

Analysis of Tetris Ballistic Deposition and the Robustness of the KPZ Universality Class

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Auburn University

Acknwolegement

NSF 2246850, NSF 2443823, & Simons Foundation Travel Grant (2022-2027)

Emerging Synergies between Stochastic Analysis and Statistical Mechanics
Banff, Alberta, Canada
October 28, 2025

Math 7820/30: Applied Stochastic Processes (2023/24):



Mauricio Montes and Ian Ruau

Plan

Tetromino Pieces

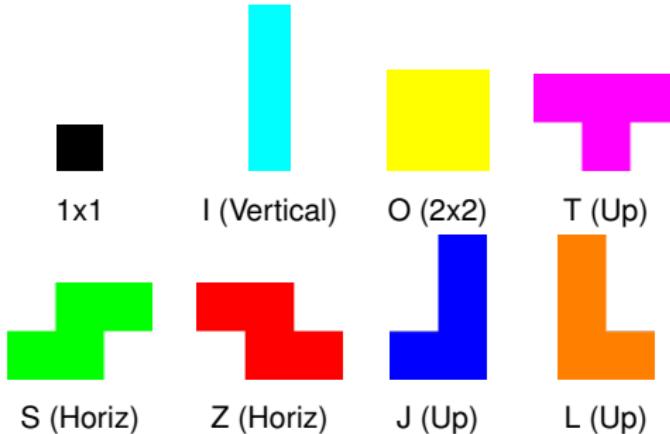
Introduction to growth model and SPDE

Plan

Tetromino Pieces

Introduction to growth model and SPDE

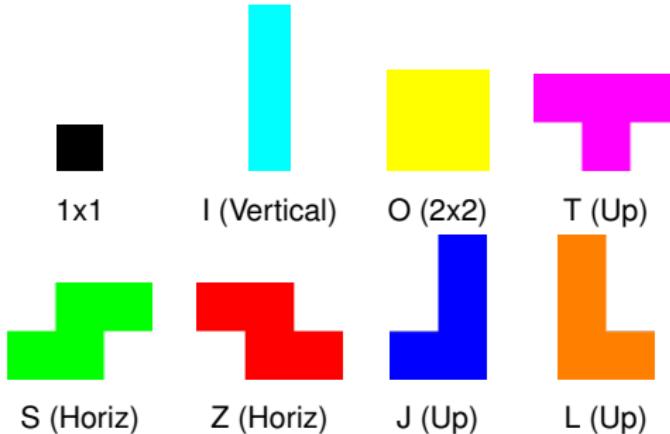
Tetrominoes



- ▶ “1x1”: Single (extra single-site particle)
- ▶ “I”: Horizontal, Vertical
- ▶ “J, L, T”: Up, Right, Down, Left
- ▶ “S, Z”: Horizontal, Vertical
- ▶ “O”: Single (2x2 square)
- ▶ Sticky
- ▶ Nonsticky

$$(1 + 1 \times 2 + 3 \times 4 + 2 \times 2 + 1) \times 2 = 20 \times 2 = 40 \text{ types of pieces}$$

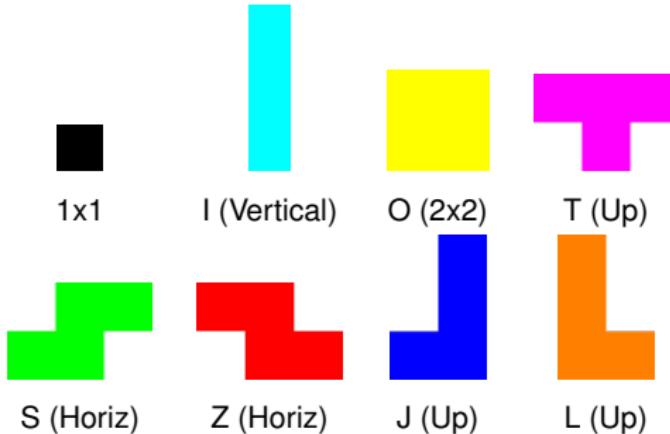
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Configure files

```
steps: 12000  
width: 100  
height: 300  
seed: 12  
Piece-00: [20, 0]  
Piece-01: [20, 0]  
Piece-02: [20, 0]  
Piece-03: [20, 0]  
Piece-04: [20, 0]  
Piece-05: [20, 0]  
Piece-06: [20, 0]  
Piece-07: [20, 0]  
Piece-08: [20, 0]  
Piece-09: [20, 0]  
Piece-10: [20, 0]  
Piece-11: [20, 0]  
Piece-12: [20, 0]  
Piece-13: [20, 0]  
Piece-14: [20, 0]  
Piece-15: [20, 0]  
Piece-16: [20, 0]  
Piece-17: [20, 0]  
Piece-18: [20, 0]  
Piece-19: [20, 0]
```

All nonsticky pieces
with equal prob.

```
steps: 12000  
width: 100  
height: 300  
seed: 12  
Piece-00: [0, 20]  
Piece-01: [0, 20]  
Piece-02: [0, 20]  
Piece-03: [0, 20]  
Piece-04: [0, 20]  
Piece-05: [0, 20]  
Piece-06: [0, 20]  
Piece-07: [0, 20]  
Piece-08: [0, 20]  
Piece-09: [0, 20]  
Piece-10: [0, 20]  
Piece-11: [0, 20]  
Piece-12: [0, 20]  
Piece-13: [0, 20]  
Piece-14: [0, 20]  
Piece-15: [0, 20]  
Piece-16: [0, 20]  
Piece-17: [0, 20]  
Piece-18: [0, 20]  
Piece-19: [0, 20]
```

All sticky pieces
with equal prob.

```
steps: 12000  
width: 100  
height: 300  
seed: 12  
Piece-00: [0, 0]  
Piece-01: [0, 0]  
Piece-02: [0, 0]  
Piece-03: [0, 0]  
Piece-04: [0, 0]  
Piece-05: [0, 0]  
Piece-06: [0, 0]  
Piece-07: [0, 0]  
Piece-08: [0, 0]  
Piece-09: [0, 0]  
Piece-10: [0, 0]  
Piece-11: [0, 0]  
Piece-12: [0, 0]  
Piece-13: [0, 0]  
Piece-14: [0, 0]  
Piece-15: [0, 0]  
Piece-16: [0, 0]  
Piece-17: [0, 0]  
Piece-18: [0, 0]  
Piece-19: [20, 80]
```

