Rutherford Scattering

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1. INTRODUCTION

Atomic Spectroscopy is one of the fundamental fields of physics where Quantum Mechanics its energy level became more robust theoretically. From atomic spectroscopy, physicists have discovered the Rydberg Formula, illustrating the transitions between electron energy levels. One of the most important of these transitions are called the Balmer Series, which depicts the transitions from n ¿ 3 states to n = 2 stages. The light from this series is visible light. Furthermore, by introducing a magnetic field to the atom, we add a new term to the Hamiltonian. This new term gives rise to Zeeman splitting. With this magnetic field, we see spectral lines splitting, due to a split in the energy levels. Using modern day optics and spectroscopic techniques, we can use these spectral lines to determine the Rydberg Constant and the Bohr Magneton.

2. THEORY

2.1. Balmer Series

To begin speaking about the Balmer Series, one must understand the Bohr Model. In this mode of an atom, the electron revolves around the nucleus of the atom, much like a planet revolves around the sun. Therefore, we have a force balance equation

$$\frac{m_{\mathrm{e}} v^2}{r} = \frac{Z k_{\mathrm{e}} e^2}{r^2} \tag{1}$$

where m_e is the electron's mass, e is the charge of the electron, k_e is Coulomb's constant and Z is the atom's atomic number. However, unlike in classical mechanics where the angular momentum is continuous, quantum mechanics exhibits discrete values of angular momentum. Specifically, the angular momentum is an integer multiple of Planck's Constant:

$$m_{\mathrm{e}} v r = n\hbar \tag{2}$$

where n is the Principle Quantum Number.

By combining Equation 1 and Equation 2, we arrive at the relationship between the Principle Quantum number and the corresponding energy:

$$E = -\frac{Z^2 (k_{\mathrm{e}} e^2)^2 m_{\mathrm{e}}}{2\hbar^2 n^2} \approx \frac{-13.6 Z^2}{n^2} \mathrm{eV} \tag{3}$$

The Balmer Series are the series of transitions where atoms end up in the $n = 2$ state. This means any transition from $n > 2$ to $n = 2$ is a Balmer Series transition. These transitions result in energy loss in the form of a photon, which is what we observed as visible spectrum lines. The energies of these photons are described by this energy difference relation, called the Rydberg Formula:

$$E = E_i - E_f = R_{\mathrm{E}} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \tag{4}$$

and since $E = hc/\lambda$, the above equation can be rewritten as

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right). \tag{5}$$

where $n_f = 2$ for the Balmer Series.

In this experiment, we use Equation 5 to determine the Rydberg Constant, R, by plotting $1/\lambda$ against $1/n^2$.

2.2. Zeeman Effect

Another quantum effect we observe in this experiment is the Zeeman Effect, where a weak magnetic field is applied to atoms, adding a term to their Hamiltonian, and thus splitting energy levels.

We begin with the Hamiltonian of an atom:

$$H = H_0 - \vec{\mu} \cdot \vec{B}, \;\; \vec{\mu} = \frac{e}{2m_e} g \vec{j} \tag{6}$$

where H_0 is the unperturbed Hamiltonian and the second term is the term resulting from the Zeeman Effect. The other terms $\vec{\mu}$ are as follows:

- e is the fundamental charge value
- m_e is the mass of the electron
- $\vec{B}$ is the magnetic field
- $\vec{j}$ is the total angular momentum resulting from a sum of the electron's orbital angular momentum $\vec{l}$ and its spin angular momentum $\vec{s}$
- g is the Lande-g Factor (a multiplicative factor that arises when a weak magnetic field is introduced to the atom.) given by:

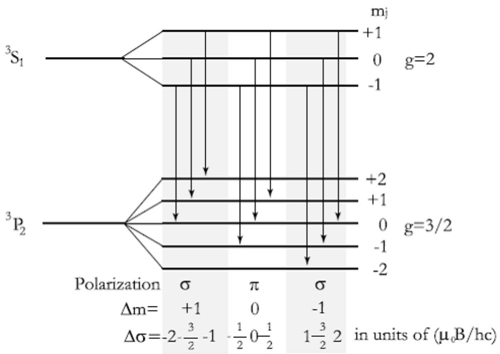


FIG. 1: Structure of the Zeeman multiplet arising in a transition from a $^3S_1$ to a $^3P_2$ level.

