University of California, Berkeley

Department of EECS

EE120: SIGNALS AND SYSTEMS (Spring 2021)

Discussion 1

Issued: January 22, 2021

Problem A. Review of complex number. Consider a complex number with magnitude $r > 0$ and phase

θ, which can be written in polar form as $z = re^{i\theta}$.

1. Express each of the following complex numbers in polar form
   (a) z^* the complex conjugate of z.
   (b) z^2
   (c) iz
   (d) zz^*
   (e) $\frac{z}{z^*}$
   (f) $\frac{1}{z}$
2. Let $r = \frac{1}{2}$ and $\theta = \frac{\pi}{3}$. For each of your results in part 1, plot the corresponding vectors in the complex plane.

Problem B. Review on delta function properties. The discrete time and continuous time delta functions are defined as follows:

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}.$$

$$(t - T) = 0, \text{ for } t \neq T$$

$$\int_{-\infty}^{\infty} \delta(t - T)dt = 1$$

1. $x[n]$ is a discrete time signal. Find an equivalent expression to $x[n]\delta[n - T]$ where $N \in \mathcal{Z}$.
2. The derivative of unit step function $\frac{du(t)}{dt}$ is delta function.
3. Express the Kronecker delta in terms of scaled and shifted unit steps.
4. $x(t)$ is a continuous time signal. Find and equivilent expression for $x(t)\delta(t - T)$ where $T \in \mathcal{R}$. What is $\int_{T-1}^{T+1} x(t)\delta(t - T)dt$?

Problem C. Even and odd part of signal. For a continuous time signal $x(t)$, derive the even part and odd part of the signal.

Problem D. Properties of signals. Determine whether or not each of the following continuous-time or discrete-time signal is periodic. If the signal is periodic, determine it's fundamental period.

1. $x(t) = [cos(2t - \frac{\pi}{3})]^2$.
2. $\mathcal{E}v\{\cos(4\pi t)u(t)\}$.
3. $\mathcal{E}v\{\sin(4\pi t)u(t)\}$.
4. $x[n] = \cos(\frac{n}{8} - \pi)$.
5. $x[n] = 2\cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{8}n) - 2\cos(\frac{\pi}{2}n + \frac{\pi}{6})$.

Problem E. Consider a periodic signal

$$x(t) \begin{cases} 1, & 0 \leq t \leq 1 \\ -2, & 1 < t < 2 \end{cases}$$

with period $T = 2$. The derivative of this signal is related to the "impulse train"

$$g(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2k)$$

with period $T = 2$. It can be shown that

$$\frac{dx(t)}{dt} = A_1 g(t - t_1) + A_2 g(t - t_2).$$

Determine the values of A_1, t_1, A_2, and t_2.

Problem F. Function drawing practice

1. $\int_{-\infty}^{\infty} \text{rect}(t - \tau)\text{rect}(\tau)d\tau$
2. $\text{sinc}(x) = \frac{\sin \pi x}{\pi x}$ (note the locations of nodes)
3. Let $x(t) = e^{2t}u(-t)$, $h(t) = u(t - 3)$. Plot $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$.