

# Math 221: LINEAR ALGEBRA

## Chapter 1. Systems of Linear Equations §1-6. Application to Chemical Reactions

Le Chen<sup>1</sup>

Emory University, 2021 Spring

(last updated on 01/12/2023)



Creative Commons License  
(CC BY-NC-SA)

<sup>1</sup>Slides are adapted from those by Karen Seyffarth from University of Calgary.



# Linear Algebra with Applications

## Lecture Notes

### Current Lecture Notes Revision: Version 2018 — Revision B

These lecture notes were originally developed by Karen Seyffarth of the University of Calgary. Edits, additions, and revisions have been made to these notes by the editorial team at Lyryx Learning to accompany their text [Linear Algebra with Applications](#) based on W. K. Nicholson's original text.

In addition we recognize the following contributors. All new content contributed is released under the same license as noted below.

- Ilijas Farah, York University

### BE A CHAMPION OF OER!

Contribute suggestions for improvements, new content, or errata:

A new topic

A new example or problem

A new or better proof to an existing theorem

Any other suggestions to improve the material

Contact Lyryx at [info@lyryx.com](mailto:info@lyryx.com) with your ideas.

### License



Attribution-NonCommercial-ShareAlike (CC BY-NC-SA)

This license lets others remix, tweak, and build upon your work non-commercially, as long as they credit you and license their new creations under the identical terms.

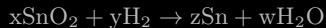
Copyright

Chemical Reactions

# Balancing Chemical Reactions

## Problem

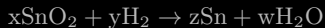
Balance the chemical reaction given below involving tin (Sn), hydrogen (H), and oxygen (O).



# Balancing Chemical Reactions

## Problem

Balance the chemical reaction given below involving tin (Sn), hydrogen (H), and oxygen (O).



## Solution

Setting up a system of equations in  $x, y, z, w$  gives

$$\text{Sn} \quad : \quad x = z \text{ or } x - z = 0$$

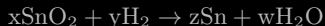
$$\text{O} \quad : \quad 2x = w \text{ or } 2x - w = 0$$

$$\text{H} \quad : \quad 2y = 2w \text{ or } 2y - 2w = 0$$

# Balancing Chemical Reactions

## Problem

Balance the chemical reaction given below involving tin (Sn), hydrogen (H), and oxygen (O).



## Solution

Setting up a system of equations in  $x, y, z, w$  gives

$$\text{Sn} \quad : \quad x = z \text{ or } x - z = 0$$

$$\text{O} \quad : \quad 2x = w \text{ or } 2x - w = 0$$

$$\text{H} \quad : \quad 2y = 2w \text{ or } 2y - 2w = 0$$

The augmented matrix is 
$$\left[ \begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 \end{array} \right]$$

## Solution (continued)

The reduced row-echelon matrix is

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{array} \right]$$



## Solution (continued)

The reduced row-echelon matrix is

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{array} \right]$$

Letting  $w = t$ , the solution is

$$x = \frac{1}{2}t$$

$$y = t$$

$$z = \frac{1}{2}t$$

$$w = t$$

### Solution (continued)

The reduced row-echelon matrix is

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{array} \right]$$

Letting  $w = t$ , the solution is

$$\begin{aligned} x &= \frac{1}{2}t \\ y &= t \\ z &= \frac{1}{2}t \\ w &= t \end{aligned}$$

We can choose any values for  $w = t$ . Suppose we choose  $w = 4$ , then  $x = 2, y = 4, z = 2$  and the balanced reaction is

