Math 362: Mathematical Statistics II

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Plan

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$$\frac{\frac{X}{n} - \frac{Y}{m} - \mathbb{E}\left(\frac{X}{n} - \frac{Y}{m}\right)}{\sqrt{\mathsf{Var}\left(\frac{X}{n} - \frac{Y}{m}\right)}} \overset{\mathsf{approx.}}{\sim} \mathsf{N}(0, 1)$$

Under $H_0: p_X = p_Y$

$$\mathbb{E}\left(\frac{X}{n} - \frac{Y}{m}\right) = 0$$

$$\operatorname{Var}\left(\frac{X}{n} - \frac{Y}{m}\right) = \frac{p(1-p)}{n} + \frac{p(1-p)}{m}$$

The MLE for p under H_0 is

$$p_e = \frac{x + y}{n + m}$$

$$\frac{\frac{X}{n} - \frac{Y}{m} - \mathbb{E}\left(\frac{X}{n} - \frac{Y}{m}\right)}{\sqrt{\mathsf{Var}\left(\frac{X}{n} - \frac{Y}{m}\right)}} \overset{\mathsf{approx.}}{\sim} \mathsf{N}(0, 1)$$

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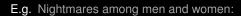
Testing
$$H_0: p_X = p_Y$$

v.s.

(at the α level of significance)

$$z = \frac{\frac{x}{n} - \frac{y}{m}}{\sqrt{p_e(1 - p_e)\left(\frac{1}{n} + \frac{1}{m}\right)}}, \qquad p_e = \frac{x + y}{n + m}$$

$$H_1: p_X < p_Y$$
: $H_1: p_X \neq p_Y$: $H_1: p_X > p_Y$: Reject H_0 if Reject H_0 if $z < -z_{\alpha}$ $|z| \geq z_{\alpha/2}$ $z > z_{\alpha}$



Is 34.4% significantly different from 31.1% ($\alpha = 0.05$)?

Sol.

E.g. Nightmares among men and women:

Table 9.4.1 Frequency of Nightmares			
	Men	Women	Total
Nightmares often Nightmares seldom Totals % often:	55 105 160 34.4	60 132 192 31.3	115 237

Is 34.4% significantly different from 31.1% ($\alpha = 0.05$)?

Sol. ...