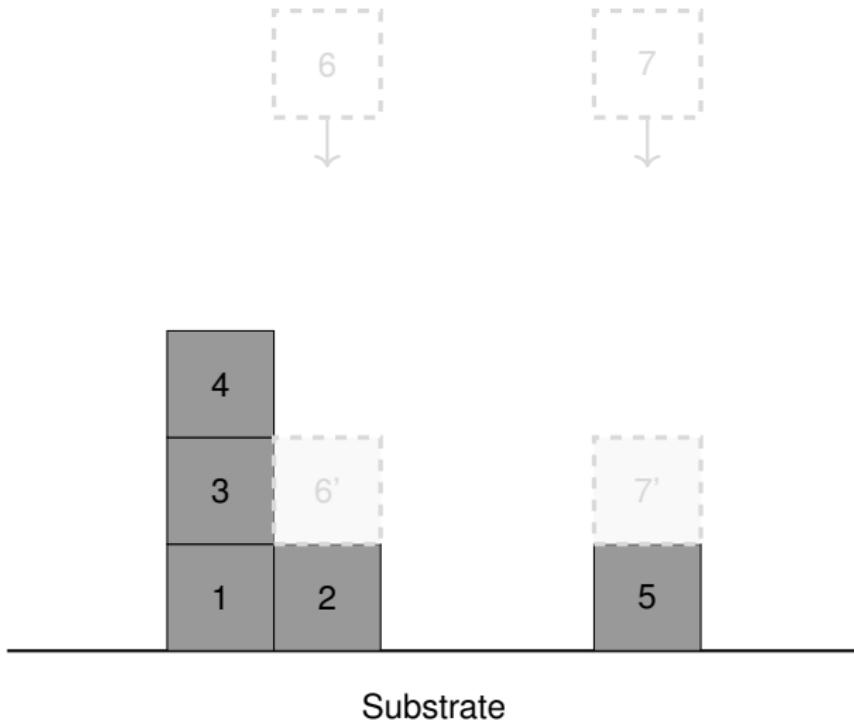
20% nonsticky
+ 80% sticky
of 1x1 piece

Plan

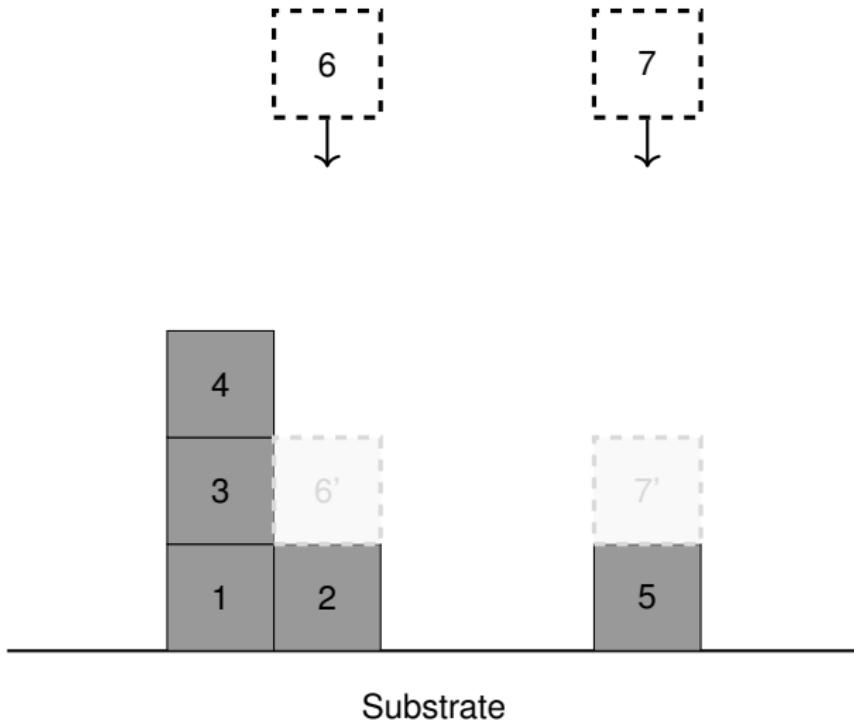
Tetromino Pieces

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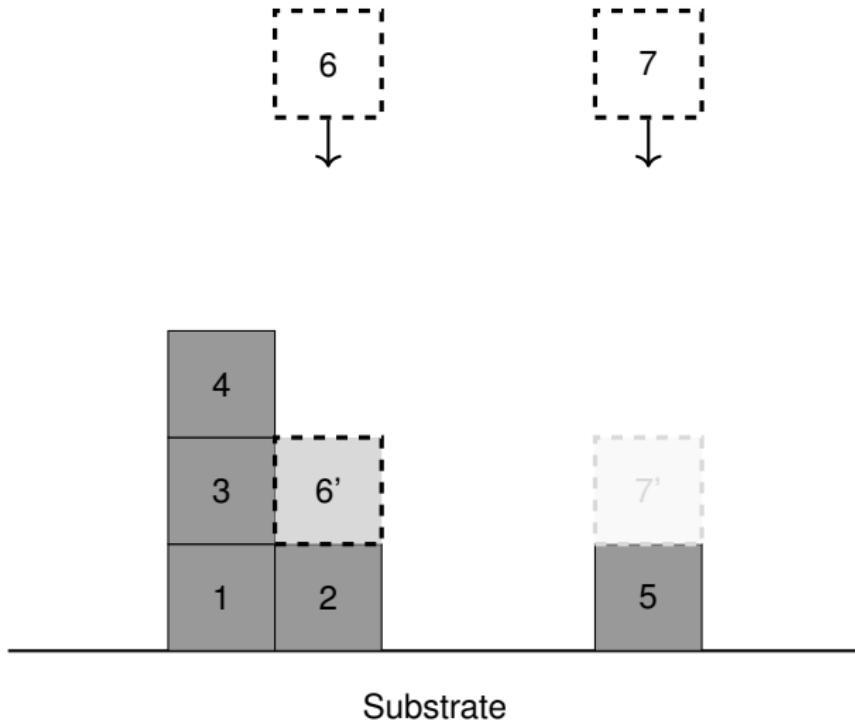
Random deposition



