

# Analysis of Tetris Ballistic Deposition and the Robustness of the KPZ Universality Class

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Auburn University

Acknwolegement

*NSF 2246850, NSF 2443823, & Simons Foundation Travel Grant (2022-2027)*

Talk available at: [github.com/chenle02](https://github.com/chenle02)

Emerging Synergies between Stochastic Analysis and Statistical Mechanics  
Banff, Alberta, Canada  
October 28, 2025

# Integration of Research, Education, and Outreach

## Outreach

- ▶ Auburn Summer Science Institute (AU-SSI): **2024, 2025**  
*Selected talented high school students*
- ▶ Destination STEM: **2023, 2024**  
*Junior middle school students*

## Education

- ▶ Graduate Student Seminars (Mathematics), Auburn: **2022–2025**
- ▶ Math 7820/7830: Applied Stochastic Processes Course project, **2023/24**

## Research

- ▶ Simulation and modeling packages (open source)
- ▶ Forming conjectures and validating results

Most materials are available at

[github.com/chenle02](https://github.com/chenle02)

## Math 7820/30: Applied Stochastic Processes (2023/24):



Mauricio Montes and Ian Ruau

Simulation package:

[https://github.com/chenle02/Simulations\\_on\\_Some\\_Surface\\_Growth\\_Models](https://github.com/chenle02/Simulations_on_Some_Surface_Growth_Models)

```
pip install tetris-ballistic
```



Image is generated by OpenAI's *DALL-E*

Random deposition



Ballistic deposition

Image is generated by OpenAI's *DALL-E*

# Plan

Introduction to growth model and SPDE

Family-Vicsek scaling and experiments

More examples

Tetromino Pieces

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Introduction to growth model and SPDE

