Financial Mathematics

MATH 5870/6870[1]
Fall 2021

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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 3. Insurance, Collars, and Other Strategies

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§ 3.1 Basic insurance strategies

§ 3.2 Put-call parity

§ 3.3 Spreads and collars

§ 3.4 Speculating on volatility

§ 3.5 Problems

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1. Used to insure long positions (floors)
2. Used to insure short positions (caps)
3. Written against asset positions (selling insurance)
   Covered call writing
   Covered put writing

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Four positions

positions w.r.t. asset	put option	call option
long	purchased (floor)	written
short	written	purchased (cap)

Buying insurance	Selling insurance
floor = buying a put option	Covered put writing
cap = buying a call option	Covered call writing

We will work under the following setup

S&S index

index price	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
premium for 1000-strike 6-month put	\$74.201

Insuring a long position – Floors

owning a home	owning a stock index
insuring the house	buying a put (floor)

Goal: to insure against a fall in the price of the underlying asset.

Example 3.1-1 Under the following scenario, compute the combined profit.

S&R index

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	\$74.201
index price at expiration	\$900

Solution.

$$\underbrace{\$900 - \$1,000 \times 1.02}_{\text{profit on S\&R index}} + \underbrace{\$1,000 - \$900 - \$74.201 \times 1.02}_{\text{profit on put}} = -\$95.68.$$

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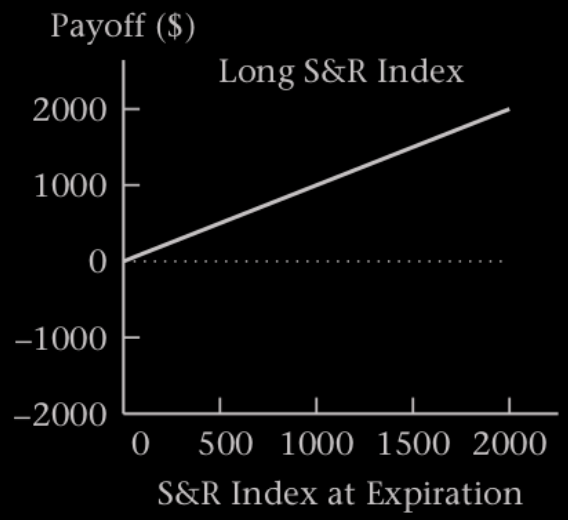
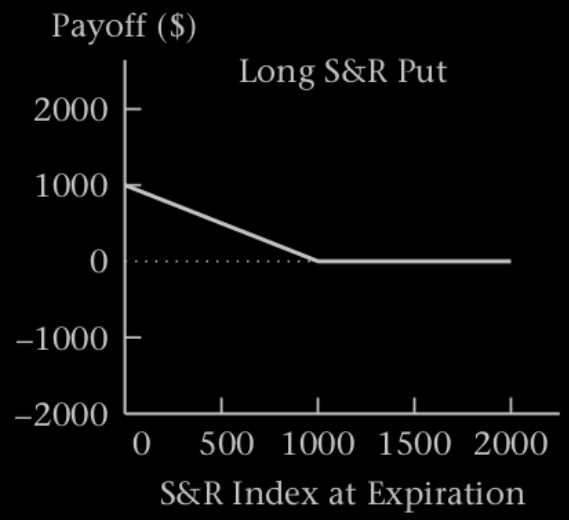
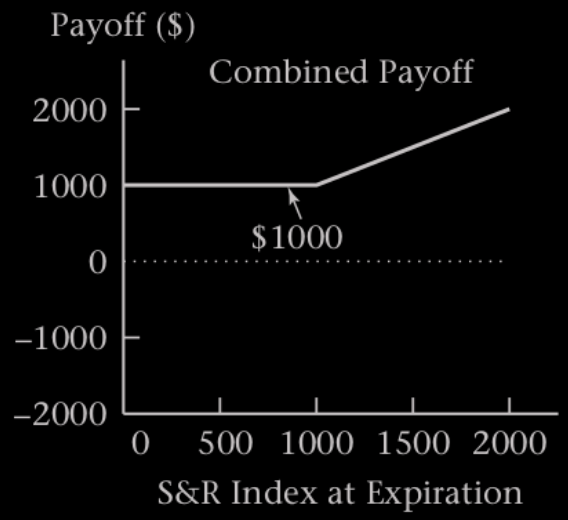

Payoff ($)
Long S&R Index
2000
1000
0
–1000
–2000
0
500
1000
1500
2000
S&R Index at Expiration

