

Analysis of Tetris Ballistic Deposition and the Robustness of the KPZ Universality Class

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Acknwolegement

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Emerging Synergies between Stochastic Analysis and Statistical Mechanics
Banff, Alberta, Canada
October 28, 2025

Math 7820/30: Applied Stochastic Processes (2023/24):



Mauricio Montes and Ian Ruau

Plan

Tetromino Pieces

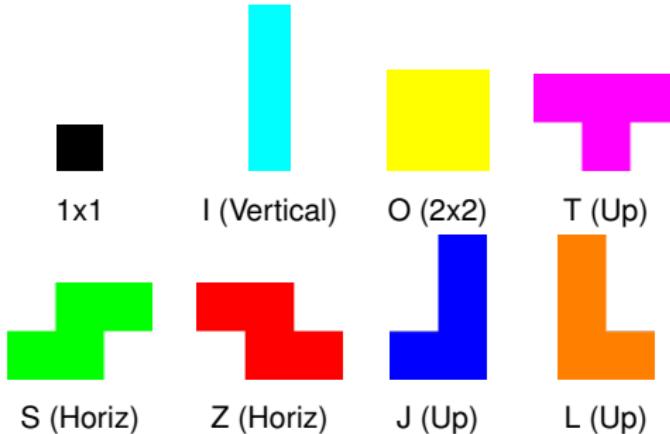
Introduction to growth model and SPDE

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Tetromino Pieces

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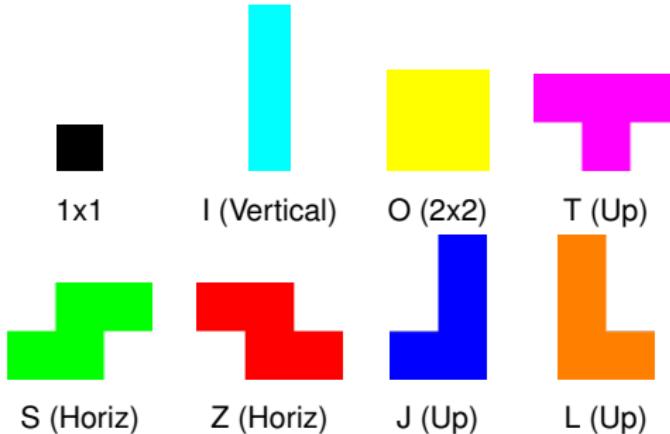
Tetrominoes



- ▶ “1x1”: Single (extra single-site particle)
- ▶ “I”: Horizontal, Vertical
- ▶ “J, L, T”: Up, Right, Down, Left
- ▶ “S, Z”: Horizontal, Vertical
- ▶ “O”: Single (2x2 square)
- ▶ Sticky
- ▶ Nonstikcy

$$(1 + 1 \times 2 + 3 \times 4 + 2 \times 2 + 1) \times 2 = 20 \times 2 = 40 \text{ types of pieces}$$

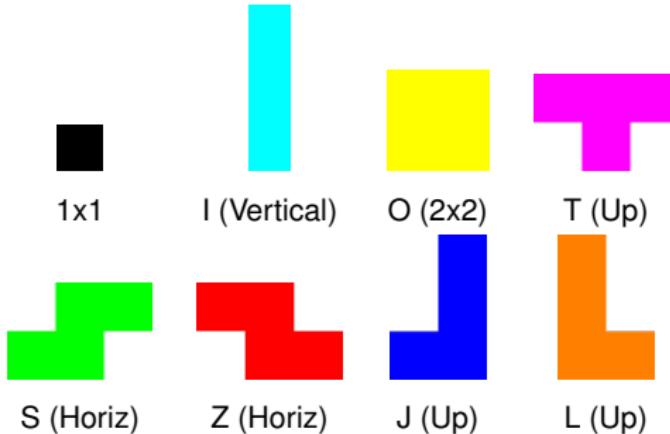
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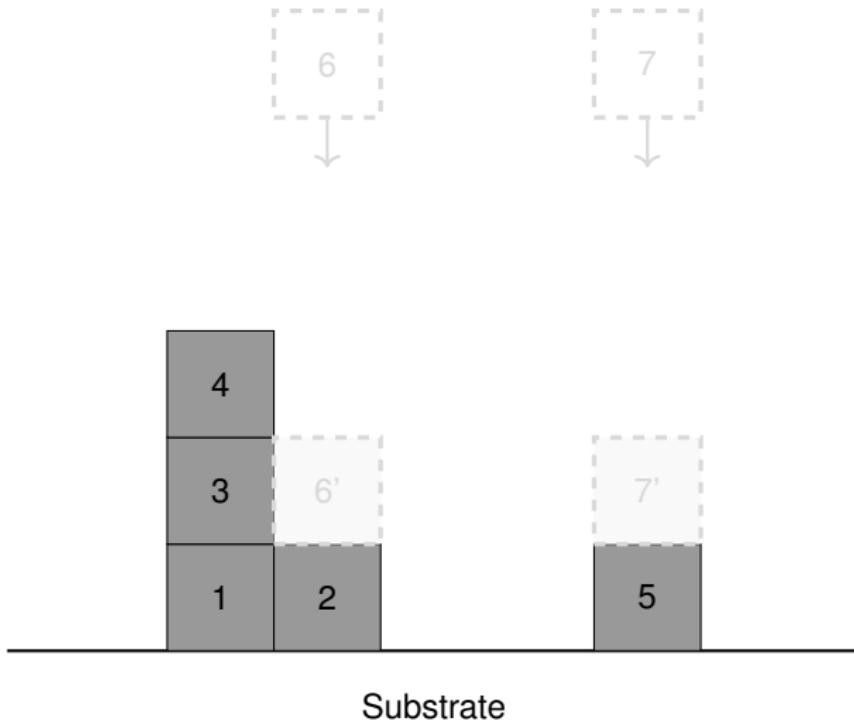
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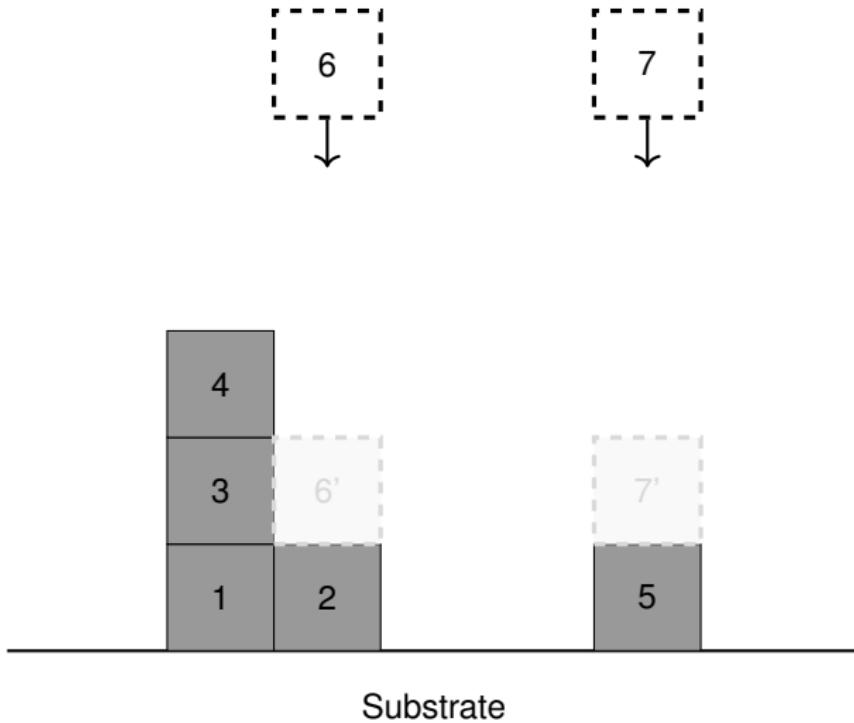
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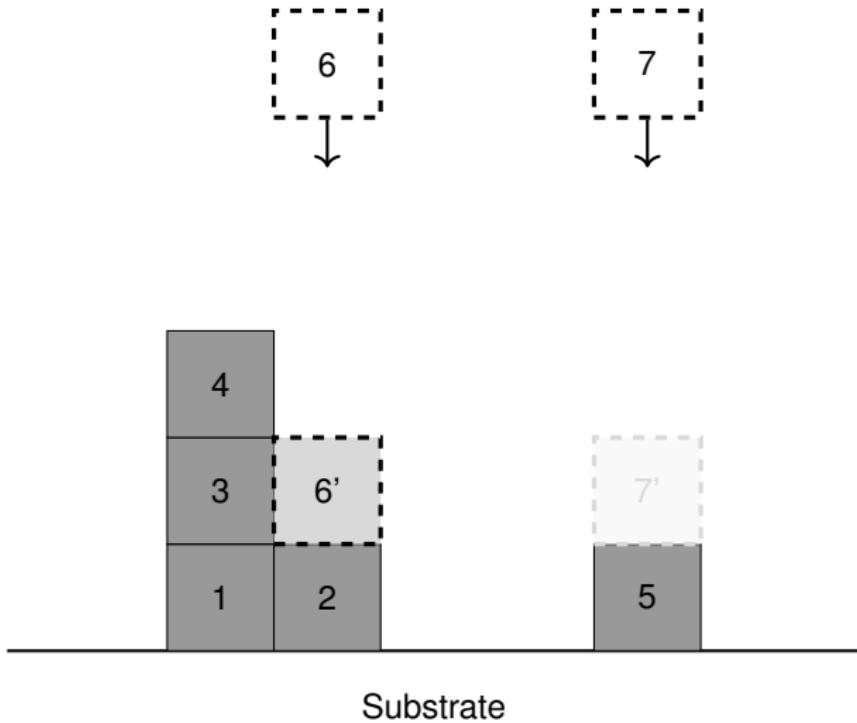
Random deposition



