Financial Mathematics

MATH 5870/6870[1]
Fall 2021

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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 3. Insurance, Collars, and Other Strategies

Chapter 3. Insurance, Collars, and Other Strategies

Options can be

1. Used to insure long positions (floors)
2. Used to insure short positions (caps)
3. Written against asset positions (selling insurance)
   Covered call writing
   Covered put writing

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Four positions

positions w.r.t. asset	put option	call option
long	purchased (floor)	written
short	written	purchased (cap)

Buying insurance

floor = buying a put option

cap = buying a call option

Selling insurance

Covered put writing

Covered call writing

We will work under the following setup

S&S index

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
premium for 1000-strike 6-month put	\$74.201

Insuring a long position – Floors

owning a home	owning a stock index
insuring the house	buying a put (floor)

Goal: to insure against a fall in the price of the underlying asset.

Example 3.1-1 Under the following scenario, compute the combined profit of insuring a long position via buying a put for the following S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	\$74.201
index price at expiration	\$900

Solution.

$$\underbrace{\$900 - \$1,000 \times 1.02}_{\text{profit on S\&R index}} + \underbrace{\$1,000 - \$900 - \$74.201 \times 1.02}_{\text{profit on put}} = -\$95.68.$$

$\square$

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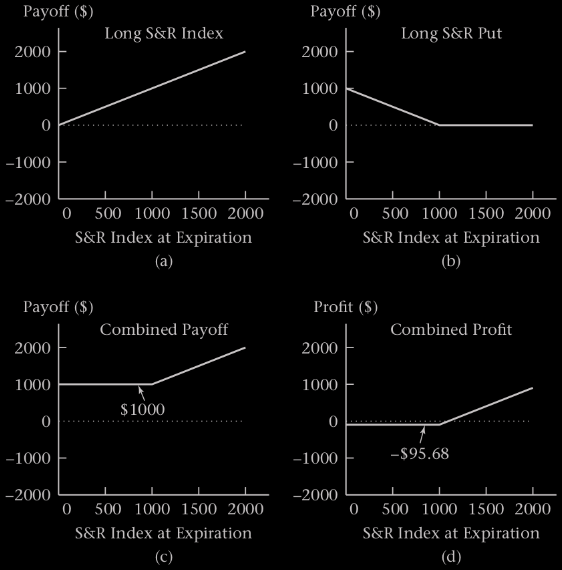
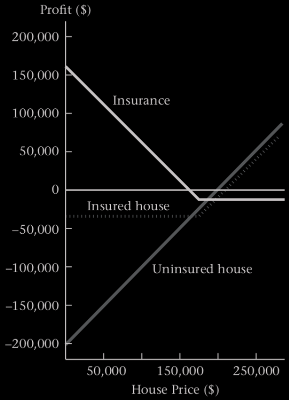


Payoff ($)
Long S&R Index
2000
1000
0
−1000
−2000
0
500
1000
1500
2000
S&R Index at Expiration
(a)
Payoff ($)
Long S&R Put
2000
1000
0
−1000
−2000
0
500
1000
1500
2000
S&R Index at Expiration
(b)
Payoff ($)
Combined Payoff
2000
1000
$1000
0
−1000
−2000
0
500
1000
1500
2000
S&R Index at Expiration
(c)
Profit ($)
Combined Profit
2000
1000
0
−$95.68
−1000
−2000
0
500
1000
1500
2000
S&R Index at Expiration
(d)


Profit ($)
200,000
150,000
100,000
50,000
0
−50,000
−100,000
−150,000
−200,000
Insurance
Insured house
Uninsured house
50,000
150,000
250,000
House Price ($)

Insuring a short position – Caps

If we have a short position in the S&R index, we experience a loss when the index rises.

We can insure a short position by purchasing a call option (cap) to protect against a higher price of repurchasing the index.

