

# **Optimal Bundling of Technological Products with Network Externality**

Ashutosh Prasad, R. Venkatesh, Vijay Mahajan\*

July 2010

Forthcoming in *Management Science*

\*Ashutosh Prasad is Associate Professor of Marketing, School of Management, The University of Texas at Dallas, Richardson, TX 75080 ([aprasad@utdallas.edu](mailto:aprasad@utdallas.edu); 972-883-2027). R. Venkatesh is Associate Professor of Marketing, Katz Graduate School of Business, University of Pittsburgh, Pittsburgh, PA 15260 ([rvenkat@katz.pitt.edu](mailto:rvenkat@katz.pitt.edu); 412-648-1725). Vijay Mahajan is John P. Harbin Centennial Chair in Business, McCombs School of Business, The University of Texas at Austin, Austin, TX 78712 ([vijay.mahajan@mcombs.utexas.edu](mailto:vijay.mahajan@mcombs.utexas.edu); 512-471-0840). The authors thank Department Editor Preyas Desai, the Associate Editor, three anonymous reviewers, colleagues Dennis Galletta, Esther Gal-Or, Chris Kemerer, Prakash Mirchandani, Ram Rao and Brian Ratchford, and seminar participants at University College London and the 2010 ALIO-INFORMS conference in Buenos Aires for their helpful comments. They further thank Oguz Alagoz, Wei Chang and Tuba Pinar Yildirim for help with the analysis. R. Venkatesh is the corresponding author.

## **Optimal Bundling of Technological Products with Network Externality**

**Abstract:** For many high-tech and Internet-related products, utility to consumers depends in part on the size of the user base, a phenomenon called network externality. A firm with a portfolio of these and other products – that are often asymmetric in their degree of network externality or marginal cost – may have to look beyond the traditional strategies of pure components, pure bundling and mixed bundling. One such strategic alternative in a two-product case would be a so-called mixed bundling-1 under which the bundle and product 1 are offered, but the other product can only be purchased in a bundled form. The purpose of this study is to compare and contrast the impact of such a/symmetry in (direct) network externality and cost on the choice of bundling strategies. We model a monopolist firm that has a product in each of two categories and faces heterogeneous consumers. Results suggest that pure bundling is more profitable when both products have low marginal costs or high network externality whereas pure components or mixed bundling-1 is more profitable when the products diverge in their costs and network externality (e.g., only one product has network externality). Traditional mixed bundling is optimal in other instances.

**Keywords:** Bundling; Network Externality; Pricing; High Technology; eCommerce

## 1. Introduction

When the seller of two or more products sells these products separately, the strategy is called *pure components* or PC. Sometimes (e.g., when marginal costs are negligible) it is more profitable to sell the products together as a bundle, a strategy called *pure bundling* or PB (Bakos and Brynjolfsson 1999). A more nuanced strategy called *mixed bundling* or MB, occurs when the products and the bundle are both sold, the premium priced individual products are targeted to consumers who value only those products, while the bundle is targeted to consumers who value all products. Researchers have compared the optimality of these three strategies under different product-market conditions (e.g., Schmalensee 1984). We re-examine the issue in the context of high technology products, such as offerings of assorted software and digital goods, computers and peripherals, and cell phone plans with cable services.

Specifically, in these cases, one or both products in the bundle may have network externality. That is, for these products, utility depends on the number of other users who are in the same network (Katz and Shapiro 1985). The cause of the network externality can be direct (e.g., for products such as phones, fax, data networks, online communities wireless calling plans, and music-sharing programs, clearly a larger network size is useful) or indirect (e.g., for products such as television, video players, game consoles a larger user base leads to greater content availability and thus increases consumer utility). We focus on direct network externality (similar to Farrell and Saloner 1986, among others).

Our study examines this interface between bundling and network externality. More specifically, we examine how asymmetry on two key factors – the degree of network externality and marginal cost of the products – impact their optimal bundling decision. Although practical examples abound of products with diverse types of asymmetry in network effects or costs, the literature is silent on how the optimal bundling decision should differ across these contexts.

We find that when one or more products have asymmetry in costs and network externality, mixed bundling needs to be further categorized. Thus for two products labeled 1 and 2, three mixed bundling strategies are feasible: The mixed bundle offerings could be  $\{1\}$ ,  $\{2\}$ , and  $\{1,2\}$  priced at  $P_1$ ,  $P_2$  and  $P_{12}$  respectively. We will call this strategy MB-12 and we distinguish it from the mixed bundling strategies MB-1, where only  $\{1\}$  and  $\{1,2\}$  are offered, and MB-2, where only  $\{2\}$  and  $\{1,2\}$  are offered. MB-1 and MB-2 remain obscured in a typical, symmetric component analysis but emerge here because the components are allowed to be asymmetric. Counting PC and PB, we thus have to compare the profitability of five product-line and pricing strategies.

While there are numerous real world manifestations of PC, PB and MB-12, the sub-types of mixed bundling are also prevalent. For example, Playchess.com is a popular online chess service that hosts over 200,000 online games and 5000 players per day ([www.playchess.com](http://www.playchess.com)). It is offered at €29.90

for a year's subscription. The service has network externality because its utility depends in part on the number of other subscribers one can potentially play with. The same seller, ChessBase GmbH, is the dominant resource for high quality chess engines, e.g., Fritz 12 software, used for individual play and analysis (i.e., has standalone value but no network externality). Fritz 12 is listed for €49.90, but as can be seen from the Fritz 12 product pages, both current and archived, this is really a bundle because a Playchess.com subscription is automatically included. Fritz 12 independent of Playchess.com is not offered. We will revisit the example in the Conclusion section.

We develop a model that has a monopolist offering two products with particular levels of marginal costs and degrees of network externality.<sup>1</sup> The reservation price for a product consists of its intrinsic valuation and its network externality. Reservation prices are heterogeneous across consumers. Given this we address:

- i. Under what product market conditions (degree of a/symmetry in network externality and/or marginal costs) should the seller prefer pure components over pure bundling (or vice versa)?
- ii. If the three types of mixed bundling are also available, how does the optimal strategy change based on the level of a/symmetry in the degree of network externality and/or marginal costs?

The relevant baseline is of two products without network externality and with symmetric marginal costs (e.g., Schmalensee 1984). We rely on a combination of analytical methods and numerical simulation.

Comparing pure components and pure bundling, we find that, roughly speaking, the former is more profitable when marginal costs of the products are symmetric and network externality is asymmetric, or when asymmetry in marginal costs of the products is high. Pure bundling is mostly optimal otherwise. When all five strategies are compared, pure bundling is favored by low, symmetric costs or high, symmetric network externality. When only one product offers network externality and has lower marginal cost, pure components or MB-1 is optimal. Traditional mixed bundling (MB-12) is optimal in most other instances. Exploring further the newcomer strategy MB-1, we find that MB-1 should especially be considered when product 1 is costless and has network externality. Then MB-1 offers product 1 to customers who primarily value 1 and the bundle to push product 1 even to those who primarily value 2. This leads to larger sales and network benefits from 1. MB-12 is suboptimal (and degenerates to MB-1) in this case because the availability of product 2 in its standalone form will reduce the demand of the bundle, thereby reducing 1's network benefit. Pure bundling is not optimal here either

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<sup>1</sup> In the above example, ChessBase's offerings, including Playchess.com and Fritz, are seen as staples for genuinely interested chess players. Other large online chess forums are FICS ([www.freechess.org](http://www.freechess.org)) which is always free, hence nonstrategic, and ICC ([www.chessclub.com](http://www.chessclub.com)) which is not free, but whose past prices (obtained from the archives at <http://web.archive.org>) show no response to Playchess.com's entry or pricing; hence it may also be treated as nonstrategic. Thus, ChessBase appears to have significant market power. That said, the examples are primarily to suggest that sellers may rely on alternatives to PC, PB and MB-12 under a/symmetry in network externality or costs. As our study is stylized and analytical in nature, our examples are for motivational purposes only.

as customers who value 1 alone may forego purchase entirely, again reducing its network benefit and the seller's profits.

We next review the literature and positioning of this study (§2). In §3, we present the model. In §4, we compare first pure components and pure bundling, then introduce mixed bundling, and present propositions and results on the optimal strategies. Section 5 provides the discussion of the theoretical and managerial implications of the study, limitations and future research directions.

## **2. Literature and Positioning**

Bundling and network externality are well-researched topics in economics and marketing. While a comprehensive review of either is beyond our scope, suggested resources are the website of Nicholas Economides ([www.stern.nyu.edu/networks/site.html](http://www.stern.nyu.edu/networks/site.html)) for an extensive bibliography on the “Economics of Networks,” and Venkatesh and Mahajan (2009) for a recent review of the bundling literature.

In the bundling literature, in the absence of network externality, pure bundling is shown to be more profitable than pure components if both products have low marginal costs, otherwise pure components is more profitable (e.g., Schmalensee 1984). Due to its flexibility, mixed bundling typically trumps pure components and pure bundling.

Several bundling articles focus only on symmetric costs (e.g., Ghosh and Balachander 2007; Venkatesh and Kamakura 2003). In the context of asymmetry in costs, studies sometimes let the costs of the products be different but do not link the degree of cost asymmetry to optimal bundling strategy (e.g., Chen 1997). Schmalensee (1984) is the only article to our knowledge to link asymmetry to bundling strategy. It finds that pure bundling is weakened vis-à-vis pure components under departures from symmetry. However, as Schmalensee's asymmetry analysis is tied to a composite parameter of which cost is one element, it is difficult to tease out the effect of cost asymmetry in his article.