[“Atomic Physics.” - Physics 111-Lab Wiki. UC Berkeley, n.d. Web. 23 Apr. 2015.]

$$g_J \approx \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}. \tag{7}$$

By taking the dot product, we get the projection of the angular momentum $\vec{j}$ against the axis of the magnetic field, which results in the following energy perturbation:

$$\Delta E = \mu_0 B \Delta(g m_j) \tag{8}$$

This m_j terms are what the energy levels split into, and can take on the integral values from $-j$ to $+j$ depending on the j value.

An important part of this experiment is the Fabry – Pérot interferometer, which is discussed in depth in Section 3.2.1. A relation that arises from this device is the Free Spectral Range, which relates the wave number to the properties of the F–P interferometer.

$$\Delta\sigma = \frac{\alpha}{2tn} \tag{9}$$

where α is a constant depending on how much Zeeman Splitting we induce, n is the index of refraction of air, t is the thickness of the interferometer, and σ is the wave number. We also know through the deBroglie relations that

$$E = hc/\lambda \tag{10}$$

and since $\lambda = 1/\sigma$, then $E = hc\sigma$ and $\Delta E = hc\Delta\sigma$. A change in energy from Equation 8 results in $\Delta E = \mu_0 B \Delta(m_j g)$. Relating this to Equation 9 and Equation 8, we get the relationship to find the Bohr Magneton:

$$\mu_0 = \frac{hc\alpha}{2tnB\Delta(g m_j)} \tag{11}$$

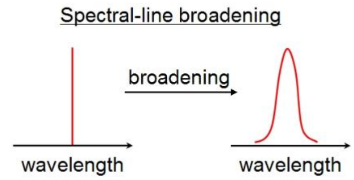


FIG. 2: Broadening, caused by several factors like natural broadening, thermal broadening, and collision broadening.

[“CV Accretion Discs.” CV Accretion Discs. N.p., n.d. Web. 24 Apr. 2015.]

2.3. Broadening

If each electron transition was just to emit a photon with a well defined energy and, thus, one wavelength, then the spectral line we ought to observe would be infinitely thin (Left side of Fig 2). However, several factors give the line a width: the uncertainty principle,

$$\Delta E \Delta t \geq \frac{\hbar}{2} \Rightarrow \Delta\nu = \frac{1}{2\pi\Delta t} \tag{12}$$

thermal broadening,

$$\Delta\nu = \sqrt{\frac{8kT\ln 2}{mc^2}}\nu_0 \tag{13}$$

and collision broadening

$$\Delta\nu = \sqrt{\frac{8kT}{m}} n\sigma \tag{14}$$

Natural broadening is naturally caused by the Heisenberg Uncertainty Principle. Second, thermal Broadening is caused by the movement of the atoms. Since the gases that are being excited are moving, some atoms will be moving towards the observer, and thus emitting blue shifted light, while some atoms will move away from the observer will emit red shifted light. This frequency shifting is caused by thermal effects, and therefore is called Thermal Broadening. Last, spectral line broadening is also caused by collision broadening. If atoms that are emitting light are constantly colliding with other atoms, then their electron energy levels get spread out.

In Table I, we see that thermal broadening has the largest effect on the spread of the spectral line by almost three orders of magnitude. Since the values we used to calculate the spreads in the table are similar to the actual values in the experiment, we can assume that the largest cause in spread and uncertainty will be through thermal broadening.

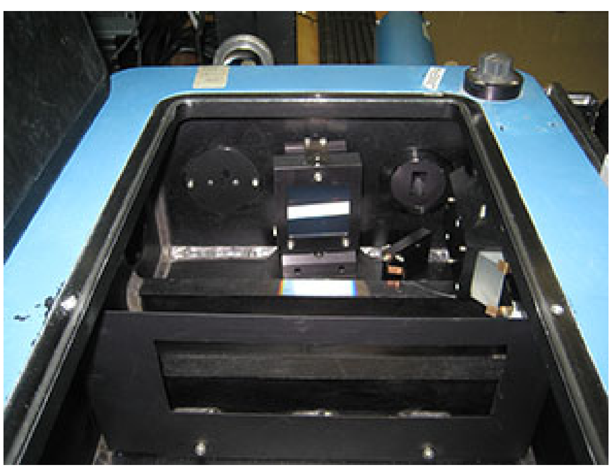
Type of Broadening	FWHM Spread
Natural	8 MHz
Thermal	5 GHz
Collision	13 MHz

TABLE I: The approximate spreads due to different types of broadening. Calculations were done with P = 5 Torr, T = 600 K, and $\Delta t = 10$ ns

3. SETUP

3.1. Balmer Series

(a)Balmer Series Setup



(b)Grating in the Monochromator

FIG. 3: Atomic Physics Experiment Setup at UC Berkeley

["Atomic Physics." - Physics 111-Lab Wiki. UC Berkeley, n.d. Web. 4 May. 2015.]

