

# Financial Mathematics

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<sup>1</sup>Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

## Chapter 5. Financial Forwards and Futures

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§ 5.1 Alternative ways to buy a stock

§ 5.2 Prepaid forward contracts on stock

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§ 5.4 Futures contracts

§ 5.5 Problems

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**Forward price** is the future value of the prepaid forward price:

- ▶ No dividends

$$F_{0,T} = \text{FV} \left( F_{0,T}^p \right)$$

- ▶ Continuous dividends

$$F_{0,T} = S_0 e^{(r-\delta)T}$$

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$$F_{0,T} = S_0 e^{(r-\delta)T}$$

$$\text{Forward premium} = \frac{F_{0,T}}{S_0}$$

$$\text{Annualized forward premium} = \frac{1}{T} \ln \left( \frac{F_{0,T}}{S_0} \right)$$

# Does the forward price predict the future spot price?

## Buying a stock

Compensation for	Earn	Buying a stock
time value of the money	interest	✓
the risk of the stock	risk premium	✓

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## Entering a forward contract

Compensation for	Earn	Entering a forward contract
time value of the money	interest	✗
the risk of the stock	risk premium	✓



The forward price is the **expected future spot price**,  
**discounted at the risk premium.**

$$F_{0,T} = e^{rT} \times \underbrace{F_{0,T}^p}_{=\mathbb{E}_0(S_T)e^{-\alpha T}} = \mathbb{E}_0(\mathbf{S}_T)\mathbf{e}^{-(\alpha-r)T}$$

## Creating a synthetic forward contract

Assuming that the dividends are continuous and paid at the rate  $\delta$ .

Recall that

Payoff of a long forward position at expiration

$$\begin{array}{c} || \\ S_T - F_{0,T} \\ || \\ S_T - S_0 e^{(r-\delta)T} \end{array}$$

Forward = Stock – Zero-coupon bond

Transaction	Cash Flows	
	Time 0	Time $T$ (expiration)
Buy $e^{-\delta T}$ units of the index	$-S_0 e^{-\delta T}$	$+ S_T$
Borrow $S_0 e^{-\delta T}$	$+ S_0 e^{-\delta T}$	$- S_0 e^{(r-\delta)T}$
<b>Total</b>	0	$S_T - S_0 e^{(r-\delta)T}$

Stock = Forward + Zero-coupon bond

Transaction	Cash Flows	
	Time 0	Time $T$ (expiration)
Long one forward	0	$S_T - F_{0,T}$
Lend $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_0 e^{(r-\delta)T}$
<b>Total</b>	$-S_0 e^{-\delta T}$	$S_T$

Zero-coupon bond = Stock – Forward

Transaction	Cash Flows	
	Time 0	Time $T$ (expiration)
Long one forward	0	$S_T - F_{0,T}$
Lend $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_0 e^{(r-\delta)T}$
<b>Total</b>	$-S_0 e^{-\delta T}$	$S_T$

**Cash-and-carry** is a transaction in which one buys the underlying asset and short the offsetting forward contract.

A cash-and-carry has no risk because  
 You have an obligation to deliver the asset  
 that you have already owned.

Transaction	Cash Flows	
	Time 0	Time $T$ (expiration)
Buy tailed position in stock, paying $S_0e^{-\delta T}$	$-S_0e^{-\delta T}$	$+S_T$
Borrow $S_0e^{-\delta T}$	$+S_0e^{-\delta T}$	$-S_0e^{(r-\delta)T}$
Short forward	0	$F_{0,T} - S_T$
<b>Total</b>	0	$F_{0,T} - S_0e^{(r-\delta)T}$

## Cash-and-carry

Transaction	Cash Flows	
	Time 0	Time $T$ (expiration)
Buy tailed position in stock, paying $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_T$
Borrow $S_0 e^{-\delta T}$	$+S_0 e^{-\delta T}$	$-S_0 e^{(r-\delta)T}$
Short forward	0	$F_{0,T} - S_T$
<b>Total</b>	0	$F_{0,T} - S_0 e^{(r-\delta)T}$

Arbitrage when  $F_{0,T} > S_0 e^{(r-\delta)T}$

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## Reverse cash-and-carry

Transaction	Cash Flows	
	Time 0	Time $T$ (expiration)
Short tailed position in stock, receiving $S_0 e^{-\delta T}$	$+S_0 e^{-\delta T}$	$-S_T$
Lend $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_0 e^{(r-\delta)T}$
Long forward	0	$S_T - F_{0,T}$
<b>Total</b>	0	$S_0 e^{(r-\delta)T} - F_{0,T}$

Arbitrage when  $F_{0,T} < S_0 e^{(r-\delta)T}$