Financial Mathematics

MATH 5870/6870[1]
Fall 2021

Le Chen

lzc0090@auburn.edu

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Auburn University
Auburn AL

[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 3. Insurance, Collars, and Other Strategies

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It is possible to mimic a long forward position on an asset by

buying a call + selling a put,

with each option having the same strike price and expiration time.

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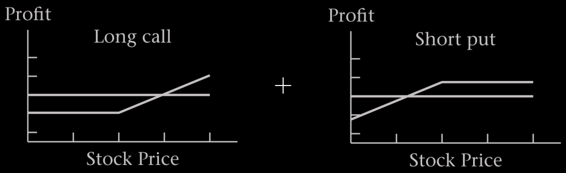
A synthetic forward

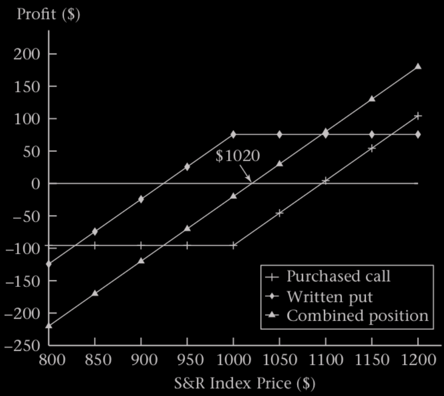
Example 3.2-1 Working with the S&R index. Suppose that

6-month interest rate	2%
premium for 1000-strike 6-month call	$93.809
premium for 1000-strike 6-month put	$74.201

Draw profit digram for the combined position of a purchased call with a written put, namely,



Solution.


Profit ($)
200
150
100
50
0
−50
−100
−150
−200
−250
$1020
Purchased call
Written put
Combined position
800 850 900 950 1000 1050 1100 1150 1200
S&R Index Price ($)

□

A synthetic long forward contract

We pay the net option premium

We pay the strike price

The actual forward

We pay zero premium

We pay the forward price

Basic Assumption

The net cost of buying the index using options

must equal

the net cost of buying the index using a forward contract.

NO ARBITRAGE!

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NO ARBITRAGE!

The Put-Call parity equation

$$\mathrm{Call}(K, T) - \mathrm{Put}(K, T) = \mathrm{PV}\left(F_{0,T} - K\right)$$

- K: strike price
- T: expiration date
- $\mathrm{Call}(\cdot, \circ)$: the premium for call.
- $\mathrm{Put}(\cdot, \circ)$: the premium for put.
- $F_{0,T}$: the forward price at time T if one enters at time 0 into a long forward position.
- $\mathrm{PV}(\cdot)$: the present value function.

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Example 3.2-2 Check Example 3.2-1 to see if the put-call parity equation is satisfied.

Solution. We need to check:

$$\$93.809 - \$74.201 \stackrel{?}{=} \mathrm{PV}(\$1,000 \times 1.02 - \$1,000)$$

Clearly, LHS $= \$19.61$. On the other hand, the RHS is equal to

$$\begin{aligned}\mathrm{PV}(\$1,000 \times 1.02 - \$1,000) &= \mathrm{PV}\left(1,000 \times (1.02 - 1)\right) \\ &= \mathrm{PV}\left(1,000 \times 0.02\right) \\ &= \frac{1,000 \times 0.02}{1.02} \\ &= \$19.61.\end{aligned}$$

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$$\text{Call}(K,T) - \text{Put}(K,T) = \text{PV}\left(F_{0,T} - K\right)$$

$$\Updownarrow$$

$$\text{PV}\left(F_{0,T}\right) + \text{Put}(K,T) = \text{Call}(K,T) + \text{PV}\left(K\right)$$

Buying the index and buying the put

generate the same payoff as

buying the call and buying a zero-coupon bond (i.e. lending) PV(K)

$$\mathrm{Call}(K, T) - \mathrm{Put}(K, T) = \mathrm{PV}\left(F_{0,T} - K\right)$$

$$\Updownarrow$$

$$\mathrm{PV}\left(F_{0,T}\right) - \mathrm{Call}(K, T) = \mathrm{PV}\left(K\right) - \mathrm{Put}(K, T)$$

Writing a covered call

has the same profit as

lending $\mathrm{PV}(K)$ and selling a put

$$\mathrm{Call}(K, T) - \mathrm{Put}(K, T) = \mathrm{PV}\left(F_{0,T}\right) - \mathrm{PV}\left(K\right)$$

Revisit four positions in Section 3.1

Position	Meaning	equivalent to
Inuring a long position (floors)		
Inuring a short position (caps)		
Covered call writing		
Covered put writing		

$$\mathrm{Call}(K,T) - \mathrm{Put}(K,T) = \mathrm{PV}\left(F_{0,T}\right) - \mathrm{PV}\left(K\right)$$

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Position	Meaning	equivalent to
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