Financial Mathematics

MATH 5870/6870[1]
Fall 2021

Le Chen

lzc0090@auburn.edu

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Auburn University
Auburn AL

[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 3. Insurance, Collars, and Other Strategies

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Options can be

1. Used to insure long positions (floors)
2. Used to insure short positions (caps)
3. Written against asset positions (selling insurance)

   Covered call writing

   Covered put writing

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Four positions

positions w.r.t. asset	put option	call option
long	purchased (floor)	written
short	written	purchased (cap)

Buying insurance	Selling insurance
floor = buying a put option	Covered put writing
cap = buying a call option	Covered call writing

We will work under the following setup

S&S index

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
premium for 1000-strike 6-month put	\$74.201

Insuring a long position – Floors

owning a home	owning a stock index
insuring the house	buying a put (floor)

Goal: to insure against a fall in the price of the underlying asset.

Example 3.1-1 Under the following scenario, compute the combined profit of insuring a long position via buying a put for the following S&R index.

index price today	$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	$74.201
index price at expiration	$900

Solution.

$$\underbrace{\$900 - \$1,000 \times 1.02}_{\text{profit on S\&R index}} + \underbrace{\$1,000 - \$900 - \$74.201 \times 1.02}_{\text{profit on put}} = -\$95.68.$$

$\square$

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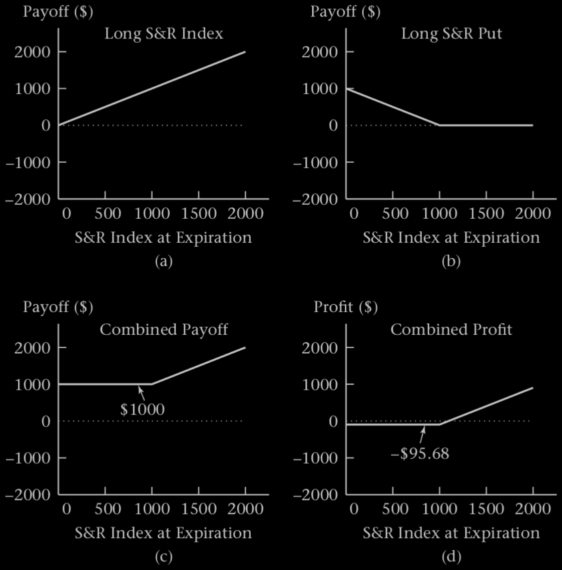
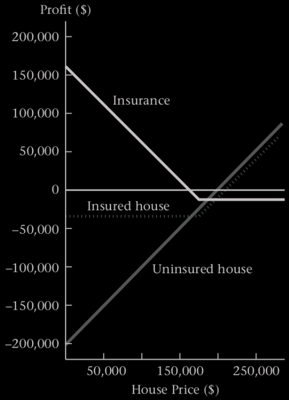

Payoff ($)
Long S&R Index
2000
1000
0
–1000
–2000
0
500
1000
1500
2000
S&R Index at Expiration
(a)
Payoff ($)
Long S&R Put
2000
1000
0
–1000
–2000
0
500
1000
1500
2000
S&R Index at Expiration
(b)
Payoff ($)
Combined Payoff
2000
1000
$1000
0
–1000
–2000
0
500
1000
1500
2000
S&R Index at Expiration
(c)
Profit ($)
Combined Profit
2000
1000
0
–$95.68
–1000
–2000
0
500
1000
1500
2000
S&R Index at Expiration
(d)


Profit ($)
200,000
150,000
100,000
50,000
0
–50,000
–100,000
–150,000
–200,000
Insurance
Insured house
Uninsured house
50,000
150,000
250,000
House Price ($)

Insuring a short position – Caps

If we have a short position in the S&R index, we experience a loss when the index rises.

We can insure a short position by purchasing a call option (cap) to protect against a higher price of repurchasing the index.

Example 3.1-2 Under the following scenario, compute the combined profit for insuring a short position via buying a call of the following S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
index price at expiration	\$1,100

Solution.

$$\underbrace{\$1,000 \times 1.02}_{\text{future value of short S\&R index}} - \underbrace{\$93.809 \times 1.02}_{\text{FV of premium for call}} - \underbrace{\$1,000}_{\text{exercise the call option}} = -\$75.685. \qquad \square$$

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index price at expiration	\$1,100

Solution.

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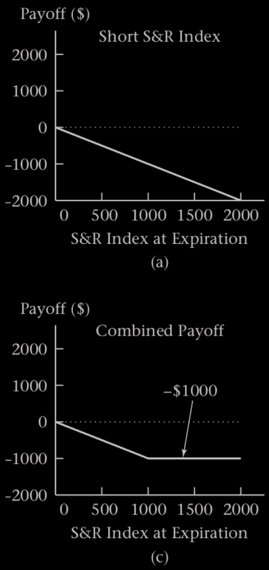
□


Payoff ($)
Short S&R Index
2000
1000
0
–1000
–2000
0 500 1000 1500 2000
S&R Index at Expiration
(a)
Payoff ($)
Long S&R Call
2000
1000
0
–1000
–2000
0 500 1000 1500 2000
S&R Index at Expiration
(b)
Payoff ($)
Combined Payoff
–$1000
2000
1000
0
–1000
–2000
0 500 1000 1500 2000
S&R Index at Expiration
(c)
Profit ($)
Combined Profit
–$75.68
2000
1000
0
–1000
–2000
0 500 1000 1500 2000
S&R Index at Expiration
(d)

Selling insurance

For every insurance buyer there must be an insurance seller

Strategies used to sell insurance

- **Covered writing** (**option overwriting** or **selling a covered call**) is writing an option when there is a corresponding long position in the underlying asset.
- **Naked writing** is writing an option when the writer does not have a position in the asset.

