Math 221: LINEAR ALGEBRA

Chapter 2. Matrix Algebra
§2-1. Matrix Addition, Scalar Multiplication and Transposition

Le Chen[1]

Emory University, 2021 Spring

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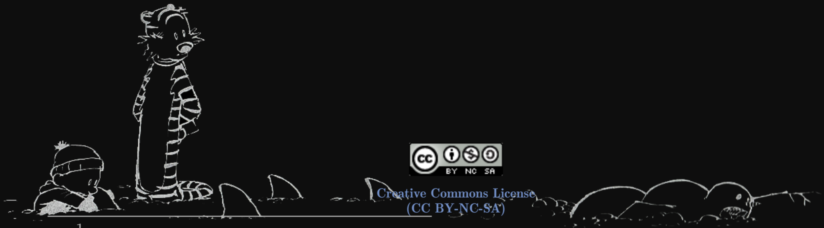

[1] Slides are adapted from those by Karen Seyffarth from University of Calgary.

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Matrices – Definitions and Basic Properties

Matrix Addition

Scalar Multiplication

The Transpose

Linear Algebra with Applications

Lecture Notes

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These lecture notes were originally developed by Karen Seyffarth of the University of Calgary. Edits, additions, and revisions have been made to these notes by the editorial team at Lyryx Learning to accompany their text Linear Algebra with Applications based on W. K. Nicholson's original text.

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- **Ilijas Farah, York University**

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General notation for an $m \times n$ matrix, A:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} = [a_{ij}]$$

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6. Subtraction: for $m \times n$ matrices A and B, $A - B = A + (-1)B$.

Matrix Addition

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Definition

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two $m \times n$ matrices. Then $A + B = C$ where C is the $m \times n$ matrix $C = [c_{ij}]$ defined by

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Example

Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}, B = \begin{bmatrix} 0 & -2 \\ 6 & 1 \end{bmatrix}$. Then,

$$\begin{aligned} A + B &= \begin{bmatrix} 1+0 & 3+-2 \\ 2+6 & 5+1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ 8 & 6 \end{bmatrix} \end{aligned}$$

Theorem (Properties of Matrix Addition)

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4. There exists an $m \times n$ matrix $-A$ such that $A + (-A) = 0$. (existence of an additive inverse).

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Then

$$\begin{aligned} 3A &= \begin{bmatrix} 3(2) & 3(0) & 3(-1) \\ 3(3) & 3(1) & 3(-2) \\ 3(0) & 3(4) & 3(5) \end{bmatrix} \\ &= \begin{bmatrix} 6 & 0 & -3 \\ 9 & 3 & -6 \\ 0 & 12 & 15 \end{bmatrix} \end{aligned}$$

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4. $1A = A$. (existence of a multiplicative identity).

Example

$$2\begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} + 4\begin{bmatrix} -2 & 1 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 6 & 8 \\ 1 & -1 \end{bmatrix} =$$

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Let A and B be $m \times n$ matrices. Simplify the expression

$$2[9(A - B) + 7(2B - A)] - 2[3(2B + A) - 2(A + 3B) - 5(A + B)]$$

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Solution

$$\begin{aligned} & 2[9(A - B) + 7(2B - A)] - 2[3(2B + A) - 2(A + 3B) - 5(A + B)] \\ = \;& 2(9A - 9B + 14B - 7A) - 2(6B + 3A - 2A - 6B - 5A - 5B) \\ = \;& 2(2A + 5B) - 2(-4A - 5B) \\ = \;& 12A + 20B \end{aligned}$$

Matrix Transpose

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Definition

If A is an $m \times n$ matrix, then its transpose, denoted A^T, is the $n \times m$ whose i^{th} row is the i^{th} column of A, $1 \leq i \leq n$; i.e., if $A = [a_{ij}]$, then

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Find the matrix A if $\left(A+3\begin{bmatrix}1 & -1 & 0\\ 1 & 2 & 4\end{bmatrix}\right)^{T}=\begin{bmatrix}2 & 1\\ 0 & 5\\ 3 & 8\end{bmatrix}$.

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Examples

$$\begin{bmatrix} 2 & -3 \\ -3 & 17 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 5 \\ 0 & 2 & 11 \\ 5 & 11 & -3 \end{bmatrix}, \begin{bmatrix} 0 & 2 & 5 & -1 \\ 2 & 1 & -3 & 0 \\ 5 & -3 & 2 & -7 \\ -1 & 0 & -7 & 4 \end{bmatrix}$$

are symmetric matrices, and each is symmetric about its main diagonal.

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We must show that $(A - A^T)^T = -(A - A^T)$.

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Problem

Show that if A is a square matrix, then $A - A^T$ is skew-symmetric.

Solution

We must show that $(A - A^T)^T = -(A - A^T)$. Using the properties of matrix addition, scalar multiplication, and transposition

$$(A - A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T).$$