Math 221: LINEAR ALGEBRA

Chapter 1. Systems of Linear Equations

§1-5. Application to Electrical Networks

Le Chen[1]

Emory University, 2021 Spring

(last updated on 01/12/2023)

[1] Slides are adapted from those by Karen Seyffarth from University of Calgary.

Copyright

Electrical Networks

Linear Algebra with Applications

Lecture Notes

Current Lecture Notes Revision: Version 2018 — Revision B

These lecture notes were originally developed by Karen Seyffarth of the University of Calgary. Edits, additions, and revisions have been made to these notes by the editorial team at Lyryx Learning to accompany their text Linear Algebra with Applications based on W. K. Nicholson's original text.

In addition we recognize the following contributors. All new content contributed is released under the same license as noted below.

- **Ilijas Farah, York University**

BE A CHAMPION OF OER!

Contribute suggestions for improvements, new content, or errata:
A new topic
A new example or problem
A new or better proof to an existing theorem
Any other suggestions to improve the material

Contact Lyryx at info@lyryx.com with your ideas.

License

Electrical Networks

Resistor Networks

Important Symbols:

Resistor:

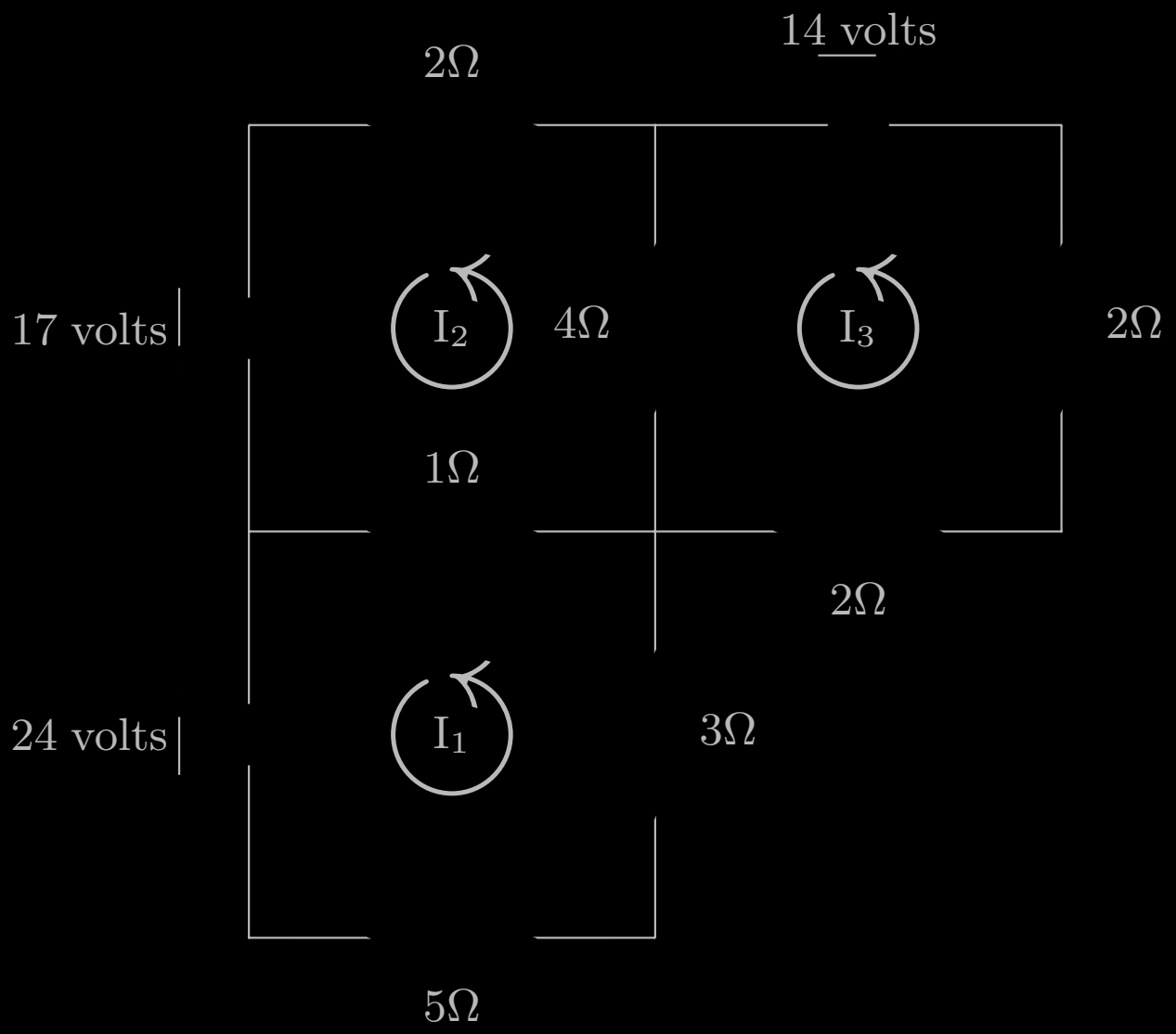
Voltage Source:

Current:

Resitance is measured in ohms, Ω. Voltage is measured in volts, V. Current is measured in amps, A.

Problem

Write an equation for each circuit and solve for each current in the following diagram.



Solution

The equation for the bottom circuit, with current I_1 is given by

$$5I_1 + 3I_1 + I_1 - I_2 = -24$$

The top left circuit, with current I_2 is

$$I_2 - I_1 + 4I_2 - 4I_3 + 2I_2 = 17$$

The top right circuit is

$$4I_3 - 4I_2 + 2I_3 + 2I_3 = -14$$

After simplifying, this system is represented by

$$\left[\begin{array}{rrr|r} 9 & -1 & 0 & -24 \\ -1 & 7 & -4 & 17 \\ 0 & -4 & 8 & -14 \end{array}\right]$$

Solution (continued)

The reduced row-echelon form of this matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & -1 \end{array}\right]$$

This gives values of the currents of

$$\begin{aligned} \mathrm{I}_1 &= -\frac{5}{2} \\ \mathrm{I}_2 &= \frac{3}{2} \\ \mathrm{I}_3 &= -1 \end{aligned}$$