Articles on network externality have examined implications of licensing (e.g., Economides 1996), compatibility and standardization (e.g., Katz and Shapiro 1985) and preannouncement behavior (e.g., Farrell and Saloner 1986), among others. We focus on a monopolist, so compatibility among competing systems and licensing are assumed away. Our intent is to examine how bundling strategies are impacted under direct network externality.

We focus the remainder of this section on the limited research at the *interface* of bundling and network externality. We will seek to highlight the distinctive aspects of the current study.

Bakos and Brynjolfsson (1999, p. 1621), show the optimality of pure bundling for a monopolist selling a large number of independently valued information goods with no marginal costs or network externality, and conjecture that the result might extend to situations with network externality. The present issue, the impact of asymmetry in network externality and marginal costs, is unrelated to their analysis.

In the context of two-sided markets with network externality (e.g., Adobe Acrobat writer and reader), Gallaugh and Wang (2002) find that a firm with a higher market share for one component is able to leverage that advantage for the complementary component (under pure components).

Articles that consider the issues of bundling and compatibility (indirectly related to network externality) include Denicolo (2000) and Matutes and Regibeau (1992). Denicolo shows that in a competition between generalist and specialist firms, where the generalist produces both products and specialists produce one product or the other, the generalist will pursue pure bundling when one product is less differentiated than the other. Matutes and Regibeau show that in a two-product duopoly with linear demand for each product, pure components is superior to pure bundling when the firms offer compatible products; otherwise, pure bundling prevails. With mixed bundling available, the optimal strategy depends on the consumers' valuation of their "ideal bundle"; pure components is better when consumers strongly prefer their ideal bundle. However, these articles do not consider network externality per se and the impact of asymmetry in costs and network externality is not examined.

### 3. Model

Our seller is a profit maximizing monopolist (as in McAfee et al. 1989 and Schmalensee 1984) and offers products 1 and 2, whose marginal costs are  $c_1$  and  $c_2$  respectively. We denote component prices as  $P_1$  and  $P_2$  for products 1 and 2 respectively, and the bundle price as  $P_{12}$ .

Prospective consumers are surplus maximizers with a demand for at most one unit of each product. Total market potential is normalized to 1. Each consumer's reservation price for a product has two parts. The first, the intrinsic valuation for product  $i$  ( $i = 1$  or  $2$ ), is denoted  $R_{ki}$  for customer  $k$ . For example, this could be the value of a personal computer without Internet connection. Following Carbajo et al. (1990), Matutes and Regibeau (1992) and Nalebuff (2004), among others, we assume that  $(R_{1k}, R_{2k})$  are uniformly distributed over the unit square  $[0,1] \times [0,1]$  to represent customer heterogeneity.

The second part of consumer  $k$ 's reservation price is the direct network externality. We specify it as  $n_i D_i$ , i.e., as a linear function of the market demand  $D_i$  for product  $i$ . Parameter  $n_i$  ( $n_i \geq 0$ ) captures the degree of network externality for product  $i$ . This linear form and product-specific network parameter assumptions are commonly used (e.g., Economides 2000; Padmanabhan, Rajiv, and Srinivasan 1997). Thus the reservation price for each consumer is the sum of the consumer-specific intrinsic valuation and the product-specific network externality. For example, consumer  $k$ 's reservation price  $RP_i$  for product  $i$ , under pure components, is given by  $R_{ki} + n_i D_i$ , and under pure bundling, is given by  $R_{ki} + n_i D_{12}$  where  $D_{12}$

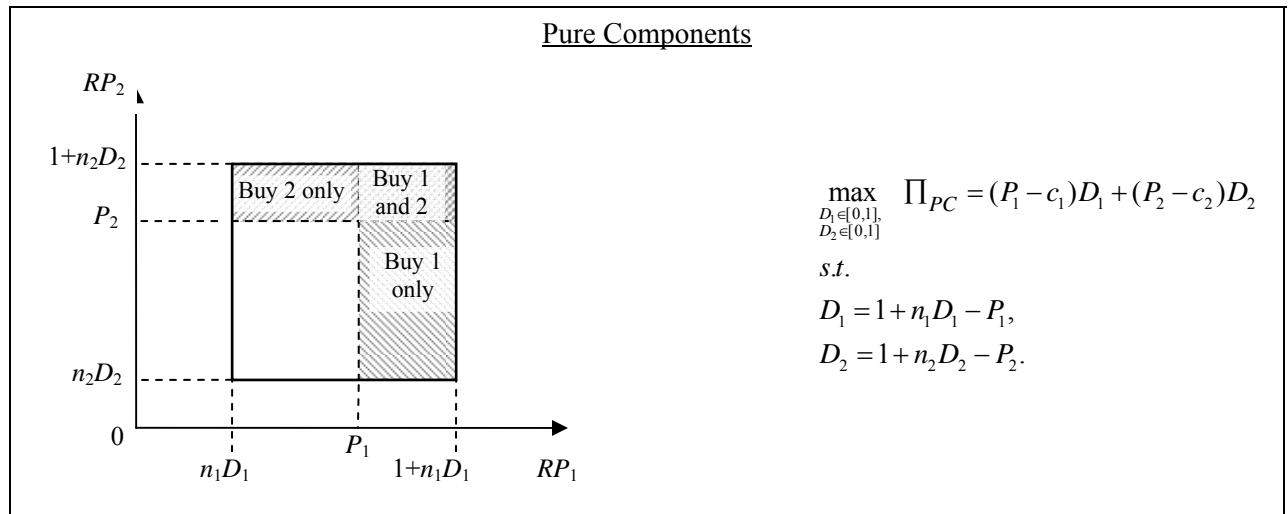
is the demand for the bundle. This additive formulation is consistent with studies such as Choi (1994, p. 169), Economides (2000, p. 4) and Farrell and Saloner (1986, p. 944).<sup>2</sup>

A final question is, what do customers know about the demand  $D_i$  when they make their decisions? We assume, as do Economides (2000), Haruvy and Prasad (1998) and Katz and Shapiro (1986), that consumers have perfect foresight (i.e., rational expectations) about equilibrium demand.

For presentation, it is helpful to scale marginal cost  $c_i$  and degree of network externality  $n_i$  in relation to the maximum intrinsic valuation for product  $i$ . Thus, we assume  $c_i, n_i \in [0, 1]$  for  $i \in \{1, 2\}$ .

From this description of the market, Table 1 provides the details of the optimization problem for PB, PC, MB-12 and MB-1 (and MB-2 is analogous, hence omitted). The graphs show how demand is obtained from the prices, following Adams and Yellen (1976). Worth noting is that demand equations are recursive, higher demand leading to greater utility for consumers yielding even higher demand.<sup>3</sup>

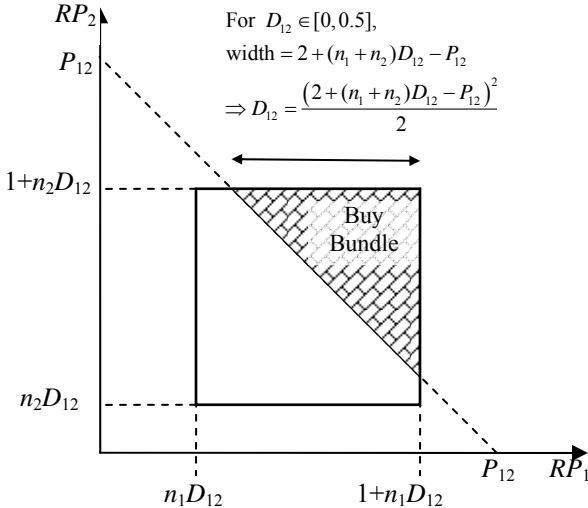
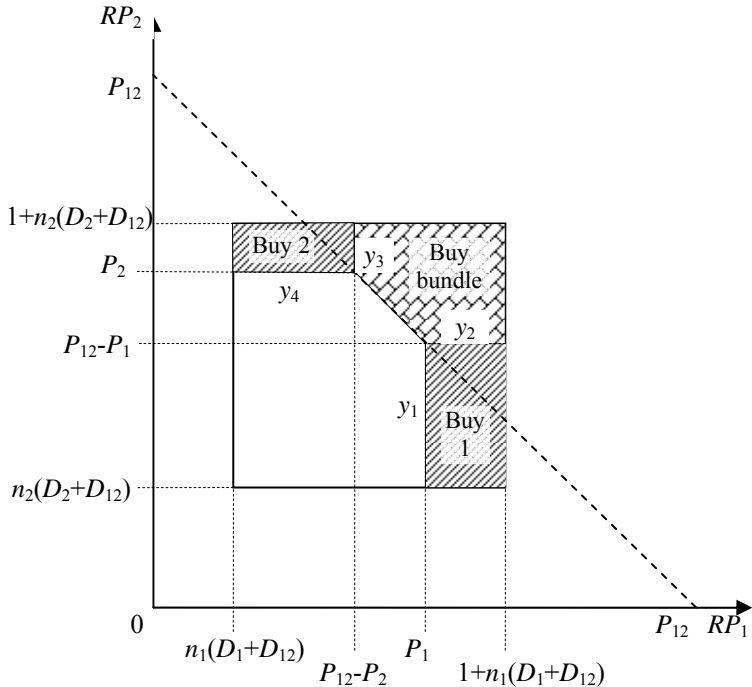
**Table 1: Demand Derivations and the Maximization Problems for PB, PC, MB-12 and MB-1**