Example 3.1-2 Under the following scenario, compute the combined profit for insuring a short position via buying a call of the following S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
index price at expiration	\$1,100

Solution.

$$\underbrace{\$1,000 \times 1.02}_{\text{future value of short S\&R index}} - \underbrace{\$93.809 \times 1.02}_{\text{FV of premium for call}} - \underbrace{\$1,000}_{\text{exercise the call option}} = -\$75.685.$$

□

Example 3.1-2 Under the following scenario, compute the combined profit for insuring a short position via buying a call of the following S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
index price at expiration	\$1,100

Solution.

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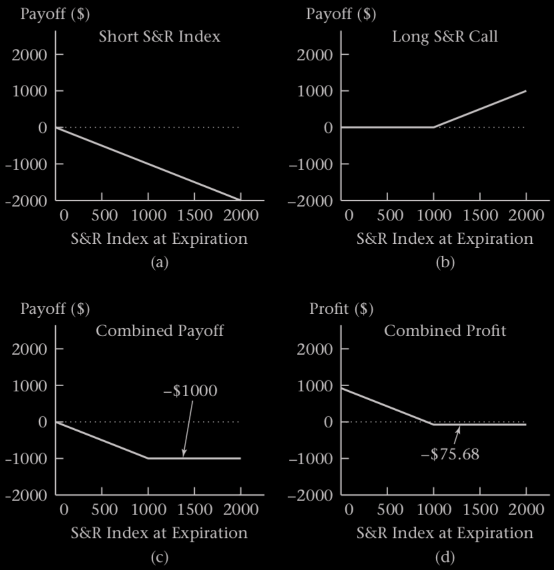
Payoff ($)
Short S&R Index
2000
1000
0
−1000
−2000
0
500
1000
1500
2000
S&R Index at Expiration
(a)
Payoff ($)
Long S&R Call
S&R Index at Expiration
(b)
Payoff ($)
Combined Payoff
−$1000
S&R Index at Expiration
(c)
Profit ($)
Combined Profit
−$75.68
S&R Index at Expiration
(d)

Selling insurance

For every insurance buyer there must be an insurance seller

Strategies used to sell insurance

- **Covered writing** (**option overwriting** or **selling a covered call**) is writing an option when there is a corresponding long position in the underlying asset.

- **Naked writing** is writing an option when the writer does not have a position in the asset.

Selling insurance

For every insurance buyer there must be an insurance seller

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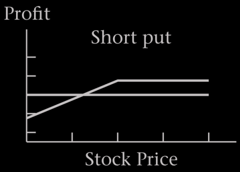
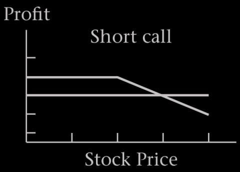
Covered call writing

Long position of the asset + Sell a call option $\sim$



Covered put writing

Short position of the asset + Sell a put option $\sim$



Covered call writing

Example 3.1-3 Under the following scenario, compute the combined profit for writing a covered call for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
index price at expiration	\$1,100

Solution.

$$\underbrace{\$1,100 - \$1,000 \times 1.02}_{\text{profit on S\&R index}} + \underbrace{\$1,000 - \$1,100 + \$93.809 \times 1.02}_{\text{profit on written call}} = \$75.68.$$

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Covered call writing

Example 3.1-3 Under the following scenario, compute the combined profit for writing a covered call for S&R index.

index price today	\$1,000
6-month interest rate	2%
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index price at expiration	\$1,100

Solution.

$$\underbrace{\$1,100 - \$1,000 \times 1.02}_{\text{profit on S\&R index}} + \underbrace{\$1,000 - \$1,100 + \$93.809 \times 1.02}_{\text{profit on written call}} = \$75.68.$$

