

Financial Mathematics

MATH 5870/6870¹
Fall 2021

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¹Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 3. Insurance, Collars, and Other Strategies

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§ 3.1 Basic insurance strategies

§ 3.2 Put-call parity

§ 3.3 Spreads and collars

§ 3.4 Speculating on volatility

§ 3.5 Problems

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§ 3.2 Put-call parity

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§ 3.4 Speculating on volatility

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Options can be

1. Used to insure long positions (floors)
2. Used to insure short positions (caps)
3. Written against asset positions (selling insurance)

Covered call writing

Covered put writing

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Four positions

positions w.r.t. asset	put option	call option
long	purchased (<i>floor</i>)	written
short	written	purchased (<i>cap</i>)

Buying insurance

floor = buying a *put* option

cap = buying a *call* option

Selling insurance

Covered *put* writing

Covered *call* writing

We will work under the following setup

S&S index

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
premium for 1000-strike 6-month put	\$74.201

Insuring a long position

– Floors

owning a home	owning a stock index
insuring the house	buying a put (floor)

Goal: to insure against a fall in the price of the underlying asset.

Example 3.1-1 Under the following scenario, compute the combined profit of insuring a long position via **buying a put** for the following S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	\$74.201
index price at expiration	\$900

Solution.

$$\underbrace{\$900 - \$1,000 \times 1.02}_{\text{profit on S\&R index}} + \underbrace{\$1,000 - \$900 - \$74.201 \times 1.02}_{\text{profit on put}} = -\$95.68.$$



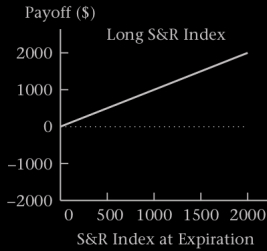
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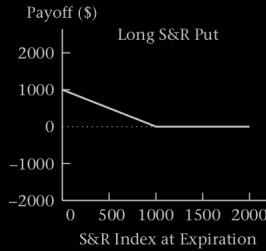
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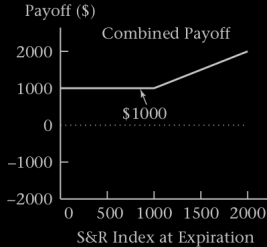




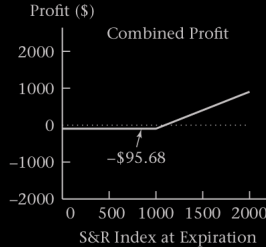
(a)



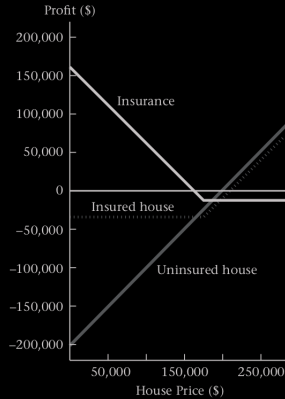
(b)



(c)



(d)



Insuring a short position

– Caps

If we have a short position in the S&R index, we experience a loss when the index rises.

We can insure a short position by purchasing a call option (cap) to protect against a higher price of repurchasing the index.

Example 3.1-2 Under the following scenario, compute the combined profit for insuring a short position via **buying a call** of the following S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
index price at expiration	\$1,100

Solution.

$$\underbrace{\$1,000 \times 1.02}_{\text{future value of short S\&R index}} - \underbrace{\$93.809 \times 1.02}_{\text{FV of premium for call}} - \underbrace{\$1,000}_{\text{exercise the call option}} = -\$75.685.$$

□

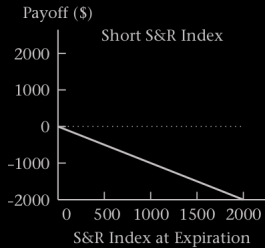
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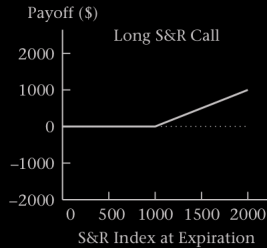
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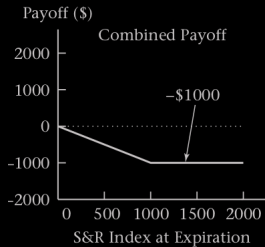




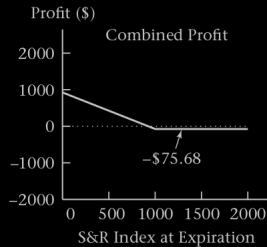
(a)



(b)



(c)



(d)

Selling insurance

For every insurance buyer there must be an insurance seller

Strategies used to sell insurance

- ▶ Covered writing (option overwriting or selling a covered call) is writing an option when there is a corresponding long position in the underlying asset.
- ▶ Naked writing is writing an option when the writer does not have a position in the asset.

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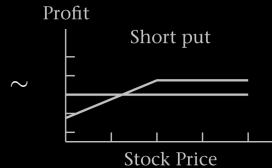
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- ▶ **Covered writing** (option overwriting or selling a covered call) is writing an option when there is a corresponding long position in the underlying asset.
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Covered call writing

Long position of the asset + Sell a **call** option



Covered put writing

Short position of the asset + Sell a **put** option



Covered call writing

Example 3.1-3 Under the following scenario, compute the combined profit for writing a **covered call** for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
index price at expiration	\$1,100

Solution.

$$\underbrace{\$1,100 - \$1,000 \times 1.02}_{\text{profit on S\&R index}} + \underbrace{\$1,000 - \$1,100 + \$93.809 \times 1.02}_{\text{profit on written call}} = \$75.68.$$



Covered call writing

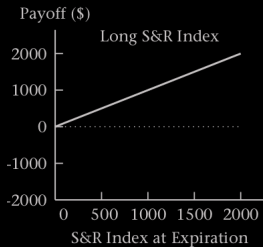
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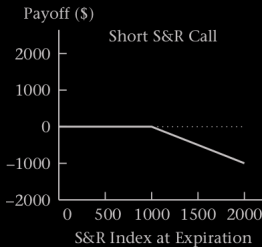
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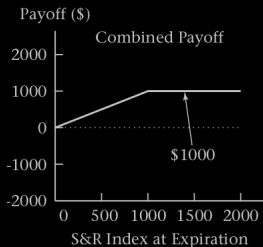




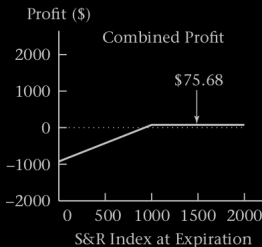
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Covered put writing

Example 3.1-4 Under the following scenario, compute the combined profit for writing a covered put for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	\$74.201
index price at expiration	\$900

Solution.

$$\underbrace{\$1,000 \times 1.02 - \$900}_{\text{profit on selling S\&R index}} + \underbrace{+\$900 - \$1,000 + \$74.201 \times 1.02}_{\text{profit on written put}} = \$95.685.$$



Covered put writing

Example 3.1-4 Under the following scenario, compute the combined profit for writing a covered put for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	\$74.201
index price at expiration	\$900

Solution.

$$\underbrace{\$1,000 \times 1.02 - \$900}_{\text{profit on selling S\&R index}} + \underbrace{+\$900 - \$1,000 + \$74.201 \times 1.02}_{\text{profit on written put}} = \$95.685.$$



