Analysis of Tetris Ballistic Deposition and the Robustness of the KPZ Universality Class

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Auburn University

Acknwolegement

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Emerging Synergies between Stochastic Analysis and Statistical Mechanics
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Math 7820/30: Applied Stochastic Processes (2023/24):

Mauricio Montes and Ian Ruau

Plan

Tetromino Pieces

Introduction to growth model and SPDE

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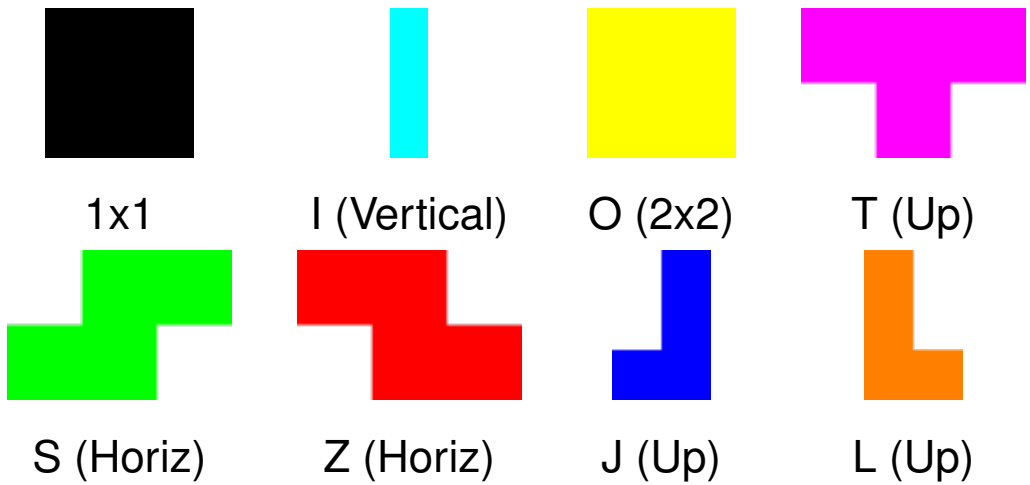
Tetromino Pieces

Introduction to growth model and SPDE

Tetrominoes



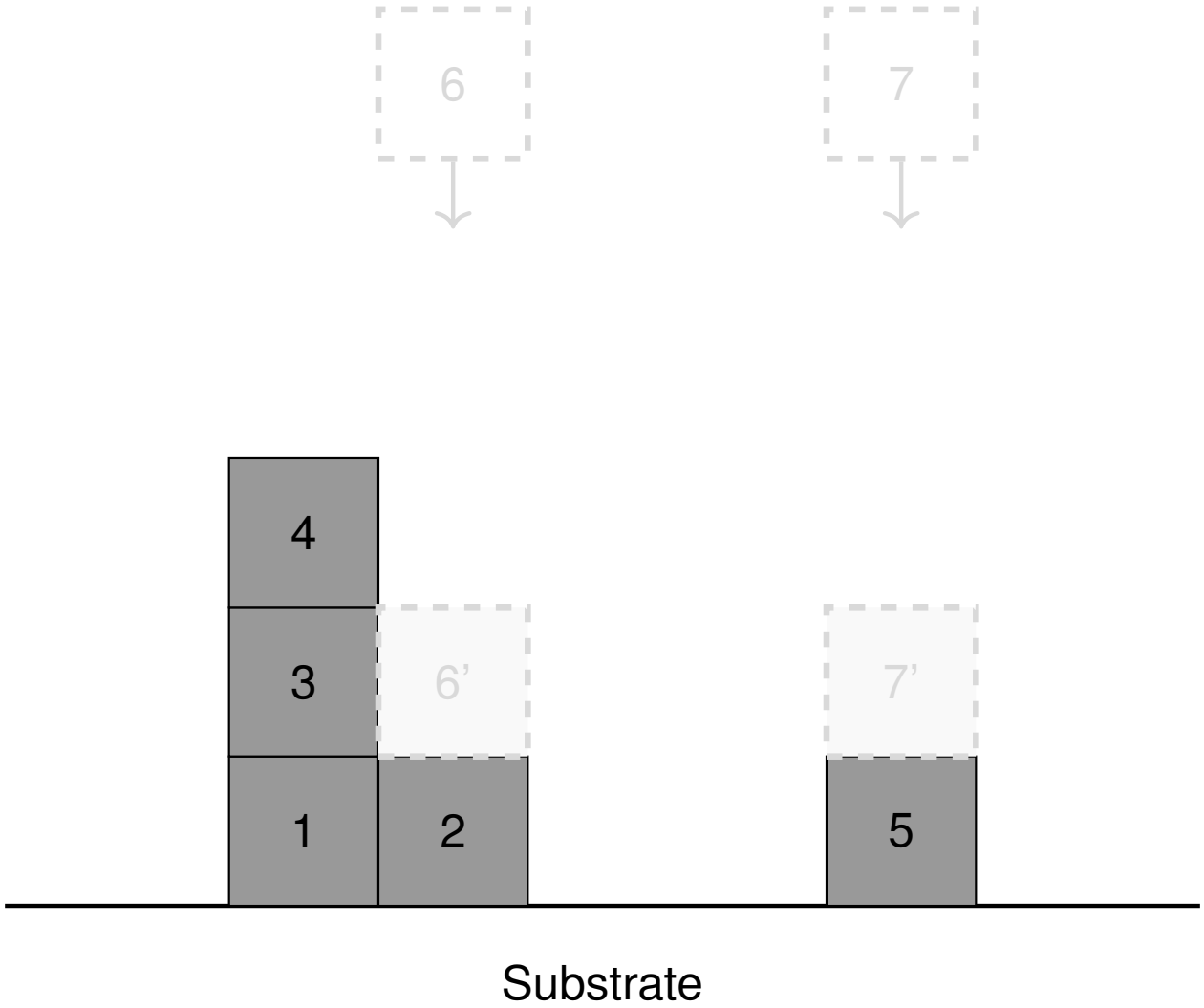
Sticky pieces are shown (non-bordered). Non-sticky variants use bordered images with the same names plus the suffix `_bordered`. All orientations are available for I, J, L, T, S, Z; O is a 2x2 square; 1x1 is an extra single-site piece.

