Embracing Randomness:
Intriguing Role of Chance in Science

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https://github.com/chenle02/2012_Science_Summer_Institute_Auburn_Probability_by_Le

Summer Science Institute 2023
Auburn University

What is chance

How to measure chance

Birthday problem

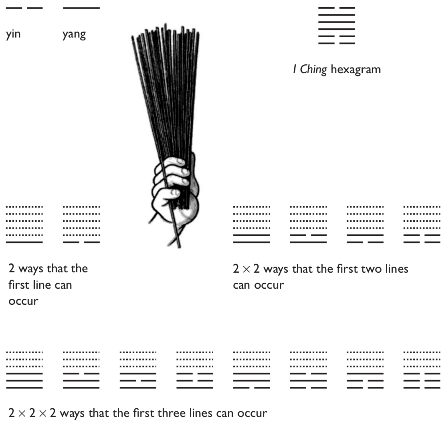
Rolling three dices

What is CHANCE?

Tyche – The Greek goddess of chance

Fortuna – The goddess of chance in Roman religion

I Ching (~ 6th century B.C.)
— A divination tool in ancient China



[1]

[1] Image is from Bennett (1998), *Randomness*, Harvard University Press.

Games of chance — using knucklebones or dice

Known to Egyptians, Babylonians, Romans, ...

There was no qualitative theory of chance in these times.

How to measure chance?

How about measure length?

The determination of a “right and lawful rood” or rod in the early sixteenth century in Germany by measuring an essentially random selection of 16 men as they leave church [2].

[2] Stephen Stigler (1996). Statistics and the Question of Standards, *Journal of Research of the National Institute of Standards and Technology*, vol. 101.

The same for chance

To measure probability,

1. we first find or make equally probable cases,
2. then we count.

The probability of an event A, denoted by $\mathbb{P}(A)$, is then

$$\mathbb{P}(A) = \frac{\text{number of cases in which } A \text{ occurs}}{\text{total number of cases}}.$$

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*Girolamo Cardano** (1501 – 1576)

* An Italian polymath, whose interests and proficiencies ranged through those of mathematician, physician, biologist, physicist, chemist, astrologer, astronomer, philosopher, writer, and gambler. He was one of the most influential mathematicians of the Renaissance, and was one of the key figures in the foundation of probability and the earliest introducer of the binomial coefficients and the binomial theorem in the Western world.

Avid and serious gambler

"*Liber de ludo aleae*" (The Book on Games of Chance)

"equally likely outcomes"

Influencing later thinkers like *Pascal* and *Fermat*

Now we see that probability has to satisfy the following properties:

1. Probability should be never negative.
2. If A occurs in all cases, then $\mathbb{P}(A) = 1$.
3. If A and B never occur in the same case, then

$$\mathbb{P}(A \text{ or } B) = \mathbb{P}(A) + \mathbb{P}(B).$$

In particular, the probability of an event not occurring is equal to

$$\mathbb{P}(\text{not } A) = 1 - \mathbb{P}(A).$$

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How to generate the equi-probable cases?

Prim sticks (variations of dice) [3] and deck of poker...

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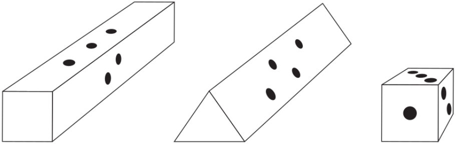

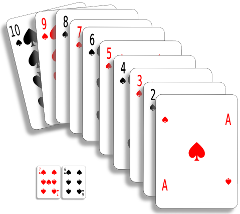

Prim sticks (variations of dice) [3] and deck of poker...

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Example one: Birthday Problem

Question: How likely do two students have the same birth day if there are

- 2
- 5
- 15
- 23
- 46
- 64
- 366

students in the class?

Assuming that

1. each year has 365 days (i.e., neglecting leap years),
2. birthdays are equi-probable,
3. birthdays are independent (no twins in the class).

