

# Analysis of Tetris Ballistic Deposition and the Robustness of the KPZ Universality Class

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Auburn University

Acknwolegement

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Emerging Synergies between Stochastic Analysis and Statistical Mechanics  
Banff, Alberta, Canada  
October 28, 2025

Math 7820/30: Applied Stochastic Processes (2023/24):



Mauricio Montes and Ian Ruau

# Plan

Tetromino Pieces

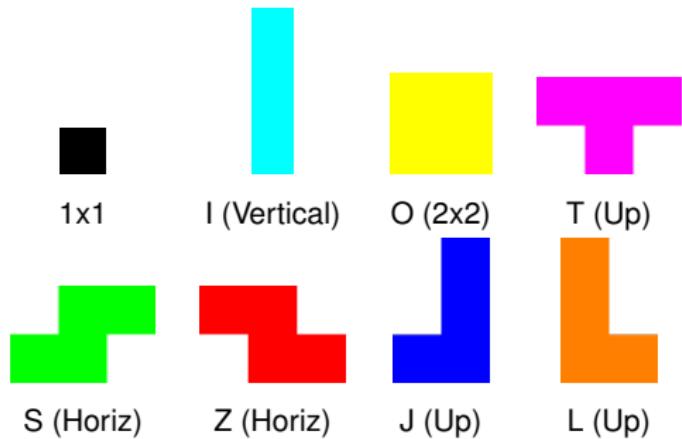
Introduction to growth model and SPDE

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Tetromino Pieces

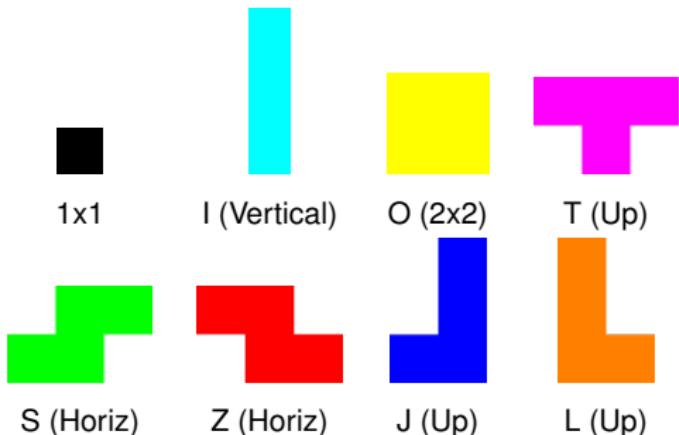
Introduction to growth model and SPDE

# Tetrominoes



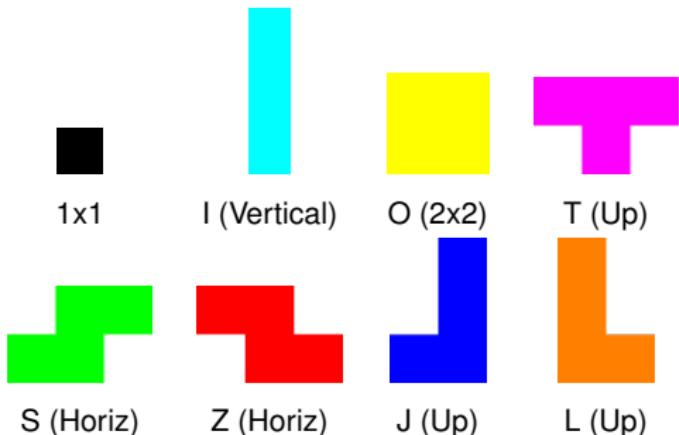
- ▶ “1x1”: Single (extra single-site particle)
  - ▶ “I”: Horizontal, Vertical
  - ▶ “J, L, T”: Up, Right, Down, Left
  - ▶ “S, Z”: Horizontal, Vertical
  - ▶ “O”: Single (2×2 square)
- × ▶ Sticky  
          ▶ Nonstikcy

# Tetrominoes



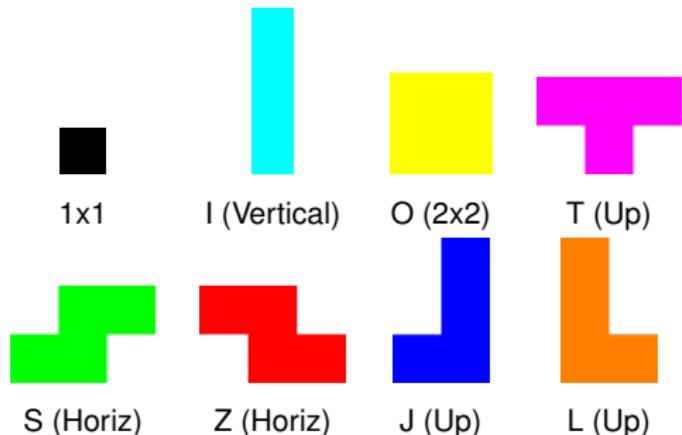
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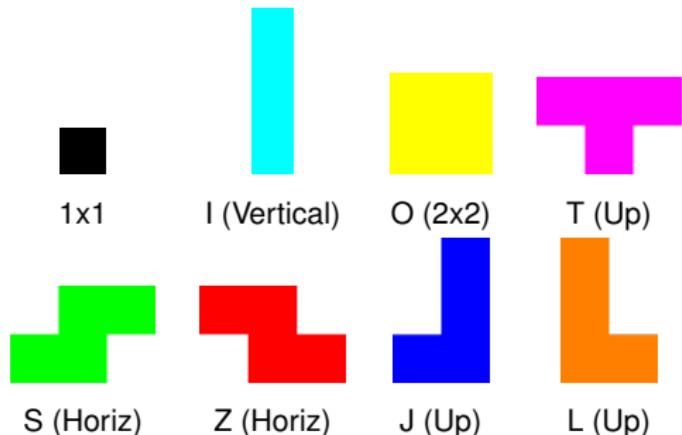
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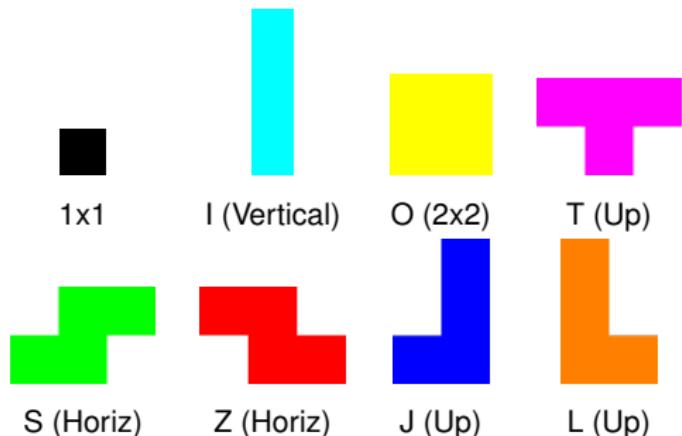
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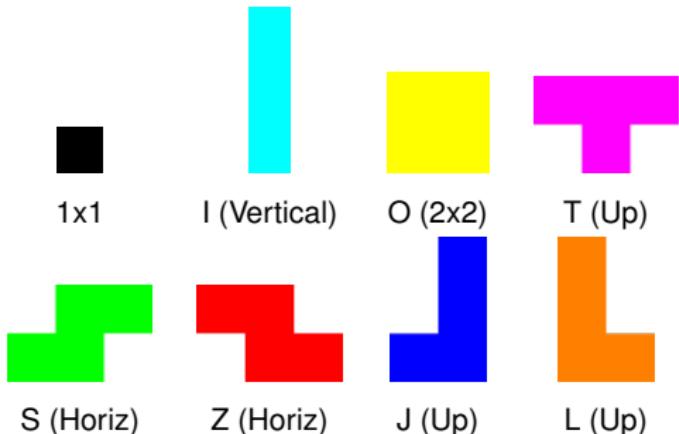
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# Tetrominoes