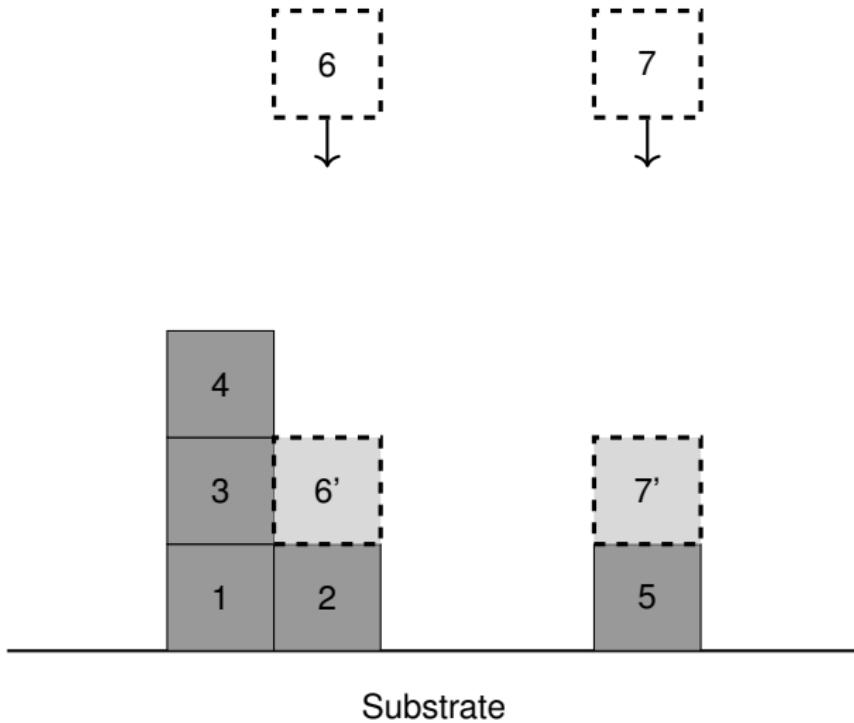
Random deposition



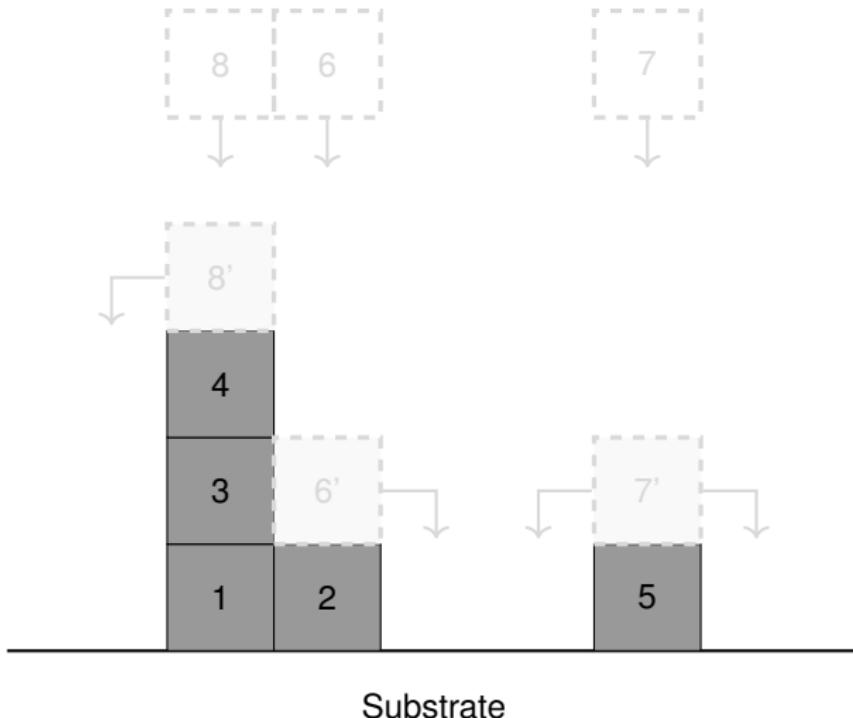
Random deposition



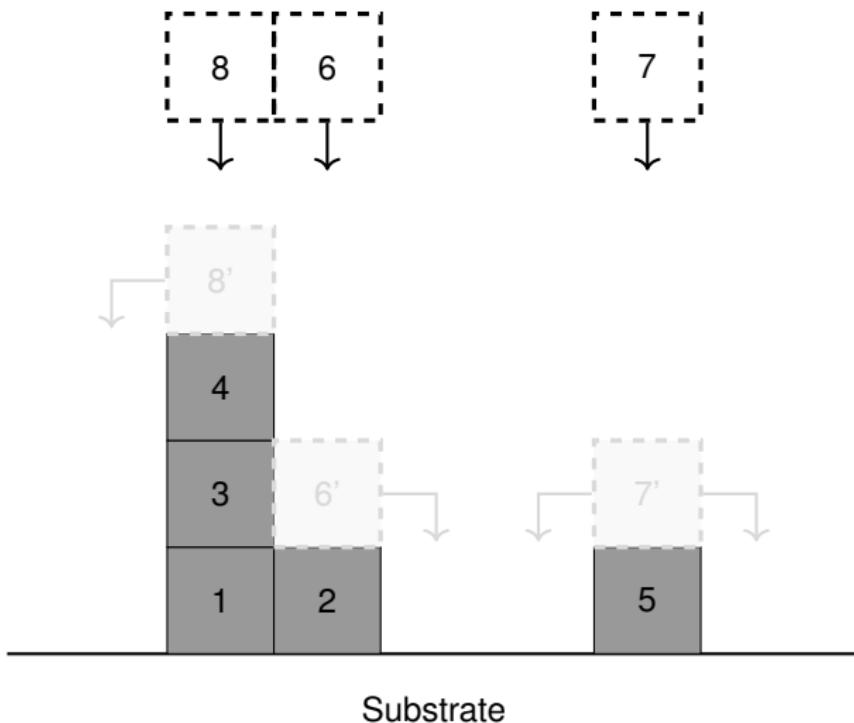
Random deposition



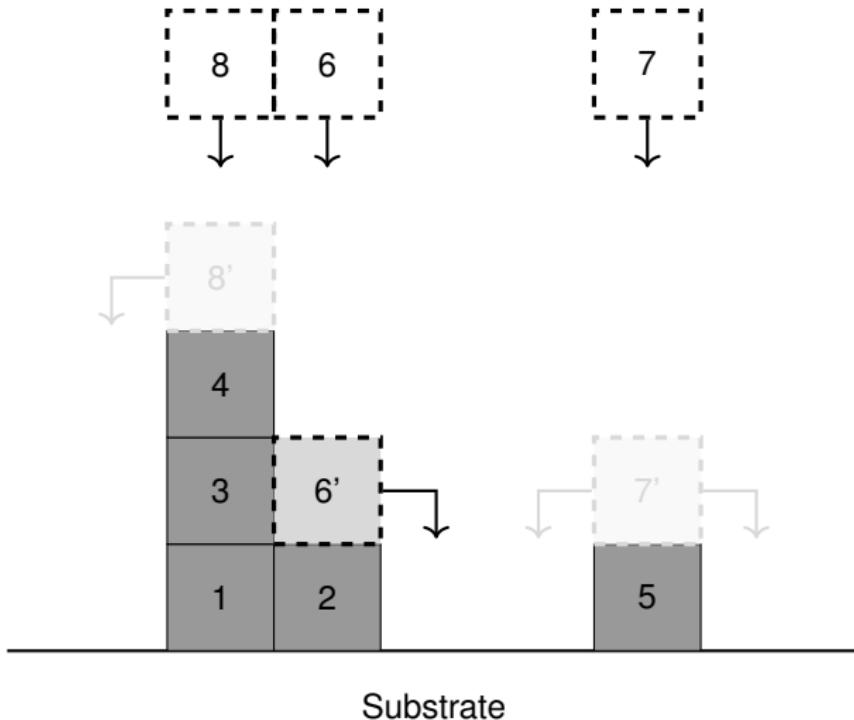
Random deposition with surface relaxation



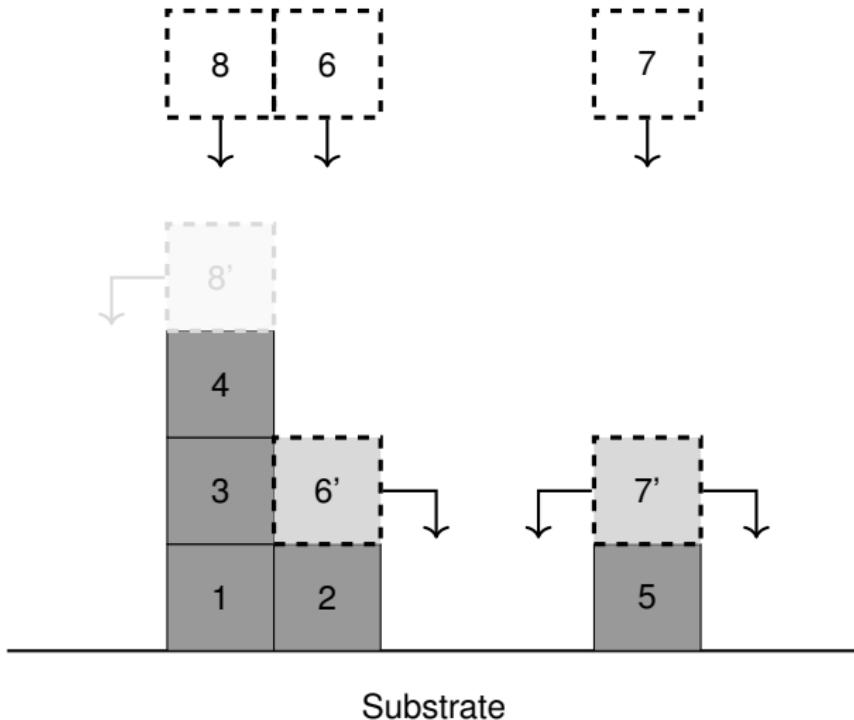
Random deposition with surface relaxation



