Analysis of Tetris Ballistic Deposition and the Robustness of the KPZ Universality Class

Le Chen
Auburn University

Acknwolegement

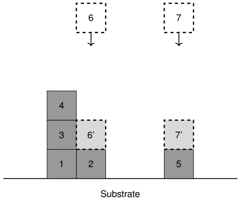
NSF 2246850, NSF 2443823, & Simons Foundation Travel Grant (2022-2027)

`Talk available at: github.com/chenle02`

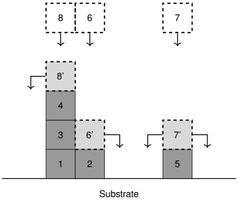
Emerging Synergies between Stochastic Analysis and Statistical Mechanics
Banff, Alberta, Canada
October 28, 2025

How do surfaces grow?

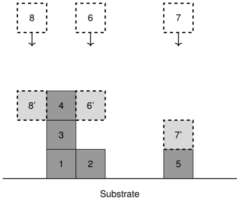
Random deposition



RD with surface relaxation



Ballistic deposition



Integration of Research, Education, and Outreach

Outreach

- Auburn Summer Science Institute (AU-SSI): **2024, 2025**
  Selected talented high school students
- Destination STEM: **2023, 2024**
  Junior middle school students

Education

- Graduate Student Seminars (Mathematics), Auburn: **2022–2025**
- Math 7820/7830: Applied Stochastic Processes
  Course project, **2023/24**

Research

- Simulation and modeling packages (open source)
- Forming conjectures and validating results

Most materials are available at

`github.com/chenle02`

Math 7820/30: Applied Stochastic Processes (2023/24):


Mauricio Montes and Ian Ruau

Simulation package:

https://github.com/chenle02/Simulations_on_Some_Surface_Growth_Models

```
pip install tetris-ballistic
```

Image is generated by OpenAI's *DALL-E*

Random deposition

Ballistic deposition

Image is generated by OpenAI's *DALL-E*

Plan

Introduction to growth model and SPDE

Family-Vicsek scaling and experiments

Tetromino Pieces

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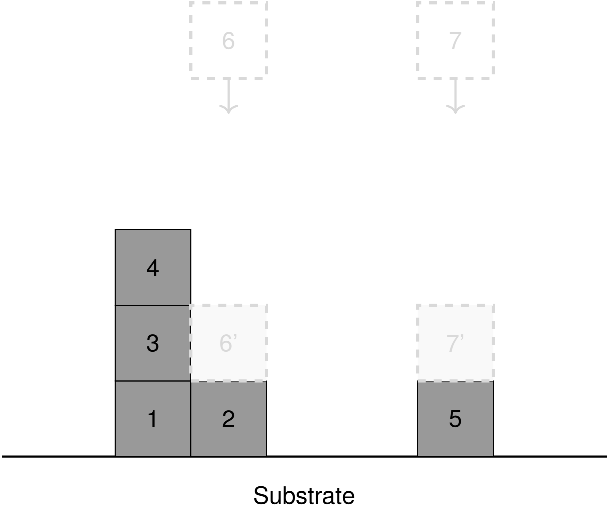

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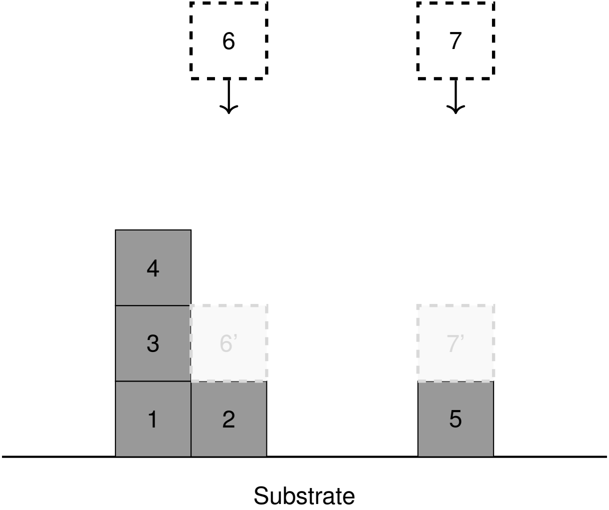

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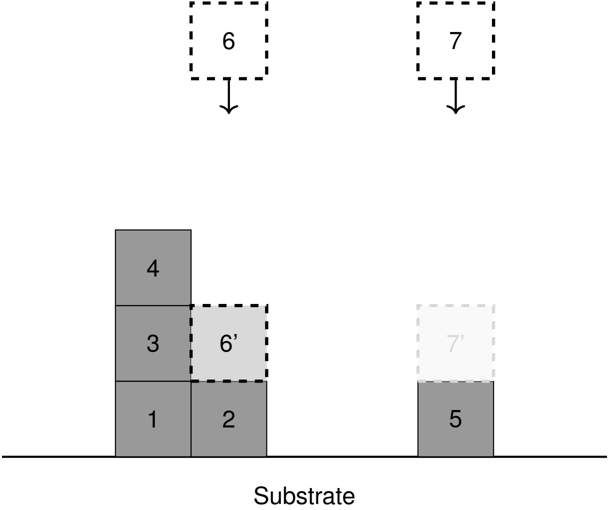