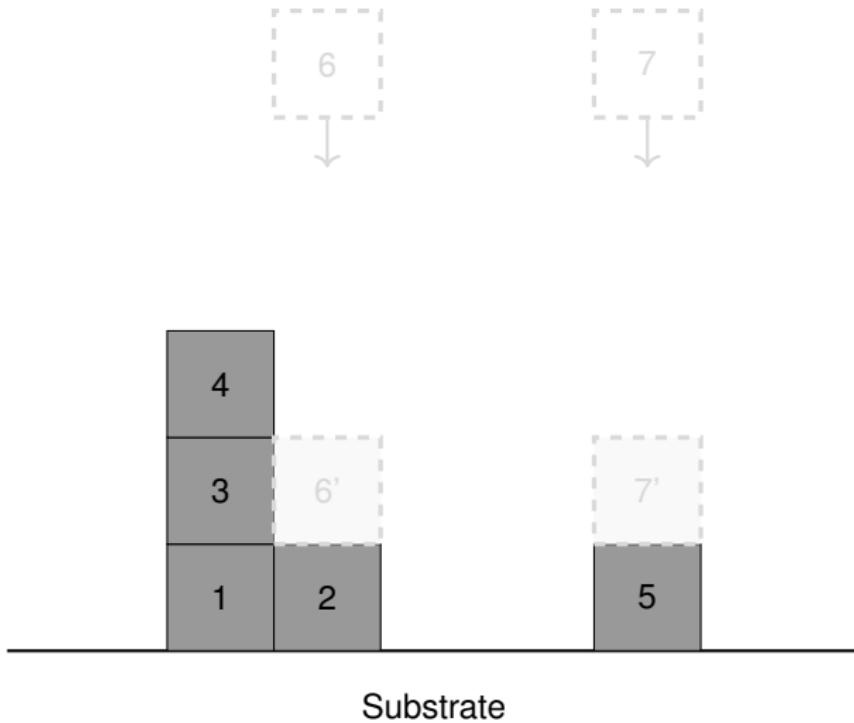
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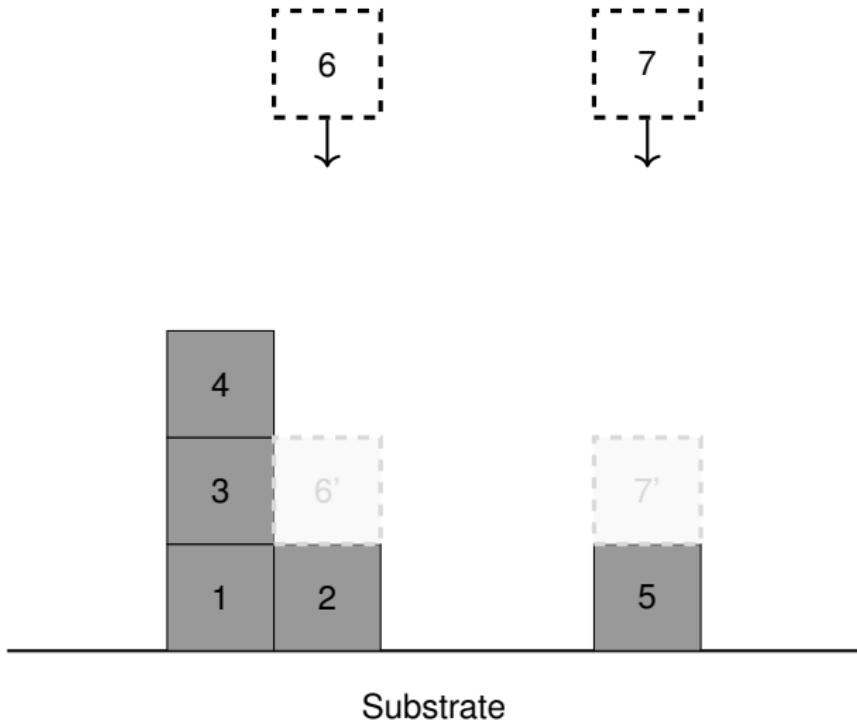
Tetromino Pieces

