Financial Mathematics

MATH 5870/6870[1]
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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 2. An Introduction to Forwards and Options

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§ 2.1 Forward contracts

§ 2.2 Call options

§ 2.3 Put options

§ 2.4 Options are insurance

§ 2.5 Summary of forward and option positions

§ 2.6 Problems

Chapter 2. An Introduction to Forwards and Options

Definition 2.1-1 Forward contract is a binding agreement (obligation) to buy or sell an underlying asset in the future, at a price set today. The time at which the contract settles is called the expiration date. A forward contract specifies

- The features and quantity of the asset to be delivered.
- The delivery logistics, such as time, date, and place.
- The price the buyer will pay at the time of delivery.

Remark 2.1-1

1. Futures contracts are the same as forwards in principle except for some institutional and pricing differences. We will study future contracts in Chapter 5.
2. A forward contract requires no initial payment or premium.

Long = buy short = sell

Definition 2.1-2 Payoff for a contract is its value at expiration. In particular, for forward contracts,

Payoff for Long forward = Spot price at expiration − Forward price

Payoff for Short forward = Forward price − Spot price at expiration

Remark 2.1-2 Payoff and profit (net payoff) are the same for forward contracts because there is no initial payment – premium.

Example 2.1-1 S&R (special and rich) index:

Today: Spot price = \$1,000
6-month forward price = \$1,020
In six months at contract expiration: Spot price = \$1,050.

What are the payoff of long/short forward?

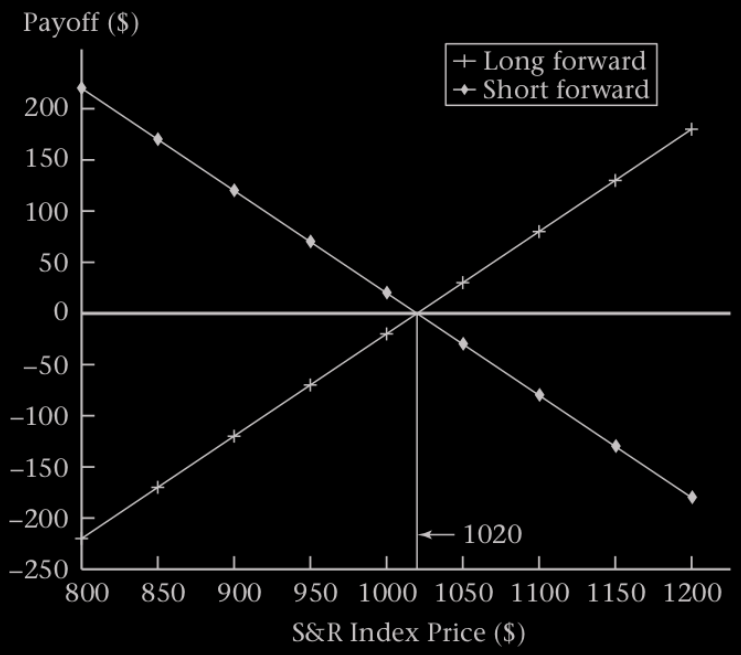
Solution.

$$\text{Long position payoff} = \$1,050 - \$1,020 = \$30,$$

$$\text{Short position payoff} = \$1,020 - \$1,050 = (\$30).$$

□

Payoff diagram for a forward price = \$1,020



Forward versus outright purchase

We will see this through the following example:

Example 2.1-2 S&R 6-month forward contract with a zero-coupon bound (e.g., Treasury bills). The 6-month interest rate is 2%. Spot price today $= \$1,000$.

$1,000 today is worth $\$1,000 \times 1.02 = \$1,020$ in 6 months.

Outright purchase[2] is equivalent to forward + bond[3]

because

$$\text{Payoff of forward+bond} = \underbrace{\text{Spot price at expiration} - \$1,020}_{\text{Forward payoff}} + \underbrace{\$1,020}_{\text{Bound payoff}}$$
$$= \text{Spot price at expiration}$$
$$= \text{Payoff of outright purchase}$$

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[2] It is also called long physical index.

[3] Invest $1,000 to bond for 6 month and enter long position of forward contract at the same time.

$1,000 today is worth $\$1,000 \times 1.02 = \$1,020$ in 6 months.

