); fplot(fit5,z,'-.'); s5 =
sprintf('2nd Known:\ny = %6.4f*x + %6.4f',P5(1),P5(2));text(0.5,0,s5);
plot(N(2:end),LErT2(2:end),'*');P6 = polyfit(N(2:end),LErT2(2:end),1);fit6 = @(r) P6(1)*r+P6(2);
fplot(fit6,z,'-.'); s6 = sprintf('2nd Unknown:\ny = %6.4f*x + %6.4f',P6(1),P6(2));text(1,0,s6);
plot(N(2:end),LErT4(2:end),'*');P7 = polyfit(N(2:end),LErT4(2:end),1);fit7 = @(r) P7(1)*r+P7(2);
fplot(fit7,z,'-.'); s7 = sprintf('4th Known:\ny = %6.4f*x + %6.4f',P7(1),P7(2));text(0.5,-3,s7);
plot(N(2:end),LErT4U(2:end),'*');P8 = polyfit(N(2:end),LErT4U(2:end),1);fit8 = @(r)
P8(1)*r+P8(2); fplot(fit8,z,'-.'); s8 = sprintf('4th Unknown:\ny = %6.4f*x +
%6.4f',P8(1),P8(2));text(1,-3,s8);
legend('2nd Known','2nd Known Fit','2nd Unknown','2nd Unknown Fit','4th Known','4th Known
Fit','4th Unknown','4th Unknown Fit');
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('P116: CONVERGENCE FOR 2ND & 4TH ORDER FDM FOR TEMPERATURE','Fontsize',18);


fprintf('\nT-prime at x = 2 (C)\ndelta-x\t\tT-prime\n');
for b = 1:16
    h = 1/b;
    eta = 1+(h/4)+((h^2)/12)+((h^3)/32);
    TdB4(b) = (T4{b,1}(b+1)-T4{b,1}(b))/(h*eta);
    Ti = TdB4(b);
    fprintf('%6.4f\t\t%12.10f\n',h,Ti);
end

LErB4 = zeros(1,15);
LErBT4 = zeros(1,15);
h4 = zeros(1,15);
for w = 2:16
    h4(w-1) = log10(w);
    Ti = TdB4(w);
    LErB4(w-1) = log10(100*abs((Ti-Td(2))/Td(2)));
    Tii = T4{w,1}(w+1);
    LErBT4(w-1) = log10(100*abs((Tii-Ta2(2))/Ta2(2)));
end

figure('name','Project 1: Temperature at Boundary');
plot(h4,LErB4,'*');set(gca,'Fontsize',14);
title('P116: T-prime at x = 2 Using 4th Order Difference','Fontsize',16);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);

figure('name','Project 1: Temperature at Boundary');
plot(h4,LErBT4,'*');set(gca,'Fontsize',14);
title('P116: Temperature at x = 2 Using 4th Order Difference','Fontsize',16);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);

%% Project 1: 2nd Order FDM P116
% Seyed Sepehr Seyedi
%
% UIN 424006176
```

```
%
% AERO 430-500
%
% October 5th, 2015

%% Function

function [A,b] = Order2FDM116(T0,xf,x0,N,f)

h = (xf-x0)/N;
A = zeros(N,N);
A(1,1) = -2*(x0 + h);
A(1,2) = x0 + 1.5*h;
for i = 2:N-1
    A(i,i-1) = x0 + h*(i - 0.5);
    A(i,i) = -2*(x0 + h*i);
    A(i,i+1) = x0 + h*(i + 0.5);
end
A(N,N-1) = xf;
A(N,N) = -xf-5*h*(xf+(h/2));
b = zeros(N,1);
b(1) = -(x0 + 0.5*h)*T0 + (h^2)*(x0+h)*(f(x0+h));


%% Project 1: 4th Order FDM P116
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% October 5th, 2015