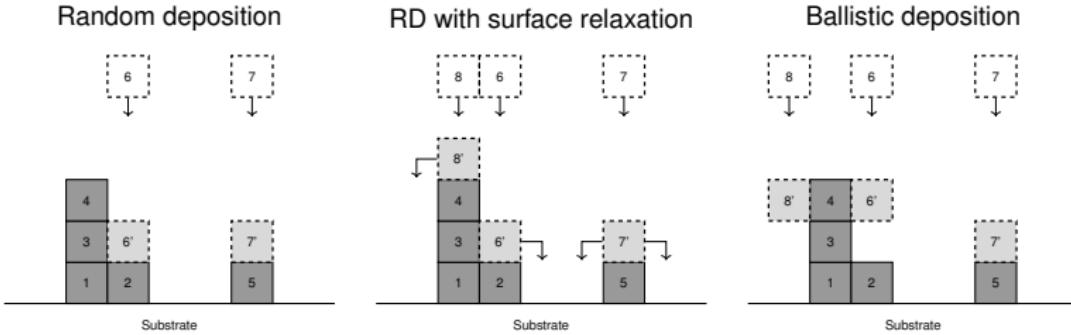
Family-Vicsek scaling and experiments

More examples

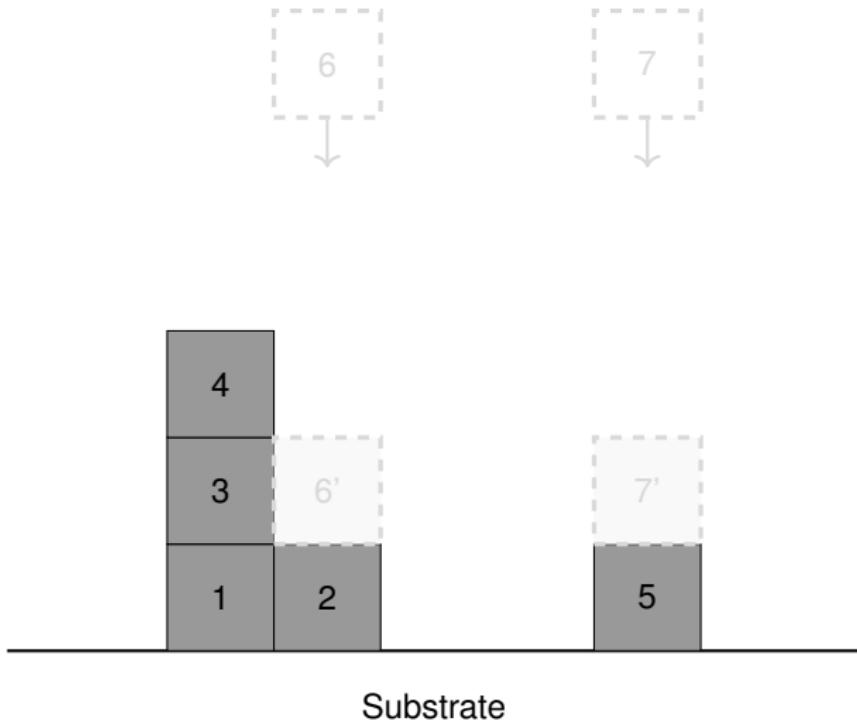
Tetromino Pieces

# How do surfaces grow?

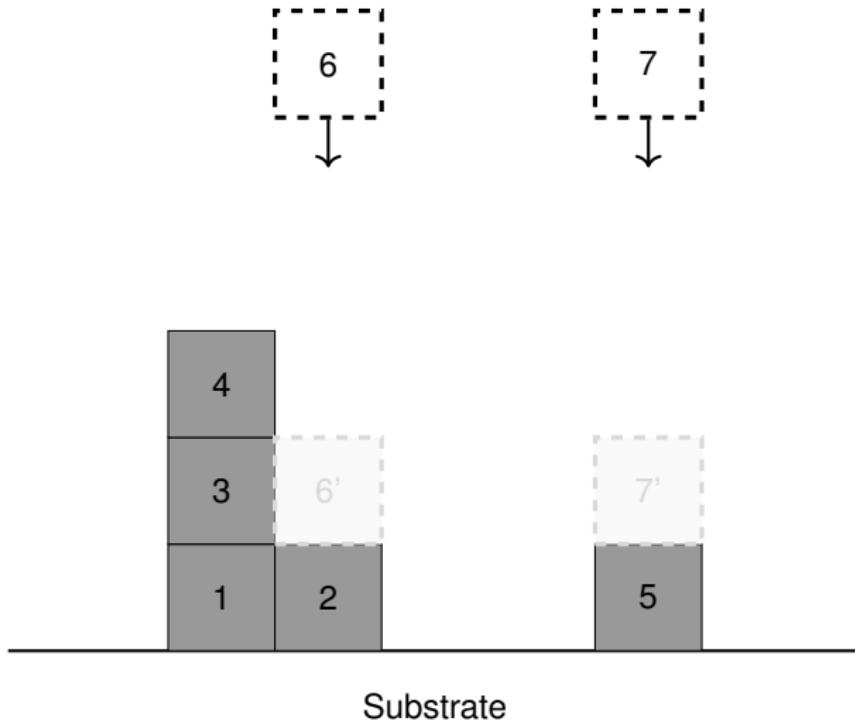
## Three models



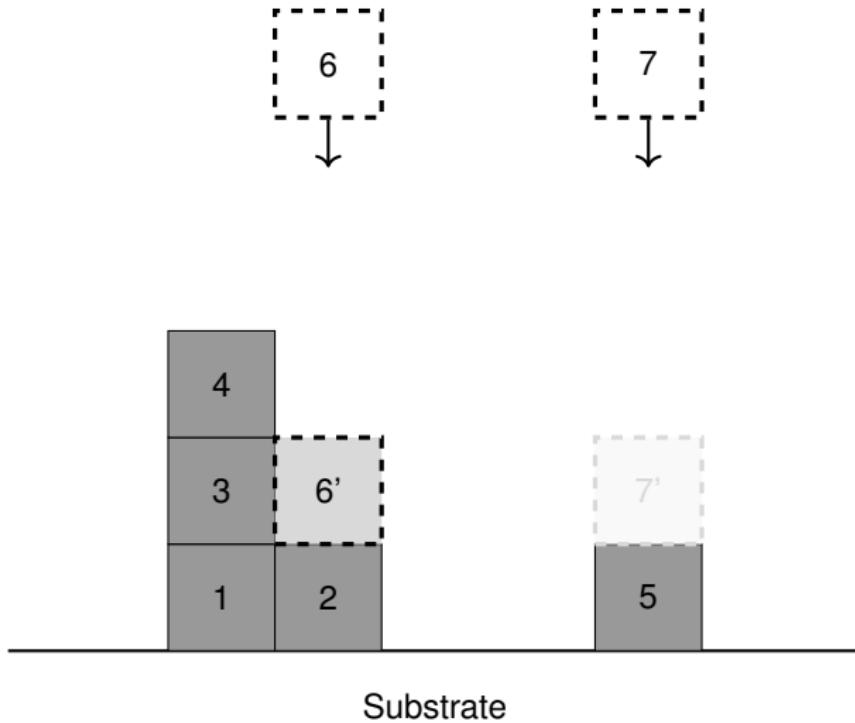
## Random deposition



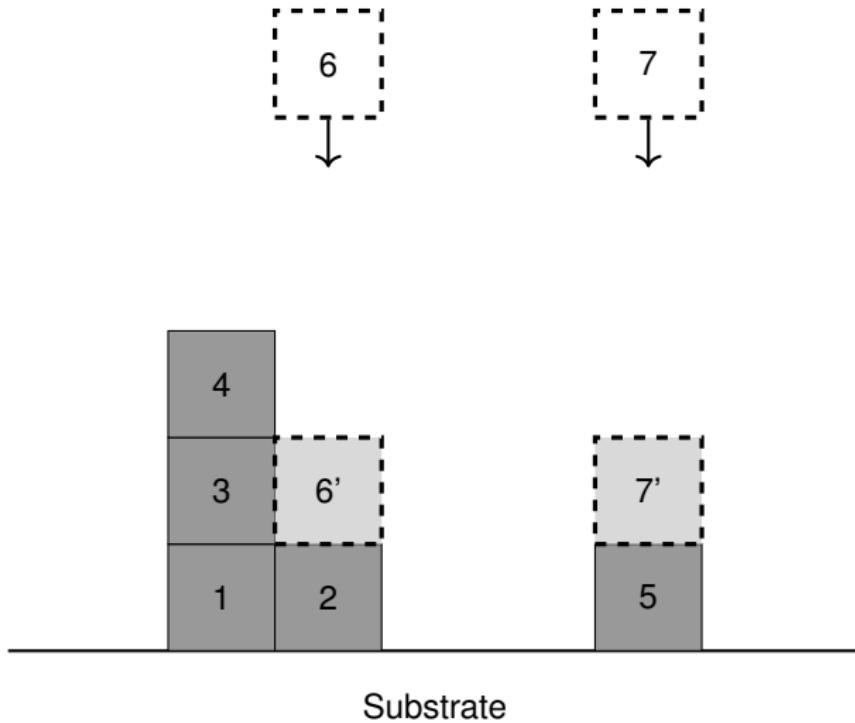
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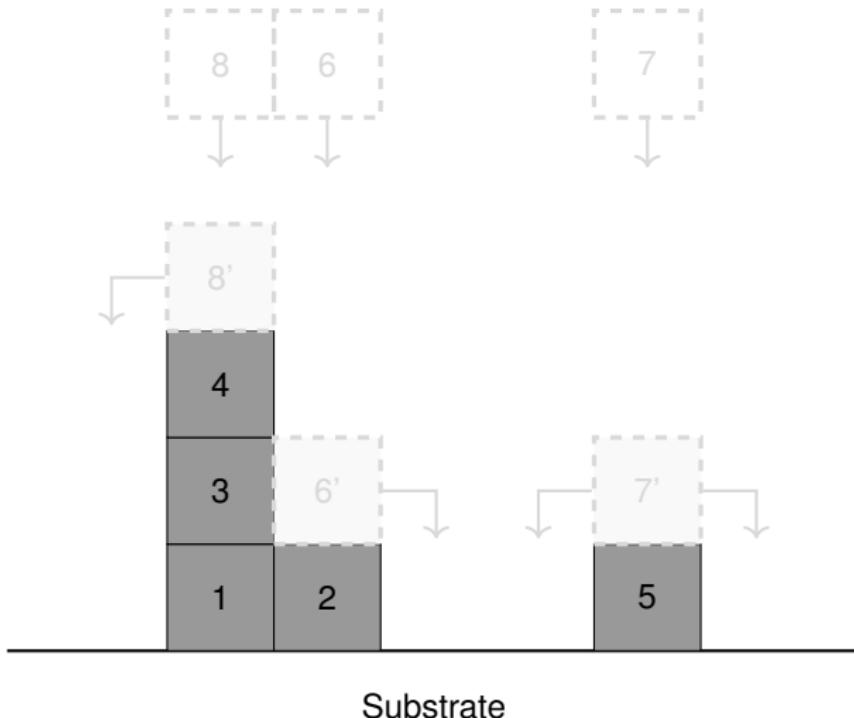
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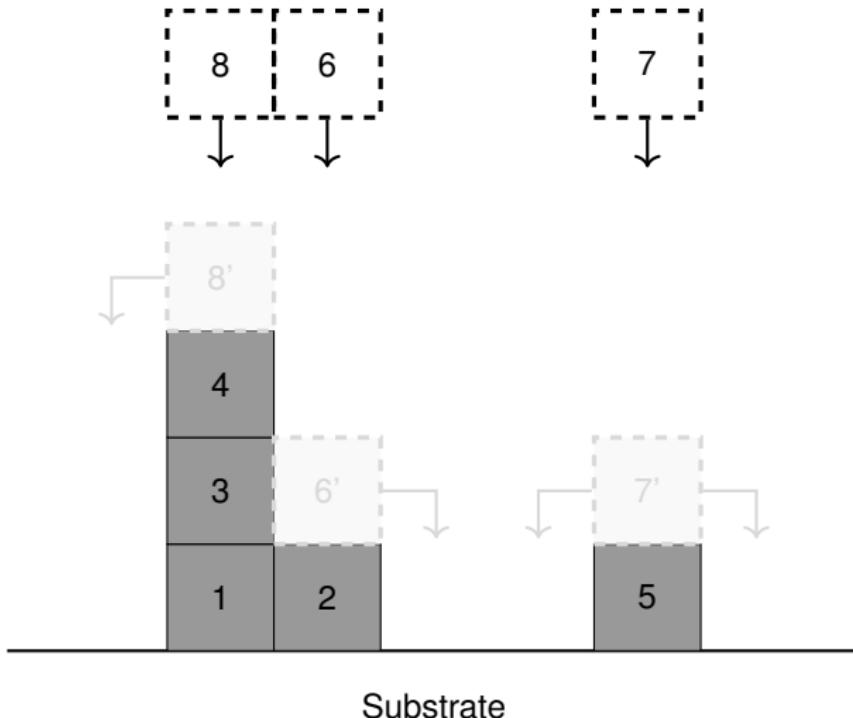
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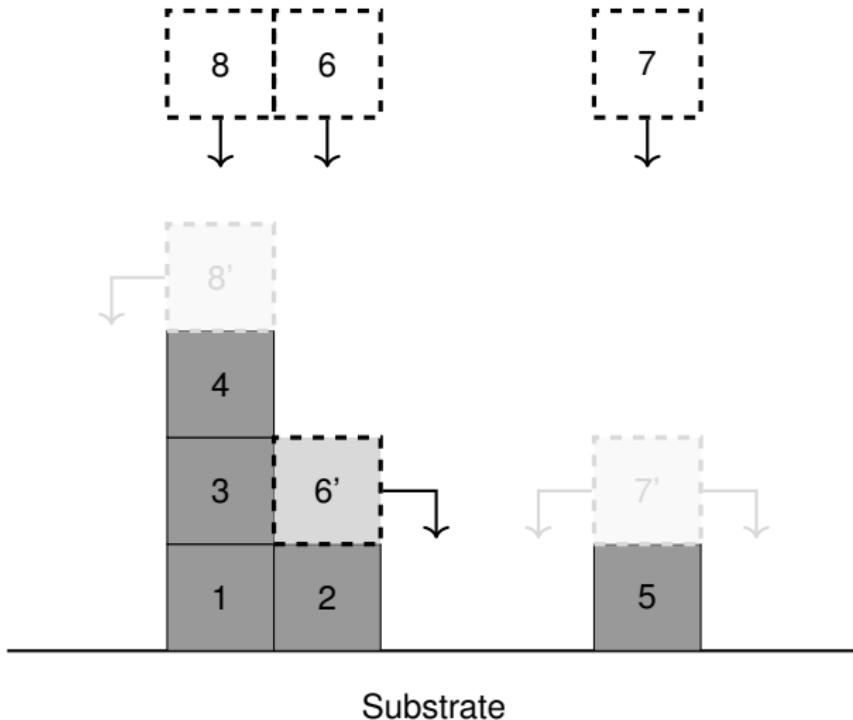
## Random deposition with surface relaxation



