From Greeks to Black–Scholes equation

In this note, we will verify that the Greeks satisfy the famous Black-Schole equatio

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First input all constants and functions

$$n[d\_] := \int_{-\infty}^{d} \frac{1}{\sqrt{2\pi}} \mathrm{Exp}\left[\frac{-x^2}{2}\right] dx$$

$$d1 = \frac{\mathrm{Log}\left[\frac{S}{K}\right] + \left(r - \delta + \frac{1}{2}\sigma^2\right)(T - t)}{\sigma\sqrt{T - t}};$$

$$d2 = d1 - \sigma\sqrt{T - t};$$

$$\mathrm{OptionCall} = S\, e^{-\delta(T-t)}\, n[d1] - K\, e^{-r(T-t)}\, n[d2];$$

$$\mathrm{OptionPut} = K\, e^{-r(T-t)}\, n[-d2] - S\, e^{-\delta(T-t)}\, n[-d1];$$

We first verify Call option

Now Compute Greeks : Δ, Γ, θ

In[∘]:=
```
Δ = D[OptionCall , S];
Γ = D[OptionCall , {S, 2}];
θ = D[OptionCall , t];
```

Now verify the Black – Scholes equation

In[∘]:=
```
Simplify[θ + 1/2 σ^2 S^2 Γ + (r - δ) S Δ - r OptionCall ,
 Assumptions → {r ≥ 0, δ ≥ 0, K > 0, T > t > 0, S > 0, σ > 0}]
```

Out[∘]= 0

Now we check the put option

Now Compute Greeks : Δ, Γ, θ

In[ ]:=
```
Δ = D[OptionPut , S];
Γ = D[OptionPut , {S, 2}];
θ = D[OptionPut , t];
```

Now verify the Black – Scholes equation

In[ ]:=
```
FullSimplify[θ + 1/2 σ^2 S^2 Γ + (r - δ) S Δ - r OptionPut ,
  Assumptions → {r ≥ 0, δ ≥ 0, K > 0, T > t > 0, S > 0, σ > 0}]
```

Out[ ]= 0