

Computations for Figure 10.5

European Call with no dividend

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Define functions

```

In[ ]:= u[r_, δ_, h_, σ_] := Exp[(r - δ) h + σ √h]
d[r_, δ_, h_, σ_] := Exp[(r - δ) h - σ √h]

Δ[r_, δ_, h_, σ_, S_, Cu_, Cd_] := Exp[-δ h]  $\frac{Cu - Cd}{S (u[r, δ, h, σ] - d[r, δ, h, σ])}$ 

B[r_, δ_, h_, σ_, Cu_, Cd_] := Exp[-r h]  $\frac{u[r, δ, h, σ] Cd - d[r, δ, h, σ] Cu}{u[r, δ, h, σ] - d[r, δ, h, σ]}$ 

OptionPrice [Δ_, B_, S_] := Δ S + B

```

Input the data first

```

In[ ]:= r = 0.08;
δ = 0;
h =  $\frac{1}{3}$ ;
σ = 0.3;
S = 41;
K = 40;

```

Construct the binomial tree

```

In[ ]:= S u[r, δ, h, σ]
S u[r, δ, h, σ]2
S u[r, δ, h, σ]3

```

```
Out[ ]:= 50.0711
```

```
Out[ ]:= 61.1491
```

```
Out[ ]:= 74.6781
```

```

In[ ]:= S d[r, δ, h, σ]
S d[r, δ, h, σ]2
S d[r, δ, h, σ]3

```

```
Out[ ]:= 35.4114
```

```
Out[ ]:= 30.5846
```

```
Out[ ]:= 26.4157
```

```
In[ ]:= S u[r,  $\delta$ , h,  $\sigma$ ] d[r,  $\delta$ , h,  $\sigma$ ]
      S u[r,  $\delta$ , h,  $\sigma$ ]2 d[r,  $\delta$ , h,  $\sigma$ ]
      S u[r,  $\delta$ , h,  $\sigma$ ] d[r,  $\delta$ , h,  $\sigma$ ]2
```

```
Out[ ]:= 43.246
```

```
Out[ ]:= 52.814
```

```
Out[ ]:= 37.3513
```

Backwards computation

Node 11

```
In[ ]:= myS = S u[r,  $\delta$ , h,  $\sigma$ ]2
      Cu = Max[myS u[r,  $\delta$ , h,  $\sigma$ ] - K, 0]
      Cd = Max[myS d[r,  $\delta$ , h,  $\sigma$ ] - K, 0]
      myDelta =  $\Delta$ [r,  $\delta$ , h,  $\sigma$ , myS, Cu, Cd]
      myB = B[r,  $\delta$ , h,  $\sigma$ , Cu, Cd]
      OptionPrice [myDelta, myB, myS]
```

```
Out[ ]:= 61.1491
```

```
Out[ ]:= 34.6781
```

```
Out[ ]:= 12.814
```

```
Out[ ]:= 1.
```

```
Out[ ]:= -38.9474
```

```
Out[ ]:= 22.2017
```

Node 10

```
In[ ]:= myS = S u[r,  $\delta$ , h,  $\sigma$ ] d[r,  $\delta$ , h,  $\sigma$ ]
      Cu = Max[myS u[r,  $\delta$ , h,  $\sigma$ ] - K, 0]
      Cd = Max[myS d[r,  $\delta$ , h,  $\sigma$ ] - K, 0]
      myDelta =  $\Delta$ [r,  $\delta$ , h,  $\sigma$ , myS, Cu, Cd]
      myB = B[r,  $\delta$ , h,  $\sigma$ , Cu, Cd]
      OptionPrice [myDelta, myB, myS]
```

```
Out[ ]:= 43.246
```

```
Out[ ]:= 12.814
```

```
Out[ ]:= 0
```

```
Out[ ]:= 0.828703
```

```
Out[ ]:= -30.1386
```

```
Out[ ]:= 5.69951
```

5.700

Node 00

```
In[ ]:= myS = S d[r,  $\delta$ , h,  $\sigma$ ]2
Cu = Max[myS u[r,  $\delta$ , h,  $\sigma$ ] - K, 0]
Cd = Max[myS d[r,  $\delta$ , h,  $\sigma$ ] - K, 0]
myDelta =  $\Delta$ [r,  $\delta$ , h,  $\sigma$ , myS, Cu, Cd]
myB = B[r,  $\delta$ , h,  $\sigma$ , Cu, Cd]
OptionPrice [myDelta, myB, myS]
```

Out[]:= 30.5846

Out[]:= 0

Out[]:= 0

Out[]:= 0.

Out[]:= 0.

Out[]:= 0.

Node 1

```
In[ ]:= myS = S u[r,  $\delta$ , h,  $\sigma$ ]
Cu = 22.202
Cd = 5.700
myDelta =  $\Delta$ [r,  $\delta$ , h,  $\sigma$ , myS, Cu, Cd]
myB = B[r,  $\delta$ , h,  $\sigma$ , Cu, Cd]
OptionPrice [myDelta, myB, myS]
```

Out[]:= 50.0711

Out[]:= 22.202

Out[]:= 5.7

Out[]:= 0.92174

Out[]:= -33.2627

Out[]:= 12.8899

Node 0

```
In[ ]:= myS = S d[r,  $\delta$ , h,  $\sigma$ ]
Cu = 5.700
Cd = 0.000
myDelta =  $\Delta$ [r,  $\delta$ , h,  $\sigma$ , myS, Cu, Cd]
myB = B[r,  $\delta$ , h,  $\sigma$ , Cu, Cd]
OptionPrice [myDelta, myB, myS]
```

```
Out[ ]:= 35.4114
```

```
Out[ ]:= 5.7
```

```
Out[ ]:= 0.
```

```
Out[ ]:= 0.450185
```

```
Out[ ]:= -13.4064
```

```
Out[ ]:= 2.53528
```

Node root

```
In[ ]:= myS = S
Cu = 12.889869559234839`
Cd = 2.535280965407516`
myDelta =  $\Delta$ [r,  $\delta$ , h,  $\sigma$ , myS, Cu, Cd]
myB = B[r,  $\delta$ , h,  $\sigma$ , Cu, Cd]
OptionPrice [myDelta, myB, myS]
```

```
Out[ ]:= 41
```

```
Out[ ]:= 12.8899
```

```
Out[ ]:= 2.53528
```

```
Out[ ]:= 0.70633
```

```
Out[ ]:= -21.8854
```

```
Out[ ]:= 7.07414
```