Financial Mathematics

MATH 5870/6870[1]
Fall 2021

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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 2. An Introduction to Forwards and Options

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§ 2.1 Forward contracts

§ 2.2 Call options

§ 2.3 Put options

§ 2.4 Summary of forward and option positions

§ 2.5 Problems

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§ 2.2 Call options

§ 2.3 Put options

§ 2.4 Summary of forward and option positions

§ 2.5 Problems

Chapter 2. An Introduction to Forwards and Options

Can one modify the forward contract so that
the buyer can walk away from the deal at expiration?

Definition 2.2-1 A call option is a contract where the buyer has the right to buy, but not the obligation to buy.

Example 2.2-1 S&R index: Buyers' perspective

- Today: call buyer acquires the right to pay \$1,020 in six months for the index, but is not obligated to do so
- In six months at contract expiration:
  if the spot price is \$1,100, call buyers payoff $= \$1,100 - \$1,020 = \$80$
  if the spot price is \$900, call buyer walks away, buyers payoff = \$0.

Example 2.2-2 S&R index: Sellers' perspective

- Today: call seller is obligated to sell the index for \$1,020 in six months, if asked to do so
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- Strike (or exercise) price: the amount paid by the option buyer for the asset if he/she decides to exercise.

- Exercise: the act of paying the strike price to buy the asset.

- Expiration: the date by which the option must be exercised or become worthless.

- Exercise style: specifies when the option can be exercised.

Style	can be exercised
European	only at expiration date
American	at any time before expiration
Bermudan	during specified periods

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Payoff of purchased call $= \max(0, \text{spot price at expiration} - \text{strike price})$

Profit of purchased call = payoff of purchased call
$-$ future value of option premium

Payoff of written call $= -\max(0, \text{spot price at expiration} - \text{strike price})$

Profit of written call = payoff of written call
$+$ future value of option premium

Example 2.2-3 S&R Index 6-month European call option

$$\begin{aligned} \text{Strike price} &= \$1,000, \\ \text{Premium} &= \$93.81, \\ \text{6-month risk-free rate} &= 2\%. \end{aligned}$$

Compute both payoff and profit of the purchased call option if the index value in six months $\$1,100$ (resp. $\$900$).

Solution.

If index value in six months = \$1,100,

$$\begin{aligned} \text{Payoff} &= \max(0, \$1,100 - \$1,000) \\ &= \$100 \\ \text{Profit} &= \$100 - \$93.81 \times 1.02 \\ &= \$4.32. \end{aligned}$$

If index value in six months = \$900,

$$\begin{aligned} \text{Payoff} &= \max(0, \$900 - \$1,000) \\ &= \$0 \\ \text{Profit} &= \$0 - \$93.81 \times 1.02 \\ &= -\$95.68. \end{aligned}$$

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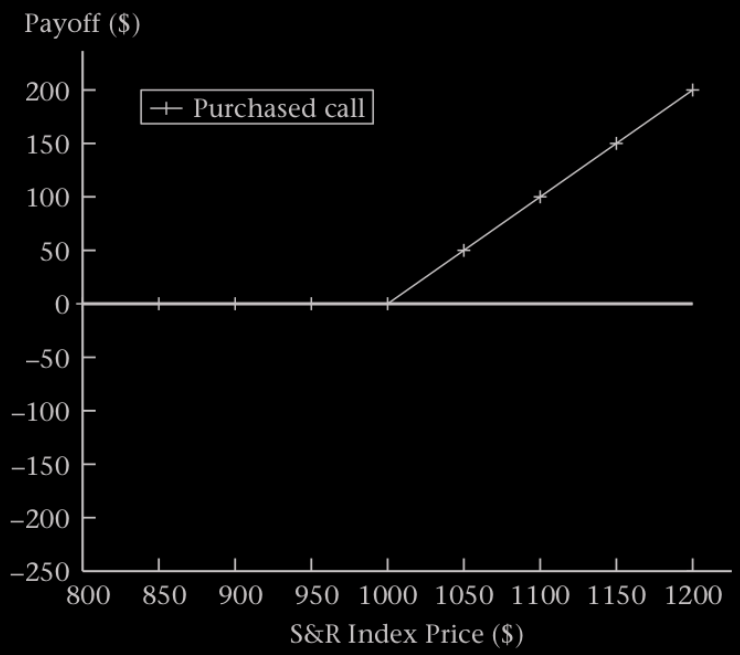
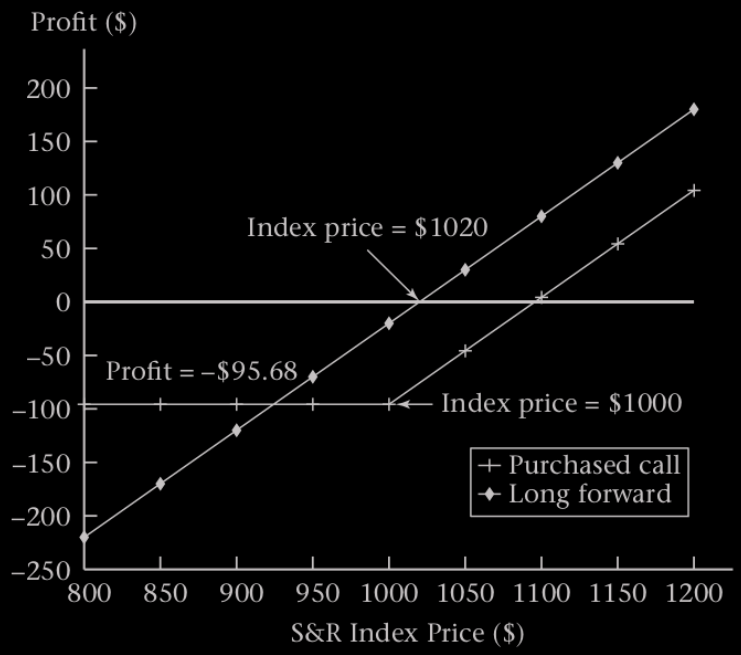

