Computations for Figure 10.7

# Ammerican Put with no dividend

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Crated on Tue 28 Sep 2021 09:45:34 AM CDT

# Define the functions

```
u[r_, δ_, h_, σ_] := Exp[(r - δ) h + σ √h]
d[r_, δ_, h_, σ_] := Exp[(r - δ) h - σ √h]
Δ[r_, δ_, h_, σ_, S_, Cu_, Cd_] := Exp[-δ h] (Cu - Cd) / (S (u[r, δ, h, σ] - d[r, δ, h, σ]))
B[r_, δ_, h_, σ_, Cu_, Cd_] := Exp[-r h] (u[r, δ, h, σ] Cd - d[r, δ, h, σ] Cu) / (u[r, δ, h, σ] - d[r, δ, h, σ])
OptionPrice[Δ_, B_, S_] := Max[K - S, Δ S + B]
```

## Input the data first

```
r = 0.08;
δ = 0;
h = 1/3;
σ = 0.3;
S = 41;
K = 40;
```

## Contruct the binomial tree

```
S u[r, δ, h, σ]
S u[r, δ, h, σ]^2
S u[r, δ, h, σ]^3
```

```
50.0711
```

```
61.1491
```

```
74.6781
```

```
S d[r, δ, h, σ]
S d[r, δ, h, σ]^2
S d[r, δ, h, σ]^3
```

```
35.4114
```

```
30.5846
```

```
26.4157
```

```
In[ ]:= S u[r, δ, h, σ] d[r, δ, h, σ]
        S u[r, δ, h, σ]^2 d[r, δ, h, σ]
        S u[r, δ, h, σ] d[r, δ, h, σ]^2
Out[ ]= 43.246
Out[ ]= 52.814
Out[ ]= 37.3513
```

## Backwards computation

### Node 11

```
In[ ]:= myS = S u[r, δ, h, σ]^2
        Cu = Max[0, K - myS u[r, δ, h, σ]]
        Cd = Max[0, K - myS d[r, δ, h, σ]]
        myDelta = Δ[r, δ, h, σ, myS, Cu, Cd]
        myB = B[r, δ, h, σ, Cu, Cd]
        OptionPrice[myDelta, myB, myS]
Out[ ]= 61.1491
Out[ ]= 0
Out[ ]= 0
Out[ ]= 0.
Out[ ]= 0.
Out[ ]= 0.
```

### Node 10

```
In[ ]:= myS = S u[r, δ, h, σ] d[r, δ, h, σ]
        Cu = Max[0, K - myS u[r, δ, h, σ]]
        Cd = Max[0, K - myS d[r, δ, h, σ]]
        myDelta = Δ[r, δ, h, σ, myS, Cu, Cd]
        myB = B[r, δ, h, σ, Cu, Cd]
        OptionPrice[myDelta, myB, myS]
Out[ ]= 43.246
Out[ ]= 0
Out[ ]= 2.64873
Out[ ]= -0.171297
Out[ ]= 8.80883
Out[ ]= 1.40091
```

## Node 00

```
In[ ]:= myS = S d[r, δ, h, σ]^2
        Cu = Max[0, K - myS u[r, δ, h, σ]]
        Cd = Max[0, K - myS d[r, δ, h, σ]]
        myDelta = Δ[r, δ, h, σ, myS, Cu, Cd]
        myB = B[r, δ, h, σ, Cu, Cd]
        OptionPrice[myDelta, myB, myS]
```

```
Out[ ]= 30.5846

Out[ ]= 2.64873

Out[ ]= 13.5843

Out[ ]= -1.

Out[ ]= 38.9474

Out[ ]= 9.41544
```

## Node 1

```
In[ ]:= myS = S u[r, δ, h, σ]
        Cu = 0.000
        Cd = 1.401
        myDelta = Δ[r, δ, h, σ, myS, Cu, Cd]
        myB = B[r, δ, h, σ, Cu, Cd]
        OptionPrice[myDelta, myB, myS]
```

```
Out[ ]= 50.0711

Out[ ]= 0.

Out[ ]= 1.401

Out[ ]= -0.0782546

Out[ ]= 4.65928

Out[ ]= 0.740988
```

## Node 0

```
In[ ]:= myS = S d[r, δ, h, σ]
        Cu = 1.401
        Cd = 9.415
        myDelta = Δ[r, δ, h, σ, myS, Cu, Cd]
        myB = B[r, δ, h, σ, Cu, Cd]
        OptionPrice[myDelta, myB, myS]
```

```
Out[ ]= 35.4114

Out[ ]= 1.401

Out[ ]= 9.415

Out[ ]= -0.632944

Out[ ]= 28.0162

Out[ ]= 5.60274
```

## Node root

```
In[ ]:= myS = S
        Cu = 0.740988360714645`
        Cd = 5.602735250023741`
        myDelta = Δ[r, δ, h, σ, myS, Cu, Cd]
        myB = B[r, δ, h, σ, Cu, Cd]
        OptionPrice[myDelta, myB, myS]
```

```
Out[ ]= 41

Out[ ]= 0.740988

Out[ ]= 5.60274

Out[ ]= -0.33164

Out[ ]= 16.8901

Out[ ]= 3.29287
```