%% Function

function [A,b] = Order4FDM116(T0,xf,x0,N,f,fd,fdd)

h = (xf-x0)/N;
A = zeros(N,N);
alpha = 1 + (h^2/(2*((x0 + h)^2)));
A(1,1) = -(2/(alpha*(h^2)))-(2/(3*alpha*((x0+h)^2)));
A(1,2) = (1/(alpha*(h^2)))+(1/(3*alpha*((x0+h)^2))) + (1/(2*h*(x0+h)));
for i = 2:N-1
    alpha2 = 1 + (h^2/(2*((x0 + i*h)^2)));
    A(i,i-1) = (1/(alpha2*(h^2)))+(1/(3*alpha2*((x0+i*h)^2))) - (1/(2*h*(x0+i*h)));
    A(i,i) = -(2/(alpha2*(h^2)))-(2/(3*alpha2*((x0+i*h)^2)));
    A(i,i+1) = (1/(alpha2*(h^2)))+(1/(3*alpha2*((x0+i*h)^2))) + (1/(2*h*(x0+i*h)));
end
alpha3 = 1 + (h^2/(2*((xf - h)^2)));
gamma = (1/(2*h))+(1/(3*alpha3*xf));
A(N,N-1) = (1/(alpha3*(h^2)))+(1/(3*alpha3*(xf^2)))-(1/(2*xf*h))+(1/gamma)*((1/(2*h))-
(1/(3*alpha3*xf)))*((1/(alpha3*(h^2)))+(1/(3*alpha3*(xf^2)))+(1/(2*xf*h)));
A(N,N) = -(2/(alpha3*(h^2)))-(2/(3*alpha3*(xf^2)))+(1/gamma)*((2/(3*alpha3*xf))-
5)*((1/(alpha3*(h^2)))+(1/(3*alpha3*(xf^2)))+(1/(2*xf*h)));
b = zeros(N,1);
b(1) = f(x0+h)*(1+(h^2/(6*((x0+h)^2))))+fd(x0+h)*((h^2)/(6*(x0+h)))+(fdd(x0+h)-
(1/(x0+h))*fd(x0+h)-(3/((x0+h)^2))*f(x0+h))*(h^2/(12*alpha))*(1+((h^2)/(3*((x0+h)^2))))-
T0*((1/(alpha*(h^2)))+(1/(3*alpha*((x0+h)^2)))-(1/(2*h*(x0+h))));
end


%% Project 1: Problem 117
% Seyed Sepehr Seyedi
%
% UIN 424006176
%
% AERO 430-500
%
% October 27th, 2015

%% Parameters - Boundary Conditions & Inhomogeneous Elements
```

```
Tf = 0;
xf = 1;
x0 = 0;
f = @(r) -100*(1+(r^2));
fd = @(r) -200*r;
fdd = @(r) -200;

% Define the exact solution function and its derivative.
Ta2 = @(r) (125/4)-100*((r^2/4)+(r^4/16));
Td = @(r) -25*r*(2+(r^2));

%% 2nd Order FDM

% Define variables as cell arrays to access data on the basis of grid
% spacing.
figure('name','Project 1: P117 2nd Order Temp')
T = cell(16,1);
x = cell(16,1);
xin = cell(16,1);
Er2 = cell(15,1);

% Evaluate T and x values using the interior equation, and plot temperature vs grid spacing.
T{1,1} = [Tf Tf];
x{1,1} = [x0 xf];
plot(x{1},T{1},'*');set(gca,'Fontsize',14);
hold on;
for i = 2:16
    [A,b] = Order2FDM117(Tf,xf,x0,i,f);
    T{i,1} = [(A\b)' Tf];
    for j = 1:i+1
        x{i,1}(j) = x0 + (j-1)*((xf-x0)/i);
    end
    for u = 1:i-1
        xin{i-1,1}(u) = x0 + (u)*((xf-x0)/i);
    end
    plot(x{i,1},T{i,1},'-*');set(gca,'Fontsize',14);
    for e = 1:i-1
        Er2{i-1,1}(e) = log10(100*abs((Ta2(x{i,1}(e+1))-T{i,1}(e+1))/(Ta2(x{i,1}(e+1)))));
    end
end
xlabel('x position','Fontsize',16);
ylabel('Temperature (C)','Fontsize',16);
title('Plot of Temperature vs Position for 2nd Order FDM For P117','Fontsize',16);
legend('N=1','N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N
= 12','N = 13','N = 14','N = 15','N = 16');