Plan

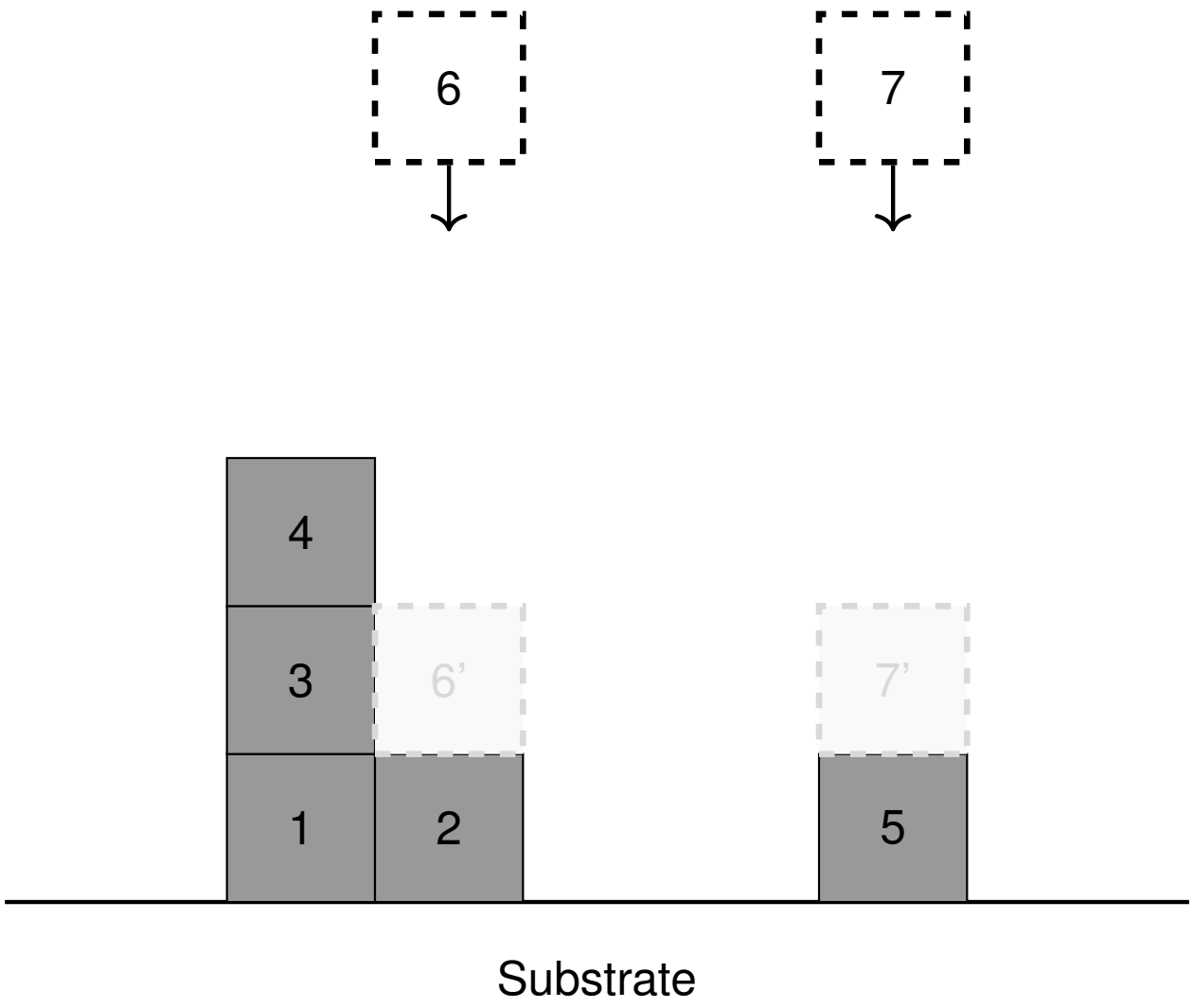
Tetromino Pieces

Introduction to growth model and SPDE

Random deposition



Random deposition



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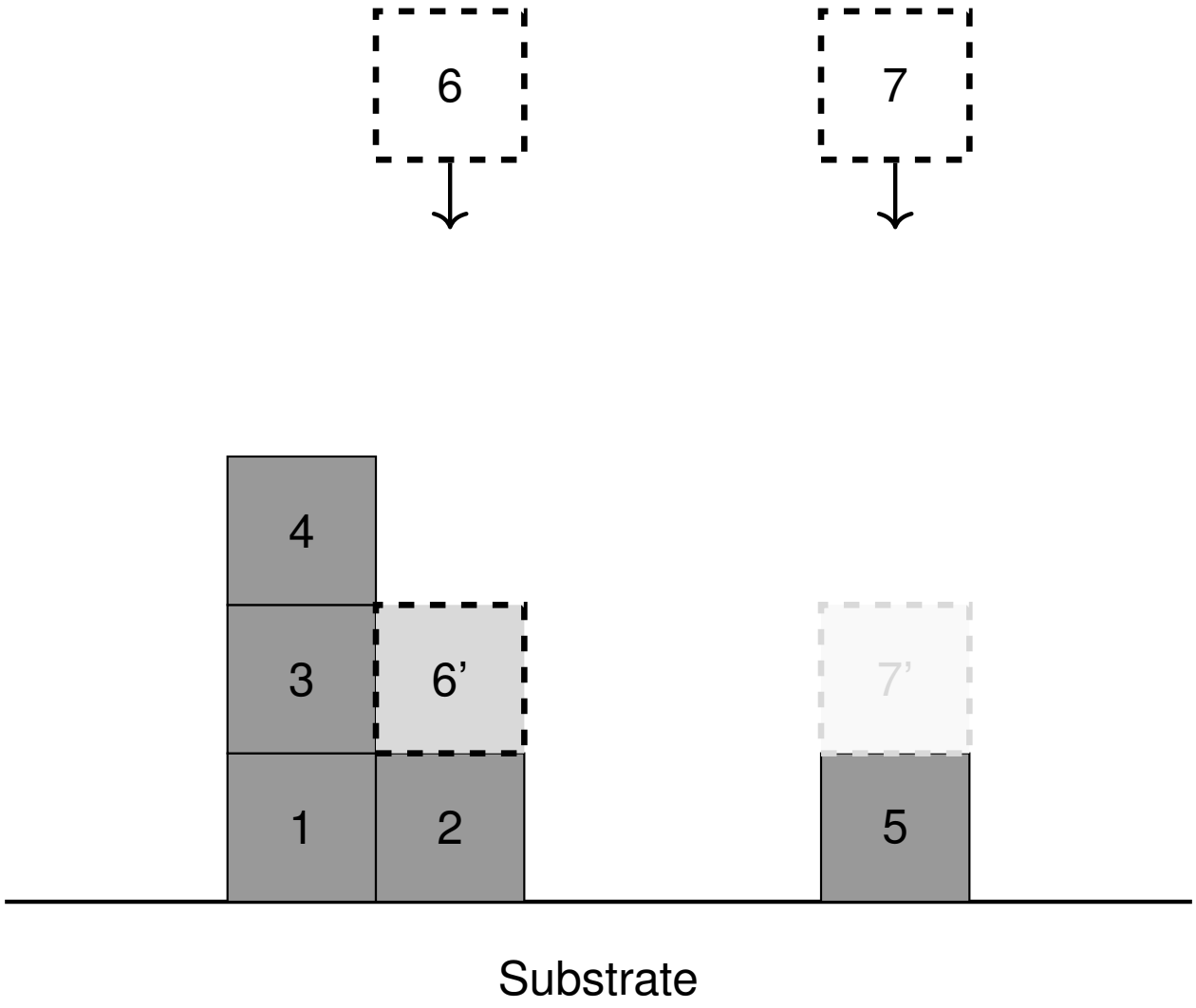
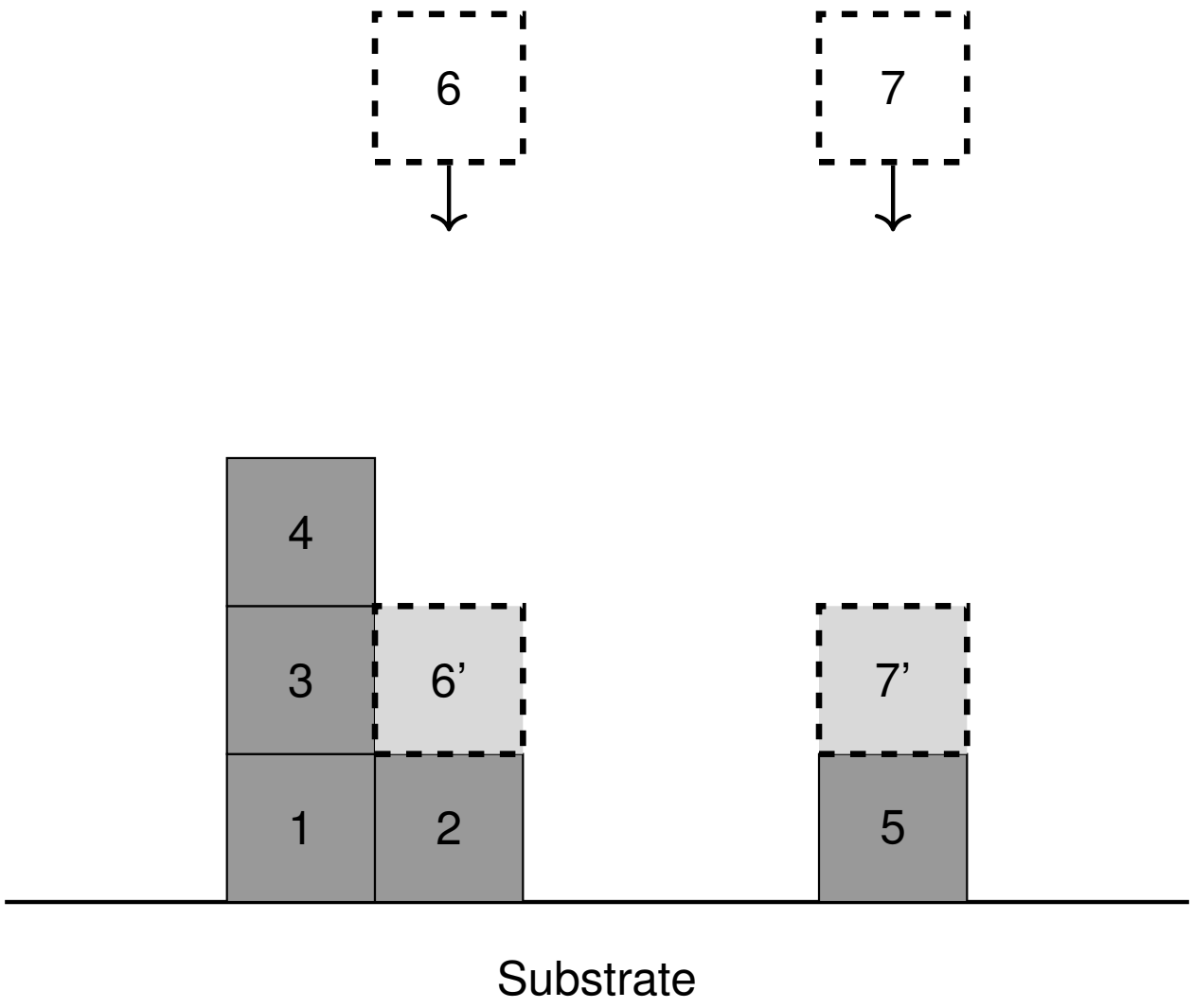
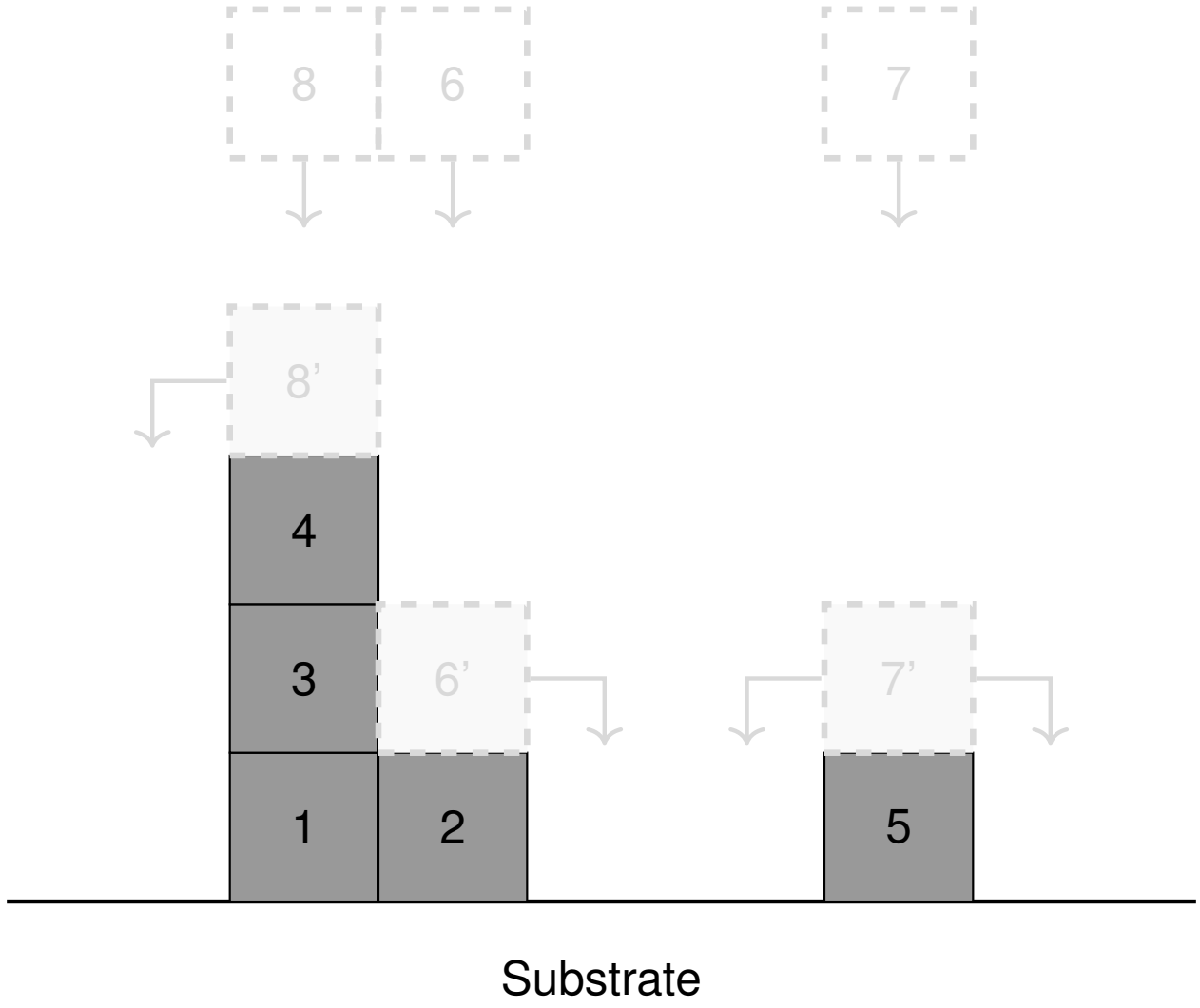


