Study on Dynamic Programming Model of Emergency Decision *

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Abstract

Based on emergency management and dynamic programming theory, a new quantity emergency decision model was built up. The study primarily focused on emergency decision when there is an emergency event, and through explaining the theory of emergency management, a dynamic programming model used in emergency decision was given. In this model, the signification of each parameter in dynamic programming was defined as factors with the characteristics of emergency management. The approach of establishing the model and the solution procedure were presented. We set up the model by using dynamic programming and two illustrative cases were presented to demonstrate the solution procedure with an interpretation of the EDDP model.

Keywords: emergency management; dynamic programming; operational research; emergency decision

1. Introduction

9.11, SARS and the Indian Ocean tsunami urged us to pay more attention on emergency management. Since accidents occur frequently in today's world, every country has paid more attention to the research of emergency management. The emergency management is a comprehensive subject and there are five main problems in this field, they are: emergency resource management, emergency evacuation management, the constitution of emergency plans, emergency decision support and training [1]. Many theories and technologies had been introduced into emergency management, among which, the method of operational research had been widely used.

During the last three decades, modeling emergency management based on operational research has attracted the interest of researchers and many research papers concerning this have been published in this field in recent years. Rahman derived the method based on linear programming of optimal values for location and allocation (P-median and MCLP) [2]. ReVelle C.S. developed this model and presented a more considerate model (LSCP) [3]. Based on MCLP model, Daskin MS set up the MEXLP model to research maximal expected coverage location problem [4]. Jenkins set up an integer programming model to instruct the constitution of emergency plans [5]. Multiple-objective shortest path problems were considered by Getachew [6]. Time dependency in multiple-objective dynamic programming was presented by Lancaster [7]. To this day, quantitative models which can be used in emergency management are rather rare. Therefore, upon further consideration, building up a quantitative model for emergency is very important.

In this paper, we focus on establishment of a quantitative model to solve emergency decision problems. Aiming at the emergency objective, we divided the emergency process into several emergency periods and through optimizing objective of each period, we got the optimal solution of the whole process. The method of dynamic programming was used in building up the model. This paper is divided into three sections. In section 2, we will introduce some concepts of emergency management and the approach of the founding of EDDP. At the same time, we will give the solution procedure of the model. In section 3, we'll show that the EDDP model can be used in emergency response decisions that cause important effects. The paper is concluded in Section 4 with some conclusions.

2. Emergency decision model based on dynamic programming (EDDP)

The construction of the dynamic programming model of emergency management is based on the analysis of emergency management and Operational Research.

2.1. Emergency management

Emergency management is often characterized by multiple objectives and multiple stages. For an emergency rescue management, it is a multistage and

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circulatory course. An integrated course of emergency rescue management is shown in figure 1.

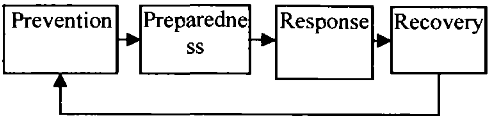

Fig 1: Multistage of emergency rescue management

As shown in figure 1, we divided the whole emergency rescue operation into four emergency periods (EP). They are: prevention, preparedness, response and recovery. The four periods may be intersectant in the real life, while each period has individual emergency objectives (EO). The factors which affect the emergency objectives are emergency decisions (ED) and emergency states (ES). Emergency decisions are the decisions made by the decision maker, who can instruct the emergency rescue operation. They can be selection of available emergency plans, preparedness of emergency resource, designation of action tactics, recovery procedure, etc. Emergency states are the information of the on-sight situation of an emergency event, and they include restrained time, resource, action scheme, etc.

Time limitation is of great importance in emergency management and dividing the emergency time into several phases is an effective means to research emergency management.

In this article, we derived the method of dividing the time into several phases to study problems through a dynamic programming way.

2.2. EDDP

By this way, we generate relationship between the EP, ES, ED and EO as shown in figure 2.

The meanings of the parameters in figure 2 are shown as follows:

k: We divided a emergency problem into several periods, *k* denotes the emergency period.

x_k is the emergency state variable. In an emergency event, the state of the on-sight situation is variational continually. The state variables describe the change of information got from the event. The information of each emergency stage includes restrained time, resource, action scheme, etc.

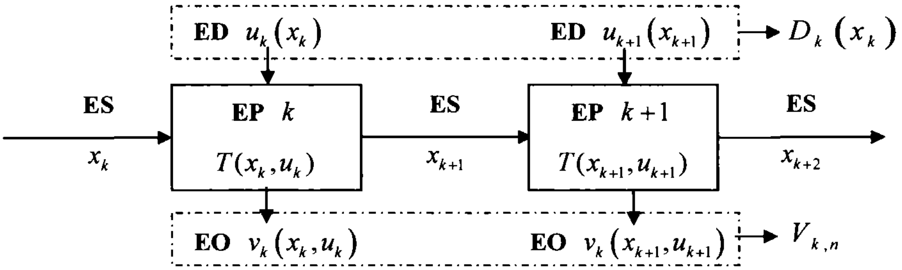

Fig 2: Relationship of multiple-stages emergency management

u_k is the emergency decision variable. From the beginning of the emergency rescue operation, the decision maker (DM) will face a problem of making a decision among several optional plans, such as preparedness of emergency resource, designation of action tactics, recovery procedure, etc.