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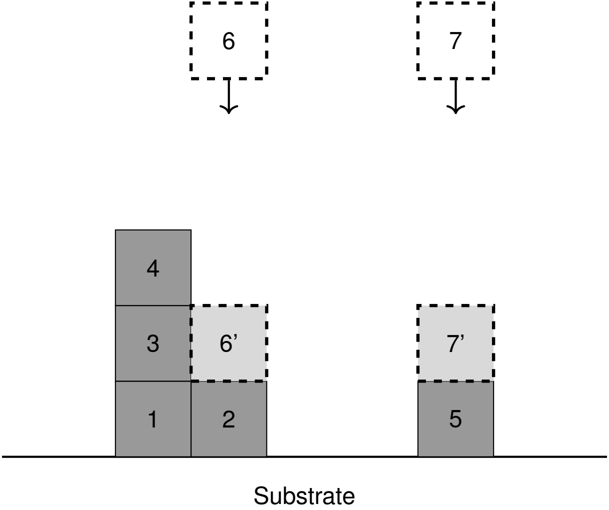
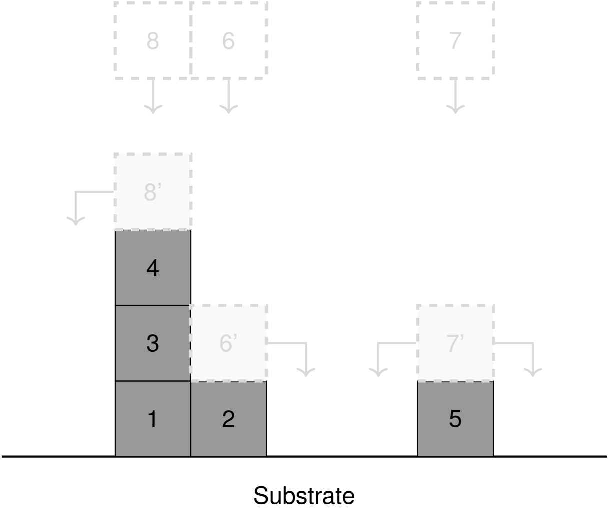


Random deposition with surface relaxation



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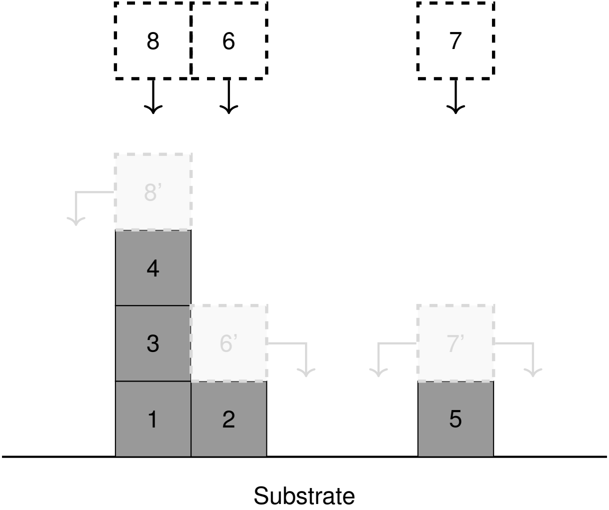

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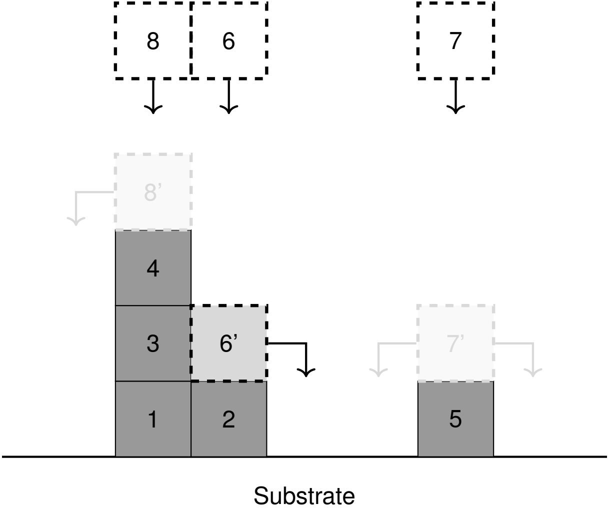

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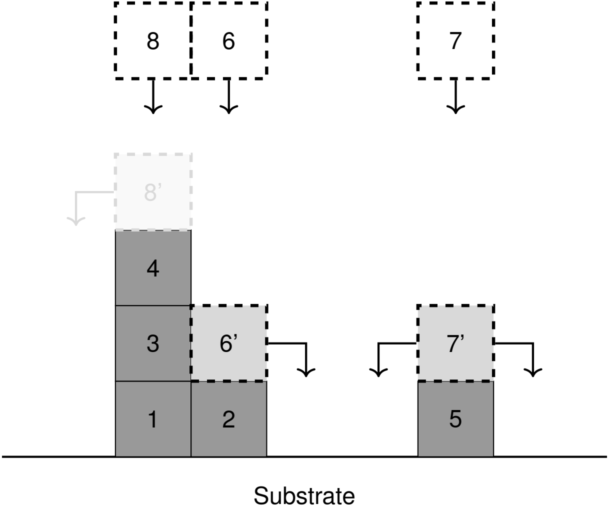