Random deposition



Random deposition with surface relaxation



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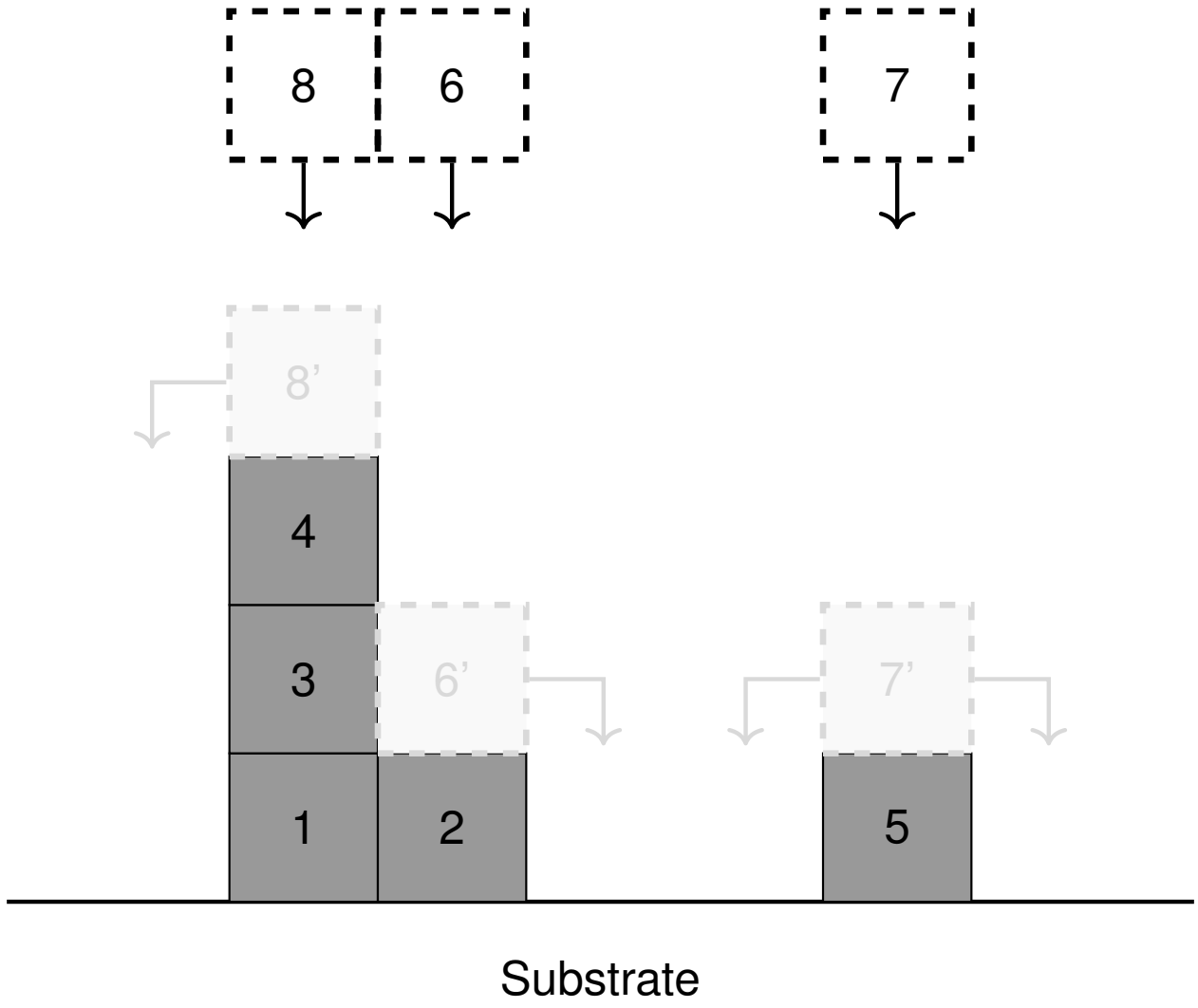
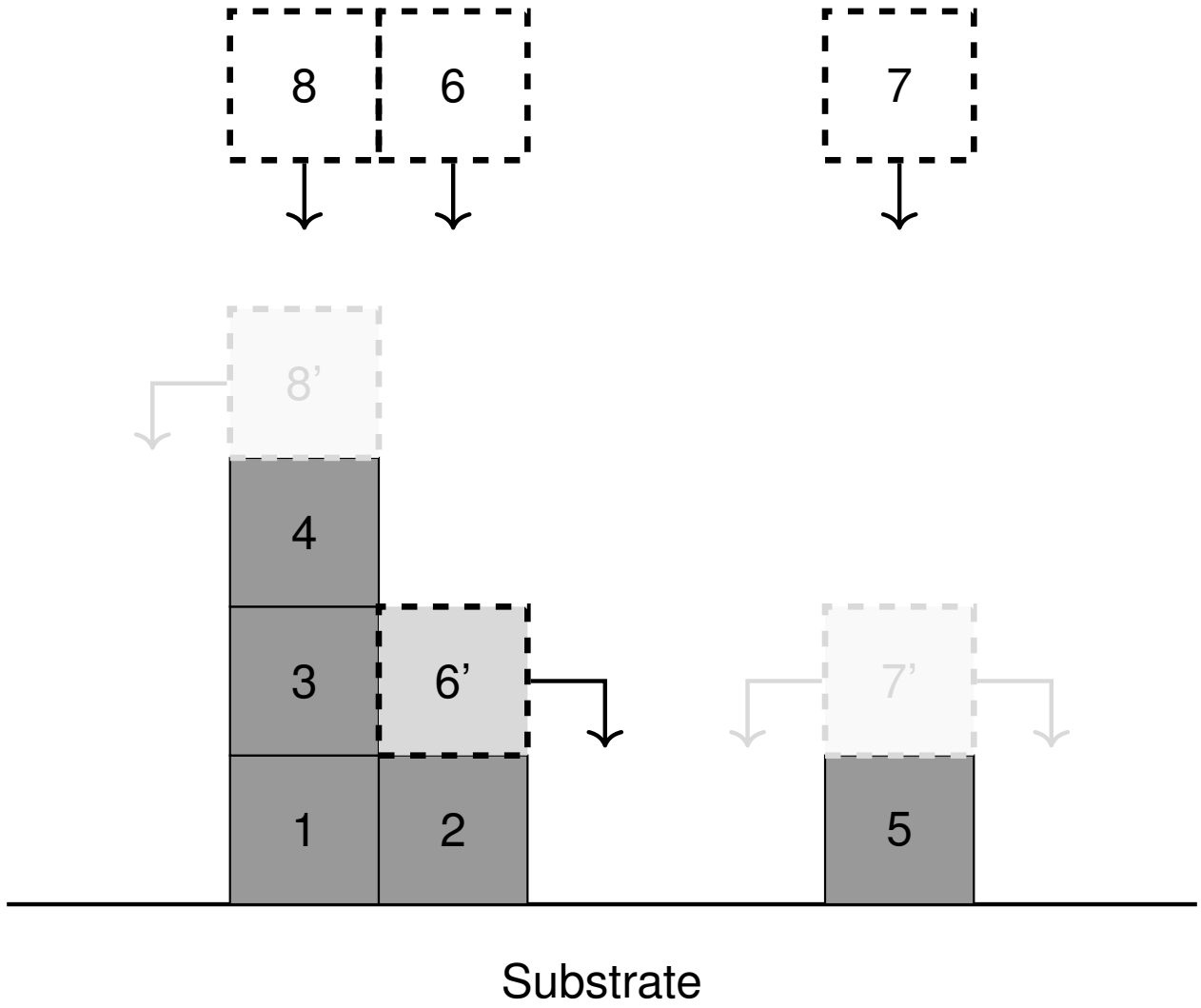
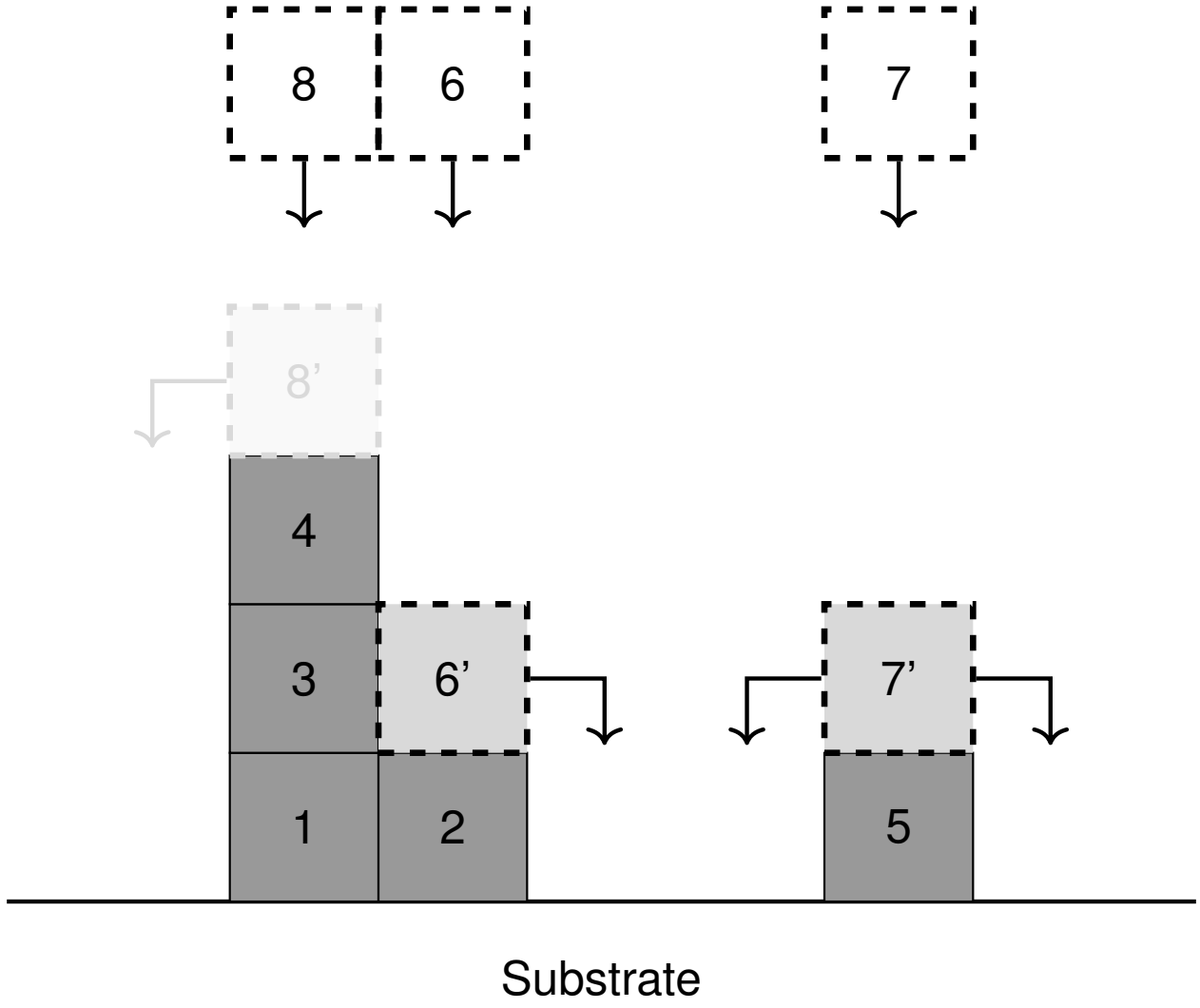


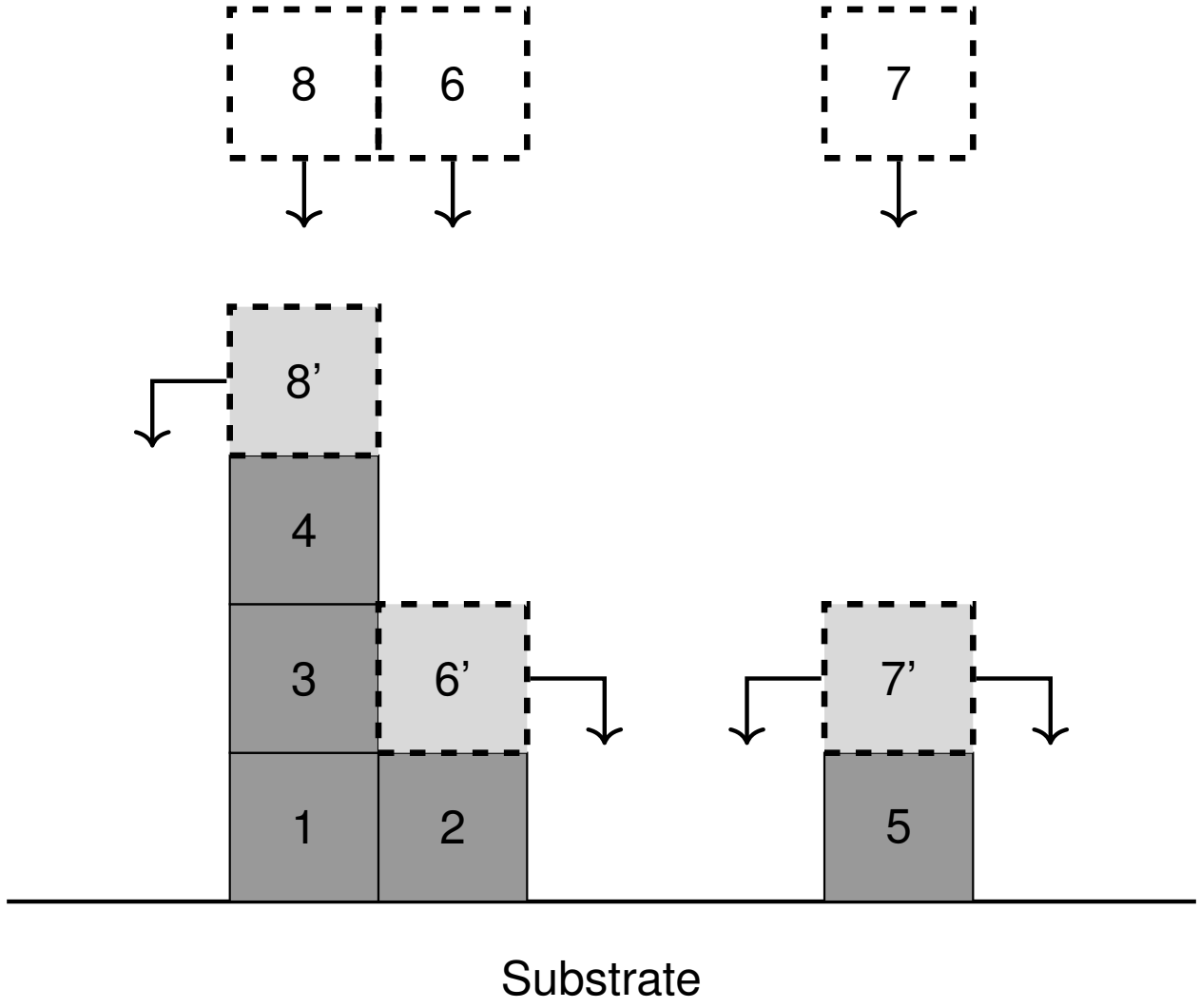
Random deposition with surface relaxation



