By Le Chen.
Crated on Wed 06 Oct 2021 10:04:27 AM CDT

In[ ]:=
```
S = 100;
K = 95;
r = 0.08;
σ = 0.30;
δ = 0.05;
h = 1/8;
u = e^((r-δ-0.5 σ^2) h + σ √h);
d = e^((r-δ-0.5 σ^2) h - σ √h);
```

Build up the tree first

First row

In[ ]:=
```
Table[S u^n, {n, 0, 8}]
```

Out[ ]=
```
{100., 110.981, 123.168, 136.694, 151.704, 168.363, 186.852, 207.371, 230.142}
```

Second row

In[ ]:=
```
Table[S d u^n, {n, 0, 7}]
```

Out[ ]=
```
{89.7681, 99.6257, 110.566, 122.707, 136.182, 151.137, 167.733, 186.153}
```

Third row

In[ ]:=
```
Table[S d^2 u^n, {n, 0, 6}]
```

Out[ ]=
```
{80.583, 89.4321, 99.2528, 110.152, 122.248, 135.672, 150.571}
```

4 th row

In[ ]:=
```
Table[S d^3 u^n, {n, 0, 5}]
```

Out[ ]=
```
{72.3378, 80.2814, 89.0973, 98.8813, 109.74, 121.79}
```

5 th row

In[ ]:=
```
Table[S d^4 u^n, {n, 0, 4}]
```

Out[ ]=
```
{64.9363, 72.0671, 79.9809, 88.7638, 98.5112}
```

6 th row

In[ ]:=
```
Table[S d^5 u^n, {n, 0, 3}]
```

Out[ ]=
```
{58.292, 64.6932, 71.7973, 79.6816}
```

7 th row

```
In[ ]:= Table[S d^6 u^n, {n, 0, 2}]
Out[ ]= {52.3276 , 58.0738 , 64.4511}
```

8 th row

```
In[ ]:= Table[S d^7 u^n, {n, 0, 1}]
Out[ ]= {46.9735 , 52.1317}
```

9 th row

```
In[ ]:= Table[S d^8 u^n, {n, 0, 0}]
Out[ ]= {42.1672}
```

European Call

Bottom leaves

```
In[ ]:= Max[S u^8 - K, 0]
Out[ ]= 135.142
```

```
In[ ]:= Table[Max[S u^(8-n) d^n - K, 0], {n, 0, 8}]
Out[ ]= {135.142 , 91.1525 , 55.5709 , 26.7905 , 3.51119 , 0, 0, 0, 0}
```

Level - 1

```
In[ ]:= n = 7;
        m = 7 - n;
        Cu = Max[S u^n d^m u - K, 0];
        Cd = Max[S u^n d^m d - K, 0];
        myS = S u^n d^m;
        Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
        B = e^(-r h) (u Cd - d Cu)/(u - d);
        OptionPrice = Δ myS + B
Out[ ]= 112.024
```

In[ ]:=
```
n = 6;
m = 7 - n;
Cu = Max[S u^(n+1) d^m - K, 0];
Cd = Max[S u^n d^(m+1) - K, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Δ myS + B
```

Out[ ]= 72.6335

In[ ]:=
```
n = 5;
m = 7 - n;
Cu = Max[S u^(n+1) d^m - K, 0];
Cd = Max[S u^n d^(m+1) - K, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Δ myS + B
```

Out[ ]= 40.7724

In[ ]:=
```
n = 4;
m = 7 - n;
Cu = Max[S u^(n+1) d^m - K, 0];
Cd = Max[S u^n d^(m+1) - K, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Δ myS + B
```

Out[ ]= 15.0012

In[ ]:=
```
n = 3;
m = 7 - n;
Cu = Max[S u^(n+1) d^m - K, 0];
Cd = Max[S u^n d^(m+1) - K, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Δ myS + B
```

Out[ ]= 1.7383

In[ ]:=
```
n = 2;
m = 7 - n;
Cu = Max[S u^(n+1) d^m - K, 0];
Cd = Max[S u^n d^(m+1) - K, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Δ myS + B
```

Out[ ]= 0.

In[ ]:=
```
n = 1;
m = 7 - n;
Cu = Max[S u^(n+1) d^m - K, 0];
Cd = Max[S u^n d^(m+1) - K, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Δ myS + B
```

Out[ ]= 0.

```
In[ ]:= n = 0;
        m = 7 - n;
        Cu = Max[S u^(n+1) d^m - K, 0];
        Cd = Max[S u^n d^(m+1) - K, 0];
        myS = S u^n d^m;
        Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
        B = e^(-r h) (u Cd - d Cu)/(u - d);
        OptionPrice = Δ myS + B
Out[ ]= 0.
```

European Put

Bottom leaves

```
In[ ]:= Table[Max[K - S d^n u^(8-n), 0], {n, 0, 8}]
Out[ ]= {0, 0, 0, 0, 0, 15.3184 , 30.5489 , 42.8683 , 52.8328}
```

Level -1

```
In[ ]:= n = 7;
        m = 7 - n;
        Cu = Max[K - S u^n d^m u, 0];
        Cd = Max[K - S u^n d^m d, 0];
        myS = S u^n d^m;
        Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
        B = e^(-r h) (u Cd - d Cu)/(u - d);
        OptionPrice = Δ myS + B
Out[ ]= 0.
```

In[ ]:=
```
n = 6;
m = 7 - n;
Cu = Max[K - S u^n d^m u, 0];
Cd = Max[K - S u^n d^m d, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Δ myS + B
```

Out[ ]= 0.

