Analysis of Tetris Ballistic Deposition and the Robustness of the KPZ Universality Class

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Auburn University

Acknwolegement

NSF 2246850, NSF 2443823, & Simons Foundation Travel Grant (2022-2027)

`Talk available at: github.com/chenle02`

Emerging Synergies between Stochastic Analysis and Statistical Mechanics
Banff, Alberta, Canada
October 28, 2025

Integration of Research, Education, and Outreach

Outreach

- Auburn Summer Science Institute (AU-SSI): **2024, 2025**
  Selected talented high school students
- Destination STEM: **2023, 2024**
  Junior middle school students

Education

- Graduate Student Seminars (Mathematics), Auburn: **2022–2025**
- Math 7820/7830: Applied Stochastic Processes
  Course project, **2023/24**

Research

- Simulation and modeling packages (open source)
- Forming conjectures and validating results

Most materials are available at

`github.com/chenle02`

Math 7820/30: Applied Stochastic Processes (2023/24):

Mauricio Montes and Ian Ruau

Simulation package:

https://github.com/chenle02/Simulations_on_Some_Surface_Growth_Models

```
pip install tetris-ballistic
```

Image is generated by OpenAI's *DALL-E*

Random deposition

Ballistic deposition

Image is generated by OpenAI's *DALL-E*

Plan

Introduction to growth model and SPDE

Family-Vicsek scaling and experiments

More examples

Tetromino Pieces

Plan

Introduction to growth model and SPDE

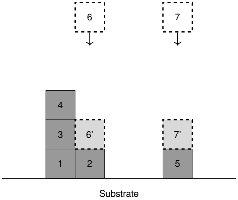
Family-Vicsek scaling and experiments

More examples

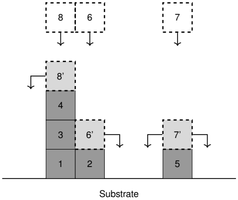
Tetromino Pieces

How do surfaces grow?
Three models

Random deposition



RD with surface relaxation



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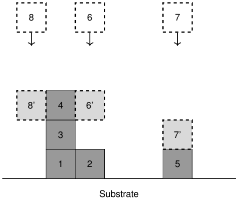

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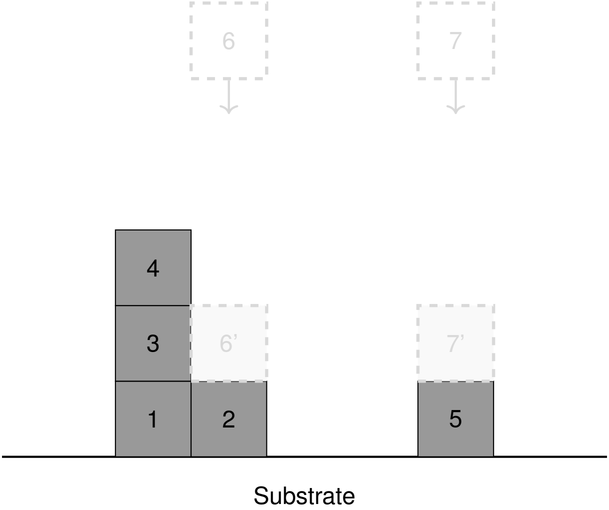

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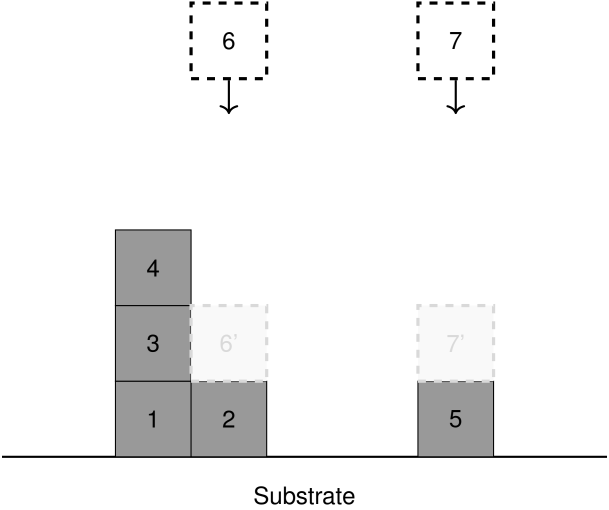

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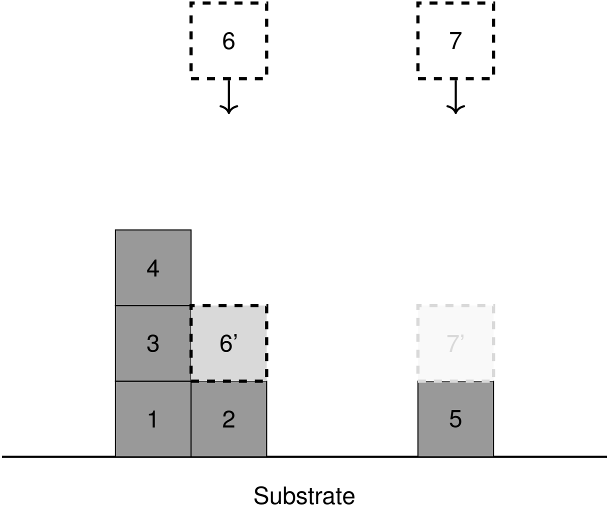