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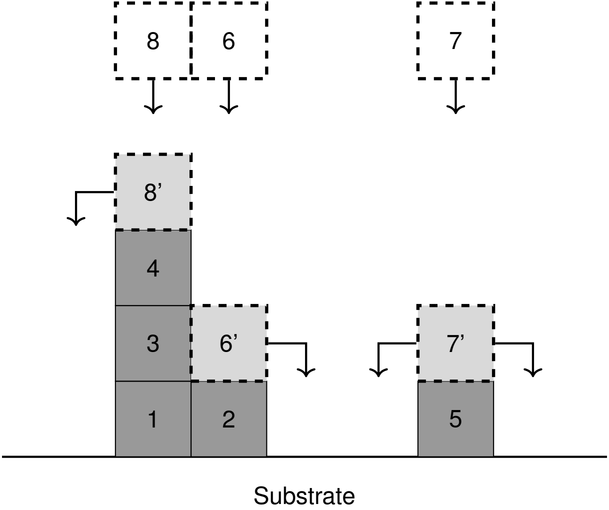
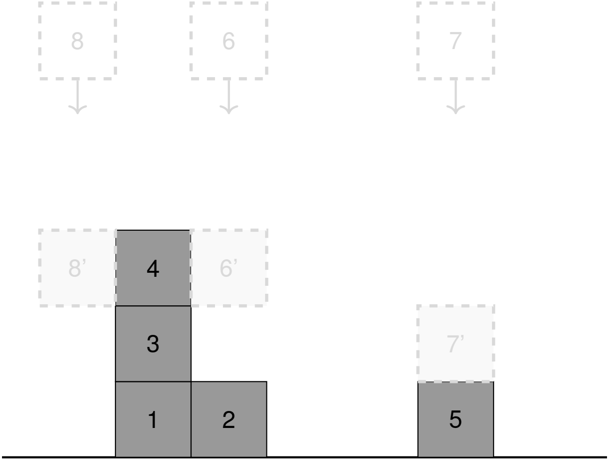
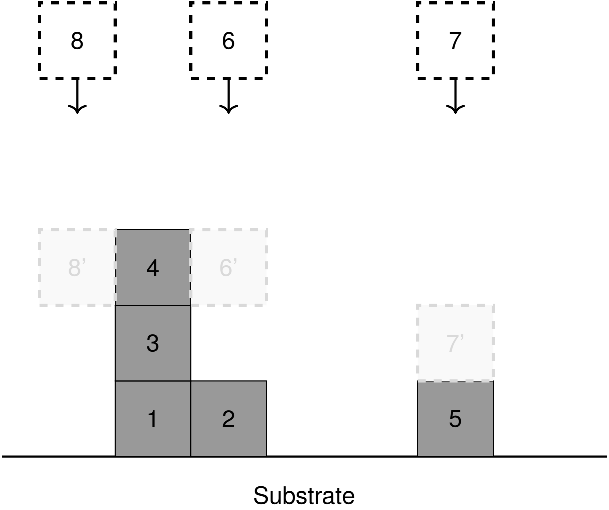


Ballistic deposition



Substrate

Ballistic deposition



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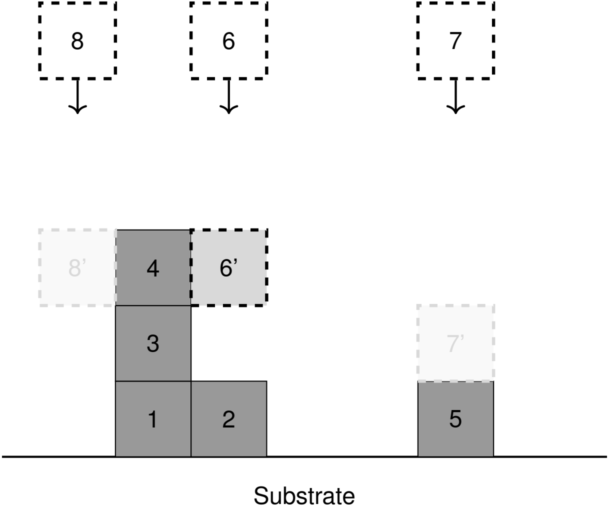

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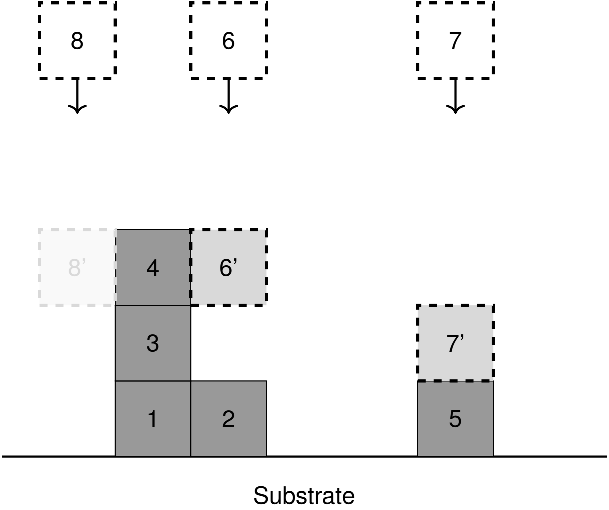