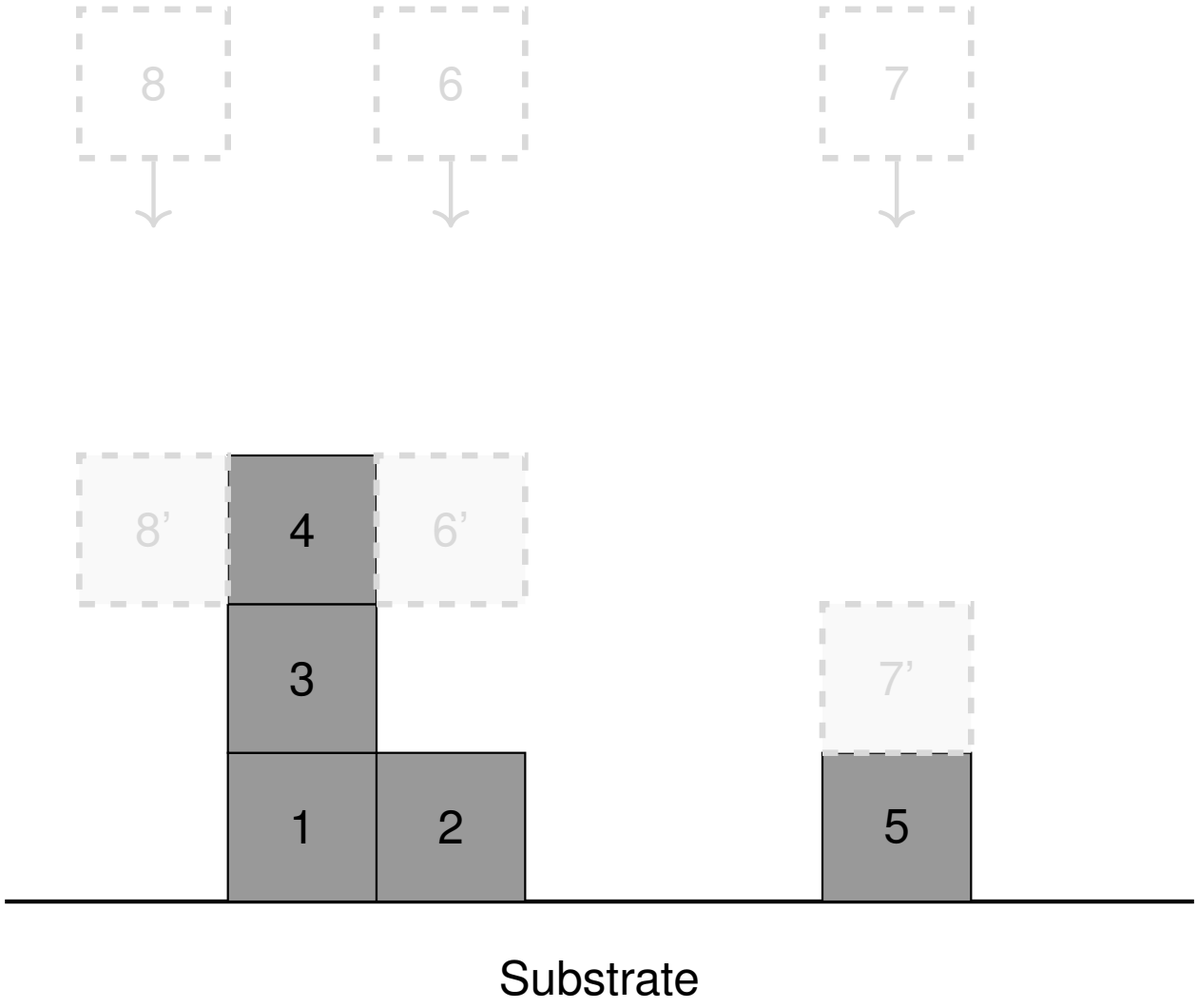
Random deposition with surface relaxation



Random deposition with surface relaxation



Ballistic deposition



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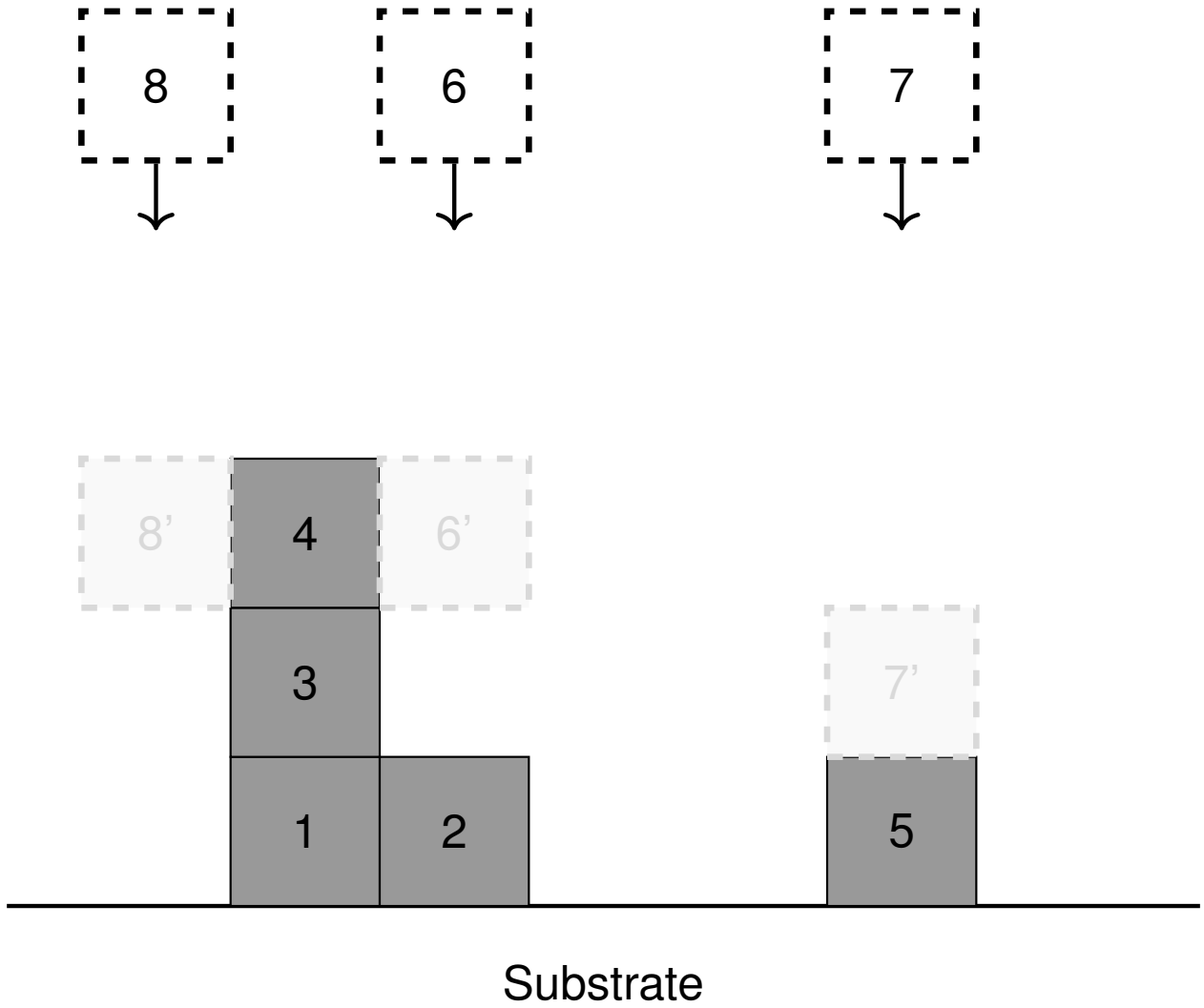
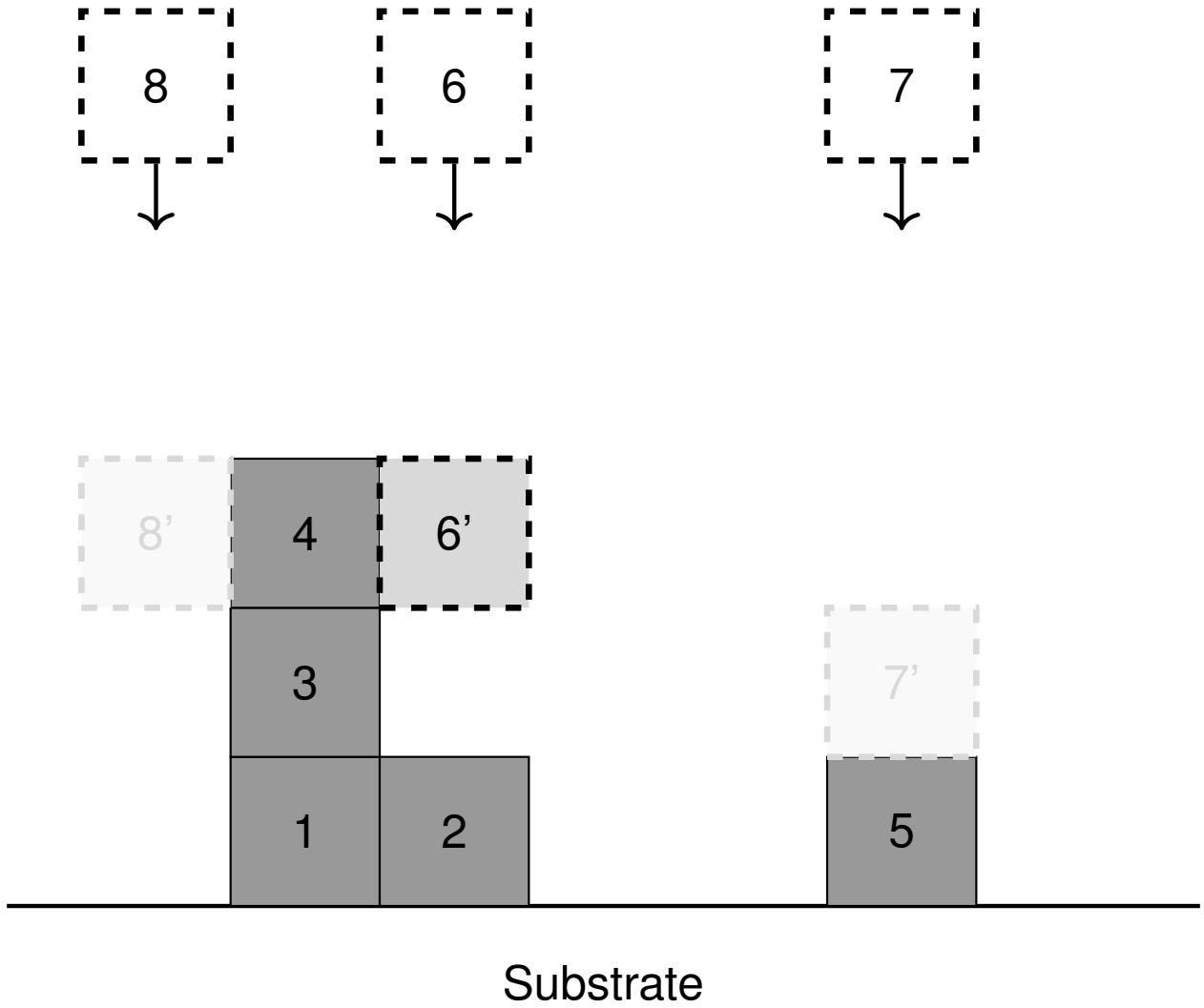
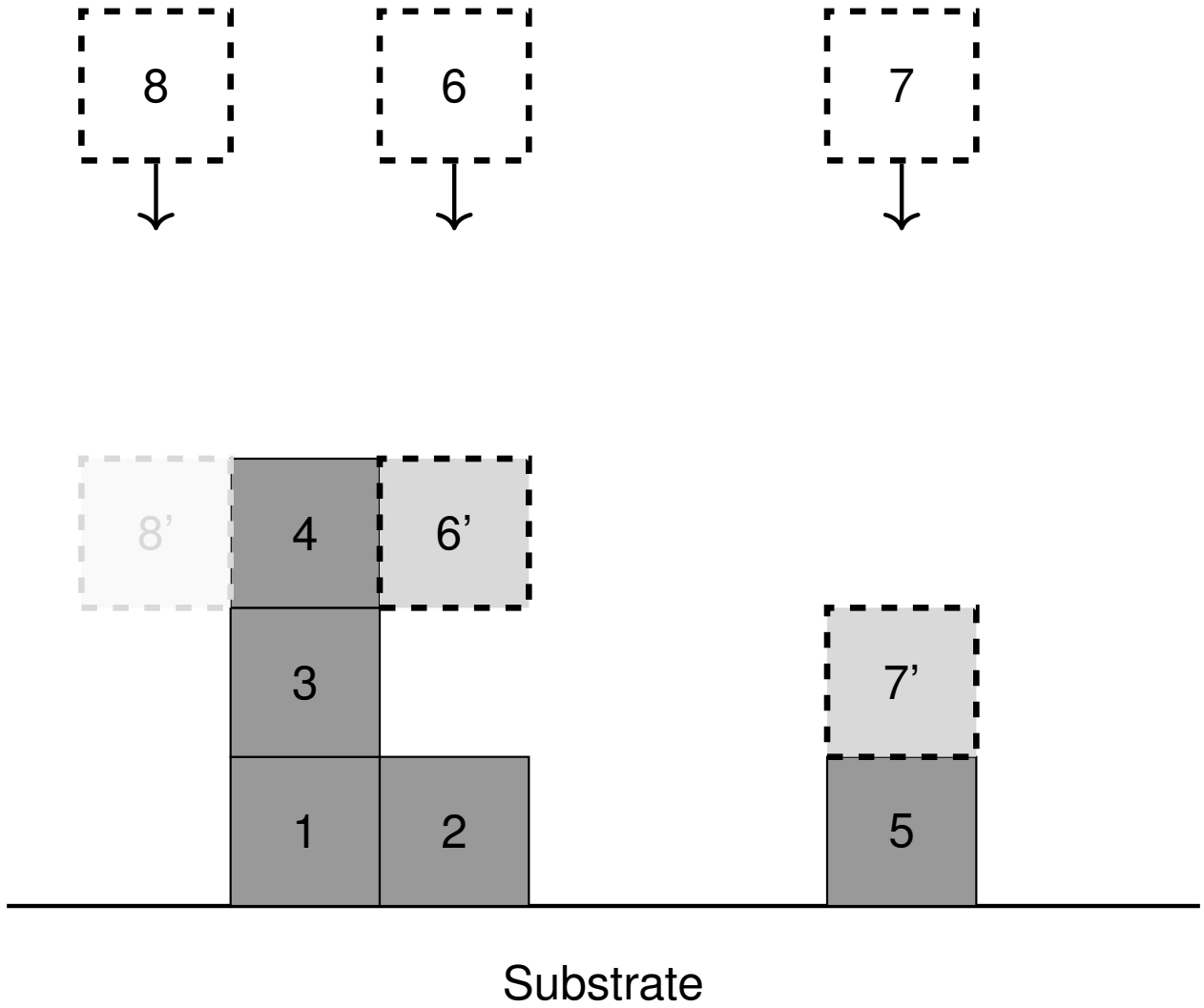