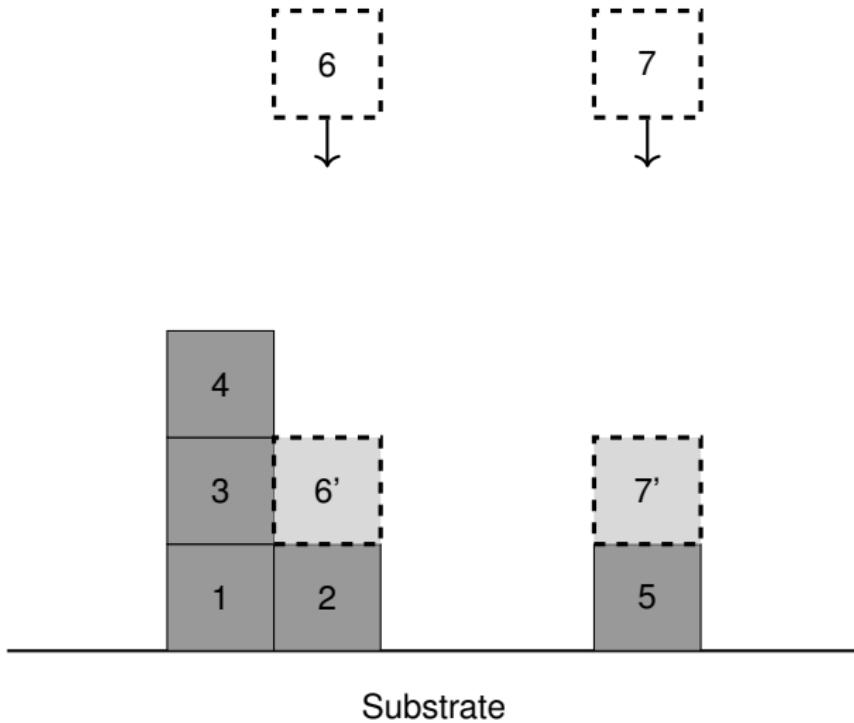
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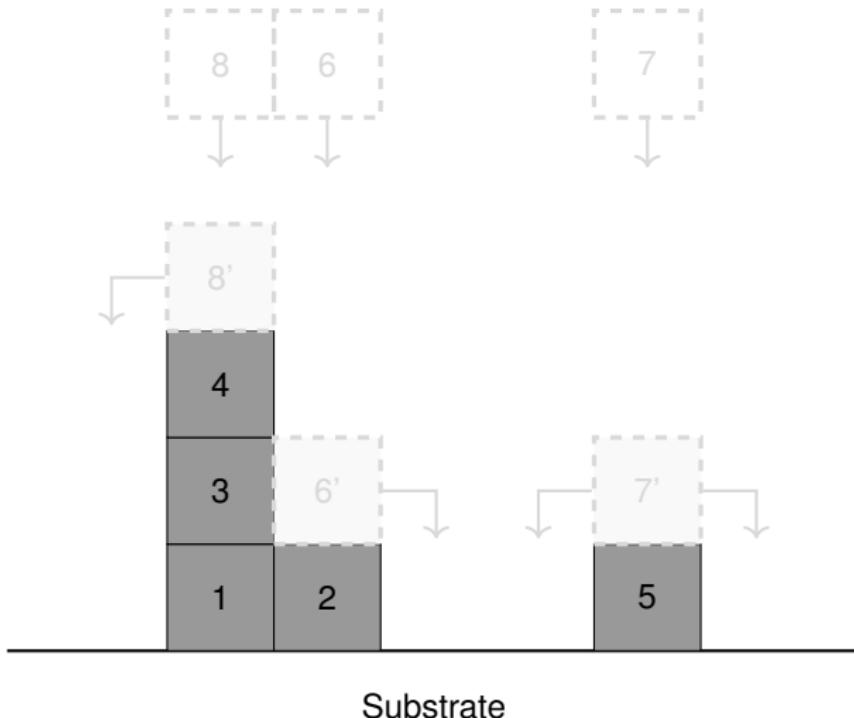
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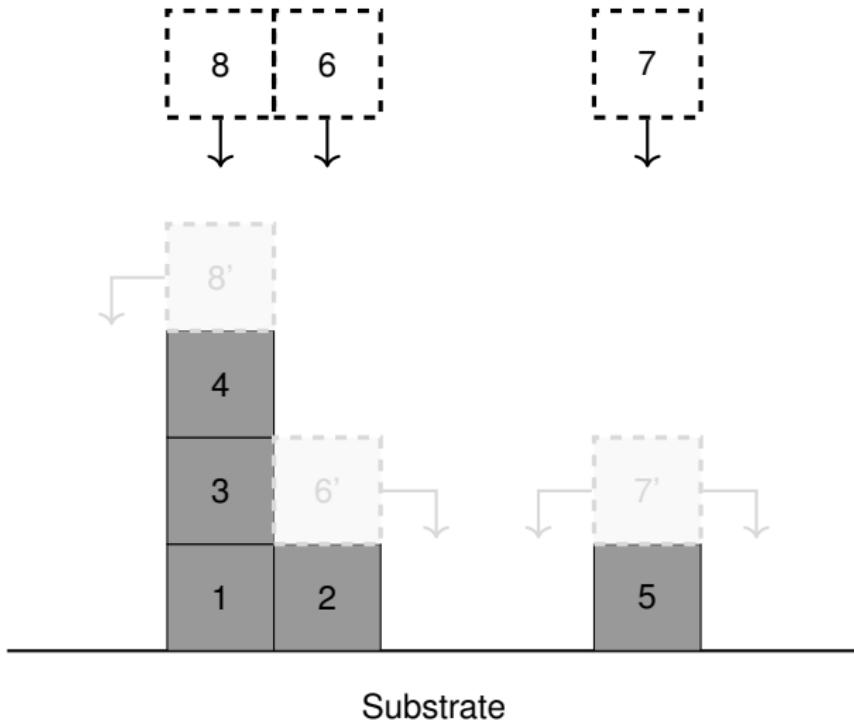
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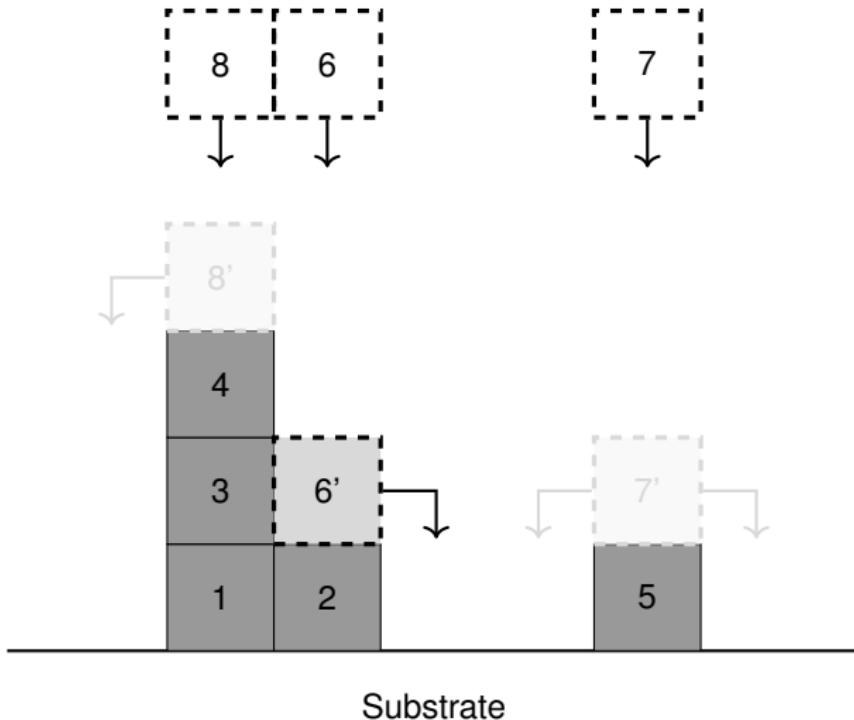
Random deposition with surface relaxation