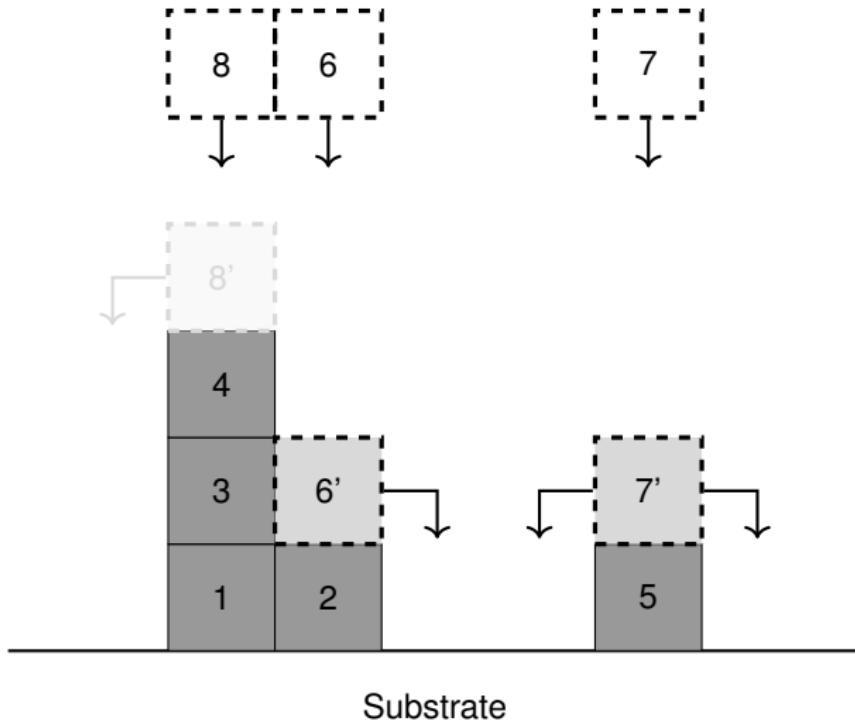
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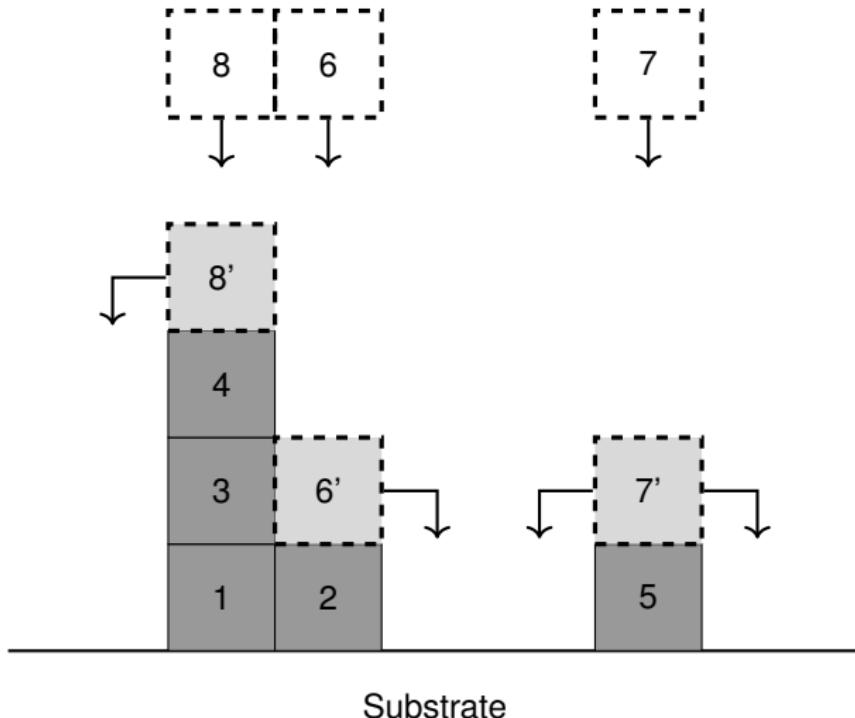
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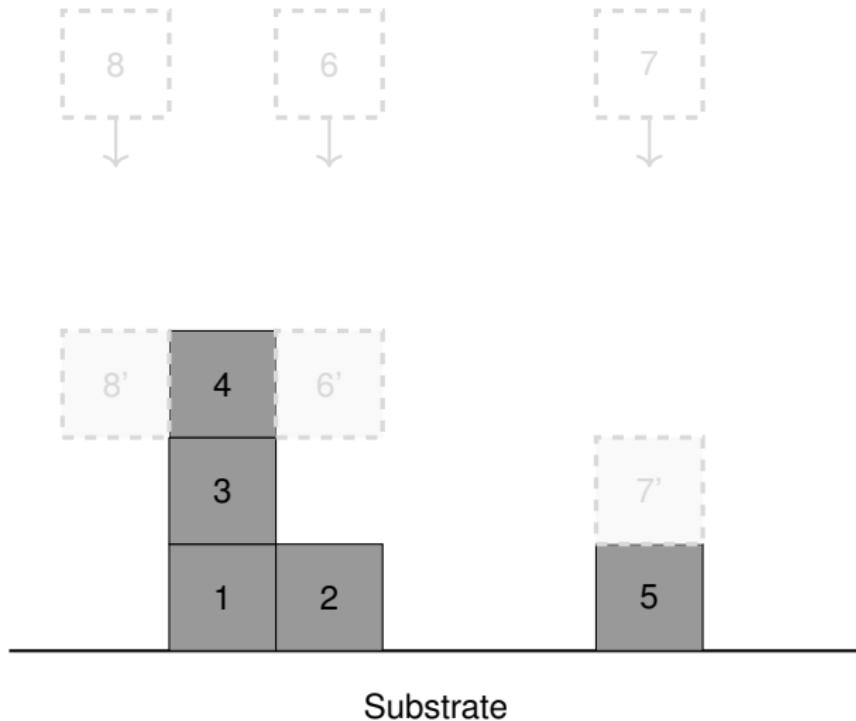
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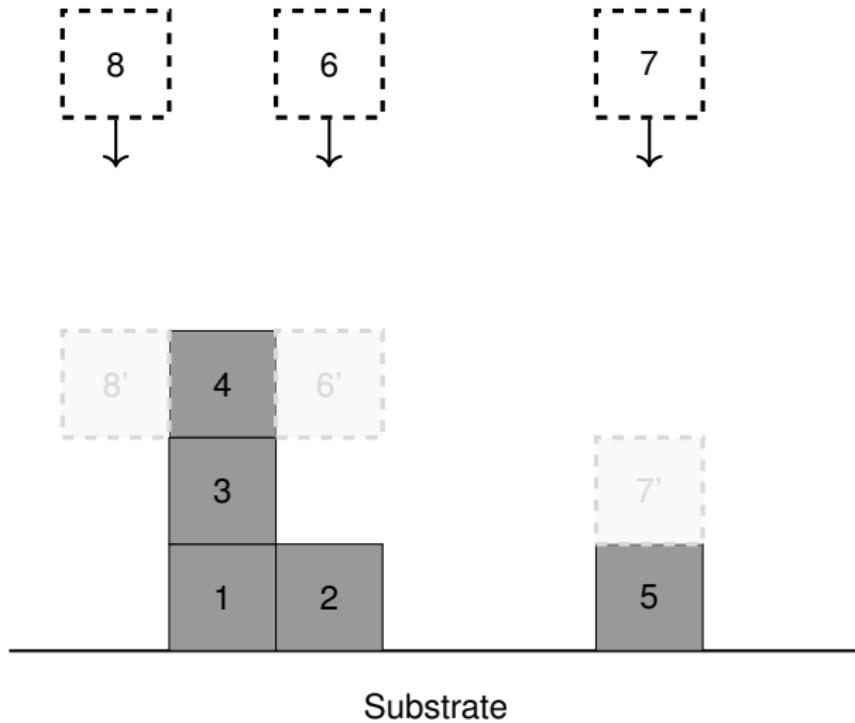
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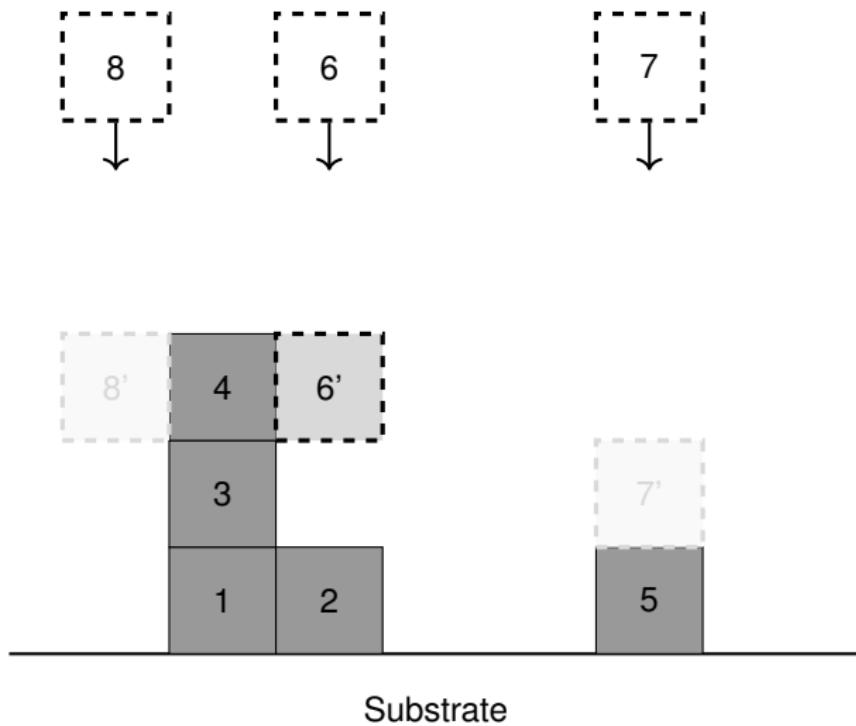
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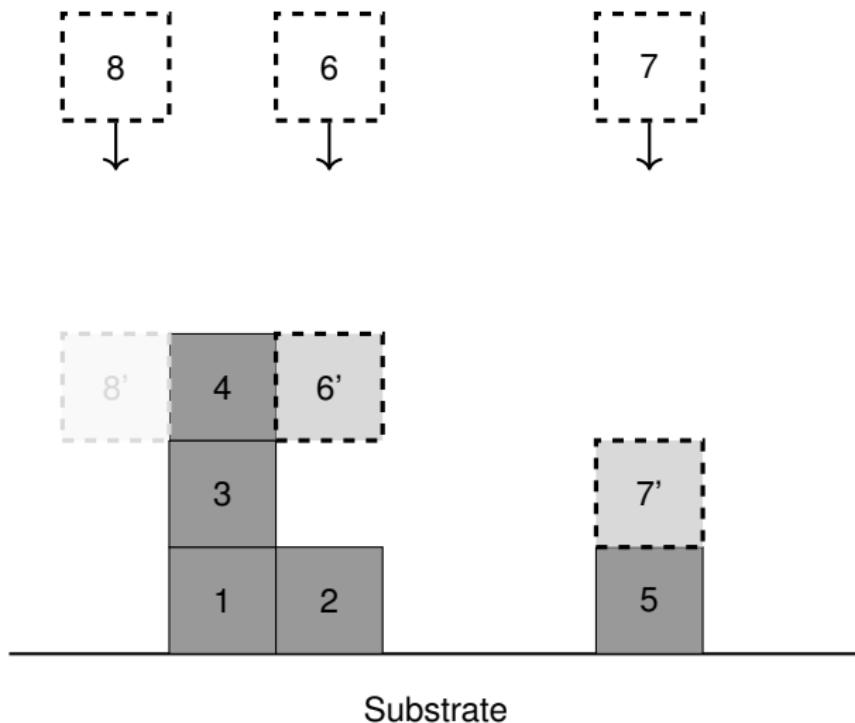
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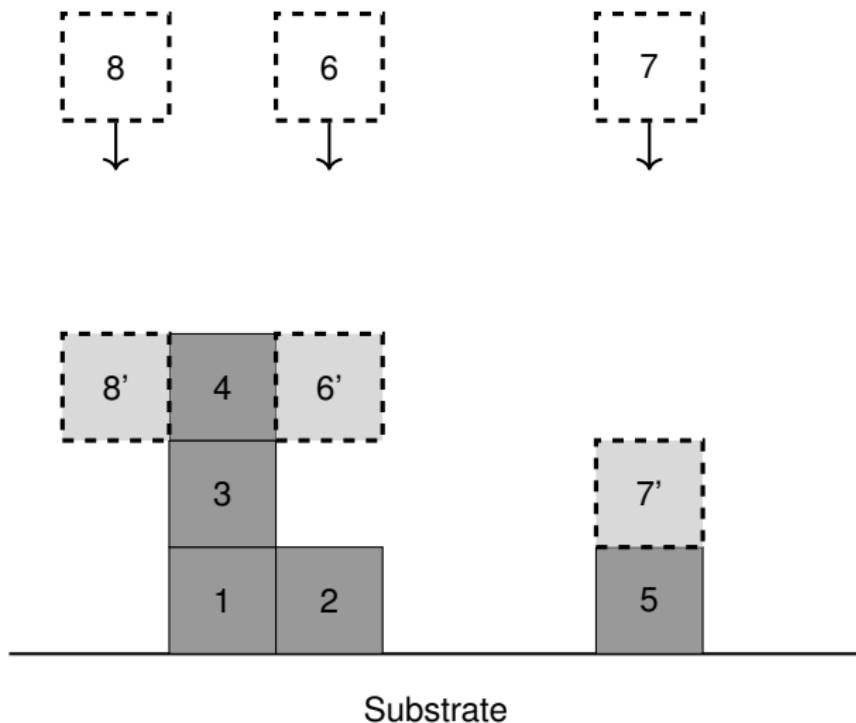
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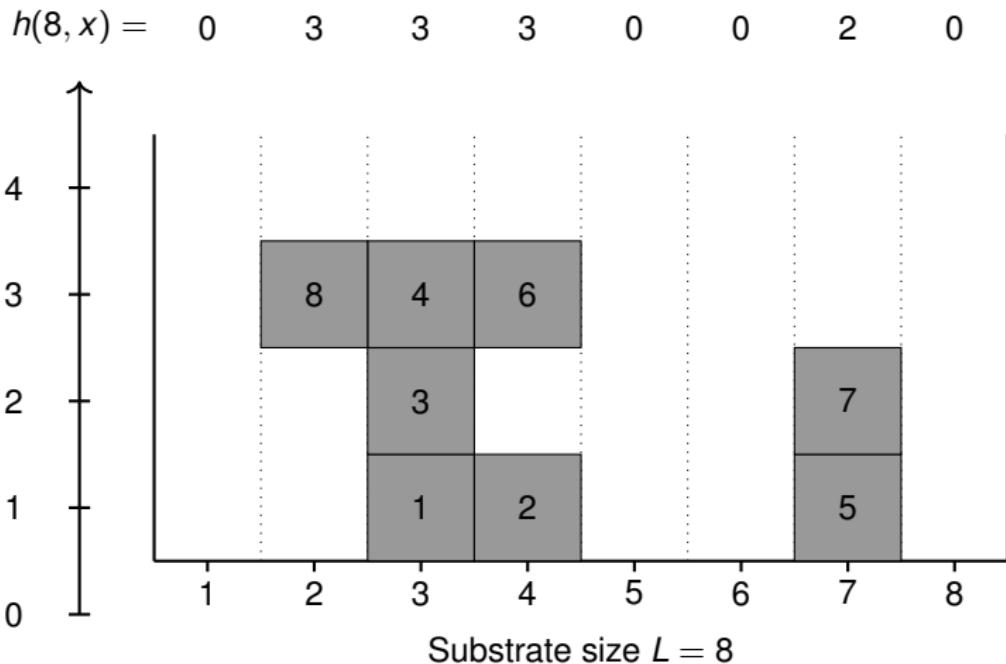
## Ballistic deposition