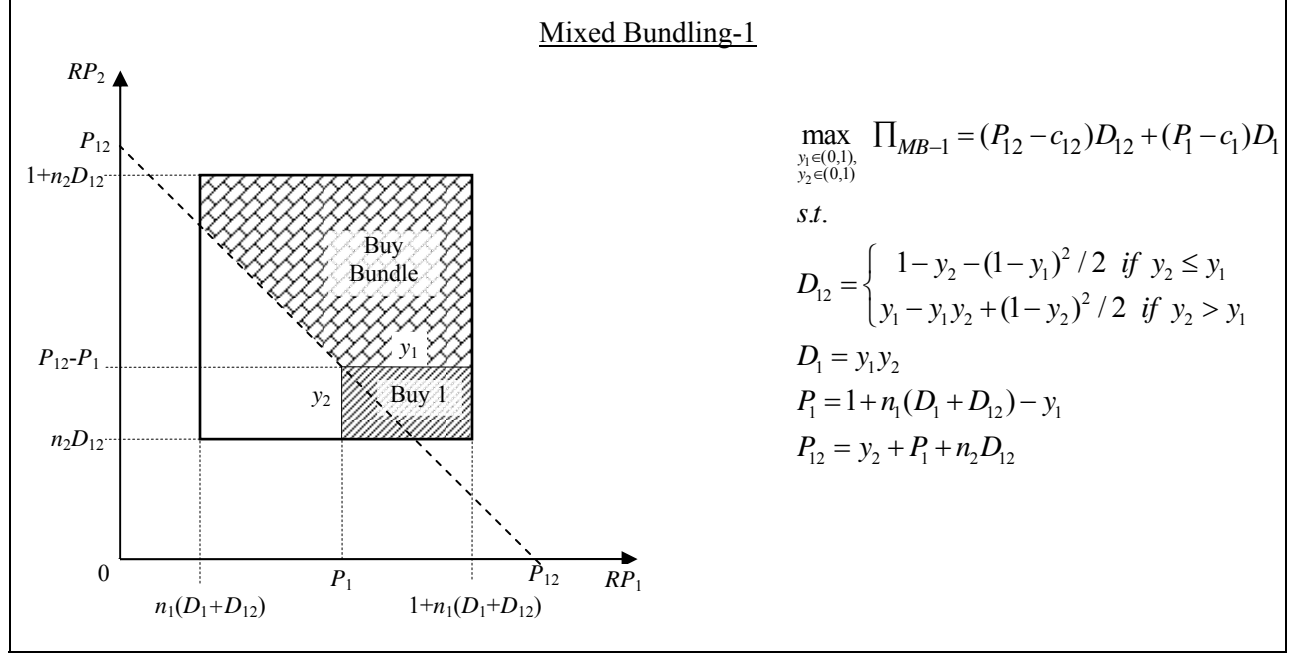
<sup>2</sup> An alternative formulation, albeit with less literature support, is to model reservation price as  $R_{ki} * n_i D_i$ ; i.e., the network benefit is larger for a consumer with a higher intrinsic valuation. Analysis with this formulation was also conducted by the authors. It did not qualitatively alter PB-PC comparisons whereas mixed bundling computations become significantly more complex.

<sup>3</sup> Network externality creates interactions between two otherwise independent markets for product 1 and 2. To see this, consider the bundle offered as part of pure bundling or MB-1 (where 1 has network externality). The bundle in these instances causes a consumer A who might otherwise just buy product 2 under MB-12 or PC to now buy the bundle; in doing so, it increases the perceived network benefit of product 1 to a potential consumer B who might not have otherwise purchased 1 in standalone or bundled form, inducing that consumer to buy product 1, and so on.

Table 1 (continued)

<p style="text-align: center;"><u>Pure Bundling</u></p>  <p>For <math>D_{12} \in [0, 0.5]</math>,  width <math>= 2 + (n_1 + n_2)D_{12} - P_{12}</math>  <math>\Rightarrow D_{12} = \frac{(2 + (n_1 + n_2)D_{12} - P_{12})^2}{2}</math></p> <p style="text-align: right;"> <math>\max_{D_{12} \in [0, 1]} \Pi_{PB} = (P_{12} - (c_1 + c_2))D_{12}</math>  s.t.  <math>D_{12} = \begin{cases} \frac{(2 + (n_1 + n_2)D_{12} - P_{12})^2}{2}, &amp; D_{12} \in [0, \frac{1}{2}], \\ 1 - \frac{(P_{12} - (n_1 + n_2)D_{12})^2}{2}, &amp; D_{12} \in [\frac{1}{2}, 1]. \end{cases}</math> </p>	<p style="text-align: center;"><u>Mixed Bundling-12</u></p>  <p style="text-align: right;"> <math>\max_{\substack{y_1 \in (0, 1), \\ y_2 \in (0, 1), \\ y_3 \in (0, 1 - y_1)}} (P_{12} - c_{12})D_{12} + \sum_{i \in \{1, 2\}} (P_i - c_i)D_i</math>  s.t.  <math>y_4 = y_1 + y_3 - y_2, D_1 = y_1 y_2, D_2 = y_3 y_4</math>  <math>D_{12} = (1 - y_1)(1 - y_4) - (1 - y_1 - y_3)^2 / 2</math>  <math>P_1 = 1 + n_1(D_1 + D_{12}) - y_2</math>  <math>P_2 = 1 + n_2(D_2 + D_{12}) - y_3</math>  <math>P_{12} = y_1 + P_1 + n_2(D_2 + D_{12})</math> </p>
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In what follows, we will compare the profits between the strategies of PC and PB, and the mixed bundling strategies MB-12, MB-1 and MB-2. Our objective is to determine the profit maximizing strategy under alternative conditions of marginal costs and network externality. For some of the maximization problems, such as for PB and PC, a complete analytical solution can be obtained; in case of mixed bundling, resort to numerical methods may be needed where a grid search is used to obtain the global maximum. Indeed, bundling articles often resort to numerical analysis for mixed bundling even without the complications of network externality (e.g., Schmalensee 1984; Venkatesh and Kamakura 2003). However, we derive some analytical insights by using the Lagrange multiplier approach to the maximization problems in MB-12 and MB-1.

#### 4. Analysis

We begin by comparing the profitability of pure components and pure bundling in §4.1, and examine the three mixed bundling strategies in §4.2. Such sequencing allows for better understanding of underlying mechanisms and has precedence in the literature. Often only the pure strategies can be examined analytically, a point well recognized in the literature (see Wilson 1993).

##### 4.1. Pure Components vs. Pure Bundling

Solving the maximization problems for PC and PB yields Propositions 1 and 2. (Also see Appendix.)

*Proposition 1: For Pure Components, the optimal prices and demands are as follows:*

(a) If  $2n_i - 1 - c_i \geq 0$  then  $P_i = n_i$  and  $D_i = 1$ ,  $\forall i \in \{1, 2\}$ .

(b) If  $2n_i - 1 - c_i < 0$  then  $P_i = \frac{1+c_i}{2}$  and  $D_i = \frac{1-c_i}{2(1-n_i)}$ ,  $\forall i \in \{1, 2\}$ .

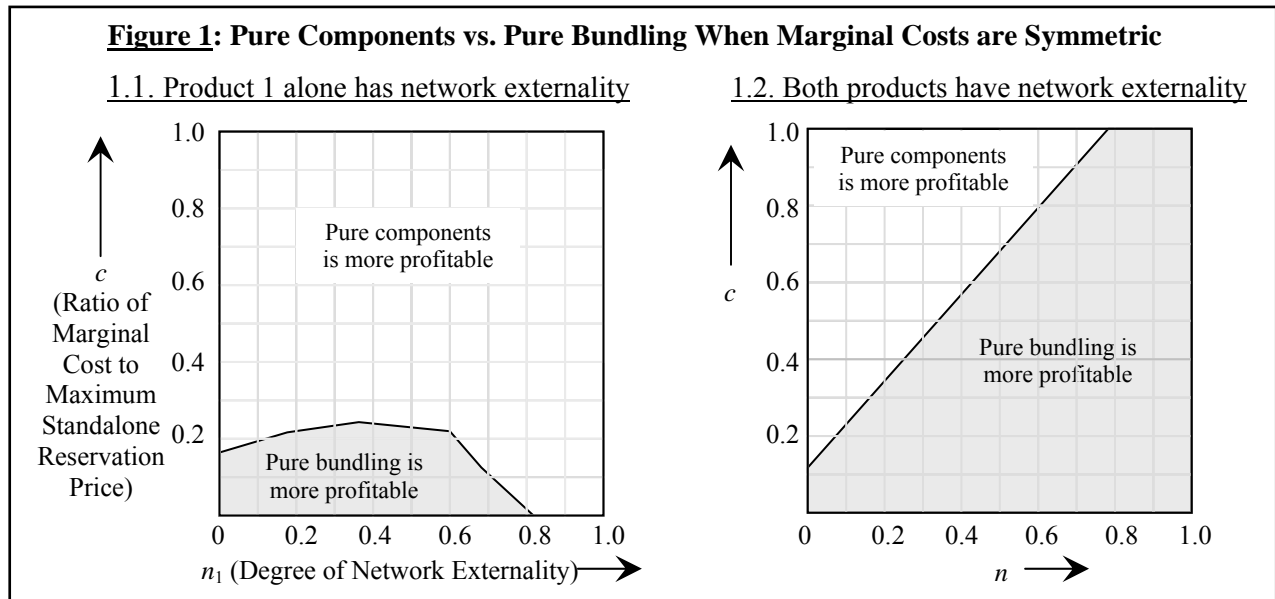
If there is no network externality, which can be obtained by setting  $n_i$  to zero, part (b) applies. Some consumers are excluded and deadweight loss occurs. However, if the network externality is sufficiently high, so that part (a) applies, then the entire market is served. Even low reservation price customers become valuable to the firm because of their contribution to network externality.

