Financial Mathematics

MATH 5870/6870[1]
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Le Chen

lzc0090@auburn.edu

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Auburn University
Auburn AL

[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 9. Parity and other option relationships

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§ 9.1 Put-call parity

§ 9.2 Generalized parity and exchange options

§ 9.3 Comparing options with respect to style, maturity, and strike

§ 9.4 Problems

European options

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\begin{aligned}
C(K,T) - P(K,T) &= \mathrm{PV}_{0,T}\left(F_{0,T} - K\right) \\
&= e^{-rT}\left(F_{0,T} - K\right)
\end{aligned}
$$

Buying a call and selling a put
with the strike both equal to the forward price (i.e., $K = F_{0,T}$)
creates a synthetic forward contract
and hence must have a zero price.

Parity generally fails for American options!

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Parity for stocks

$$C(K, T) = P(K, T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT} K$$

Example 9.1-1 Suppose that the price of a non-dividend-paying stock is \$40, the continuously compounded interest rate is 8%, and options have 3 months to expiration. If a 40-strike European call sells for \$2.78, find the price for a 40-strike European put sells.

Solution. Let the price for put be y. Then

$$\$2.78 = y + \$40 - \$40e^{-0.08\times 0.25}$$

Hence,

$$y = \$1.99.$$

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Why is a call more expensive than a put?

When $S_0 = K$ and $\mathrm{Div} = 0$, then

$$C(K,T) - P(K,T) = K\left(1 - e^{-rT}\right)$$

The difference of a call and put is the time value of money.

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Example 9.1-2 Make the same assumptions as in Example 9.1-1, except suppose that the stock pays a \$5 dividend just before expiration. If the price of the European call is \$0.74, what would be the price of the European put?

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$$\$0.74 = y + \left(\$40 - \$5e^{-0.08\times0.25}\right) - \$40e^{-0.08\times0.25}$$

Hence,

$$y = \$4.85.$$

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Synthetic securities

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

- Synthetic stock

$$S_0 = C(K,T) - P(K,T) + \mathrm{PV}_{0,T}(\mathrm{Div}) + e^{-rT}K$$

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- Synthetic Treasury bill (T-bill)

$$\underbrace{S_0 - C(K,T) + P(K,T)}_{\text{a conversion}} = \mathrm{PV}_{0,T}(\mathrm{Div}) + e^{-rT}K$$

Motivation:

A hedged position that has no risk but requires investment.

T-bills are taxed differently than stocks.

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- Synthetic options

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

$$P(K,T) = C(K,T) - (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) + e^{-rT}K$$