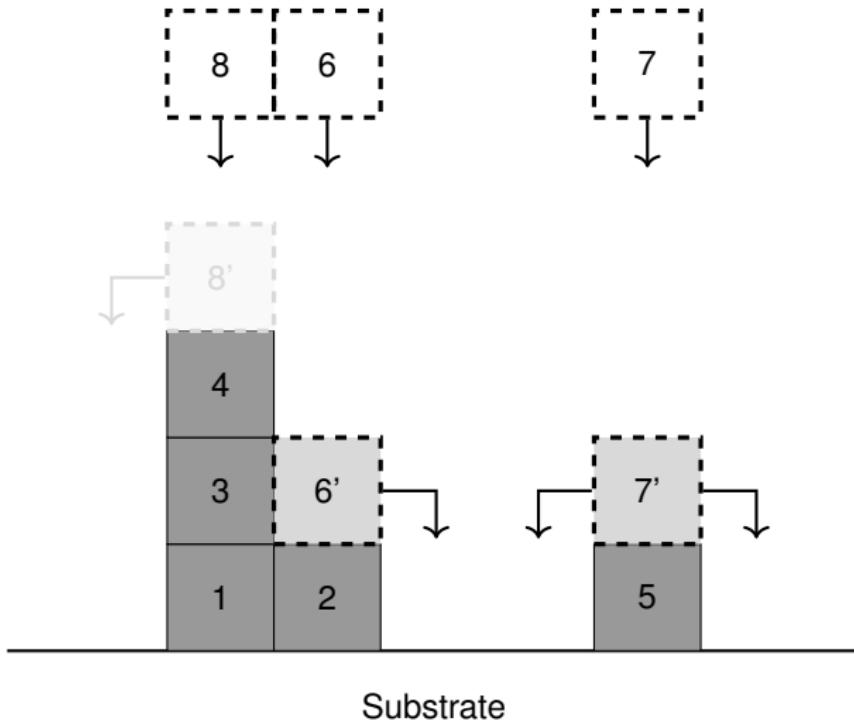
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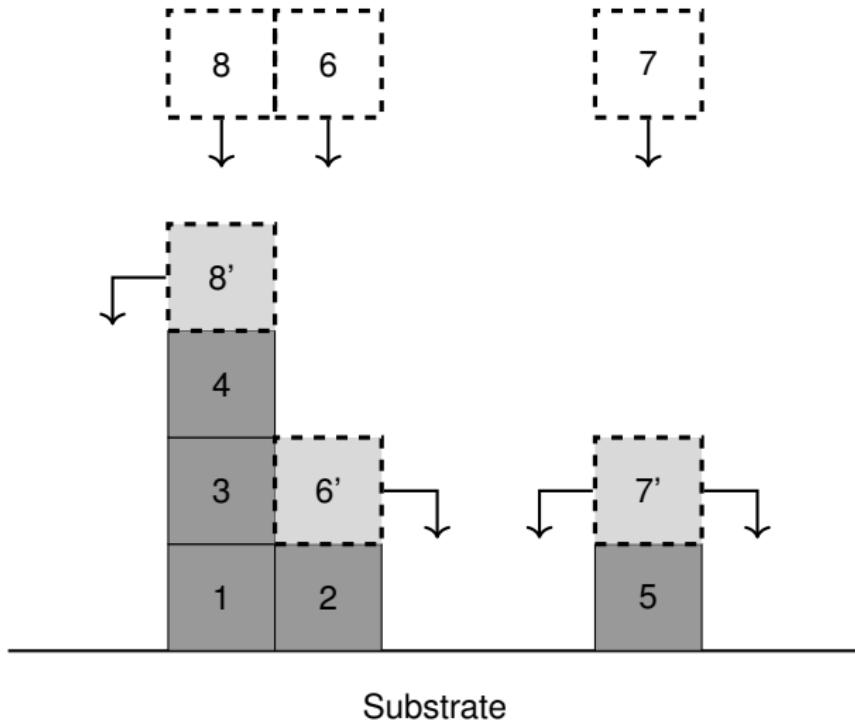
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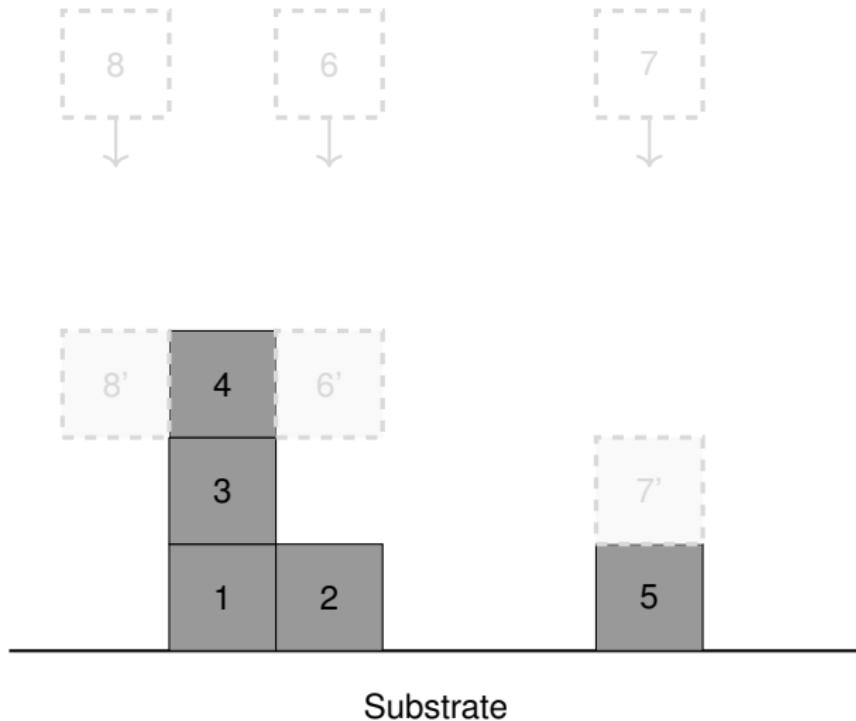
