

Financial Mathematics

MATH 5870/6870¹
Fall 2021

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¹Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 3. Insurance, Collars, and Other Strategies

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§ 3.1 Basic insurance strategies

§ 3.2 Put-call parity

§ 3.3 Spreads and collars

§ 3.4 Speculating on volatility

§ 3.5 Problems

Chapter 3. Insurance, Collars, and Other Strategies

§ 3.1 Basic insurance strategies

§ 3.2 Put-call parity

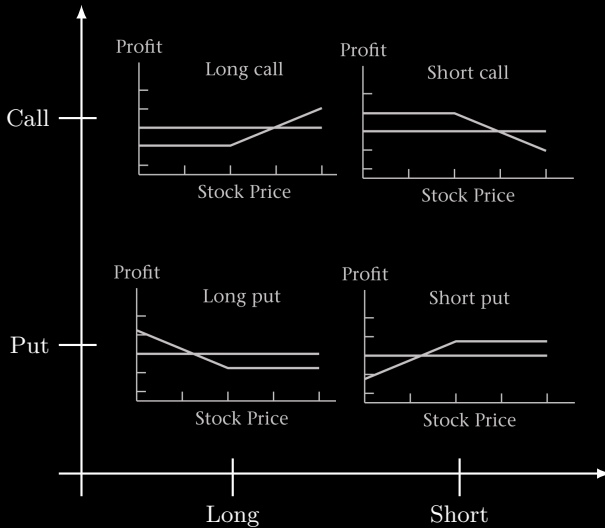
§ 3.3 Spreads and collars

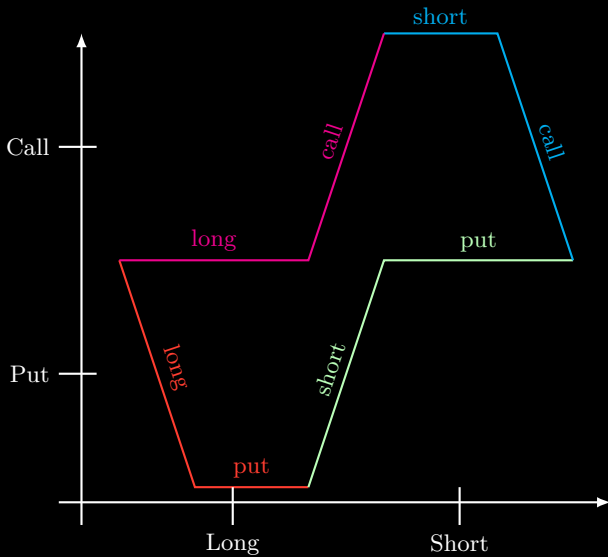
§ 3.4 Speculating on volatility

§ 3.5 Problems

It is always possible
to
lower the cost of a position
by
reducing its payoff!

By combining two or more options, we find many well-known strategies.





An **option spread** is a position consisting of only calls or only puts, in which some options are purchased and some written.