% Output temperature values for each grid size.
fprintf('TEMPERATURE VS RADIUS\n------------------------------------');
for k = 1:16
    delta = 1/(k);
    fprintf('\ndeltaX = %6.4f\nx\t\tTemperature(C)\n',delta);
    for m = 1:k+1
        xi = x{k,1}(m);
        Ti = T{k,1}(m);
        fprintf('%6.4f\t\t%8.6f\n',xi,Ti);
    end
end


% Define derivative variables as cell arrays to access data on the basis of grid spacing.
Td2 = cell(15,1);
Er = cell(15,1);
ErP = cell(15,1);
lx = cell(15,1);

% Evaluate and output derivative values using 2nd order difference, and plot error vs grid
spacing.
figure('name','Project 1: P117 2nd Order Error Plot')
fprintf('\nFirst Derivative vs Position');
for a = 2:16
```

```
    delta = 1/(a);
    fprintf('\ndeltaX = %6.4f\nx\t\tT prime(C)\n',delta);
    for b = 1:a-1
        Td2{a-1,1}(b) = (T{a,1}(b+2)-T{a,1}(b))/(2*delta);
        xi = x{a,1}(b+1);
        Ti = Td2{a-1,1}(b);
        fprintf('%6.4f\t\t%8.6f\n',xi,Ti);
        Er{a-1,1}(b) = log(abs(Ti-Td(x{a,1}(b+1))));
        ErP{a-1,1}(b) = log10(100*abs((Ti-Td(x{a,1}(b+1)))/Td(x{a,1}(b+1))));
    end
    plot(xin{a-1,1},ErP{a-1,1},'-*');set(gca,'Fontsize',14);
    hold on;
end
xlabel('x position','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of T-prime Error vs Position of 2nd Order FDM For P117 Known
Solution','Fontsize',14);
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N =
12','N = 13','N = 14','N = 15','N = 16');

figure('name','P117: T-prime 2nd order')
for b = 1:15
    plot(xin{b,1},Td2{b,1},'-*');set(gca,'Fontsize',14);
    hold on;
end
xlabel('x position','Fontsize',16);
ylabel('T''(x)  (C/m)','Fontsize',16);
title('P117: 2ND ORDER FDM FIRST DERIVATIVE OF TEMP.','Fontsize',18);
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N =
12','N = 13','N = 14','N = 15','N = 16');
hold off;

% Convergence Analysis of Derivative - Known Solution at x = 0.5.
N = zeros(1,8);
LEr2 = zeros(1,8);
LEr = zeros(1,8);
for w = 2:2:16
    temp = 0.5*w;
    N(temp) = log10(w);
    LEr2(temp) = Er2{w-1,1}(w/2);
    LEr(temp) = ErP{w-1,1}(w/2);
end

% Plot convergence of derivative vs grid size on log-log plot.
figure('name','Project 1: P117 Convergence')
plot(N,LEr,'*');set(gca,'Fontsize',14);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Convergence of 2nd Order FDM For T-prime P117 - Known Solution','Fontsize',14);

% Extrapolation for Temperature Values.
figure('name','Project 1: P116 2nd Order Extrapolation');
xQ2 = [1.125:0.125:1.875];
for y = 1:7
    T2Q2(y) = (4*(T{16,1}(2*y+1))-(T{8,1}(y+1)))/3;
    ErQ2(y) = log10(100*abs((T2Q2(y)-Ta2(xQ2(y)))/(Ta2(xQ2(y)))));
end