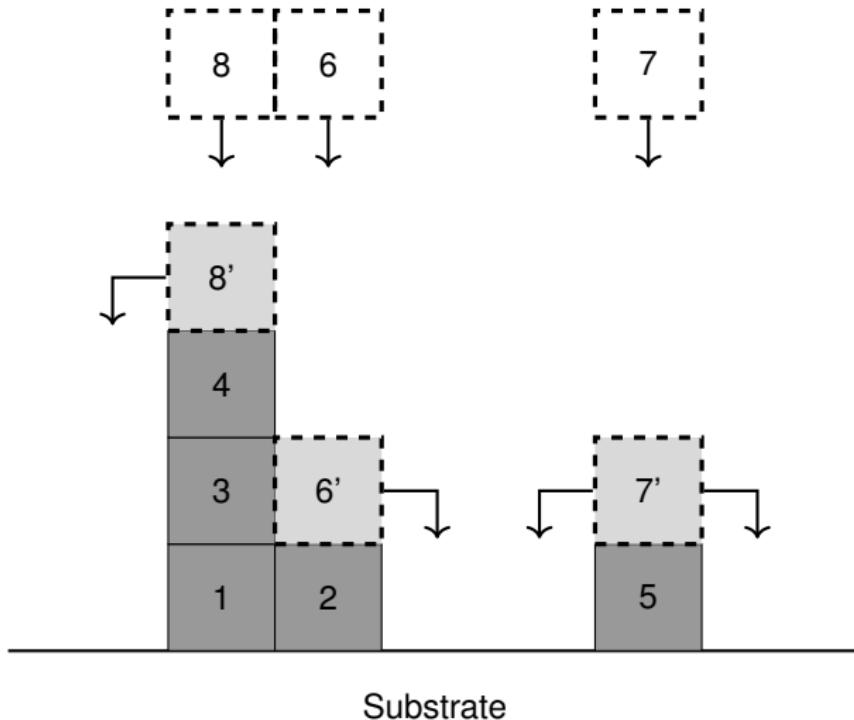
Random deposition with surface relaxation



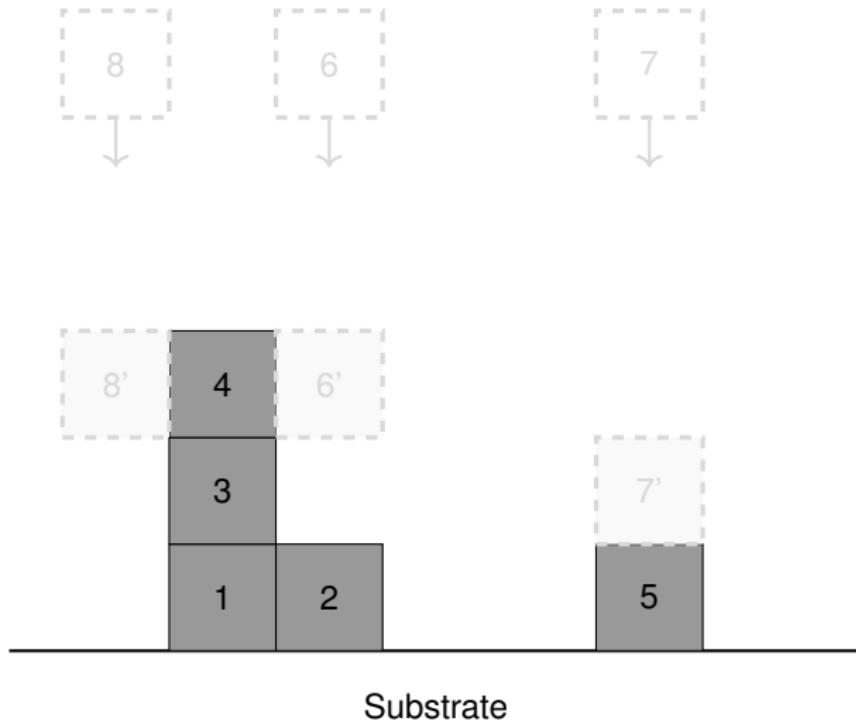
Random deposition with surface relaxation



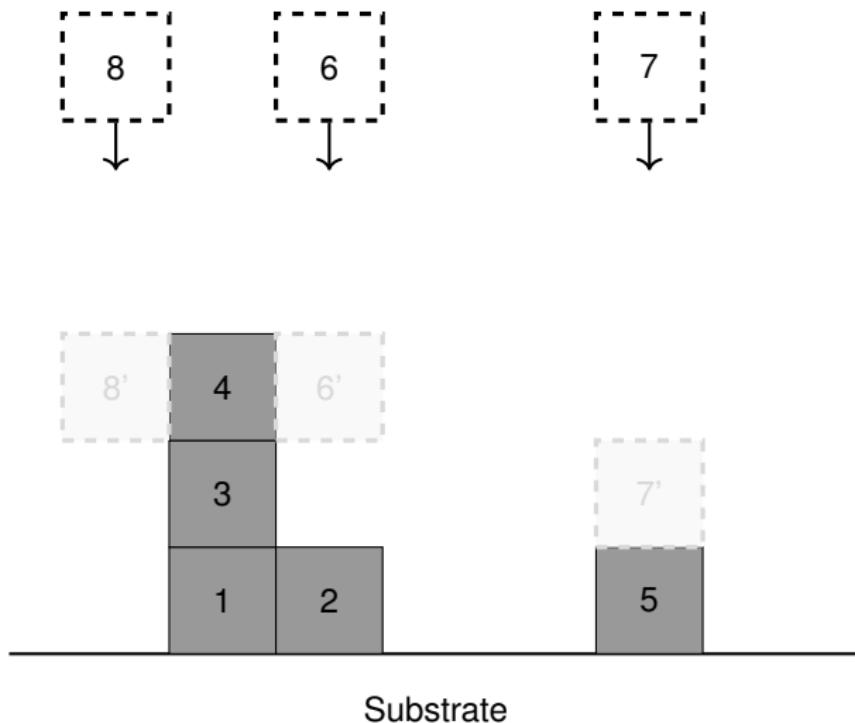
Random deposition with surface relaxation