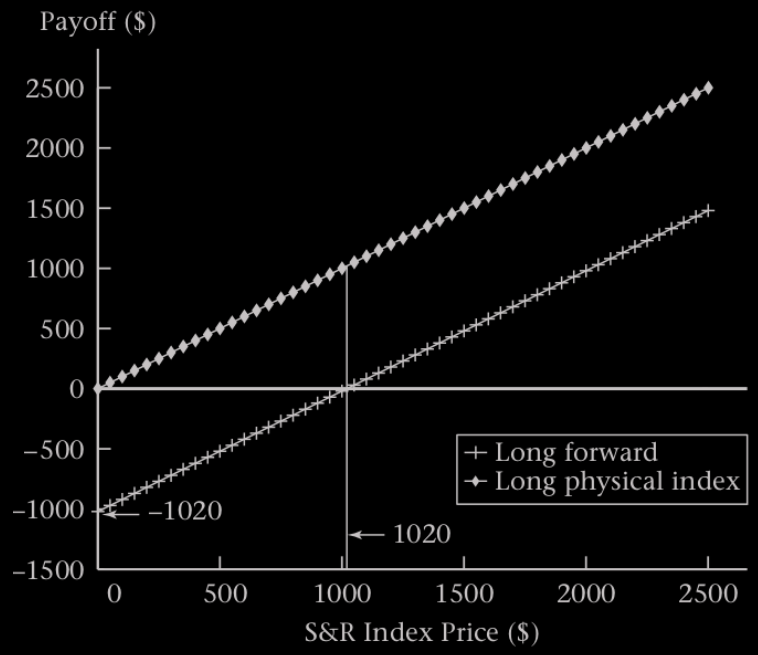
Long forward is equivalent to borrow-to-buy[4]

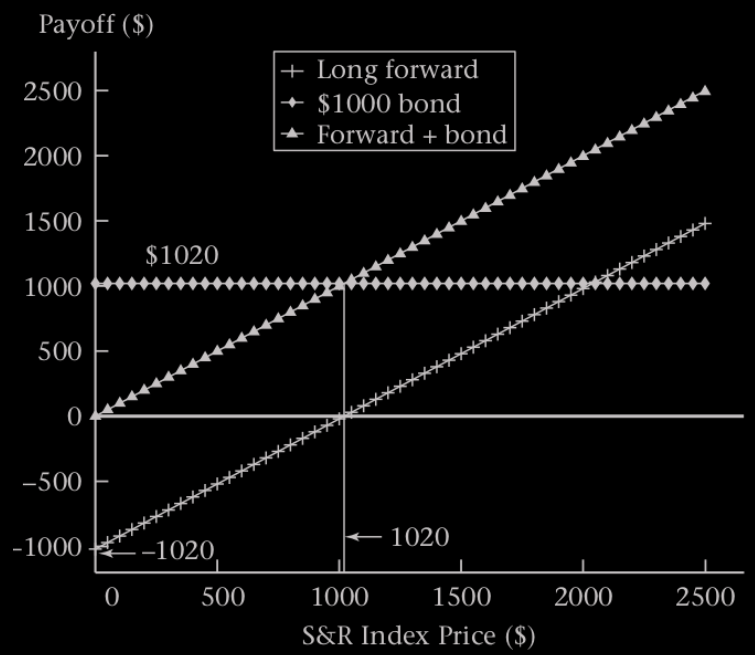
because

$$\text{Payoff of borrow-to-buy} = \underbrace{\text{Spot price at expiration}}_{\text{Payoff for outright buy}} - \underbrace{\$1,020}_{\text{Return borrowed money}}$$

$$= \text{Payoff of long forward}.$$

[4] Borrow money ($1,000) to outright buy physical index and at expiration pay back the money ($1,020).


Payoff ($)
2500
2000
1500
1000
500
0
−500
−1000
−1500
0
500
1000
1500
2000
2500
S&R Index Price ($)
Long forward
Long physical index
−1020
1020


Payoff ($)
2500
2000
1500
1000
500
0
−500
-1000
0
500
1000
1500
2000
2500
S&R Index Price ($)
Long forward
$1000 bond
Forward + bond
$1020
−1020
1020

Cash settlement versus physical delivery

– Type of settlement

- Cash settlement: less costly and more practical
- Physical delivery: often avoided due to significant costs

Example 2.1-3 Consider the S&R index with the forward price \$1,020.

- Suppose that the S&R index at expiration is \$1,040.
- The long position has a payoff of \$20.
- Similarly, the short position loses \$20.

With cash settlement, the short simply pays \$20 to the long, with no transfer of the physical asset, and hence no transaction costs. It is as if the long paid \$1,020, acquired the index worth \$1,040, and then immediately sold it with no transaction costs.

- Suppose that the S&R index price at expiration had instead been \$960.
- The long position would have a payoff of $-\$60$.
- The short would have a payoff of \$60.

Cash settlement in this case entails the long paying \$60 to the short.

Credit risk

All derivatives contracts have credit risk, which is the possibility that the counterparty who owes money fails to make a payment.

- Major issue for over-the-counter (OTC) contracts

  Credit check
  Credit protections such as collateral and bank letter of credit

- Less severe for exchange-traded contracts

  Exchange guarantees transactions, requires collateral