% Convergence of Temperature - Unknown Solution using Quadratic
% Extrapolation
LErT2 = zeros(1,8);
for w = 1:8
    LErT2(w) = log10(100*abs((T{2*w,1}(w+1)-T2Q2(4))/T2Q2(4)));
end

% Quadratic extrapolation and output of derivative values and plot of error vs x-position.
figure('name','Project 1: P117 2nd Order Extrapolation');
fprintf('\nQuadratic Extrapolation\nx\t\tT prime\n');
xQ2 = [0.125:0.125:0.875];
for y = 1:7
    TQ2(y) = (4*(Td2{15,1}(2*y))-(Td2{7,1}(y)))/3;
```

```
    fprintf('%6.4f\t\t%8.6f\n',xQ2(y),TQ2(y));
    ErPQ2(y) = log10(100*abs((TQ2(y)-Td(xQ2(y)))/(Td(xQ2(y)))));
end
plot(xQ2,ErPQ2,'-d');set(gca,'Fontsize',14);
hold on;

% Richardson's extrapolation and output of derivative values and plot of error vs x-position.
fprintf('\nRichardson''s Extrapolation\nx\t\tT prime\n');
xR2 = [0.25 0.5 0.75];
for z = 1:3
    TR2(z) = (((Td2{7,1}(2*z))^2)-(Td2{3,1}(z))*(Td2{15,1}(4*z)))/(2*(Td2{7,1}(2*z))-
(Td2{3,1}(z))-Td2{15,1}(4*z));
    fprintf('%6.4f\t\t%8.6f\n',xR2(z),TR2(z));
    ErPR2(z) = log10(100*abs((TR2(z)-Td(xR2(z)))/(Td(xR2(z)))));
end
plot(xR2,ErPR2,'-o');set(gca,'Fontsize',14);
xlabel('x position','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Extrapolated T-prime Error vs Position of 2nd Order FDM For P117','Fontsize',13);
legend('QuadEx','RichEx');

% Convergence of derivative values - Unknown solution
LEr2U = zeros(1,8);
for w = 1:8
    Ti = Td2{2*w-1,1}(w);
    LEr2U(w) = log10(100*abs((Ti-TQ2(4))/TQ2(4)));
end

% Plot convergence of derivative values vs grid spacing - Unknown Solution.
figure('name','Project 1: P117 2nd Order Convergence - Extrapolation')
plot(N,LEr2U,'*');set(gca,'Fontsize',14);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Convergence of 2nd Order FDM For T-prime P117 - Unknown Solution (Quadratic
Extrapolation)','Fontsize',12);


fprintf('\nT-prime at x = 1 (C)\ndelta-x\t\tT-prime\n');
for b = 1:16
    h = 1/b;
    eta = 1+(h/2);
    TdB2(b) = (((T{b,1}(b+1)-T{b,1}(b))/h)+(h/2)*f(1))/eta;
    Ti = TdB2(b);
    fprintf('%6.4f\t\t%8.6f\n',h,Ti);
end

% Evaluate derivative values at the boundary.
LErB2 = zeros(1,15);
LErBT2 = zeros(1,15);
h2 = zeros(1,15);

for w = 2:16
    h2(w-1) = log10(w);
    Ti = TdB2(w);
    LErB2(w-1) = log10(100*abs((Ti-Td(x{w,1}(w+1)))/Td(x{w,1}(w+1))));
    Tii = T{w,1}(1);
    LErBT2(w-1) = log10(100*abs((Tii-Ta2(0))/Ta2(0)));
end

figure('name','Project 1: Temperature at Boundary');
plot(h2,LErB2,'*');set(gca,'Fontsize',14);
title('P117: T-prime at x = 1 Using 2nd Order Difference','Fontsize',16);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);

figure('name','Project 1: Temperature at Boundary');
plot(h2,LErBT2,'*');set(gca,'Fontsize',14);
title('P117: Temperature at x = 0 Using 2nd Order Difference','Fontsize',16);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
```