Payoff ($)
Purchased call
200
150
100
50
0
–50
–100
–150
–200
–250
800 850 900 950 1000 1050 1100 1150 1200
S&R Index Price ($)


Profit ($)
Index price = $1020
Profit = –$95.68
Index price = $1000
Purchased call
Long forward
200
150
100
50
0
–50
–100
–150
–200
–250
800 850 900 950 1000 1050 1100 1150 1200
S&R Index Price ($)

Example 2.2-4 S&R Index 6-month European call option

$$\begin{aligned} \text{Strike price} &= \$1,000, \\ \text{Premium} &= \$93.81, \\ \text{6-month risk-free rate} &= 2\%. \end{aligned}$$

Compute both payoff and profit of the written call option if the index value in six months $\$1,100$ (resp. $\$900$).

Solution.

If index value in six months = \$1,100,

$$\begin{aligned} \text{Payoff} &= -\max(0, \$1,100 - \$1,000) \\ &= -\$100 \\ \text{Profit} &= -\$100 + \$93.81 \times 1.02 \\ &= -\$4.32. \end{aligned}$$

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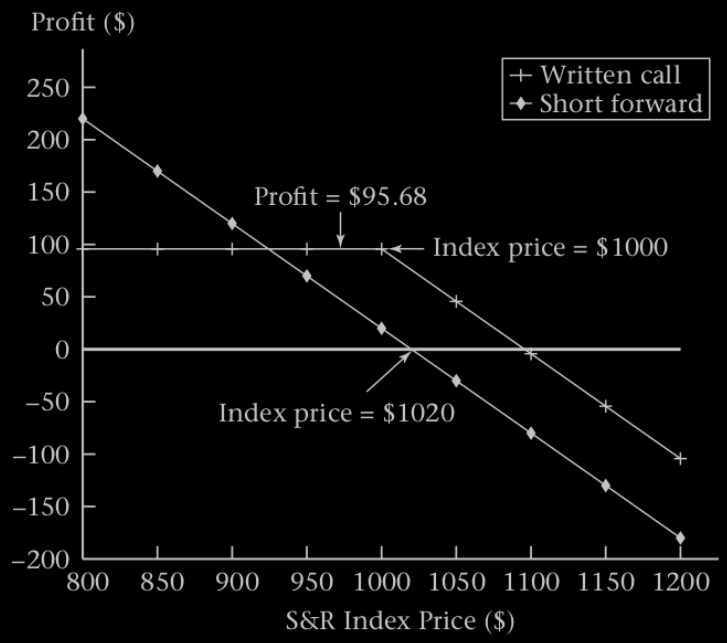
Profit ($)
Written call
Short forward
250
200
150
100
50
0
−50
−100
−150
−200
Profit = $95.68
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Index price = $1020
800
850
900
950
1000
1050
1100
1150
1200
S&R Index Price ($)