Math 362: Mathematical Statistics II

Le Chen

le.chen@emory.edu chenle02@gmail.com

> Emory University Atlanta, GA

Last updated on Spring 2021 Last compiled on January 15, 2023

2021 Spring

Creative Commons License (CC By-NC-SA)

Chapter 9. Two-Sample Inferences

- § 9.1 Introduction
- § 9.2 Testing $H_0: \mu_X = \mu_Y$
- § 9.3 Testing $H_0: \sigma_X^2 = \sigma_Y^2$
- § 9.4 Binomial Data: Testing $H_0: p_X = p_Y$
- § 9.5 Confidence Intervals for the Two-Sample Problem

1

Plan

- § 9.1 Introduction
- § 9.2 Testing $H_0: \mu_X = \mu_Y$
- § 9.3 Testing $H_0: \sigma_X^2 = \sigma_Y^2$
- § 9.4 Binomial Data: Testing $H_0: p_X = p_Y$
- § 9.5 Confidence Intervals for the Two-Sample Problem

Chapter 9. Two-Sample Inferences

- § 9.1 Introduction
- § 9.2 Testing $H_0: \mu_X = \mu_Y$
- § 9.3 Testing $H_0: \sigma_X^2 = \sigma_Y^2$
- § 9.4 Binomial Data: Testing $H_0: p_X = p_Y$
- § 9.5 Confidence Intervals for the Two-Sample Problem

Similar to the hypothesis test ...

- 1. Let X_1, \dots, X_n be a random sample of size n from $N(\mu_X, \sigma_X^2)$.
- 2. Let Y_1, \dots, Y_m be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2 , or σ_X/σ_Y

- 1. Let X_1, \dots, X_n be a random sample of size n from $N(\mu_X, \sigma_X^2)$.
- **2.** Let Y_1, \dots, Y_m be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2 , or $\sigma_{X,I}$

- 1. Let X_1, \dots, X_n be a random sample of size n from $N(\mu_X, \sigma_X^2)$.
- **2**. Let Y_1, \dots, Y_m be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2 , or σ_X

- 1. Let X_1, \dots, X_n be a random sample of size n from $N(\mu_X, \sigma_X^2)$.
- **2.** Let Y_1, \dots, Y_m be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2 , or σ_X

- 1. Let X_1, \dots, X_n be a random sample of size n from $N(\mu_X, \sigma_X^2)$.
- **2**. Let Y_1, \dots, Y_m be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

Prob. 2 Find the $100(1-\alpha)\%$ C.l. for σ_V^2/σ_V^2 , or σ_X

- **1.** Let X_1, \dots, X_n be a random sample of size n from $N(\mu_X, \sigma_X^2)$.
- **2**. Let Y_1, \dots, Y_m be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

- **1.** Let X_1, \dots, X_n be a random sample of size n from $N(\mu_X, \sigma_X^2)$.
- **2**. Let Y_1, \dots, Y_m be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

- **1.** Let X_1, \dots, X_n be a random sample of size n from $N(\mu_X, \sigma_X^2)$.
- **2.** Let Y_1, \dots, Y_m be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2 , or σ_X/σ_Y

Sol

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \sim N(0, 1)$$

$$\mathbb{P}\left(-z_{\alpha/2} \le \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \le z_{\alpha/2}\right) = 1 - \epsilon$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \le \mu_X - \mu_Y \le (\overline{X} - \overline{Y}) + z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}\right)$$

$$\left((\overline{X} - \overline{Y}) - Z_{\alpha/2} \sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \right), \quad (\overline{X} - \overline{Y}) + Z_{\alpha/2} \sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \right)$$

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \sim \textit{N}(0, 1)$$

$$\mathbb{P}\left(-z_{\alpha/2} \le \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \le z_{\alpha/2}\right) = 1 - \alpha$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \le \mu_X - \mu_Y \le (\overline{X} - \overline{Y}) + z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}\right)$$

$$\left((\overline{X} - \overline{Y}) - Z_{\alpha/2} \sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \right), \quad (\overline{X} - \overline{Y}) + Z_{\alpha/2} \sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}$$

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \sim \textit{N}(0, 1)$$

$$\mathbb{P}\left(-\mathbf{z}_{\alpha/2} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \leq \mathbf{z}_{\alpha/2}\right) = 1 - \alpha$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \le \mu_X - \mu_Y \le (\overline{X} - \overline{Y}) + z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}\right)$$

$$\left((\overline{X} - \overline{Y}) - Z_{\alpha/2} \sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \right), \quad (\overline{X} - \overline{Y}) + Z_{\alpha/2} \sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}$$

Sol.

