#### Financial Mathematics

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<sup>&</sup>lt;sup>1</sup>Based on Robert L. McDonald's *Derivatives Markets*. 3rd Ed. Pearson. 2013.

# Chapter 2. An Introduction to Forwards and Options

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- § 2.1 Forward contracts
- § 2.2 Call options
- § 2.3 Put options
- § 2.4 Summary of forward and option positions
- § 2.5 Problems

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Call option: Buyer can walk away.

???? option : Seller can walk away.

Definition 2.3-1 A put option gives the owner the right but not the obligation to sell the underlying asset at a predetermined price during a predetermined time period.

Remark 2.3-1 Similar to the call option case, a premium paid by the put buyer at the time the option is purchased is needed in order to compensate the put seller for being in a disadvantage position.

of put option	someone needs to		premium
seller	buy	has to buy if asked	receive
buyer	sell	can walk away	pay

Payoff of purchased put =  $\max(0, \text{strike price} - \text{spot price at expiration})$ 

Profit of purchased put = payoff of purchased put - future value of option premium

Payoff of written put =  $-\max(0, \text{strike price} - \text{spot price at expiration})$ 

Profit of written put = payoff of written put + future value of option premium

### Example 2.3-1 S&R Index 6-month European put option

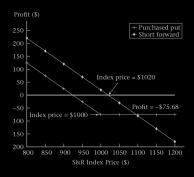
$$\begin{aligned} \text{Strike price} &=\$1,000,\\ \text{Premium} &=\$74.20,\\ \text{6-month risk-free rate} &=2\%. \end{aligned}$$

Compute both payoff and profit of the purchased put option if the index value in six months \$1,100 (resp. \$900).

#### Solution.

If index value in six months = $$1,100$ ,	If index value in six months = $$900$ ,
Payoff = $\max(0, \$1,000 - \$1, 100)$	Payoff = $\max(0, \$1,000 - \$900)$
= \$0	= \$100
$Profit = \$0 - \$74.20 \times 1.02$	$Profit = \$100 - \$74.20 \times 1.02$
=-\$75.68.	= \$24.32.

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### Example 2.3-2 S&R Index 6-month European put option

$$\begin{aligned} \text{Strike price} &=\$1,000,\\ \text{Premium} &=\$74.20,\\ \text{6-month risk-free rate} &=2\%. \end{aligned}$$

Compute both payoff and profit of the written put option if the index value in six months \$1,100 (resp. \$900).

#### Solution.

If index value in six months = $$1,100$ ,	If index value in six months = $$900$ ,
Payoff = $-\max(0, \$1,000 - \$1, 100)$	Payoff = $-\max(0, \$1,000 - \$900)$
= \$0	= -\$100
$Profit = \$0 + \$74.20 \times 1.02$	$Profit = -\$100 + \$74.20 \times 1.02$
= \$75.68.	=-\$24.32.

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#### A call option becomes more profitable when the underlying asset appreciates in value

A put option becomes more profitable when the underlying asset depreciates in value Definition 2.3-2 Moneyness of an option describes whether the option payoff would be positive if the option were exercised immediately.

## In particular, one has

Moneyness	payoff if exercised immediately
In-the-money option	> 0
At-the-money option	=0
Out-of-the money option	< 0

## Options are insurance

Example 2.3-3 Homeowner's insurance is a put option