```matlab
%% 4th Order FDM

figure('name','Project 1: P117 4th Order Temp')
T4 = cell(16,1);
Er4 = cell(15,1);
T4{1,1} = [(Tf+25) Tf];
plot(x{1,1},T4{1,1},'*');set(gca,'Fontsize',14);
hold on;
for i = 2:16
    [A,b] = Order4FDM117(Tf,xf,x0,i,f,fd,fdd);
    T4{i,1} = [(A\b)' Tf];
    plot(x{i,1},T4{i,1},'-*');set(gca,'Fontsize',14);
    for e = 1:i-1
        Er4{i-1,1}(e) = log10(100*abs((Ta2(x{i,1}(e+1))-T4{i,1}(e+1))/(Ta2(x{i,1}(e+1)))));
    end
end
xlabel('x position','Fontsize',16);
ylabel('Temperature (C)','Fontsize',16);
title('Plot of Temperature vs Position for 4th Order FDM For P117','Fontsize',15);
legend('N = 1','N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N =
11','N = 12','N = 13','N = 14','N = 15','N = 16');

fprintf('TEMPERATURE VS RADIUS\n-----------------------------------');
for k = 1:16
    delta = 1/(k);
    fprintf('\ndeltaX = %6.4f\nx\t\tTemperature(C)\n',delta);
    for m = 1:k+1
        xi = x{k,1}(m);
        Ti = T4{k,1}(m);
        fprintf('%6.4f\t\t%12.10f\n',xi,Ti);
    end
end

T4d2 = cell(15,1);
ErP4 = cell(15,1);

figure('name','Project 1: P117 4th Order Error Plot')
fprintf('\nFirst Derivative vs Position');
for a = 2:16
    delta = 1/a;
    fprintf('\ndeltaX = %6.4f\nx\t\tT prime(C)\n',delta);
    for b = 1:a-1
        xi = x{a,1}(b+1);
        alpha2 = 1 + ((delta^2)/(2*(xi^2)));
        T4d2{a-1,1}(b) = (T4{a,1}(b+2)-T4{a,1}(b))/(2*delta) +
((delta^2)/6)*((2/xi)*(((T4{a,1}(b+2)-
2*T4{a,1}(b+1)+T4{a,1}(b))/(alpha2*(delta^2)))+(delta^2/(12*alpha2))*((3/(xi^2))*(f(xi))+(1/xi)*f
d(xi)-fdd(xi)))-fd(xi)-(1/xi)*f(xi));
        Ti = T4d2{a-1,1}(b);
        fprintf('%6.4f\t\t%8.6f\n',xi,Ti);
        ErP4{a-1,1}(b) = log10(100*abs((Ti-Td(x{a,1}(b+1)))/Td(x{a,1}(b+1))));
    end
    plot(xin{a-1,1},ErP4{a-1,1},'-*');set(gca,'Fontsize',14);
    hold on;
end
xlabel('x position','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of T-prime Error vs Position of 4th Order FDM For P117 Known
Solution','Fontsize',14);
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N =
12','N = 13','N = 14','N = 15','N = 16');

figure('name','P117: T-prime 4th order')
for b = 1:15
    plot(xin{b,1},T4d2{b,1},'-*');set(gca,'Fontsize',14);
    hold on;
end
xlabel('x position','Fontsize',16);
ylabel('T''(x) (C/m)','Fontsize',16);
title('P117: 4TH ORDER FDM FIRST DERIVATIVE OF TEMP.','Fontsize',18);
```

```
legend('N = 2','N = 3','N = 4','N = 5','N = 6','N = 7','N = 8','N = 9','N = 10','N = 11','N =
12','N = 13','N = 14','N = 15','N = 16');
hold off;