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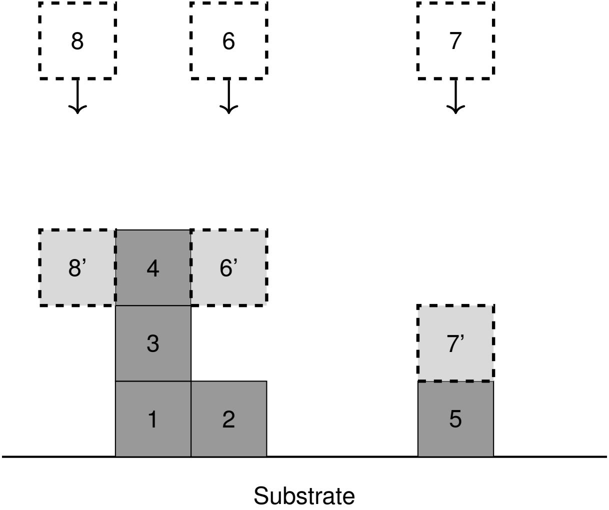
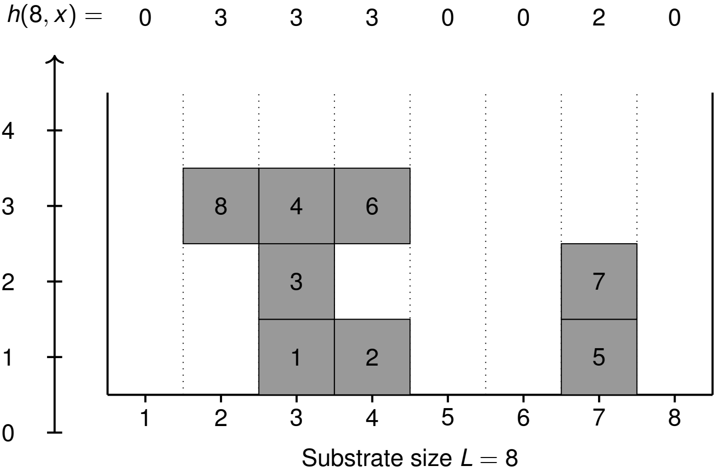


Average height and fluctuation

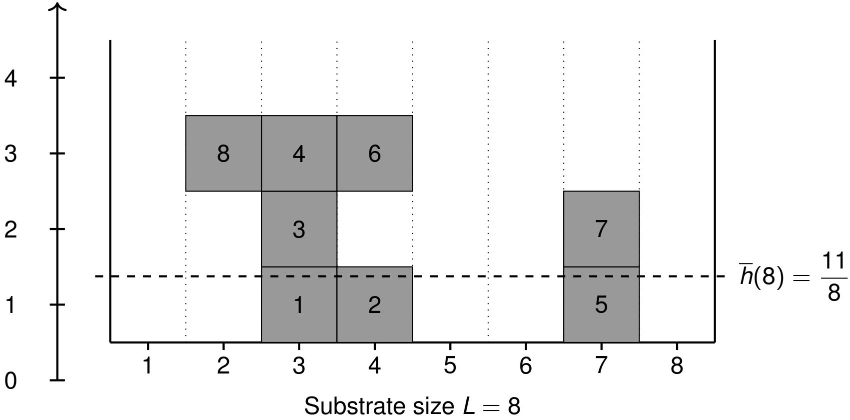


Average height and fluctuation

$$\overline{h}(t) = \frac{1}{L}\sum_{x=1}^{L} h(t,x)$$

Fluctuation $W(L,t) = \sqrt{\frac{1}{L}\sum_{x=1}^{L}\left[h(t,x) - \overline{h}(t)\right]^2}$

$h(8,x) =$ 0 3 3 3 0 0 2 0



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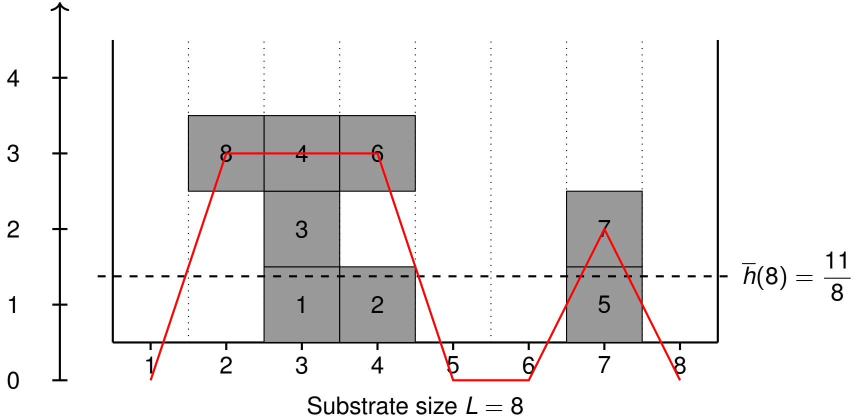

Random Deposition (independent columns, nonsticky)