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## Average height and fluctuation

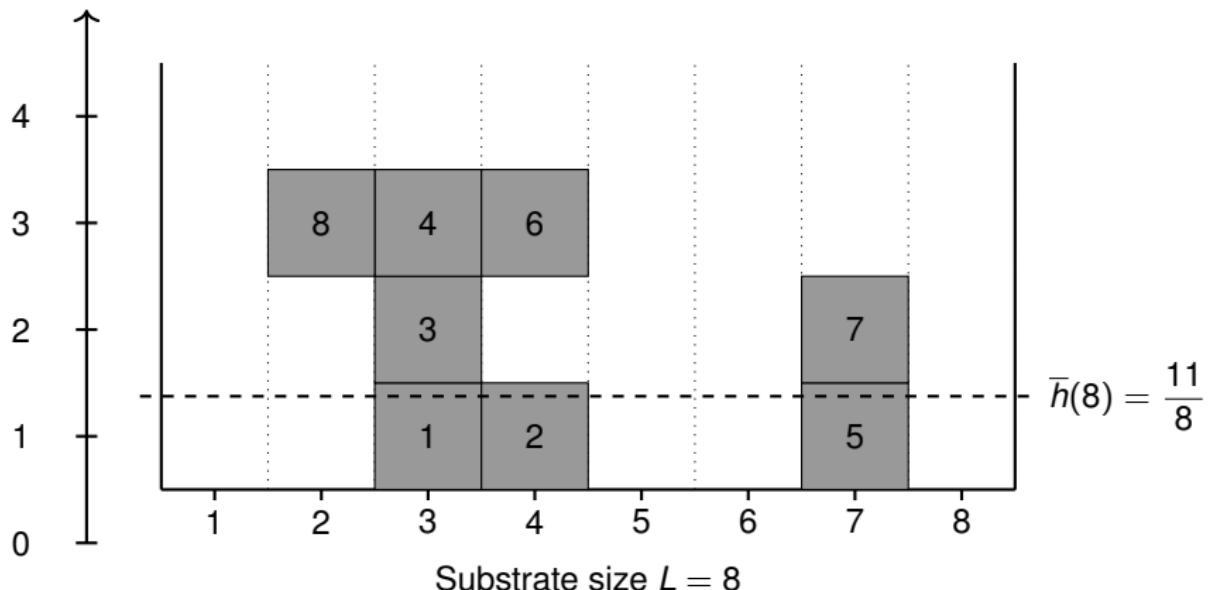


## Average height and fluctuation

$$\bar{h}(t) = \frac{1}{L} \sum_{x=1}^L h(t, x)$$

$$h(8, x) = \begin{array}{ccccc} 0 & 3 & 3 & 3 & 0 \end{array}$$

$$\text{Fluctuation } W(L, t) = \sqrt{\frac{1}{L} \sum_{x=1}^L [h(t, x) - \bar{h}(t)]^2}$$