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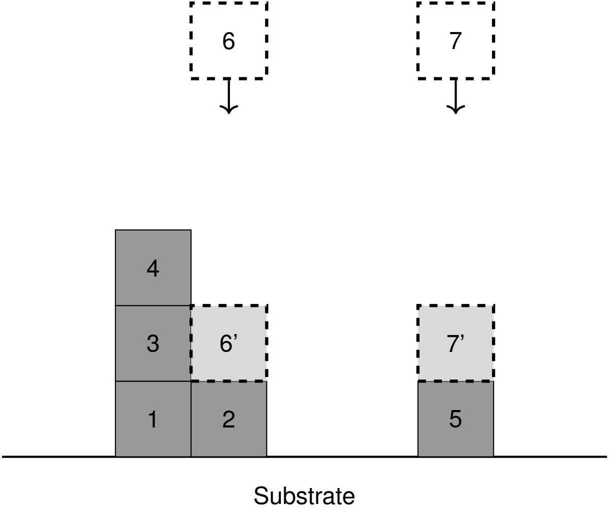
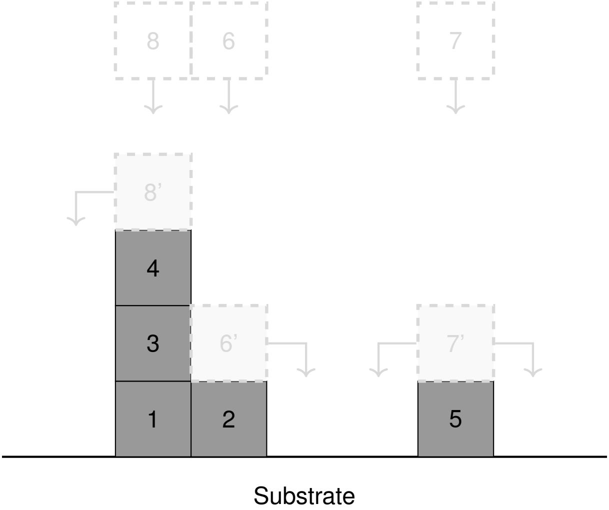


Random deposition with surface relaxation



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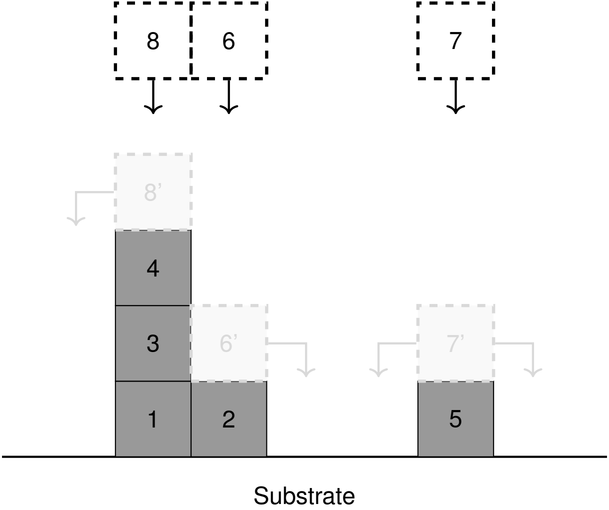

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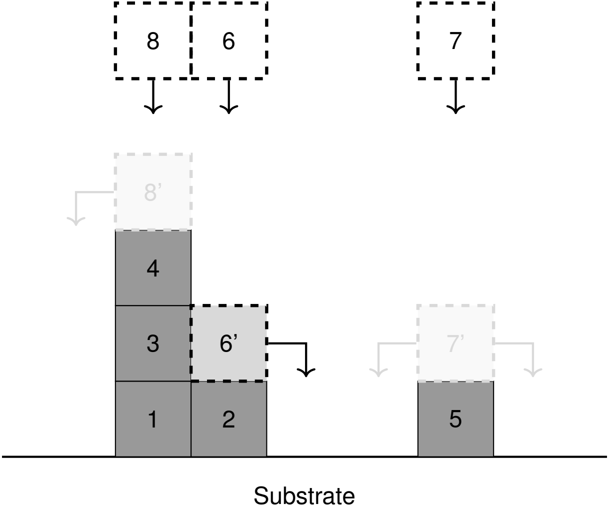

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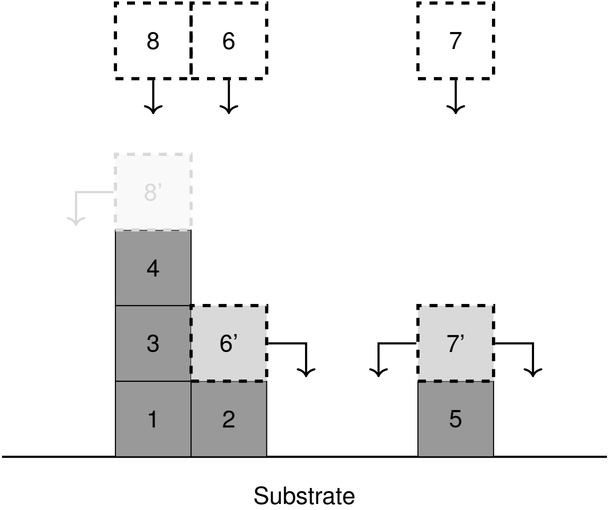

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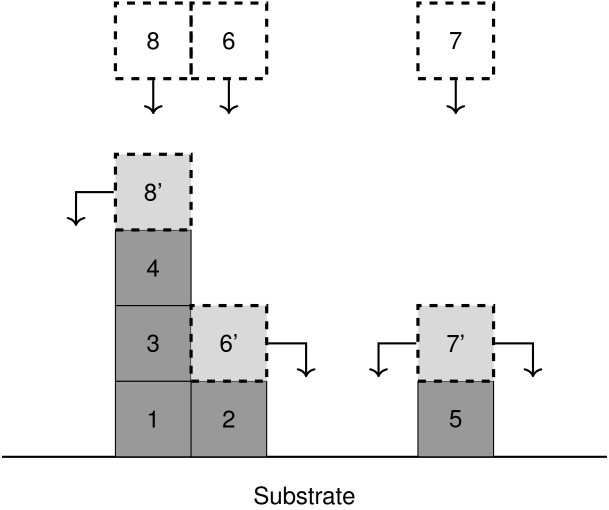

