

# Math 362: Mathematical Statistics II

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# Chapter 7. Inference Based on The Normal Distribution

## § 7.1 Introduction

## § 7.2 Comparing $\frac{\bar{Y}-\mu}{\sigma/\sqrt{n}}$ and $\frac{\bar{Y}-\mu}{S/\sqrt{n}}$

## § 7.3 Deriving the Distribution of $\frac{\bar{Y}-\mu}{S/\sqrt{n}}$

## § 7.4 Drawing Inferences About $\mu$

## § 7.5 Drawing Inferences About $\sigma^2$

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## § 7.5 Drawing Inferences About $\sigma^2$

Let  $Y_1, \dots, Y_n$  be a random sample from  $N(\mu, \sigma^2)$ .

Question Find a test statistic  $\Lambda$  in order to test  $H_0 : \mu = \mu_0$  v.s.  $H_1 : \mu \neq \mu_0$ .

Case I.  $\sigma^2$  is known:

$$\Lambda = \frac{\bar{Y} - \mu_0}{\sigma/\sqrt{n}}$$

Case II.  $\sigma^2$  is unknown:

$$\Lambda = ? \quad \Lambda \stackrel{?}{=} \frac{\bar{Y} - \mu_0}{s/\sqrt{n}} \sim ?$$

# Summary

A random sample of size  $n$  from  
a normal distribution  $N(\mu, \sigma^2)$

	$\sigma^2$ known	$\sigma^2$ unknown
Statistic	$Z = \frac{\bar{Y} - \mu}{\sigma/\sqrt{n}}$	$T_{n-1} = \frac{\bar{Y} - \mu}{S/\sqrt{n}}$
Score	$z = \frac{\bar{y} - \mu}{\sigma/\sqrt{n}}$	$t = \frac{\bar{y} - \mu}{s/\sqrt{n}}$
Table	$z_\alpha$	$t_{\alpha, n-1}$
100(1 - $\alpha$ )% C.I.	$\left( \bar{y} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{y} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right)$	$\left( \bar{y} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}, \bar{y} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \right)$
Test $H_0 : \mu = \mu_0$		
$H_1 : \mu > \mu_0$	Reject $H_0$ if $z \geq z_\alpha$	Reject $H_0$ if $t \geq t_{\alpha, n-1}$
$H_1 : \mu < \mu_0$	Reject $H_0$ if $z \leq -z_\alpha$	Reject $H_0$ if $t \leq -t_{\alpha, n-1}$
$H_1 : \mu \neq \mu_0$	Reject $H_0$ if $ z  \geq z_{\alpha/2}$	Reject $H_0$ if $ t  \geq t_{\alpha/2, n-1}$

## Computing $s$ from data

Step 1  $a = \sum_{i=1}^n y_i$

Step 2.  $b = \sum_{i=1}^n y_i^2$

Step 3.  $s = \sqrt{\frac{nb - a^2}{n(n-1)}}$

Proof.

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2 = \frac{n(\sum_{i=1}^n y_i^2) - (\sum_{i=1}^n y_i)^2}{n(n-1)}$$

□

Case 7.4.1 How far apart are the bat and the insect when the bat first senses that insect is there?

Or, what is the effective range of a bat's echolocation system?

Table 7.4.1	
Catch Number	Detection Distance (cm)
1	62
2	52
3	68
4	23
5	34
6	45
7	27
8	42
9	83
10	56
11	40

Answer the question by construct a 95% C.I.

Sol. ...



```

1 # Case7_4_1.py
2 import numpy as np
3 import scipy.stats as st
4
5
6 # returns confidence interval of mean
7 def confIntMean(a, conf=0.95):
8     mean, sem, m = np.mean(a), st.sem(a), st.t.ppf((1+conf)/2., len(a)-1)
9     return mean - m*sem, mean + m*sem
10
11
12 def main():
13     alpha = 5
14     data = np.array([62, 52, 68, 23, 34, 45, 27, 42, 83, 56, 40])
15     lower, upper = confIntMean(data, 1-alpha/100)
16     print( """ \
17
18     The {alpha}% confidence interval is ({lower:.2f},{upper:.2f})
19
20     """ .format(**locals() ) )
21
22
23 if __name__ == "__main__":
24     main()

```

```

1 In [83]: run Case7_4_1.py

```

```

2
3 The 95% confidence interval is (36.21,60.51)

```



Eg. 7.4.2 Bank approval rates for inner-city residents v.s. rural ones.

Approval rate for rural residents is 62%.

Do bank treat two groups equally?  $\alpha = 0.05$

Table 7.4.3			
Bank	Location	Affiliation	Percent Approved
1	3rd & Morgan	AU	59
2	Jefferson Pike	TU	65
3	East 150th & Clark	TU	69
4	Midway Mall	FT	53
5	N. Charter Highway	FT	60
6	Lewis & Abbot	AU	53
7	West 10th & Lorain	FT	58
8	Highway 70	FT	64
9	Parkway Northwest	AU	46
10	Lanier & Tower	TU	67
11	King & Tara Court	AU	51
12	Bluedot Corners	FT	59

Sol.

$$H_0 : \mu = 62 \quad v.s. \quad H_1 : \mu \neq 62.$$

**Table 7.4.4**

Banks	$n$	$\bar{y}$	$s$	$t$ Ratio	Critical Value	Reject $H_0$ ?
All	12	58.667	6.946	-1.66	$\pm 2.2010$	No

**Table 7.4.5**

Banks	$n$	$\bar{y}$	$s$	$t$ Ratio	Critical Value	Reject $H_0$ ?
American United	4	52.25	5.38	-3.63	$\pm 3.1825$	Yes
Federal Trust	5	58.80	3.96	-1.81	$\pm 2.7764$	No
Third Union	3	67.00	2.00	+4.33	$\pm 4.3027$	Yes

```

1 # Eg7_4_2.py
2 import numpy as np
3 import scipy.stats as st
4
5 data = np.array([59, 65, 69, 53, 60, 53, 58, 64, 46, 67, 51, 59])
6 alpha = 5
7 mean, sem = np.mean(data), st.sem(data)
8 n = len(data)
9 s = sem * np.sqrt(n)
10 cv = st.t.ppf(1-alpha/200., len(data)-1)
11 tRatio = (mean-62)/sem
12
13
14 print (""" \
15
16     n={n}, sample mean={mean:.3f}, s={s:.3f}, t Ratio={tRatio:.2 f}, Critical values={cv:.4f}
17     """ .format(**locals() ))

```

```

1 In [113]: run Eg7_4_2.py
2
3     n=12, sample mean=58.667, s=6.946, t Ratio=-1.66, Critical values=2.2010

```