Model. L independent columns. At each integer time $t = 1, 2, \ldots$, drop *one* particle on a uniformly random column. Heights $h(t,x)$, mean $\overline{h}(t) = \frac{1}{L}\sum_{x=1}^{L} h(t,x) = \frac{t}{L}$, width

$$W^2(L,t) = \frac{1}{L}\sum_{x=1}^{L}\left(h(t,x) - \overline{h}(t)\right)^2.$$

Single–column law: After t drops total,

$$h(t,x) \sim \mathrm{Binomial}\left(t, \frac{1}{L}\right), \qquad \mathbb{E}[h(t,x)] = \frac{t}{L}, \quad \mathrm{Var}(h(t,x)) = t\,\frac{1}{L}\left(1 - \frac{1}{L}\right).$$

Fluctuation: By i.i.d. columns,

$$\mathbb{E}\left[W^2(L,t)\right] = \frac{1}{L}\sum_{x=1}^{L}\mathbb{E}\left[h(t,x)^2\right] - \mathbb{E}\left[\overline{h}^2(t)\right] = \mathbb{E}\left[h(t,1)^2\right] - \left(\frac{t}{L}\right)^2 = \left(1 - \frac{1}{L}\right)\mathrm{Var}(h(t,1)).$$

Hence

$$\boxed{\mathbb{E}\left[W^2(L,t)\right] = \left(1 - \frac{1}{L}\right) t\,\frac{1}{L}\left(1 - \frac{1}{L}\right) = \frac{t}{L}\left(1 - \frac{1}{L}\right)^2}$$

and

$$\boxed{W(L,t) \simeq \left(1 - \frac{1}{L}\right)\left(\frac{t}{L}\right)^{1/2}}$$

Scaling. Growth exponent $\beta = \frac{1}{2}$.

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For ballistic deposition (sticky), lateral interactions appear, can we view it as a perturbation of random deposition (nonsticky)?

Plan

What we measure

Discrete interface with heights $h(x,t)$, $x = 1, \dots, L$. Mean $\bar{h}(t) = \frac{1}{L}\sum_{x=1}^{L} h(x,t)$.

$$w(L,t) := \sqrt{\frac{1}{L}\sum_{x=1}^{L}\big(h(x,t) - \bar{h}(t)\big)^2} \quad \text{(interface width / roughness).}$$

Empirically (log–log):

- Early time: $w(L,t) \sim t^{\beta}$ (growth exponent β).
- Late time: $w(L,t) \to w_{\mathrm{sat}}(L) \sim L^{\alpha}$ (roughness exponent α).
- Crossover time: $t_{\times}(L) \sim L^{z}$ (dynamic exponent z).

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Family–Vicsek scaling ansatz

$$\boxed{w(L,t) \;\sim\; L^{\alpha}\, f\!\left(\frac{t}{L^{z}}\right)}$$

with

$$f(u) \sim \begin{cases} u^{\beta}, & u \ll 1, \\ \text{const}, & u \gg 1. \end{cases} \qquad \Rightarrow \qquad \boxed{\beta = \frac{\alpha}{z}}$$

Immediate consequences:

$$w_{\text{sat}}(L) \sim L^{\alpha}, \qquad t_{\times}(L) \sim L^{z}.$$

Interpretation: dynamic renormalization $x \to bx$, $t \to b^{z} t$, $h \to b^{\alpha} h$ leaves $w(L,t)/L^{\alpha}$ invariant as a function of t/L^{z}.

How to verify FV scaling (data collapse)

Recipe

1. Simulate/measure $w(L, t)$ for several L.
2. Estimate α from $w_{\text{sat}}(L) \sim L^{\alpha}$ (fit saturated widths).
3. Estimate z from $t_{\times}(L) \sim L^{z}$ or use $\beta = \alpha/z$ with early-time slope.
4. Plot $w(L, t)/L^{\alpha}$ vs. t/L^{z}; curves collapse to the master $f(u)$.

Pitfalls

- Transient window before true power law; finite-size drifts in w_{sat}.
- Use consistent fitting ranges; report uncertainty from range variation.

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Continuum viewpoint: EW and KPZ

EW (linear diffusion + noise):

$$\partial_t h = \nu \nabla^2 h + \eta, \qquad \langle \eta\eta \rangle \propto \delta(x)\delta(t).$$

Scale invariance gives $z = 2$, $\alpha = \frac{1}{2}$ in 1D $\Rightarrow \beta = \frac{1}{4}$.

KPZ (nonlinear growth):

$$\partial_t h = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta.$$

Galilean/tilt invariance $\Rightarrow \alpha + z = 2$. In 1D (exact):

$$\boxed{\alpha = \tfrac{1}{2}, \quad z = \tfrac{3}{2}, \quad \beta = \tfrac{1}{3}.}$$

Many lattice models (BD, RSOS, Eden) fall into KPZ in 1+1D.

Reading plots (as in Barabási–Stanley)

- On w vs. t (log–log) for several L: early slopes $\approx \beta$; plateaus $\propto L^{\alpha}$; horizontal shift of crossovers $\propto L^{z}$.
- After rescaling: plot w/L^{α} vs. t/L^{z}; all curves collapse to $f(u)$ with $f(u) \sim u^{\beta}$ ($u \ll 1$) and $f(u) \to \text{const}$ ($u \gg 1$).

