

# Financial Mathematics

MATH 5870/6870<sup>1</sup>  
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<sup>1</sup>Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

## Chapter 3. Insurance, Collars, and Other Strategies

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§ 3.1 Basic insurance strategies

§ 3.2 Put-call parity

§ 3.3 Spreads and collars

§ 3.4 Speculating on volatility

§ 3.5 Problems

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§ 3.1 Basic insurance strategies

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§ 3.4 Speculating on volatility

§ 3.5 Problems

It is possible to mimic a long forward position on an asset by  
buying a call + selling a put,  
with each option having the same strike price and expiration time.

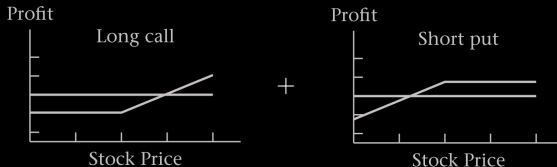
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A synthetic forward

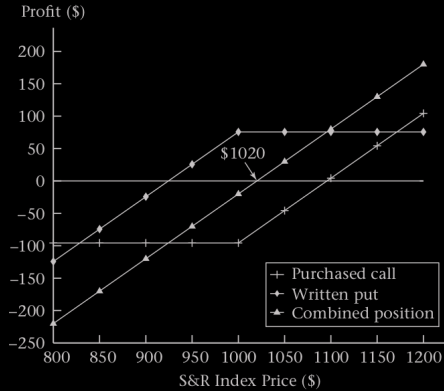
Example 3.2-1 Working with the S&R index. Suppose that

6-month interest rate	2%
premium for 1000-strike 6-month <b>call</b>	\$93.809
premium for 1000-strike 6-month <b>put</b>	\$74.201

Draw profit diagram for the combined position of a purchased call with a written put, namely,



Solution.



## A synthetic long forward contract

We pay the net option premium

We pay the strike price

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## The actual forward

We pay zero premium

We pay the forward price



## Basic Assumption

The net cost of buying the index using options  
must equal  
the net cost of buying the index using a forward contract.

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## The Put-Call parity equation

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T} - K)$$

- ▶  $K$ : strike price
- ▶  $T$ : expiration date
- ▶  $\text{Call}(c, K)$ : the premium for call
- ▶  $\text{Put}(p, K)$ : the premium for put
- ▶  $F_{0,T}$ : the forward price at time  $T$  if one enters at time 0 into a long forward position
- ▶  $\text{PV}(\cdot)$ : the present value function

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**Example 3.2-2** Check Example 3.2-1 to see if the put-call parity equation is satisfied.

Solution. We need to check:

$$\$93.809 - \$74.201 \stackrel{?}{=} PV(\$1,000 \times 1.02 - \$1,000)$$

Clearly, LHS = \$19.61. On the other hand, the RHS is equal to

$$\begin{aligned} PV(\$1,000 \times 1.02 - \$1,000) &= PV(1,000 \times (1.02 - 1)) \\ &= PV(1,000 \times 0.02) \\ &= \frac{1,000 \times 0.02}{1.02} \\ &= \$19.61. \end{aligned}$$

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$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T} - K)$$



$$\text{PV}(F_{0,T}) + \text{Put}(K, T) = \text{Call}(K, T) + \text{PV}(K)$$

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Buying the index and buying the put

generate the same payoff as

buying the call and buying a zero-coupon bond (i.e. lending)  $\text{PV}(K)$

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T} - K)$$



$$\text{PV}(F_{0,T}) - \text{Call}(K, T) = \text{PV}(K) - \text{Put}(K, T)$$

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Writing a covered call

has the same profit as

lending  $\text{PV}(K)$  and selling a put



$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T}) - \text{PV}(K)$$


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Revisit four positions in Section 3.1

Position	Meaning	equivalent to
Inuring a long position (floors)		
Inuring a short position (caps)		
Covered call writing		
Covered put writing		

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T}) - \text{PV}(K)$$


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Revisit four positions in Section 3.1

Position	Meaning	equivalent to
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Position	Meaning	equivalent to
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