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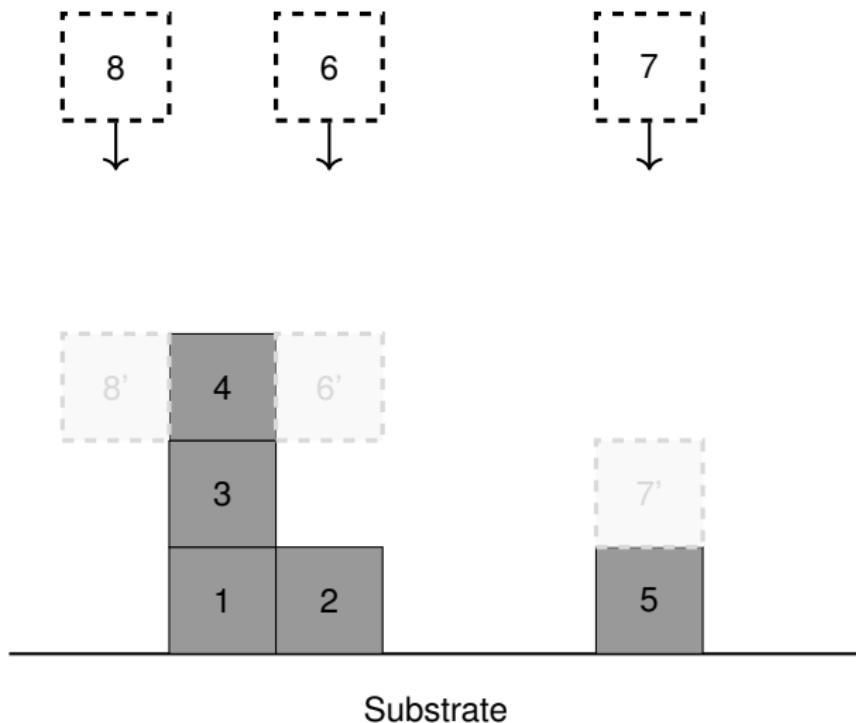
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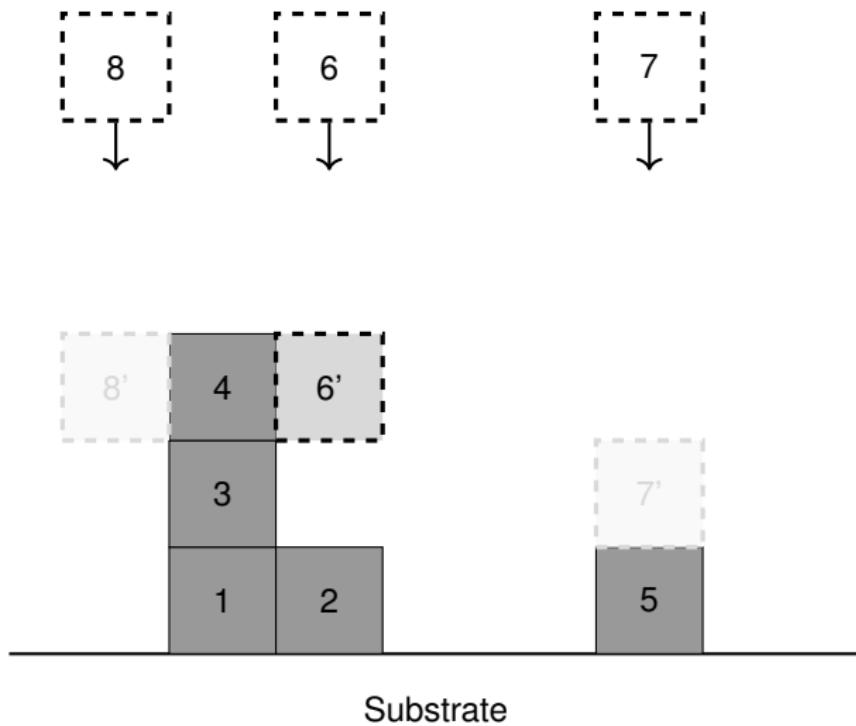
Ballistic deposition