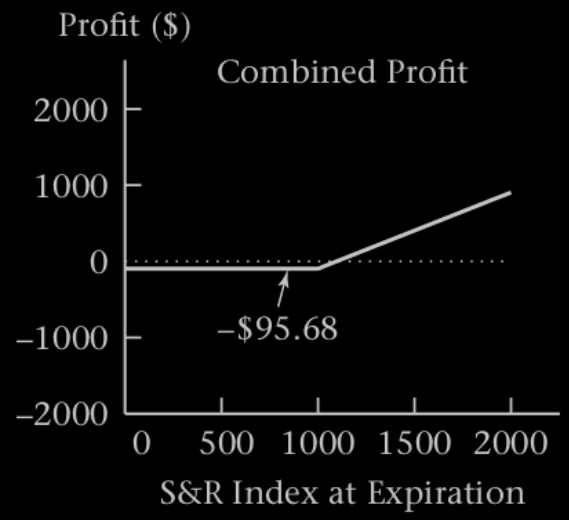
(a)


Payoff ($)
Long S&R Put
2000
1000
0
–1000
–2000
0
500
1000
1500
2000
S&R Index at Expiration

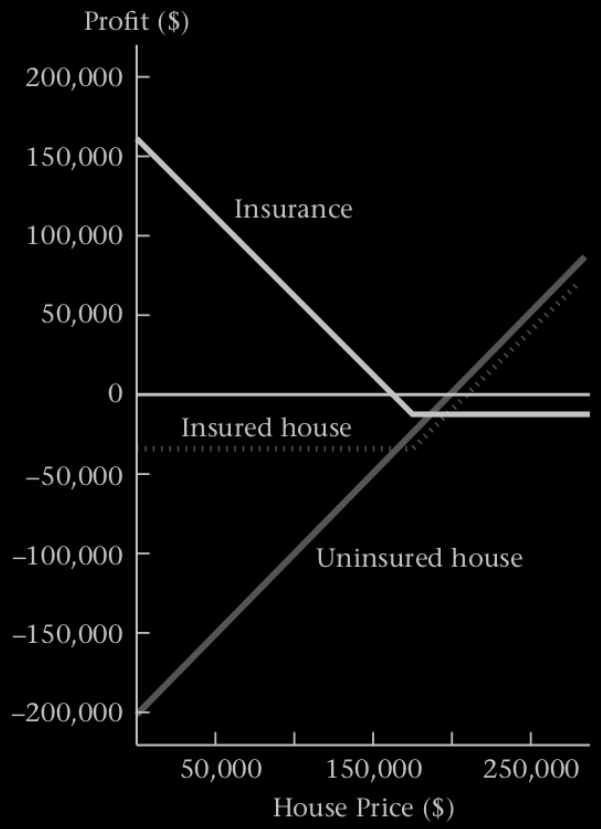
(b)


Payoff ($)
Combined Payoff
2000
1000
$1000
0
–1000
–2000
0
500
1000
1500
2000
S&R Index at Expiration

(c)


Profit ($)
Combined Profit
2000
1000
0
–$95.68
–1000
–2000
0
500
1000
1500
2000
S&R Index at Expiration

(d)


Profit ($)
200,000
150,000
100,000
50,000
0
–50,000
–100,000
–150,000
–200,000
Insurance
Insured house
Uninsured house
50,000
150,000
250,000
House Price ($)

Insuring a short position – Caps

If we have a short position in the S&R index, we experience a loss when the index rises.

We can insure a short position by purchasing a call option (cap) to protect against a higher price of repurchasing the index.

Example 3.1-2 Under the following scenario, compute the combined profit.

S&R index

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
index price at expiration	\$1,100

Solution.

$$\underbrace{\$1,000 \times 1.02}_{\text{future value of short S\&R index}} - \underbrace{\$93.809 \times 1.02}_{\text{FV of premium for call}} - \underbrace{\$1,000}_{\text{exercise the call option}} = -\$75.685.$$