**Proposition 2:** For Pure Bundling, the optimal prices and demands are as follows:

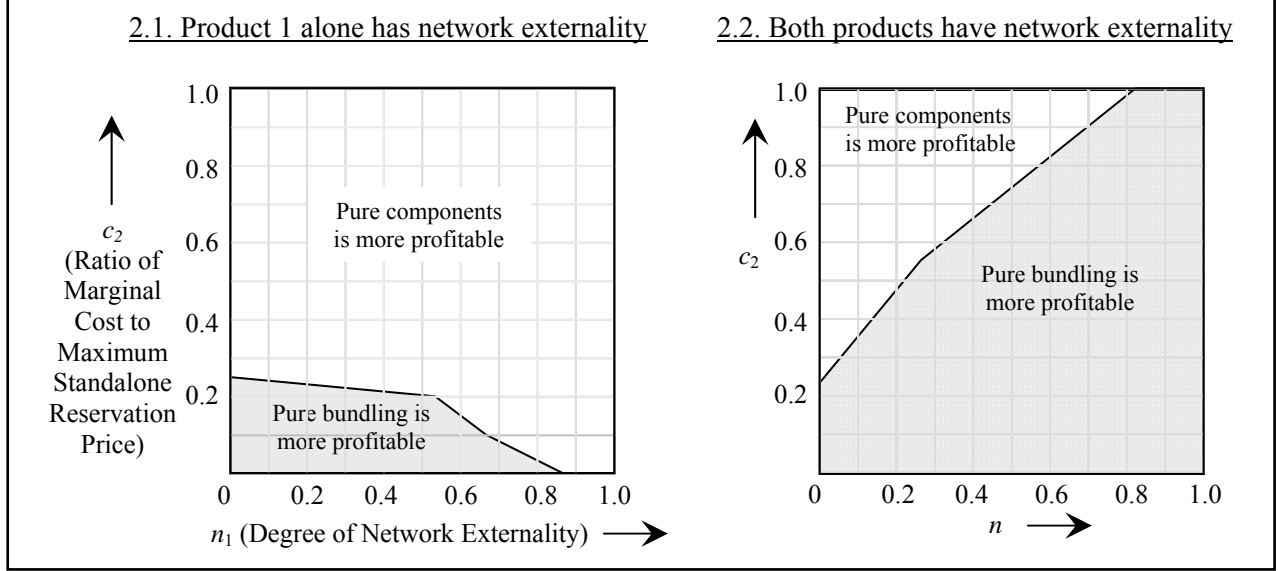
- (a) If  $(n_1 + n_2 + 0.5) < (c_1 + c_2)$ , then  $D_{12} \in [0, 0.5]$ .  $P_{12} = \frac{2 + 2(c_1 + c_2) - (n_1 + n_2)D_{12}}{3}$  and  $D_{12} = \left( \frac{2\sqrt{2}(2 - c_1 - c_2)}{3 + \sqrt{9 - 16(n_1 + n_2)(2 - c_1 - c_2)}} \right)^2$ .
- (b) If  $(n_1 + n_2 + 0.5) = (c_1 + c_2)$ , then  $P_{12} = 1 + \frac{(n_1 + n_2)}{2}$  and  $D_{12} = 0.5$ .
- (c) If  $(n_1 + n_2 + 0.5) > (c_1 + c_2)$ , then  $P_{12} = (n_1 + n_2)D_{12} + \sqrt{2}\sqrt{1 - D_{12}}$  and  $D_{12}$  is the solution of  $2\sqrt{2}(n_1 + n_2)(1 - D_{12})^{3/2} - 3(1 - D_{12}) - \sqrt{2}(2(n_1 + n_2) - (c_1 + c_2))(1 - D_{12})^{1/2} + 1 = 0$  which exists and is unique in  $D_{12} \in (0.5, 1]$ .

Using these results, we can compare the profits of pure components and pure bundling for any set of marginal cost and network externality parameters to yield normative guidelines. Instead of using lengthy formulae, the results are best seen by plotting whether PB or PC is more profitable for different values of the parameter space  $(c_1, c_2, n_1, n_2)$ . This route is also taken by Bakos and Brynjolfsson (1999, p. 1617) and Salinger (1995, p. 95) for example.

Figures 1 and 2 compare pure components and pure bundling. When the two products have symmetric marginal costs, the vertical axis is labeled  $c$  ( $= c_1 = c_2$ ). Likewise the horizontal axis is labeled  $n$  ( $= n_1 = n_2$ ) when network externality of the products is symmetric. Otherwise the suffixes are shown.



**Figure 2: Pure Components vs. Pure Bundling when Product 1 is Costless**



Based on the profit comparison in the graphs the following observations can be made.

*Result 1(A). When both products have network externality, PB is more profitable than PC when approximately  $c < n + 0.1$ . The domain of PB is larger if product 1 is costless.*

*Result 1(B). When only product 1 has network externality, PB is more profitable only when marginal cost (for product 2 or both products) is about 0.2 or less and the degree of externality (for product 1 or both products) is about 0.8 or less. Otherwise, PC is more profitable.*

We now proceed to examine the intuition behind these results. The profits of both PB and PC (and other strategies) increase when  $n_1$  or  $n_2$  increases or  $c_1$  or  $c_2$  decreases. Thus we are interested in how the relative profitability is affected by parameter changes. A starting point is that in the absence of network externality, pure bundling is more profitable if both products have low marginal costs, because then the bundle price can be lowered to capture sales from consumers who have low valuations for one product but high valuation for the other, or moderate valuations for both products. The increase in demand compensates for the lower bundle price. Intuitively, the presence of network externality should enhance this result, i.e., if PB is more profitable due to high demand in the absence of network externality, the high demand should be rewarded even more by its presence. Indeed, Result 1(A) shows clear support for the increase in optimality of PB with increase in the network parameter. However, Result 1(B), where only one product has network externality, shows a more complicated pattern.

The difference in Results 1A and 1B can be understood by considering the *effective costs* of the products. Specifically, an increase in the network externality of a product can be thought to decrease its effective marginal cost since all consumers valuations are increased while the actual marginal cost remains unchanged. This effect explains the tradeoff  $c < n + 0.1$  and hence the Result 1(A). Consistent

with the notion that lower costs support PB, the domain of optimality of PB is slightly larger when  $c_1$  is restricted to be zero in Figure 2.2 than in Figure 1.2.

The same effect is at work in Result 1(B) with the difference that because only product 1 has network externality, only its effective marginal cost is reduced. We thus have a relatively stronger product 1 with network externality effect and lower effective marginal cost, compared to product 2. The difference becomes more acute as the network parameter increases.

In this case, under PC, product 2 contributes the same amount of profit for any  $n_1$  value. However, because PC uncouples the products' pricing, product 1 can be priced quite high. On the other hand, under PB, demand and price increase slowly with an increase in  $n_1$  because the bundle must be priced to sell the (relatively) weak product 2. When  $n_1$  is sufficiently high, the result in Proposition 1(a) occurs, which is that the PC price  $P_1$  increases without a corresponding reduction in demand  $D_1$  because the whole market is being served. Thus, at high  $n_1$  the profitability of the PC strategy accelerates and ultimately it overtakes pure bundling.

To follow the above description, Table 2 shows the demand, prices and profits for Figure 2.1 at values of  $c_1=0$ ,  $n_2=0$ ,  $c_2=0.1$ , and gradually increasing  $n_1$ .

**Table 2: Optimal Demands ( $D$ 's) and Prices ( $P$ 's) for PB and PC when  $c_1=0$ ,  $n_2=0$ ,  $c_2=0.1$**

$n_1$	Pure Components					Pure Bundling		
	$D_1$	$D_2$	$P_1$	$P_2$	$Profit_{PC}$	$D_{12}$	$P_{12}$	$Profit_{PB}$
0.00	0.50	0.45	0.50	0.55	0.453	0.638	0.851	0.479
0.15	0.588	0.45	0.50	0.55	0.479	0.695	0.885	0.546
0.30	0.714	0.45	0.50	0.55	0.56	0.749	0.993	0.624
0.45	0.909	0.45	0.50	0.55	0.657	0.798	0.995	0.714
0.60	1.0	0.45	0.60	0.55	0.802	0.838	1.072	0.815
0.75	1.0	0.45	0.75	0.55	0.953	0.871	1.161	0.924
0.90	1.0	0.45	0.90	0.55	1.103	0.896	1.262	1.042

One can generalize that when asymmetry in network externality is pronounced, pure bundling's appeal is diminished. Uncoupling the products (i.e., pure components) is desirable under such asymmetry.

#### 4.2. Analysis of Mixed Bundling

Mixed bundling is an effective price discrimination device, combining the advantages of pure components and pure bundling. However, despite the seeming generality of mixed bundling, it can take the form at optimality of MB-1, MB-2 or a pure strategy, and we say that MB-12 has converged to one of those strategies. Looking at the maximization problem for MB-12, the price and demand constraints can all be substituted into the objective function for numerical optimization and one can check ex post whether  $y_1$ ,

$y_2, y_3$  are in the interior. If they are not, it means the MB-12 solution has converged to another strategy, for example, if  $y_3=0$ , to MB-1.

