Financial Mathematics

MATH 5870/6870[1]
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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 18. The Lognormal Distribution

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- Let $R(t, s)$ be the continuously compounded return from time t to a later time s.
- For $t_0 < t_1 < t_2$, $R(\cdot, \cdot)$ has to satisfy the additivity property:

$$R(t_0, t_2) = R(t_0, t_1) + R(t_1, t_2)$$

- For time interval $[0, T]$ divided into n subintervals of equal length T/n, we have

$$R(0, T) = R(0, h) + R(h, 2h) + \cdots + R((n-1)h, T)$$

  Assume that

$$\mathbb{E}\left(R((i-1)h, ih)\right) = \alpha_h \quad \text{and} \quad \mathrm{Var}\left(R((i-1)h, ih)\right) = \sigma_h^2$$

  Then

$$\mathbb{E}\left(R(0, T)\right) = n\alpha_h \quad \text{and} \quad \mathrm{Var}\left(R(0, T)\right) = n\sigma_h^2$$

- By central limit limit theorem, as $n \to \infty$, one can assume that

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$$\ln(S_t/S_0) \sim N\left([\alpha - \delta - 0.5\sigma^2]t, \sigma^2 t\right)$$

$$\ln(S_t/S_0) = [\alpha - \delta - 0.5\sigma^2]t + \sigma\sqrt{t}\, Z$$

$$S_t = S_0 e^{[\alpha-\delta-0.5\sigma^2]t} e^{\sigma\sqrt{t}\, Z}$$

$$\mathbb{E}[S_t] = S_0 e^{[\alpha-\delta]t} \quad \text{and} \quad \text{Median stock price} = e^{[\alpha-\delta-0.5\sigma^2]t}$$

$$\text{One standard deviation} \begin{cases} \text{move up} = e^{[\alpha-\delta-0.5\sigma^2]t+\sigma\sqrt{t}\times 1} \\ \text{move down} = e^{[\alpha-\delta-0.5\sigma^2]t-\sigma\sqrt{t}\times 1} \end{cases}$$

Go over examples 18.4 and 18.5 on textbook on p. 555.