$D_k(x_k)$ denotes the set of all feasible decisions. For the whole emergency process, all the emergency decision variables should be included. That is: $u_k(x_k) \in D_k(x_k)$

T is the transformation of the emergency states, which describes emergency state variable's transformation between one period and the next period. The transformation has the form: $x_{k+1}=T(x_k, u_k)$.

v_k is the aspiration value variable of emergency objective. For example, the emergency objectives of emergency rescue operation include the control the situation of a disaster, salvage of the victim and evacuation of the personnel. v_k denotes the shortest time, the minimal injured people, the minimal risk, etc.

$V_{k,n}$ is the sum of all aspiration emergency objective value variables. It denotes quantitative index of our emergency management. For the whole emergency process, we can learn that $V_{k,n}=\sum v_{k,n}$ and for each period, it will follow $V_{k+1,n}=V_{k,n}+v_k(x_k, u_k)$.

Based on the analysis of emergency management, the model of EDDP can be presented as follows:

$$EDDP \begin{cases} f_k(x_{k+1}) = \underset{u_k \in D_1(x_1)}{opt} \sum_{k=1}^{n} V_k = \underset{u_k \in D_k(x_k)}{opt} \{v_k(x_{k+1}, u_k) + f_{k-1}(x_k)\} \\ f_o(x_1) = 0 \end{cases} \tag{1}$$

where

opt denotes optimization, it could be *max* or *min* according to the real situation.

$f_0(x_1)=0$ means that in the beginning of an emergency event, no actions has been taken to solve the problem.

Thus the optimization problem of operational research becomes an optimization problem containing the parameters in the equations of emergency objective functions and constraints. It can be used in emergency management.

The algorithm of this model is based on operational research [9]. Suppose there is an emergency event, an emergency decision is to be made by the decision maker, and he can use the EDDP model in the following way:

- Step 1. Confirm the EP: that is to make sure of what the emergency event is going on and the period of the emergency event and to define EP as k.
- Step 2. Confirm the EO: that is to make sure of the acceptable emergency objective, to define EO as v_k, and the sum of EO is $V_{k,n}$. The objective function $f_k(x_k)=opt\ V_{k,n}$.
- Step 3. Confirm the ES: that is to make sure of the emergency states. In this step, the emergency state variable x_k should be defined. For a complex event, there may be several emergency state variables.
- Step 4. Confirm the ED: that is to make sure of the available emergency plans and other emergency decisions and to define the emergency decision variable as u_k
- Step 5. Confirm the transformation (T): From one EP to the next EP, the ES and ED will have a change, we use T to describe the changes of x_k and u_k.
- Step 6. Compute the formula: put the parameters which are given above into the formula and find the optimal decision and solution.

In the next section, we will use two scenarios to illustrate the establishment of the model and the procedure to have an optimal solution.

3. Applications of EDDP

In this section, we will show some real-world applications which will make it possible to follow the model. Its solution is basically put forward by application of the described technique above. The applications presented in this section will include two scenarios:

- rescue resource allocation
- rescue routing problem

3.1. Optimal emergency rescue resource allocation problem in hazard situation

The emergency rescue resources are comprised of manpower, rescue material and self-protect equipments, etc. The optimal allocation of emergency rescue resources is a kind of very important emergency management. Whether in period of emergency prevention or in the emergency rescue operation period, the allocation of emergency rescue resources plays an important role. Now, we will introduce how to use the EDDP method to solve the optimal allocation of emergency resources.

Suppose that S is an available quantity of rescue resources. We wish to find an optimal allocation of these resources for four hazard situations and the available allocation quantities are from i to k for each situation, the risk value are known to us as shown in table 1. Our objective is to minimize the risk value with limited resources. (In this example, $S=12$, $i=2$, $k=4$)

Table 1: The allocation of limited emergency rescue resources among four hazard situations

Risk area / Risk value / Rescue resource	A	B	C	D
2	20	31	27	38
3	16	26	25	35
4	11	21	23	33

We used the procedure given above to solve this problem:

Step 1. We consider that this allocation has four emergency periods (EP), that is $k=4$.

Step 2. For each stage, the EO is v_k,. The objective of this allocation is the minimized risk value, so the objective function is $f_k(x_k)=min\ V_{k,n}$ and $V_{k+1,n}=V_{k,n}+v_k(x_k,u_k)$.

Step 3. There is only one state variable in this example, so x_k is the quantity of the emergency resource and $x_5=12$.

Step 4. The quantities of available decisions are from 2 to 4, so the decision variable u_k will follow: $2\le u_k\le 4$.

Step 5. From this example, we can get that the transformation following this rule: $x_k=x_{k+1}-u_k$.

