University of California, Berkeley

Department of EECS

EE120: SIGNALS AND SYSTEMS (Spring 2021)

Homework 0 (Grade by completion)

Issued: January 22, 2021 **Due:** 11:59 PM, January 29, 2021

Collaborators:

Problem A. Euler formula. The Euler's formula is

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

1. Derive the following identities using Euler's formula
   (a) $\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$
   (b) $\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$
2. Derive **de Moivre's theorem**: for any real number θ, integer n,

$$(\cos(\theta) + i\sin(\theta))^n = \cos(n\theta) + i\sin(n\theta)$$

3. Show that any linear combination of a set of sine waves of frequency ω is always a sine wave of the same frequency, *even if* each sine wave has a distinct phase. In particular, show that

$$\sum_{k=1}^{N} A_k \cos(\omega t + \phi_k) = A\cos(\omega t + \phi).$$

Problem B. Periodicity of signals. Determine whether or not each of the following continuous-time or discrete-time signals is periodic. If the signal is periodic, determine it's fundamental period.

1. $x(t) = 3\cos(4t + \frac{\pi}{3})$
2. $x(t) = e^{i(\pi t-1)}$
3. $x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)}u(2t-n)$
4. $x[n] = \sin(\frac{n}{8} - \pi)$
5. $x[n] = \cos(\frac{\pi}{8}n^2)$

6. $x[n] = \cos(\frac{6\pi}{7}n + 1)$

Problem C. Signal transformation. For a continuous-time signal $x(t)$ in Figure 1, carefully sketch the following:

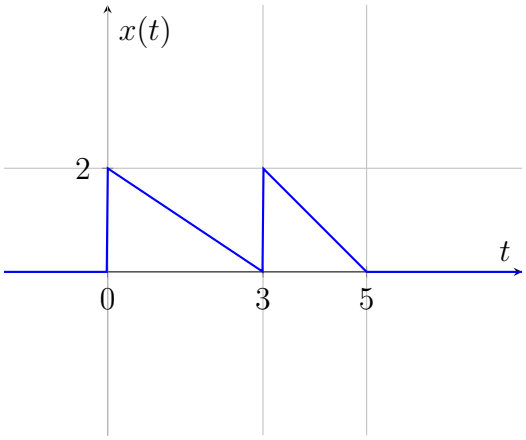

Figure 1

1. $x(-t)$
2. $x(2t)$
3. $x(t + 2)$
4. $x(\frac{x}{2} - 1)$
5. $x(1 - 3t)$

Problem D. Integral review. Evaluate the following integrals.

1. $y(t) = \int_{-1}^{\infty} e^{-2t} dt$
2. $y(t) = \int_{-\infty}^{\infty} u(\tau) e^{-a(t-\tau)} u(t - \tau) d\tau$
3. $X(\omega) = \int_{-\infty}^{\infty} e^{-a|t|} e^{-i\omega t} dt$

Problem E. Functions and signals. Sketch the following functions or signals as described. Label your sketches carefully.

1. Discrete delta function (a.k.a. Kronecker delta) $\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$.

2. Comb function (a.k.a. Shah function) $\text{Ш}_T(t) = \frac{1}{T}\text{Ш}(\frac{t}{T}) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$.

3. Rectangular function rect$(\frac{t+2.5}{3})$

4. Sinusoidal function $x(t) = \sin(\pi t + \frac{\pi}{4})$

5. $x(t) = e^{2t}u(-t)$

6. A discrete signal: $x[n] = e^{-n}u[n]$