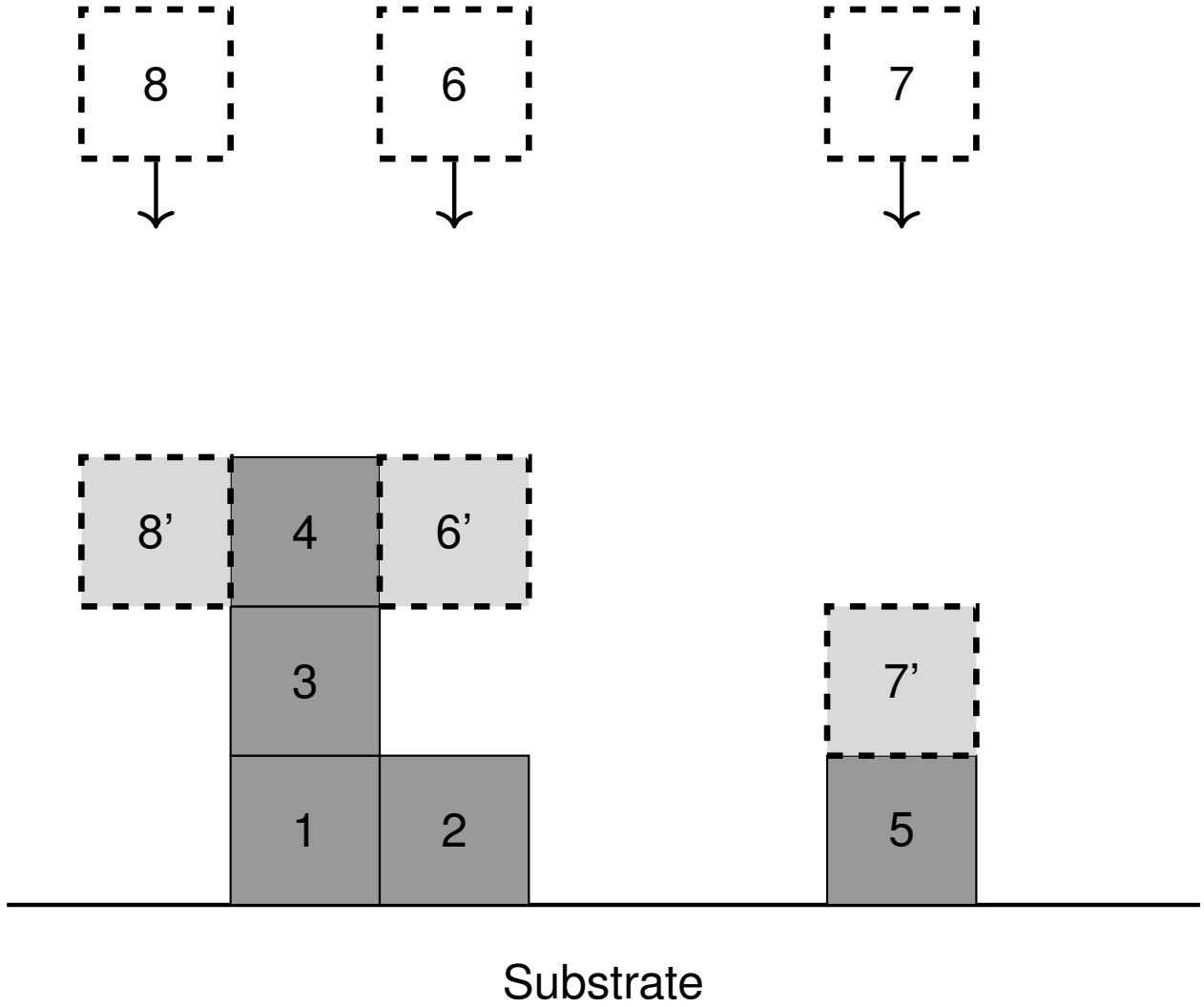
Ballistic deposition



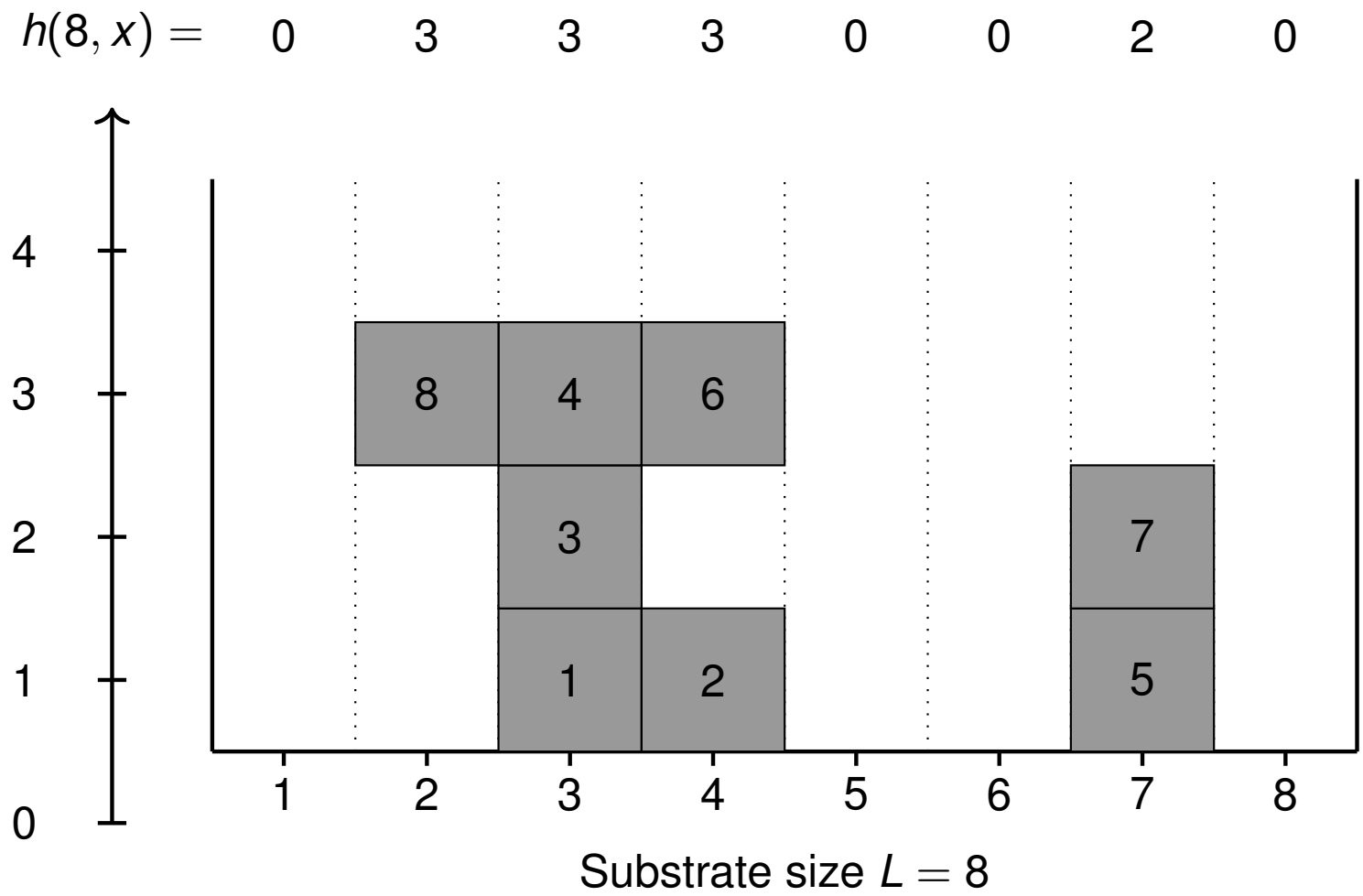
Ballistic deposition



Ballistic deposition

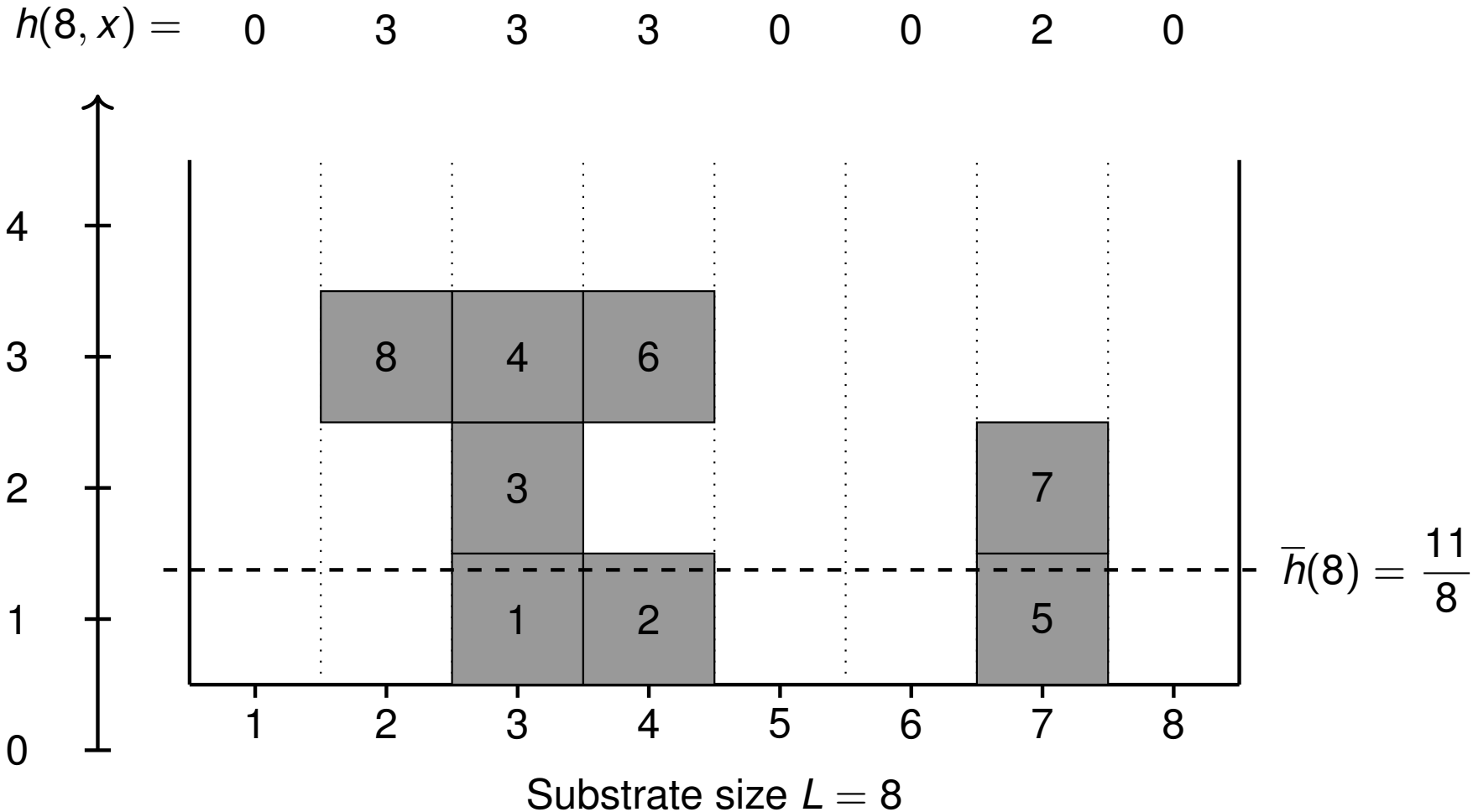


Average height and fluctuation



Average height and fluctuation

$$\overline{h}(t) = \frac{1}{L}\sum_{x=1}^{L} h(t,x) \qquad \text{Fluctuation } W(L,t) = \sqrt{\frac{1}{L}\sum_{x=1}^{L}\left[h(t,x) - \overline{h}(t)\right]^2}$$



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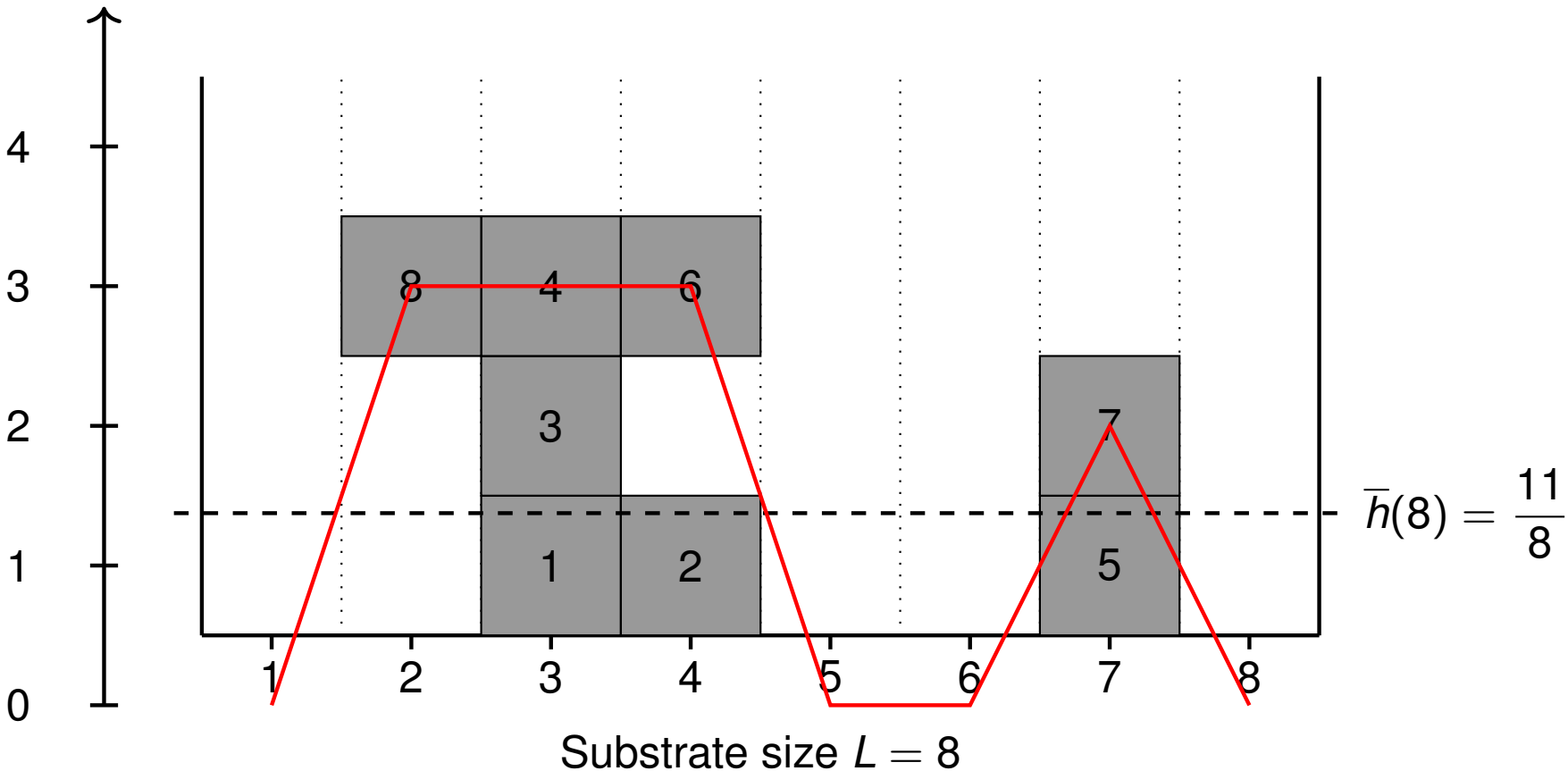
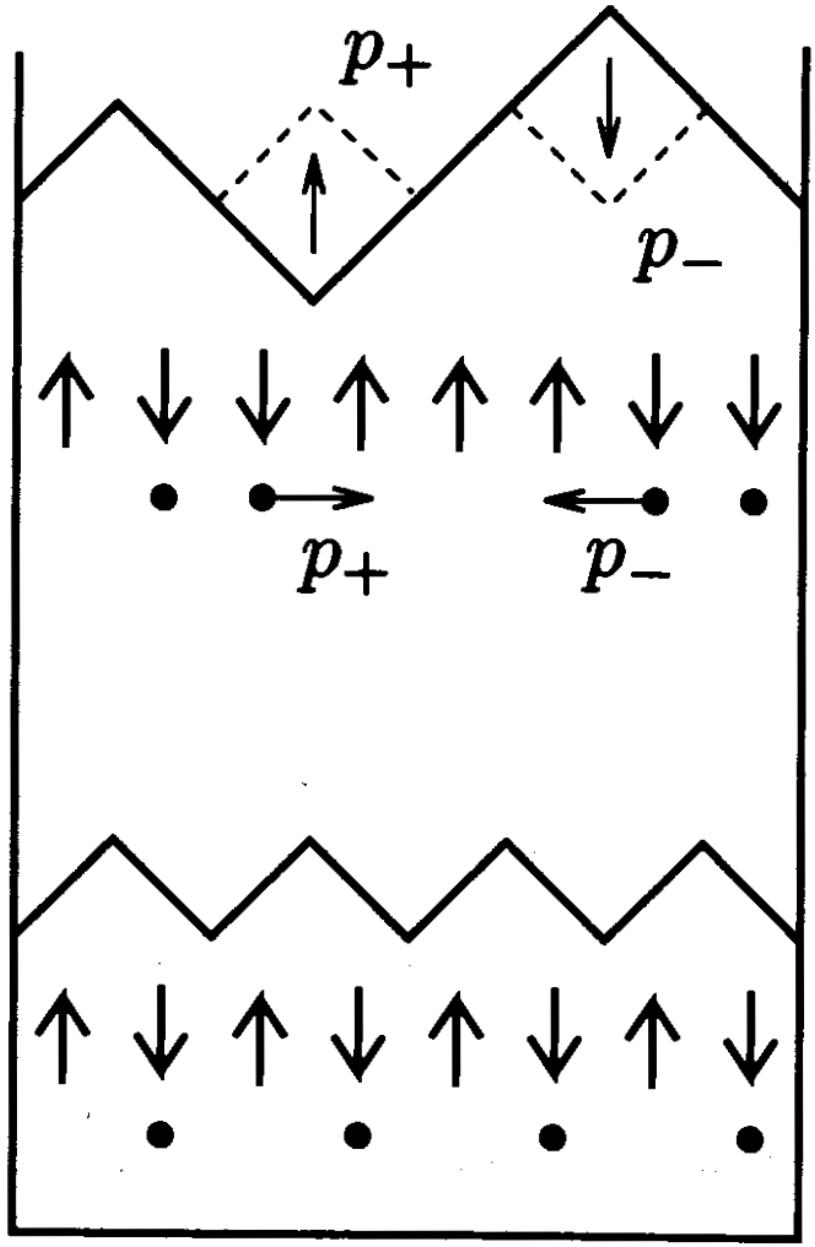
$h(8,x) =$ 0 3 3 3 0 0 2 0



Simulations on

Random deposition vs. Ballistic decomposition

More models? Even more simpler?