$$\frac{\overline{\mathbf{X}} - \overline{\mathbf{Y}} - (\mu_{\mathbf{X}} - \mu_{\mathbf{Y}})}{\sqrt{\frac{\sigma_{\mathbf{X}}^2}{n} + \frac{\sigma_{\mathbf{Y}}^2}{m}}} \sim \mathbf{N}(0, 1)$$

$$\mathbb{P}\left(-z_{\alpha/2} \le \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \le z_{\alpha/2}\right) = 1 - \alpha$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \le \mu_X - \mu_Y \le (\overline{X} - \overline{Y}) + z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}\right)$$

$$\left((\overline{\mathbf{X}}-\overline{\mathbf{y}})-\mathbf{Z}_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}\right.,\quad (\overline{\mathbf{X}}-\overline{\mathbf{y}})+\mathbf{Z}_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}\right)$$

Γ

Sol

$$rac{\overline{X}-\overline{Y}-(\mu_X-\mu_Y)}{S_p\sqrt{rac{1}{n}+rac{1}{m}}}\sim ext{Student t-distribution }(n+m-2)$$

$$\mathbb{P}\left(-t_{\alpha/2,n+m-2} \le \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{S_p \sqrt{\frac{1}{n} + \frac{1}{m}}} \le t_{\alpha/2,n+m-2}\right) = 1 - \|$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - t_{\alpha/2, n+m-2}S_p\sqrt{\frac{1}{n}} + \frac{1}{m} \le \mu_X - \mu_Y \le (\overline{X} - \overline{Y}) + t_{\alpha/2, n+m-2}S_p\sqrt{\frac{1}{n}} + \frac{1}{m}\right)$$

$$\left((\overline{X}-\overline{y})-t_{\alpha/2,n+m-2}s_{p}\sqrt{\frac{1}{n}+\frac{1}{m}}\right),\quad (\overline{X}-\overline{y})+t_{\alpha/2,n+m-2}s_{p}\sqrt{\frac{1}{n}+\frac{1}{m}}\right)$$

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{S_p \sqrt{\frac{1}{n} + \frac{1}{m}}} \sim \text{Student t-distribution } (n + m - 2)$$

$$S_p\sqrt{rac{1}{n}+rac{1}{m}}$$
) \parallel $S_p\sqrt{rac{1}{n}+rac{1}{m}}$ $S_p\sqrt{rac{1}{n}+rac{1}{m}}$ $S_p\sqrt{rac{1}{n}+rac{1}{n}}$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - t_{\alpha/2, n+m-2}S_{p}\sqrt{\frac{1}{n}} + \frac{1}{m} \le \mu_{X} - \mu_{Y} \le (\overline{X} - \overline{Y}) + t_{\alpha/2, n+m-2}S_{p}\sqrt{\frac{1}{n}} + \frac{1}{m}\right)$$

$$\left((\overline{x}-\overline{y})-t_{\alpha/2,n+m-2}s_p\sqrt{\frac{1}{n}+\frac{1}{m}}\right),\quad (\overline{x}-\overline{y})+t_{\alpha/2,n+m-2}s_p\sqrt{\frac{1}{n}+\frac{1}{m}}\right)$$



$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{S_p \sqrt{\frac{1}{n} + \frac{1}{m}}} \sim \text{Student t-distribution } (n + m - 2)$$

$$\mathbb{P}\left(-t_{\alpha/2,n+m-2} \le \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{S_\rho \sqrt{\frac{1}{n} + \frac{1}{m}}} \le t_{\alpha/2,n+m-2}\right) = 1 - \alpha$$

$$\parallel$$

$$-t_{\alpha/2,n+m-2}S_{\alpha \lambda} \sqrt{\frac{1}{n} + \frac{1}{m}} \le \mu_X - \mu_X \le (\overline{X} - \overline{Y}) + t_{\alpha/2,n+m-2}S_{\alpha \lambda} \sqrt{\frac{1}{n} + \frac{1}{m}}$$

$$\mathbb{P}\left((\overline{X}-\overline{Y})-t_{\alpha/2,n+m-2}S_{p}\sqrt{\frac{1}{n}+\frac{1}{m}}\leq\mu_{X}-\mu_{Y}\leq(\overline{X}-\overline{Y})+t_{\alpha/2,n+m-2}S_{p}\sqrt{\frac{1}{n}+\frac{1}{m}}\right)$$

$$\left((\overline{x}-\overline{y})-t_{\alpha/2,n+m-2}s_p\sqrt{\frac{1}{n}+\frac{1}{m}}\right.,\quad (\overline{x}-\overline{y})+t_{\alpha/2,n+m-2}s_p\sqrt{\frac{1}{n}+\frac{1}{m}}\right)$$

$$rac{\overline{X}-\overline{Y}-(\mu_X-\mu_Y)}{S_{0}\sqrt{rac{1}{a}+rac{1}{m}}}\sim ext{Student t-distribution }(n+m-2)$$