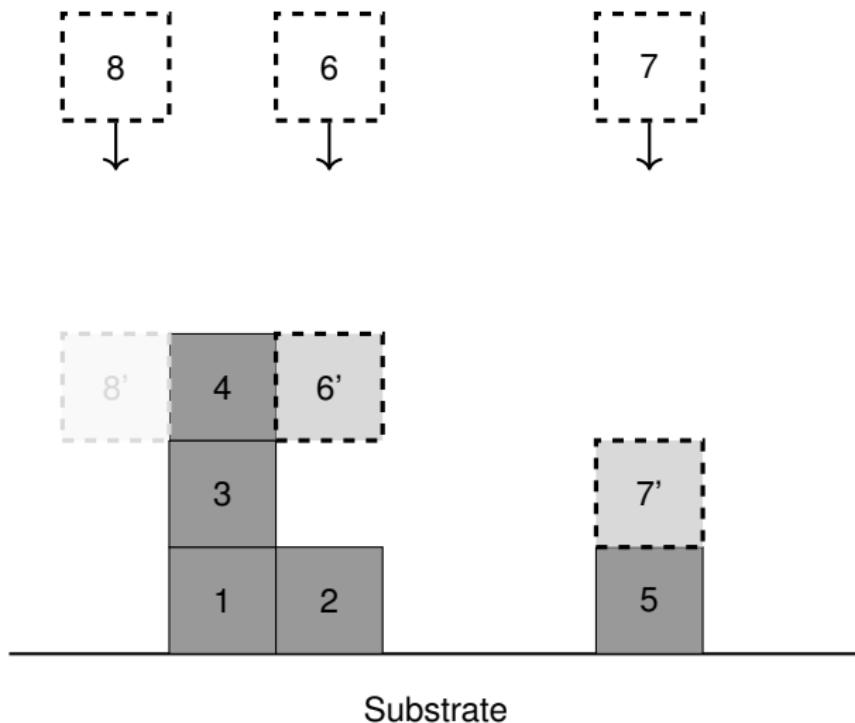
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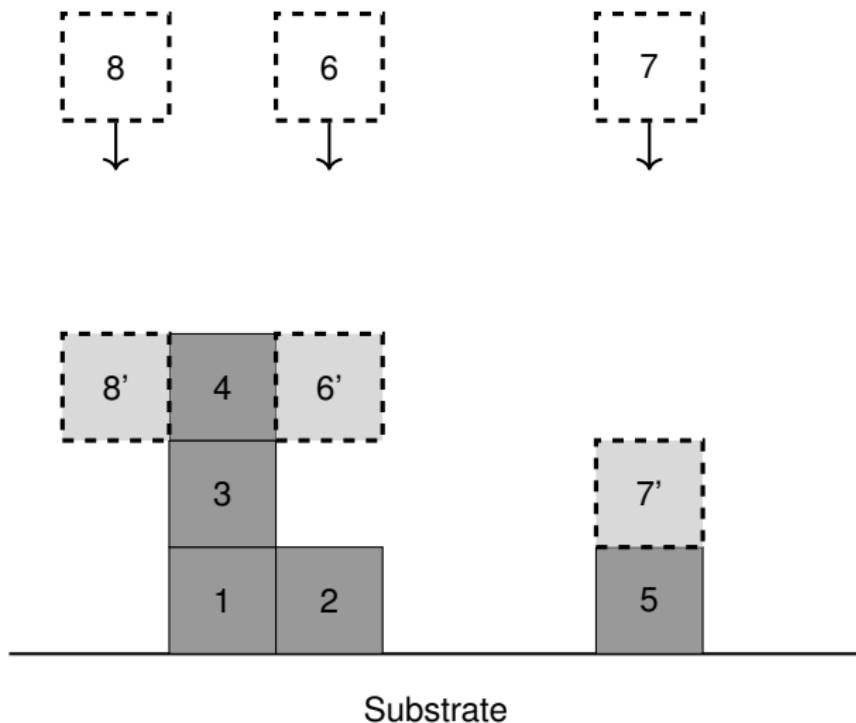
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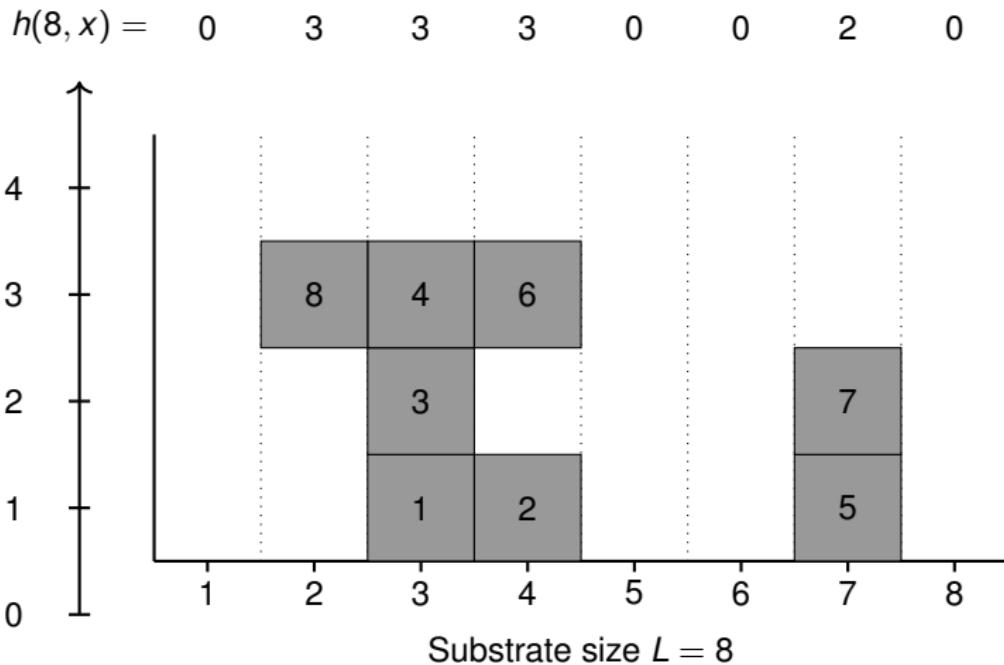
Ballistic deposition