- ▶ Bull and bear spreads

- ▶ Box spreads

- ▶ Ratio spreads

- ▶ Collars

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Example for this section

Black-Scholes option prices

Stock price = \$40

Volatility = 30%

Effective annual risk-free rate = 8.33%

Dividend yield = \$0

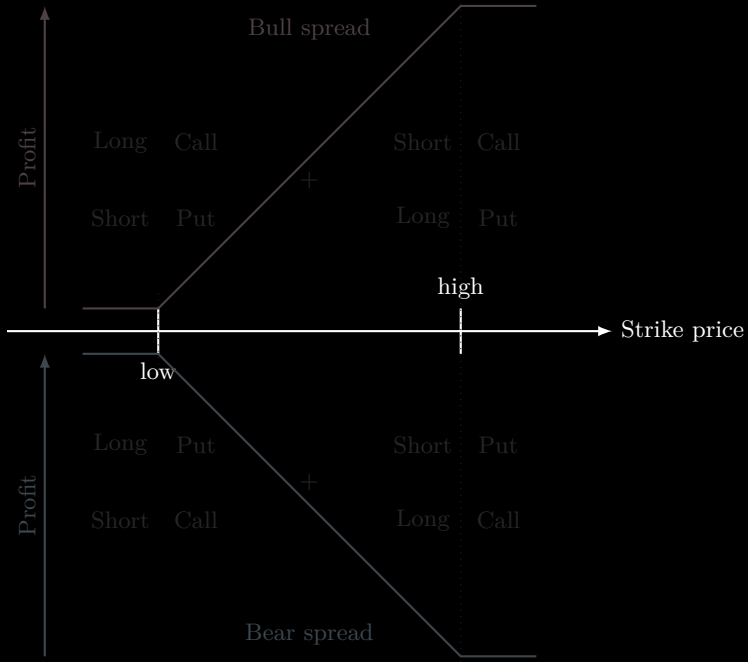
Expiration days = 91 days

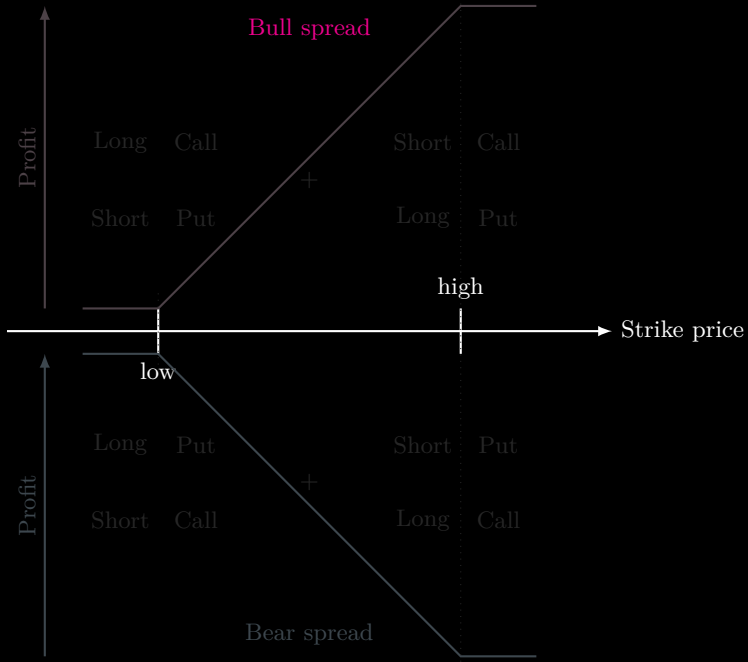
Strike	Call	Put
35	6.13	0.44
40	2.78	1.99
45	0.97	5.08

