

Financial Mathematics

MATH 5870/6870¹
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¹Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 3. Insurance, Collars, and Other Strategies

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§ 3.1 Basic insurance strategies

§ 3.2 Put-call parity

§ 3.3 Spreads and collars

§ 3.4 Speculating on volatility

§ 3.5 Problems

Chapter 3. Insurance, Collars, and Other Strategies

§ 3.1 Basic insurance strategies

§ 3.2 Put-call parity

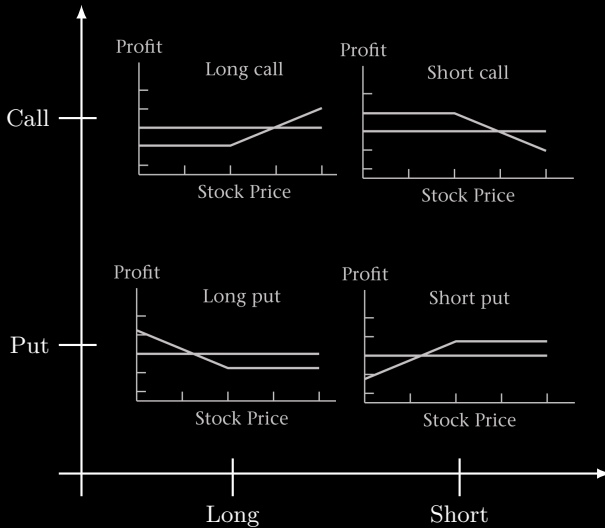
§ 3.3 Spreads and collars

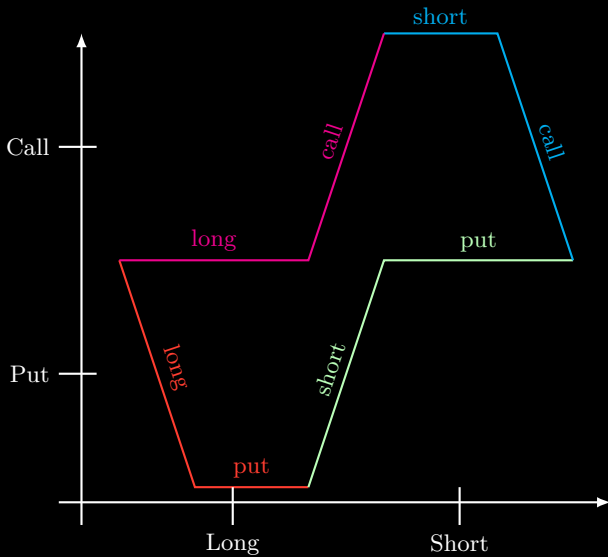
§ 3.4 Speculating on volatility

§ 3.5 Problems

It is always possible
to
lower the cost of a position
by
reducing its payoff!

By combining two or more options, we find many well-known strategies.





An **option spread** is a position consisting of only calls or only puts, in which some options are purchased and some written.