Ballistic deposition



Average height and fluctuation

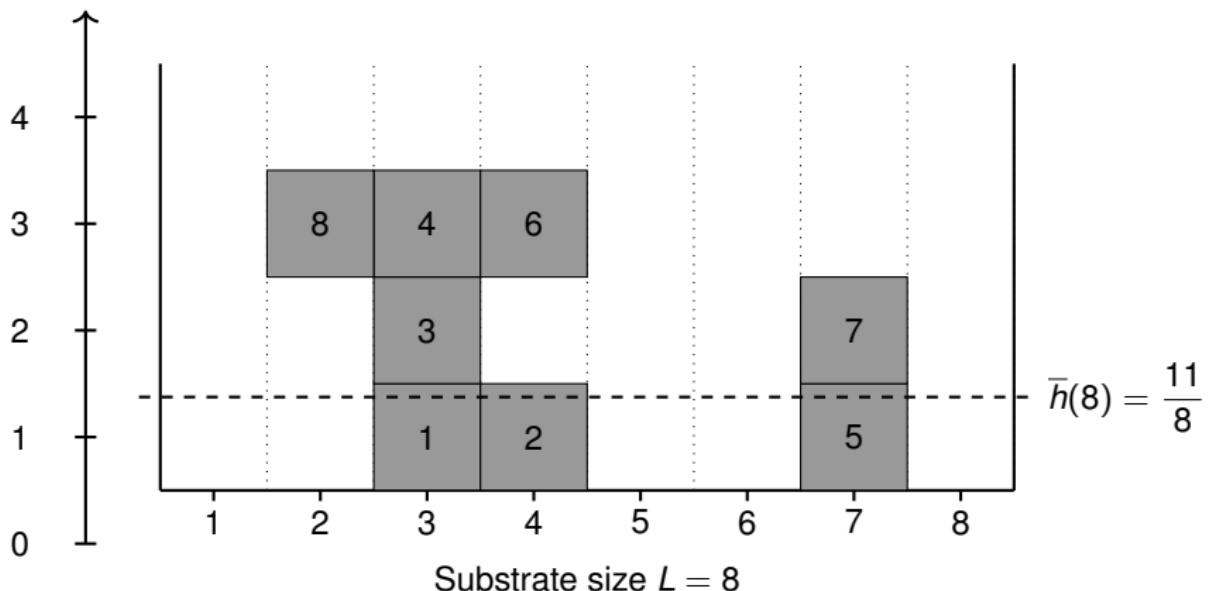


Average height and fluctuation

$$\bar{h}(t) = \frac{1}{L} \sum_{x=1}^L h(t, x)$$

$$\text{Fluctuation } W(L, t) = \sqrt{\frac{1}{L} \sum_{x=1}^L [h(t, x) - \bar{h}(t)]^2}$$