Introduction to growth model and SPDE

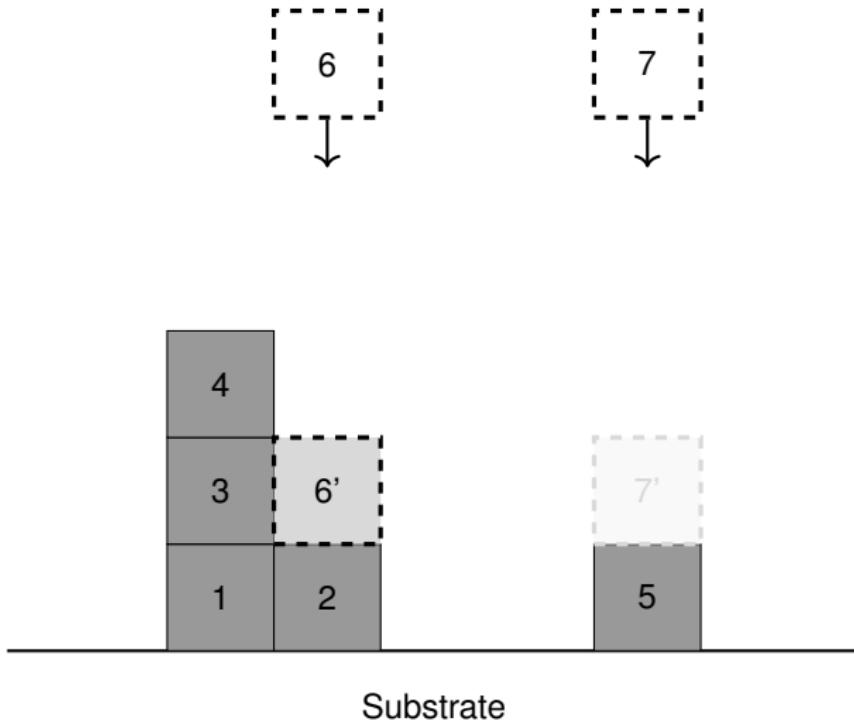
# Random deposition



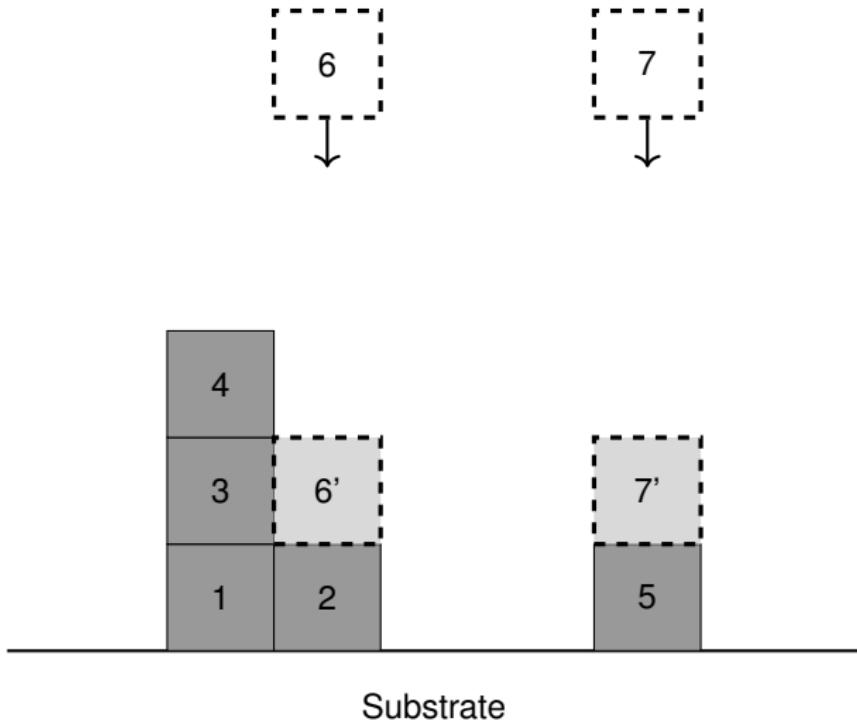
## Random deposition



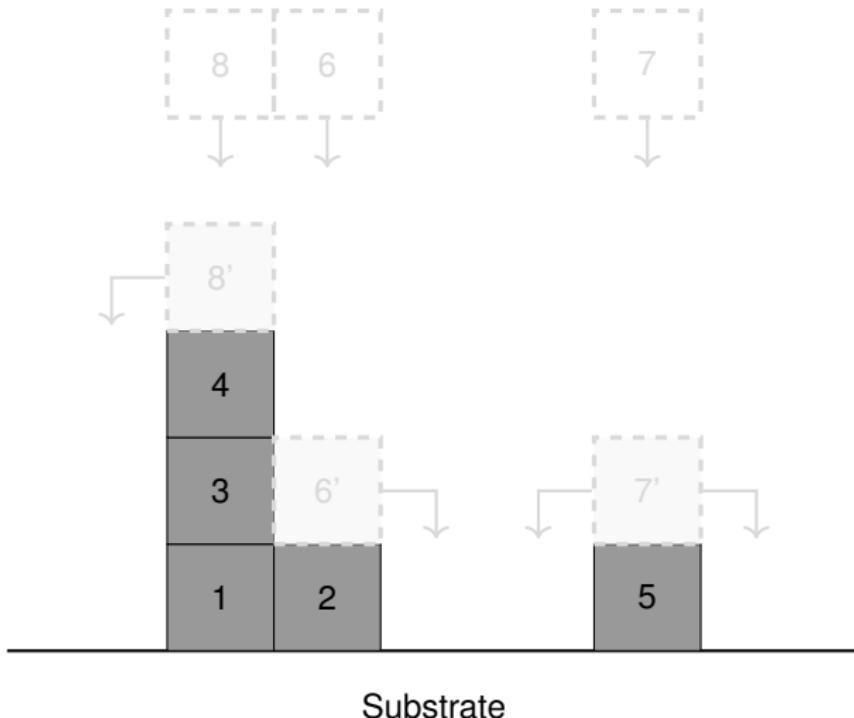
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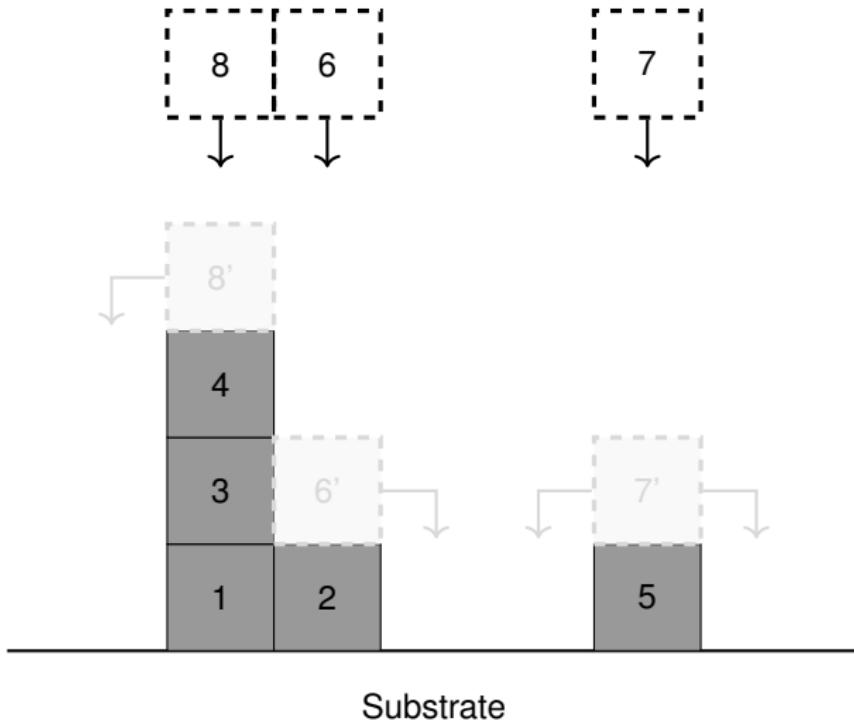
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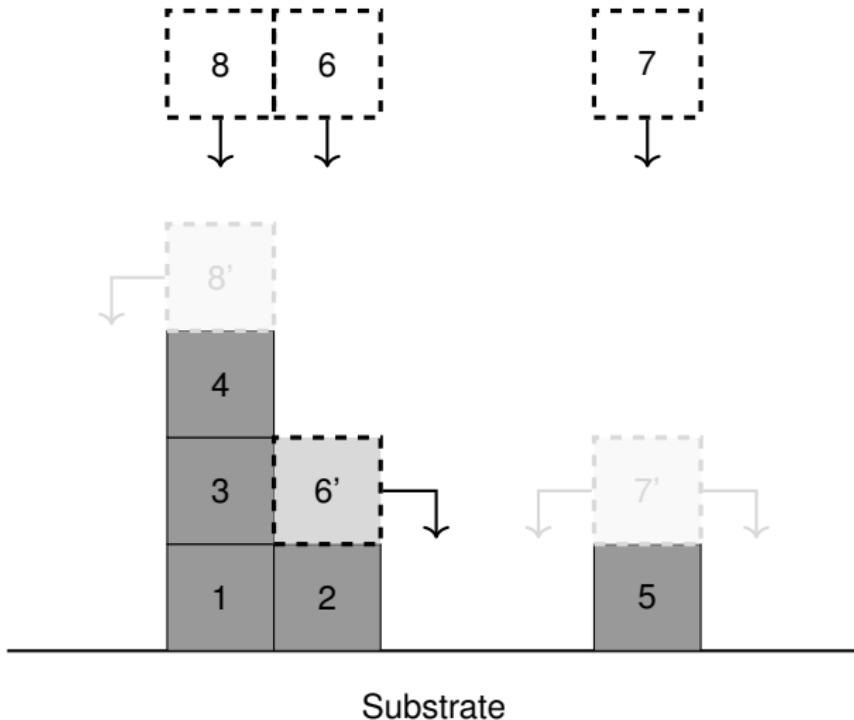
## Random deposition with surface relaxation