Thus it is useful to know under what parameter values a corner solution occurs, because these parameters values define the profit indifference boundary between two strategies and thereby delineate their domains of optimality. Notably, we find that it is possible to obtain many of the boundaries in the mixed bundling phase diagrams analytically because, while the explicit solution of the MB-12 and MB- $i$  problems is intractable, if we only want to find the corner solutions (e.g., parameters when  $D_1=D_2=0$  for MB-12, which is the boundary between MB-12 and PB), the analysis is simpler. We use the Lagrange-multiplier method (see Chiang 1984, Ch. 12) to obtain the optimal strategy analytically. This approach could be useful for other bundling studies as well. The following analytical results were obtained (and the proofs are available from the authors):

*Proposition 3: The following results hold in the  $(c_1, c_2, n_1, n_2)$  parameter space (see figures in Table 1):*

(a) *There is no common boundary between MB-12 and PC, i.e., MB-12 does not converge to PC for any parameter values.*

(b) *The boundary between MB-12 and PB is given by the pair of conditions  $4n_1 - 3c_1 = 0$  and*

*$4n_2 - 3c_2 = 0$ . The boundary between MB-1 and PB is*

$$\frac{n_1 c_2}{n_2} + \sqrt{2} \sqrt{1 - c_2 / 2n_2} - c_1 - \frac{c_2}{2\sqrt{2n_2} \sqrt{1 - c_2 / 2n_2}} = 0.$$

(c) *The boundary between MB-12 and MB-1 when  $n_2=0$  is given by the simultaneous equations*

$$\frac{8n_1}{3} - \frac{2n_1 c_2}{3} - c_1 - 2 + \sqrt{c_2^2 + 6(1 - c_2)} - n_1 \left( 2 + \frac{2c_2}{3} \right) \left( \frac{-c_2 + \sqrt{c_2^2 + 6(1 - c_2)}}{3} \right) = 0, \text{ and}$$

$$\delta^3 - \frac{2}{3} \delta^2 - \frac{2(3 + c_2)}{9} \delta + \frac{4}{9} = 0 \text{ where } \delta = \frac{-3c_1 + 4n_1}{2c_2 n_1}.$$

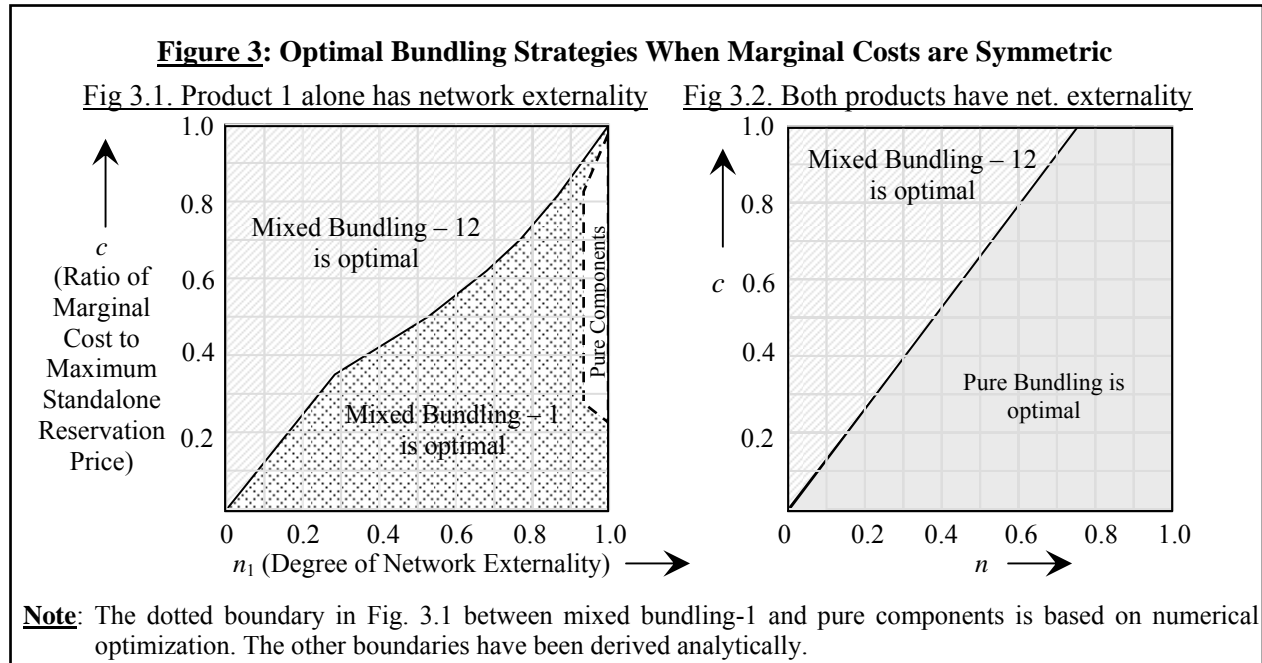
McAfee et al. (1989) showed that mixed bundling dominates pure components and this result carries over as shown in Part (a) of the Proposition given that MB-12 does not converge to PC as the parameters change smoothly. A look ahead at the figures confirms this to be the case, with the caveat that PC can be optimal in other parts of the phase diagram. Indeed, with two products, traditional mixed bundling (MB-12) has been shown to be optimal in the baseline case with zero marginal cost and the absence of network externality (see Venkatesh and Kamakura 2003). This means the convergence of MB-12 to a more basic strategy can only occur, at most, for non-zero values of marginal cost and network externality.

Part (b) shows some simple expressions define that boundary. Revelation of such relationships is a benefit of the analytical approach. On the other hand, part (c) shows that the analytical expressions for the boundaries can be quite complicated. Given that the equations are nonlinear, there are multiple solutions and numerical comparison is required to pick the correct solution.

The analytical approach has the limitation that it can delineate the domains of optimality of nested strategies but not non-nested strategies. Specifically, the points of convergence for MB-12 to MB-1, MB-12 to PB, MB-12 to PC, MB-1 (or MB-2) to PB can be determined. This is because in each of these pairs the latter strategy is a special case of the former. In addition, while they are not nested, the profit indifference boundary between PB and PC has already been obtained in §4.1. This leaves the convergence of MB-1 (or MB-2) to PC which cannot be determined with the above approach as PC is not a special case of MB-1 (say) because product 2 which is offered under PC is not part of the MB-1 strategy. As noted earlier in this section, numerical optimization yields the profit indifference boundary between MB- $i$  and pure components. We now proceed to comparison of optimal strategies.

*Result 2. When the products have comparable marginal costs (i.e.,  $c_1 = c_2 = c$ ):*

- (A) When product 1 alone has network externality (i.e.,  $n_2 = 0$ ): (i) Mixed bundling-1 is optimal when the degree of network externality is higher and the marginal costs are lower ( $n_1 \geq c$ , approx.); (ii) Converse conditions to part (i) favor mixed bundling-12.
- (B) When both products have network externality (i.e.,  $n_1 = n_2 = n$ ): (i) Pure bundling is optimal when the degree of network externality is higher and the marginal costs are lower ( $n \geq 0.75c$ , approx.); (ii) Converse conditions favor mixed bundling-12.



R2(A) is based on Figure 3.1 and R2(B) on Figure 3.2. The upper left halves of the figures are similar. This region is characterized by higher values of marginal cost and lower values of the network externality parameter, and MB-12 is the optimal strategy. For the converse conditions, a simpler strategy, MB-1 or PB is optimal. In Figure 3.2, note that only three strategies are relevant since the symmetry of the problem precludes the MB-1 and MB-2 strategies from being optimal.

The optimality of MB-12 under low to no network externality is basically a known result. The question is why MB-12 – the “be all and end all” of bundling strategies – is not superior under higher levels of the network externality parameter. Unique MB-12 is a constrained pricing problem: each product’s price is constrained to fall below the bundle price and the sum total of the product prices must exceed the bundle price. Maintaining these constraints can result in a profit penalty. Specifically, MB-12 allows the marketer to target those customers who strongly prefer one product but not the other with its “component” sales. But for these customers to not instead buy the bundle, the bundle price must be kept higher, excluding more customers in the middle. With network externality present, the penalty for excluding potential customers of the networked product can be quite high. Accordingly, when the network externality is higher, MB-12 is not as profitable as MB-1 or PB. If we look at Figure 3.2, we would expect that as each constraint binds (e.g.,  $P_2 \rightarrow P_{12}$ ), MB-12 would converge to MB-1 and then PB (in fact, this is what happens later in Figure 4.2) but given that the products are symmetric in Figure 3.2, the MB-1 strategy is not observed.

