Embracing Randomness:
Intriguing Role of Chance in Science

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Science Center Auditorium

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https://github.com/chenle02/2012_Science_Summer_Institute_Auburn_Probability_by_Le

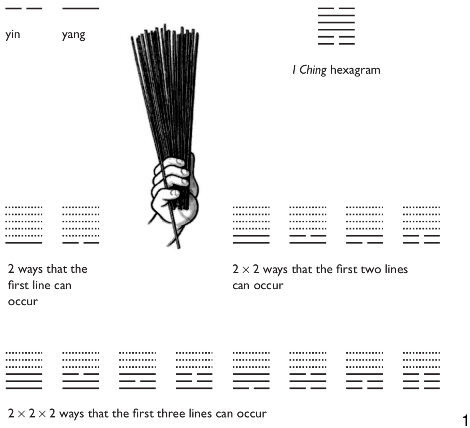
Summer Science Institute 2023
Auburn University

What is CHANCE?

Tyche – The Greek goddess of chance

Fortuna – The goddess of chance in Roman religion

I Ching (~ 6th century B.C.) — A divination tool in ancient China



[1] Image is from Bennett (1998), *Randomness*, Harvard University Press.

Games of chance — using knucklebones or dice

Known to Egyptians, Babylonians, Romans, ...

There was no qualitative theory of chance in these times.

How to measure chance?

How about measure length?

The determination of a "right and lawful rood" or rod in the early sixteenth century in Germany by measuring an essentially random selection of 16 men as they leave church [2].

[2]Stephen Stigler (1996). Statistics and the Question of Standards, *Journal of Research of the National Institute of Standards and Technology*, vol. 101.

The same for chance

To measure probability,

1. we first find or make equally probable cases,
2. then we count.

The probability of an event A, denoted by $\mathbb{P}(A)$, is then

$$\mathbb{P}(A) = \frac{\text{number of cases in which } A \text{ occurs}}{\text{total number of cases}}.$$

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*Girolamo Cardano** (1501 – 1576)

* An Italian polymath, whose interests and proficiencies ranged through those of mathematician, physician, biologist, physicist, chemist, astrologer, astronomer, philosopher, writer, and gambler. He was one of the most influential mathematicians of the Renaissance, and was one of the key figures in the foundation of probability and the earliest introducer of the binomial coefficients and the binomial theorem in the Western world.

Avid and serious gambler

"*Liber de ludo aleae*" (The Book on Games of Chance)

"equally likely outcomes"

Influencing later thinkers like *Pascal* and *Fermat*

Now we see that probability has to satisfy the following properties:

1. Probability should be never negative.
2. If A occurs in all cases, then $\mathbb{P}(A) = 1$.
3. If A and B never occur in the same case, then

$$\mathbb{P}(A \text{ or } B) = \mathbb{P}(A) + \mathbb{P}(B).$$

In particular, the probability of an event not occurring is equal to

$$\mathbb{P}(\text{not } A) = 1 - \mathbb{P}(A).$$

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How to generate the equi-probable cases?

Prim sticks (variations of dice) [3] and deck of poker...

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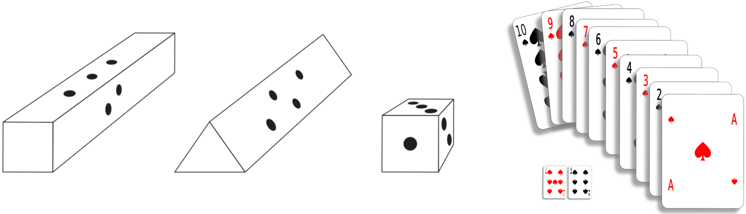

Prim sticks (variations of dice) [3] and deck of poker...

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Example one: Birthday Problem

Question: How likely do two students have the same birth day if there are

- 2
- 5
- 15
- 23
- 46
- 64
- 366

students in the class?

Assuming that

1. each year has 365 days (i.e., neglecting leap years),
2. birthdays are equi-probable,
3. birthdays are independent (no twins in the class).