Rule of thumb (1D):

$$\text{EW: } (\alpha, \beta, z) = \left(\tfrac{1}{2}, \tfrac{1}{4}, 2\right), \qquad \text{KPZ/BD: } (\alpha, \beta, z) = \left(\tfrac{1}{2}, \tfrac{1}{3}, \tfrac{3}{2}\right).$$

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Exactly solvable baseline: Random Deposition (RD)

One particle per step, random column; after t steps heights are multinomial:

$$\mathbb{E}\left[w^2(L,t)\right] = \frac{t}{L}\Big(1 - \frac{1}{L}\Big) \ \sim \ \frac{t}{L}.$$

Hence $w \sim (t/L)^{1/2}$ ($\beta = \frac{1}{2}$), *no lateral correlations*, no finite z and no true saturation. RD serves as a growth-only sanity check; FV scaling requires correlated dynamics (EW/KPZ).

Structure factor (optional but handy)

Fourier modes $h_k(t)$, structure factor $S(k,t) = \langle |h_k(t)|^2 \rangle$ obey

$$S(k,t) \sim k^{-(2\alpha+1)}\, g(k^z t), \qquad g(y) \sim \begin{cases} y^{2\beta+1} & y \ll 1, \\ \text{const} & y \gg 1. \end{cases}$$

Useful when $w(L,t)$ is noisy; fit (α, z) from spectral slopes and collapse $S(k,t)$.

Cheat sheet / takeaways

$$\boxed{w(L,t) \sim L^{\alpha} f\left(\frac{t}{L^{z}}\right), \quad f(u) \sim u^{\beta}\ (u \ll 1),\ f(u) \to \text{const}\ (u \gg 1),\ \beta = \alpha/z}$$

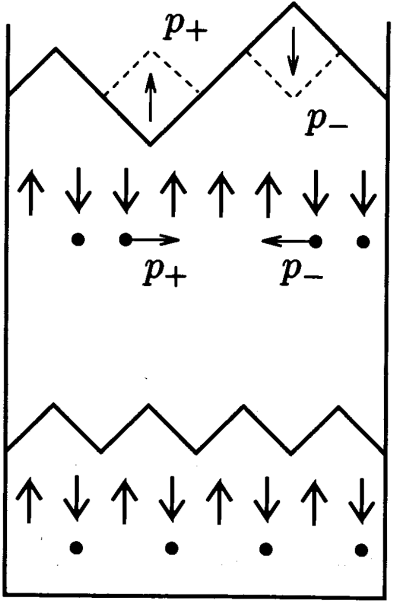
1D benchmarks: EW $(\alpha, \beta, z) = (\frac{1}{2}, \frac{1}{4}, 2)$; KPZ/BD $(\frac{1}{2}, \frac{1}{3}, \frac{3}{2})$.

Workflow: measure $w(L,t) \Rightarrow$ fit α and $\beta \Rightarrow$ infer/fit $z \Rightarrow$ collapse w/L^{α} vs. t/L^{z}.

Simulations on

Random deposition vs. Ballistic decomposition

More models? Even more simpler?

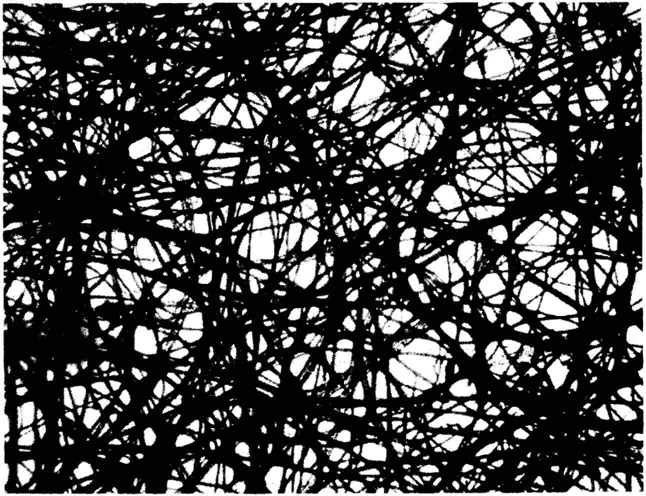


Solid-on-Solid

Ising

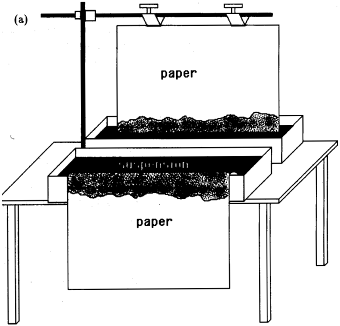
Lattice Gas

Paper – a random environment



Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

Paper wetting experiment



Barabási, A.-L., Stanley, H. E., 1995

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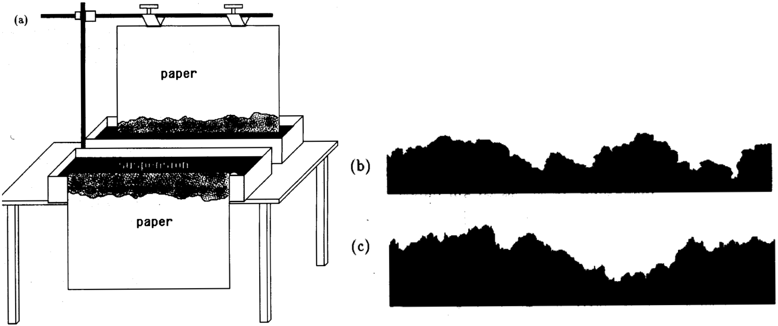