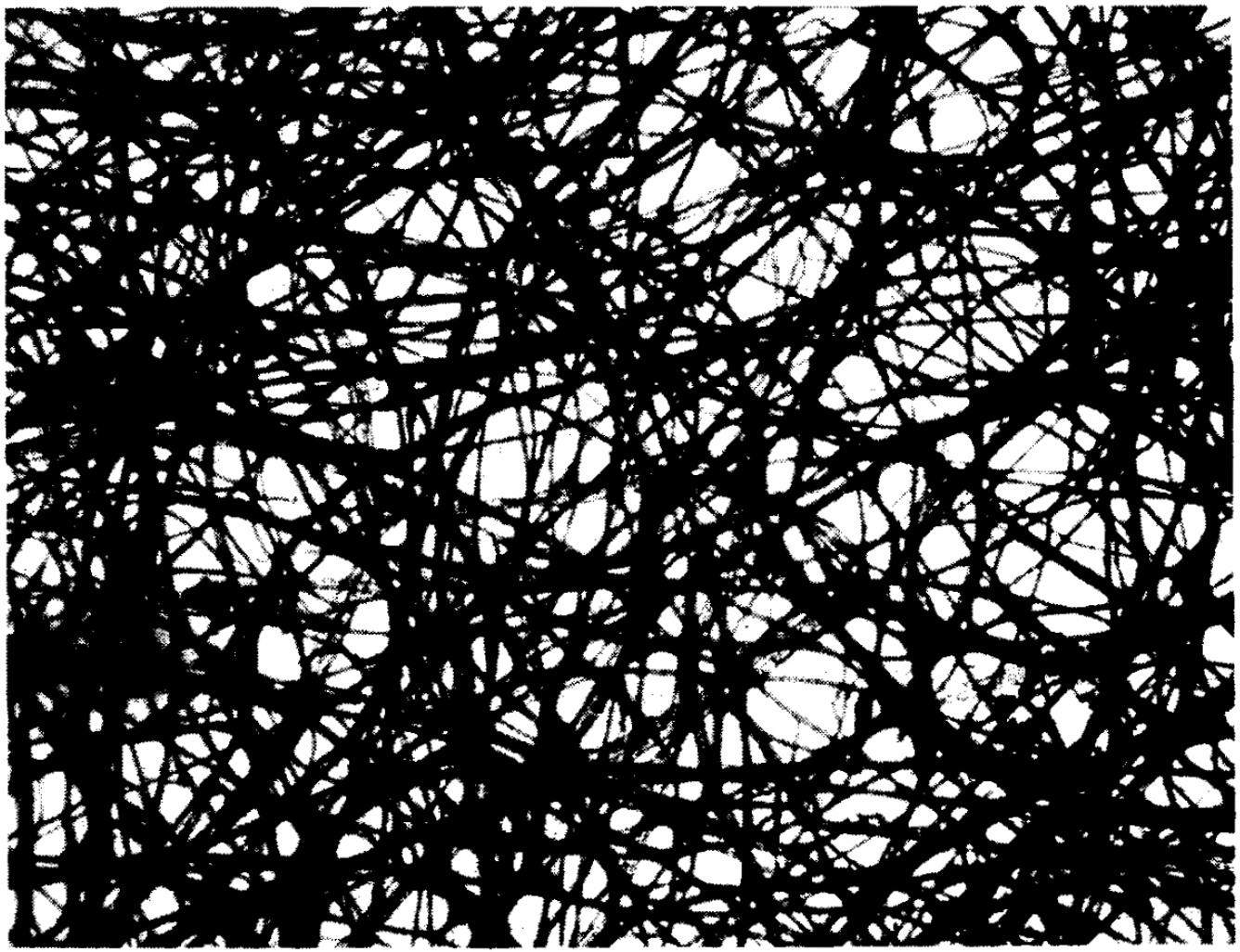


Solid-on-Solid

Ising

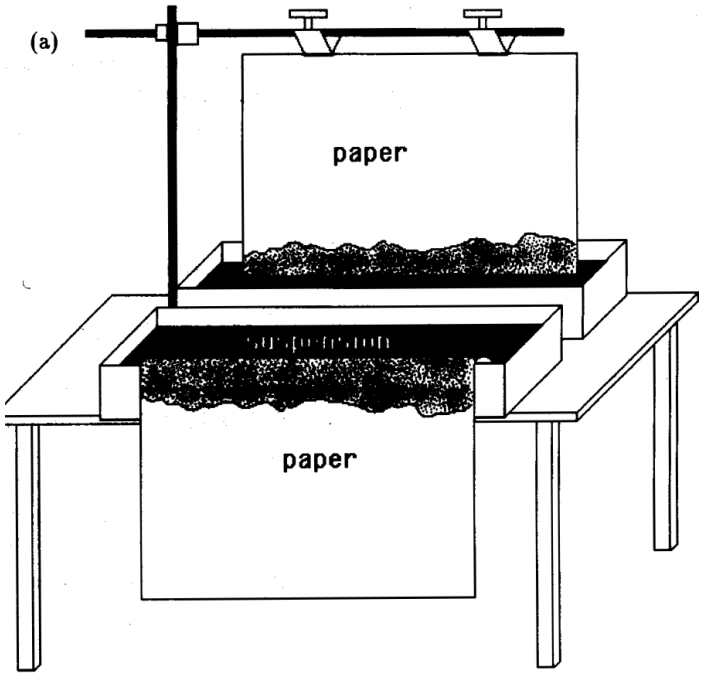
Lattice Gas

Paper – a random environment



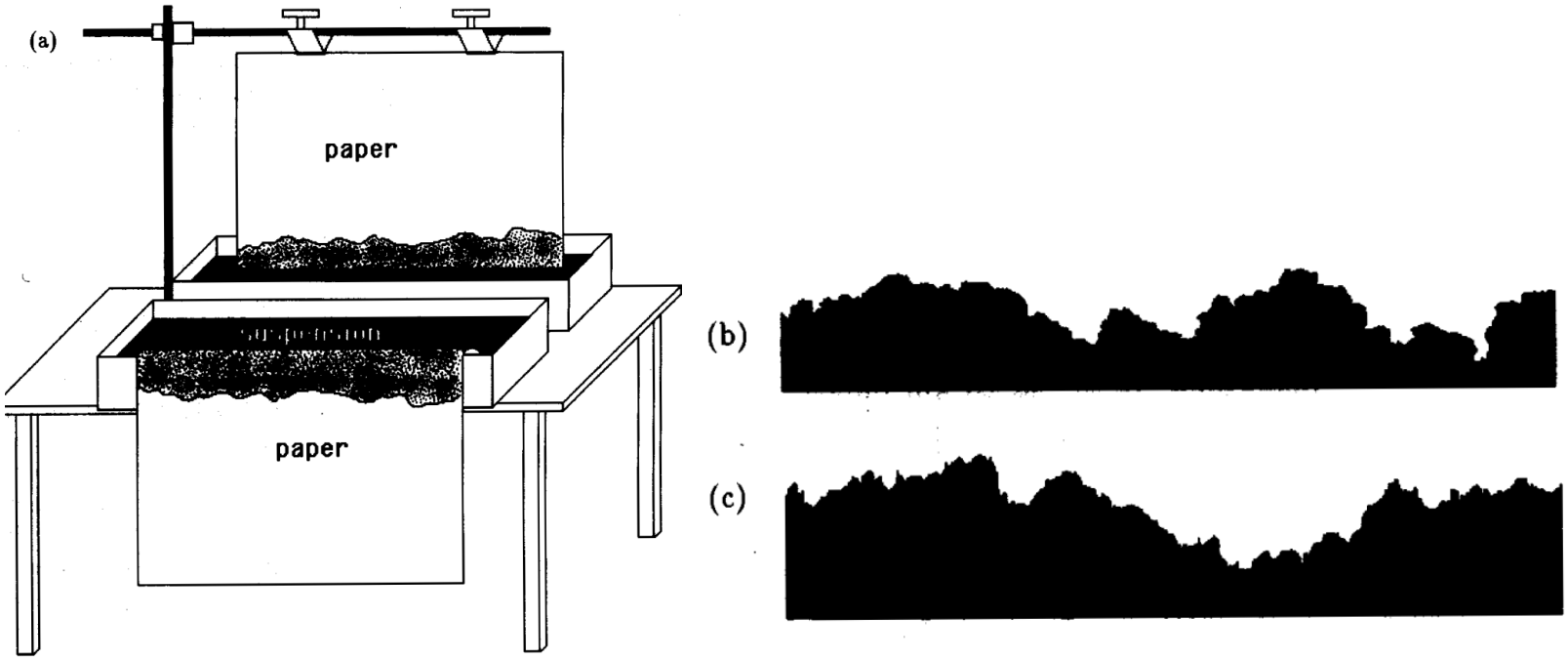
Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