Step 6. Put these parameters into the formula, we can get:

$$EDDP\begin{cases} f_k(x_{k+1})=\min\limits_{2\le u_k\le 4}\sum\limits_{k=1}^{n}V_k=\min\limits_{2\le u_k\le 4}\{v_k(x_{k+1},u_k)+f_{k-1}(x_k)\} \\ f_0(x_1)=0 \end{cases} \tag{2}$$

The method of calculation is derived from operational research [9] and the results are shown in table 2 to table 5.

Table 2

x_2 \ u_1	$\sum V_1$: 2	3	4	$min \sum V_1$	u_1^*
2	20+0	——	——	20	2
3	20+0	16+0	——	16	3
4	20+0	16+0	11+0	11	4
5	20+0	16+0	11+0	11	4
6	20+0	16+0	11+0	11	4

Table 3

x_3 \ u_2	$\sum V_2$: 2	3	4	$min \sum V_2$	u_2^*
4	31+20	——	——	51	2
5	31+16	26+20	——	46	3
6	31+11	26+16	21+20	42	4
7	31+11	26+11	21+16	37	3,4
8	31+11	26+11	21+11	32	4

Table 4

x_4 \ u_3	$\sum V_3$: 2	3	4	$min \sum V_3$	u_3^*
8	27+42	25+46	23+51	69	2
9	27+37	25+42	23+46	64	2
10	27+32	25+37	23+42	59	2

Table 5

x_5 \ u_4	$\sum V_4$: 2	3	4	$min \sum V_4$	u_4^*
12	38+59	35+64	33+69	97	2

According to the results, we get the optimal allocation of the limited resources among the four risk areas, that is: 4 in A, 4 in B, 2 in C and 2 in D. The total risk values are minimized to 97.

3.2. Optimal rescue routing problem

When there is a disaster, the rescue action should be taken as soon as possible. To minimize and prevent more loss caused by the disaster, the equipment and manpower must be dispatched to the disaster location as quickly as possible. Then, the decision maker will face the problem of rescue routing.

Let's assume that a fire happened in a building. The fire engines and firemen must arrive at the disaster location as soon as possible. As time is the most important factor for rescue action, how to arrive there in the shortest time is in consideration.

We assume such a routing problem with Markovian property. We wish to use the shortest time to arrive E from A. The time used on each road is shown in figure 3. We use the EDDP to solve this problem. The process is the same as what is described in 3.1, and we get the formula as follows:

$$EDDP\begin{cases} f_k(x_{k+1}) = \min\limits_{2\le u_k\le 3} \sum\limits_{k=1}^{n} V_k = \min\limits_{2\le u_k\le 3} \{v_k(x_{k+1}, u_k) + f_{k-1}(x_k)\} \\ f_n(x_1) = 0 \end{cases} \tag{3}$$

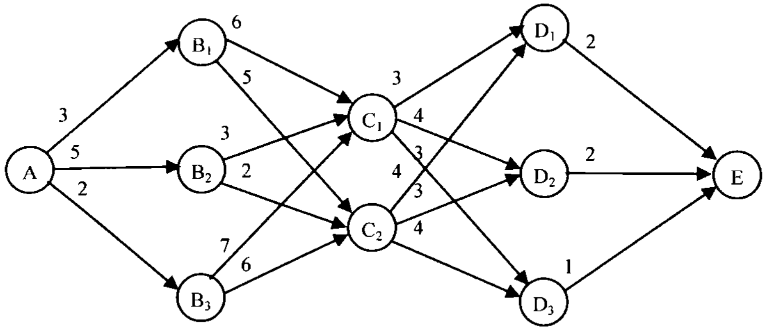

Fig 3: The route and needed time to pass

The results gotten from WinQSB are shown in figure 4.

04-27-2007 15:18:43	Stage	From Input State	To Output State	Distance	Distance to Node10	Status
1	1	Node1	Node3	5	12	Optimal
2	2	Node2	Node5	6	10	
3	2	Node3	Node5	3	7	Optimal
4	2	Node4	Node5	7	11	
5	3	Node5	Node9	3	4	Optimal
6	3	Node6	Node9	4	5	
7	4	Node7	Node10	2	2	
8	4	Node8	Node10	2	2	
9	4	Node9	Node10	1	1	Optimal
	From Node1	To Node10	Minimum	Distance =	12	CPU = 0

Fig 4: The results based on WinQSB

So the optimal route is $A \to B_2 \to C_2 \to D_2 \to E$. The optimal total spent time is minimized to 12.

4. Conclusions

In this paper an emergency decisions model (EDDP) that can be used in emergency management was introduced to a decision-maker. This model was set up based on the theory of emergency management and Operational Research. The parameters in this model characterize emergency management and such concepts as emergency period, emergency state and emergency objective were defined. The algorithm of the model is also introduced and in this way, the decision maker can deal with the complex emergency management problems through mathematics. At the same time, we used this model to analyze and calculate two typical examples of emergency management which are known to us as optimal resource allocation and optimal routing problems.

The method discussed in the papers is not fully mature yet and further research work for improvement still need to be carried out.

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