

# Financial Mathematics

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<sup>1</sup>Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

# Chapter 11. Binomial Option Pricing: Selected Topics

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§ 11.1 Understanding Early Exercise

§ 11.2 Understanding risk-neutral pricing

§ 11.3 The Binomial tree and lognormality

§ 11.4 Problems

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## Options may be rationally exercised prior to expiration

By exercising, the option holder

- ▶ Receives the stock and thus receives dividends
- ▶ Pays the strike price prior to expiration (this has an interest cost)
- ▶ Loses the insurance implicit in the call against the possibility that the stock price will be less than the strike price at expiration

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If **volatility is zero**, the value of insurance is zero. Then, it is optimal to defer exercise as long as interest savings on the strike exceed dividends lost

$$rK > \delta S$$



$$\text{It is optimal to exercise} \iff S > \frac{rK}{\delta}$$

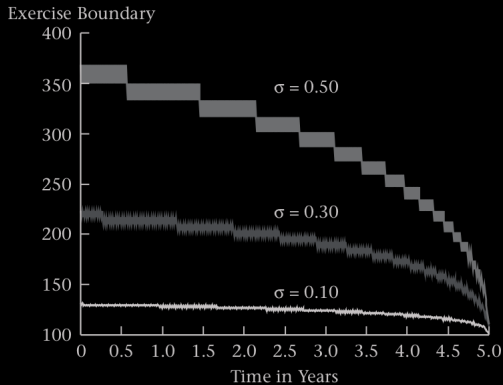
E.g. When  $r = \delta$  and  $\sigma = 0$ , any in-the-money option should be exercised immediately.

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When **volatility is positive**, the implicit insurance has value that varies with time to expiration.

**FIGURE 11.1**

Early-exercise boundaries for volatilities of 10%, 30%, and 50% for a 5-year American call option. In all cases,  $K = \$100$ ,  $r = 5\%$ , and  $\delta = 5\%$ .



**FIGURE 11.2**

Early-exercise boundaries for volatilities of 10%, 30%, and 50% for a 5-year American put option. In all cases,  $K = \$100$ ,  $r = 5\%$ , and  $\delta = 5\%$ .