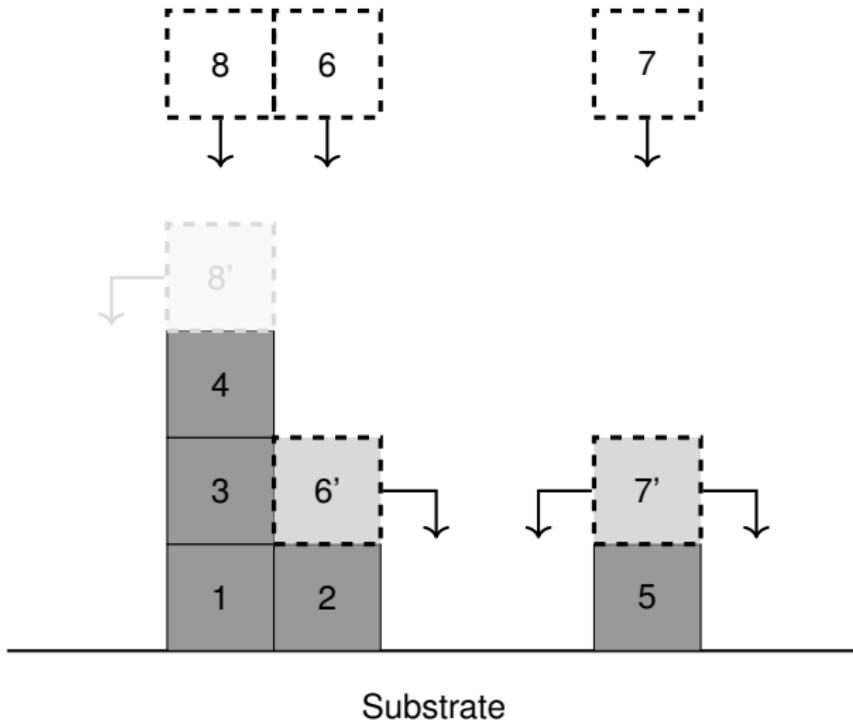
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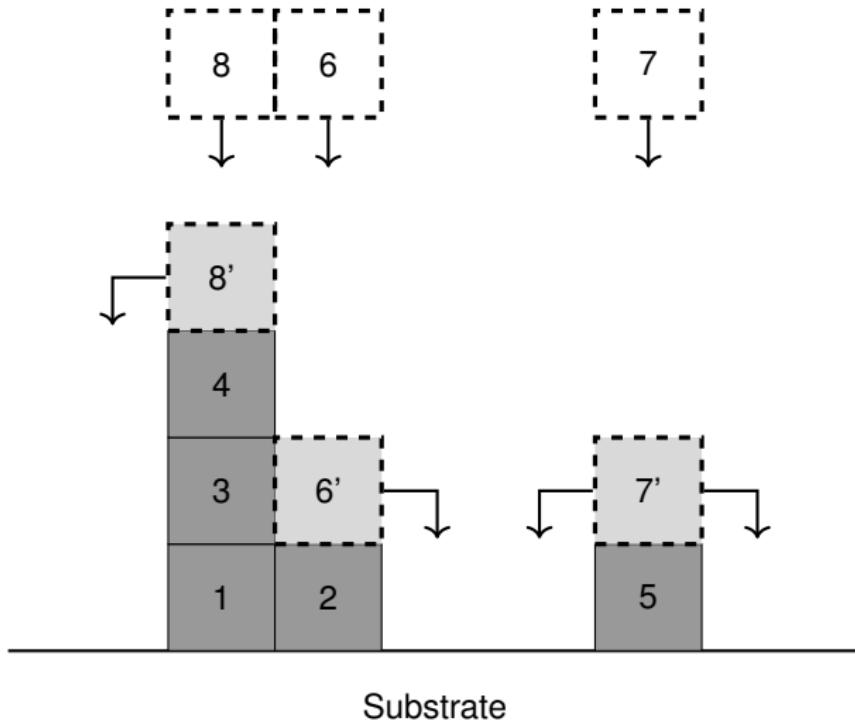
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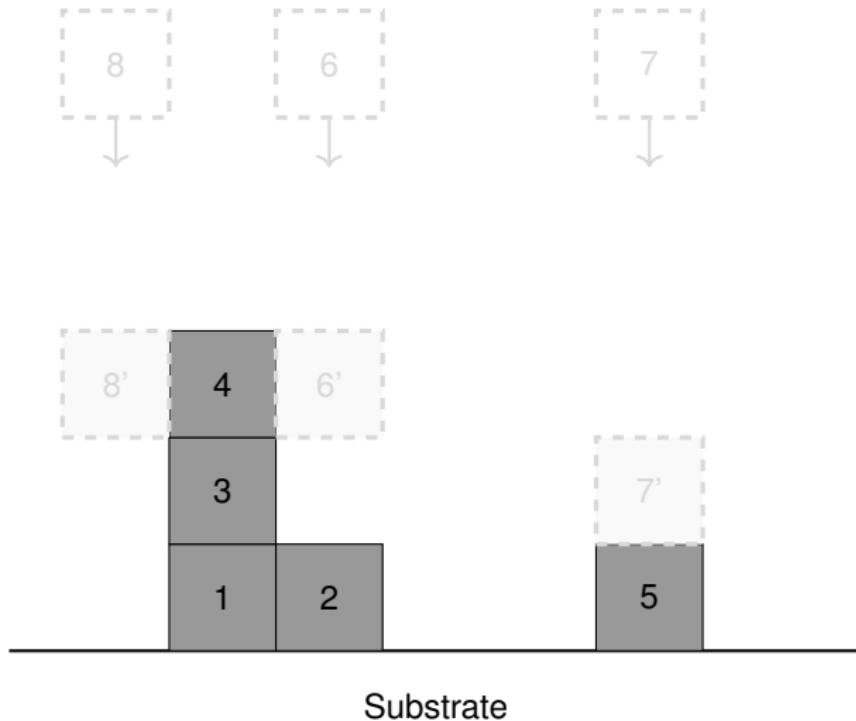
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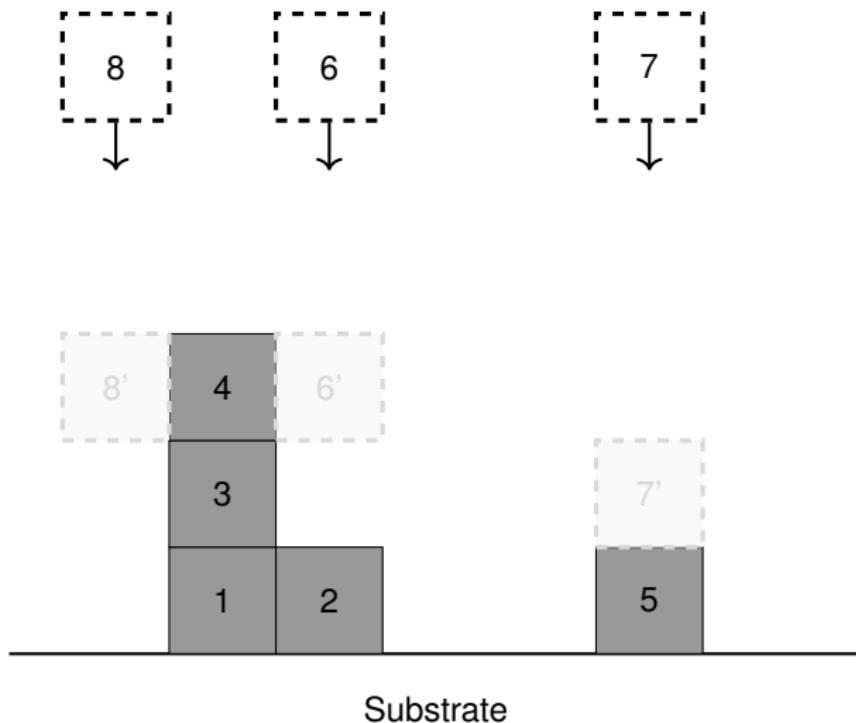
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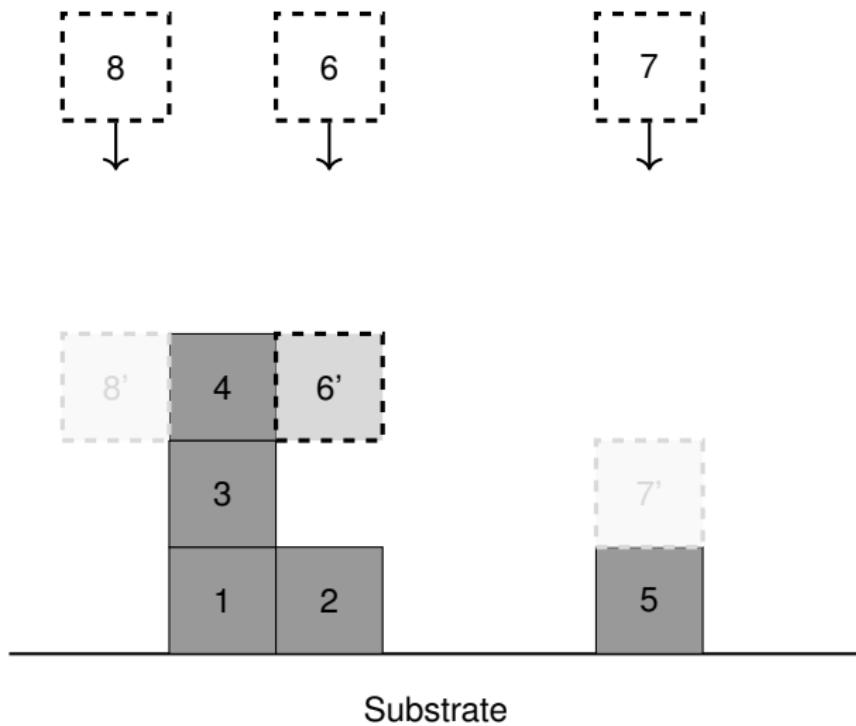
# Ballistic deposition