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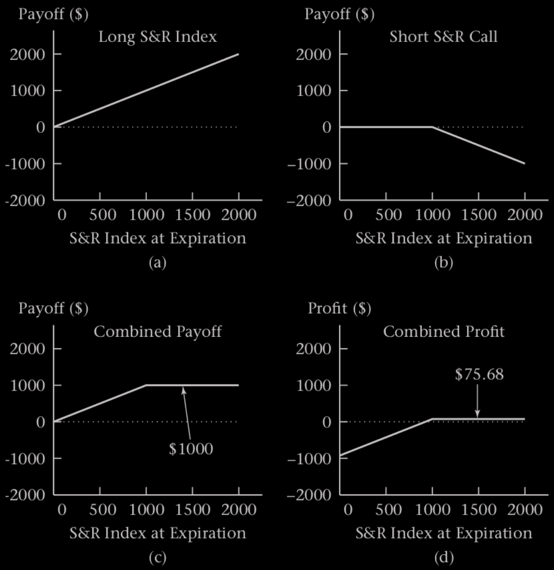

Payoff ($)
Long S&R Index
Payoff ($)
Short S&R Call
Payoff ($)
Combined Payoff
$1000
Profit ($)
Combined Profit
$75.68
2000
1000
0
-1000
-2000
0 500 1000 1500 2000
S&R Index at Expiration
(a)
(b)
(c)
(d)

Covered put writing

Example 3.1-4 Under the following scenario, compute the combined profit for writing a covered put for S&R index.

index price today	$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	$74.201
index price at expiration	$900

Solution.

$$\underbrace{\$1,000 \times 1.02 - \$900}_{\text{profit on selling S\&R index}} + \underbrace{\$900 - \$1,000 + \$74.201 \times 1.02}_{\text{profit on written put}} = \$95.685.$$

□

Covered put writing

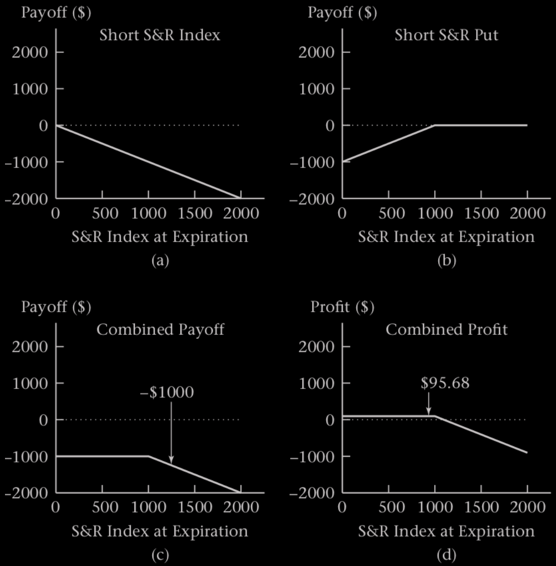
Example 3.1-4 Under the following scenario, compute the combined profit for writing a covered put for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	\$74.201
index price at expiration	\$900

Solution.

$$\underbrace{\$1,000 \times 1.02 - \$900}_{\text{profit on selling S\&R index}} + \underbrace{\$900 - \$1,000 + \$74.201 \times 1.02}_{\text{profit on written put}} = \$95.685.$$

$\square$



Payoff ($)
Short S&R Index
2000
1000
0
−1000
−2000
0
500
1000
1500
2000
S&R Index at Expiration
(a)
Payoff ($)
Short S&R Put
2000
1000
0
−1000
−2000
0
500
1000
1500
2000
S&R Index at Expiration
(b)
Payoff ($)
Combined Payoff
−$1000
2000
1000
0
−1000
−2000
0
500
1000
1500
2000
S&R Index at Expiration
(c)
Profit ($)
Combined Profit
$95.68
2000
1000
0
−1000
−2000
0
500
1000
1500
2000
S&R Index at Expiration
(d)

It is possible to <u>mimic a long forward position</u> on an asset by

buying a call + selling a put,

with each option having the same strike price and expiration time.