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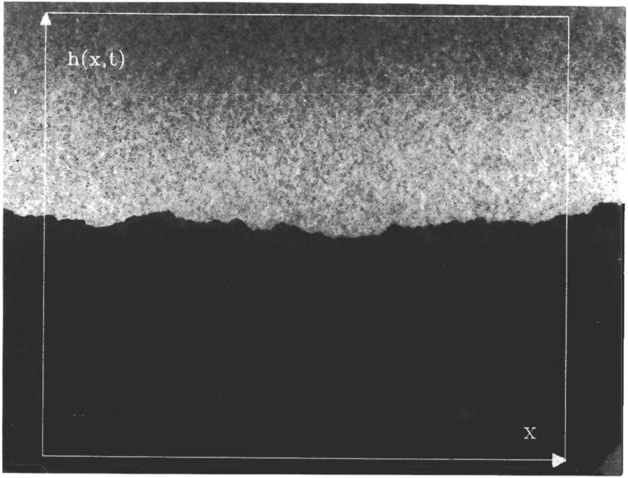
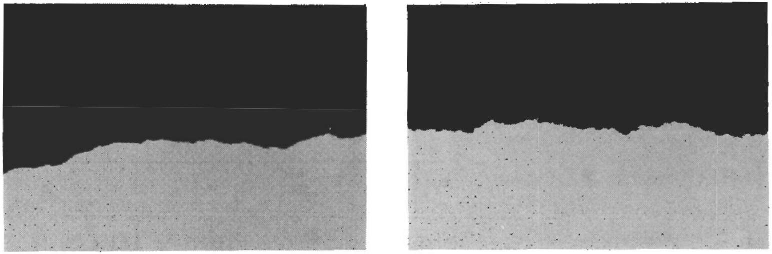
Paper burning experiment



Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

Paper rupture experiment



Kertész, J., Horváth, V. k., Weber, F., *Fractals*, 1993

Study of growing interfaces in a thin film

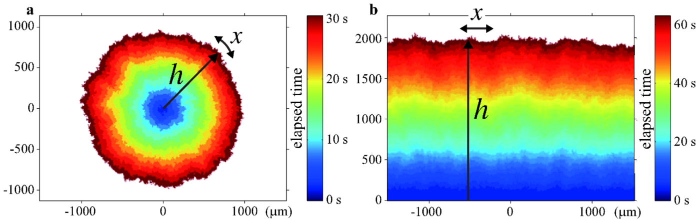
— Convection of nematic liquid crystal*

Show movies !

Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

Study of growing interfaces in a thin film

— Convection of nematic liquid crystal*



Prediction from KPZ equation:

$$h \asymp v_\infty t + (\Gamma t)^{1/3}\,\xi$$

Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

Study of growing interfaces in a thin film

— Convection of nematic liquid crystal*

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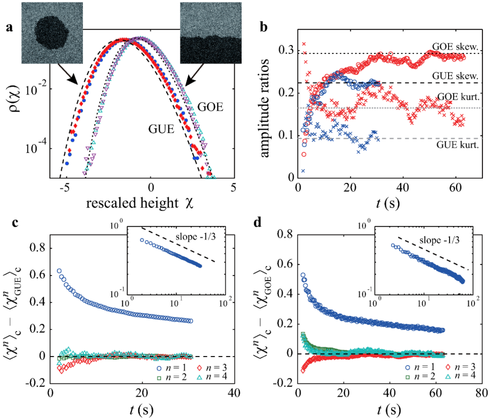


Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

KPZ Equation '86

$$\frac{\partial}{\partial t} h(t,x) = \frac{1}{2}\Delta h(t,x) + \frac{\lambda}{2}(\nabla h)^2 + \dot{W}(t,x) \qquad \text{(KPZ)}$$

Mehran Kardar (1957 –)

Giorgio Parisi (1948 –)

Yicheng Zhang

Kardar, M., Parisi, G., Zhang, Y.-C., *Phys. Rev. Lett.*, 1986

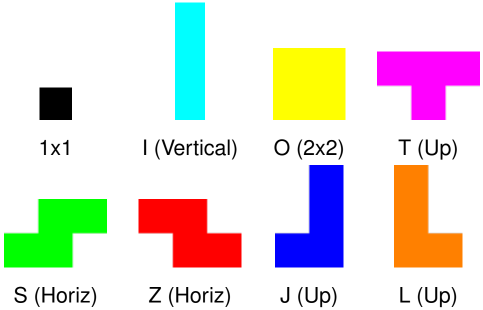
Plan

Introduction to growth model and SPDE

Family-Vicsek scaling and experiments

Tetromino Pieces

Tetrominoes

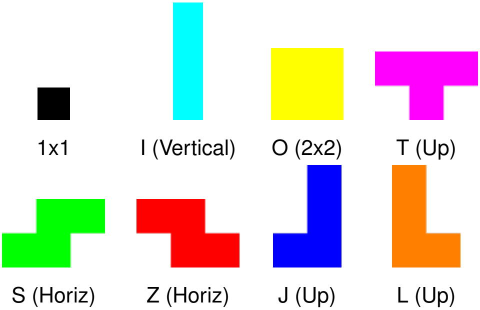


- “1x1”: Single (extra single-site particle)
- “I”: Horizontal, Vertical
- “J, L, T”: Up, Right, Down, Left
- “S, Z”: Horizontal, Vertical
- “O”: Single (2x2 square)