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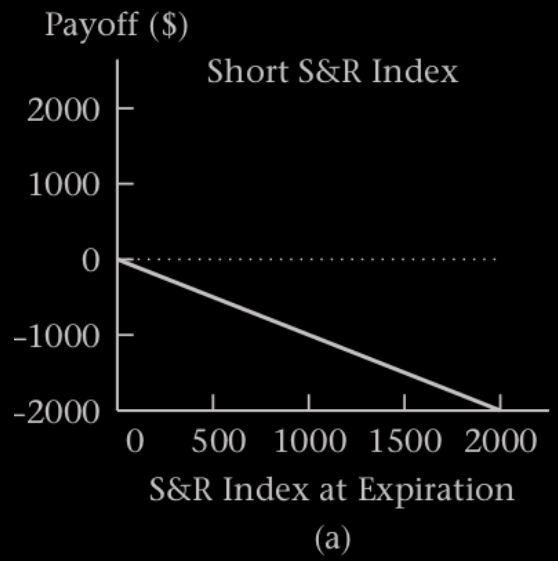
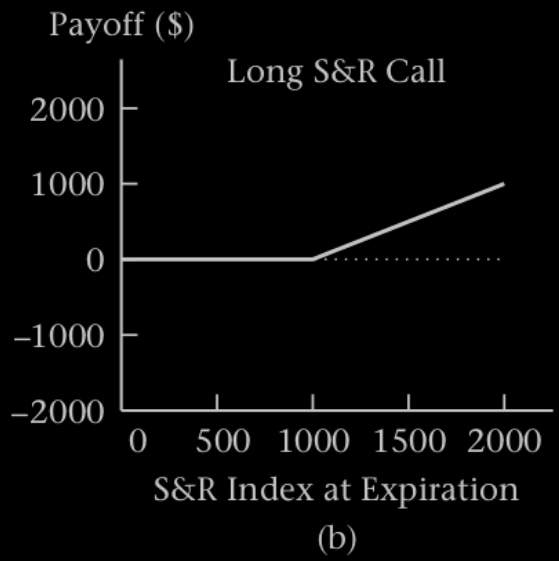
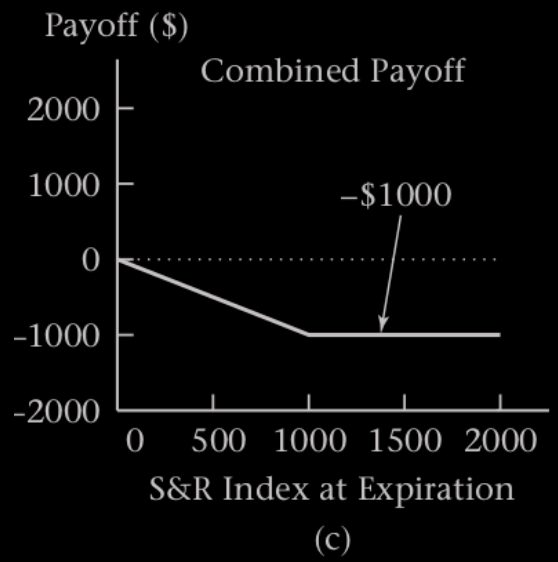
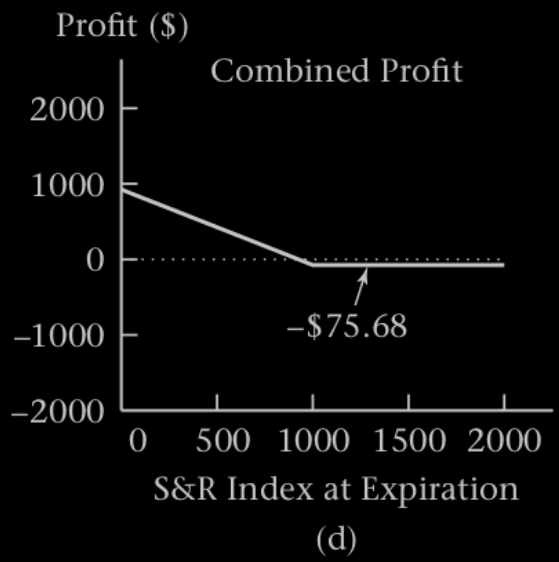

Payoff ($)
Short S&R Index
2000
1000
0
-1000
-2000
0 500 1000 1500 2000
S&R Index at Expiration
(a)


Payoff ($)
Long S&R Call
2000
1000
0
–1000
–2000
0 500 1000 1500 2000
S&R Index at Expiration
(b)


Payoff ($)
Combined Payoff
2000
1000
0
-1000
-2000
–$1000
0 500 1000 1500 2000
S&R Index at Expiration
(c)


Profit ($)
Combined Profit
2000
1000
0
–1000
–2000
–$75.68
0 500 1000 1500 2000
S&R Index at Expiration
(d)

Selling insurance

For every insurance buyer there must be an insurance seller

Strategies used to sell insurance

- Covered writing (option overwriting or selling a covered call) is writing an option when there is a corresponding long position in the underlying asset.
- Naked writing is writing an option when the writer does not have a position in the asset.