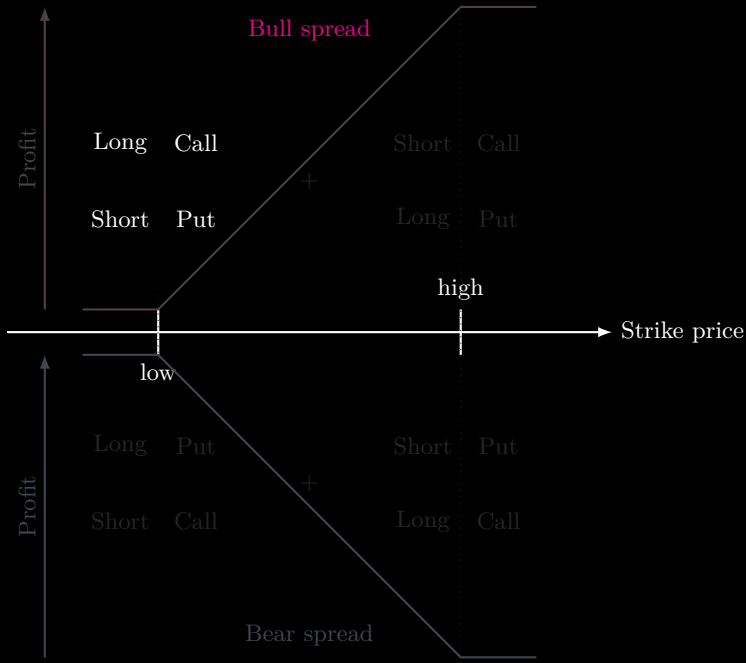
Bull and bear spreads

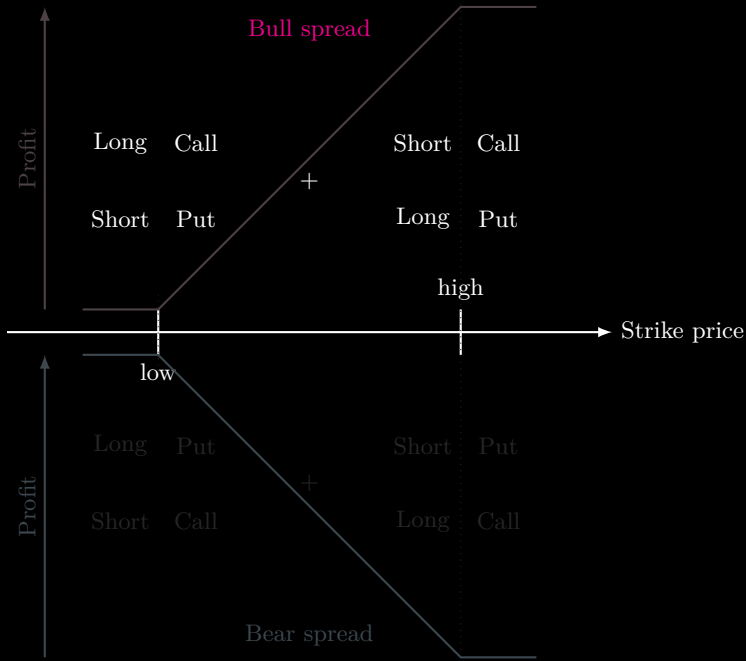
A position in which you buy a call and sell an otherwise identical call with a higher strike price is an example of a **bull spread**. Bull spreads can also be constructed using puts.

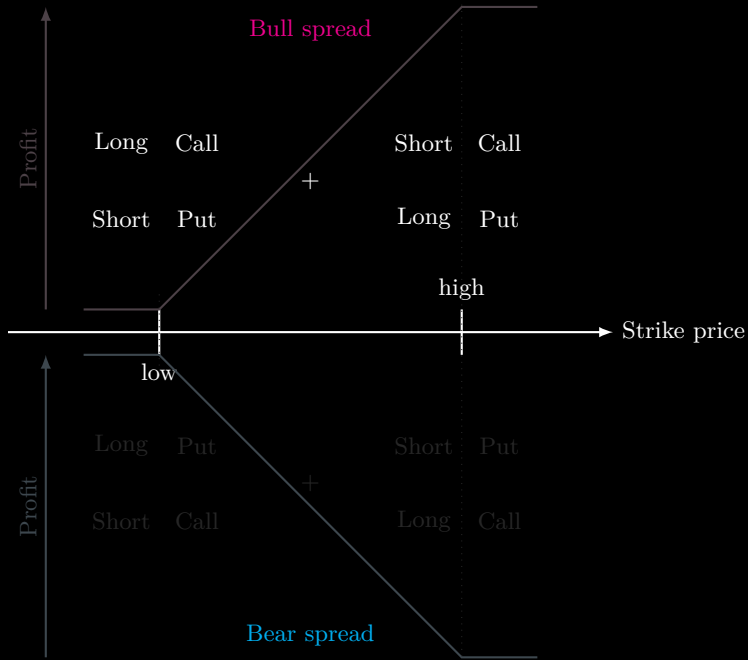
The opposite of a bull spread is a **bear spread**.

