Binomail trees
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```
u[r_, δ_, h_, σ_] := Exp[(r - δ) h + σ √h]
d[r_, δ_, h_, σ_] := Exp[(r - δ) h - σ √h]
Δ[r_, δ_, h_, σ_, S_, Cu_, Cd_] := Exp[-δ h] (Cu - Cd) / (S (u[r, δ, h, σ] - d[r, δ, h, σ]))
B[r_, δ_, h_, σ_, Cu_, Cd_] := Exp[-r h] (u[r, δ, h, σ] Cd - d[r, δ, h, σ] Cu) / (u[r, δ, h, σ] - d[r, δ, h, σ])
OptionPrice[Δ_, B_, S_] := Δ S + B
```

Figure 10.5

Input the data first

```
r = 0.08;
δ = 0;
h = 1/3;
σ = 0.3;
S = 41;
K = 40;
```

Contruct the binomial tree

```
S u[r, δ, h, σ]
S u[r, δ, h, σ]^2
S u[r, δ, h, σ]^3
```

```
50.0711
```

```
61.1491
```

```
74.6781
```

```
S d[r, δ, h, σ]
S d[r, δ, h, σ]^2
S d[r, δ, h, σ]^3
```

```
35.4114
```

```
30.5846
```

```
26.4157
```

```
In[ ]:= S u[r, δ, h, σ] d[r, δ, h, σ]
        S u[r, δ, h, σ]^2 d[r, δ, h, σ]
        S u[r, δ, h, σ] d[r, δ, h, σ]^2
```

```
Out[ ]= 43.246
```

```
Out[ ]= 52.814
```

```
Out[ ]= 37.3513
```

Backwards computation

Node 11

```
In[ ]:= Cu = Max[S u[r, δ, h, σ]^3 - K, 0]
        Cd = Max[S u[r, δ, h, σ]^2 d[r, δ, h, σ] - K, 0]
        myS = S u[r, δ, h, σ]^2
        myDelta = Δ[r, δ, h, σ, myS, Cu, Cd]
        myB = B[r, δ, h, σ, Cu, Cd]
        OptionPrice[myDelta, myB, myS]
```

```
Out[ ]= 34.6781
```

```
Out[ ]= 12.814
```

```
Out[ ]= 61.1491
```

```
Out[ ]= 1.
```

```
Out[ ]= -38.9474
```

```
Out[ ]= 22.2017
```

Node 10

```
In[ ]:= Cu = Max[S u[r, δ, h, σ]^2 d[r, δ, h, σ] - K, 0]
        Cd = Max[S u[r, δ, h, σ] d[r, δ, h, σ]^2 - K, 0]
        myS = S u[r, δ, h, σ] d[r, δ, h, σ]
        myDelta = Δ[r, δ, h, σ, myS, Cu, Cd]
        myB = B[r, δ, h, σ, Cu, Cd]
        OptionPrice[myDelta, myB, myS]
```

```
Out[ ]= 12.814
```

```
Out[ ]= 0
```

```
Out[ ]= 43.246
```

```
Out[ ]= 0.828703
```

```
Out[ ]= -30.1386
```

```
Out[ ]= 5.69951
```

```
5.700
```

Node 00

```
In[ ]:= Cu = Max[S u[r, δ, h, σ] d[r, δ, h, σ]^2 - K, 0]
        Cd = Max[S d[r, δ, h, σ]^3 - K, 0]
        myS = S d[r, δ, h, σ] d[r, δ, h, σ]
        myDelta = Δ[r, δ, h, σ, myS, Cu, Cd]
        myB = B[r, δ, h, σ, Cu, Cd]
        OptionPrice[myDelta, myB, myS]
Out[ ]= 0
Out[ ]= 0
Out[ ]= 30.5846
Out[ ]= 0.
Out[ ]= 0.
Out[ ]= 0.
```

Node 1

```
In[ ]:= Cu = 22.202
        Cd = 5.700
        myS = S u[r, δ, h, σ]
        myDelta = Δ[r, δ, h, σ, myS, Cu, Cd]
        myB = B[r, δ, h, σ, Cu, Cd]
        OptionPrice[myDelta, myB, myS]
Out[ ]= 22.202
Out[ ]= 5.7
Out[ ]= 50.0711
Out[ ]= 0.92174
Out[ ]= -33.2627
Out[ ]= 12.8899
```

Node 0

```
In[ ]:= Cu = 5.700
        Cd = 0.000
        myS = S d[r, δ, h, σ]
        myDelta = Δ[r, δ, h, σ, myS, Cu, Cd]
        myB = B[r, δ, h, σ, Cu, Cd]
        OptionPrice[myDelta, myB, myS]
Out[ ]= 5.7
Out[ ]= 0.
Out[ ]= 35.4114
Out[ ]= 0.450185
Out[ ]= -13.4064
Out[ ]= 2.53528
```

Node root

```
In[ ]:= Cu = 12.889869559234839`
        Cd = 2.535280965407516`
        myS = S
        myDelta = Δ[r, δ, h, σ, myS, Cu, Cd]
        myB = B[r, δ, h, σ, Cu, Cd]
        OptionPrice[myDelta, myB, myS]
Out[ ]= 12.8899
Out[ ]= 2.53528
Out[ ]= 41
Out[ ]= 0.70633
Out[ ]= -21.8854
Out[ ]= 7.07414
```