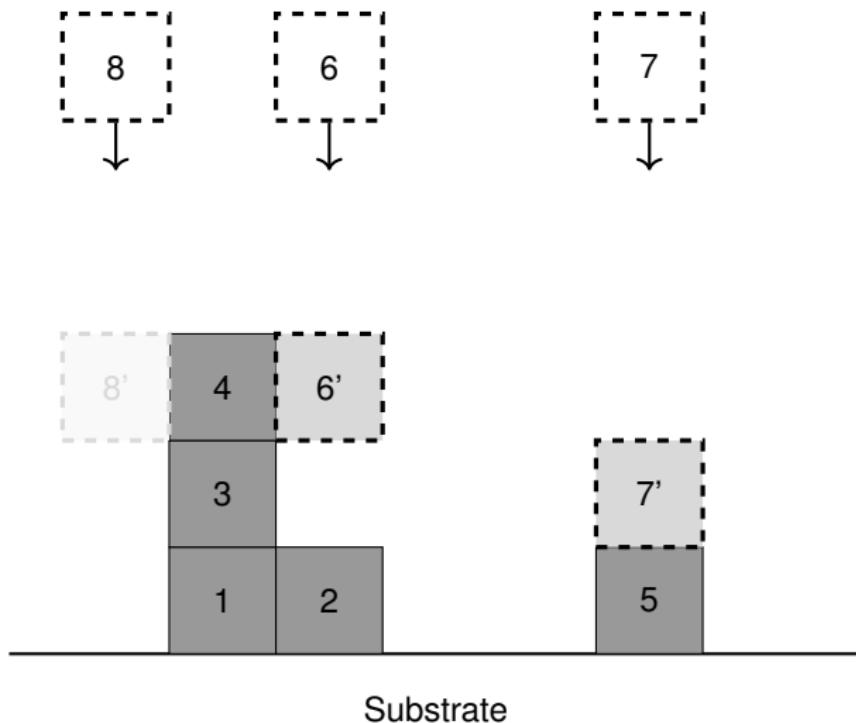
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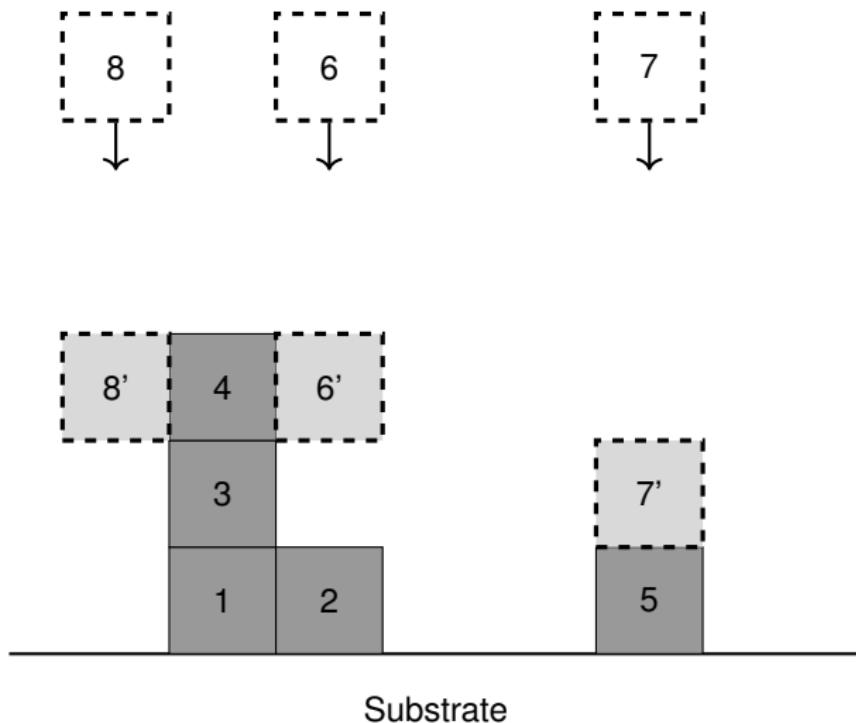
## Ballistic deposition