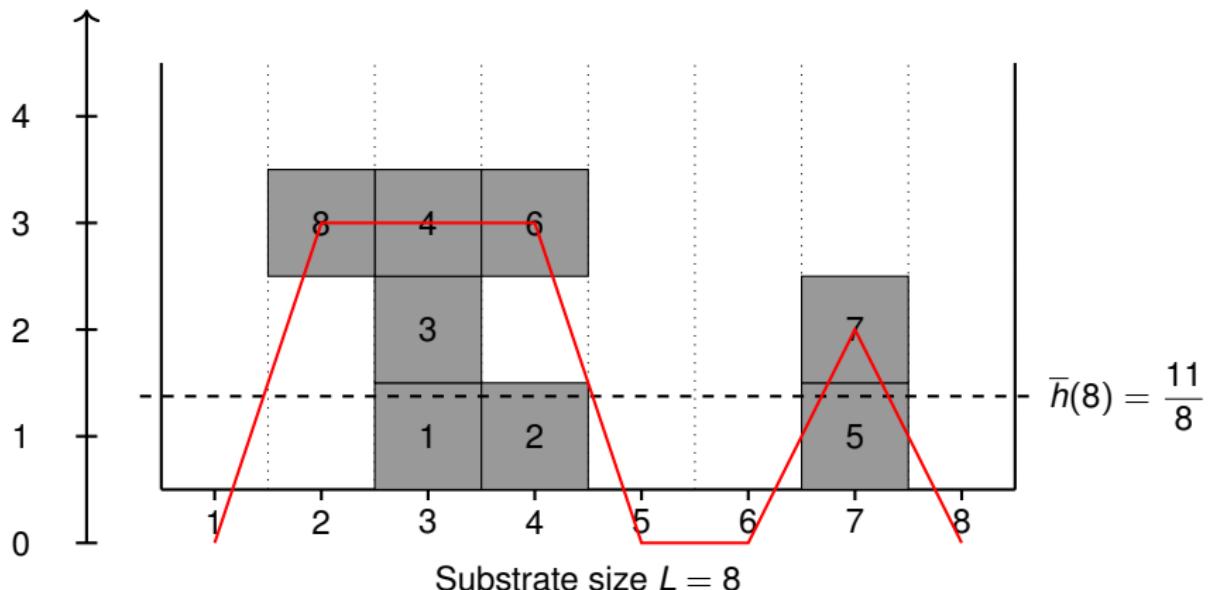


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## Random Deposition (independent columns, nonsticky)

**Model.**  $L$  independent columns. At each integer time  $t = 1, 2, \dots$ , drop *one* particle on a uniformly random column. Heights  $h(t, x)$ , mean  $\bar{h}(t) = \frac{1}{L} \sum_{x=1}^L h(t, x) = \frac{t}{L}$ , width

$$W^2(L, t) = \frac{1}{L} \sum_{x=1}^L (h(t, x) - \bar{h}(t))^2.$$

**Single-column law:** After  $t$  drops total,

$$h(t, x) \sim \text{Binomial}\left(t, \frac{1}{L}\right), \quad \mathbb{E}[h(t, x)] = \frac{t}{L}, \quad \text{Var}(h(t, x)) = t \frac{1}{L} \left(1 - \frac{1}{L}\right).$$

**Fluctuation:** By i.i.d. columns,

$$\mathbb{E}[W^2(L, t)] = \frac{1}{L} \sum_{x=1}^L \mathbb{E}[h(t, x)^2] - \mathbb{E}[\bar{h}^2(t)] = \mathbb{E}[h(t, 1)^2] - \left(\frac{t}{L}\right)^2 = \left(1 - \frac{1}{L}\right) \text{Var}(h(t, 1)).$$

Hence

$$\boxed{\mathbb{E}[W^2(L, t)] = \left(1 - \frac{1}{L}\right) t \frac{1}{L} \left(1 - \frac{1}{L}\right) = \frac{t}{L} \left(1 - \frac{1}{L}\right)^2}$$

and

$$\boxed{W(L, t) \simeq \left(1 - \frac{1}{L}\right) \left(\frac{t}{L}\right)^{1/2}}$$

**Scaling.** Growth exponent  $\beta = \frac{1}{2}$ .

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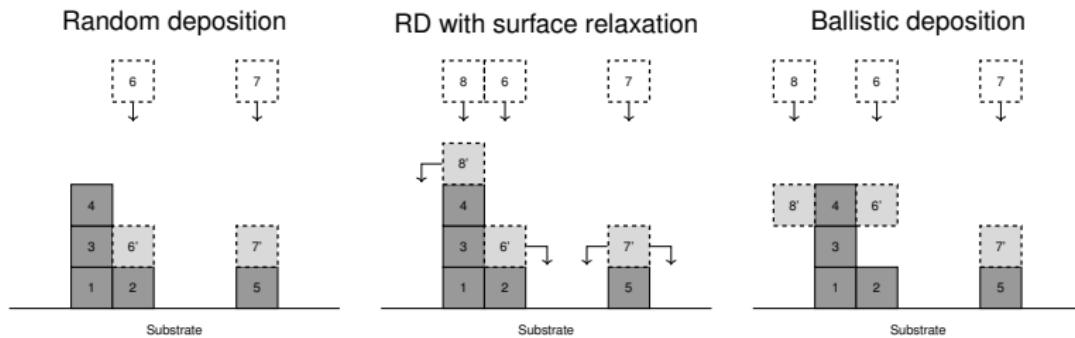
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# Questions

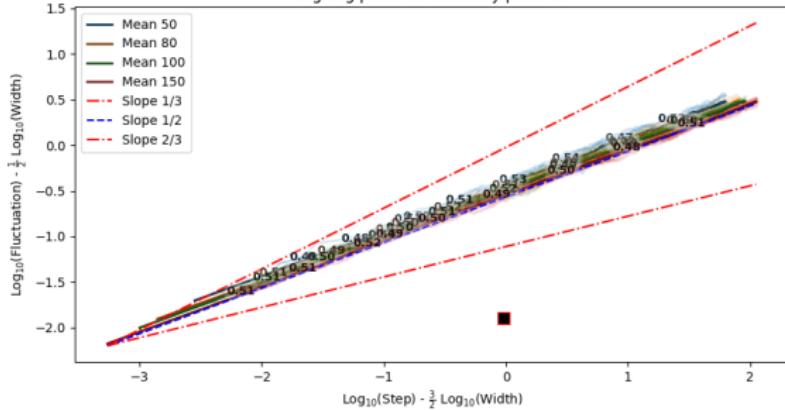


$$W(t, x) \sim t^{1/2}$$

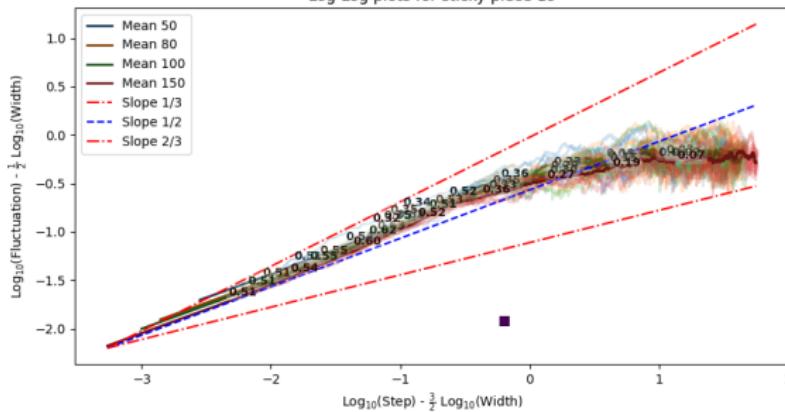
$$W(t, x) \sim t^{??}$$

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Log-Log plots for nonsticky piece 19



Log-Log plots for sticky piece 19



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## Family–Vicsek scaling theory

Discrete interface with heights  $h(x, t)$ ,  $x = 1, \dots, L$ . Mean  $\bar{h}(t) = \frac{1}{L} \sum_{x=1}^L h(x, t)$ .

$$w(L, t) := \sqrt{\frac{1}{L} \sum_{x=1}^L (h(x, t) - \bar{h}(t))^2} \quad (\text{interface width / roughness}).$$

Empirically (log–log):

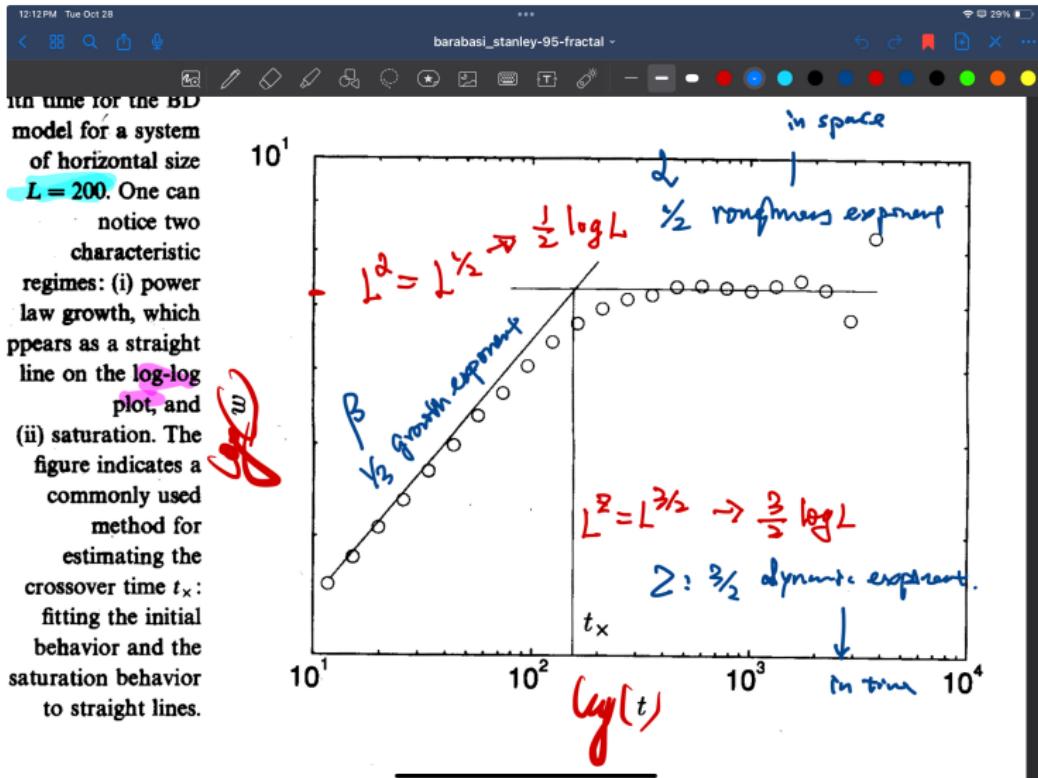
- ▶ Early time:  $w(L, t) \sim t^\beta$  (growth exponent  $\beta$ ).
- ▶ Late time:  $w(L, t) \rightarrow w_{\text{sat}}(L) \sim L^\alpha$  (roughness exponent  $\alpha$ ).
- ▶ Crossover time:  $t_x(L) \sim L^z$  (dynamic exponent  $z$ ).