- ▶ Bull and bear spreads

- ▶ Box spreads

- ▶ Ratio spreads

- ▶ Collars

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Example for this section

Black-Scholes option prices

Stock price = \$40

Volatility = 30%

Effective annual risk-free rate = 8.33%

Dividend yield = \$0

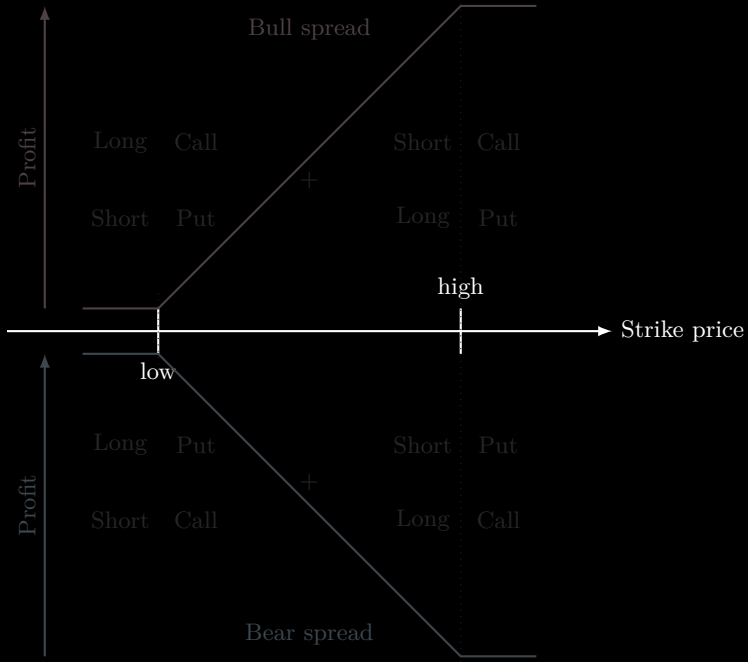
Expiration days = 91 days

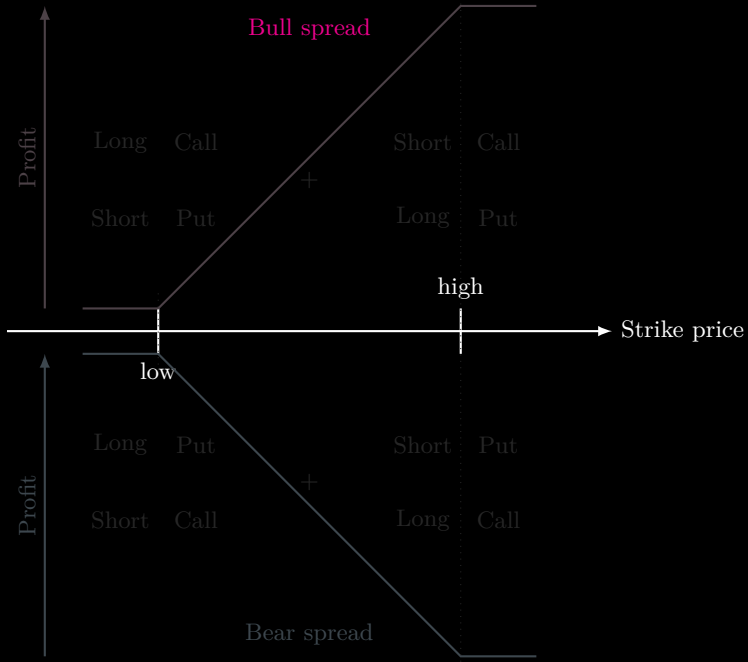
Strike	Call	Put
35	6.13	0.44
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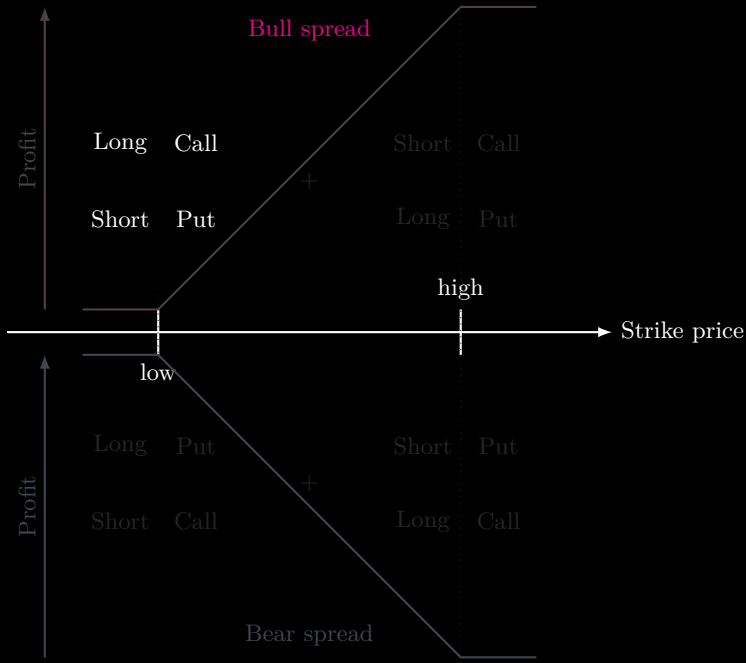
Bull and bear spreads