- Suppose $n = 5$.
- Let A be the event that there is a shared birthday among these n students.
- It is not easy to compute $\mathbb{P}(A)$ directly.
- However, one can compute $\mathbb{P}(\text{not } A)$ by counting:

the probability that no two students have the same birthday, or

all students have different birthday,

is equal to

$$\mathbb{P}(\text{not } A) = \frac{365}{365}\frac{364}{365}\frac{363}{365}\frac{362}{365}\frac{361}{365}$$

- Hence,

$$\mathbb{P}(A) = 1 - \mathbb{P}(\text{not } A) = 1 - \frac{365}{365}\frac{364}{365}\frac{363}{365}\frac{362}{365}\frac{361}{365} \approx 0.027.$$

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$$\mathbb{P}(A) = 1 - \frac{365 \times 364 \times \cdots \times (365 - n + 1)}{365^n}$$

n	2	5	15	23	46	64	366
$\mathbb{P}(A)$							1.0

$$\mathbb{P}(A) = 1 - \frac{365 \times 364 \times \cdots \times (365 - n + 1)}{365^n}$$

n	2	5	15	23	46	64	366
$\mathbb{P}(A)$	0.0027						1.0

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n	2	5	15	23	46	64	366
$\mathbb{P}(A)$	0.0027	0.0271					1.0

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n	2	5	15	23	46	64	366
$\mathbb{P}(A)$	0.0027	0.0271	0.2529				1.0

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n	2	5	15	23	46	64	366
$\mathbb{P}(A)$	0.0027	0.0271	0.2529	0.5073			1.0

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n	2	5	15	23	46	64	366
$\mathbb{P}(A)$	0.0027	0.0271	0.2529	0.5073	0.9483		1.0

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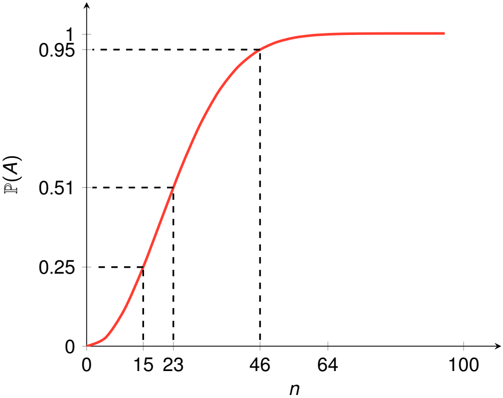


Number of generations	Size of population to have about even odds	Examples (as of 2023)
1	23	A typical class
2	438	—
3	$8,368$	Princeton University[4]
4	$159,870$	Tuscaloosa[5]; Auburn[6]
5	$3,054,312$	Atlanta[7]; Chicago[8]

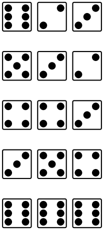
Population rank of 500 cities in the United States:

https://en.wikipedia.org/wiki/List_of_United_States_urban_areas

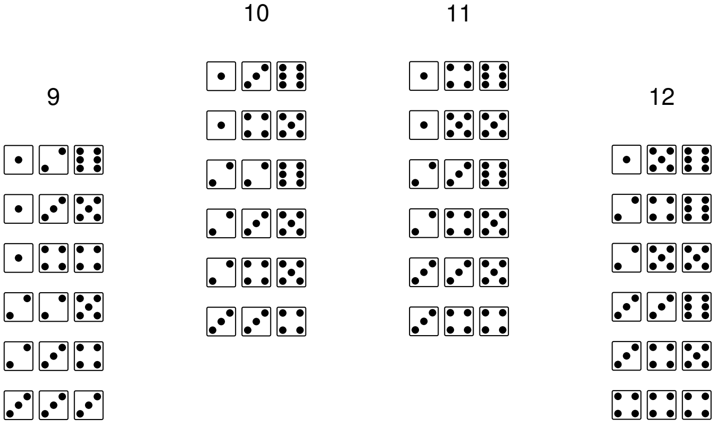
[4] 8,478
[5] Urban: 139,114, Metro: 235,628
[6] Metro: 158,991
[7] Urban: 4,999,259; Metro: 6,144,050
[8] City: 2,746,388; Urban: 8,671,746; Metro: 9,618,502

Another example: a question by *Grand Duke of Tuscany* posed to *Galileo* in early seventh century

Three dice are thrown, such as



Counting combinations of numbers, 10 and 11 can be made in 6 ways, as can 9 and 12:



Yet it is known that long observation has made dice-players consider 10 *and* 11 *to be more advantageous than* 9 *and* 12*. How can this be?*