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Family, F., Vicsek, T., *Journal of Physics A: Mathematical and General*, 1985

# Family-Vicsek scaling theory

$\alpha$ : Roughness exp.;  $\beta$ : Growth exp.;  $z$ : Dynamic exp.



(Image from Barabási-Stanley's book 95)

# Family–Vicsek scaling theory

$\alpha$ : Roughness exp.;  $\beta$ : Growth exp.;  $z$ : Dynamic exp.

$$w(L, t) \sim L^\alpha f\left(\frac{t}{L^z}\right)$$

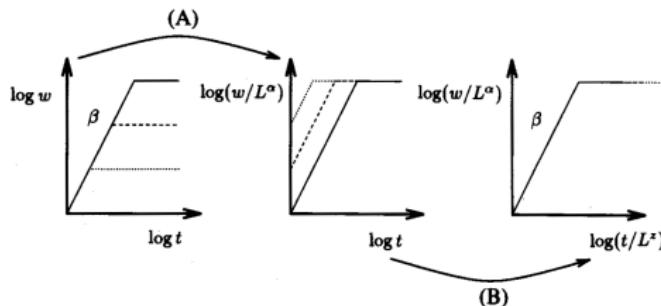
with

$$f(u) \sim \begin{cases} u^\beta, & u \ll 1, \\ \text{const}, & u \gg 1. \end{cases} \Rightarrow \beta = \frac{\alpha}{z}$$

Immediate consequences:

$$w_{\text{sat}}(L) \sim L^\alpha, \quad t_x(L) \sim L^z.$$

Interpretation: dynamic renormalization  $x \rightarrow bx$ ,  $t \rightarrow b^z t$ ,  $h \rightarrow b^\alpha h$  leaves  $w(L, t)/L^\alpha$  invariant as a function of  $t/L^z$ .



(Image from Barabási-Stanley's book 95)

Family and Vicsek reported from their simulation that

$$\beta = 0.30 \pm 0.02 \approx \frac{1}{3}.$$

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Family, F., Vicsek, T., *Journal of Physics A: Mathematical and General*, 1985

## Continuum viewpoint: EW and KPZ

**EW (linear diffusion + noise):**

$$\partial_t h = \nu \nabla^2 h + \eta, \quad \langle \eta \eta \rangle \propto \delta(x) \delta(t).$$

Scale invariance gives  $z = 2$ ,  $\alpha = \frac{1}{2}$  in 1D  $\Rightarrow \beta = \frac{1}{4}$ .

**KPZ (nonlinear growth):**

$$\partial_t h = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta.$$

Galilean/tilt invariance  $\Rightarrow \alpha + z = 2$ . In 1D (exact):

$$\boxed{\alpha = \frac{1}{2}, \quad z = \frac{3}{2}, \quad \beta = \frac{1}{3}.}$$

Many lattice models (BD, RSOS, Eden) fall into KPZ in 1+1D.

## Reading plots (as in Barabási–Stanley)

- ▶ On  $w$  vs.  $t$  (log–log) for several  $L$ : early slopes  $\approx \beta$ ; plateaus  $\propto L^\alpha$ ; horizontal shift of crossovers  $\propto L^z$ .
- ▶ After rescaling: plot  $w/L^\alpha$  vs.  $t/L^z$ ; all curves collapse to  $f(u)$  with  $f(u) \sim u^\beta$  ( $u \ll 1$ ) and  $f(u) \rightarrow \text{const}$  ( $u \gg 1$ ).

Rule of thumb (1D):

$$\text{EW: } (\alpha, \beta, z) = \left(\frac{1}{2}, \frac{1}{4}, 2\right), \quad \text{KPZ/BD: } (\alpha, \beta, z) = \left(\frac{1}{2}, \frac{1}{3}, \frac{3}{2}\right).$$

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## Exactly solvable baseline: Random Deposition (RD)

One particle per step, random column; after  $t$  steps heights are multinomial:

$$\mathbb{E}[w^2(L, t)] = \frac{t}{L} \left(1 - \frac{1}{L}\right) \sim \frac{t}{L}.$$

Hence  $w \sim (t/L)^{1/2}$  ( $\beta = \frac{1}{2}$ ), *no lateral correlations*, no finite  $z$  and no true saturation. RD serves as a growth-only sanity check; FV scaling requires correlated dynamics (EW/KPZ).

## Structure factor (optional but handy)

Fourier modes  $h_k(t)$ , structure factor  $S(k, t) = \langle |h_k(t)|^2 \rangle$  obey

$$S(k, t) \sim k^{-(2\alpha+1)} g(k^z t), \quad g(y) \sim \begin{cases} y^{2\beta+1} & y \ll 1, \\ \text{const} & y \gg 1. \end{cases}$$

Useful when  $w(L, t)$  is noisy; fit  $(\alpha, z)$  from spectral slopes and collapse  $S(k, t)$ .

## Cheat sheet / takeaways

$$w(L, t) \sim L^\alpha f\left(\frac{t}{L^z}\right), \quad f(u) \sim u^\beta \text{ (} u \ll 1 \text{), } f(u) \rightarrow \text{const (} u \gg 1 \text{), } \beta = \alpha/z$$

1D benchmarks: EW ( $\alpha, \beta, z$ ) = ( $\frac{1}{2}, \frac{1}{4}, 2$ ); KPZ/BD ( $\frac{1}{2}, \frac{1}{3}, \frac{3}{2}$ ).

**Workflow:** measure  $w(L, t)$   $\Rightarrow$  fit  $\alpha$  and  $\beta$   $\Rightarrow$  infer/fit  $z$   $\Rightarrow$  collapse  $w/L^\alpha$  vs.  $t/L^z$ .

Simulations on  
Random deposition vs. Ballistic decomposition

# Study of growing interfaces in a thin film

— Convection of nematic liquid crystal\*

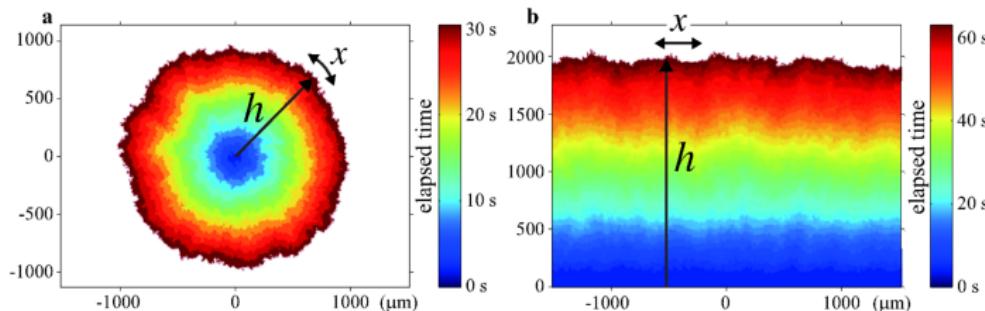
Show movies !