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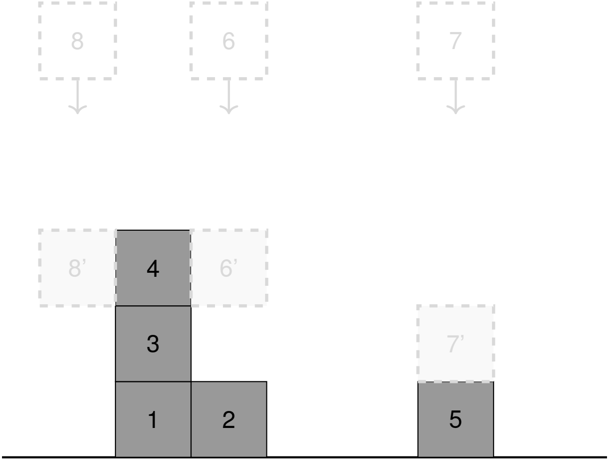
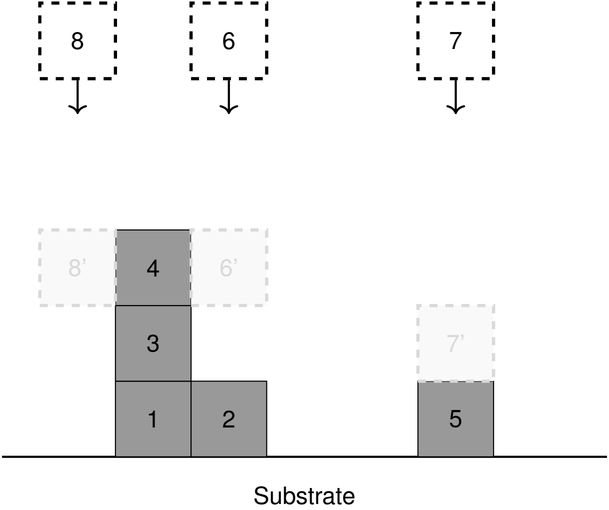


Substrate

Ballistic deposition



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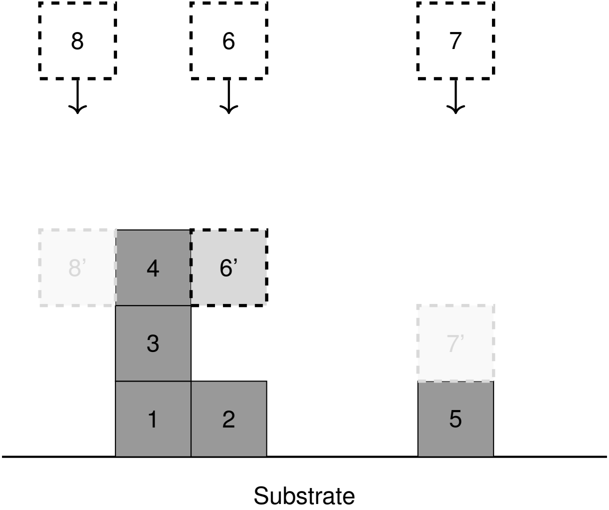

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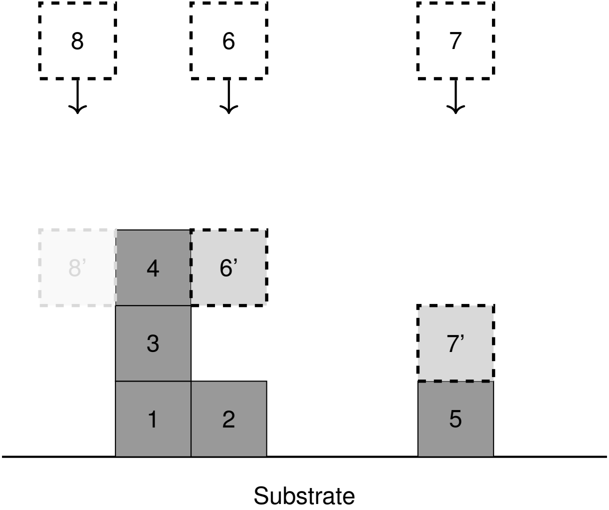

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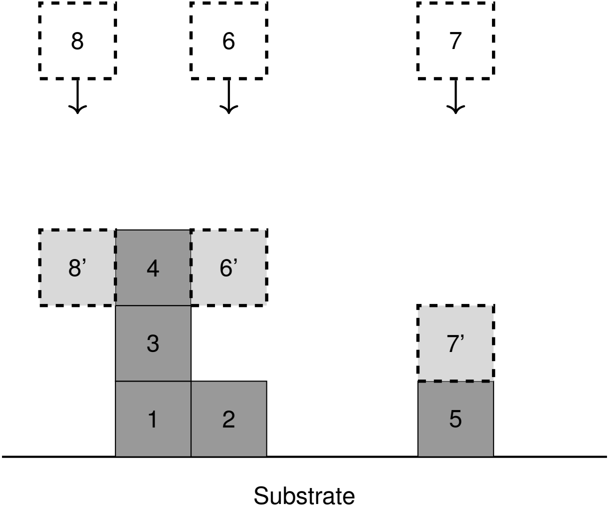
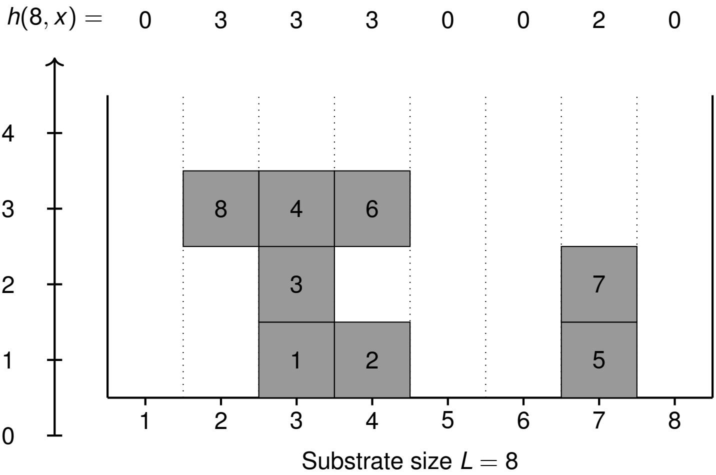


