

Financial Mathematics

MATH 5870/6870¹
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¹Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 19. Monte Carlo Valuation

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§ 19.1 Computing the option price as a discounted expected value

§ 19.2 Computing random numbers

§ 19.3 Simulating lognormal stock prices

§ 19.4 Monte Carlo valuation

§ 19.5 Efficient Monte Carlo valuation

§ 19.6 Valuation of American options

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§ 19.6 Valuation of American options

$$V(S_0, 0) = \frac{1}{n} e^{-rT} \sum_{n=1}^n V(S_T^i, T)$$

where

- S_T^1, \dots, S_T^n are n randomly drawn time- T stock prices.
- For European Call:

$$V(S_T^i, T) = \max(0, S_T^i - K)$$

Similarly one finds the expression for European put.

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Example 19.4-1 Carry out the Monte Carlo valuation of the European call under the setting of the following table:

TABLE 19.2

Results of Monte Carlo valuation of European call with $S = \$40$, $K = \$40$, $\sigma = 30\%$, $r = 8\%$, $t = 91$ days, and $\delta = 0$. The Black-Scholes price is \$2.78. Each trial uses 500 random draws.

Trial	Computed Price (\$)
1	2.98
2	2.75
3	2.63
4	2.75
5	2.91
Average	2.804

Solution. Check

codes/Table_19-2.py



Example 19.4-2 Carry out the Monte Carlo valuation of the Asian call under the setting of the following table:

TABLE 19.3

Prices of arithmetic average-price Asian options estimated using Monte Carlo and exact prices of geometric average price options. Assumes option has 3 months to expiration and average is computed using equal intervals over the period. Each price is computed using 10,000 trials, assuming $S = \$40$, $K = \$40$, $\sigma = 30\%$, $r = 8\%$, $T = 0.25$, and $\delta = 0$. In each row, the same random numbers were used to compute both the geometric and arithmetic average price options. σ_n is the standard deviation of the estimated arithmetic option prices, divided by $\sqrt{10,000}$.

Number of Averages	Monte Carlo Prices (\$)		Exact	
	Arithmetic	Geometric	Geometric Price (\$)	σ_n
1	2.79	2.79	2.78	0.0408
3	2.03	1.99	1.94	0.0291
5	1.78	1.74	1.77	0.0259
10	1.70	1.66	1.65	0.0241
20	1.66	1.61	1.59	0.0231
40	1.63	1.58	1.56	0.0226

Solution. Check

codes/Table_19-3.py