% Convergence Analysis
LErT4 = zeros(1,8);
LEr4 = zeros(1,8);
for w = 2:2:16
    temp = 0.5*w;
    LEr4(temp) = ErP4{w-1,1}(w/2);
    LErT4(temp) = Er4{w-1,1}(w/2);
end

figure('name','Project 1: P117 Convergence')
plot(N,LEr4,'*');set(gca,'Fontsize',14);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Convergence of 4th Order FDM For T-prime P117 - Known Solution','Fontsize',14);

% Extrapolation for Temperature Values
figure('name','Project 1: P115 4th Order Extrapolation');
for y = 1:7
    T4Q2(y) = (16*(T4{16,1}(2*y+1))-(T4{8,1}(y+1)))/15;
    ErQ4(y) = log10(100*abs((T4Q2(y)-Ta2(xQ2(y)))/(Ta2(xQ2(y)))));
end

% Convergence of Temperature - Unknown Solution using Quadratic
% Extrapolation
LErT4U = zeros(1,8);
for w = 1:8
    LErT4U(w) = log10(100*abs((T4{2*w,1}(w+1)-T4Q2(4))/T4Q2(4)));
end

% Extrapolation for Derivative values
figure('name','Project 1: P117 4th Order Extrapolation');
fprintf('\nQuartic Extrapolation\nx\t\tT prime\n');
for y = 1:7
    TQ4(y) = (16*(T4d2{15,1}(2*y))-(T4d2{7,1}(y)))/15;
    fprintf('%6.4f\t\t%12.10f\n',xQ2(y),TQ4(y));
    ErPQ4(y) = log10(100*abs((TQ4(y)-Td(xQ2(y)))/(Td(xQ2(y)))));
end
plot(xQ2,ErPQ4,'-d');set(gca,'Fontsize',14);
hold on;

fprintf('\nRichardson''s Extrapolation\nx\t\tT prime\n');
for z = 1:3
    TR4(z) = (((T4d2{7,1}(2*z))^2)-(T4d2{3,1}(z))*(T4d2{15,1}(4*z)))/(2*(T4d2{7,1}(2*z))-
(T4d2{3,1}(z))-T4d2{15,1}(4*z));
    fprintf('%6.4f\t\t%12.10f\n',xR2(z),TR4(z));
    ErPR4(z) = log10(100*abs((TR4(z)-Td(xR2(z)))/(Td(xR2(z)))));
end
plot(xR2,ErPR4,'-o');set(gca,'Fontsize',14);
xlabel('x position','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('Plot of Extrapolated T-prime Error vs Position of 4th Order FDM For P117','Fontsize',13);
legend('QuartEx','RichEx');


% Convergence - Unknown solution

LEr4U = zeros(1,8);
for w = 1:8
    Ti = T4d2{2*w-1,1}(w);
    LEr4U(w) = log10(100*abs((Ti-TQ4(4))/TQ4(4)));
end

figure('name','Project 1: P117 4th Order Convergence - Extrapolation')
plot(N,LEr4U,'*');set(gca,'Fontsize',14);
z = [N(1) N(8)];
hold on;
```

```
P1 = polyfit(N,LEr,1); fit1 = @(r) P1(1)*r+P1(2); fplot(fit1,z,'-.'); s1 = sprintf('2nd Known:\ny
= %6.4f*x + %6.4f',P1(1),P1(2));text(0.5,0,s1);
plot(N,LEr2U,'*');P2 = polyfit(N,LEr2U,1);fit2 = @(r) P2(1)*r+P2(2); fplot(fit2,z,'-.'); s2 =
sprintf('2nd Unknown:\ny = %6.4f*x + %6.4f',P2(1),P2(2));text(1,0,s2);
plot(N,LEr4,'*');P3 = polyfit(N,LEr4,1);fit3 = @(r) P3(1)*r+P3(2); fplot(fit3,z,'-.'); s3 =
sprintf('4th Known:\ny = %6.4f*x + %6.4f',P3(1),P3(2));text(0.5,-3,s3);
plot(N,LEr4U,'*');P4 = polyfit(N,LEr4U,1);fit4 = @(r) P4(1)*r+P4(2); fplot(fit4,z,'-.'); s4 =
sprintf('4th Unknown:\ny = %6.4f*x + %6.4f',P4(1),P4(2));text(1,-3,s4);
legend('2nd Known','2nd K. Fit','2nd Unk.','2nd Unk. Fit','4th Known','4th K. Fit','4th
Unk.','4th Unk. Fit');
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('P117: CONVERGENCE OF 2ND & 4TH ORDER FDM FOR T-PRIME','Fontsize',18);