- Suppose $n = 5$.
- Let A be the event that there is a shared birthday among these n students.
- It is not easy to compute $\mathbb{P}(A)$ directly.
- However, one can compute $\mathbb{P}(\text{not } A)$ by counting:

the probability that no two students have the same birthday, or

all students have different birthday,

is equal to

$$\mathbb{P}(\text{not } A) = \frac{365}{365}\frac{364}{365}\frac{363}{365}\frac{362}{365}\frac{361}{365}$$

- Hence,

$$\mathbb{P}(A) = 1 - \mathbb{P}(\text{not } A) = 1 - \frac{365}{365}\frac{364}{365}\frac{363}{365}\frac{362}{365}\frac{361}{365} \approx 0.027.$$

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$$\mathbb{P}(A) = 1 - \frac{365 \times 364 \times \cdots \times (365 - n + 1)}{365^n}$$

n	2	5	15	23	46	64	366
$\mathbb{P}(A)$							1.0

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n	2	5	15	23	46	64	366
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n	2	5	15	23	46	64	366
$\mathbb{P}(A)$	0.0027	0.0271	0.2529				1.0

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n	2	5	15	23	46	64	366
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$\mathbb{P}(A)$	0.0027	0.0271	0.2529	0.5073	0.9483		1.0

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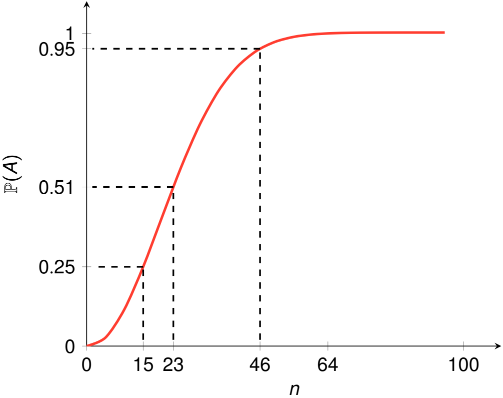

Number of generations	Size of population to have about even odds	Examples (as of 2023)
1	23	A typical class
2	438	—
3	$8,368$	Princeton University[4]
4	$159,870$	Tuscaloosa[5]; Auburn[6]
5	$3,054,312$	Atlanta[7]; Chicago[8]

Population rank of 500 cities in the United States:

https://en.wikipedia.org/wiki/List_of_United_States_urban_areas

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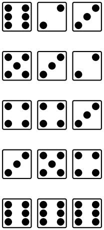
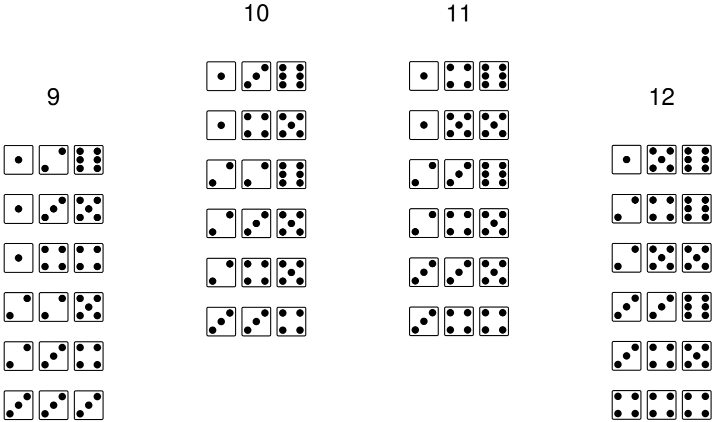
[4] 8,478
[5] Urban: 139,114, Metro: 235,628
[6] Metro: 158,991
[7] Urban: 4,999,259; Metro: 6,144,050
[8] City: 2,746,388; Urban: 8,671,746; Metro: 9,618,502

Another example: a question by *Grand Duke of Tuscany* posed to *Galileo* in early seventh century

Three dice are thrown, such as



Counting combinations of numbers, 10 and 11 can be made in 6 ways, as can 9 and 12:



Yet it is known that long observation has made dice-players consider 10 *and* 11 *to be more advantageous than* 9 *and* 12. *How can this be?*

