# Financial Mathematics

MATH 5870/6870[1]
Fall 2021

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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

# Chapter 3. Insurance, Collars, and Other Strategies

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§ 3.1 Basic insurance strategies

§ 3.2 Put-call parity

§ 3.3 Spreads and collars

§ 3.4 Speculating on volatility

§ 3.5 Problems

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§ 3.4 Speculating on volatility

§ 3.5 Problems

It is possible to <u>mimic a long forward position</u> on an asset by

buying a call + selling a put,

with each option having the same strike price and expiration time.

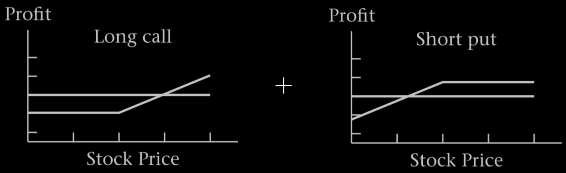
$$||$$

A synthetic forward

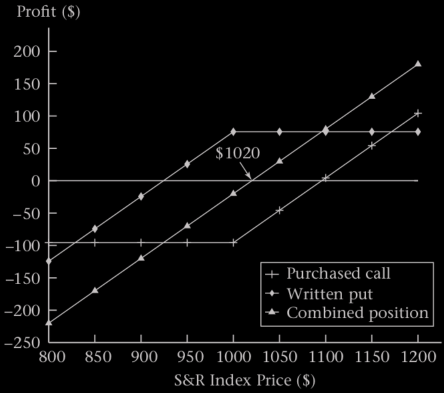
Example 3.2-1 Working with the S&R index. Suppose that

| | |
|---|---|
| 6-month interest rate | 2% |
| premium for 1000-strike 6-month call | \$93.809 |
| premium for 1000-strike 6-month put | \$74.201 |

Draw profit digram for the combined position of a purchased call with a written put, namely,



Solution.


Profit ($)
200
150
100
50
0
–50
–100
–150
–200
–250
800
850
900
950
1000
1050
1100
1150
1200
S&R Index Price ($)
$1020
Purchased call
Written put
Combined position

□

## A synthetic long forward contract

We pay the net option premium

We pay the strike price

---

## The actual forward

We pay zero premium

We pay the forward price

## Basic Assumption

The net cost of buying the index using options

must equal

the net cost of buying the index using a forward contract.

**NO ARBITRAGE!**

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## The Put-Call parity equation

$$\mathrm{Call}(K, T) - \mathrm{Put}(K, T) = \mathrm{PV}\left(F_{0,T} - K\right)$$

- $K$: strike price
- $T$: expiration date
- $\mathrm{Call}(\cdot, \circ)$: the premium for call.
- $\mathrm{Put}(\cdot, \circ)$: the premium for put.
- $F_{0,T}$: the forward price at time $T$ if one enters at time 0 into a long forward position.
- $\mathrm{PV}(\cdot)$: the present value function.

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Example 3.2-2 Check Example 3.2-1 to see if the put-call parity equation is satisfied.

Solution. We need to check:

$$\$93.809 - \$74.201 \stackrel{?}{=} \mathrm{PV}(\$1,000 \times 1.02 - \$1,000)$$

Clearly, LHS $= \$19.61$. On the other hand, the RHS is equal to

$$\begin{aligned}\mathrm{PV}(\$1,000 \times 1.02 - \$1,000) &= \mathrm{PV}\left(1,000 \times (1.02 - 1)\right)\\ &= \mathrm{PV}\left(1,000 \times 0.02\right)\\ &= \frac{1,000 \times 0.02}{1.02}\\ &= \$19.61.\end{aligned}$$

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Hence, the put-call parity equation is satisfied. $\square$

$$\mathrm{Call}(K, T) - \mathrm{Put}(K, T) = \mathrm{PV}\left(F_{0,T} - K\right)$$

$$\Updownarrow$$

$$\mathrm{PV}\left(F_{0,T}\right) + \mathrm{Put}(K, T) = \mathrm{Call}(K, T) + \mathrm{PV}\left(K\right)$$

---

Buying the index and buying the put

generate the same payoff as

buying the call and buying a zero-coupon bond (i.e. lending) PV($K$)

$$\mathrm{Call}(K, T) - \mathrm{Put}(K, T) = \mathrm{PV}\left(F_{0,T} - K\right)$$

$$\Updownarrow$$

$$\mathrm{PV}\left(F_{0,T}\right) - \mathrm{Call}(K, T) = \mathrm{PV}\left(K\right) - \mathrm{Put}(K, T)$$

---

Writing a covered call

has the same profit as

lending $\mathrm{PV}(K)$ and selling a put

$$\mathrm{Call}(K,T) - \mathrm{Put}(K,T) = \mathrm{PV}\left(F_{0,T}\right) - \mathrm{PV}\left(K\right)$$

Revisit four positions in Section 3.1

| Position | Meaning | equivalent to |
|---|---|---|
| Inuring a long position (floors) | | |
| Inuring a short position (caps) | | |
| Covered call writing | | |
| Covered put writing | | |

$$\mathrm{Call}(K,T) - \mathrm{Put}(K,T) = \mathrm{PV}\left(F_{0,T}\right) - \mathrm{PV}\left(K\right)$$

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