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Assignment 3 Part 2: ShiDong Xu 1006521890 Liang Chen 1005735126

Q1

(a) Closures:
L+ = LNOSMQRP
M+ = MP
N+ = NMQRP
O+ = OS

DFs that violate BCNF:
$M \rightarrow P$
$N \rightarrow MQR$
$O \rightarrow S$

(b) First level of split using $N \rightarrow MQR$
Left relation $NMQRP$ with FD $N \rightarrow MQRP$ and $M \rightarrow P$
Right relation $NLOS$ with FD $O \rightarrow S$ and $L \rightarrow NO$

Second level of split on the left relation on $M \rightarrow P$
Left side we have MP with FD $M \rightarrow P$
Right side we have $MNQR$ with FD $N \rightarrow MQR$

Second second of split on the right relation on $O \rightarrow S$
Left side we have OS with FD $O \rightarrow S$
Right side we have OLN with FD $L \rightarrow NO$

So in the end, we have:
relation LNO with $L \rightarrow NO$
relation $MNQR$ with $N \rightarrow MQR$
relation MP with $M \rightarrow P$
relation OS with $O \rightarrow S$

(c) The schema does preserve dependencies, because every single FD inside the original set of FDs are preserved in the final schema, no more, no less.

(d) We start with:

L	M	N	O	P	Q	R	S
l	10	n	o	11	12	13	14
15	m	n	16	17	q	r	18
19	m	20	21	p	22	23	24
25	26	27	o	28	29	30	s

Because we have $N \rightarrow MQR$, we make these changes:

L	M	N	O	P	Q	R	S
l	~~10~~m	n	o	11	~~12~~q	~~13~~r	14
15	m	n	16	17	q	r	18
19	m	20	21	p	22	23	24
25	26	27	o	28	29	30	s

Because we have $O \rightarrow S$, we make these changes:

L	M	N	O	P	Q	R	S
l	~~10~~m	n	o	11	~~12~~q	~~13~~r	~~14~~s
15	m	n	16	17	q	r	18
19	m	20	21	p	22	23	24
25	26	27	o	28	29	30	s

Lastly, Because we have $M \rightarrow P$, we make these changes:

L	M	N	O	P	Q	R	S
l	~~10~~m	n	o	~~11~~p	~~12~~q	~~13~~r	~~14~~s
15	m	n	16	17	q	r	18
19	m	20	21	p	22	23	24
25	26	27	o	28	29	30	s

We observe that the tuple $< l, m, n, o, p, q, r, s >$ does occur (first row). The Chase Test has succeeded.

Q2

(a) **Step 1**: Split the RHSs to get our initial set of FDs, S1:
(a) $ACD \rightarrow E$
(b) $B \rightarrow C$
(c) $B \rightarrow D$
(d) $BE \rightarrow A$
(e) $BE \rightarrow C$
(f) $BE \rightarrow F$
(g) $D \rightarrow A$
(h) $D \rightarrow B$
(i) $E \rightarrow A$
(j) $E \rightarrow C$

Step 2: For each FD, try to reduce the LHS:
(a) No sublist of ACD can yield E, Therefore, cannot reduce.
(b) LHS has only one element. Therefore, cannot reduce.
(c) LHS has only one element. Therefore, cannot reduce.
(d) B+ = BCDAB, since we only need to consider B to yield A, we can reduce this to new FD $B \rightarrow A$
(e) from above we know that B+ = BCDAB, we can reduce this to new FD $B \rightarrow C$
(f) E+ = EAC. Therefore, cannot be reduced.
(g) LHS has only one element. Therefore, cannot reduce.

(h) LHS has only one element. Therefore, cannot reduce.
(i) LHS has only one element. Therefore, cannot reduce.
(j) LHS has only one element. Therefore, cannot reduce.

Therefore, we have a new set of FDs, S2:
(a) $ACD \to E$
(b) $B \to C$
(c) $B \to D$
(d) $B \to A$
(e) $B \to C$
(f) $BE \to F$
(g) $D \to A$
(h) $D \to B$
(i) $E \to A$
(j) $E \to C$

Step 3: Try to eliminate each FD.
(a) $ACD+_{S2-a} = ACDB$, We need this FD.
(b) $B+_{S2-b} = BDAC$, We got C, therefore, we DO NOT need this FD.
(c) $B+_{S2-b-c} = BAC$, We need this FD.
(d) $B+_{S2-b-d} = BDA$, We got A, therefore, we DO NOT need this FD.
(e) $B+_{S2-b-d-e} = BDA$, We need this FD.
(f) $BE+_{S2-b-d-f} = BEACD$, We need this FD.
(g) $D+_{S2-b-d-g} = DBC$, We need this FD.
(h) $D+_{S2-b-d-h} = DA$, We need this FD.
(i) $E+_{S2-b-d-i} = EC$, We need this FD.
(j) $E+_{S2-b-d-j} = EA$, We need this FD.

Our final set of FDs is:
(a) $ACD \to E$
(b) $B \to C$
(c) $B \to D$
(d) $BE \to F$
(e) $D \to A$
(f) $D \to B$
(g) $E \to A$
(h) $E \to C$

(b) We first organize all the attributes into 4 categories.

Attribute	appears on LHS	Appears on RHS	Conclusion
G, H	-	-	must be in every key
none	✓	-	must be in every key
F	-	✓	is not in any key
A, B, C, D, E	✓	✓	must check

Next, we examine all combinations of A, B, C, D, E, and each combination must include G and H.
$AGH+ = AGH$, not a a key. $BGH+ = BGHCDAEF$, Therefore, BGH is a key.
$CGH+ = CGH$, not a key.

$DGH+ = DGHABCEF$, Therefore, DGH is a key.
$EGH+ = EGHAC$, not a key.

All other possibilities include BGH or DGH, because we can never get B or D with only A, C, E.
2 keys for relation: BGH and DGH.

(c) Firstly, with the set of minimal basis FDs, we combine all FDs with the same LHS:
(a) $ACD \to E$
(b) $B \to CD$
(c) $BE \to F$
(d) $D \to AB$
(e) $E \to AC$

Second, For each FD, construct a relation:
(a) $ACDE$
(b) BCD
(c) BEF
(d) DAB
(e) EAC

Third, for each relation, remove ones that are a sublist of another:
(a) $ACDE$
(b) BCD
(c) BEF
(d) DAB

Next, Check if any relation is a superkey:
This is not possible, because attribute G and H did not appear in any FDs, neither LHS nor RHS.

Final, add another relation that is a key (BGH):
(a) $ACDE$
(b) BCD
(c) BEF
(d) DAB
(e) BGH

(d) This schema does allow redundancy, relation (a) is the one that has an FD that violates BCNF:
Projection of the FDs onto (a), Consider the original FD $E \to AC$, find closure of E we have: $E+ = EAC$. Therefore, the projection is $E \to AC$, which violates BCNF. E does not yield every attribute in the relation $ACDE$.
Therefore, This schema allows redundancy.