3 = ⚀ + ⚀ + ⚀

4 = ⚀ + ⚀ + ⚁

5 = ⚀ + ⚀ + ⚂ = ⚀ + ⚁ + ⚁

6 = ⚀ + ⚀ + ⚃ = ⚀ + ⚁ + ⚂ = ⚁ + ⚁ + ⚁

7 = ⚀ + ⚀ + ⚄ = ⚀ + ⚁ + ⚃ = ⚁ + ⚁ + ⚂ = ⚂ + ⚂ + ⚀

8 = ⚀ + ⚀ + ⚅ = ⚀ + ⚁ + ⚄ = ⚀ + ⚂ + ⚃ = ⚁ + ⚁ + ⚃ = ⚁ + ⚂ + ⚂

9 = ⚀ + ⚁ + ⚅ = ⚀ + ⚂ + ⚄ = ⚀ + ⚃ + ⚃ = ⚁ + ⚁ + ⚄ = ⚁ + ⚂ + ⚃ = ⚂ + ⚂ + ⚂

10 = ⚀ + ⚂ + ⚅ = ⚀ + ⚃ + ⚄ = ⚁ + ⚁ + ⚅ = ⚁ + ⚂ + ⚄ = ⚁ + ⚃ + ⚃ = ⚂ + ⚂ + ⚃

11 = ⚀ + ⚃ + ⚅ = ⚀ + ⚄ + ⚄ = ⚁ + ⚂ + ⚅ = ⚁ + ⚃ + ⚄ = ⚂ + ⚂ + ⚄ = ⚂ + ⚃ + ⚃

12 = ⚀ + ⚄ + ⚅ = ⚁ + ⚃ + ⚅ = ⚁ + ⚄ + ⚄ = ⚂ + ⚂ + ⚅ = ⚂ + ⚃ + ⚄ = ⚃ + ⚃ + ⚃

13 = ⚀ + ⚅ + ⚅ = ⚁ + ⚄ + ⚅ = ⚂ + ⚃ + ⚅ = ⚂ + ⚄ + ⚄ = ⚃ + ⚃ + ⚄

14 = ⚁ + ⚅ + ⚅ = ⚂ + ⚄ + ⚅ = ⚃ + ⚃ + ⚅ = ⚃ + ⚄ + ⚄

15 = ⚂ + ⚅ + ⚅ = ⚃ + ⚄ + ⚅ = ⚄ + ⚄ + ⚄

16 = ⚃ + ⚅ + ⚅ = ⚄ + ⚄ + ⚅

17 = ⚄ + ⚅ + ⚅

18 = ⚅ + ⚅ + ⚅

$$
\begin{aligned}
3 &= 1+1+1\\
4 &= 1+1+2\\
5 &= 1+1+3 = 1+2+2\\
6 &= 1+1+4 = 1+2+3 = 2+2+2\\
7 &= 1+1+5 = 1+2+4 = 2+2+3 = 3+3+1\\
8 &= 1+1+6 = 1+2+5 = 1+3+4 = 2+2+4 = 2+3+3\\
9 &= 1+2+6 = 1+3+5 = 1+4+4 = 2+2+5 = 2+3+4 = 3+3+3\\
10 &= 1+3+6 = 1+4+5 = 2+2+6 = 2+3+5 = 2+4+4 = 3+3+4\\
11 &= 1+4+6 = 1+5+5 = 2+3+6 = 2+4+5 = 3+3+5 = 3+4+4\\
12 &= 1+5+6 = 2+4+6 = 2+5+5 = 3+3+6 = 3+4+5 = 4+4+4\\
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18 &= 6+6+6
\end{aligned}
$$

k	Probability of a sum of k	$\approx$
3	1/216	0.5%
4	3/216	1.4%
5	6/216	2.8%
6	10/216	4.6%
7	15/216	7.0%
8	21/216	9.7%
9	25/216	11.6%
10	27/216	12.5%
11	27/216	12.5%
12	25/216	11.6%
13	21/216	9.7%
14	15/216	7.0%
15	10/216	4.6%
16	6/216	2.8%
17	3/216	1.4%
18	1/216	0.5%

Game of rolling three dices dated back to Roman Empire.