3 = ⚀ + ⚀ + ⚀

4 = ⚀ + ⚀ + ⚁

5 = ⚀ + ⚀ + ⚂ = ⚀ + ⚁ + ⚁

6 = ⚀ + ⚀ + ⚃ = ⚀ + ⚁ + ⚂ = ⚁ + ⚁ + ⚁

7 = ⚀ + ⚀ + ⚄ = ⚀ + ⚁ + ⚃ = ⚁ + ⚁ + ⚂ = ⚂ + ⚂ + ⚀

8 = ⚀ + ⚀ + ⚅ = ⚀ + ⚁ + ⚄ = ⚀ + ⚂ + ⚃ = ⚁ + ⚁ + ⚃ = ⚁ + ⚂ + ⚂

9 = ⚀ + ⚁ + ⚅ = ⚀ + ⚂ + ⚄ = ⚀ + ⚃ + ⚃ = ⚁ + ⚁ + ⚄ = ⚁ + ⚂ + ⚃ = ⚂ + ⚂ + ⚂

10 = ⚀ + ⚂ + ⚅ = ⚀ + ⚃ + ⚄ = ⚁ + ⚁ + ⚅ = ⚁ + ⚂ + ⚄ = ⚁ + ⚃ + ⚃ = ⚂ + ⚂ + ⚃

11 = ⚀ + ⚃ + ⚅ = ⚀ + ⚄ + ⚄ = ⚁ + ⚂ + ⚅ = ⚁ + ⚃ + ⚄ = ⚂ + ⚂ + ⚄ = ⚂ + ⚃ + ⚃

12 = ⚀ + ⚄ + ⚅ = ⚁ + ⚃ + ⚅ = ⚁ + ⚄ + ⚄ = ⚂ + ⚂ + ⚅ = ⚂ + ⚃ + ⚄ = ⚃ + ⚃ + ⚃

13 = ⚀ + ⚅ + ⚅ = ⚁ + ⚄ + ⚅ = ⚂ + ⚃ + ⚅ = ⚂ + ⚄ + ⚄ = ⚃ + ⚃ + ⚄

14 = ⚁ + ⚅ + ⚅ = ⚂ + ⚄ + ⚅ = ⚃ + ⚃ + ⚅ = ⚃ + ⚄ + ⚄

15 = ⚂ + ⚅ + ⚅ = ⚃ + ⚄ + ⚅ = ⚄ + ⚄ + ⚄

16 = ⚃ + ⚅ + ⚅ = ⚄ + ⚄ + ⚅

17 = ⚄ + ⚅ + ⚅

18 = ⚅ + ⚅ + ⚅

$$
\begin{aligned}
3 &= 1+1+1 \\
4 &= 1+1+2 \\
5 &= 1+1+3 = 1+2+2 \\
6 &= 1+1+4 = 1+2+3 = 2+2+2 \\
7 &= 1+1+5 = 1+2+4 = 2+2+3 = 3+3+1 \\
8 &= 1+1+6 = 1+2+5 = 1+3+4 = 2+2+4 = 2+3+3 \\
9 &= 1+2+6 = 1+3+5 = 1+4+4 = 2+2+5 = 2+3+4 = 3+3+3 \\
10 &= 1+3+6 = 1+4+5 = 2+2+6 = 2+3+5 = 2+4+4 = 3+3+4 \\
11 &= 1+4+6 = 1+5+5 = 2+3+6 = 2+4+5 = 3+3+5 = 3+4+4 \\
12 &= 1+5+6 = 2+4+6 = 2+5+5 = 3+3+6 = 3+4+5 = 4+4+4 \\
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15 &= 3+6+6 = 4+5+6 = 5+5+5 \\
16 &= 4+6+6 = 5+5+6 \\
17 &= 5+6+6 \\
18 &= 6+6+6
\end{aligned}
$$

k	Probability of a sum of k	$\approx$
3	1/216	0.5%
4	3/216	1.4%
5	6/216	2.8%
6	10/216	4.6%
7	15/216	7.0%
8	21/216	9.7%
9	25/216	11.6%
10	27/216	12.5%
11	27/216	12.5%
12	25/216	11.6%
13	21/216	9.7%
14	15/216	7.0%
15	10/216	4.6%
16	6/216	2.8%
17	3/216	1.4%
18	1/216	0.5%

Game of rolling three dices dated back to Roman Empire.

3	18	Punctatura	1	Cadentia	1
4	17	Punctatura	1	Cadentiæ	3
5	16	Punctaturæ	2	Cadentiæ	6
6	15	Punctaturæ	3	Cadentiæ	10
7	14	Punctaturæ	4	Cadentiæ	15
8	13	Punctaturæ	5	Cadentiæ	21
9	12	Punctaturæ	6	Cadentiæ	25
10	11	Punctaturæ	6	Cadentiæ	27

Richard de Fournival discovered in 13th century the summary of 216 possible sequences [9] in his poem, *De Vetula*, written between 1220 to 1250.

[9] Image is from Bennett (1998), *Randomness*, Harvard University Press.

Thank you for your listening and participating !

References:

- Persi Diaconis and Brian Skyrms (2017). *The great ideas about chance.* Princeton University Press.
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