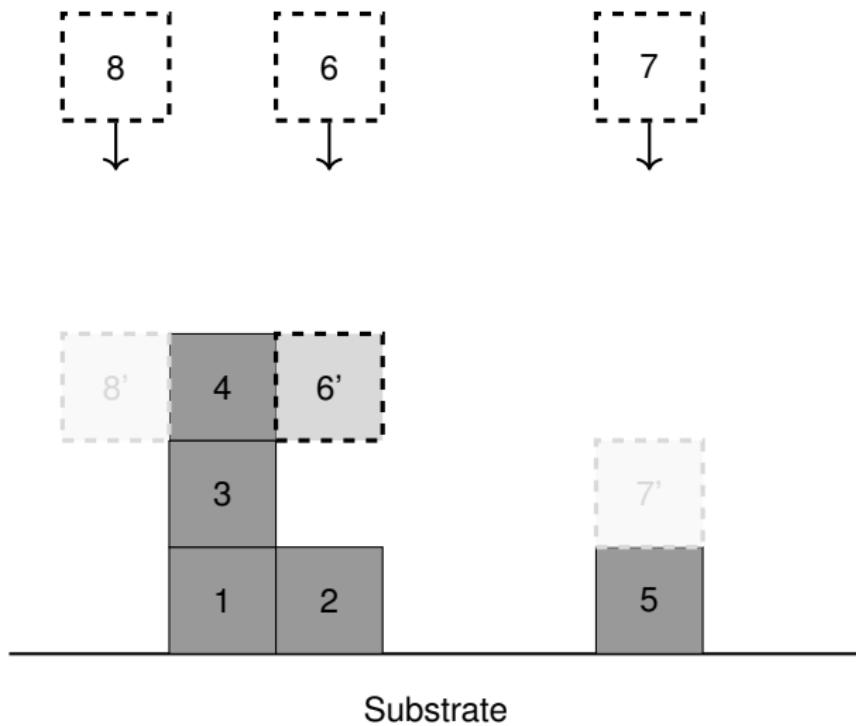
Ballistic deposition



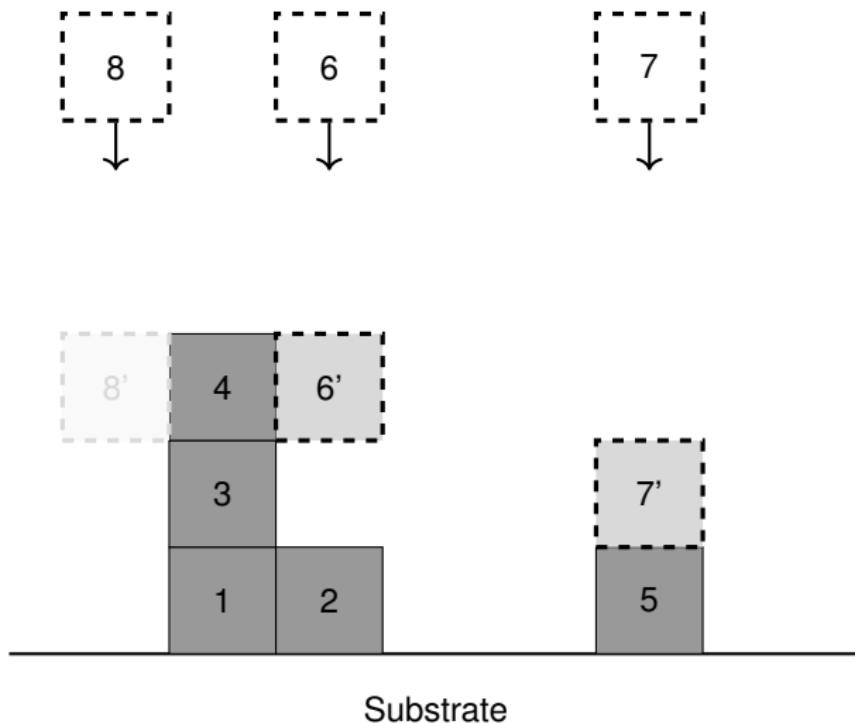
Ballistic deposition



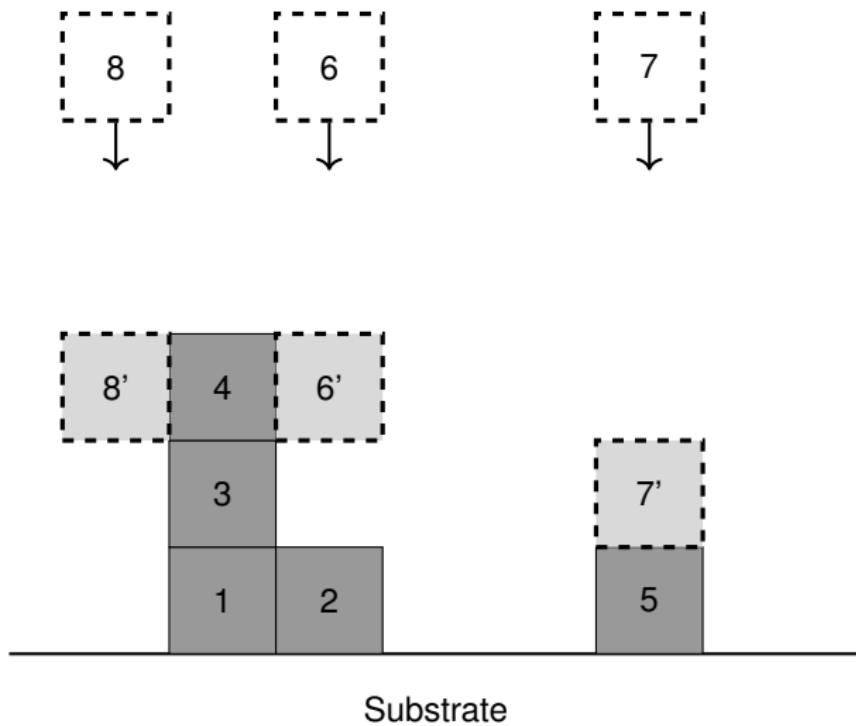
Ballistic deposition



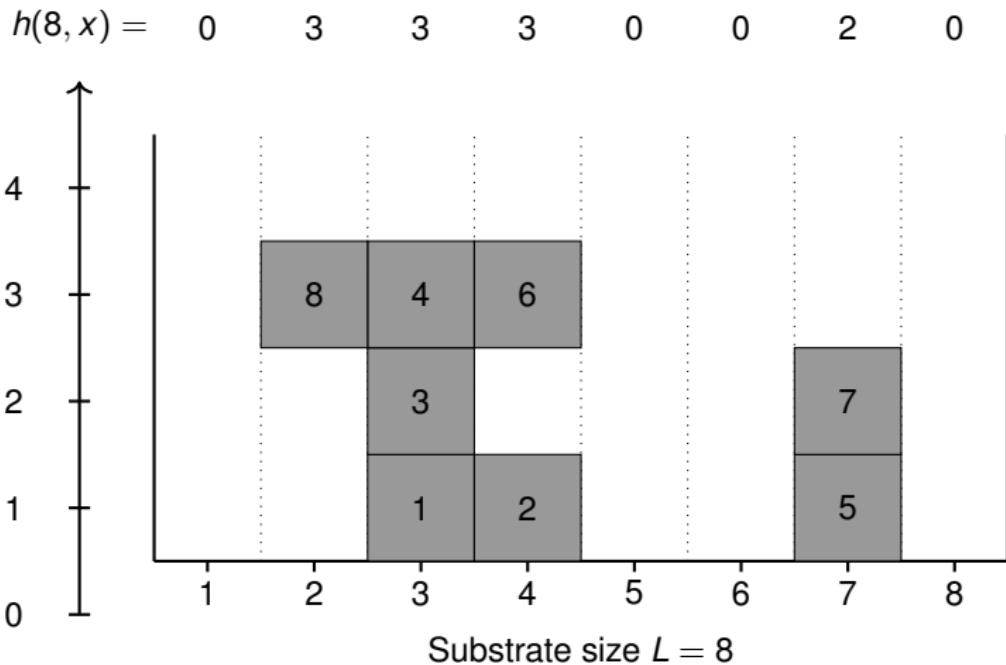
Ballistic deposition