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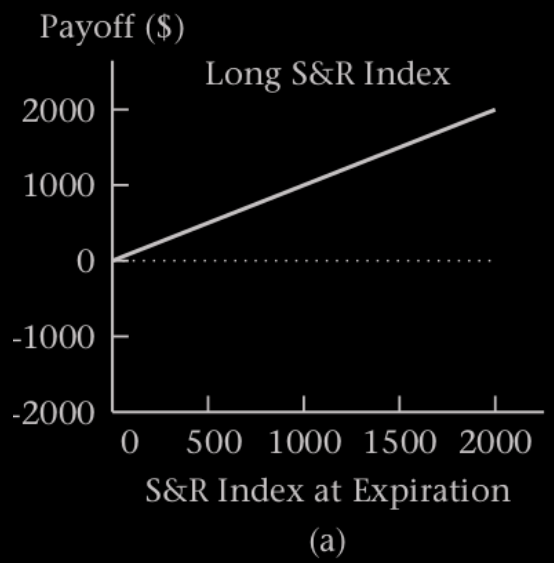
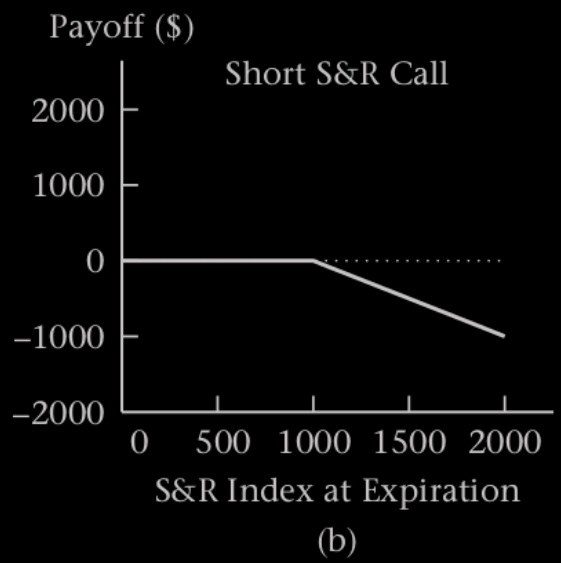
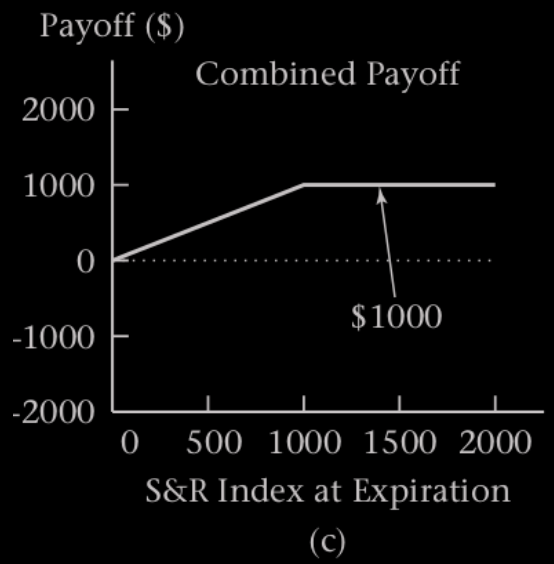
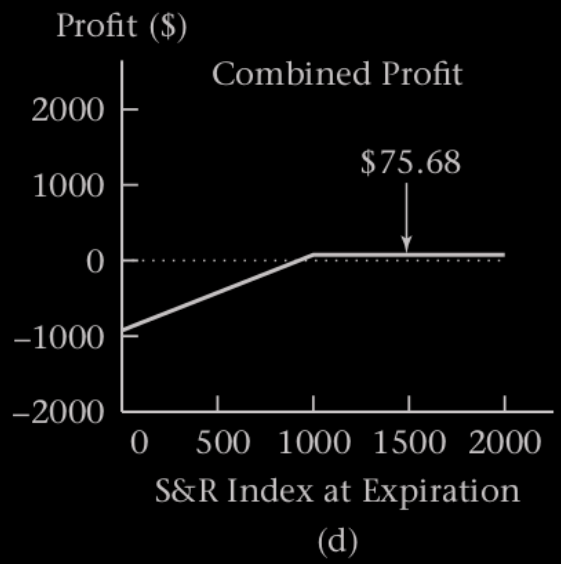
Covered call writing

