

Financial Mathematics

MATH 5870/6870¹
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¹Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 9. Parity and other option relationships

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§ 9.1 Put-call parity

§ 9.2 Generalized parity and exchange options

§ 9.3 Comparing options with respect to style, maturity, and strike

§ 9.4 Problems

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European options

$$\begin{aligned}C(K, T) - P(K, T) &= \text{PV}_{0,T}(F_{0,T} - K) \\ &= e^{-rT}(F_{0,T} - K)\end{aligned}$$

Buying a call and selling a put
with the strike both equal to the forward price (i.e., $K = F_{0,T}$)
creates a synthetic forward contract
and hence must have a zero price.

Parity generally fails for American options!

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Parity for stocks

$$C(K, T) = P(K, T) + (S_0 - \text{PV}_{0,T}(\text{Div})) - e^{-rT} K$$

Example 9.1-1 Suppose that the price of a non-dividend-paying stock is \$40, the continuously compounded interest rate is 8%, and options have 3 months to expiration. If a 40-strike European call sells for \$2.78, find the price for a 40-strike European put sells.

Solution. Let the price for put be y . Then

$$\$2.78 = y + \$40 - \$40e^{-0.08 \times 0.25}$$

Hence,

$$y = \$1.99.$$



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Why is a call more expensive than a put?

When $S_0 = K$ and $\text{Div} = 0$, then

$$C(K, T) - P(K, T) = K \left(1 - e^{-rT} \right)$$

The difference of a call and put is
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Example 9.1-2 Make the same assumptions as in Example 9.1-1, except suppose that the stock pays a \$5 dividend just before expiration. If the price of the European call is \$0.74, what would be the price of the European put?

Solution. Let the price for put be y . Then

$$\$0.74 = y + (\$40 - \$5e^{-0.08 \times 0.25}) - \$40e^{-0.08 \times 0.25}$$

Hence,

$$y = \$4.85.$$



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Synthetic securities

$$C(K, T) = P(K, T) + (S_0 - \text{PV}_{0,T}(\text{Div})) - e^{-rT}K$$

► Synthetic stock

$$S_0 = C(K, T) - P(K, T) + \text{PV}_{0,T}(\text{Div}) + e^{-rT}K$$

$$C(K, T) = P(K, T) + (S_0 - \text{PV}_{0,T}(\text{Div})) - e^{-rT} K$$

► Synthetic Treasury bill (T-bill)

$$\underbrace{S_0 - C(K, T) + P(K, T)}_{\text{a conversion}} = \text{PV}_{0,T}(\text{Div}) + e^{-rT} K$$

Motivation:

A hedged position that has no risk but requires investment.

T-bills are taxed differently than stocks.

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► Synthetic options

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$$P(K, T) = C(K, T) - (S_0 - PV_{0,T}(\text{Div})) + e^{-rT} K$$

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