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Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

# Study of growing interfaces in a thin film

— Convection of nematic liquid crystal\*



Prediction from KPZ equation:

$$h \asymp v_\infty t + (\Gamma t)^{1/3} \xi$$

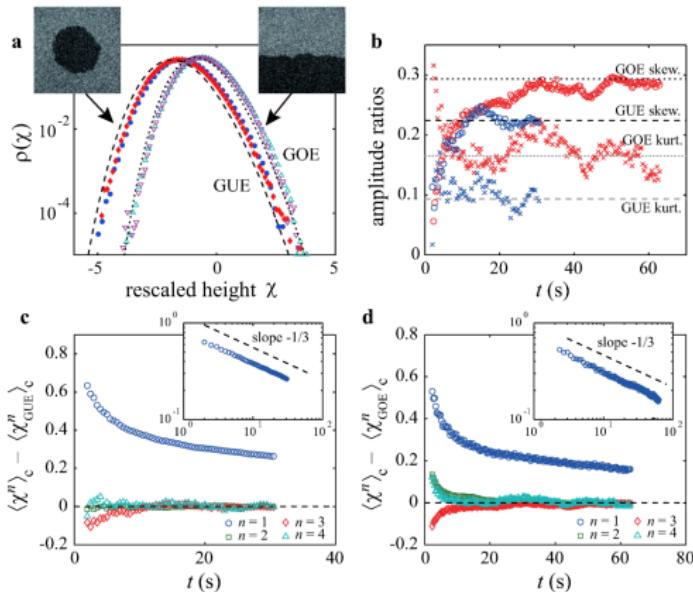
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Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

# Study of growing interfaces in a thin film

## — Convection of nematic liquid crystal\*

$$h \asymp v_\infty t + (\Gamma t)^{1/3} \xi$$



# KPZ Equation '86

$$\frac{\partial}{\partial t} h(t, x) = \frac{1}{2} \Delta h(t, x) + \frac{\lambda}{2} (\nabla h)^2 + \dot{W}(t, x) \quad (\text{KPZ})$$



Mehran Kardar (1957 –) Giorgio Parisi (1948 –)



Yicheng Zhang

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Kardar, M., Parisi, G., Zhang, Y.-C., *Phys. Rev. Lett.*, 1986

# Plan

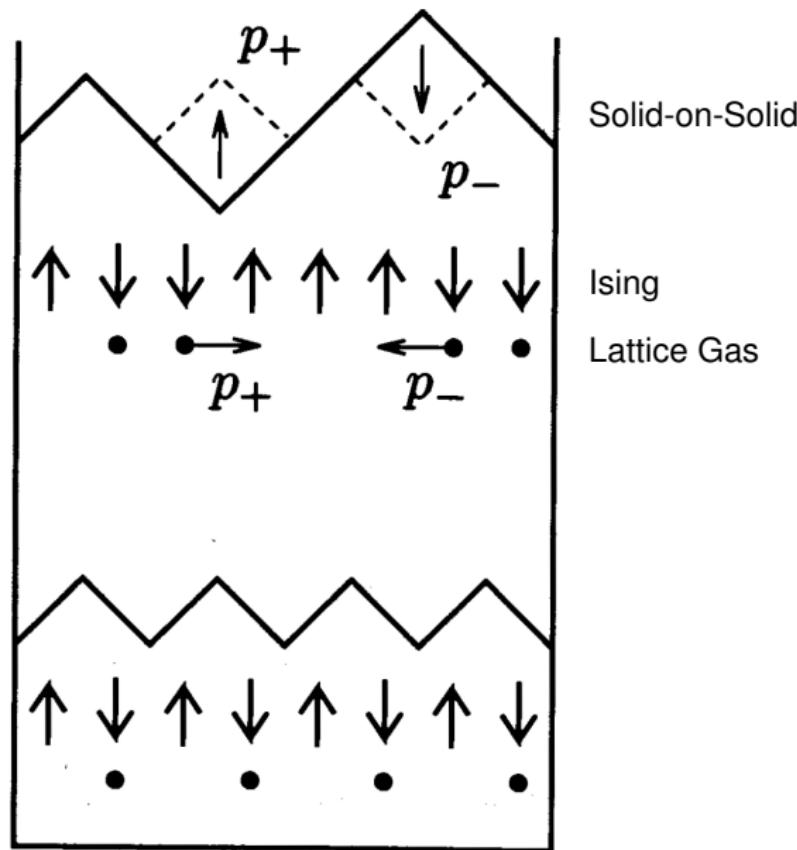
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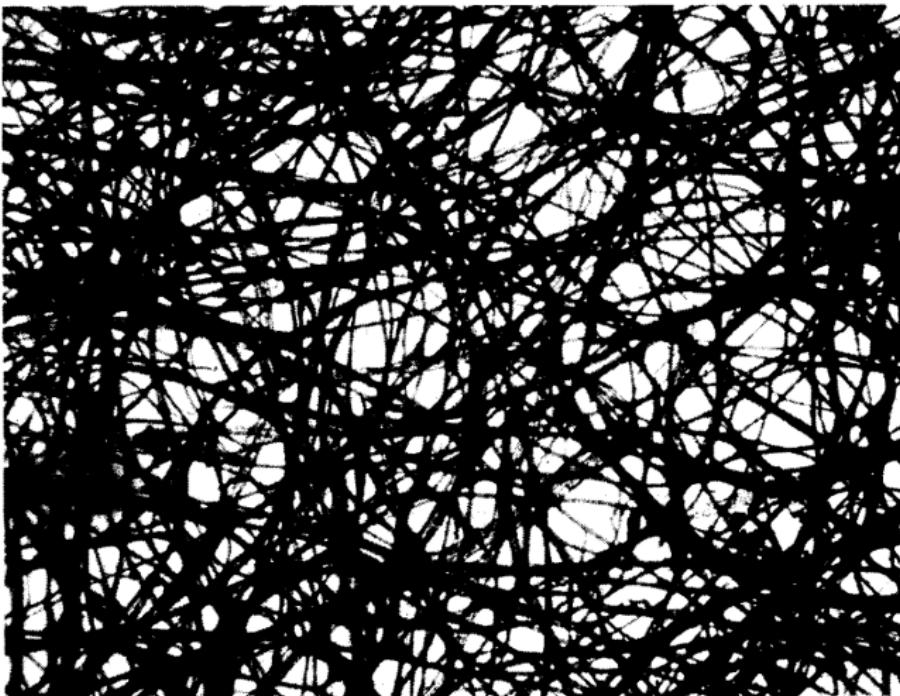
More examples

Tetromino Pieces

## More models? Even more simpler?



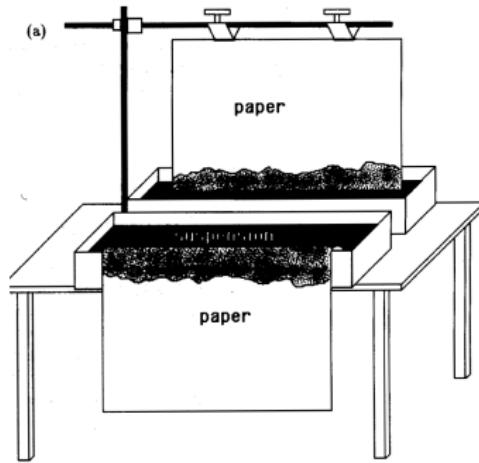
## Paper – a random environment



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Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

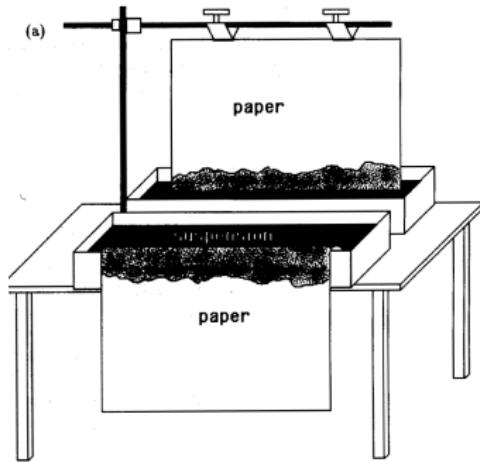
# Paper wetting experiment



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Barabási, A.-L., Stanley, H. E., 1995

# Paper wetting experiment



(b)



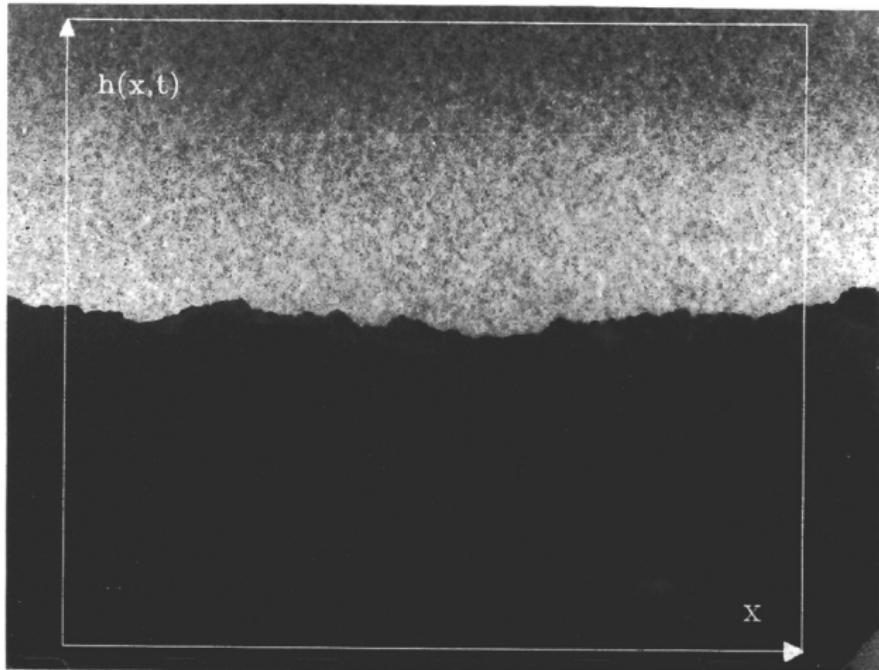
(c)