- Sticky
- Nonstikcy

$(1 + 1 \times 2 + 3 \times 4 + 2 \times 2 + 1) \times 2 = 20 \times 2 = 40$ types of pieces

Tetrominoes



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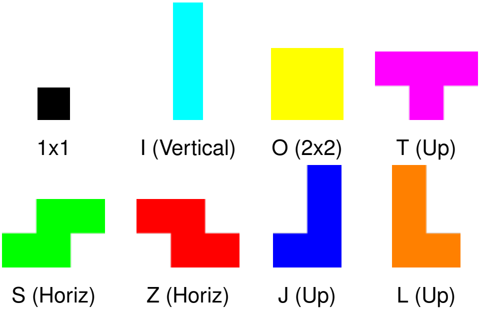

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- "O": Single (2x2 square)

- Sticky
- Nonstikcy

$$(1 + 1 \times 2 + 3 \times 4 + 2 \times 2 + 1) \times 2 = 20 \times 2 = 40 \text{ types of pieces}$$

Configure files

```
steps: 12000
width: 100
height: 300
seed: 12
Piece-00: [20, 0]
Piece-01: [20, 0]
Piece-02: [20, 0]
Piece-03: [20, 0]
Piece-04: [20, 0]
Piece-05: [20, 0]
Piece-06: [20, 0]
Piece-07: [20, 0]
Piece-08: [20, 0]
Piece-09: [20, 0]
Piece-10: [20, 0]
Piece-11: [20, 0]
Piece-12: [20, 0]
Piece-13: [20, 0]
Piece-14: [20, 0]
Piece-15: [20, 0]
Piece-16: [20, 0]
Piece-17: [20, 0]
Piece-18: [20, 0]
Piece-19: [20, 0]
```

All nonsticky pieces
with equal prob.

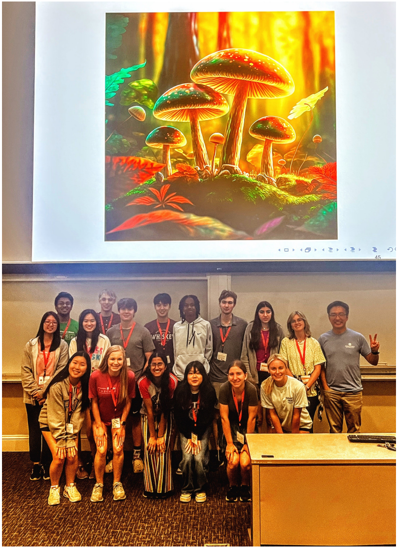
```
steps: 12000
width: 100
height: 300
seed: 12
Piece-00: [0, 20]
Piece-01: [0, 20]
Piece-02: [0, 20]
Piece-03: [0, 20]
Piece-04: [0, 20]
Piece-05: [0, 20]
Piece-06: [0, 20]
Piece-07: [0, 20]
Piece-08: [0, 20]
Piece-09: [0, 20]
Piece-10: [0, 20]
Piece-11: [0, 20]
Piece-12: [0, 20]
Piece-13: [0, 20]
Piece-14: [0, 20]
Piece-15: [0, 20]
Piece-16: [0, 20]
Piece-17: [0, 20]
Piece-18: [0, 20]
Piece-19: [0, 20]
```

All sticky pieces
with equal prob.

```
steps: 12000
width: 100
height: 300
seed: 12
Piece-00: [0, 0]
Piece-01: [0, 0]
Piece-02: [0, 0]
Piece-03: [0, 0]
Piece-04: [0, 0]
Piece-05: [0, 0]
Piece-06: [0, 0]
Piece-07: [0, 0]
Piece-08: [0, 0]
Piece-09: [0, 0]
Piece-10: [0, 0]
Piece-11: [0, 0]
Piece-12: [0, 0]
Piece-13: [0, 0]
Piece-14: [0, 0]
Piece-15: [0, 0]
Piece-16: [0, 0]
Piece-17: [0, 0]
Piece-18: [0, 0]
Piece-19: [20, 80]
```

20% nonsticky
+ 80% sticky
of 1x1 piece

Outreach Highlights



AU-SSI 2023

AU-SSI 2024

Destination STEM 2023

Main References:

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Zhang, J., Zhang, Y.-C., Alstrøm, P., & Levinsen, M. (1992).Modeling forest fire by a paper-burning experiment, a realization of the interface growth mechanism. *Phys. A: Stat. Mech. Appl.*, *189*(3), 383–389.

Thank you!

Questions?