Financial Mathematics

MATH 5870/6870¹ Fall 2021

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¹Based on Robert L. McDonald's *Derivatives Markets*. 3rd Ed. Pearson. 2013.

Chapter 20. Brownian Motion and Ito Lemma

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- § 20.1 The Black-Scholes assumption about stock prices
- § 20.2 Brownian motion
- § 20.3 Geometric Brownian motion
- § 20.4 The Ito formula
- § 20.5 The Sharpe ratio
- § 20.6 Risk-neutral valuation
- § 20.7 Problems

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The vast majority of technical option pricing discussions, including the original paper by Black and Scholes, assume that the price of the underlying asset follows a process determined by

$$dS(t) = (\alpha - \delta)dt + \sigma dZ(t), \quad S(0) = S_0.$$
 (1)

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$$dS(t) = (\alpha - \delta)dt + \sigma dZ(t), \quad S(0) = S_0$$

- ▶ S(t) is the stock price. dS(t) is the instantaneous change in the stock price. S_0 is the initial asset value.
- \triangleright α is the continuously compound expected return on the stock;
- $ightharpoonup \sigma$ is the volatility, i.e., the standard deviation of the instantaneous return:
- \triangleright Z(t) is the standard Brownian motion.
- ightharpoonup dZ(t) requires rigorous justification.

- ► Equation of this type is called stochastic differential equation.
- ► Solution to this specific equation is the geometric Brownian motion.

Remark 20.1-1 We will see in this chapter that solution to this equation is lognormally distributed:

$$\ln(\mathcal{S}(t)) \sim \mathcal{N}\left(\ln(\mathcal{S}_0) + \left(\alpha - \delta - \frac{1}{2}\sigma^2\right)t, \ \sigma^2 \ t\right), \quad ext{for all } t > 0.$$

Remark 20.1-2 Note that Remark 20.1-1 is valid for all t > 0. It works for the terminal time t = T. It can also help us solve path-dependent options.

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