3	18	Punctatura	1	Cadentia	1
4	17	Punctatura	1	Cadentiæ	3
5	16	Punctaturæ	2	Cadentiæ	6
6	15	Punctaturæ	3	Cadentiæ	10
7	14	Punctaturæ	4	Cadentiæ	15
8	13	Punctaturæ	5	Cadentiæ	21
9	12	Punctaturæ	6	Cadentiæ	25
10	11	Punctaturæ	6	Cadentiæ	27

Richard de Fournival discovered in 13th century the summary of 216 possible sequences [9] in his poem, *De Vetula*, written between 1220 to 1250.

[9] Image is from Bennett (1998), *Randomness*, Harvard University Press.

Additive v.s. Multiplicative

Let X_n be independent Bernoulli random variables with

$$\mathbb{P}(X_n = 4) = \mathbb{P}(X_n = 0) = 1/2.$$

$$\boxed{\Sigma_n = X_1 + X_2 + \cdots + X_n}$$

$$\boxed{S_n = X_1 \times X_2 \times \cdots \times X_n}$$

$$\mathbb{P}\left(S_n = 2^{2n}\right) = \frac{1}{2^n}$$

$$\frac{\Sigma_n - \mathbb{E}(\Sigma_n)}{\sqrt{Var(\Sigma_n)}} \to N(0,1)$$

$$\mathbb{P}\left(S_n = 0\right) = 1 - \frac{1}{2^n}$$

$$\mathbb{E}\left[S_n\right] = 2^{2n} \times \frac{1}{2^n} + 0 \times \left(1 - \frac{1}{2^n}\right) = 2^n$$

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Embracing Randomness via Simulations

— Part I without interactions

1. Brownian motion on $\mathbb{R}$
2. Exponential Brownian motion on $\mathbb{R}$
3. Random walk on $\mathbb{Z}^2$
4. Random walk on $\mathbb{R}^2$

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Embracing Randomness via Simulations

— Part II with interactions

1. Self avoiding random walk on $\mathbb{Z}^2$

2. Random walk in a random environment on $\mathbb{Z}^2$

Embracing Randomness via Simulations

— Part II with interactions

1. Self avoiding random walk on $\mathbb{Z}^2$
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Matthew Effect

The rich get richer and the poor get poorer.

For to every one who has will more be given, and he will have abundance; but from him who has not, even what he has will be taken away.

—Matthew 25:29, RSV.

Intermittency

Intermittency is a very universal phenomenon which occurs practically irrespective of detailed properties of the background instability in a random medium provided only that the random field is of multiplicative type, ...

Zeldovich et al. **The almighty chance***. 1990*

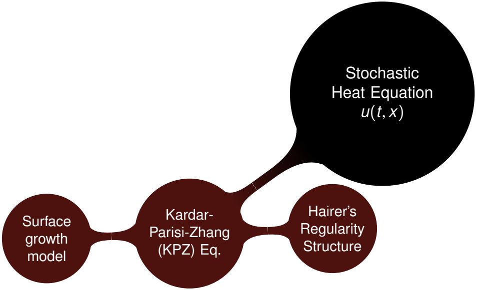
High peaks + highly concentrated on small islands | multiplicative noise

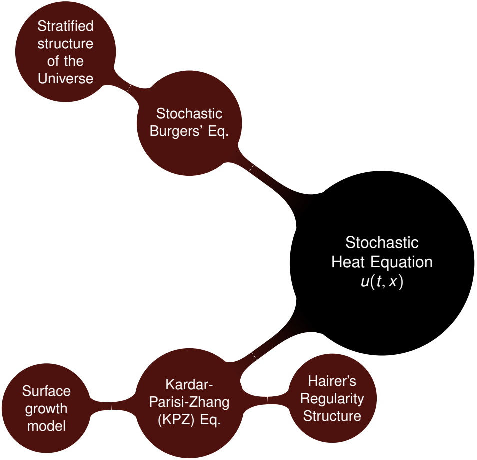
Tug war for stochastic heat equation (SHE)

$$\left(\frac{\partial}{\partial t} - \frac{1}{2}\Delta\right) u(t,x) = \rho(u(t,x))\dot{W}(t,x)$$

Δ	$\rho(u)\dot{W}$
Smoothing	Roughening

Stochastic
Heat Equation
$u(t, x)$


Stochastic
Heat Equation
$u(t, x)$
Surface
growth
model
Kardar-
Parisi-Zhang
(KPZ) Eq.
Hairer's
Regularity
Structure