Average height and fluctuation

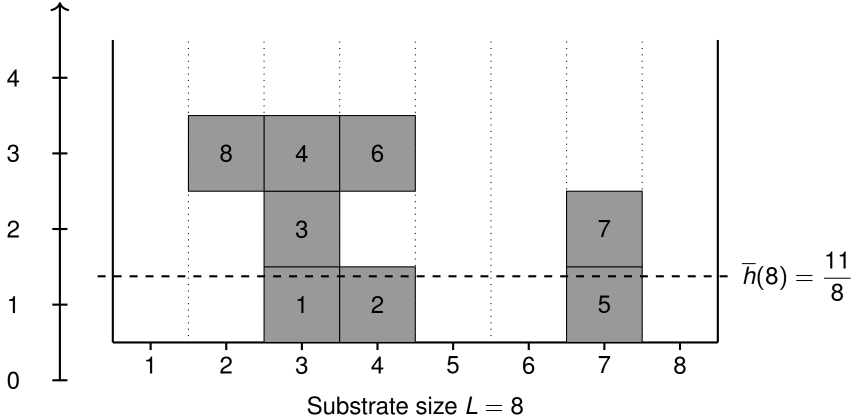


Average height and fluctuation

$$\overline{h}(t) = \frac{1}{L}\sum_{x=1}^{L} h(t,x)$$

Fluctuation $W(L,t) = \sqrt{\frac{1}{L}\sum_{x=1}^{L}\left[h(t,x) - \overline{h}(t)\right]^2}$

$h(8,x) =$ 0 3 3 3 0 0 2 0



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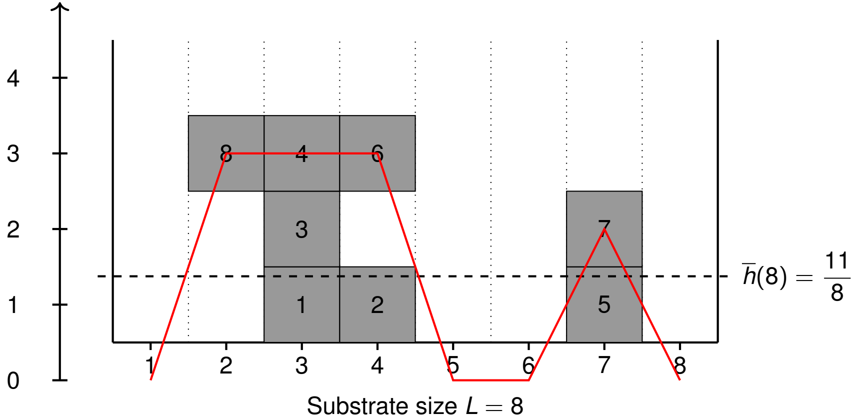

Random Deposition (independent columns, nonsticky)

Model. L independent columns. At each integer time $t = 1, 2, \ldots$, drop *one* particle on a uniformly random column. Heights $h(t,x)$, mean $\overline{h}(t) = \frac{1}{L}\sum_{x=1}^{L} h(t,x) = \frac{t}{L}$, width

$$W^2(L,t) = \frac{1}{L}\sum_{x=1}^{L}\left(h(t,x) - \overline{h}(t)\right)^2.$$

Single–column law: After t drops total,

$$h(t,x) \sim \mathrm{Binomial}\left(t, \frac{1}{L}\right), \qquad \mathbb{E}[h(t,x)] = \frac{t}{L}, \quad \mathrm{Var}(h(t,x)) = t\,\frac{1}{L}\left(1 - \frac{1}{L}\right).$$

Fluctuation: By i.i.d. columns,

$$\mathbb{E}\left[W^2(L,t)\right] = \frac{1}{L}\sum_{x=1}^{L}\mathbb{E}\left[h(t,x)^2\right] - \mathbb{E}\left[\overline{h}^2(t)\right] = \mathbb{E}\left[h(t,1)^2\right] - \left(\frac{t}{L}\right)^2 = \left(1 - \frac{1}{L}\right)\mathrm{Var}\big(h(t,1)\big).$$

Hence

$$\boxed{\mathbb{E}\left[W^2(L,t)\right] = \left(1 - \frac{1}{L}\right) t\,\frac{1}{L}\left(1 - \frac{1}{L}\right) = \frac{t}{L}\left(1 - \frac{1}{L}\right)^2}$$

and

$$\boxed{W(L,t) \simeq \left(1 - \frac{1}{L}\right)\left(\frac{t}{L}\right)^{1/2}}$$

Scaling. Growth exponent $\beta = \frac{1}{2}$.