Ballistic deposition



Average height and fluctuation

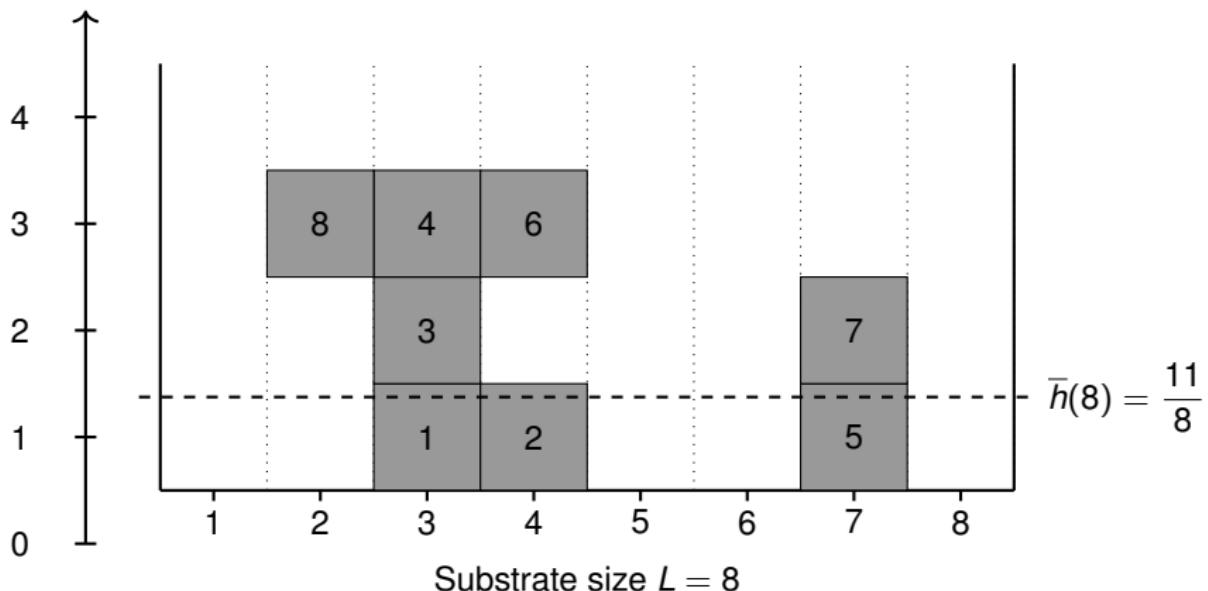


Average height and fluctuation

$$\bar{h}(t) = \frac{1}{L} \sum_{x=1}^L h(t, x)$$

$$\text{Fluctuation } W(L, t) = \sqrt{\frac{1}{L} \sum_{x=1}^L [h(t, x) - \bar{h}(t)]^2}$$