Stratified structure of the Universe
Stochastic Burgers' Eq.
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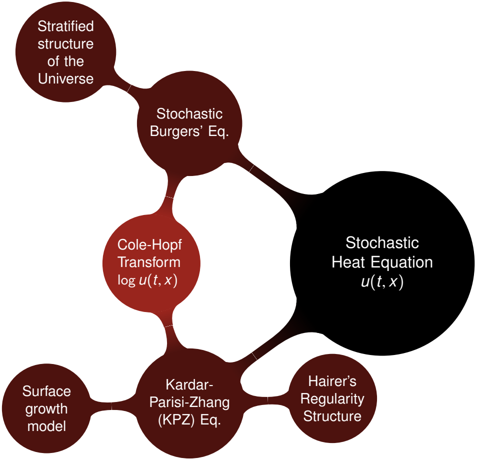
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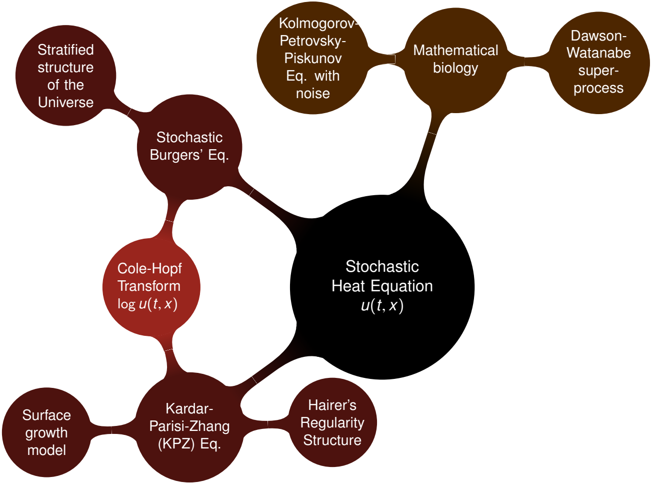
Stratified structure of the Universe
Kolmogorov-Petrovsky-Piskunov Eq. with noise
Mathematical biology
Dawson-Watanabe super-process
Stochastic Burgers' Eq.
Cole-Hopf Transform $\log u(t,x)$
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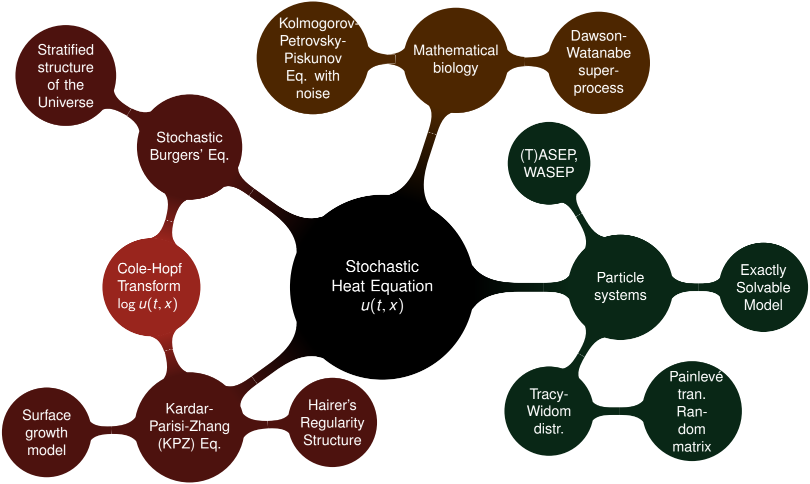
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Tracy-Widom distr.
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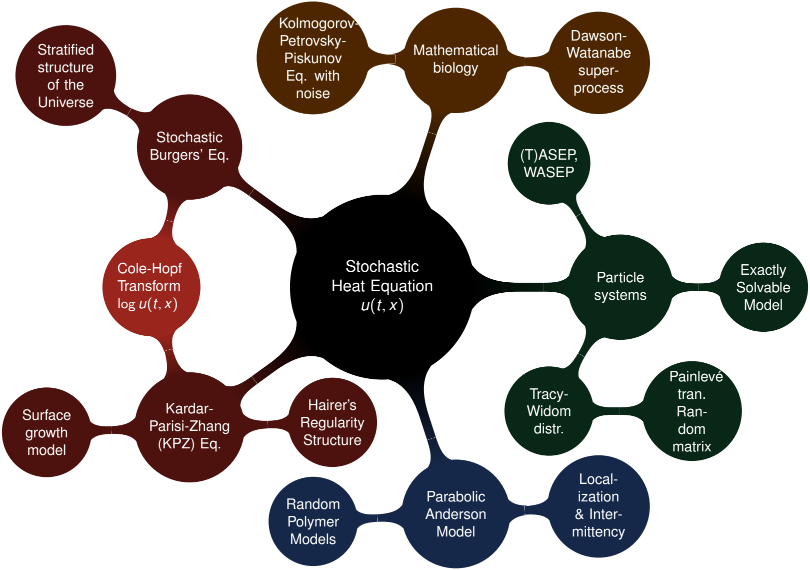

