

# Financial Mathematics

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<sup>1</sup>Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

## Chapter 18. The Lognormal Distribution

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§ 18.1 The normal distribution

§ 18.2 The lognormal distribution

§ 18.3 A lognormal model of stock prices

§ 18.4 Lognormal probability calculations

§ 18.5 Problems

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§ 18.5 Problems

- Let  $R(t, s)$  be the continuously compounded return from time  $t$  to a later time  $s$ .
- For  $t_0 < t_1 < t_2$ ,  $R(\cdot, \cdot)$  has to satisfy the additivity property:

$$R(t_0, t_2) = R(t_0, t_1) + R(t_1, t_2)$$

- For time interval  $[0, T]$  divided into  $n$  subintervals of equal length  $T/n$ , we have

$$R(0, T) = R(0, h) + R(h, 2h) + \cdots + R((n-1)h, T)$$

Assume that

$$\mathbb{E}(R((i-1)h, ih)) = \alpha_h \quad \text{and} \quad \text{Var}(R((i-1)h, ih)) = \sigma_h^2$$

Then

$$\mathbb{E}(R(0, T)) = n\alpha_h \quad \text{and} \quad \text{Var}(R(0, T)) = n\sigma_h^2$$


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- By central limit theorem, as  $n \rightarrow \infty$ , one can assume that

$$R(0, T) \sim N$$

$$\ln(\mathcal{S}_t/\mathcal{S}_0) \sim N([\alpha - \delta - 0.5\sigma^2]t, \sigma^2 t)$$

$$\ln(\mathcal{S}_t/\mathcal{S}_0) = [\alpha - \delta - 0.5\sigma^2]t + \sigma\sqrt{t}Z$$

$$\mathcal{S}_t = \mathcal{S}_0 e^{[\alpha - \delta - 0.5\sigma^2]t} e^{\sigma\sqrt{t}Z}$$

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$$\mathbb{E}[\mathcal{S}_t] = \mathcal{S}_0 e^{[\alpha - \delta]t} \quad \text{and} \quad \text{Median stock price} = e^{[\alpha - \delta - 0.5\sigma^2]t}$$

$$\text{One standard deviation} \begin{cases} \text{move up} = e^{[\alpha - \delta - 0.5\sigma^2]t + \sigma\sqrt{t} \times 1} \\ \text{move down} = e^{[\alpha - \delta - 0.5\sigma^2]t - \sigma\sqrt{t} \times 1} \end{cases}$$

Go over examples 18.4 and 18.5 on textbook on p. 555.