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Questions

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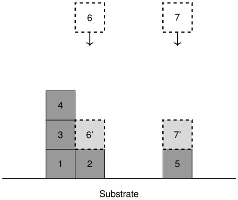

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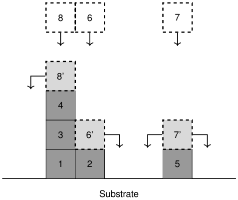
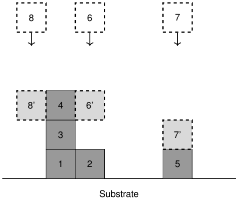


Ballistic deposition



$$W(t, x) \sim t^{1/2}$$

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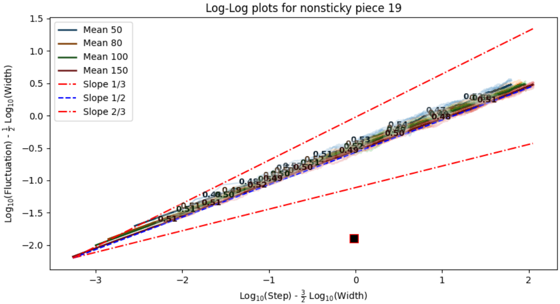
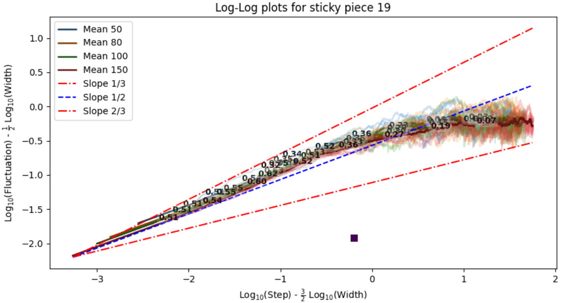

Log-Log plots for nonsticky piece 19
Mean 50
Mean 80
Mean 100
Mean 150
Slope 1/3
Slope 1/2
Slope 2/3
$\mathrm{Log}_{10}(\mathrm{Step}) - \frac{3}{2}\mathrm{Log}_{10}(\mathrm{Width})$
$\mathrm{Log}_{10}(\mathrm{Fluctuation}) - \frac{1}{2}\mathrm{Log}_{10}(\mathrm{Width})$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Plan

Family–Vicsek scaling theory

Recall height function $h(x,t)$, $x = 1, \ldots, L$. Mean $\bar{h}(t) = \frac{1}{L}\sum_{x=1}^{L} h(x,t)$.

$$w(L,t) := \sqrt{\frac{1}{L}\sum_{x=1}^{L}\big(h(x,t) - \bar{h}(t)\big)^2} \quad \text{(Fluctuation).}$$

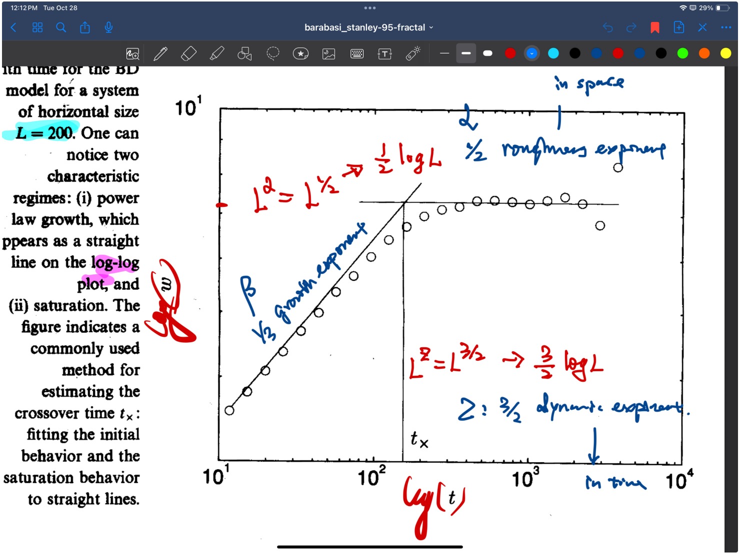
Empirical scaling (log–log)

Early time	$w(L,t) \sim t^{\beta}$	β: growth exponent
Late time	$w(L,t) \to w_{\text{sat}}(L) \sim L^{\alpha}$	α: roughness exponent
Crossover time	$t_{\times}(L) \sim L^{z}$	z: dynamic exponent

Family, F., Vicsek, T., *Journal of Physics A: Mathematical and General*, 1985

Family–Vicsek scaling theory

α: Roughness exp.; β: Growth exp.; z: Dynamic exp.



(Image from Barabási-Stanley's book 95)

Family–Vicsek scaling theory

α: Roughness exp.; β: Growth exp.; z: Dynamic exp.

$$\boxed{w(L,t) \sim L^{\alpha} f\left(\frac{t}{L^z}\right)}$$

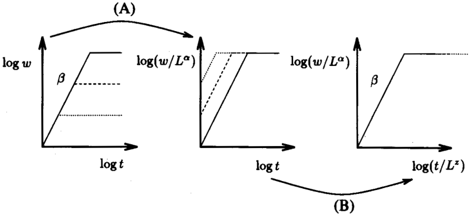
with

$$f(u) \sim \begin{cases} u^{\beta}, & u \ll 1, \\ \text{const}, & u \gg 1. \end{cases} \quad \Rightarrow \quad \boxed{\beta = \frac{\alpha}{z}}$$

Immediate consequences:

$$w_{\text{sat}}(L) \sim L^{\alpha}, \qquad t_{\times}(L) \sim L^{z}.$$

Interpretation: dynamic renormalization $x \to bx$, $t \to b^z t$, $h \to b^{\alpha} h$ leaves $w(L,t)/L^{\alpha}$ invariant as a function of t/L^z.



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Family, F., Vicsek, T., *Journal of Physics A: Mathematical and General*, 1985

Family and Vicsek reported from their simulation that

$$\beta = 0.30 \pm 0.02 \approx \frac{1}{3}.$$

Difficulty: Robust way to compute the slope in order to obtain an estimate of β.

