Section 13.3: Delta–Hedging

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First define functions

In[ ]:=

```
Clear["Global`*"]
```

$$n[d\_] := \frac{1}{2} \times \left(1 + \text{Erf}\left[\frac{d}{\sqrt{2}}\right]\right)$$

$$d1 = \frac{\text{Log}\left[\frac{S}{K}\right] + \left(r - \delta + \frac{1}{2}\,\sigma^2\right)(T - t)}{\sigma\,\sqrt{T - t}};$$

$$d2 = d1 - \sigma\,\sqrt{T - t};$$

$$\text{OptionCall}[S\_,\ t\_] = S\,e^{-\delta\,(T-t)}\,n[d1] - K\,e^{-r\,(T-t)}\,n[d2];$$

$$\text{OptionPut}[S\_,\ t\_] = K\,e^{-r\,(T-t)}\,n[-d2] - S\,e^{-\delta\,(T-t)}\,n[-d1];$$

```
Δ[S_,t_]= D[OptionCall[S,t],S];
Γ[S_,t_]= D[OptionCall[S,t],{S,2}];
θ[S_,t_]= D[OptionCall[S,t],t];
```

Then define the constants (Setup)

```
K = 40;
T = 1;
```

$$t = \text{Table}\left[T - \frac{91 - i}{365},\ \{i,\ 0,\ 5\}\right];$$

```
r = 0.08;
σ = 0.30;
δ = 0;
n = 100;
```

First work out Table 13.2

In[ ]:=
```
S = {40, 40.50, 39.25, 38.75, 40, 40};
```

Compute Option Call prices with 100 shares

In[ ]:=
```
Table[{100 OptionCall[S⟦i⟧, t⟦i⟧], S⟦i⟧}, {i, 1, 6}]
```

Out[ ]=
```
{{278.04, 40}, {306.21, 40.5}, {232.822, 39.25},
 {205.462, 38.75}, {271.04, 40}, {269.271, 40}}
```

Compute Deltas with 100 shares

In[ ]:=
```
Table[{100 Δ[S⟦i⟧, t⟦i⟧], S⟦i⟧}, {i, 1, 6}]
```

Out[ ]=
```
{{58.2404, 40}, {61.4203, 40.5}, {53.1077, 39.25},
 {49.5635, 38.75}, {58.0598, 40}, {58.014, 40}}
```

Compute Investment

In[ ]:=
```
Investment = Table[100 Δ[S⟦i⟧, t⟦i⟧] S⟦i⟧ - 100 OptionCall[S⟦i⟧, t⟦i⟧], {i, 1, 6}];
Table[{Investment⟦i⟧, S⟦i⟧}, {i, 1, 6}]
```

Out[ ]=
```
{{2051.58, 40}, {2181.31, 40.5}, {1851.65, 39.25},
 {1715.12, 38.75}, {2051.35, 40}, {2051.29, 40}}
```

Compute the Interest charge

In[ ]:=
```
InterestCharge =
  Table[-(100 Δ[S⟦i⟧, t⟦i⟧] S⟦i⟧ - 100 OptionCall[S⟦i⟧, t⟦i⟧]) (e^(r/365) - 1), {i, 1, 5}];
Table[{InterestCharge⟦i⟧, S⟦i⟧}, {i, 1, 5}]
```

Out[ ]=
```
{{-0.44971, 40}, {-0.478149, 40.5}, {-0.405887, 39.25}, {-0.375959, 38.75}, {-0.449661, 40}}
```

Compute the capital gain

In[ ]:=
```
CapitalGain = 100 Table[(S⟦i + 1⟧ - S⟦i⟧) Δ[S⟦i⟧, t⟦i⟧] -
     (OptionCall[S⟦i + 1⟧, t⟦i + 1⟧] - OptionCall[S⟦i⟧, t⟦i⟧]), {i, 1, 5}];
Table[{CapitalGain⟦i⟧, S⟦i⟧}, {i, 1, 5}]
```

Out[ ]=
```
{{0.950387, 40}, {-3.38731, 40.5}, {0.806177, 39.25}, {-3.62403, 38.75}, {1.7693, 40}}
```

Compute the daily profit

In[ ]:=
```
Profits = Table[CapitalGain⟦i⟧ + InterestCharge⟦i⟧, {i, 1, 5}]
```

Out[ ]=
```
{0.500677, -3.86546, 0.400291, -3.99998, 1.31964}
```

Now work out Table 13.3

In[ ]:=
```
S = {40, 40.642, 40.018, 39.403, 38.797, 39.420};
```

Compute Option Call prices with 100 shares

```
In[ ]:= Table[{100 OptionCall[S[[i]], t[[i]]], S[[i]]}, {i, 1, 6}]
Out[ ]= {{278.04, 40}, {314.995, 40.642}, {275.603, 40.018},
  {239.308, 39.403}, {206.121, 38.797}, {236.764, 39.42}}
```

Compute Deltas with 100 shares

```
In[ ]:= Table[{100 Δ[S[[i]], t[[i]]], S[[i]]}, {i, 1, 6}]
Out[ ]= {{58.2404, 40}, {62.3158, 40.642}, {58.2692, 40.018},
  {54.0822, 39.403}, {49.7979, 38.797}, {54.0601, 39.42}}
```

Compute Investment

```
In[ ]:= Investment = Table[100 Δ[S[[i]], t[[i]]] S[[i]] - 100 OptionCall[S[[i]], t[[i]]], {i, 1, 6}];
Table[{Investment[[i]], S[[i]]}, {i, 1, 6}]
Out[ ]= {{2051.58, 40}, {2217.64, 40.642}, {2056.21, 40.018},
  {1891.69, 39.403}, {1725.89, 38.797}, {1894.29, 39.42}}
```

Compute the Interest charge

```
In[ ]:= InterestCharge =
  Table[-(100 Δ[S[[i]], t[[i]]] S[[i]] - 100 OptionCall[S[[i]], t[[i]]]) (e^(r/365) - 1), {i, 1, 5}];
Table[{InterestCharge[[i]], S[[i]]}, {i, 1, 5}]
Out[ ]= {{-0.44971, 40}, {-0.486112, 40.642},
  {-0.450727, 40.018}, {-0.414663, 39.403}, {-0.378318, 38.797}}
```

Compute the capital gain

```
In[ ]:= CapitalGain = 100 Table[(S[[i + 1]] - S[[i]]) Δ[S[[i]], t[[i]]] -
    (OptionCall[S[[i + 1]], t[[i + 1]]] - OptionCall[S[[i]], t[[i]]]), {i, 1, 5}];
Table[{CapitalGain[[i]], S[[i]]}, {i, 1, 5}]
Out[ ]= {{0.435145, 40}, {0.507079, 40.642},
  {0.460054, 40.018}, {0.412299, 39.403}, {0.381772, 38.797}}
```

Compute the daily profit

```
In[ ]:= Profits = Table[CapitalGain[[i]] + InterestCharge[[i]], {i, 1, 5}]
Out[ ]= {-0.0145647, 0.0209669, 0.00932785, -0.00236389, 0.00345348}
```