

Source This source is from (2.9.17) of [KS04]:

$$H_{q+1,p}^{p,1} \left(z \left| \begin{array}{c} (1,1), (b_j,1)_{1,q} \\ (a_i,1)_{1,p} \end{array} \right. \right) = E(a_1, \dots, a_p; b_1, \dots, b_q; z)$$

Notes: $E(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the MacRobert E-function (see [Erd+81], Section 5.2).

$${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z) = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n}{(b_1)_n \cdots (b_q)_n} \frac{z^n}{n!},$$

where $(a)_n$ denotes the Pochhammer symbol, which represents the rising factorial $\frac{\Gamma(a+n-1)}{\Gamma(a)} = a(a+1)(a+2)\cdots(a+n-1)$ for $n > 0$, and $(a)_0 = 1$.

The MacRobert E-function is an generalization of the generalized hypergeometric series

$${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z) = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n}{(b_1)_n \cdots (b_q)_n} \frac{z^n}{n!},$$

for the case of $p > q + 1$, when the generalized hypergeometric series diverges for any $x \neq 0$. For $p \leq q + 1$ we have

$$E(p; a_r : q; \rho_s : x) = \frac{\Gamma(a_1) \cdots (a_p)}{\Gamma(\rho_1) \cdots (\rho_q)} \times {}_pF_q(a_1, \dots, a_p; \rho_1, \dots, \rho_q; -1/x)$$

where $x \neq 0$ if $p < q$ and $|x| > 1$ if $p = q + 1$.

For $p \geq q + 1$ we have

$$\begin{aligned} E(p; a_r : q; \rho_s : x) &= \sum_{r=1}^p \frac{\prod_{s=1, s \neq r}^p \Gamma(a_s - a_r)}{\prod_{t=1}^q \Gamma(\rho_t - a_r)} \Gamma(a_r) x^{a_r} \\ &\times {}_{q+1}F_{p-1}[a_r, a_r - \rho_1 + 1, \dots, a_r - \rho_q + 1; \\ &a_r - a_1, \dots, a_r - a_{r-1} + 1, a_r - a_{r+1} + 1, \dots, a_r - a_p + 1; (-1)^{p+q}x] \end{aligned}$$

where $a_r \neq 0, -1, -2 \dots$

The MacRobert E-function can also be represented as a particular case of Meijer G-function.

$$E(p; a_r : q; \rho_s : x) = G_{q+1,p}^{p,1} \left(x \left| \begin{array}{c} 1, \beta_1, \dots, \beta_q \\ a_1, \dots, a_p \end{array} \right. \right)$$

References

[Erd+81] Arthur Erdélyi, Wilhelm Magnus, Fritz Oberhettinger, and Francesco G. Tricomi. *Higher transcendental functions. Vol. I*. Based on notes left by Harry Bateman, Reprint of the 1953 original. Robert E. Krieger Publishing Co., Inc., Melbourne, Fla., 1981, pp. xviii+396. ISBN: 0-89874-069-X.

[KS04] Anatoly A. Kilbas and Megumi Saigo. *H-transforms*. Vol. 9. Analytical Methods and Special Functions. Theory and applications. Chapman & Hall/CRC, Boca Raton, FL, 2004, pp. xii+389. ISBN: 0-415-29916-0. DOI: 10.1201/9780203487372. URL: <https://doi.org/10.1201/9780203487372>.