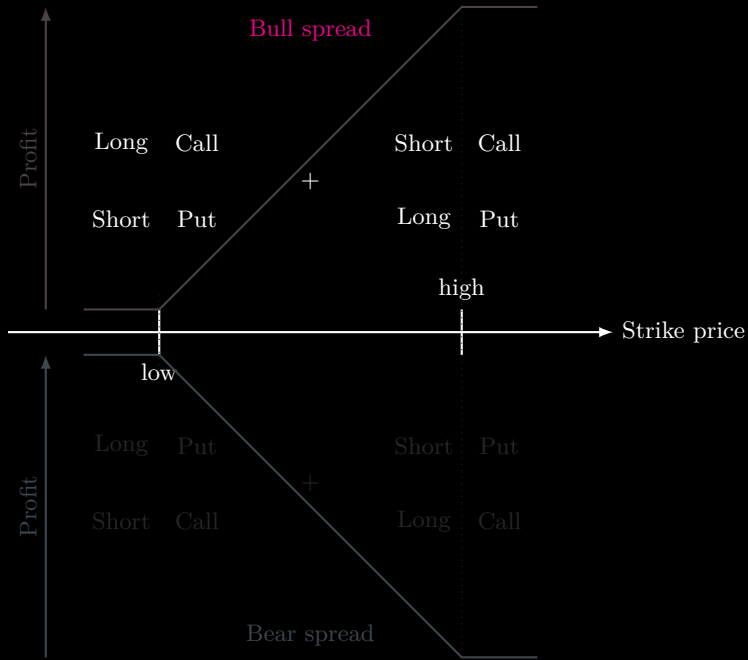
A position in which you buy a call and sell an otherwise identical call with a higher strike price is an example of a **bull spread**. Bull spreads can also be constructed using puts.

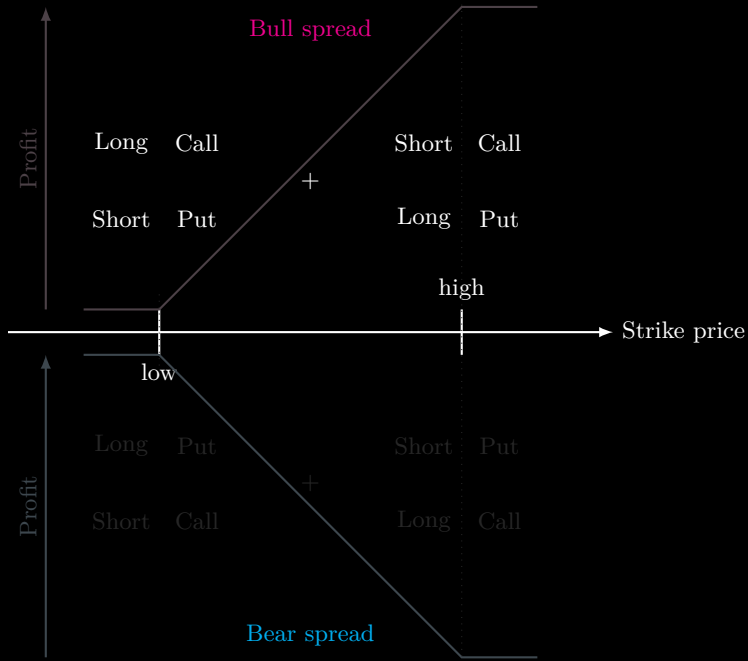
The opposite of a bull spread is a **bear spread**.