$$h(8, x) = \begin{array}{cccccccc} 0 & 3 & 3 & 3 & 0 & 0 & 2 & 0 \end{array}$$

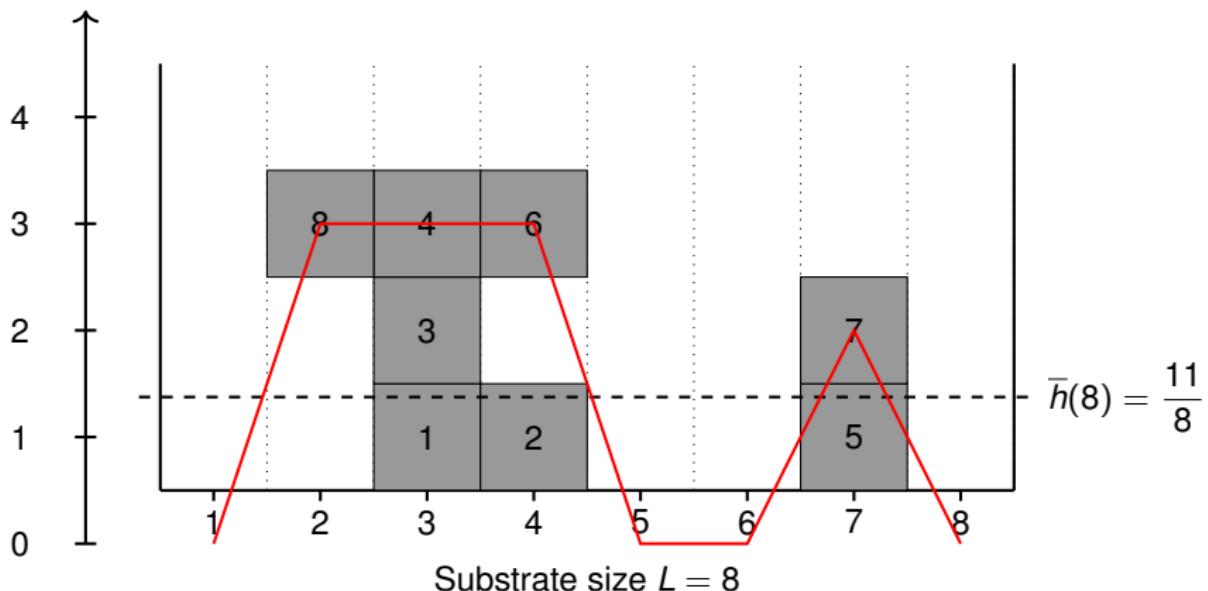


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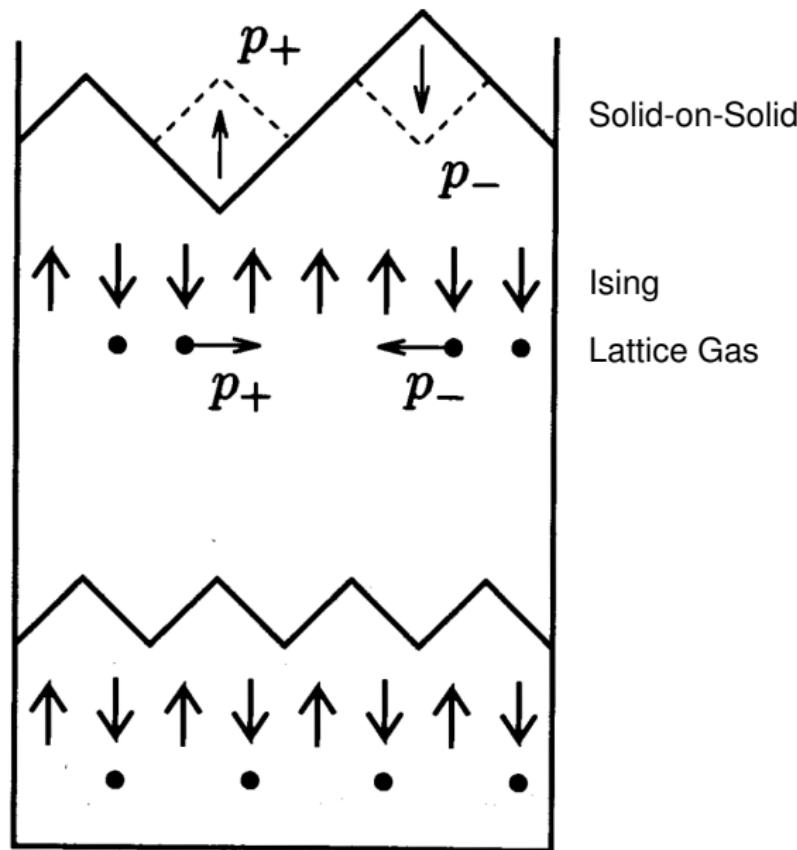
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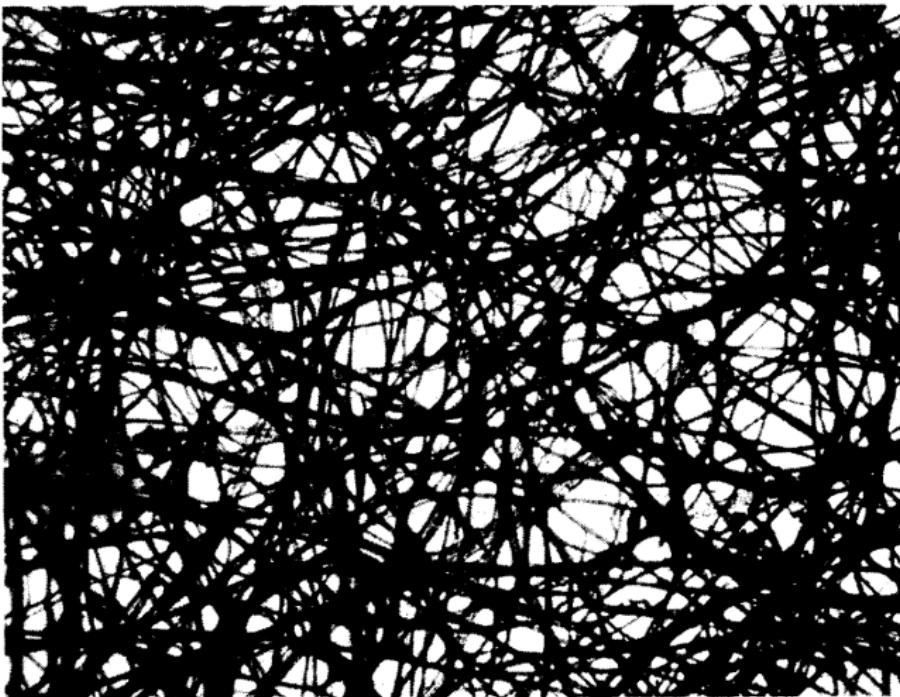


Simulations on
Random deposition vs. Ballistic decomposition

More models? Even more simpler?

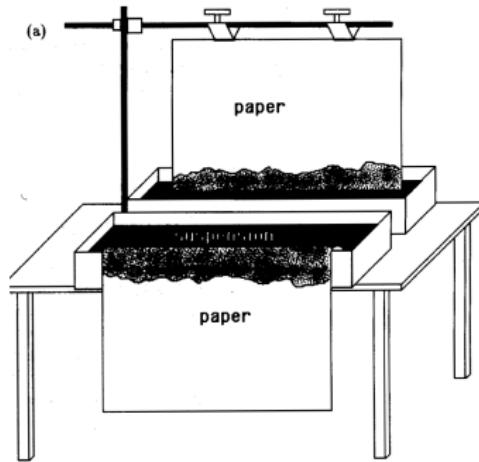


Paper – a random environment



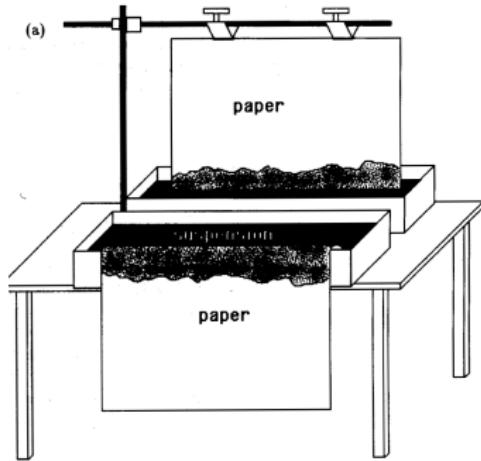
Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