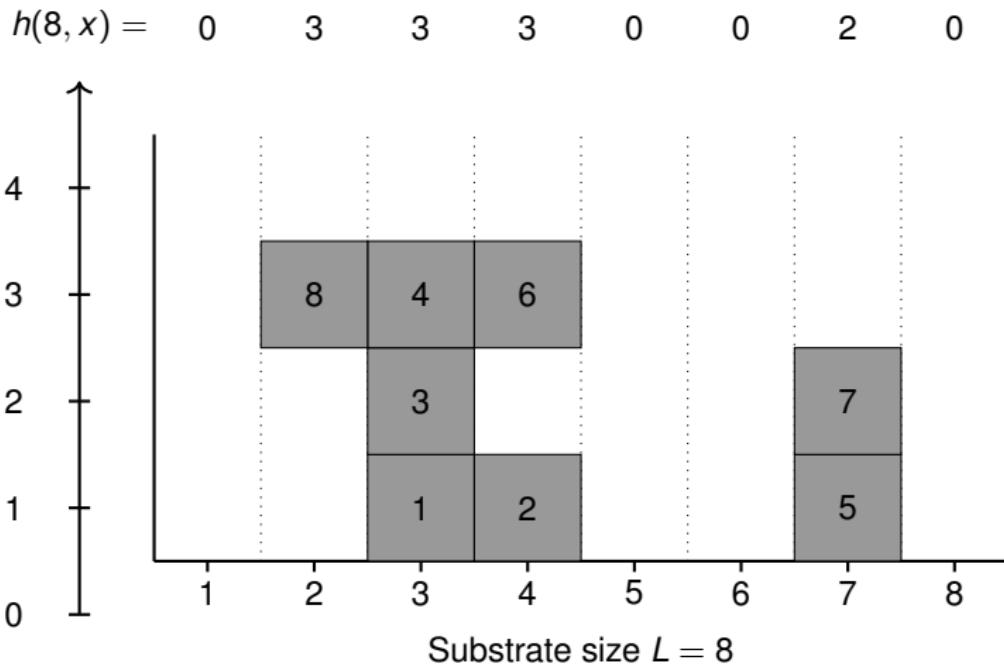
## Ballistic deposition



## Ballistic deposition



## Average height and fluctuation

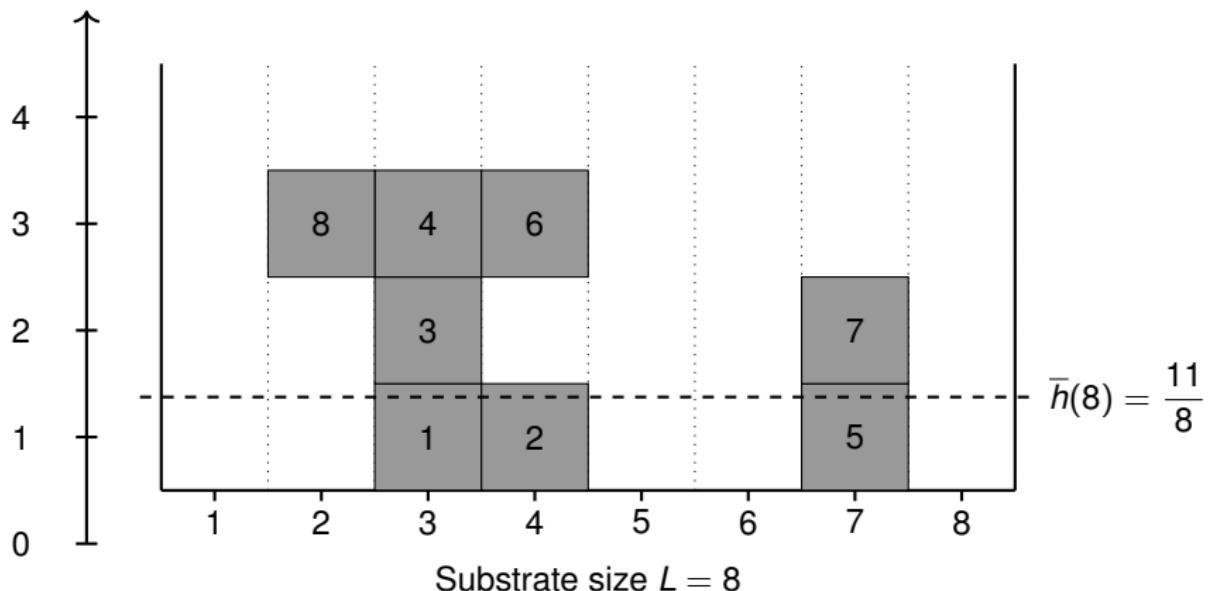


## Average height and fluctuation

$$\bar{h}(t) = \frac{1}{L} \sum_{x=1}^L h(t, x)$$

$$\text{Fluctuation } W(L, t) = \sqrt{\frac{1}{L} \sum_{x=1}^L [h(t, x) - \bar{h}(t)]^2}$$

