Financial Mathematics

MATH 5870/6870[1]
Fall 2021

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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 9. Parity and other option relationships

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§ 9.1 Put-call parity

§ 9.2 Generalized parity and exchange options

§ 9.3 Comparing options with respect to style, maturity, and strike

§ 9.4 Problems

European options

$$C(K,T) - P(K,T) = \mathrm{PV}_{0,T}\,(F_{0,T} - K)$$
$$= e^{-rT}\,(F_{0,T} - K)$$

Buying a call and selling a put
with the strike both equal to the forward price (i.e., $K = F_{0,T}$)
creates a synthetic forward contract
and hence must have a zero price.

Parity generally fails for American options!

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Parity for stocks

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

Example 9.1-1 Suppose that the price of a non-dividend-paying stock is \$40, the continuously compounded interest rate is 8%, and options have 3 months to expiration. If a 40-strike European call sells for \$2.78, find the price for a 40-strike European put sells.

Solution. Let the price for put be y. Then

$$\$2.78 = y + \$40 - \$40e^{-0.08\times 0.25}$$

Hence,

$$y = \$1.99.$$

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Why is a call more expensive than a put?

When $S_0 = K$ and $\text{Div} = 0$, then

$$C(K,T) - P(K,T) = K\left(1 - e^{-rT}\right)$$

The difference of a call and put is the time value of money.

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Example 9.1-2 Make the same assumptions as in Example 9.1-1, except suppose that the stock pays a \$5 dividend just before expiration. If the price of the European call is \$0.74, what would be the price of the European put?

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$$\$0.74 = y + \left(\$40 - \$5e^{-0.08\times0.25}\right) - \$40e^{-0.08\times0.25}$$

Hence,

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Synthetic securities

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

- Synthetic stock

$$S_0 = C(K,T) - P(K,T) + \mathrm{PV}_{0,T}(\mathrm{Div}) + e^{-rT}K$$

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

- Synthetic Treasury bill (T-bill)

$$\underbrace{S_0 - C(K,T) + P(K,T)}_{\text{a conversion}} = \mathrm{PV}_{0,T}\,(\mathrm{Div}) + e^{-rT}K$$

Motivation:

A hedged position that has no risk but requires investment.

T-bills are taxed differently than stocks.

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- Synthetic options

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

$$P(K,T) = C(K,T) - (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) + e^{-rT}K$$

Generalize the parity to apply to the case where the strike asset is not necessarily cash but could be any other asset.

We will skip this section and leave it for motivated students.

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Chapter 9. Parity and other option relationships

European versus American options

$$C_{\mathrm{Amer}}(S, K, T) \geq C_{\mathrm{Eur}}(S, K, T)$$

$$P_{\mathrm{Amer}}(S, K, T) \geq P_{\mathrm{Eur}}(S, K, T)$$

Maximum and minimum option prices

$$S \geq C_{\mathrm{Amer}}(S, K, T) \geq C_{\mathrm{Eur}}(S, K, T) \geq \max\left(0, \mathrm{PV}_{0,T}(F_{0,T}) - \mathrm{PV}_{0,T}(K)\right)$$

$$K \geq P_{\mathrm{Amer}}(S, K, T) \geq P_{\mathrm{Eur}}(S, K, T) \geq \max\left(0, \mathrm{PV}(K) - \mathrm{PV}_{0,T}(F_{0,T})\right)$$

Early exercise for American options

$$C_{\text{Amer}} \geq C_{\text{Eur}} > S_t - K$$

$$K - \mathrm{PV}_{t,T}(K) > \mathrm{PV}_{t,T}(\mathrm{Div})$$

See p. 277 for the proof of the first set of inequalities.

Unfinished...

Problems: 9.1, 9.2, 9.3, 9.4, 9.8, 9.9, 9.15.

Due Date: TBA