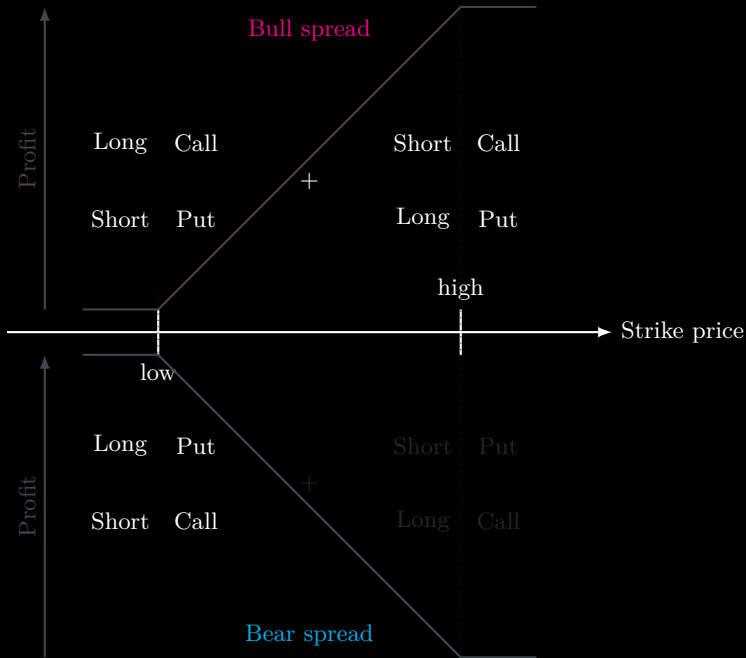


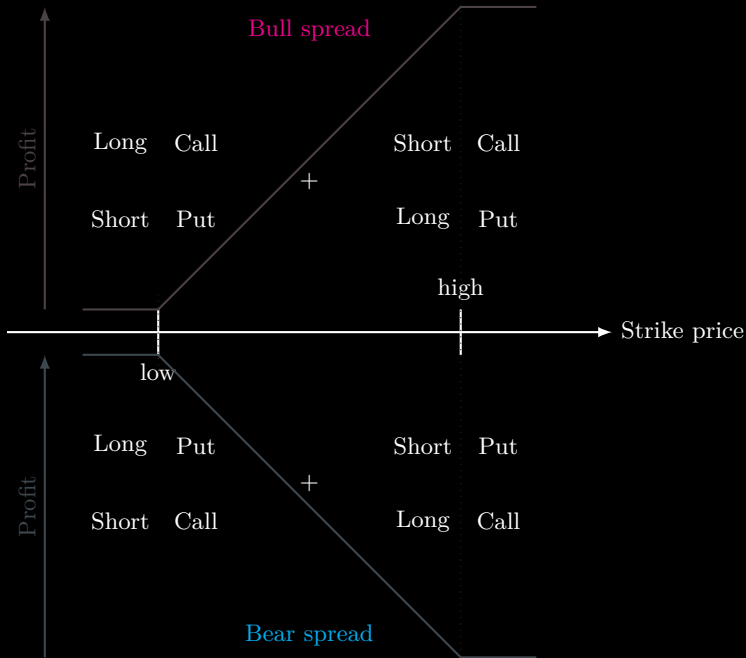


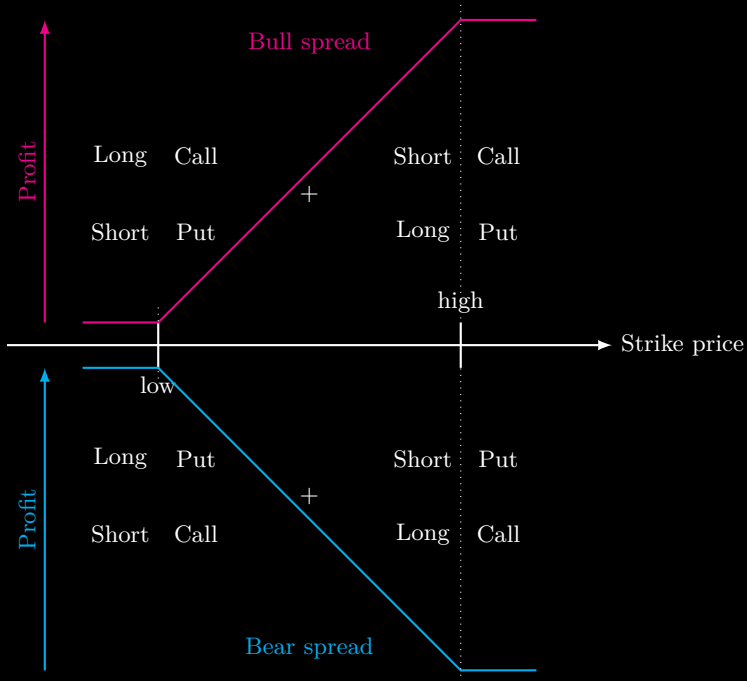












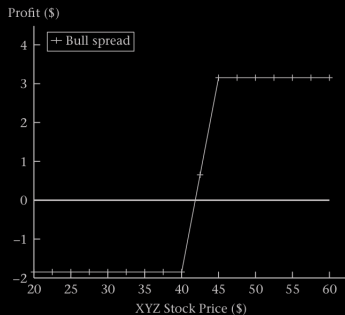
Example 3.3-1 Draw profit diagram for a 40-45 bull spread, namely, buying a 40-strike call and selling a 45-strike call.