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Parabolic Anderson Model
Random Polymer Models
Local-ization & Inter-mittency

Embracing Randomness via Simulations

— Part III SHE

1. Stochastic heat equation in an interval
2. Stochastic heat equation on $\mathbb{R}$ with localized initial data

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1. Stochastic heat equation in an interval
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Study of growing interfaces in a thin film

— Convection of nematic liquid crystal [10]

Show movies !

[10] K. Takeuchi, M. Sano, T. Sasamoto *et al. Sci. Rep. (Nature)*, 1, 34 (2011).

Study of growing interfaces in a thin film

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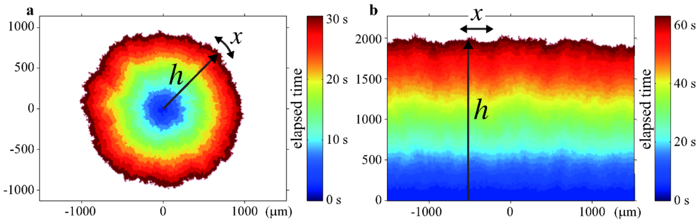

Prediction from KPZ equation:

$$h \asymp v_\infty t + (\Gamma t)^{1/3} \xi$$

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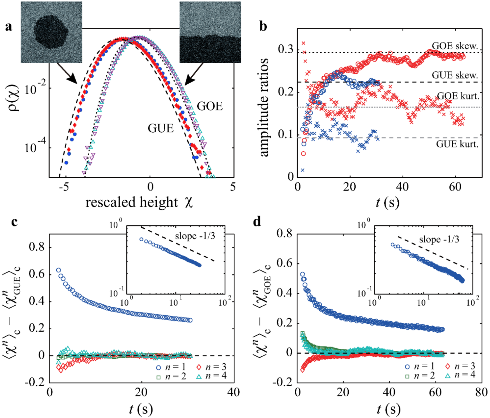

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KPZ Equation ’86 [11]



Mehran Kardar (1957 –) Giorgio Parisi (1948 –) Yicheng Zhang

[11] "Dynamic Scaling of Growing Interfaces". Physical Review Letters. 56 (9): 889–892, 1986.

Giorgio Parisi
Facts

Ill. Niklas Elmehed © Nobel Prize Outreach

Giorgio Parisi
The Nobel Prize in Physics 2021

Born: 4 August 1948, Rome, Italy

Affiliation at the time of the award: Sapienza University of Rome, Rome, Italy

Prize motivation: "for the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales."

Prize share: 1/2

https://www.nobelprize.org/prizes/physics/2021/parisi/facts/

Sir
Martin Hairer
KBE FRS

Hairer at the Royal Society admissions day in London, July 2014

Born	14 November 1975 (age 47) Geneva, Switzerland
Citizenship	Austrian British
Education	College Claparede, Geneva
Alma mater	University of Geneva
Spouse	Xue-Mei Li (m. 2003)[1][2]
Awards	Whitehead Prize (2008) Philip Leverhulme Prize (2008) Wolfson Research Merit Award (2009) Fermat Prize (2013) Fröhlich Prize (2014) Fields Medal (2014) Breakthrough Prize in Mathematics (2021) King Faisal Prize (2022)
Scientific career	
Fields	Probability theory[3] Analysis[3]
Institutions	École Polytechnique Fédérale de Lausanne Imperial College London University of Warwick New York University[1]
Thesis	*Comportement Asymptotique d'Équations à Dérivées Partielles Stochastiques* (2001)
Doctoral advisor	Jean-Pierre Eckmann[4]
Website	hairer.org

- M. Hairer. Solving the KPZ equation. *Annals of Mathematics*, vol. 178, 2013.
- M. Hairer. A theory of regularity structures. *Inventiones mathematicae*, vol. 198, 2014.