$$\mathbb{P}\left(-t_{\alpha/2,n+m-2} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}} \leq t_{\alpha/2,n+m-2}\right) = 1 - \alpha$$

$$\mathbb{P}\left((\overline{X}-\overline{Y})-t_{\alpha/2,n+m-2}\mathcal{S}_{p}\sqrt{\frac{1}{n}+\frac{1}{m}}\leq\mu_{X}-\mu_{Y}\leq(\overline{X}-\overline{Y})+t_{\alpha/2,n+m-2}\mathcal{S}_{p}\sqrt{\frac{1}{n}+\frac{1}{m}}\right)$$

$$\left((\overline{x}-\overline{y})-t_{\alpha/2,n+m-2}s_p\sqrt{\frac{1}{n}+\frac{1}{m}}\right),\quad (\overline{x}-\overline{y})+t_{\alpha/2,n+m-2}s_p\sqrt{\frac{1}{n}+\frac{1}{m}}\right)$$

Sol

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \sim \text{Student t-distribution } (\nu)$$

$$\mathbb{P}\left(-t_{\alpha/2,\nu} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \leq t_{\alpha/2,\nu}\right) \approx 1 - \alpha$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}} \le \mu_X - \mu_Y \le (\overline{X} - \overline{Y}) + t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}\right)$$

$$\left((\overline{X}-\overline{Y})-t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n}+\frac{s_Y^2}{m}}\right.,\quad (\overline{X}-\overline{Y})+t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n}+\frac{s_Y^2}{m}}\right)$$

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \sim \text{Student t-distribution } (\nu)$$

$$\mathbb{P}\left(-t_{\alpha/2,\nu} \le \frac{X - Y - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \le t_{\alpha/2,\nu}\right) \approx 1 - c$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}} \le \mu_X - \mu_Y \le (\overline{X} - \overline{Y}) + t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}\right)$$

$$\left((\overline{X}-\overline{Y})-t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n}+\frac{s_Y^2}{m}}\right.,\quad (\overline{X}-\overline{Y})+t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n}+\frac{s_Y^2}{m}}\right)$$

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \sim \text{Student t-distribution } (\nu)$$

$$\mathbb{P}\left(-t_{\alpha/2,\nu} \le \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \le t_{\alpha/2,\nu}\right) \approx 1 - \alpha$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}} \leq \mu_X - \mu_Y \leq (\overline{X} - \overline{Y}) + t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}\right)$$

$$\left((\overline{x}-\overline{y})-t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n}+\frac{s_Y^2}{m}}\right.,\quad (\overline{x}-\overline{y})+t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n}+\frac{s_Y^2}{m}}\right)$$

Sol.

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \sim \text{Student t-distribution } (\nu)$$

$$\mathbb{P}\left(-t_{\alpha/2,\nu} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \leq t_{\alpha/2,\nu}\right) \approx 1 - \alpha$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}} \leq \mu_X - \mu_Y \leq (\overline{X} - \overline{Y}) + t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}\right)$$

$$\left((\overline{\mathbf{x}}-\overline{\mathbf{y}})-t_{\alpha/2,\nu}\sqrt{\frac{\mathbf{s}_\chi^2}{n}+\frac{\mathbf{s}_Y^2}{m}}\right.,\quad (\overline{\mathbf{x}}-\overline{\mathbf{y}})+t_{\alpha/2,\nu}\sqrt{\frac{\mathbf{s}_\chi^2}{n}+\frac{\mathbf{s}_Y^2}{m}}\right)$$

Γ

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2

Sol 1.

$$\begin{split} \frac{S_\chi^2/\sigma_\chi^2}{S_\gamma^2/\sigma_\gamma^2} \sim & \text{F-disribution } (n-1,m-1) \\ \mathbb{P}\left(F_{\alpha/2,n-1,m-1} \leq \frac{S_\chi^2/\sigma_\chi^2}{S_\gamma^2/\sigma_\gamma^2} \leq F_{1-\alpha/2,n-1,m-1}\right) = 1-\alpha \\ & || \\ \mathbb{P}\left(\frac{S_\chi^2}{S_\gamma^2} \frac{1}{F_{1-\alpha/2,n-1,m-1}} \leq \frac{\sigma_\chi^2}{\sigma_\gamma^2} \leq \frac{S_\chi^2}{S_\gamma^2} \frac{1}{F_{\alpha/2,n-1,m-1}}\right) \end{split}$$

$$\left(rac{s_X^2}{s_Y^2}rac{1}{F_{1-lpha/2,n-1,m-1}}
ight. , rac{s_X^2}{s_Y^2}rac{1}{F_{lpha/2,n-1,m-1}}
ight)$$

E0

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2

Sol 1.

