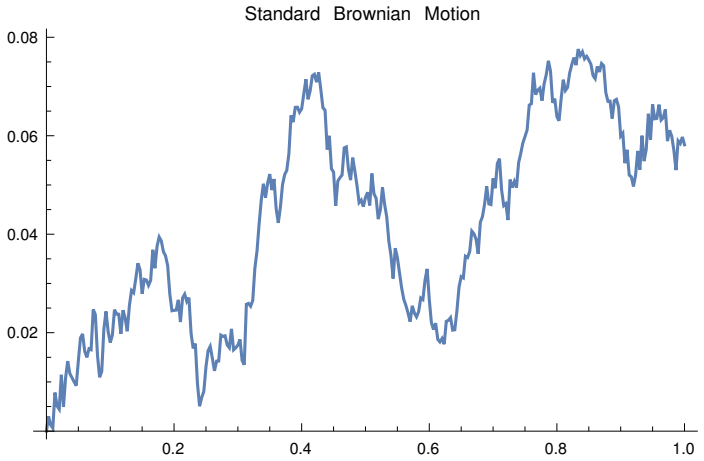
Simulate Brownian motion and geometric Brownian motion sample path
By Le Chen.
Crated on Wed 17 Nov 2021 09:14:01 AM CST

Simulate the standard Brownian path

In[ ]:=
```
n = 300; (* This is the number of steps in t ∈ [0,1] *)
p = RandomVariate[NormalDistribution[0, 1/n], n];
(* Generate independent Brownian increments *)
f = {{0, 0}};(* Brownian motion starts from (0,0) *)
For[i = 0, i < n, i++;
  f = Append[f, {i/n, p[[i]] + f[[i]][[2]]}]]; (* Add all increments to form Brownian path. *)
ListLinePlot[f, PlotLabel → "Standard Brownian Motion"](* Plot the Brownian path. *)
```

Out[ ]=



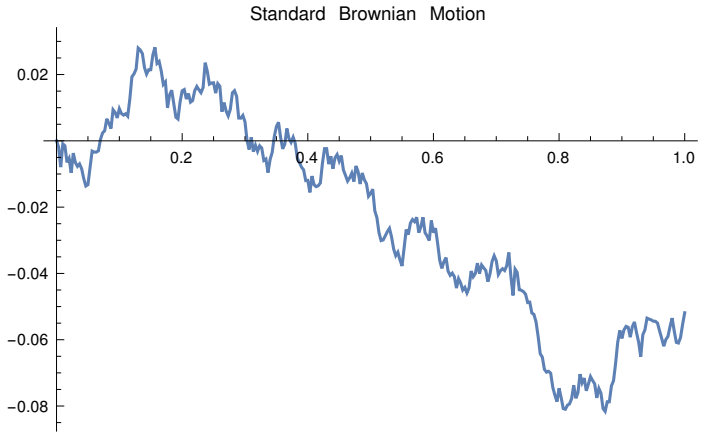
Simulate the geometric Brownian motion

$$d S_t = \mu S_t dt + \sigma S_t d W_t$$

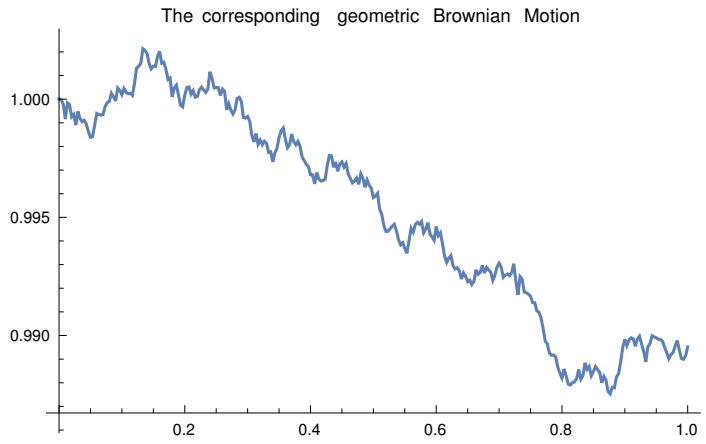
$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t}$$

```
(* Setting parameters *)
μ = 0;
σ = 0.1;
S0 = 1;
(* We first generate the Brownian path. *)
n = 300; (* This is the number of steps in t ∈ [0,1] *)
p = RandomVariate[NormalDistribution[0, 1/n], n];
(* Generate independent Brownian increments *)
f = {{0, 0}};(* Brownian motion starts from (0,0) *)
For[i = 0, i < n, i++; f = Append[f, {i/n, p[[i]] + f[[i]][[2]]}]]
(* Mow we can form the corresponding geometric Brownian path. *)
g = {{0, S0}};(* Geometric Brownian motion starts from (0,S0) *)
For[i = 0, i < n, i++; g = Append[g, {i/n, S0 Exp[(μ - σ^2/2) i/n + σ f[[i]][[2]]]}]];
(* Now we can plot two paths. *)
ListLinePlot[{f}, PlotRange → Full, PlotLabel → "Standard Brownian Motion"]
ListLinePlot[{g}, PlotRange → Full,
 PlotLabel → "The corresponding geometric Brownian Motion"]
 (* Plot the Brownian path. *)
```

Out[ ]=



Out[ ]=


The corresponding geometric Brownian Motion
1.000
0.995
0.990
0.2
0.4
0.6
0.8
1.0