Financial Mathematics

MATH 5870/6870¹ Fall 2021

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Last updated on August 6, 2021

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¹Based on Robert L. McDonald's *Derivatives Markets*. 3rd Ed. Pearson. 2013.

- § 3.1 Basic insurance strategies
- § 3.2 Put-call parity
- \S 3.3 Spreads and collars
- § 3.4 Speculating on volatility
- § 3.5 Problems

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- 1. Used to insure long positions (floors)
- 2. Used to insure short positions (caps)
- 3. Written against asset positions (selling insurance)

Covered call writing

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Four positions

positions w.r.t. asset	put option	call option
long	purchased (floor)	written
short	written	purchased (cap)

Buying insurance	Selling insurance
floor = buying a put option	Covered put writing
cap = buying a call option	Covered call writing

We will work under the following setup

${\rm S\&S}$ index

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
premium for 1000-strike 6-month put	\$74.201

Insuring a long position – Floors

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owning a home owning a stock index insuring the house buying a put (floor)
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Goal: to insure against a fall in the price of the underlying asset.

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Example 3.1-1 Under the following scenario, compute the combined profit of insuring a long position via buying a put for the following S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	\$74.201
index price at expiration	\$900

Solution

$$\underbrace{\$900 - \$1,000 \times 1.02}_{\$900} + \underbrace{\$1,000 - \$900 - \$74.201 \times 1.02}_{\$900} = -\$95.68$$

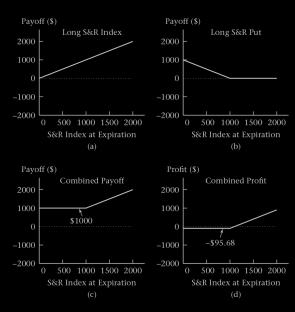
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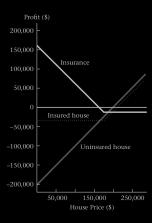
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index price at expiration	\$900

Solution.

$$\underbrace{\$900 - \$1,000 \times 1.02}_{\text{profit on S\&R index}} + \underbrace{\$1,000 - \$900 - \$74.201 \times 1.02}_{\text{profit on put}} = -\$95.68.$$





Insuring a short position — Caps

If we have a short position in the S&R index, we experience a loss when the index rises.

We can insure a short position by purchasing a call option (cap) to protect against a higher price of repurchasing the index.

Example 3.1-2 Under the following scenario, compute the combined profit for insuring a short position via buying a call of the following S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
index price at expiration	\$1,100

Solution

$$$1,000 \times 1.02$$$
 - $$93.809 \times 1.02$$ - $$1,000$ = -\$75.685

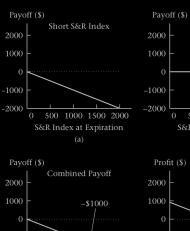
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Solution.

$$$1,000 \times 1.02$$$
 - $$93.809 \times 1.02$$ - $$1,000$ = -\$75.685. future value of short S&R index FV of premium for call exercise the call option

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500 1000 1500 2000

S&R Index at Expiration

-1000

-2000



Long S&R Call

500 1000 1500 2000

S&R Index at Expiration

For every insurance buyer there must be an insurance seller

Strategies used to sell insurance

- Covered writing (option overwriting or selling a covered call) is writing an option when there is a corresponding long position in the underlying asset
- Naked writing is writing an option when the writer does not have a

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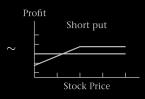
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Strategies used to sell insurance

- ► Covered writing (option overwriting or selling a covered call) is writing an option when there is a corresponding long position in the underlying asset.
- ▶ Naked writing is writing an option when the writer does not have a position in the asset.

Covered call writing

Long position of the asset + Sell a call option



Covered put writing

Short position of the asset + Sell a put option



Covered call writing

Example 3.1-3 Under the following scenario, compute the combined profit for writing a covered call for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
index price at expiration	\$1,100

Solution

$$\underbrace{\$1,100-\$1,000\times1.02}_{\text{profit on }\$\text{-}P\text{-}index} + \underbrace{\$1,000-\$1,100+\$93.809\times1.02}_{\text{profit on written call}} = \$75.68$$

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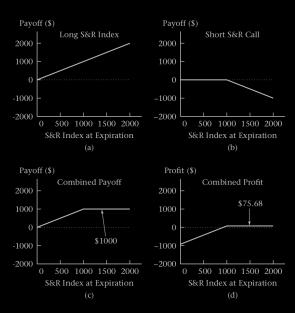
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Solution.

$$\underbrace{\$1,100-\$1,000\times1.02}_{\text{profit on S\&R index}} + \underbrace{\$1,000-\$1,100+\$93.809\times1.02}_{\text{profit on written call}} = \$75.68.$$



Covered put writing

Example 3.1-4 Under the following scenario, compute the combined profit for writing a covered put for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	\$74.201
index price at expiration	\$900

Solution

$$\$1,000 \times 1.02 - \$900 + \$900 - \$1,000 + \$74.201 \times 1.02 = \$95.685$$

profit on selling S&R inde

profit on written pu

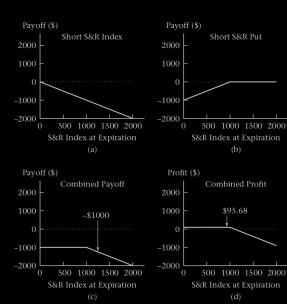
Covered put writing

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index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	\$74.201
index price at expiration	\$900

Solution.

$$\underbrace{\$1,000\times 1.02 -\$900}_{\text{profit on selling S&R index}} + \underbrace{\$900 -\$1,000 +\$74.201\times 1.02}_{\text{profit on written put}} = \$95.685.$$



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