$$\begin{split} \frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \sim & \text{F-disribution } (n-1,m-1) \\ \mathbb{P}\left(F_{\alpha/2,n-1,m-1} \leq \frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \leq F_{1-\alpha/2,n-1,m-1}\right) = 1-\alpha \\ & \qquad \qquad || \\ \mathbb{P}\left(\frac{S_X^2}{S_Y^2} \frac{1}{F_{1-\alpha/2,n-1,m-1}} \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq \frac{S_X^2}{S_Y^2} \frac{1}{F_{\alpha/2,n-1,m-1}}\right) \end{split}$$

$$\left(\frac{s_X^2}{s_Y^2} \frac{1}{F_{1-\alpha/2,n-1,m-1}} \ , \ \frac{s_X^2}{s_Y^2} \frac{1}{F_{\alpha/2,n-1,m-1}}\right)$$

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2

Sol 1.

$$\begin{split} \frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \sim & \text{F-disribution } (n-1,m-1) \\ \mathbb{P}\left(F_{\alpha/2,n-1,m-1} \leq \frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \leq F_{1-\alpha/2,n-1,m-1}\right) = 1-\alpha \\ & \qquad \qquad || \\ \mathbb{P}\left(\frac{S_X^2}{S_Y^2} \frac{1}{F_{1-\alpha/2,n-1,m-1}} \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq \frac{S_X^2}{S_Y^2} \frac{1}{F_{\alpha/2,n-1,m-1}}\right) \end{split}$$

$$\left(\frac{s_X^2}{s_Y^2} \frac{1}{F_{1-\alpha/2, n-1, m-1}} , \frac{s_X^2}{s_Y^2} \frac{1}{F_{\alpha/2, n-1, m-1}}\right)$$

E0

Sol 2. Or equivalently,

$$\frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \sim \text{F-disribution } (\textit{m}-1,\textit{n}-1)$$

$$\mathbb{P}\left(F_{\alpha/2,\textit{m}-1,\textit{n}-1} \leq \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \leq F_{1-\alpha/2,\textit{m}-1,\textit{n}-1}\right) = 1-\alpha$$

$$||$$

$$\mathbb{P}\left(\frac{S_X^2}{S_Y^2}F_{\alpha/2,\textit{m}-1,\textit{n}-1} \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq \frac{S_X^2}{S_Y^2}F_{1-\alpha/2,\textit{m}-1,\textit{n}-1}\right)$$

Recall:
$$F_{\alpha,m,n} = \frac{1}{F_{1,\alpha,n,n}}$$

Sol 2. Or equivalently,

$$\begin{split} \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \sim & \text{F-disribution } (\textit{m}-1,\textit{n}-1) \\ \mathbb{P}\left(F_{\alpha/2,\textit{m}-1,\textit{n}-1} \leq \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \leq F_{1-\alpha/2,\textit{m}-1,\textit{n}-1}\right) = 1-\alpha \\ & \qquad \qquad || \\ \mathbb{P}\left(\frac{S_X^2}{S_Y^2}F_{\alpha/2,\textit{m}-1,\textit{n}-1} \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq \frac{S_X^2}{S_Y^2}F_{1-\alpha/2,\textit{m}-1,\textit{n}-1}\right) \\ & \left(\frac{s_X^2}{s_Y^2}F_{\alpha/2,\textit{m}-1,\textit{n}-1} \right. , \quad \frac{s_X^2}{s_Y^2}F_{1-\alpha/2,\textit{m}-1,\textit{n}-1}\right) \end{split}$$

Recall:
$$F_{\alpha,m,n} = \frac{1}{F_{1-\alpha,n,n}}$$

E 4

Sol 2. Or equivalently,

$$\begin{split} \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} &\sim \text{F-disribution } (\textit{m}-1,\textit{n}-1) \\ \mathbb{P}\left(F_{\alpha/2,\textit{m}-1,\textit{n}-1} \leq \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \leq F_{1-\alpha/2,\textit{m}-1,\textit{n}-1}\right) = 1-\alpha \\ & \qquad \qquad || \\ \mathbb{P}\left(\frac{S_X^2}{S_Y^2}F_{\alpha/2,\textit{m}-1,\textit{n}-1} \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq \frac{S_X^2}{S_Y^2}F_{1-\alpha/2,\textit{m}-1,\textit{n}-1}\right) \\ & \left(\frac{s_X^2}{s_Y^2}F_{\alpha/2,\textit{m}-1,\textit{n}-1} \right. , \quad \frac{s_X^2}{s_Y^2}F_{1-\alpha/2,\textit{m}-1,\textit{n}-1}\right) \end{split}$$

$$F_{\alpha,m,n} = \frac{1}{F_{1-\alpha,n,m}}$$

_

Examples from the book...