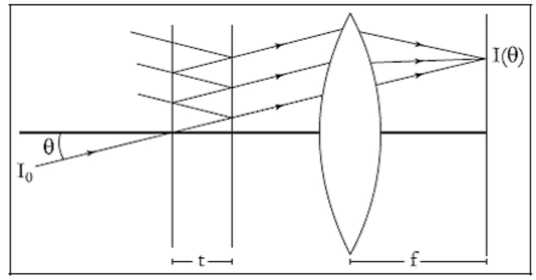


FIG. 4: A Fabry – Pérot interferometer. The light enters the interferometer and then exists, interfering with itself and creating Airy Disks.

["Atomic Physics." - Physics 111-Lab Wiki. UC Berkeley, n.d. Web. 23 Apr. 2015.]

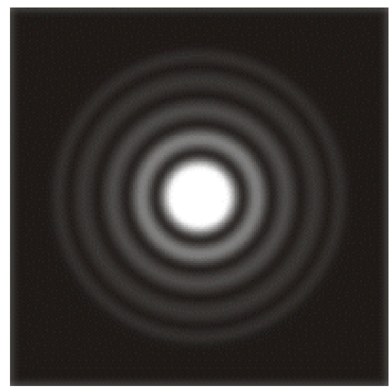


FIG. 5: An airy disk, showing interference patterns.

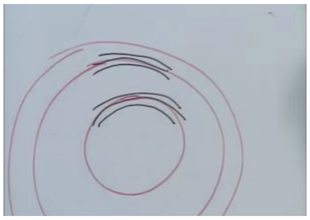
["Telescope Equations.": Resolving Power. N.p., n.d. Web. 23 Apr. 2015.]

3.2. Zeeman Series

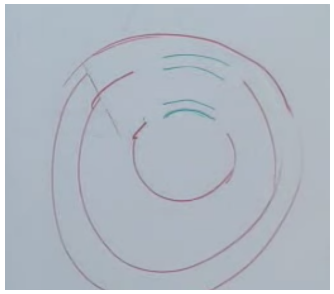
3.2.1. Fabry – Pérot interferometer

A crucial piece of equipment in the Zeeman Effect portion of this experiment is the Fabry – Pérot interferometer (Fig 4). This interferometer takes incoming light, reflects it multiple times within its internal mirrors, and then sends the light back out, causing the light to interfere with itself. When one observes the outgoing light from an F–P interferometer, one sees Airy Disks(Fig 5).

In this experiment, we observe a discharge tube a Helium gas. By passing the light through a red filter and then through the FP interferometer, we are able to see the Airy Disks. These disks are separated by a quantity that we will call $\Delta\sigma$, which has units of cm^{-1}. What is more powerful about this interferometer is that when a magnetic field is applied to the Helium gas, we see the Airy disks split, thus resembling the Zeeman splitting of the energy levels. In Fig 1, we see that there are two different kind of transitions, σ and π transitions.



(a)With both σ and π components. The top and bottom black lines are the σ components while the middle line is the π component.



(b)With the polarizer, eliminating the π component.

FIG. 6: Zeeman Splitting showing σ and π Components
[“Physics 111: Atomic Physics (ATM) Part 2. Zeeman Effect.” YouTube. YouTube, n.d. Web. 23 Apr. 2015.]

These transitions denote the change in angular momentum. Since m_j is an indicator of the angular momentum of the electron, a transition that yields a change in m_j is a σ transition and yields a change in angular momentum. π transitions, on the other hand, do not have changes in angular momentum.

We can measure these splittings independently with a polarizer. By setting the polarizer to filter out any light that is not circularly polarized, we end up seeing only the σ transitions. Then we adjusted the magnetic field such that the airy disks were evenly spaced out. This means we had the σ lines move $1/4\Delta\sigma$.