$$h(8, x) = \begin{array}{cccccccc} 0 & 3 & 3 & 3 & 0 & 0 & 2 & 0 \end{array}$$

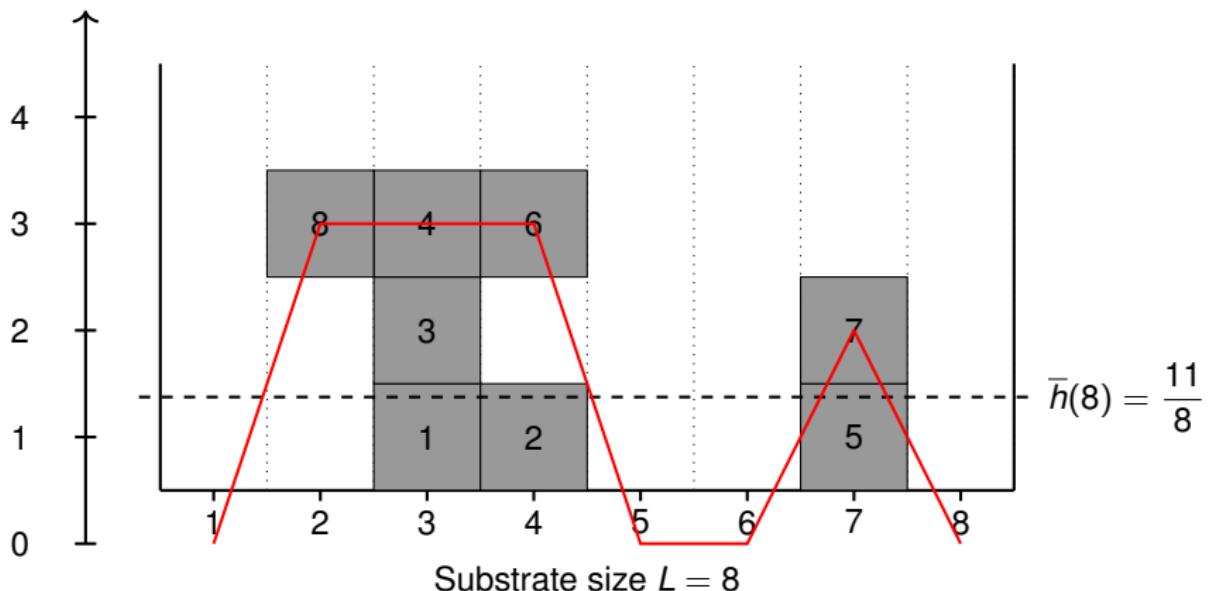


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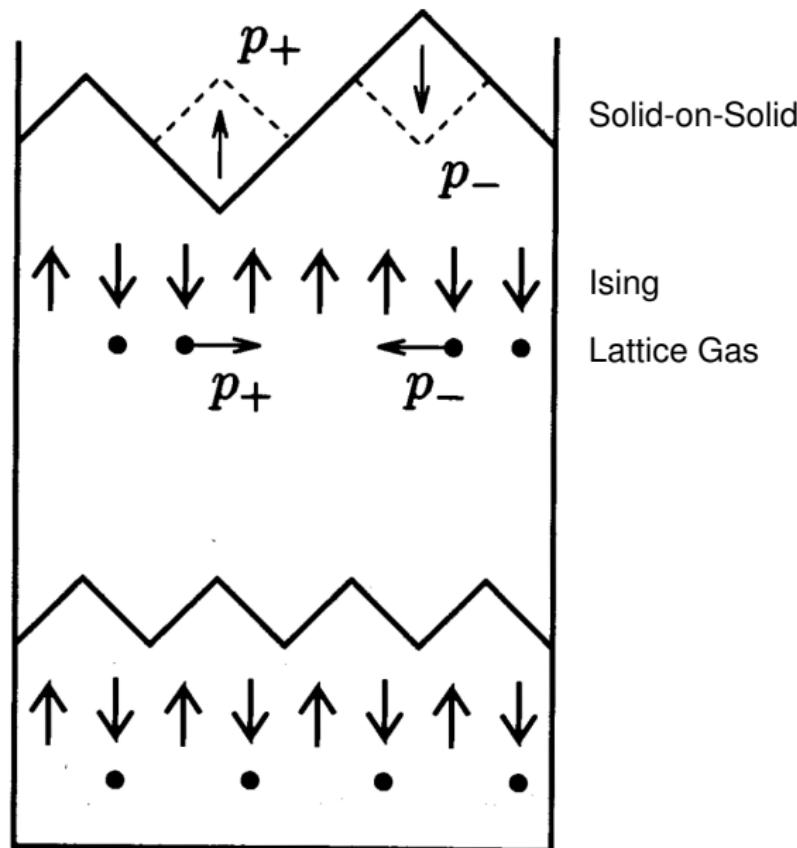
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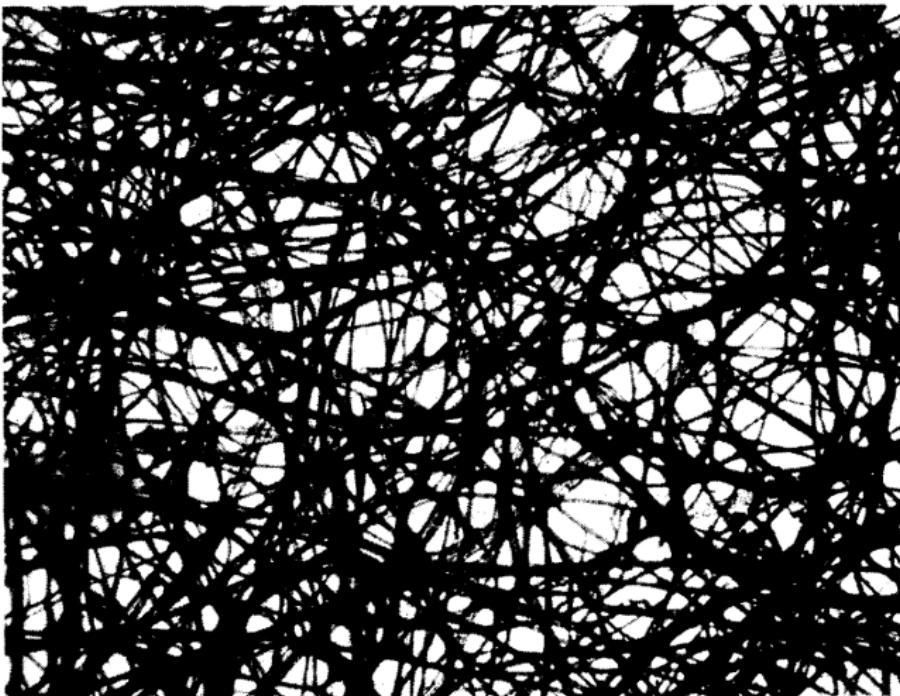


Simulations on  
Random deposition vs. Ballistic decomposition

## More models? Even more simpler?



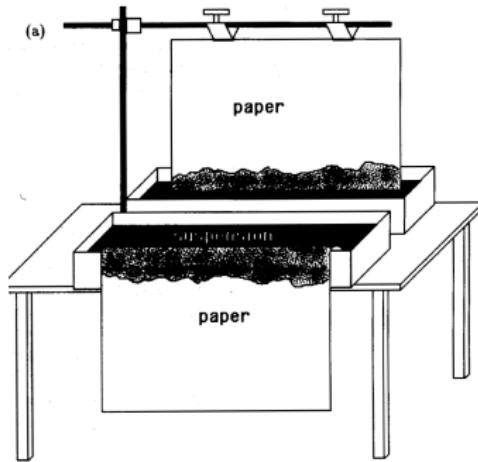
## Paper – a random environment



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Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

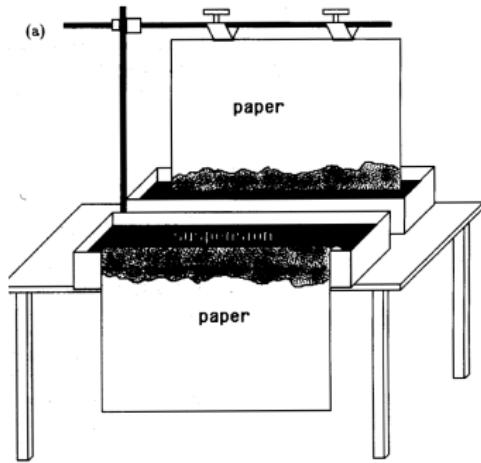
# Paper wetting experiment



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Barabási, A.-L., Stanley, H. E., 1995

# Paper wetting experiment



(b)



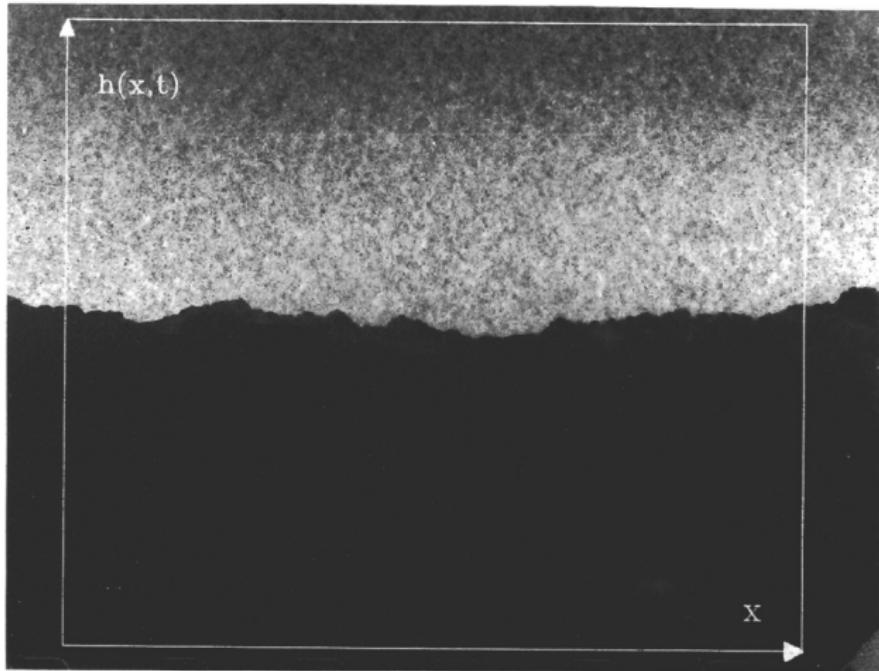
(c)