Solution.

We only need to determine the two levels.

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Solution(Continued).

(a) Suppose that the index price is \$ 30 at the expiration:

$$(\$2.78 - \$0.97) \times (1 + 0.0833)^{1/4} = \$1.81.$$

(b) Suppose that the index price is \$50 at the expiration:

$$(\$50 - \$40) - (\$40 - \$45) - \$1.81 = \$3.15.$$



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Box spreads

A **box spread** is accomplished by using options to create a **synthetic long forward** at one price and a **synthetic short forward** at a different price.

This strategy guarantees a cash flow in the future.

Hence, it is an option spread that is purely a means of borrowing or lending money. It is costly but has no stock price risk.

Example 3.3-2 Suppose we simultaneously enter into the following two transactions:

1. Buy a 40-strike call and sell a 40-strike put.
2. Sell a 45-strike call and buy a 45-strike put.

Explain why there is no free lunch. Draw the profit diagram.

Solution. The profit is

$$5 + \underbrace{(1.99 - 2.78) \times (1.0833)^{1/4}}_{\text{Synthetic long forward}} + \underbrace{(0.97 - 5.08) \times (1.0833)^{1/4}}_{\text{Synthetic short forward}} = \$0.0099851.$$



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Buy a \$40-strike call

Sell a \$40-strike call

Synthetic long forward

Sell a \$45-strike call

Buy a \$40-strike call

Synthetic short forward

Bull spread

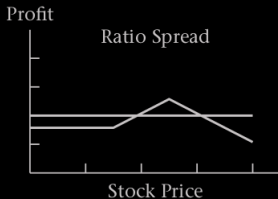


Bear spread



Ratio spreads

A **ratio spread** is constructed by buying m options at one strike and selling n options at a different strike, with all options having the same type (call or put), same time to maturity, and same underlying asset.



Example 3.3-3 (Problem 3.15) Compute profit diagrams for the following ratio spreads:

- a Buy 950-strike call, sell two 1050-strike calls.
- b Buy two 950-strike calls, sell three 1050-strike calls.
- c Consider buying n 950-strike calls and selling m 1050-strike calls so that the premium of the position is zero. Considering your analysis in (a) and (b), what can you say about n/m ? What exact ratio gives you a zero premium?

Strike	Call	Put
\$950	\$120.405	\$51.777
1000	93.809	74.201
1020	84.470	84.470
1050	71.802	101.214
1107	51.873	137.167

Solution. ...



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Collars

A **collar** is the purchase of a put option and the sale of a call option with a higher strike price, with both options having the same underlying asset and having the same expiration date.

If the position is reversed, i.e., sale of a put and purchase of a call, the collar is written.

The **collar width** is the difference between the call and put strikes.

Example 3.3-4 Draw the profit diagram for a purchased collar:
selling a 45-strike call + buying a 40-strike put.

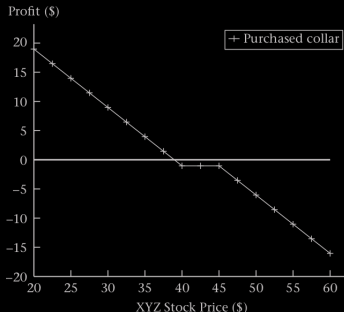
Solution. One can easily draw the profit graph. We only need to determine the level when the curve is flat. Hence, suppose the price is \$43. Then the profit is

$$(0.97 - 1.99) \times (1.083)^{1/4} = -\$1.0405.$$

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It is possible to find strike prices for the put and call such that the two premiums exactly offset one another. This position is called a **zero-cost collar**.

Example 3.3-5 Consider XYZ:

Strike	Call	Put
35	6.13	0.44
40	2.78	1.99
41.72	1.99	—
45	0.97	5.08

where we need to use **Black-Scholes formula** to find out the strike price, which is **41.72**, when the put premium is **\$1.99**. This gives a zero-cost collar.