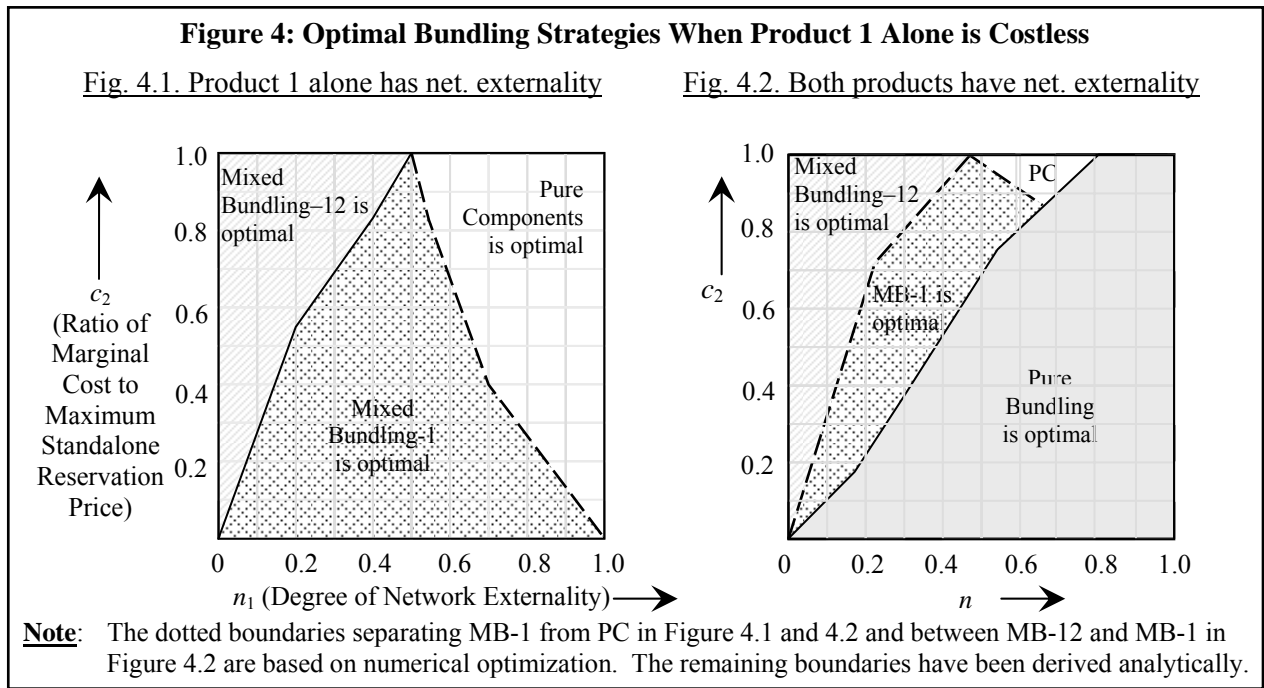
Now to contrast Figures 3.1 and 3.2: Whereas symmetry in network externality enhances the power of pure bundling under lower marginal costs, asymmetry (Figure 3.1) – when product 1 alone triggers network externality – requires that product 1 in its standalone be part of the optimal product line. Pure bundling should be avoided under asymmetry as it would overlook customers who care only for the product that offers the network benefit. MB-12 should also be avoided when the asymmetry in network externality is more pronounced. Given that the network externality is from product 1 alone, offering product 2 separately will sacrifice a part of the penetration of product 1 via the bundle, reducing the network benefit from 1. In other words, the seller gains from withholding product 2 as with MB-1.

Finally, PC is optimal in a small domain in Figure 3.1. When costs of both products are moderate/high, the bundle price becomes high enough that the bundle is unappealing to most consumers. Further, when 1’s degree of network externality is very high, it is in the seller’s interest not to compromise the sale of “1”. These two factors account for MB-1’s convergence to PC.

The next set of observations is about the effect of asymmetry in both costs and network externality.

*Result 3. When marginal costs are asymmetric (i.e., product 1 alone is costless,  $c_1=0$ ):*

- (A) When product 1 alone triggers network externality ( $n_2 = 0$ ): (i) As the degree of network externality increases, optimal strategy changes from mixed bundling-12 to mixed bundling-1 to pure components; (ii) Lower marginal costs augment the domain of optimality of mixed bundling-1.
- (B) When both products trigger network externality ( $n_1 = n_2 = n$ ): (i) As the degree of network externality increases, optimal strategy changes from mixed bundling-12 to mixed bundling-1 to pure bundling; (ii) Lower marginal costs augment the domain of optimality of pure bundling.



The optimality of mixed bundling-12 in R3(A) and (B) for low levels of network externality is essentially the baseline result. We will focus on the optimality of the other strategies.

Figure 4.1 corresponds to R3(A). When product 1 alone offers a higher degree of network externality, product 2's higher marginal cost relative to willingness to pay makes the disparity between the two products even higher. With product 1 being so attractive, the seller gains from offering it individually irrespective of the decision on offering the bundle. In this case, even the consumers with lower standalone reservation prices for product 1, but boosted by its network benefit, can still be tapped through the bundle if they have high enough standalone reservation prices for 2. With the bundle capable of serving such a role, offering product 2 separately is undesirable; doing so would curtail bundle sales and diminish 1's network benefit. MB-1 is optimal. However, when product 1's degree of network



externality is even higher, product 1 by itself can be priced high enough and still penetrate the entire market. When product 2's marginal cost burden is relatively high, the bundle price has to be increased and its ability to push 2 decreases. Indeed even when  $c_2$  is low, using the bundle implies using the more attractive product 1 to cross-subsidize 2 (and not vice versa). The bundle is relatively inefficient here. Pure components takes over from MB-1 as the optimal strategy.

Figure 4.2 corresponds to  $R3(B)$ . Pure bundling becomes optimal under high network externality as the network benefit more than offsets the marginal cost burden of product 2 (product 1 is costless anyway). At the higher levels of symmetric network externality, price discrimination through high priced components is rendered meaningless because the bundle itself is priced high enough to tap the network benefit realized by customers. The domain of pure bundling is the largest when product 2's marginal cost is low; each product enjoys the twin benefit of low cost (in relation to willingness to pay) and moderate to high network benefit, the convergence of benefits facilitating pure bundling.

In  $R3(B)$ , it is noteworthy that MB-12 converges to mixed bundling-1. When both products have network externality, using each to sell the other is in the seller's interest. Thus the role of the bundle (as in MB-12, MB-1 or PB) is typically very important. For higher levels of product 2's marginal cost and lower yet positive levels of the degree of network externality, it becomes optimal not to offer 2 alone, because component prices of 2 are high. The loss in component 2's individual sale is limited compared to the gain from supply of the bundle. MB-12 converges to MB-1.

In both Figures 3 and 4 it is noteworthy that when marginal costs are zero, even a small amount of network externality in either product is enough to push the firm towards using a strategy other than MB-12. This alternative strategy is MB-1 when only one product yields network externality, and pure bundling when network externality comes from both products.

Beyond the four scenarios described in the results, we examined a fifth scenario in which the product 1 (with network externality  $n_1$ ) has marginal cost  $c_1$  whereas product 2 is costless and has no network externality. Between pure components and pure bundling, the latter is more profitable under higher  $n_1$  or lower cost  $c_1$  because either of these makes the two products "symmetric". Also including variants of mixed bundling, the phase diagram of optimal strategies is closest to Figure 3.1 (for  $n_1 > 0$  and  $c_1 = c_2$ ). The earlier results favoring MB-1 under asymmetry in network externality again prevails (for largely the same reasons).

## 5. Discussion

While the areas of bundling and network externality have long been researched, the growth in high technology and eCommerce has given rise to new and important problems at their interface. We have

examined one such problem, specifically by clarifying how asymmetry among products in network externality and marginal cost impacts the choice of bundling strategy.

### 5.1. Contributions to the Literature

Our research complements the work of several previous bundling articles. For example, while Bakos and Brynjolfsson (1999) speculated that pure bundling becomes more powerful under network externality, we show that to be the case only when network externality is symmetric and marginal costs are low. While Schmalensee (1984) notes – for products without network externality – that asymmetry in product market conditions reduces the significance of pure bundling, we underscore this finding for cost-based asymmetry. Network externality and its interaction with cost are aspects on which our study builds on Schmalensee’s work. Another contribution of our study comes from pointing out that the familiar triad of pure components, pure bundling and mixed bundling may not suffice. Our contribution of “new” insights is in the highlighted cells below.

	Symmetric Marginal Costs	Asymmetric Marginal Costs ( $c_1=0$ )
No network externality	<u>Baseline Result</u> <ul style="list-style-type: none"> <li>Mixed bundling-12 is optimal</li> </ul>	<ul style="list-style-type: none"> <li>Mixed bundling-12 remains optimal.</li> </ul>
Both products have network externality	<ul style="list-style-type: none"> <li>When marginal costs are low or network benefits high, pure bundling is optimal.</li> <li>Mixed bundling-12 is optimal otherwise.</li> </ul>	<ul style="list-style-type: none"> <li>When the network benefit is high, pure bundling is optimal.</li> <li>When the network benefit is moderate and the cost asymmetry high, MB-1 is optimal</li> <li>Mixed bundling-12 is optimal otherwise.</li> </ul>
Only product 1 has network externality	<ul style="list-style-type: none"> <li>As the asymmetry in network benefit between products increases, mixed bundling-1 more widely optimal.</li> <li>Mixed bundling-12 is optimal otherwise.</li> </ul>	<ul style="list-style-type: none"> <li>Mixed bundling-1 is optimal, unless cost asymmetry is high.</li> <li>When network benefit and cost asymmetry are high, pure components is optimal</li> <li>Mixed bundling-12 is optimal otherwise.</li> </ul>

### 5.2. Managerial Implications: Linkages between Results and Real World Examples

Although our study is normative and the model abstracts in several ways from real world contexts, we will speculate here on how our results might correspond to actual examples. Let us consider four scenarios:

- Both products are “costless” and yield significant network externality: Our result 3(B), that pure bundling is optimal, resonates with the practice of Microsoft offering its Outlook Professional email system bundled with calendar management program.

- Only one product yields network externality and both products have low marginal cost: Per result 2A(i), MB-1 is optimal, 1 being the product yielding network externality. Consistently, ChessBase GmbH's Playchess online subscription can be bought and used to play with other subscribers. Yet the company's Fritz (a single player PC game without network effect, that offers value independent of the Playchess system) is offered only as a bundle with Playchess' annual subscription.
- Only one of the two products is "costless" and yields significant network externality: When the other product has sizable marginal cost, result 3A(i) points to pure components. Consider IEEE. The value of any of its conferences to a participant increase with the number of other participants (i.e., has network externality) and IEEE's marginal cost to host a new attendee is negligible. Any book in IEEE's archived list of titles has significant marginal cost. IEEE does not bundle its conference registration with its sales of archived book titles.
- Both products have significant marginal cost and neither yields network externality: Per result 2B(ii), mixed bundling-12 would be the normative recommendation. Amazon.com sells books separately but also offers a discount on a bundle of two books (or more).