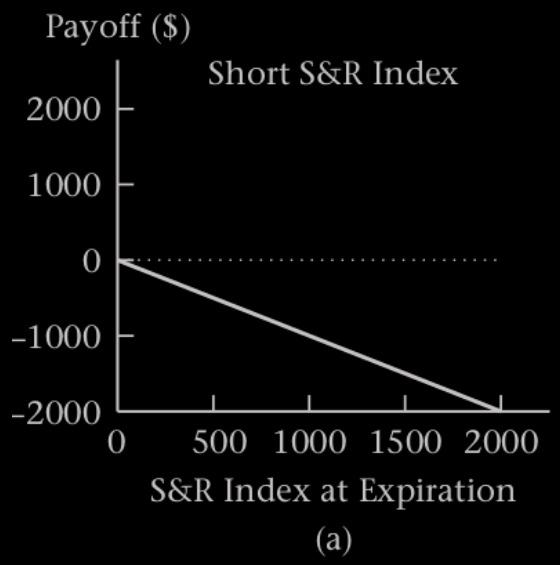


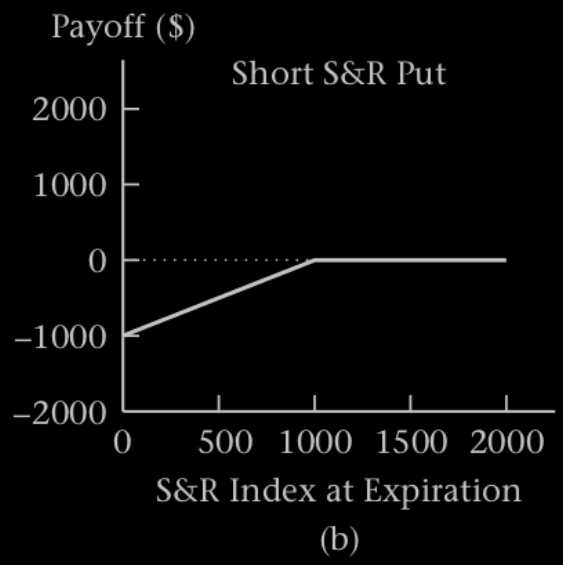


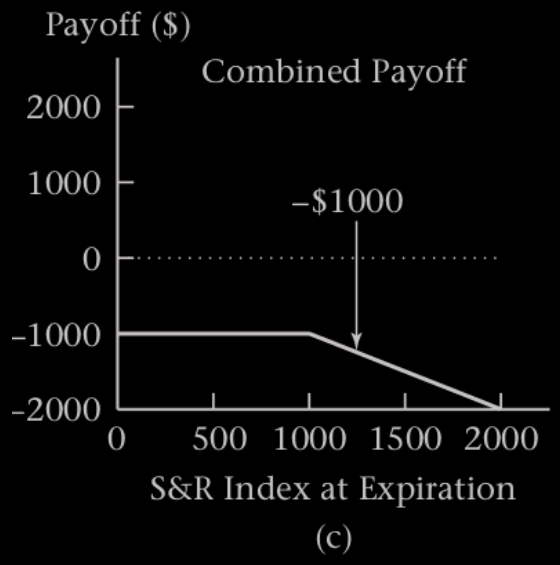


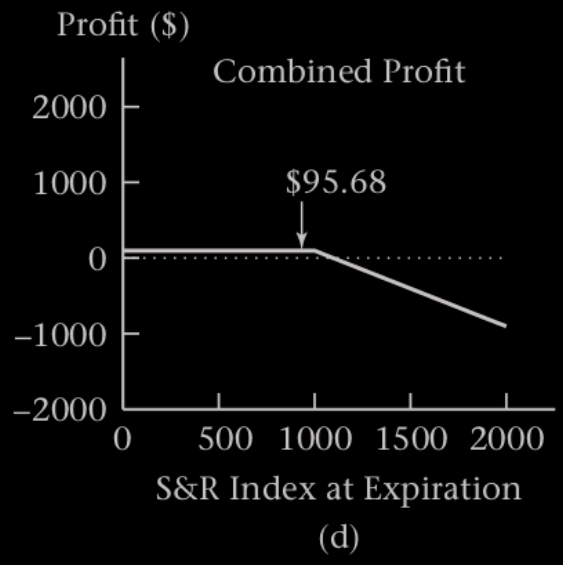


Covered put writing









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