Paper wetting experiment



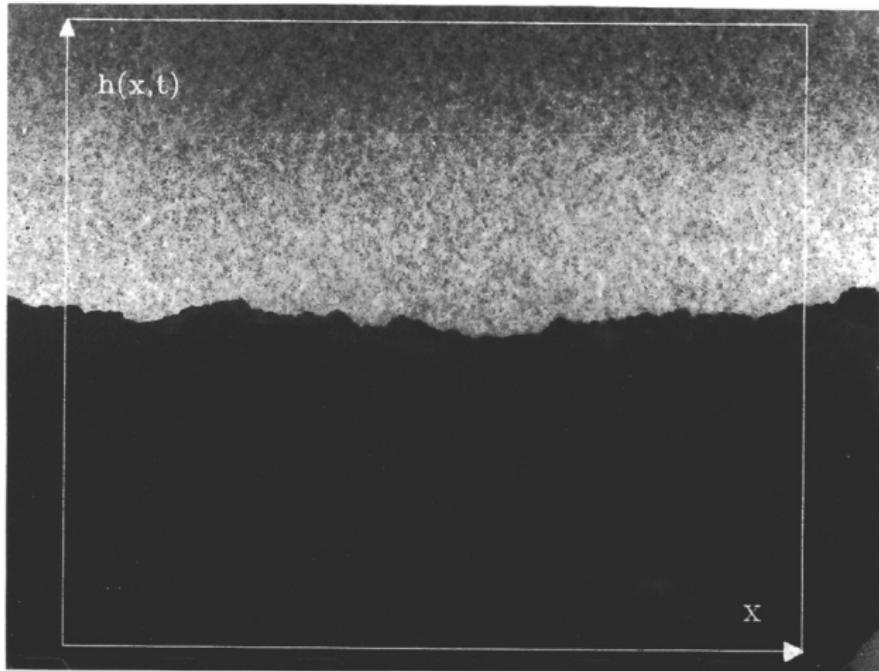
Barabási, A.-L., Stanley, H. E., 1995

Paper wetting experiment



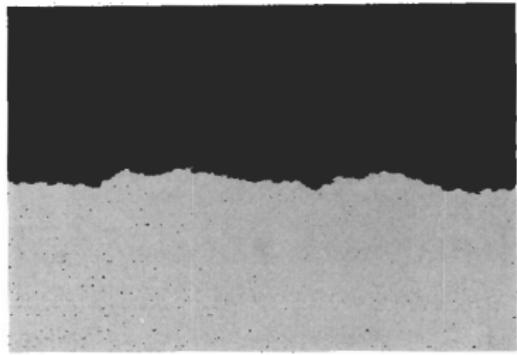
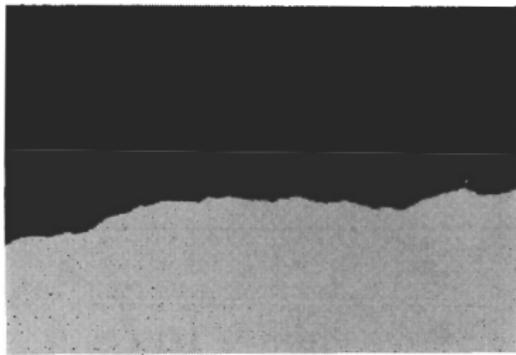
Barabási, A.-L., Stanley, H. E., 1995

Paper burning experiment



Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

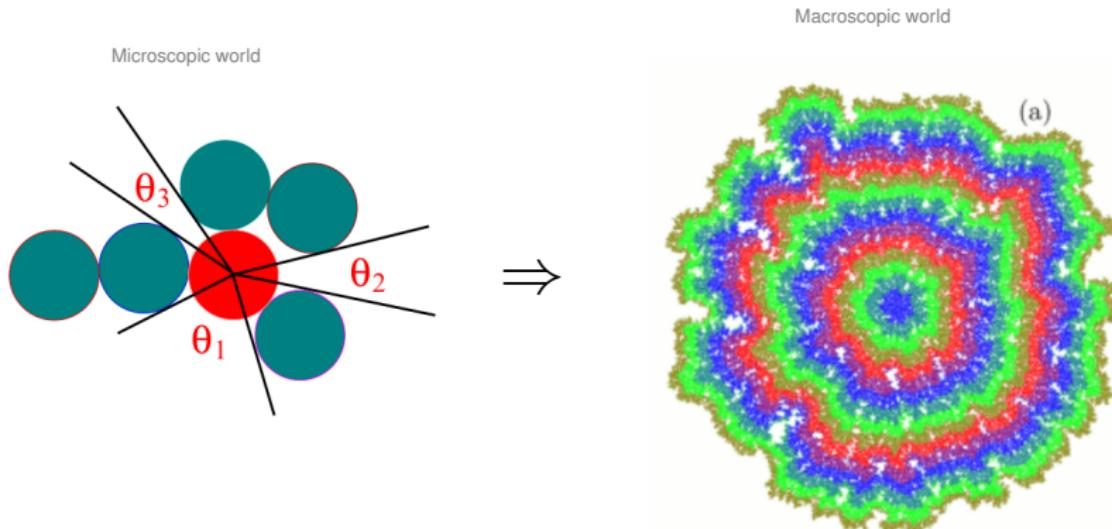
Paper rupture experiment



Kertész, J., Horváth, V. k., Weber, F., *Fractals*, 1993

Rule of replication of **cells**

Replication probability \propto Aperture angle θ_i



Study of growing interfaces in a thin film

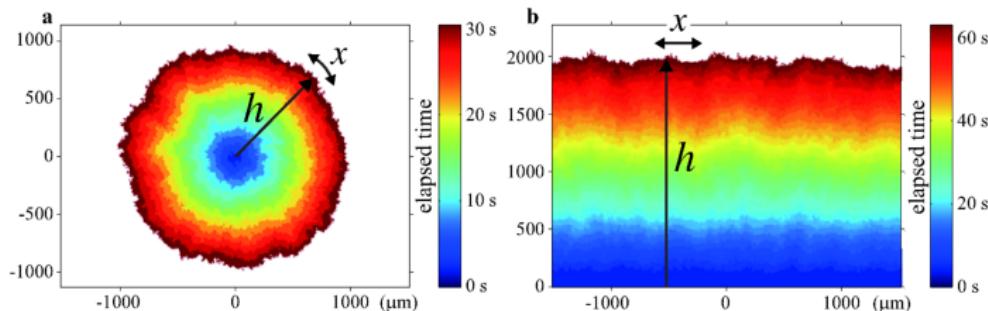
— Convection of nematic liquid crystal*

Show movies !

Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

Study of growing interfaces in a thin film

— Convection of nematic liquid crystal*



Prediction from KPZ equation:

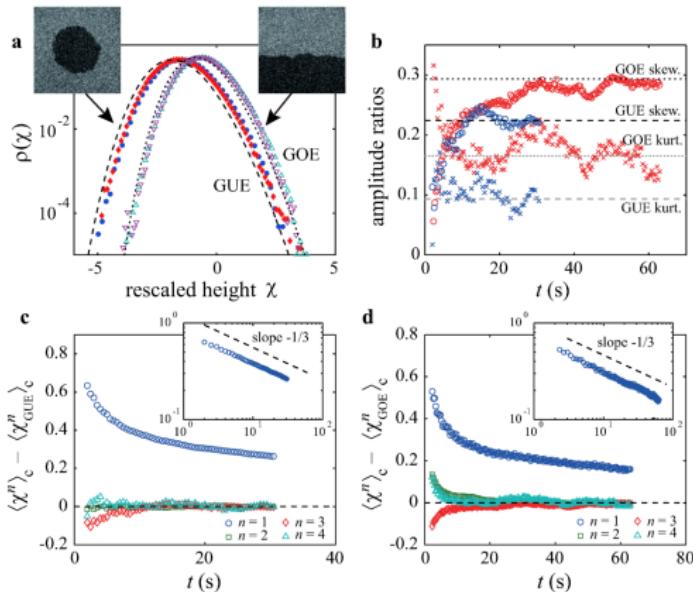
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$$h \asymp v_\infty t + (\Gamma t)^{1/3} \xi$$



KPZ Equation '86

$$\frac{\partial}{\partial t} h(t, x) = \frac{1}{2} \Delta h(t, x) + \frac{\lambda}{2} (\nabla h)^2 + \dot{W}(t, x) \quad (\text{KPZ})$$



Mehran Kardar (1957 –) Giorgio Parisi (1948 –)



Yicheng Zhang

Kardar, M., Parisi, G., Zhang, Y.-C., *Phys. Rev. Lett.*, 1986

Main References*:

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* Download the bib file: <https://github.com/chenle02/SPDEs-Bib/blob/main/All.bib>