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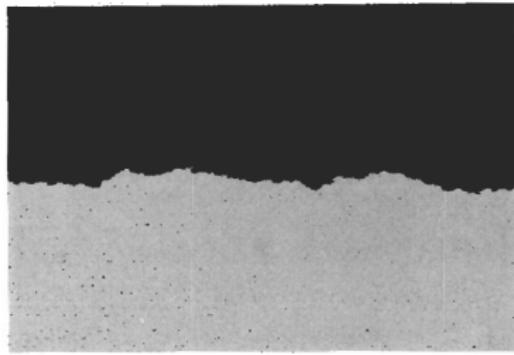
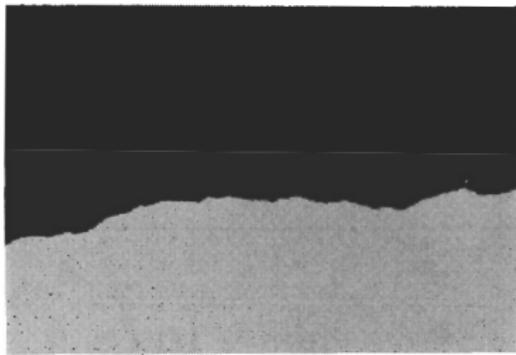
Barabási, A.-L., Stanley, H. E., 1995

# Paper burning experiment



Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

# Paper rupture experiment



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Kertész, J., Horváth, V. k., Weber, F., *Fractals*, 1993

# Plan

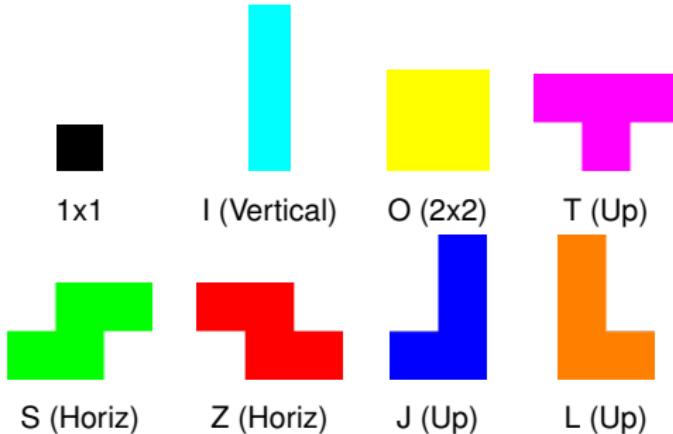
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**Tetromino Pieces**

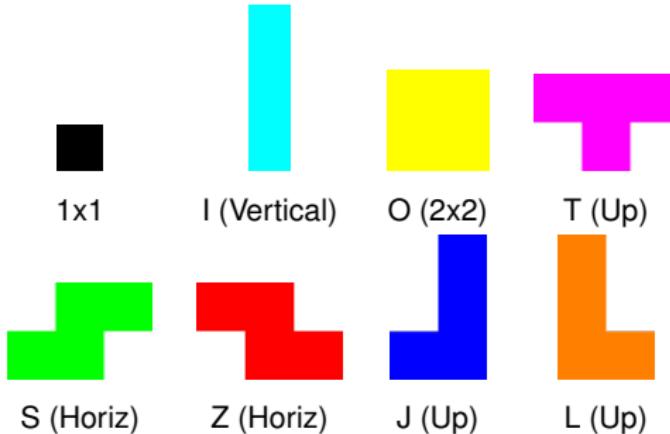
# Tetrominoes



- ▶ “1x1”: Single (extra single-site particle)
- ▶ “I”: Horizontal, Vertical
- ▶ “J, L, T”: Up, Right, Down, Left
- ▶ “S, Z”: Horizontal, Vertical
- ▶ “O”: Single (2x2 square)
- ▶ Sticky
- ▶ Nonsticky

$$(1 + 1 \times 2 + 3 \times 4 + 2 \times 2 + 1) \times 2 = 20 \times 2 = 40 \text{ types of pieces}$$

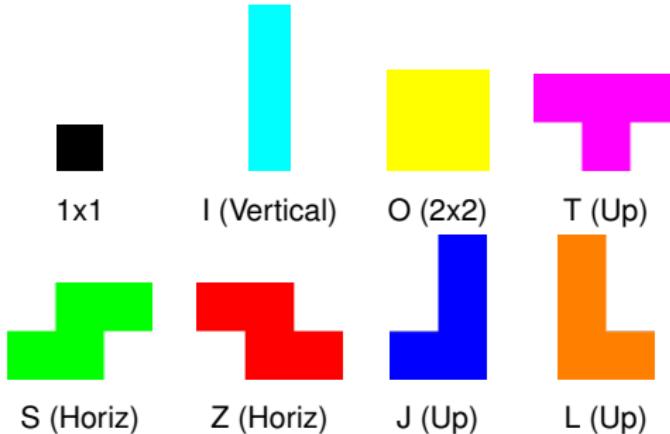
# Tetrominoes



- ▶ “1x1”: Single (extra single-site particle)
- ▶ “I”: Horizontal, Vertical
- ▶ “J, L, T”: Up, Right, Down, Left
- ▶ “S, Z”: Horizontal, Vertical
- ▶ “O”: Single (2x2 square)
- ▶ Sticky
- ▶ Nonsticky

$$(1 + 1 \times 2 + 3 \times 4 + 2 \times 2 + 1) \times 2 = 20 \times 2 = 40 \text{ types of pieces}$$

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$$(1 + 1 \times 2 + 3 \times 4 + 2 \times 2 + 1) \times 2 = 20 \times 2 = 40 \text{ types of pieces}$$

# Configure files

```
steps: 12000  
width: 100  
height: 300  
seed: 12  
Piece-00: [20, 0]  
Piece-01: [20, 0]  
Piece-02: [20, 0]  
Piece-03: [20, 0]  
Piece-04: [20, 0]  
Piece-05: [20, 0]  
Piece-06: [20, 0]  
Piece-07: [20, 0]  
Piece-08: [20, 0]  
Piece-09: [20, 0]  
Piece-10: [20, 0]  
Piece-11: [20, 0]  
Piece-12: [20, 0]  
Piece-13: [20, 0]  
Piece-14: [20, 0]  
Piece-15: [20, 0]  
Piece-16: [20, 0]  
Piece-17: [20, 0]  
Piece-18: [20, 0]  
Piece-19: [20, 0]
```

All nonsticky pieces  
with equal prob.

```
steps: 12000  
width: 100  
height: 300  
seed: 12  
Piece-00: [0, 20]  
Piece-01: [0, 20]  
Piece-02: [0, 20]  
Piece-03: [0, 20]  
Piece-04: [0, 20]  
Piece-05: [0, 20]  
Piece-06: [0, 20]  
Piece-07: [0, 20]  
Piece-08: [0, 20]  
Piece-09: [0, 20]  
Piece-10: [0, 20]  
Piece-11: [0, 20]  
Piece-12: [0, 20]  
Piece-13: [0, 20]  
Piece-14: [0, 20]  
Piece-15: [0, 20]  
Piece-16: [0, 20]  
Piece-17: [0, 20]  
Piece-18: [0, 20]  
Piece-19: [0, 20]
```

All sticky pieces  
with equal prob.

```
steps: 12000  
width: 100  
height: 300  
seed: 12  
Piece-00: [0, 0]  
Piece-01: [0, 0]  
Piece-02: [0, 0]  
Piece-03: [0, 0]  
Piece-04: [0, 0]  
Piece-05: [0, 0]  
Piece-06: [0, 0]  
Piece-07: [0, 0]  
Piece-08: [0, 0]  
Piece-09: [0, 0]  
Piece-10: [0, 0]  
Piece-11: [0, 0]  
Piece-12: [0, 0]  
Piece-13: [0, 0]  
Piece-14: [0, 0]  
Piece-15: [0, 0]  
Piece-16: [0, 0]  
Piece-17: [0, 0]  
Piece-18: [0, 0]  
Piece-19: [20, 80]
```

20% nonsticky  
+ 80% sticky  
of 1x1 piece

## Question

For various configurations of Tetromino pieces, do the resulting surface robustly exhibit Family–Vicsek scaling?

Will the scaling exponent  $\beta$  be always close to  $\frac{1}{3}$ ?

## Setup

Consider a mixture of  $\ell\%$  ( $\ell \in [0, 100]$ ) nonsticky pieces and  $(100-\alpha)\%$  sticky pieces, where only the  $1 \times 1$  piece is sticky.

## Question

How does  $\alpha$  influence the scaling exponents, in particular,  $\beta \stackrel{?}{\approx} \frac{1}{3}$ ?

## Simulations and log-log plots:

[https://chenle02.github.io/2025-10-28\\_Emerging\\_Synergies\\_Banff\\_Le/exp13/videos\\_and\\_images\\_display.html](https://chenle02.github.io/2025-10-28_Emerging_Synergies_Banff_Le/exp13/videos_and_images_display.html)

## Setup

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## Simulations and log-log plots:

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# Outreach Highlights



AU-SSI 2023



AU-SSI 2024



Destination STEM 2023

## Main References:

- Barabási, A.-L., & Stanley, H. E. (1995). *Fractal concepts in surface growth*. Cambridge University Press, Cambridge.
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Thank you!

Questions?