Family, F., Vicsek, T., *Journal of Physics A: Mathematical and General*, 1985

Continuum viewpoint: EW and KPZ

$\Rightarrow$

Edwards–Wilkinson (EW)

Linear diffusion + noise

$$\partial_t h = \nu \nabla^2 h + \eta, \qquad \langle \eta\eta \rangle \propto \delta(x)\,\delta(t).$$

Scale invariance (1D): $z = 2$, $\alpha = \frac{1}{2}$ $\Rightarrow \beta = \frac{1}{4}$.

$$\boxed{\alpha = \tfrac{1}{2},\ z = 2,\ \beta = \tfrac{1}{4}}$$

Kardar–Parisi–Zhang (KPZ)

Nonlinear growth

$$\partial_t h = \nu \nabla^2 h + \tfrac{\lambda}{2}(\nabla h)^2 + \eta.$$

Galilean/tilt invariance: $\alpha + z = 2$. In 1D (exact):

$$\boxed{\alpha = \tfrac{1}{2},\ z = \tfrac{3}{2},\ \beta = \tfrac{1}{3}}$$

BD, RSOS, Eden $\Rightarrow$ KPZ universality in 1+1D.

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Reading plots (as in Barabási–Stanley)

- On w vs. t (log–log) for several L: early slopes $\approx \beta$; plateaus $\propto L^{\alpha}$; horizontal shift of crossovers $\propto L^{z}$.
- After rescaling: plot w/L^{α} vs. t/L^{z}; all curves collapse to $f(u)$ with $f(u) \sim u^{\beta}$ $(u \ll 1)$ and $f(u) \to \text{const}$ $(u \gg 1)$.

Rule of thumb (1D):

$$\text{EW: } (\alpha, \beta, z) = \left(\tfrac{1}{2}, \tfrac{1}{4}, 2\right), \qquad \text{KPZ/BD: } (\alpha, \beta, z) = \left(\tfrac{1}{2}, \tfrac{1}{3}, \tfrac{3}{2}\right).$$

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Exactly solvable baseline: Random Deposition (RD)

One particle per step, random column; after t steps heights are multinomial:

$$\mathbb{E}\big[w^2(L,t)\big] = \frac{t}{L}\Big(1 - \frac{1}{L}\Big) \;\sim\; \frac{t}{L}.$$

Hence $w \sim (t/L)^{1/2}$ ($\beta = \frac{1}{2}$), *no lateral correlations*, no finite z and no true saturation. RD serves as a growth-only sanity check; FV scaling requires correlated dynamics (EW/KPZ).

Structure factor (optional but handy)

Fourier modes $h_k(t)$, structure factor $S(k,t) = \langle |h_k(t)|^2 \rangle$ obey

$$S(k,t) \sim k^{-(2\alpha+1)}\, g(k^z t), \qquad g(y) \sim \begin{cases} y^{2\beta+1} & y \ll 1, \\ \text{const} & y \gg 1. \end{cases}$$

Useful when $w(L,t)$ is noisy; fit (α, z) from spectral slopes and collapse $S(k,t)$.

Cheat sheet / takeaways

$$\boxed{w(L,t) \sim L^{\alpha} f\left(\frac{t}{L^z}\right), \quad f(u) \sim u^{\beta}\ (u \ll 1),\ f(u) \to \text{const}\ (u \gg 1),\ \beta = \alpha/z}$$

1D benchmarks: EW $(\alpha, \beta, z) = (\frac{1}{2}, \frac{1}{4}, 2)$; KPZ/BD $(\frac{1}{2}, \frac{1}{3}, \frac{3}{2})$.

Workflow: measure $w(L,t) \Rightarrow$ fit α and $\beta \Rightarrow$ infer/fit $z \Rightarrow$ collapse w/L^{α} vs. t/L^z.

Simulations on

Random deposition vs. Ballistic decomposition

Study of growing interfaces in a thin film

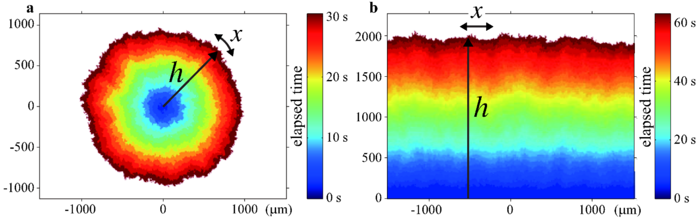
— Convection of nematic liquid crystal*

Show movies !

Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

Study of growing interfaces in a thin film

— Convection of nematic liquid crystal*



Prediction from KPZ equation:

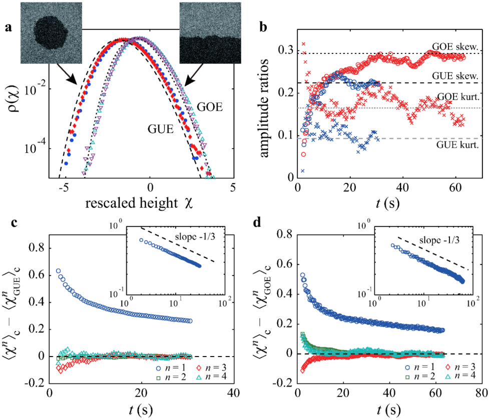
$$h \asymp v_\infty t + (\Gamma t)^{1/3} \xi$$

Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

Study of growing interfaces in a thin film

— Convection of nematic liquid crystal*

$$h \asymp v_\infty t + (\Gamma t)^{1/3}\,\xi$$



Takeuchi, K. A., Sano, M., Sasamoto, T., Spohn, H., *Sci. Rep.*, 2011

KPZ Equation '86

$$\frac{\partial}{\partial t} h(t,x) = \frac{1}{2}\Delta h(t,x) + \frac{\lambda}{2}(\nabla h)^2 + \dot{W}(t,x) \tag{KPZ}$$

Mehran Kardar (1957 –)

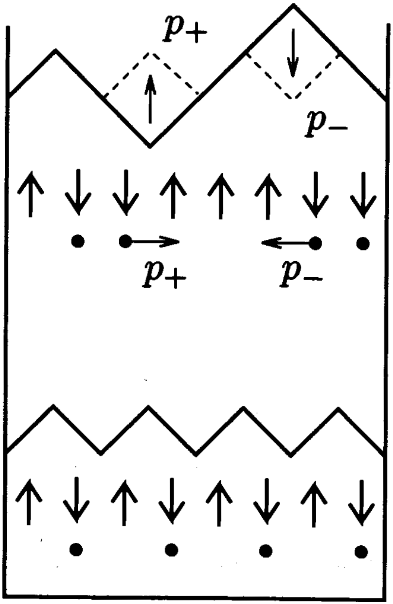
Giorgio Parisi (1948 –)

Yicheng Zhang

Kardar, M., Parisi, G., Zhang, Y.-C., *Phys. Rev. Lett.*, 1986

Plan

More models? Even more simpler?

