#### Financial Mathematics

MATH 5870/6870<sup>1</sup> Fall 2021

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Last updated on October 19, 2021

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<sup>&</sup>lt;sup>1</sup>Based on Robert L. McDonald's *Derivatives Markets*. 3rd Ed. Pearson. 2013.

Chapter 19. Monte Carlo Valuation

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- § 19.1 Computing the option price as a discounted expected value
- § 19.2 Computing random numbers
- § 19.3 Simulating lognormal stock prices
- § 19.4 Monte Carlo valuation
- § 19.5 Efficient Monte Carlo valuation
- § 19.6 Valuation of American options
- § 19.7 The Poisson distribution
- § 19.8 Simulating jumps with the Poisson distribution
- § 19.9 Simulating correlated stock prices
- § 19.10 Problems

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$$V(S_0, 0) = \frac{1}{n}e^{-rT}\sum_{n=1}^{n}V\left(S_T^i, T\right)$$

where

- $\triangleright$   $S_T^1, \dots, S_T^n$  are n randomly drawn time-T stock prices.
- ► For European Call:

$$V(S_T^i, T) = \max\left(0, S_T^i - K\right)$$

Similarly one finds the expression for European put.

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Similarly one finds the expression for European put.

# Example 19.4-1 Carry out the Monte Carlo valuation of the European call under the setting of the following table:

TABLE 19.2	Results of Monte Carlo valuation of European call with $S = \$40$ , $K = \$40$ , $\sigma = 30\%$ , $r = 8\%$ , $t = 91$ days, and $\delta = 0$ . The Black-Scholes price is \$2.78. Each trial use 500 random draws.		
	Trial	Computed Price (\$)	
		2.98	

Trial	Computed Price (\$)
	2.98
2	2.75
3	2.63
4	2.75
5	2.91
Average	2.804

Solution. Check

codes/Table\_19-2.py

# Example 19.4-2 Carry out the Monte Carlo valuation of the Asian call under the setting of the following table:

#### **TABLE 19.3**

Prices of arithmetic average-price Asian options estimated using Monte Carlo and exact prices of geometric average price options. Assumes option has 3 months to expiration and average is computed using equal intervals over the period. Each price is computed using 10,000 trials, assuming S=\$40, K=\$40,  $\sigma=30\%$ , r=8%, T=0.25, and  $\delta=0$ . In each row, the same random numbers were used to compute both the geometric and arithmetic average price options.  $\sigma_n$  is the standard deviation of the estimated arithmetic option prices, divided by  $\sqrt{10,000}$ .

Number of	ber of Monte Carlo Prices (\$)		Exact	
Averages	Arithmetic	Geometric	Geometric Price (\$)	$\sigma_n$
	2.79	2.79	2.78	0.0408
3	2.03	1.99	1.94	0.0291
5	1.78	1.74	1.77	0.0259
10	1.70	1.66	1.65	0.0241
20	1.66	1.61	1.59	0.0231
40	1.63	1.58	1.56	0.0226

Solution. Check

codes/Table\_19-3.py