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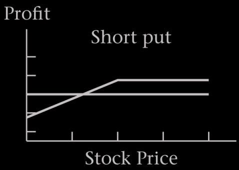
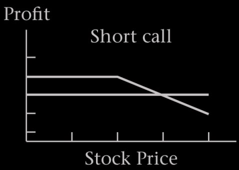
Covered call writing

Long position of the asset + Sell a call option $\sim$



Covered put writing

Short position of the asset + Sell a put option $\sim$



Covered call writing

Example 3.1-3 Under the following scenario, compute the combined profit for writing a covered call for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
index price at expiration	\$1,100

Solution.

$$\underbrace{\$1,100 - \$1,000 \times 1.02}_{\text{profit on S\&R index}} + \underbrace{\$1,000 - \$1,100 + \$93.809 \times 1.02}_{\text{profit on written call}} = \$75.68.$$

Covered call writing

Example 3.1-3 Under the following scenario, compute the combined profit for writing a covered call for S&R index.

index price today	\$1,000
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index price at expiration	\$1,100

Solution.

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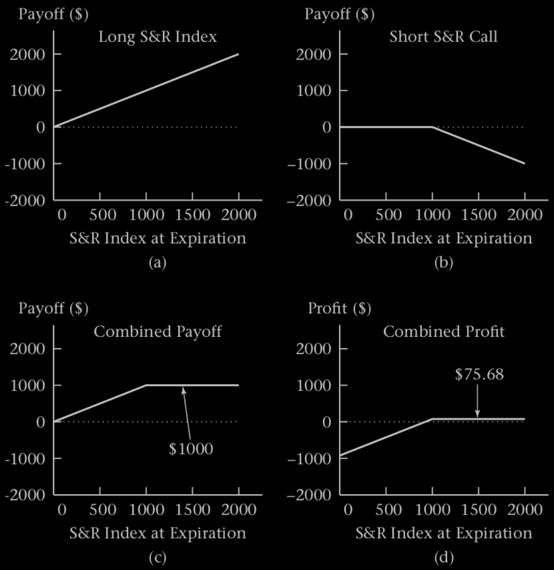
$\square$


Payoff ($)
Long S&R Index
2000
1000
0
-1000
-2000
0
500
1000
1500
2000
S&R Index at Expiration
(a)
Payoff ($)
Short S&R Call
2000
1000
0
−1000
−2000
0
500
1000
1500
2000
S&R Index at Expiration
(b)
Payoff ($)
Combined Payoff
2000
1000
0
-1000
-2000
$1000
0
500
1000
1500
2000
S&R Index at Expiration
(c)
Profit ($)
Combined Profit
2000
1000
0
−1000
−2000
$75.68
0
500
1000
1500
2000
S&R Index at Expiration
(d)

Covered put writing

Example 3.1-4 Under the following scenario, compute the combined profit for writing a covered put for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	\$74.201
index price at expiration	\$900

Solution.

$$\underbrace{\$1,000 \times 1.02 - \$900}_{\text{profit on selling S\&R index}} + \underbrace{\$900 - \$1,000 + \$74.201 \times 1.02}_{\text{profit on written put}} = \$95.685.$$

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Covered put writing

Example 3.1-4 Under the following scenario, compute the combined profit for writing a covered put for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	\$74.201
index price at expiration	\$900

Solution.

$$\underbrace{\$1,000 \times 1.02 - \$900}_{\text{profit on selling S\&R index}} + \underbrace{\$900 - \$1,000 + \$74.201 \times 1.02}_{\text{profit on written put}} = \$95.685.$$

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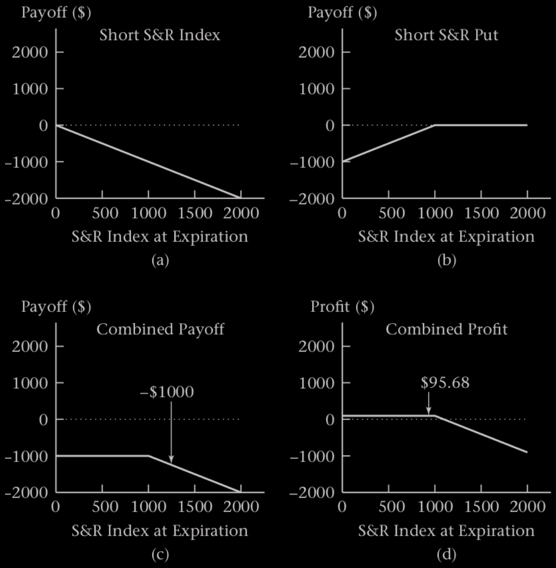
Payoff ($)
Short S&R Index
2000
1000
0
−1000
−2000
0
500
1000
1500
2000
S&R Index at Expiration
(a)
Payoff ($)
Short S&R Put
2000
1000
0
−1000
−2000
0
500
1000
1500
2000
S&R Index at Expiration
(b)
Payoff ($)
Combined Payoff
−$1000
2000
1000
0
−1000
−2000
0
500
1000
1500
2000
S&R Index at Expiration
(c)
Profit ($)
Combined Profit
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S&R Index at Expiration
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