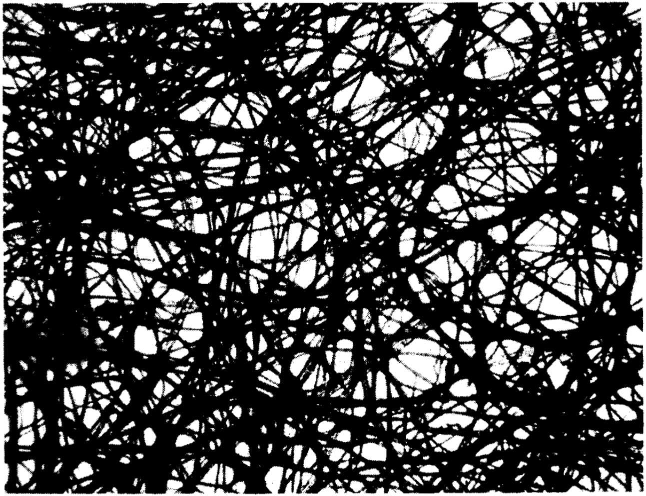


Solid-on-Solid

Ising

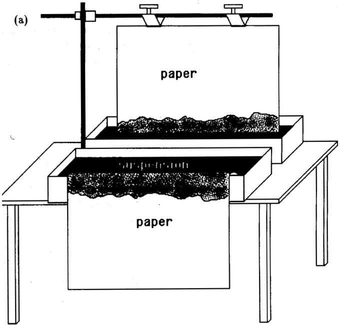
Lattice Gas

Paper – a random environment



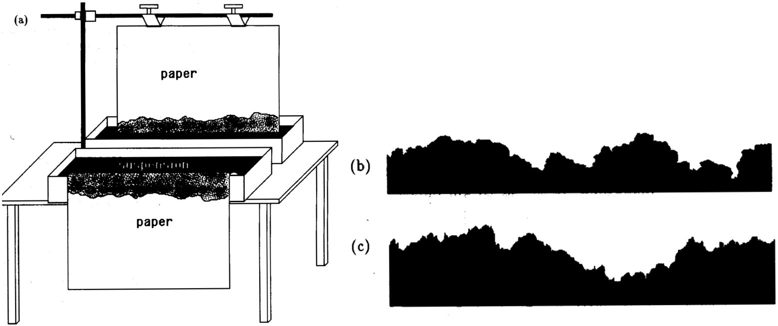
Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

Paper wetting experiment



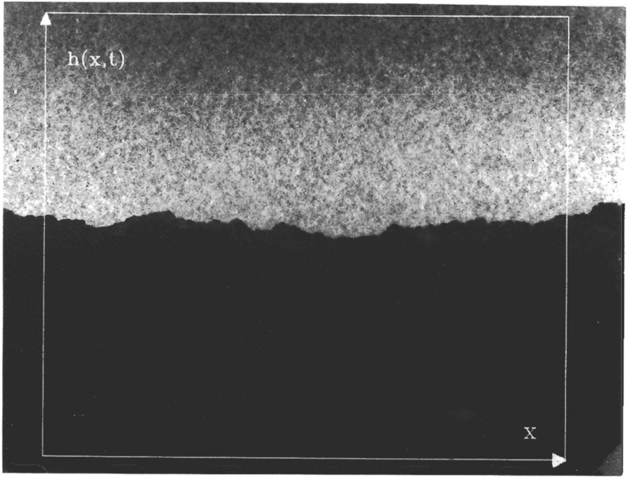
Barabási, A.-L., Stanley, H. E., 1995

Paper wetting experiment



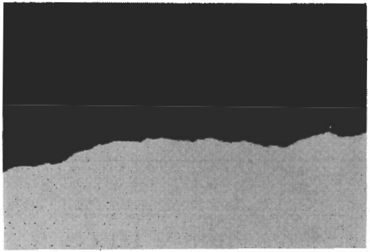
Barabási, A.-L., Stanley, H. E., 1995

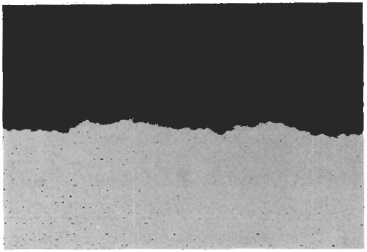
Paper burning experiment



Zhang, J., Zhang, Y.-C., Alstrøm, P., Levinsen, M., *Phys. A: Stat. Mech. Appl.*, 1992

