

for  $\forall n > 1, \in \mathcal{N}$ , we have:

$$f(n) = f(n-1) + f(n-2)$$

Suppose  $f(n) = ax^n$ , thus for  $\forall n > 1$ , we have:

$$ax^n = ax^{n-1} + ax^{n-2}$$

$$\Leftrightarrow 0 = x^2 - x - 1$$

$$\Leftrightarrow x = \begin{cases} \frac{1+\sqrt{5}}{2} \\ \frac{1-\sqrt{5}}{2} \end{cases}$$

Thus, we have:

$$f(n) = a \left( \frac{1+\sqrt{5}}{2} \right)^n + b \left( \frac{1-\sqrt{5}}{2} \right)^n$$

As  $f(0) = 0$  and  $f(1) = 1$ , we have:

$$a = b = 1$$

Thus:

$$f(n) = \left( \frac{1+\sqrt{5}}{2} \right)^n + \left( \frac{1-\sqrt{5}}{2} \right)^n$$