$$h(8, x) = \begin{array}{cccccccc} 0 & 3 & 3 & 3 & 0 & 0 & 2 & 0 \end{array}$$

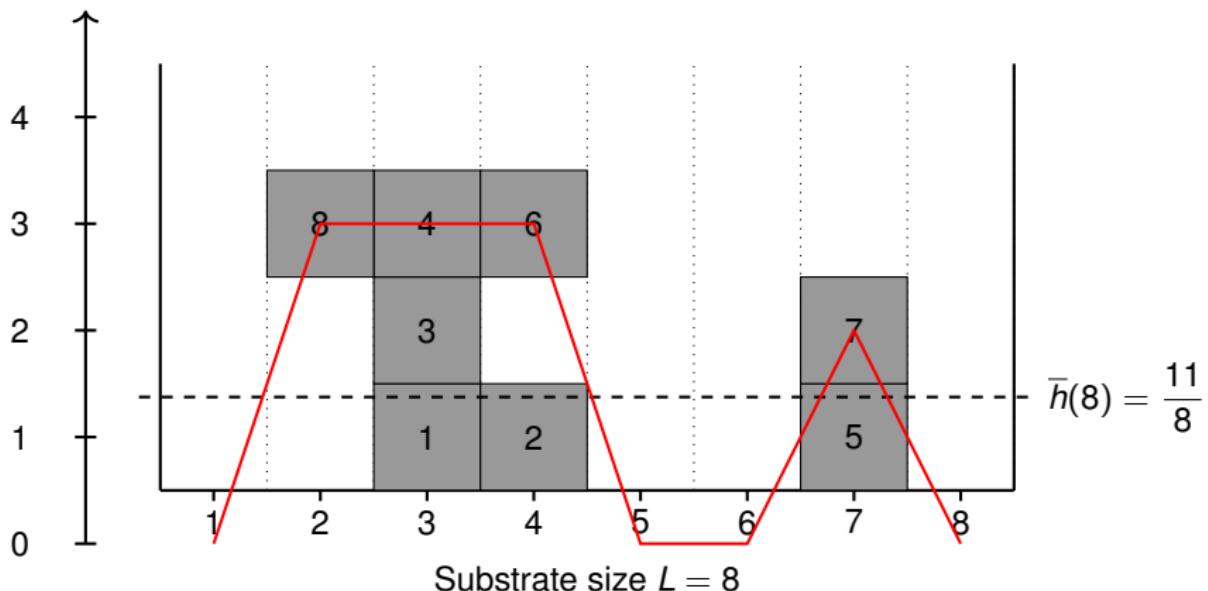


Average height and fluctuation

$$\bar{h}(t) = \frac{1}{L} \sum_{x=1}^L h(t, x)$$

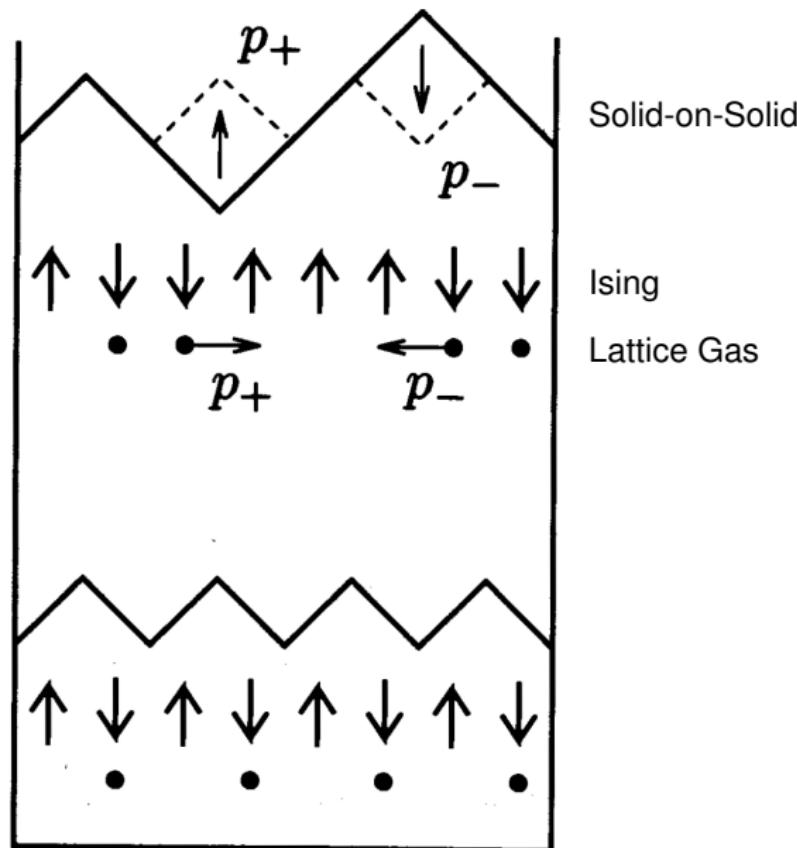
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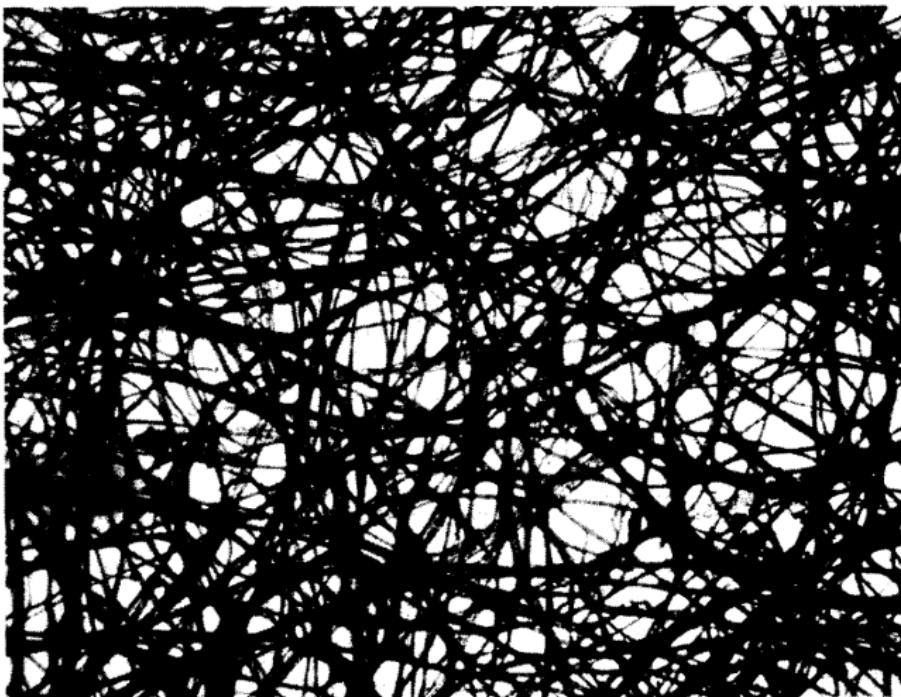


Simulations on
Random deposition vs. Ballistic decomposition

More models? Even more simpler?

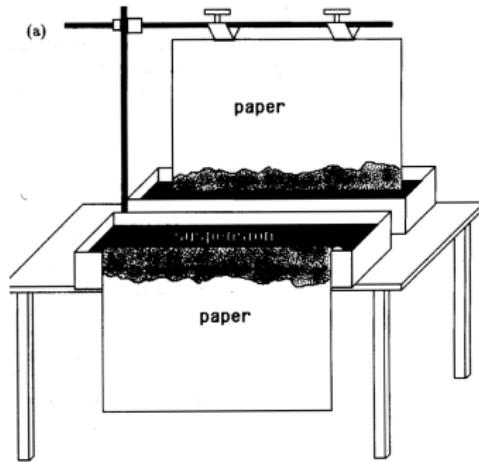


Paper – a random environment



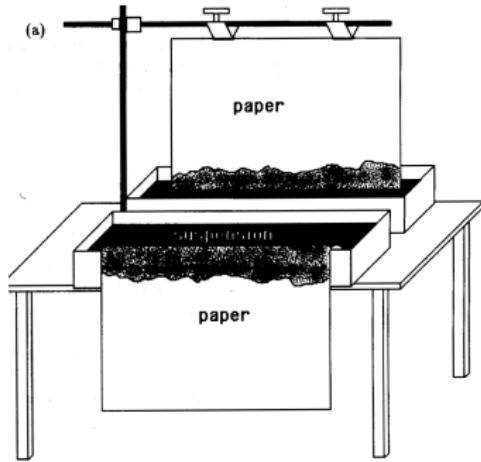
Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

Paper wetting experiment



Barabási, A.-L., Stanley, H. E., 1995

Paper wetting experiment



(b)

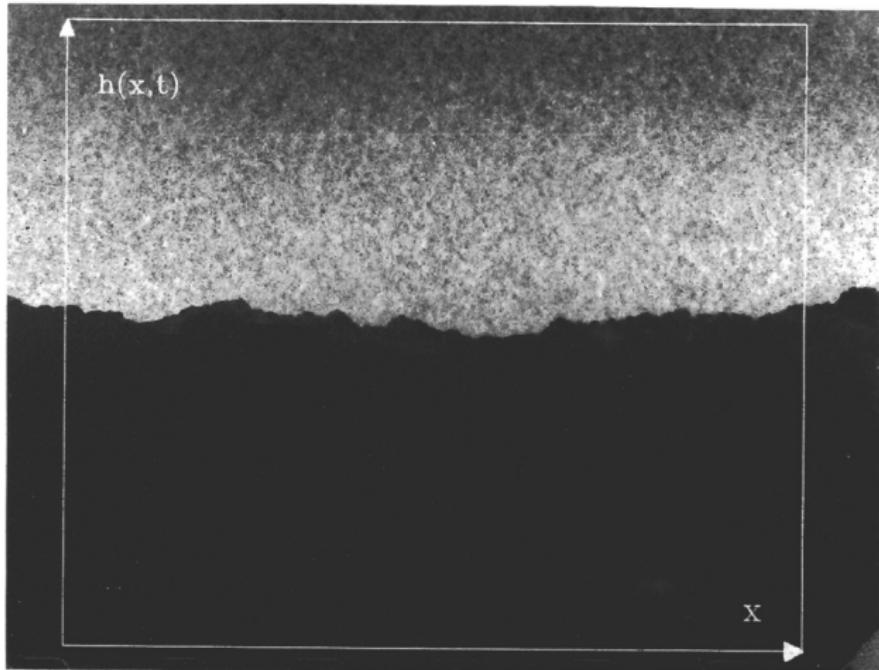


(c)