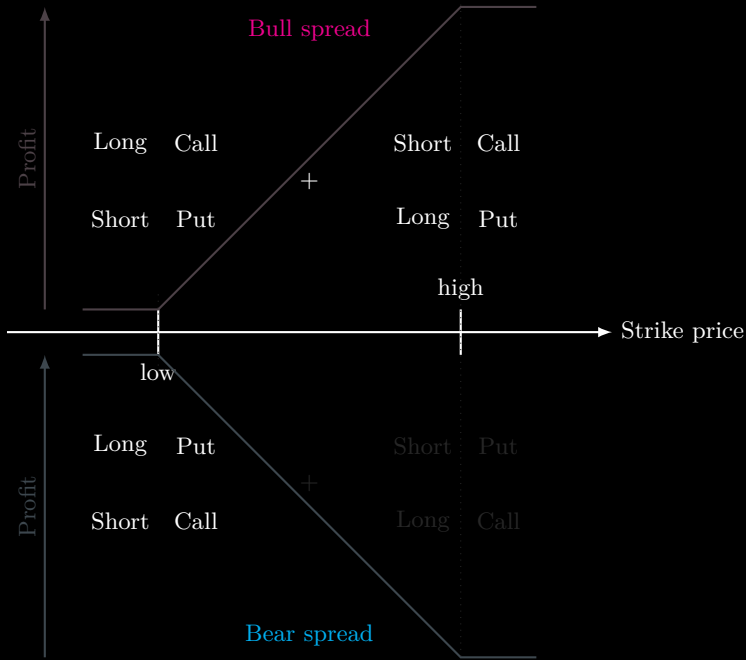


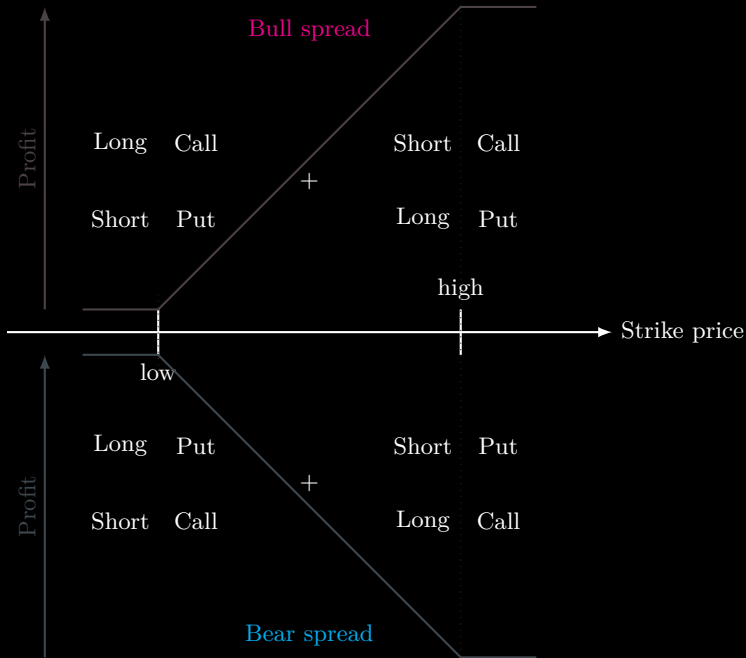


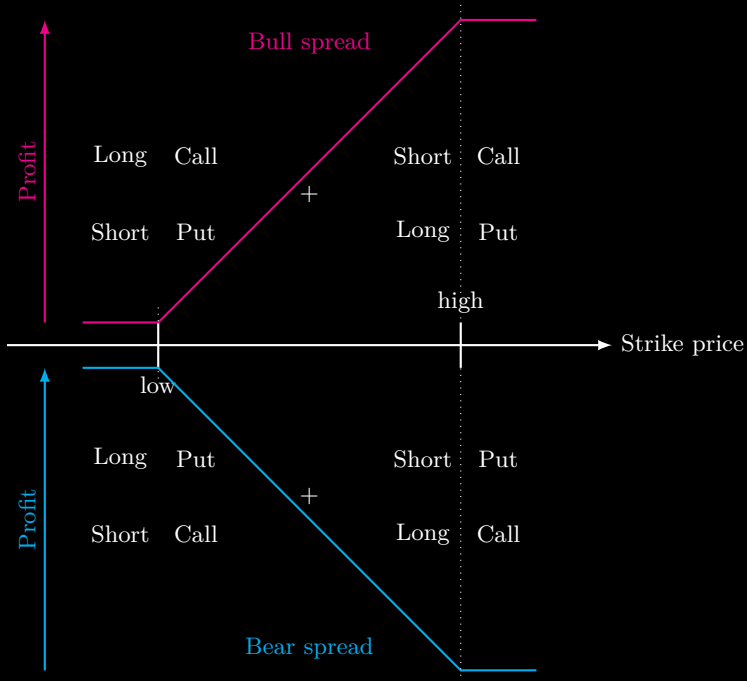












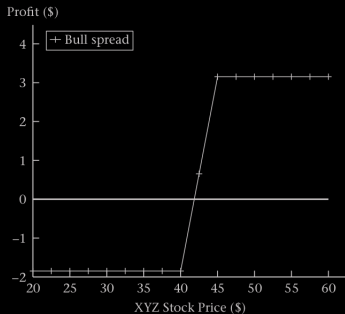
Example 3.3-1 Draw profit diagram for a 40-45 bull spread, namely, buying a 40-strike call and selling a 45-strike call.

Solution.

We only need to determine the two levels.

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Solution(Continued).

(a) Suppose that the index price is \$ 30 at the expiration:

$$(\$2.78 - \$0.97) \times (1 + 0.0833)^{1/4} = \$1.81.$$

(b) Suppose that the index price is \$50 at the expiration:

$$(\$50 - \$40) - (\$40 - \$45) - \$1.81 = \$3.15.$$



Solution(Continued).

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Box spreads

A **box spread** is accomplished by using options to create a **synthetic long forward** at one price and a **synthetic short forward** at a different price.

This strategy guarantees a cash flow in the future.

Hence, it is an option spread that is purely a means of borrowing or lending money. It is costly but has no stock price risk.

Example 3.3-2 Suppose we simultaneously enter into the following two transactions:

1. Buy a 40-strike call and sell a 40-strike put.
2. Sell a 45-strike call and buy a 45-strike put.

Explain why there is no free lunch. Draw the profit diagram.

Solution. Check book 74.



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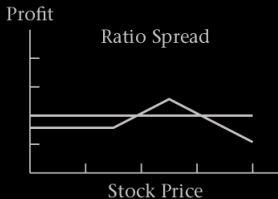
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Ratio spreads

A **ratio spread** is constructed by buying m options at one strike and selling n options at a different strike, with all options having the same type (call or put), same time to maturity, and same underlying asset.



