

# Financial Mathematics

MATH 5870/6870<sup>1</sup>  
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<sup>1</sup>Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

## Chapter 9. Parity and other option relationships

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## § 9.1 Put-call parity

## § 9.2 Generalized parity and exchange options

## § 9.3 Comparing options with respect to style, maturity, and strike

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## European options

$$\begin{aligned}C(K, T) - P(K, T) &= \text{PV}_{0,T}(F_{0,T} - K) \\ &= e^{-rT}(F_{0,T} - K)\end{aligned}$$

Buying a call and selling a put  
with the strike both equal to the forward price (i.e.,  $K = F_{0,T}$ )  
creates a synthetic forward contract  
and hence must have a zero price.

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Parity generally fails for American options!

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## Parity for stocks

$$C(K, T) = P(K, T) + (S_0 - \text{PV}_{0,T}(\text{Div})) - e^{-rT} K$$

**Example 9.1-1** Suppose that the price of a non-dividend-paying stock is \$40, the continuously compounded interest rate is 8%, and options have 3 months to expiration. If a 40-strike European call sells for \$2.78, find the price for a 40-strike European put sells.

Solution. Let the price for put be  $y$ . Then

$$\$2.78 = y + \$40 - \$40e^{-0.08 \times 0.25}$$

Hence,

$$y = \$1.99.$$



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Why is a call more expensive than a put?

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**Example 9.1-2** Make the same assumptions as in Example 9.1-1, except suppose that the stock pays a \$5 dividend just before expiration. If the price of the European call is \$0.74, what would be the price of the European put?

Solution. Let the price for put be  $y$ . Then

$$\$0.74 = y + (\$40 - \$5e^{-0.08 \times 0.25}) - \$40e^{-0.08 \times 0.25}$$

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## Synthetic securities

$$C(K, T) = P(K, T) + (S_0 - \text{PV}_{0,T}(\text{Div})) - e^{-rT} K$$

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► Synthetic stock

$$S_0 = C(K, T) - P(K, T) + \text{PV}_{0,T}(\text{Div}) + e^{-rT} K$$

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► Synthetic Treasury bill (T-bill)

$$\underbrace{S_0 - C(K, T) + P(K, T)}_{\text{a conversion}} = \text{PV}_{0,T}(\text{Div}) + e^{-rT} K$$

Motivation:

A hedged position that has no risk but requires investment.

T-bills are taxed differently than stocks.

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### ► Synthetic options

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$$P(K, T) = C(K, T) - (S_0 - PV_{0,T}(\text{Div})) + e^{-rT} K$$