figure('name','Project 1: P117 4th Order Convergence - Extrapolation')
plot(N(2:end),LEr2(2:end),'*');set(gca,'Fontsize',14);
z = [N(2) N(8)];
hold on;
P5 = polyfit(N(2:end),LEr2(2:end),1); fit5 = @(r) P5(1)*r+P5(2); fplot(fit5,z,'-.'); s5 =
sprintf('2nd Known:\ny = %6.4f*x + %6.4f',P5(1),P5(2));text(0.5,0,s5);
plot(N(2:end),LErT2(2:end),'*');P6 = polyfit(N(2:end),LErT2(2:end),1);fit6 = @(r) P6(1)*r+P6(2);
fplot(fit6,z,'-.'); s6 = sprintf('2nd Unknown:\ny = %6.4f*x + %6.4f',P6(1),P6(2));text(1,0,s6);
plot(N(2:end),LErT4(2:end),'*');P7 = polyfit(N(2:end),LErT4(2:end),1);fit7 = @(r) P7(1)*r+P7(2);
fplot(fit7,z,'-.'); s7 = sprintf('4th Known:\ny = %6.4f*x + %6.4f',P7(1),P7(2));text(0.5,-3,s7);
plot(N(2:end),LErT4U(2:end),'*');P8 = polyfit(N(2:end),LErT4U(2:end),1);fit8 = @(r)
P8(1)*r+P8(2); fplot(fit8,z,'-.'); s8 = sprintf('4th Unknown:\ny = %6.4f*x +
%6.4f',P8(1),P8(2));text(1,-3,s8);
legend('2nd Known','2nd Known Fit','2nd Unknown','2nd Unknown Fit','4th Known','4th Known
Fit','4th Unknown','4th Unknown Fit');
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);
title('P117: CONVERGENCE FOR 2ND & 4TH ORDER FDM FOR TEMPERATURE','Fontsize',18);


fprintf('\nT-prime at x = 1 (C)\ndelta-x\t\tT-prime\n');
for b = 1:16
    h = 1/b;
    eta = 1+(h/2)+(h^2/3)+(h^3/4);
    TdB4(b) = (((T4{b,1}(b+1)-T4{b,1}(b))/h)+f(1)*((h/2)+(h^2/6)+(h^3/8))-
fd(1)*((h^2/6)+(h^3/24))+fdd(1)*(h^3/24))/eta;
    Ti = TdB4(b);
    fprintf('%6.4f\t\t%12.10f\n',h,Ti);
end

LErB4 = zeros(1,15);
LErBT4 = zeros(1,15);
h4 = zeros(1,15);

for w = 2:16
    h4(w-1) = log10(w);
    Ti = TdB4(w);
    LErB4(w-1) = log10(100*abs((Ti-Td(1))/Td(1)));
    Tii = T4{w,1}(1);
    LErBT4(w-1) = log10(100*abs((Tii-Ta2(0))/Ta2(0)));
end

figure('name','Project 1: Temperature at Boundary');
plot(h4,LErB4,'*');set(gca,'Fontsize',14);
title('P117: T-prime at x = 1 Using 4th Order Difference','Fontsize',16);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);

figure('name','Project 1: Temperature at Boundary');
plot(h4,LErBT4,'*');set(gca,'Fontsize',14);
title('P117: Temperature at x = 0 Using 4th Order Difference','Fontsize',16);
xlabel('-LOG10(\Deltax)','Fontsize',16);
ylabel('LOG10(% RELATIVE ERROR)','Fontsize',16);