Example 3.3-3 (Problem 3.15) Compute profit diagrams for the following ratio spreads:

- a Buy 950-strike call, sell two 1050-strike calls.
- b Buy two 950-strike calls, sell three 1050-strike calls.
- c Consider buying n 950-strike calls and selling m 1050-strike calls so that the premium of the position is zero. Considering your analysis in (a) and (b), what can you say about n/m ? What exact ratio gives you a zero premium?

Solution. Homework.



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Collars

A **collar** is the purchase of a put option and the sale of a call option with a higher strike price, with both options having the same underlying asset and having the same expiration date.

If the position is reversed, i.e., sale of a put and purchase of a call, the collar is written.

The **collar width** is the difference between the call and put strikes.

Example 3.3-4 Draw the profit diagram for a purchased collar:

selling a 45-strike call + buying a 40-strike put.

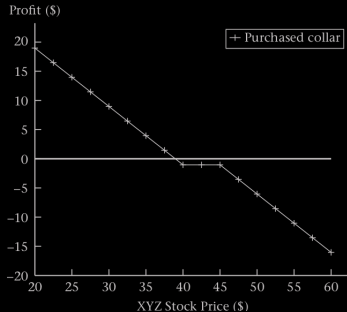
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Example 3.3-4 Draw the profit diagram for a purchased collar:

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Solution.



It is possible to find strike prices for the put and call such that the two premiums exactly offset one another. This position is called a **zero-cost collar**.

Example 3.3-5 Consider XYZ:

Strike	Call	Put
35	6.13	0.44
40	2.78	1.99
41.72	1.99	—
45	0.97	5.08

Show that the following gives a zero-cost collar

buying XYZ at \$40 + buying a 40-strike put + selling a 41.72-strike call

Draw the profit diagram.

Solution. Check book p. 77.



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