Paper rupture experiment

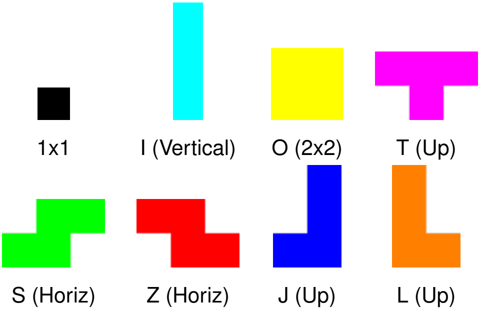




Kertész, J., Horváth, V. k., Weber, F., *Fractals*, 1993

Plan

Tetrominoes

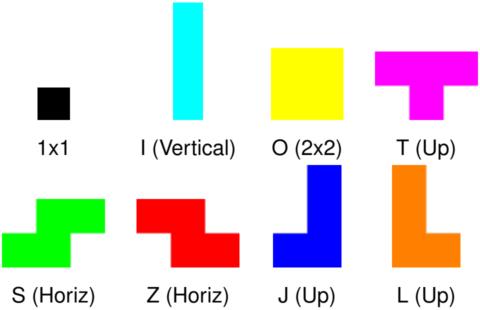


- “1x1”: Single (extra single-site particle)
- “I”: Horizontal, Vertical
- “J, L, T”: Up, Right, Down, Left
- “S, Z”: Horizontal, Vertical
- “O”: Single (2x2 square)

- Sticky
- Nonstikcy

$(1 + 1 \times 2 + 3 \times 4 + 2 \times 2 + 1) \times 2 = 20 \times 2 = 40$ types of pieces

Tetrominoes



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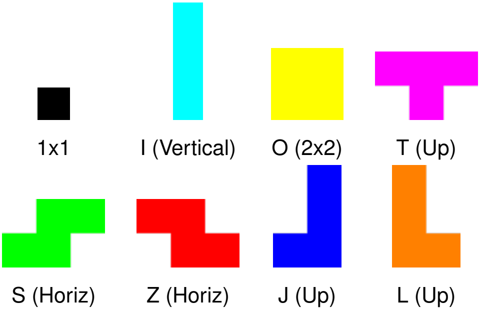

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$(1 + 1 \times 2 + 3 \times 4 + 2 \times 2 + 1) \times 2 = 20 \times 2 = 40$ types of pieces

Configure files

```
steps: 12000
width: 100
height: 300
seed: 12
Piece-00: [20, 0]
Piece-01: [20, 0]
Piece-02: [20, 0]
Piece-03: [20, 0]
Piece-04: [20, 0]
Piece-05: [20, 0]
Piece-06: [20, 0]
Piece-07: [20, 0]
Piece-08: [20, 0]
Piece-09: [20, 0]
Piece-10: [20, 0]
Piece-11: [20, 0]
Piece-12: [20, 0]
Piece-13: [20, 0]
Piece-14: [20, 0]
Piece-15: [20, 0]
Piece-16: [20, 0]
Piece-17: [20, 0]
Piece-18: [20, 0]
Piece-19: [20, 0]
```

All nonsticky pieces with equal prob.

```
steps: 12000
width: 100
height: 300
seed: 12
Piece-00: [0, 20]
Piece-01: [0, 20]
Piece-02: [0, 20]
Piece-03: [0, 20]
Piece-04: [0, 20]
Piece-05: [0, 20]
Piece-06: [0, 20]
Piece-07: [0, 20]
Piece-08: [0, 20]
Piece-09: [0, 20]
Piece-10: [0, 20]
Piece-11: [0, 20]
Piece-12: [0, 20]
Piece-13: [0, 20]
Piece-14: [0, 20]
Piece-15: [0, 20]
Piece-16: [0, 20]
Piece-17: [0, 20]
Piece-18: [0, 20]
Piece-19: [0, 20]
```

All sticky pieces with equal prob.

```
steps: 12000
width: 100
height: 300
seed: 12
Piece-00: [0, 0]
Piece-01: [0, 0]
Piece-02: [0, 0]
Piece-03: [0, 0]
Piece-04: [0, 0]
Piece-05: [0, 0]
Piece-06: [0, 0]
Piece-07: [0, 0]
Piece-08: [0, 0]
Piece-09: [0, 0]
Piece-10: [0, 0]
Piece-11: [0, 0]
Piece-12: [0, 0]
Piece-13: [0, 0]
Piece-14: [0, 0]
Piece-15: [0, 0]
Piece-16: [0, 0]
Piece-17: [0, 0]
Piece-18: [0, 0]
Piece-19: [20, 80]
```

20% nonsticky + 80% sticky of 1x1 piece

Question

For various configurations of Tetromino pieces, do the resulting surface robustly exhibit Family–Vicsek scaling?

Will the scaling exponent β be always close to $\frac{1}{3}$?

Mixture Parameter ℓ and Scaling

Setup

Consider a mixture of ℓ% ($\ell \in [0, 100]$) nonsticky pieces and $(100-\alpha)$% sticky pieces, where only the 1×1 piece is sticky.

Question

How does α influence the scaling exponents, in particular, $\beta \overset{?}{\approx} \frac{1}{3}$?

Simulations and log–log plots:

https://chenle02.github.io/2025-10-28_Emerging_Synergies_Banff_Le/exp13/videos_and_images_display.html

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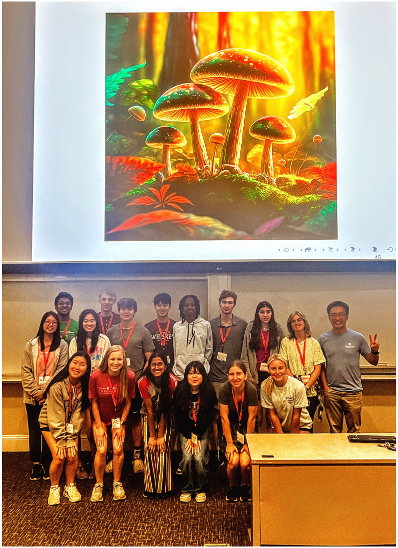
Outreach Highlights



AU-SSI 2023

AU-SSI 2024

Destination STEM 2023

Main References:

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Kardar, M., Parisi, G., & Zhang, Y.-C. (1986).Dynamic scaling of growing interfaces. *Phys. Rev. Lett.*, *56*(9), 889.

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Thank you!

Questions?