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Computation for Figure 11.4 (partial)
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Let ' s compute the node with stock price 43.246

First input the data

In[∘]:=

```
α = 0.30;
r = 0.08;
h = 1/3;
S = 43.246;
Cu = 12.814;
Cd = 0;
δ = 0;
σ = 0.30;
u = Exp[(r - δ) h + σ Sqrt[h]];
d = Exp[(r - δ) h - σ Sqrt[h]];
```

$$\alpha = 0.30;\quad r = 0.08;\quad h = \frac{1}{3};\quad S = 43.246;\quad Cu = 12.814;\quad Cd = 0;\quad \delta = 0;\quad \sigma = 0.30;$$

$$u = \mathrm{Exp}\left[(r - \delta)\, h + \sigma \sqrt{h}\right];$$

$$d = \mathrm{Exp}\left[(r - \delta)\, h - \sigma \sqrt{h}\right];$$

Now compute p

In[∘]:= $p = \dfrac{\mathrm{Exp}[\alpha\, h] - d}{u - d}$

Out[∘]= 0.675363

Now compute the expected payoff

In[∘]:= $X = p\, Cu + (1 - p)\, Cd$

Out[∘]= 8.6541

Now compute Δ and B

In[∘]:=

$$\Delta = \mathrm{Exp}[-\delta\, h]\, \frac{Cu - Cd}{S\,(u - d)}$$

$$B = \mathrm{Exp}[-r\, h]\, \frac{u\, Cd - d\, Cu}{u - d}$$

Out[∘]= 0.828701

Out[∘]= -30.1385

Now compute the discounted rate γ

In[∘]:= $\gamma = \dfrac{1}{h}\, \mathrm{Log}\left[\dfrac{\Delta\, S}{\Delta\, S + B}\, e^{\alpha\, h} + \dfrac{B}{\Delta\, S + B}\, e^{r\, h}\right]$

Out[∘]= 1.25297

Finally, compute the discounted expected payoff

In[ ]:= $X e^{-\gamma h}$

Out[ ]= 5.69949

We want to compare it with the other formula $\Delta S + B$

In[ ]:= $\Delta S + B$

Out[ ]= 5.69949

They produce the same value!

Now let's try another α

In[ ]:= `α = 0.40;`

Now compute p

In[ ]:= $p = \frac{\text{Exp}[\alpha h] - d}{u - d}$

Out[ ]= 0.78013

Now compute the expected payoff

In[ ]:= $X = p\,Cu + (1 - p)\,Cd$

Out[ ]= 9.99659

Now compute Δ and B has been computed in the previous case.

Now compute the discounted rate γ

In[ ]:= $\gamma = \frac{1}{h} \text{Log}\left[\frac{\Delta S}{\Delta S + B} e^{\alpha h} + \frac{B}{\Delta S + B} e^{r h}\right]$

Out[ ]= 1.6856

Finally, compute the discounted expected payoff

In[ ]:= $X e^{-\gamma h}$

Out[ ]= 5.69949

They produce the same value!
Different pairs of (α, γ) can produce the same option price,
these pairs are called consistent pairs.