In[ ]:=
```
n = 5;
m = 7 - n;
Cu = Max[K - S u^n d^m u, 0];
Cd = Max[K - S u^n d^m d, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Δ myS + B
```

Out[ ]= 0.

In[ ]:=
```
n = 4;
m = 7 - n;
Cu = Max[K - S u^n d^m u, 0];
Cd = Max[K - S u^n d^m d, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Δ myS + B
```

Out[ ]= 0.

In[ ]:=
```
n = 3;
m = 7 - n;
Cu = Max[K - S u^n d^m u, 0];
Cd = Max[K - S u^n d^m d, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Δ myS + B
```

Out[ ]= 7.58226

In[ ]:=
```
n = 2;
m = 7 - n;
Cu = Max[K - S u^n d^m u, 0];
Cd = Max[K - S u^n d^m d, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Δ myS + B
```

Out[ ]= 22.7047

In[ ]:=
```
n = 1;
m = 7 - n;
Cu = Max[K - S u^n d^m u, 0];
Cd = Max[K - S u^n d^m d, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Δ myS + B
```

Out[ ]= 36.3427

```
In[ ]:= n = 0;
        m = 7 - n;
        Cu = Max[K - S u^n d^m u, 0];
        Cd = Max[K - S u^n d^m d, 0];
        myS = S u^n d^m;
        Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
        B = e^(-r h) (u Cd - d Cu)/(u - d);
        OptionPrice = Δ myS + B
Out[ ]= 47.3739
```

American Put

Bottom leaves

```
In[ ]:= Table[Max[K - S d^n u^(8-n), 0], {n, 0, 8}]
Out[ ]= {0, 0, 0, 0, 0, 15.3184 , 30.5489 , 42.8683 , 52.8328}
```

Level -1

```
In[ ]:= n = 7;
        m = 7 - n;
        Cu = Max[K - S u^n d^m u, 0];
        Cd = Max[K - S u^n d^m d, 0];
        myS = S u^n d^m;
        Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
        B = e^(-r h) (u Cd - d Cu)/(u - d);
        OptionPrice = Max[Δ myS + B, K - myS]
Out[ ]= 0.
```

In[ ]:=
```
n = 6;
m = 7 - n;
Cu = Max[K - S u^n d^m u, 0];
Cd = Max[K - S u^n d^m d, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Max[Δ myS + B, K - myS]
```

Out[ ]= 0.

In[ ]:=
```
n = 5;
m = 7 - n;
Cu = Max[K - S u^n d^m u, 0];
Cd = Max[K - S u^n d^m d, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Max[Δ myS + B, K - myS]
```

Out[ ]= 0.

In[ ]:=
```
n = 4;
m = 7 - n;
Cu = Max[K - S u^n d^m u, 0];
Cd = Max[K - S u^n d^m d, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Max[Δ myS + B, K - myS]
```

Out[ ]= 0.

In[ ]:=
```
n = 3;
m = 7 - n;
Cu = Max[K - S u^n d^m u, 0];
Cd = Max[K - S u^n d^m d, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Max[Δ myS + B, K - myS]
```

Out[ ]= 7.58226

In[ ]:=
```
n = 2;
m = 7 - n;
Cu = Max[K - S u^n d^m u, 0];
Cd = Max[K - S u^n d^m d, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Max[Δ myS + B, K - myS]
```

Out[ ]= 23.2027

In[ ]:=
```
n = 1;
m = 7 - n;
Cu = Max[K - S u^n d^m u, 0];
Cd = Max[K - S u^n d^m d, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Max[Δ myS + B, K - myS]
```

Out[ ]= 36.9262

In[ ]:=
```
n = 0;
m = 7 - n;
Cu = Max[K - S u^n d^m u, 0];
Cd = Max[K - S u^n d^m d, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Max[Δ myS + B, K - myS]
```

Out[ ]= 48.0265

American Call

Level - 1

In[ ]:=
```
n = 7;
m = 7 - n;
Cu = Max[S u^n d^m u - K, 0];
Cd = Max[S u^n d^m d - K, 0];
myS = S u^n d^m;
Δ = e^(-δ h) (Cu - Cd)/(myS (u - d));
B = e^(-r h) (u Cd - d Cu)/(u - d);
OptionPrice = Max[Δ myS + B, myS - K]
```

Out[ ]= 112.371