%% Project 1: 2nd Order FDM P117
% Seyed Sepehr Seyedi
%
```

```
% UIN 424006176
%
% AERO 430-500
%
% October 5th, 2015

%% Function

function [A,b] = Order2FDM117(Tf,xf,x0,N,f)

h = (xf-x0)/N;
A = zeros(N,N);
A(1,1) = 1;
A(1,2) = -1;
for i = 1:N-2
    A(i+1,i) = x0 + h*(i - 0.5);
    A(i+1,i+1) = -2*(x0 + h*i);
    A(i+1,i+2) = x0 + h*(i + 0.5);
end
A(N,N-1) = x0 + h*(N-1.5);
A(N,N) = -2*(x0 + h*(N-1));
b = zeros(N,1);
b(1) = 0;
b(N) = -(x0 + h*(N-0.5))*Tf + (h^2)*(xf-h)*(f(xf-h));
for j = 2:N-1
    b(j) = (h^2)*(x0+(j-1)*h)*(f(x0+(j-1)*h));
end
```

```
%% Project 1: 4th Order FDM P117
% Seyed Sepehr Seyedi
%
% UIN 424006176
%
% AERO 430-500
%
% October 5th, 2015

%% Function

function [A,b] = Order4FDM117(Tf,xf,x0,N,f,fd,fdd)

h = (xf-x0)/N;
A = zeros(N,N);
A(1,1) = 1;
A(1,2) = -1;
for i = 1:N-2
    alpha2 = 1 + (h^2/(2*((x0 + i*h)^2)));
    A(i+1,i) = (1/(alpha2*(h^2)))+(1/(3*alpha2*((x0+i*h)^2))) - (1/(2*h*(x0+i*h)));
    A(i+1,i+1) = -(2/(alpha2*(h^2)))-(2/(3*alpha2*((x0+i*h)^2)));
    A(i+1,i+2) = (1/(alpha2*(h^2)))+(1/(3*alpha2*((x0+i*h)^2))) + (1/(2*h*(x0+i*h)));
end
alpha3 = 1 + (h^2/(2*((xf - h)^2)));
A(N,N-1) = (1/(alpha3*(h^2)))+(1/(3*alpha3*((xf-h)^2))) - (1/(2*h*(xf-h)));
A(N,N) = -(2/(alpha3*(h^2)))-(2/(3*alpha3*((xf-h)^2)));
b = zeros(N,1);
b(1) = 25*(h^2);
b(N) = f(xf-h)*(1+(h^2/(6*((xf-h)^2))))+fd(xf-h)*((h^2)/(6*(xf-h)))+(fdd(xf-h)-(1/(xf-h))*fd(xf-
h)-(3/((xf-h)^2))*f(xf-h))*(h^2/(12*alpha3))*(1+((h^2)/(3*((xf-h)^2))))-
Tf*((1/(alpha3*(h^2)))+(1/(3*alpha3*((x0+h)^2)))+(1/(2*h*(x0+h))));
for j = 1:N-1
    alpha2 = 1 + (h^2/(2*((x0 + j*h)^2)));
    b(j+1) = f(x0+j*h)*(1+(h^2/(6*((x0+j*h)^2))))+fd(x0+j*h)*((h^2)/(6*(x0+j*h)))+(fdd(x0+j*h)-
(1/(x0+j*h))*fd(x0+j*h)-
(3/((x0+j*h)^2))*f(x0+j*h))*(h^2/(12*alpha2))*(1+((h^2)/(3*((x0+j*h)^2))));
end
```