We must underscore that we do not have empirical data to support our conjectures of the levels of marginal cost and network externality in the above examples. However, there is some face validity to the scenarios we consider. The real world outcomes seem to mesh with our conclusions.

### **5.3. Study's Limitations and Future Research Directions**

The model is based on several assumptions. The paper's application is restricted to monopoly markets, similar to Bakos and Brynjolfsson (1999) or McAfee et al. (1989), but likely extending to monopolistic competition, where a firm due to differentiation, nonstrategic competitors, or a head start, has some market power. However, more work needs to be done with regard to competitive interactions in bundling studies because many situations cannot adequately be approximated by the monopoly assumption. With competition, network externality introduces another dimension, the scale, and thus changes the nature of competition. Compatibility and standards wars become issues. But presently, even in the absence of network externalities, there are few analytical studies of competition between sellers who do mixed bundling. A starting point might be Ghosh and Balachander (2007).

Our reservation prices are assumed to be uniformly distributed. What would be the impact of non-uniform distributions? While some articles have considered two distributions from the same family (e.g., Schmalensee 1984, who considers two normal distributions, and Kopalle et al. 1999, who partly examine a two-stage nested logit formulation with different error terms), this effort is preliminary.

Our model assumes that the network externality benefit is linear in the number of users. While this approach has precedents in the literature, it could be argued that later adopters are less avid users than the early adopters, and the marginal boost of network externality is diminishing in the size of the user pool. Separately, the network benefit can be seen as a function of both the network size and the standalone reservation price of the individual users, the argument here being those with higher standalone reservation prices enjoy higher network benefit. Our preliminary analysis suggests that heterogeneity in valuations is higher under this approach. While the domain of optimality of traditional mixed bundling is enlarged and that of pure bundling is reduced, all four bundling strategies that are shown to optimal in our study continue to be optimal.

A natural extension is to consider a firm with more than two products in its portfolio. However, as the number of products increases, product line design with bundling becomes quite difficult. We can only speculate what impact network externality might have in this case. If all products have network externality then the results from symmetric network externality (the right panels of Figures 1-4), suggest Bakos and Brynjolfsson's (1999) result might strengthen, since pure bundling dominates even for non-zero costs. When only a subset of products has network externality and not too high cost, then based on the optimality for MB-1 (left panel of Figures 3-4), these products should arguably be included in the bundle as well as offered in standalone form.<sup>4</sup>

Finally, this study is analytical and we urge future work tied to data. Relative to bundling, the network externality literature appears to have several empirical studies (e.g., Shankar and Bayus 2003). More such work at the confluence of the two streams should be insightful.

## Appendix

Proof of Proposition 1: We rewrite the constraint  $D_i = (1 + n_i D_i) - P_i$  as  $P_i = 1 - (1 - n_i) D_i$  and substitute it into the objective function. This yields,

$$\max_{D_1 \in [0,1], D_2 \in [0,1]} \left\{ \Pi = \sum_{i \in \{1,2\}} (P_i(D_i) - c_i) D_i = \sum_{i \in \{1,2\}} (1 - (1 - n_i) D_i - c_i) D_i \right\}.$$

The derivative is  $1 - 2(1 - n_i) D_i - c_i$  from which it can be seen that a corner solution  $D_i = 1$  occurs if  $2n_i - 1 - c_i \geq 0$ . At this corner solution  $P_i = n_i$ . An interior solution is obtained by setting the derivative to 0. This yields  $D_i = \frac{1 - c_i}{2(1 - n_i)}$ . At this demand, the optimal price is  $P_i = (1 + c_i) / 2$ .  $\square$

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<sup>4</sup> We thank an anonymous reviewer for raising these issues.

Proof of Proposition 2: Let  $c_1 + c_2 = C$  and  $n_1 + n_2 = N$ . Thus,  $C \in [0, 2]$  and  $N \geq 0$ .

Part (a): For  $D_{12} \in [0, 0.5]$ , demand is given by  $D_{12} = \frac{(2 + ND_{12} - P_{12})^2}{2}$ . We rewrite this as

$P_{12} = 2 + ND_{12} - \sqrt{2}\sqrt{D_{12}}$  and substitute it in  $\Pi_{PB} = (P_{12} - C)D_{12}$ . Thus, the objective is

$$\max_{D_{12}} \Pi_{PB} = (2 + ND_{12} - \sqrt{2}\sqrt{D_{12}} - C)D_{12}.$$

Examining the derivative of the profit function at the corner points, the solution is bounded away from  $D_{12} = 0$ , but  $D_{12} = 0.5$  occurs if  $0.5 - C + N \geq 0$ . The interior solution is given by the first order condition

$$2ND_{12} - 1.5\sqrt{2}\sqrt{D_{12}} + 2 - C = 0.$$

This can be solved as a quadratic to yield  $\sqrt{D_{12}} = (1.5\sqrt{2} \pm \sqrt{4.5 - 8N(2 - C)}) / 4N$ . The second order condition for maximum requires the negative root to be chosen. The solution can be re-written as,

$$\sqrt{D_{12}} = \frac{2\sqrt{2}(2 - C)}{3 + \sqrt{9 - 16N(2 - C)}}.$$

Squaring this yields the optimal demand. Now recalling that  $P_{12} = 2 + ND_{12} - \sqrt{2}\sqrt{D_{12}}$ , we multiply it by 1.5 and add it to the first order condition. This yields the optimal price  $P_{12} = (2 + 2C - ND_{12}) / 3$ .

Part (b): From the corner solutions for parts (a) or (c) of this Proposition,  $D_{12} = 0.5$  occurs when  $0.5 - C + N = 0$ . Using the price solution from either part (a) or part (c) and inserting  $D_{12} = 0.5$  and  $C = N + 0.5$  into it gives the optimal price  $P_{12} = 1 + N / 2$ .

Part (c): For  $D_{12} \in [0.5, 1]$ , the demand is given by  $D_{12} = 1 - \frac{(P_{12} - ND_{12})^2}{2}$ . We rewrite this as

$P_{12} = ND_{12} + \sqrt{2}\sqrt{1 - D_{12}}$ . The objective is thus,

$$\max_{D_{12}} \Pi_{PB} = (ND_{12} + \sqrt{2}\sqrt{1 - D_{12}} - C)D_{12}$$

Examining the derivative of the profit function

$$2ND_{12} + \sqrt{2}\sqrt{1 - D_{12}} - \frac{D_{12}}{\sqrt{2}\sqrt{1 - D_{12}}} - C$$

at the corner points,  $D_{12} = 1$  can be ruled out but  $D_{12} = 0.5$  occurs if  $0.5 - C + N \leq 0$ . The first order condition can be written as

$$2\sqrt{2}N(1 - D_{12})^{3/2} - 3(1 - D_{12}) - (2N - C)\sqrt{2}\sqrt{1 - D_{12}} + 1 = 0.$$

This is a cubic in  $\sqrt{1-D_{12}}$ . As cubic equations have lengthy solutions, we omit listing it here, but the solution is explicit.

To show existence, we show that the sign of the cubic function is negative at  $D_{12}=0.5$  (the function value is  $-0.5+C-N$  whereas for this scenario to exist the condition is  $N+0.5-C>0$ ) and positive at  $D_{12}=1$  (the function value is 1) implying by continuity that there is at least one point where the function crosses the x-axis.  $\square$

### Proof of Proposition 3:

Due to the length considerations, the proof is omitted here and is available directly from the authors.

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# Optimal Bundling of Technological Products with Network Externality

Ashutosh Prasad, R. Venkatesh, Vijay Mahajan\*

July 2010

Forthcoming in *Management Science*

## Companion Appendix: Outline of the Proof of Proposition 3(a)

Intuition of the solution methodology: Consider maximizing a function subject to constraints. The variables are denoted by a vector  $\mathbf{P}$ , and parameters by a vector  $\mathbf{A}$ . Whether using a Lagrangian multiplier method or not, the solution can be written as a system of first order conditions,  $f(\mathbf{P})=0$ . But obtaining an explicit solution  $\mathbf{P}^*(\mathbf{A})$  from  $f(\mathbf{P})=0$  may prove intractable.

However, our interest is often in another relationship  $g(\mathbf{P}^*)=0$ . (for example, for  $\mathbf{P}=\{P_1, P_2, P_{12}\}$  the relationship  $g(\mathbf{P}^*)=0$  might be  $P_1^* + P_2^* = P_{12}^*$ , that defines the parameter values when MB-12 is reduced to PC). Then, instead of explicitly solving  $f(\mathbf{P})=0$  for  $\mathbf{P}^*(\mathbf{A})$  first, we solve  $f(\mathbf{P})=0$  simultaneously with  $g(\mathbf{P})=0$ . The solution is  $g(\mathbf{P}^*)$ . Importantly, it may be possible to obtain this even though obtaining  $\mathbf{P}^*(\mathbf{A})$  was intractable, because the simplicity of the relationship  $g(\mathbf{P})=0$  eases solving the entire system of equations.