Paper wetting experiment



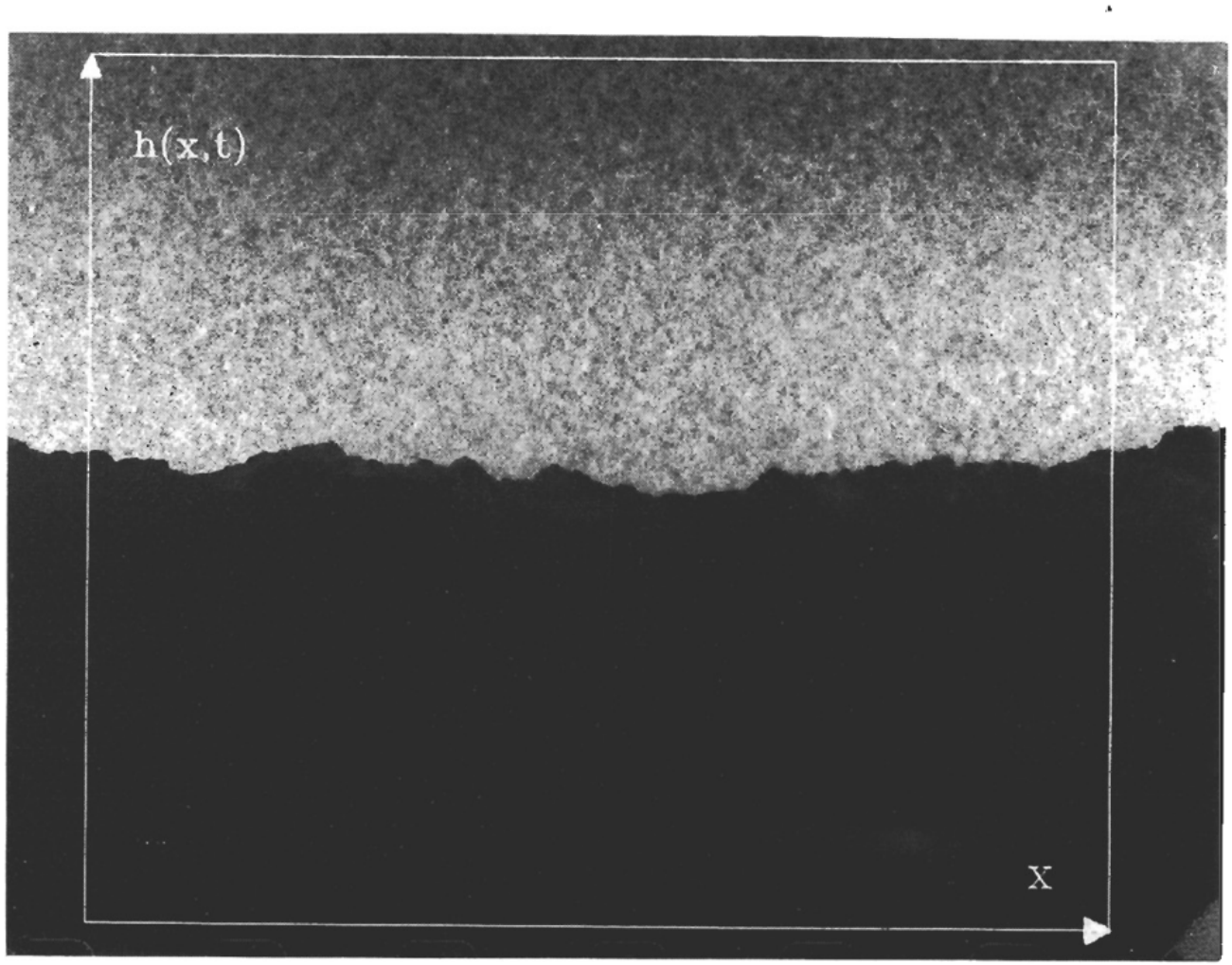
Barabási, A.-L., Stanley, H. E., 1995

Paper wetting experiment



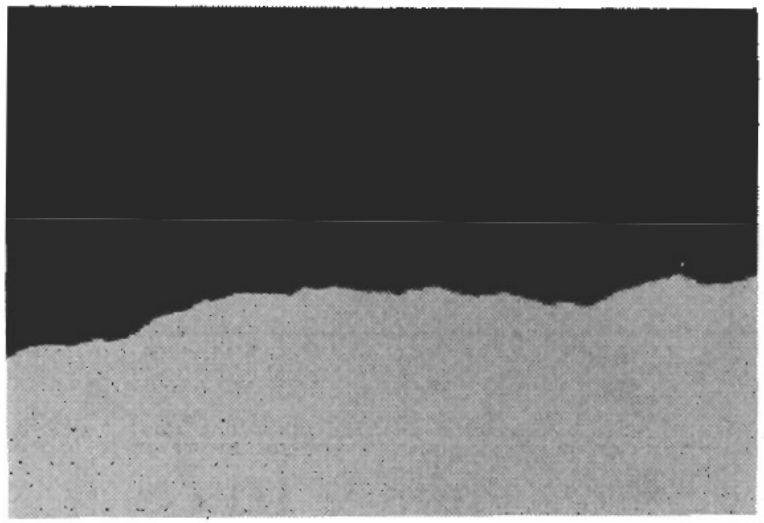
Barabási, A.-L., Stanley, H. E., 1995

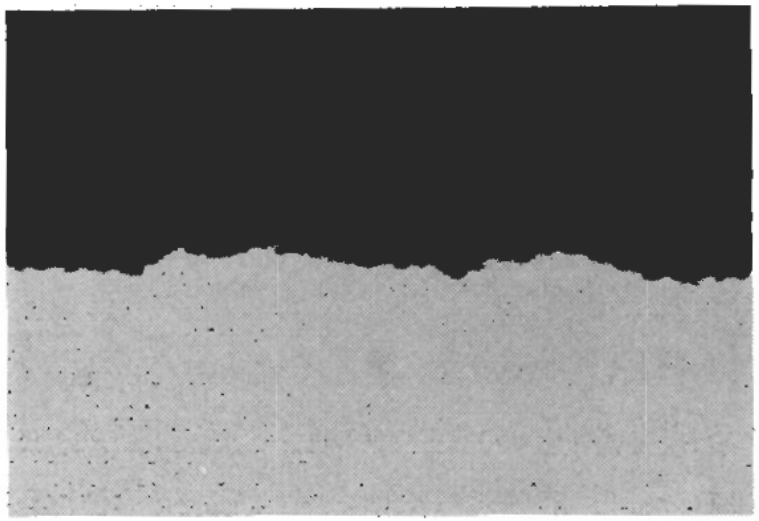
Paper burning experiment



Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

Paper rupture experiment



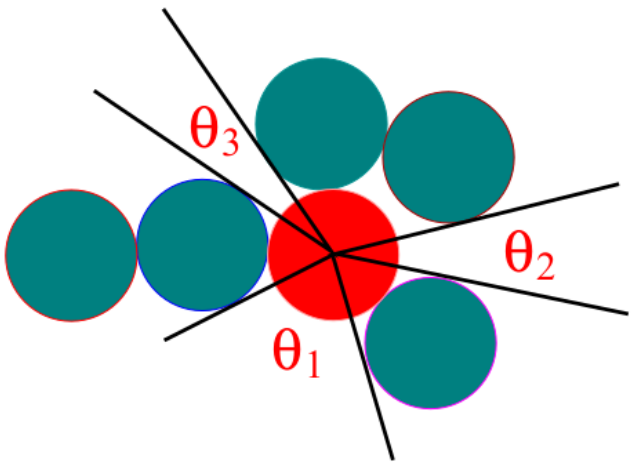


Kertész, J., Horváth, V. k., Weber, F., *Fractals*, 1993

Rule of replication of **cells**

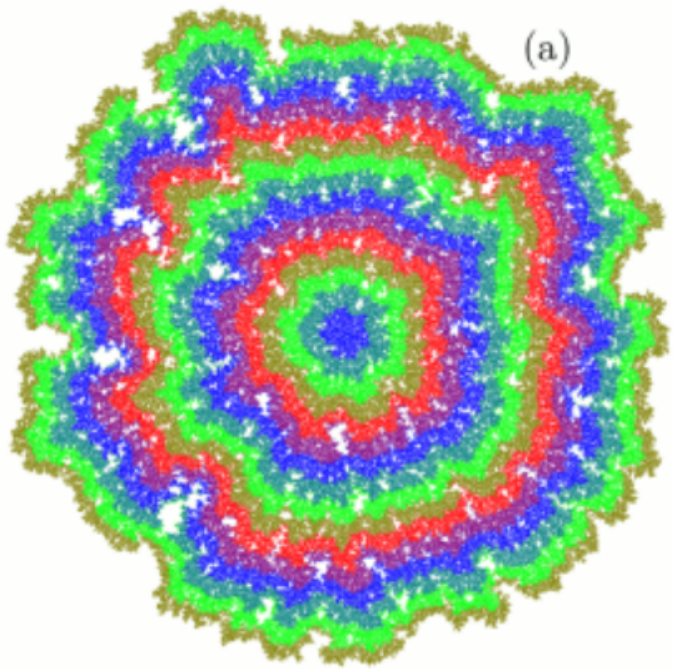
Replication probability $\propto$ Aperture angle θ_i

Microscopic world



$\Rightarrow$

Macroscopic world



Santalla, S. N., Ferreira, S. C., *Phys. Rev. E*, 2018

Study of growing interfaces in a thin film

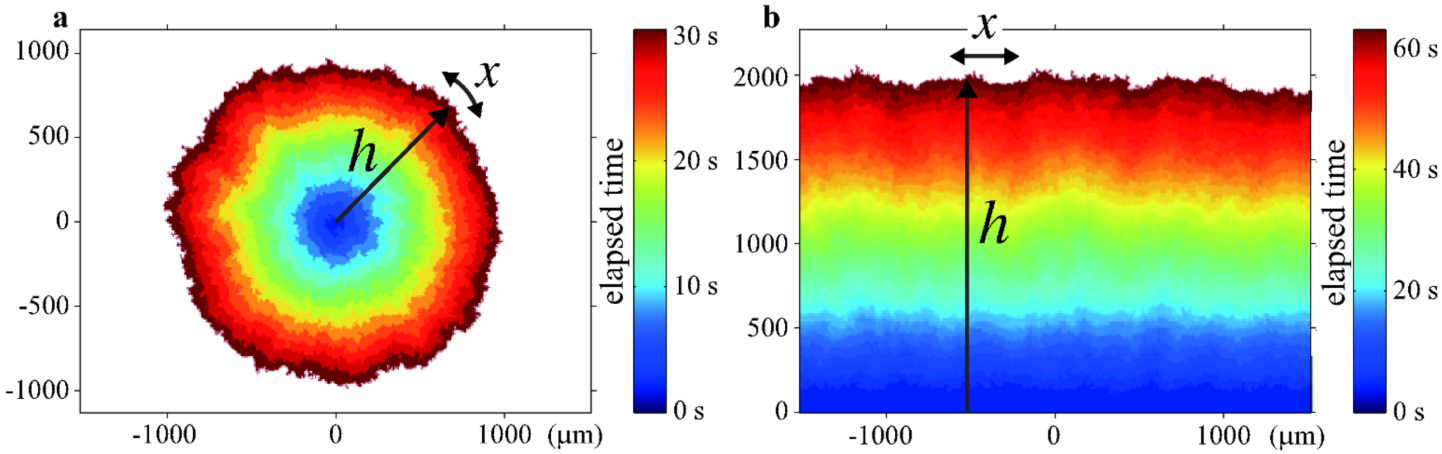
— Convection of nematic liquid crystal*

Show movies !

Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

Study of growing interfaces in a thin film

— Convection of nematic liquid crystal*



Prediction from KPZ equation:

$$h \asymp v_\infty t + (\Gamma t)^{1/3} \xi$$

Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

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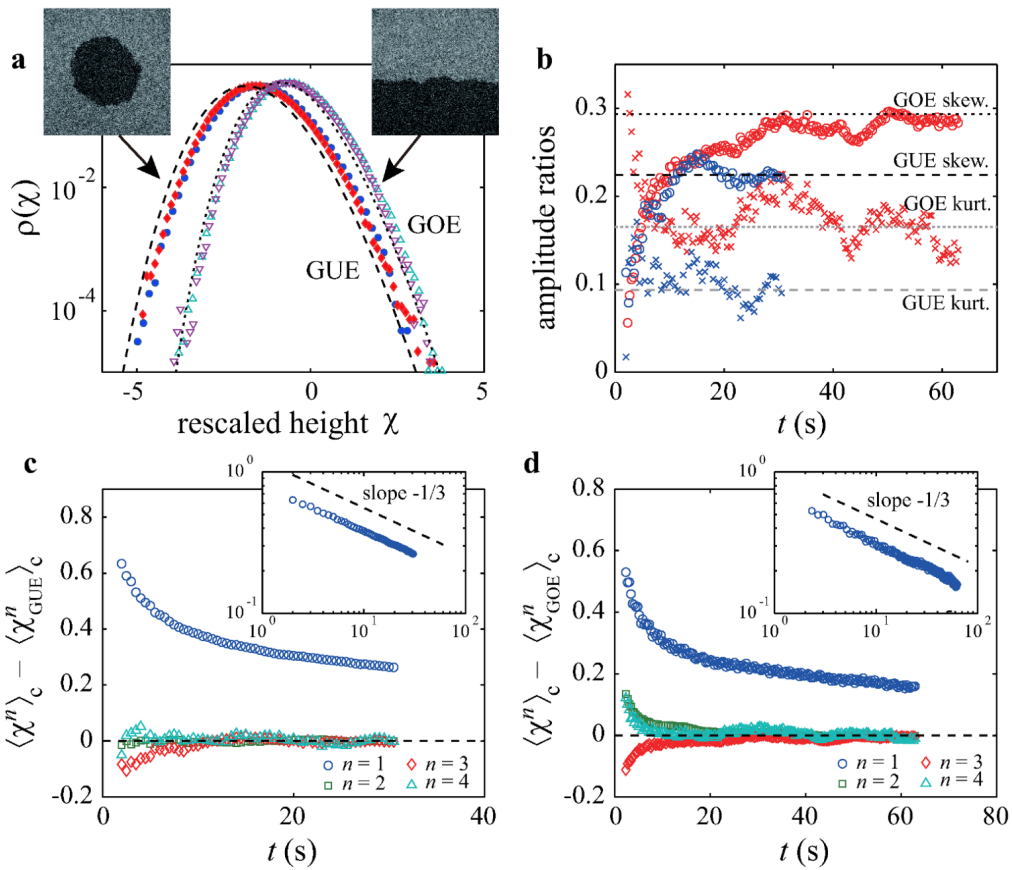

Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

KPZ Equation '86

$$\frac{\partial}{\partial t} h(t,x) = \frac{1}{2}\Delta h(t,x) + \frac{\lambda}{2}(\nabla h)^2 + \dot{W}(t,x) \qquad \text{(KPZ)}$$

Mehran Kardar (1957 –)

Giorgio Parisi (1948 –)

Yicheng Zhang

Kardar, M., Parisi, G., Zhang, Y.-C., *Phys. Rev. Lett.*, 1986

Main References*:

Chen, L., & Xia, P. (2023).Asymptotic properties of stochastic partial differential equations in the sublinear regime. *Preprint arXiv:2306.06761, to appear in Annals of Probability*.

Conus, D., Joseph, M., Khoshnevisan, D., & Shiu, S.-Y. (2013).On the chaotic character of the stochastic heat equation, II. *Probab. Theory Related Fields*, *156*(3-4), 483–533.

Zel′dovich, Y. B., Molchanov, S. A., Ruzmauikin, A. A., & Sokoloff, D. D. (1987).Self-excitation of a nonlinear scalar field in a random medium. *Proc. Nat. Acad. Sci. U.S.A.*, *84*(18), 6323–6325.

Zel′dovich, Y. B., Ruzmauikin, A. A., & Sokoloff, D. D. (1990). *The almighty chance* (Vol. 20) [Translated from the Russian by Anvar Shukurov]. World Scientific Publishing Co., Inc., River Edge, NJ.

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* References are produced from *SPDEs-Bib*: https://github.com/chenle02/SPDEs-Bib

* Download the bib file: https://github.com/chenle02/SPDEs-Bib/blob/main/All.bib