

Computations for Figure 10.6

European Put with no dividend

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Define the functions

```
In[ ]:=
u[r_, δ_, h_, σ_] := Exp[(r - δ) h + σ √h]
d[r_, δ_, h_, σ_] := Exp[(r - δ) h - σ √h]

Δ[r_, δ_, h_, σ_, S_, Cu_, Cd_] := Exp[-δ h]  $\frac{Cu - Cd}{S (u[r, δ, h, σ] - d[r, δ, h, σ])}$ 

B[r_, δ_, h_, σ_, Cu_, Cd_] := Exp[-r h]  $\frac{u[r, δ, h, σ] Cd - d[r, δ, h, σ] Cu}{u[r, δ, h, σ] - d[r, δ, h, σ]}$ 

OptionPrice [Δ_, B_, S_] := Δ S + B
```

Input the data first

```
In[ ]:=
r = 0.08;
δ = 0;
h =  $\frac{1}{3}$ ;
σ = 0.3;
S = 41;
K = 40;
```

Construct the binomial tree

```
In[ ]:=
S u[r, δ, h, σ]
S u[r, δ, h, σ]2
S u[r, δ, h, σ]3
```

```
Out[ ]:= 50.0711
```

```
Out[ ]:= 61.1491
```

```
Out[ ]:= 74.6781
```

```
In[ ]:=
S d[r, δ, h, σ]
S d[r, δ, h, σ]2
S d[r, δ, h, σ]3
```

```
Out[ ]:= 35.4114
```

```
Out[ ]:= 30.5846
```

```
Out[ ]:= 26.4157
```

```
In[ ] := S u[r,  $\delta$ , h,  $\sigma$ ] d[r,  $\delta$ , h,  $\sigma$ ]
      S u[r,  $\delta$ , h,  $\sigma$ ]2 d[r,  $\delta$ , h,  $\sigma$ ]
      S u[r,  $\delta$ , h,  $\sigma$ ] d[r,  $\delta$ , h,  $\sigma$ ]2
```

```
Out[ ] := 43.246
```

```
Out[ ] := 52.814
```

```
Out[ ] := 37.3513
```

Backwards computation

Node 11

```
In[ ] := myS = S u[r,  $\delta$ , h,  $\sigma$ ]2
      Cu = Max[0, K - myS u[r,  $\delta$ , h,  $\sigma$ ]]
      Cd = Max[0, K - myS d[r,  $\delta$ , h,  $\sigma$ ]]
      myDelta =  $\Delta$ [r,  $\delta$ , h,  $\sigma$ , myS, Cu, Cd]
      myB = B[r,  $\delta$ , h,  $\sigma$ , Cu, Cd]
      OptionPrice[myDelta, myB, myS]
```

```
Out[ ] := 61.1491
```

```
 $\theta$  u[r,  $\delta$ , h,  $\sigma$ ]
```

```
Out[ ] := 0
```

```
Out[ ] := 0.
```

```
Out[ ] := 0.
```

```
Out[ ] := 0.
```

Node 10

```
In[ ] := myS = S u[r,  $\delta$ , h,  $\sigma$ ] d[r,  $\delta$ , h,  $\sigma$ ]
      Cu = Max[0, K - myS u[r,  $\delta$ , h,  $\sigma$ ]]
      Cd = Max[0, K - myS d[r,  $\delta$ , h,  $\sigma$ ]]
      myDelta =  $\Delta$ [r,  $\delta$ , h,  $\sigma$ , myS, Cu, Cd]
      myB = B[r,  $\delta$ , h,  $\sigma$ , Cu, Cd]
      OptionPrice[myDelta, myB, myS]
```

```
Out[ ] := 43.246
```

```
Out[ ] := 0
```

```
Out[ ] := 2.64873
```

```
Out[ ] := -0.171297
```

```
Out[ ] := 8.80883
```

```
Out[ ] := 1.40091
```

Node 00

```
In[ * ]:= myS = S d[r,  $\delta$ , h,  $\sigma$ ]2
Cu = Max[0, K - myS u[r,  $\delta$ , h,  $\sigma$ ]]
Cd = Max[0, K - myS d[r,  $\delta$ , h,  $\sigma$ ]]
myDelta =  $\Delta$ [r,  $\delta$ , h,  $\sigma$ , myS, Cu, Cd]
myB = B[r,  $\delta$ , h,  $\sigma$ , Cu, Cd]
OptionPrice [myDelta, myB, myS]
```

Out[*]= 30.5846

Out[*]= 2.64873

Out[*]= 13.5843

Out[*]= -1.

Out[*]= 38.9474

Out[*]= 8.36287

Node 1

```
In[ * ]:= myS = S u[r,  $\delta$ , h,  $\sigma$ ]
Cu = 0.000
Cd = 1.401
myDelta =  $\Delta$ [r,  $\delta$ , h,  $\sigma$ , myS, Cu, Cd]
myB = B[r,  $\delta$ , h,  $\sigma$ , Cu, Cd]
OptionPrice [myDelta, myB, myS]
```

Out[*]= 50.0711

Out[*]= 0.

Out[*]= 1.401

Out[*]= -0.0782546

Out[*]= 4.65928

Out[*]= 0.740988

Node 0

```
In[ ]:= myS = S d[r,  $\delta$ , h,  $\sigma$ ]
      Cu = 1.401
      Cd = 8.363
      myDelta =  $\Delta$ [r,  $\delta$ , h,  $\sigma$ , myS, Cu, Cd]
      myB = B[r,  $\delta$ , h,  $\sigma$ , Cu, Cd]
      OptionPrice [myDelta, myB, myS]
```

```
Out[ ]:= 35.4114
```

```
Out[ ]:= 1.401
```

```
Out[ ]:= 8.363
```

```
Out[ ]:= -0.549857
```

```
Out[ ]:= 24.5175
```

```
Out[ ]:= 5.04633
```

Node root

```
In[ ]:= myS = S
      Cu = 0.740988360714645`
      Cd = 5.046332854968924`
      myDelta =  $\Delta$ [r,  $\delta$ , h,  $\sigma$ , myS, Cu, Cd]
      myB = B[r,  $\delta$ , h,  $\sigma$ , Cu, Cd]
      OptionPrice [myDelta, myB, myS]
```

```
Out[ ]:= 41
```

```
Out[ ]:= 0.740988
```

```
Out[ ]:= 5.04633
```

```
Out[ ]:= -0.293686
```

```
Out[ ]:= 15.0397
```

```
Out[ ]:= 2.99858
```