Outline of the proof: For analysis, we set up the MB-12 maximization problem anew from the diagram in the text. Let  $x_i = D_i + D_{12}$ . The three demand equations can now be written (from observation) as functions of  $x_1$  and  $x_2$ :

$$\begin{aligned} D_1 &= (P_{12} - P_1 - n_2 x_2)(1 + n_1 x_1 - P_1) \\ D_2 &= (P_{12} - P_2 - n_1 x_1)(1 + n_2 x_2 - P_2) \\ D_{12} &= (1 + n_1 x_1 - (P_{12} - P_2))(1 + n_2 x_2 - (P_{12} - P_1)) - (P_1 + P_2 - P_{12})^2 / 2 \end{aligned}$$

where  $x_1$  and  $x_2$  can be obtained by adding the first and third and then the second and third equations above, and simplifying. Thus, they solve the simultaneous equations

$$\begin{aligned} x_1 &= (1 + n_1 x_1 - P_1) + (P_1 + P_2 - P_{12})(1 + n_2 x_2 - P_2) + (P_1 + P_2 - P_{12})^2 / 2 \\ x_2 &= (1 + n_2 x_2 - P_2) + (P_1 + P_2 - P_{12})(1 + n_1 x_1 - P_1) + (P_1 + P_2 - P_{12})^2 / 2 \end{aligned}$$



Next we define the boundary conditions:

- (a) If  $P_{12} \geq P_1 + P_2$  then mixed bundling-12 will converge to pure components (PC).
- (b) If  $x_i \geq 0.5$  and  $P_{12} - P_i \leq n_{3-i}x_{3-i}$  ( $i = 1, 2$ ) or if  $x_i \leq 0.5$  and  $P_i \geq 1 + n_i x_i$  then the product  $i$  will not be offered separately as part of mixed bundling solution because the bundle will dominate the component. This is the convergence from  $MB_{12}$  to the  $MB_{3-i}$  strategy. If the condition holds for products 1 and 2, mixed bundling-12 will converge to pure bundling (PB).

These conditions hold with equality at the phase boundary defining the convergence from MB to PB or from MB to PC. Profit is  $\Pi_{MB} = (P_{12} - c_1 - c_2)D_{12} + (P_1 - c_1)D_1 + (P_2 - c_2)D_2$ , which can be rewritten as,  $\Pi_{MB} = (P_{12} - P_1 - P_2)D_{12} + (P_1 - c_1)(D_1 + D_{12}) + (P_2 - c_2)(D_2 + D_{12})$ . Thus the maximization problem is:

$$\begin{aligned} & \max_{P_1, P_2, P_{12}, x_1, x_2} \Pi_{MB} \\ & = (P_{12} - P_1 - P_2)(1 + n_1 x_1 + P_2 - P_{12})(1 + n_2 x_2 + P_1 - P_{12}) + (P_1 + P_2 - P_{12})^3 / 2 + (P_1 - c_1)x_1 + (P_2 - c_2)x_2 \end{aligned}$$

subject to the two simultaneous equations relating  $x_1$  and  $x_2$ .

The Lagrangian is:

$$\begin{aligned} L(P_1, P_2, P_{12}, x_1, x_2, \lambda_1, \lambda_2) = & (P_{12} - P_1 - P_2)(1 + n_1 x_1 + P_2 - P_{12})(1 + n_2 x_2 + P_1 - P_{12}) + (P_1 + P_2 - P_{12})^3 / 2 + (P_1 - c_1)x_1 + (P_2 - c_2)x_2 \\ & + \lambda_1 \left[ -x_1 + (1 + n_1 x_1 - P_1) + (P_1 + P_2 - P_{12})(1 + n_2 x_2 - P_2) + (P_1 + P_2 - P_{12})^2 / 2 \right] \\ & + \lambda_2 \left[ -x_2 + (1 + n_2 x_2 - P_2) + (P_1 + P_2 - P_{12})(1 + n_1 x_1 - P_1) + (P_1 + P_2 - P_{12})^2 / 2 \right] \end{aligned}$$

The Lagrangian yields the following first order conditions:

$$\begin{aligned} \partial L / \partial \lambda_1 = 0 & = -x_1 + (1 + n_1 x_1 - P_1) + (P_1 + P_2 - P_{12})(1 + n_2 x_2 - P_2) + (P_1 + P_2 - P_{12})^2 / 2 \\ \partial L / \partial \lambda_2 = 0 & = -x_2 + (1 + n_2 x_2 - P_2) + (P_1 + P_2 - P_{12})(1 + n_1 x_1 - P_1) + (P_1 + P_2 - P_{12})^2 / 2 \\ \partial L / \partial x_1 = 0 & = n_1(P_{12} - P_1 - P_2)(1 + n_2 x_2 + P_1 - P_{12}) + P_1 - c_1 + \lambda_1(n_1 - 1) + \lambda_2 n_1(P_1 + P_2 - P_{12}) \\ \partial L / \partial x_2 = 0 & = n_2(P_{12} - P_1 - P_2)(1 + n_1 x_1 + P_2 - P_{12}) + P_2 - c_2 + \lambda_1 n_2(P_1 + P_2 - P_{12}) + \lambda_2(n_2 - 1) \\ \partial L / \partial P_1 = 0 & = (1 + n_1 x_1 + P_2 - P_{12})(-2P_1 + 2P_{12} - P_2 - 1 - n_2 x_2) + 3(P_1 + P_2 - P_{12})^2 / 2 + x_1 \\ & \quad + \lambda_1(n_2 x_2 + P_1 - P_{12}) + \lambda_2(1 + n_1 x_1 - P_1) \\ \partial L / \partial P_2 = 0 & = (1 + n_2 x_2 + P_1 - P_{12})(-2P_2 + 2P_{12} - P_1 - 1 - n_1 x_1) + 3(P_1 + P_2 - P_{12})^2 / 2 + x_2 \\ & \quad + \lambda_1(-P_2 + 1 + n_2 x_2) + \lambda_2(n_1 x_1 + P_2 - P_{12}) \\ \partial L / \partial P_{12} = 0 & = (1 + n_1 x_1 + P_2 - P_{12})(1 + n_2 x_2 + P_1 - P_{12}) + (P_{12} - P_1 - P_2)(2P_{12} - P_1 - P_2 - 2 - n_1 x_1 - n_2 x_2) \\ & \quad - 3(P_1 + P_2 - P_{12})^2 / 2 - \lambda_1(1 + n_2 x_2 + P_1 - P_{12}) - \lambda_2(1 + n_1 x_1 + P_2 - P_{12}) \end{aligned} \tag{A1}$$

These expressions are simpler if we define  $\delta = P_1 + P_2 - P_{12}$  and  $\theta_i = 1 + n_i x_i - P_i$ ,  $\forall i$ . The first two equations become  $x_i = \theta_i + \delta \theta_{3-i} + \delta^2 / 2$ ,  $\forall i$ , and inserting these into the remaining equations as well, they can be rewritten as,

$$\begin{aligned}
\partial L / \partial x_1 &= 0 = -n_1 \delta^2 / 2 + 1 + (n_1 - 1) \theta_1 - c_1 + \lambda_1 (n_1 - 1) + \lambda_2 n_1 \delta \\
\partial L / \partial x_2 &= 0 = -n_2 \delta^2 / 2 + 1 + (n_2 - 1) \theta_2 - c_2 + \lambda_1 n_2 \delta + \lambda_2 (n_2 - 1) \\
\partial L / \partial P_1 &= 0 = -\theta_1 (\theta_2 + 2\delta - 1) + \lambda_1 (\theta_2 + \delta - 1) + \lambda_2 \theta_1 \\
\partial L / \partial P_2 &= 0 = -\theta_2 (\theta_1 + 2\delta - 1) + \lambda_1 \theta_2 + \lambda_2 (\theta_1 + \delta - 1) \\
\partial L / \partial P_{12} &= 0 = (\theta_1 + \delta)(\theta_2 + \delta) + \delta(\theta_1 + \theta_2) + \delta^2 / 2 - \lambda_1 (\theta_2 + \delta) - \lambda_2 (\theta_1 + \delta)
\end{aligned} \tag{A2}$$

### Convergence of MB-12 to PC

We now examine the boundary condition where mixed bundling converges to pure components. The condition is  $P_{12} = P_1 + P_2$ , i.e.,  $\delta = 0$ . Inserting this into the first order conditions we get:

$$\begin{aligned}
0 &= 1 + (n_1 - 1) \theta_1 - c_1 + \lambda_1 (n_1 - 1) \\
0 &= 1 + (n_2 - 1) \theta_2 - c_2 + \lambda_2 (n_2 - 1) \\
0 &= -\theta_1 (\theta_2 - 1) + \lambda_1 (\theta_2 - 1) + \lambda_2 \theta_1 \\
0 &= -\theta_2 (\theta_1 - 1) + \lambda_1 \theta_2 + \lambda_2 (\theta_1 - 1) \\
0 &= \theta_1 \theta_2 - \lambda_1 \theta_2 - \lambda_2 \theta_1
\end{aligned}$$

Adding the third and fifth equations gives  $\lambda_1 = \theta_1$  and adding the fourth and fifth gives  $\lambda_2 = \theta_2$ . Now inserting these into the first two equations, we get  $\lambda_i = \theta_i = (1 - c_i) / 2(1 - n_i)$ ,  $\forall i$ , and inserting it into the fifth equation we get  $\theta_1 \theta_2 = 0$ . Thus, the parameters  $\{\lambda_i = \theta_i = 0, c_i = 1\}$  coupled with  $\{\lambda_{3-i} = \theta_{3-i} = \frac{1 - c_{3-i}}{2(1 - n_{3-i})}\}$  constitute the boundary condition, and here  $x_i = \theta_i$ ,  $\forall i$  so demand is zero.

To conclude, for MB-12 to converge to PC, at least one product must be so costly ( $c_i = 1$ ) that its demand is always zero, i.e., bundling is moot. Other than this degenerate solution MB-12 does not converge to PC.