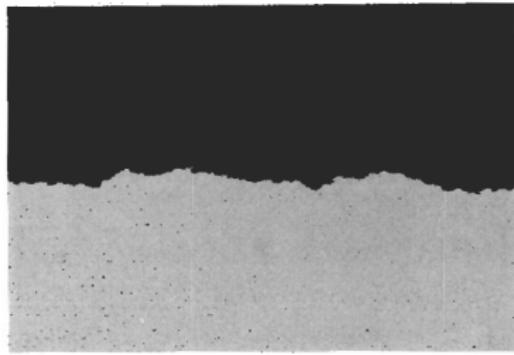
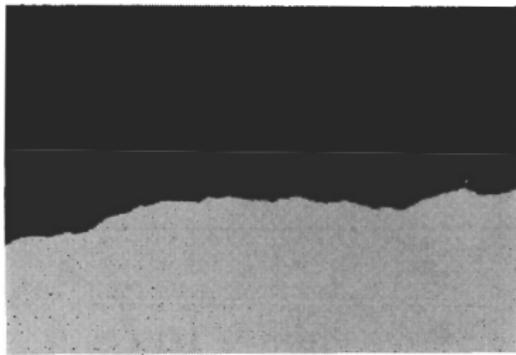
Barabási, A.-L., Stanley, H. E., 1995

Paper burning experiment



Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

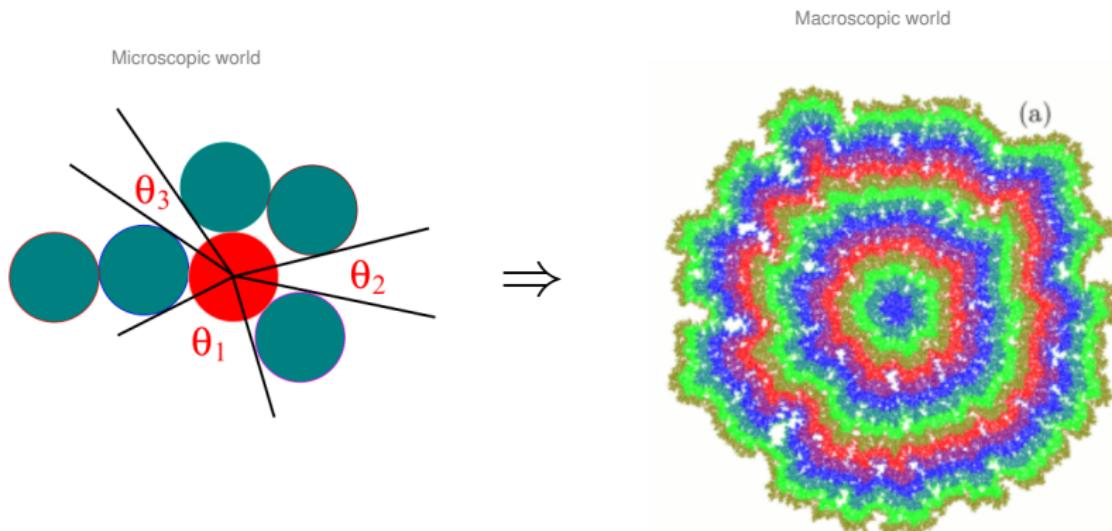
Paper rupture experiment



Kertész, J., Horváth, V. k., Weber, F., *Fractals*, 1993

Rule of replication of **cells**

Replication probability \propto Aperture angle θ_i



Study of growing interfaces in a thin film

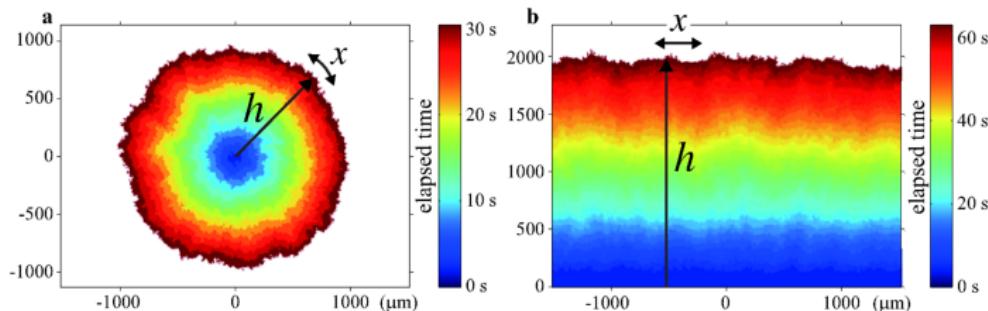
— Convection of nematic liquid crystal*

Show movies !

Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

Study of growing interfaces in a thin film

— Convection of nematic liquid crystal*



Prediction from KPZ equation:

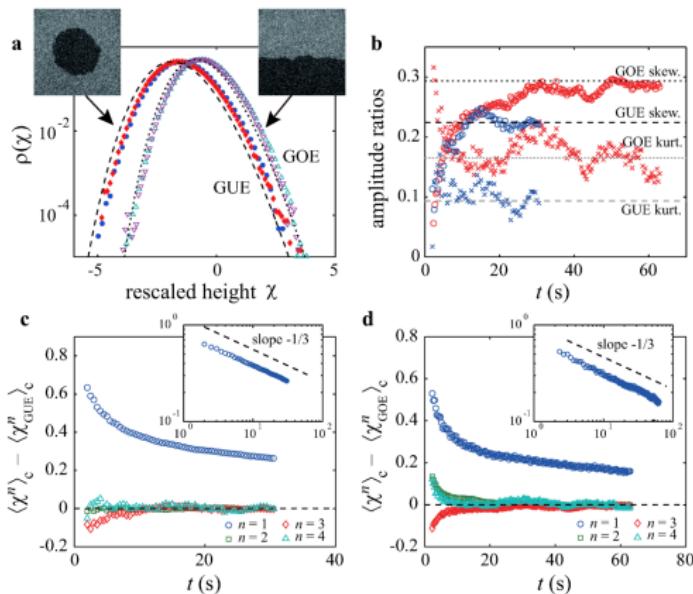
$$h \asymp v_\infty t + (\Gamma t)^{1/3} \xi$$

Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

Study of growing interfaces in a thin film

— Convection of nematic liquid crystal*

$$h \asymp v_\infty t + (\Gamma t)^{1/3} \xi$$



KPZ Equation '86

$$\frac{\partial}{\partial t} h(t, x) = \frac{1}{2} \Delta h(t, x) + \frac{\lambda}{2} (\nabla h)^2 + \dot{W}(t, x) \quad (\text{KPZ})$$



Mehran Kardar (1957 –) Giorgio Parisi (1948 –)



Yicheng Zhang

Kardar, M., Parisi, G., Zhang, Y.-C., *Phys. Rev. Lett.*, 1986

Main References*:

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- Zel'dovich, Y. B., Molchanov, S. A., Ruzmaikin, A. A., & Sokoloff, D. D. (1987). Self-excitation of a nonlinear scalar field in a random medium. *Proc. Nat. Acad. Sci. U.S.A.*, 84(18), 6323–6325.
- Zel'dovich, Y. B., Ruzmaikin, A. A., & Sokoloff, D. D. (1990). *The almighty chance* (Vol. 20) [Translated from the Russian by Anvar Shukurov]. World Scientific Publishing Co., Inc., River Edge, NJ.

* References are produced from *SPDEs-Bib*: <https://github.com/chenle02/SPDEs-Bib>

* Download the bib file: <https://github.com/chenle02/SPDEs-Bib/blob/main/All.bib>