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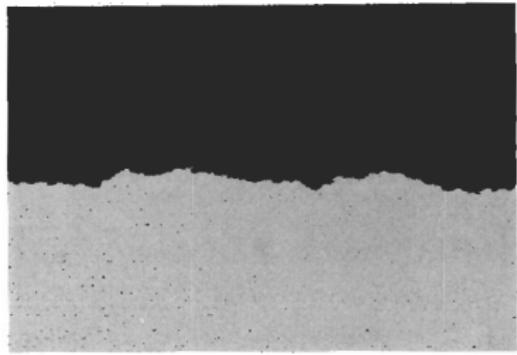
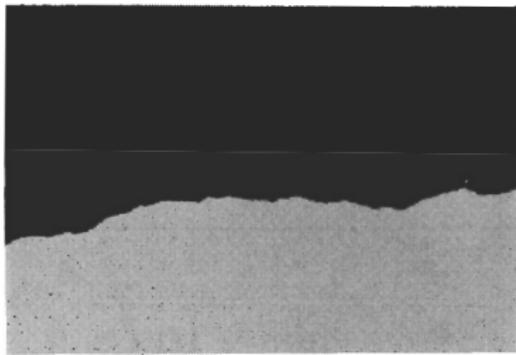
Barabási, A.-L., Stanley, H. E., 1995

# Paper burning experiment



Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

# Paper rupture experiment

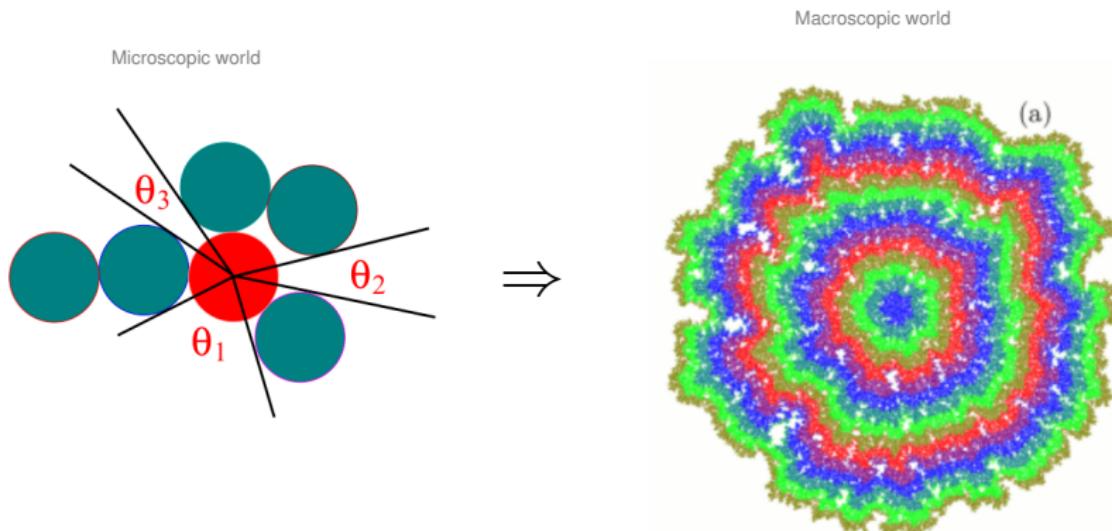


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Kertész, J., Horváth, V. k., Weber, F., *Fractals*, 1993

## Rule of replication of **cells**

Replication probability  $\propto$  Aperture angle  $\theta_i$



# Study of growing interfaces in a thin film

— Convection of nematic liquid crystal\*

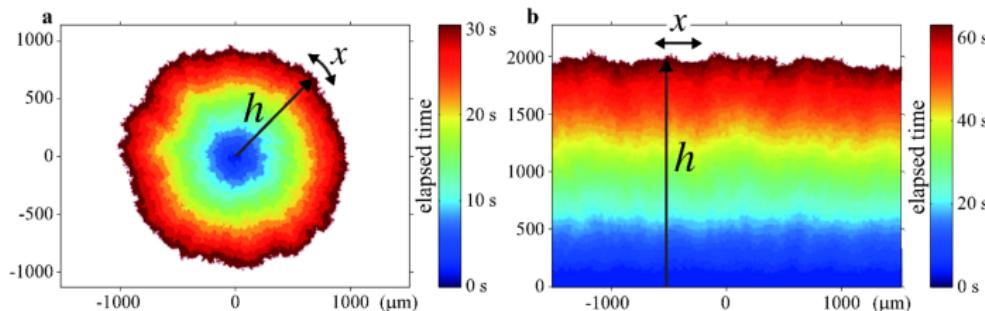
Show movies !

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Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

# Study of growing interfaces in a thin film

— Convection of nematic liquid crystal\*



Prediction from KPZ equation:

$$h \asymp v_\infty t + (\Gamma t)^{1/3} \xi$$

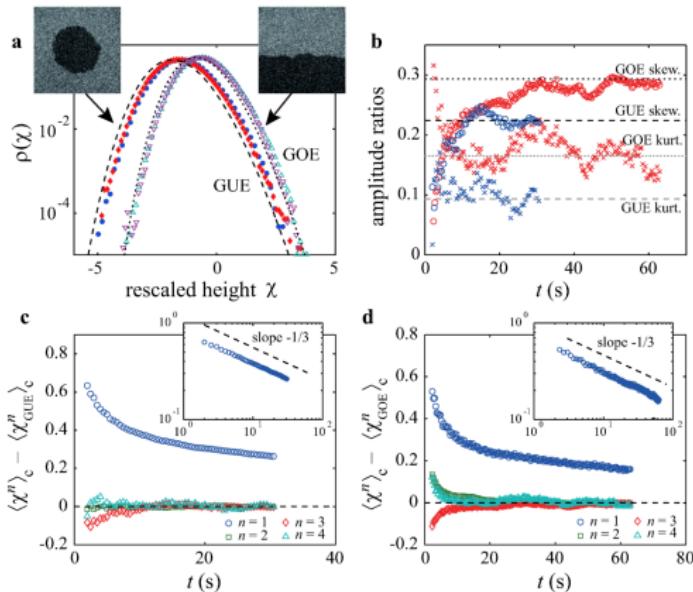
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Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

# Study of growing interfaces in a thin film

## — Convection of nematic liquid crystal\*

$$h \asymp v_\infty t + (\Gamma t)^{1/3} \xi$$



# KPZ Equation '86

$$\frac{\partial}{\partial t} h(t, x) = \frac{1}{2} \Delta h(t, x) + \frac{\lambda}{2} (\nabla h)^2 + \dot{W}(t, x) \quad (\text{KPZ})$$



Mehran Kardar (1957 –) Giorgio Parisi (1948 –)



Yicheng Zhang

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Kardar, M., Parisi, G., Zhang, Y.-C., *Phys. Rev. Lett.*, 1986

## Main References\*:

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- Zel'dovich, Y. B., Ruzmaikin, A. A., & Sokoloff, D. D. (1990). *The almighty chance* (Vol. 20) [Translated from the Russian by Anvar Shukurov]. World Scientific Publishing Co., Inc., River Edge, NJ.

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\* References are produced from *SPDEs-Bib*: <https://github.com/chenle02/SPDEs-Bib>

\* Download the bib file: <https://github.com/chenle02/SPDEs-Bib/blob/main/All.bib>