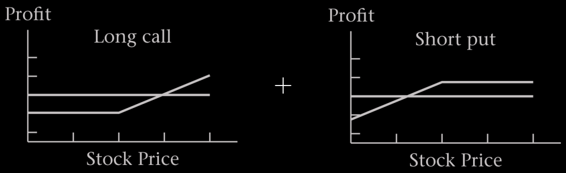
$$||$$

A synthetic forward

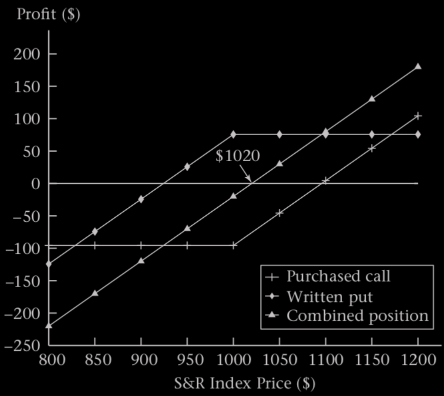
Example 3.2-1 Working with the S&R index. Suppose that

6-month interest rate	2%
premium for 1000-strike 6-month call	$93.809
premium for 1000-strike 6-month put	$74.201

Draw profit digram for the combined position of a purchased call with a written put, namely,



Solution.



Profit ($)
200
150
100
50
0
–50
–100
–150
–200
–250
$1020
Purchased call
Written put
Combined position
800
850
900
950
1000
1050
1100
1150
1200
S&R Index Price ($)

□

A synthetic long forward contract

We pay the net option premium

We pay the strike price

The actual forward

We pay zero premium

We pay the forward price

Basic Assumption

The net cost of buying the index using options

must equal

the net cost of buying the index using a forward contract.

NO ARBITRAGE!

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NO ARBITRAGE!

The Put-Call parity equation

$$\mathrm{Call}(K, T) - \mathrm{Put}(K, T) = \mathrm{PV}\left(F_{0,T} - K\right)$$

- K: strike price
- T: expiration date
- $\mathrm{Call}(\cdot, \circ)$: the premium for call.
- $\mathrm{Put}(\cdot, \circ)$: the premium for put.
- $F_{0,T}$: the forward price at time T if one enters at time 0 into a long forward position.
- $\mathrm{PV}(\cdot)$: the present value function.

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Example 3.2-2 Check Example 3.2-1 to see if the put-call parity equation is satisfied.

Solution. We need to check:

$$\$93.809 - \$74.201 \stackrel{?}{=} \mathrm{PV}(\$1,000 \times 1.02 - \$1,000)$$

Clearly, LHS $= \$19.61$. On the other hand, the RHS is equal to

$$\begin{aligned}\mathrm{PV}(\$1,000 \times 1.02 - \$1,000) &= \mathrm{PV}\left(1,000 \times (1.02 - 1)\right) \\ &= \mathrm{PV}\left(1,000 \times 0.02\right) \\ &= \frac{1,000 \times 0.02}{1.02} \\ &= \$19.61.\end{aligned}$$

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Hence, the put-call parity equation is satisfied. $\square$

$$\mathrm{Call}(K,T) - \mathrm{Put}(K,T) = \mathrm{PV}\left(F_{0,T} - K\right)$$

$$\Updownarrow$$

$$\mathrm{PV}\left(F_{0,T}\right) + \mathrm{Put}(K,T) = \mathrm{Call}(K,T) + \mathrm{PV}\left(K\right)$$

Buying the index and buying the put

generate the same payoff as

buying the call and buying a zero-coupon bond (i.e. lending) PV(K)

$$\mathrm{Call}(K, T) - \mathrm{Put}(K, T) = \mathrm{PV}\left(F_{0,T} - K\right)$$

$$\Updownarrow$$

$$\mathrm{PV}\left(F_{0,T}\right) - \mathrm{Call}(K, T) = \mathrm{PV}\left(K\right) - \mathrm{Put}(K, T)$$

Writing a covered call

has the same profit as

lending $\mathrm{PV}(K)$ and selling a put

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