Fields Medal in 2014

Thank you for your listening and participating !

References:

- Persi Diaconis and Brian Skyrms (2017). *The great ideas about chance.* Princeton University Press.
- Deborah J. Bennett (1998). *Randomness.* Harvard University Press.
- John D. McGervey (1986). *Probabilities in everyday life*. Nelson Hall Publishers.
- Ya. B. Zeldovich, A. A. Ruzmaikin, and D. D. Sokoloff (1987). *The almighty chance*. World Scientific.

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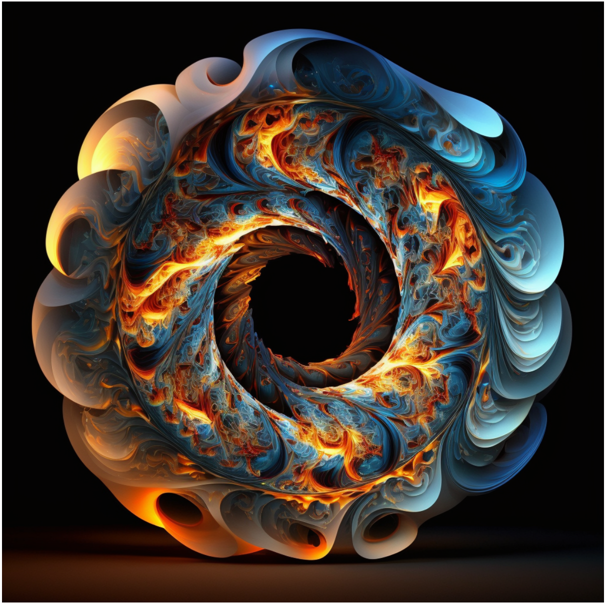
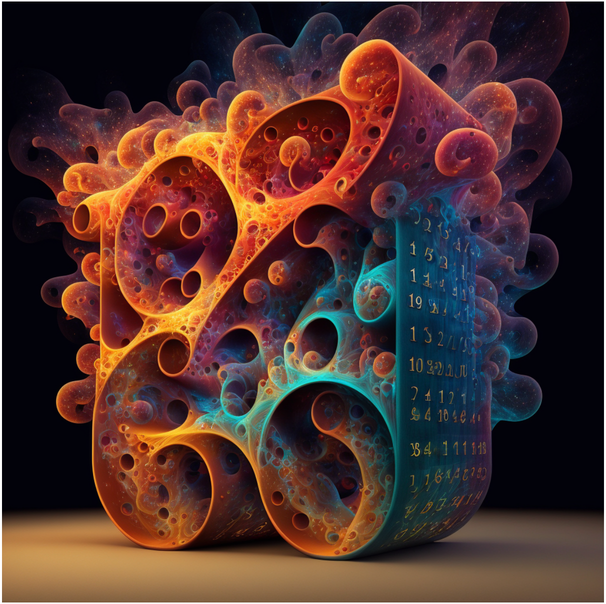
Chaotic stochastic heat flow on a torus...



Law of large numbers and central limit theorem...

Stochastic heat equation...



Intermittency parabolic Anderson model Brownian motion...

A gitar placed in the sand storm in a desert show...

Show rain drops on a small pond in a foggy raining day...

Lightning clouds dramatic light zigzag highly detailed...

Mushrooms in clusters in a forest with sun shades...

Was it nuclear explosionor cosmic forces beyond comprehension...

Standing on the planet Mars looking at home the earth...

A homesick guy looking back to earth from Mars...

A person in front of the whole universe gallaxies...

Cosmic forces beyond comprehension...