problems

1) A discrete-time signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1+\frac{n}{3}, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

a) Determine its value and sketch the signal $x(n)$.

$1+\frac{n}{3}$, $-3 \le n \le -1 \Rightarrow$ at $n=-3$; 0

$n=-2$; $+\frac{1}{3}$

$n=-1$; $\frac{2}{3}$.

from $0 \le n \le 3$; 1

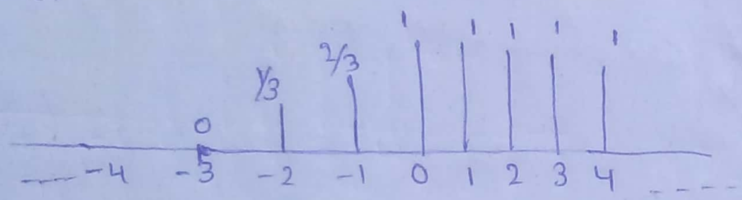
elsewhere ; 0

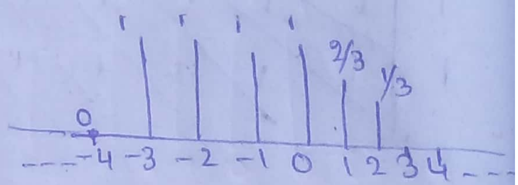
so; $x(n) = \{ \text{-----} \; 0, \frac{1}{3}, \frac{2}{3}, 1, 1, 1, 1, 0 \text{-----} \}$

b) Sketch it's values and the signals that result if we:

i) First fold $x(n)$ and then delay the resulting by 4 samples.

$x(n)$



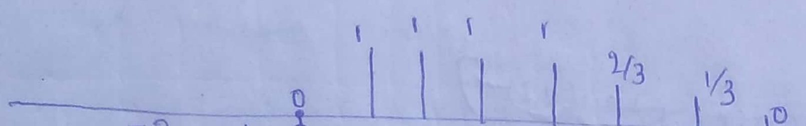


$x(-n+4) \rightarrow$ after delaying

$(-3 \le -n+4 \le 2)$

$-7 \le -n \le -2$

$(+7 \ge n \ge -2)$.

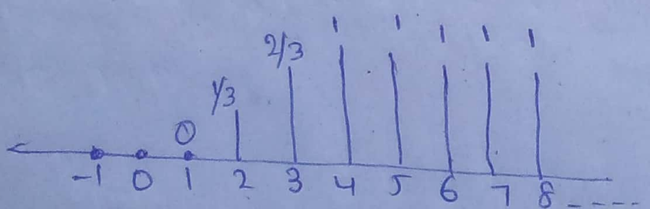


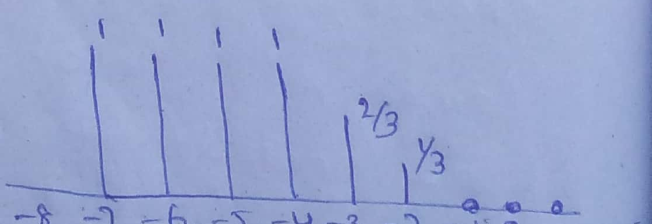
2) First delay $x(+n)$ by four samples & then fold the resulting signal.

folding $x(n-4) \Rightarrow x(-n-4)$

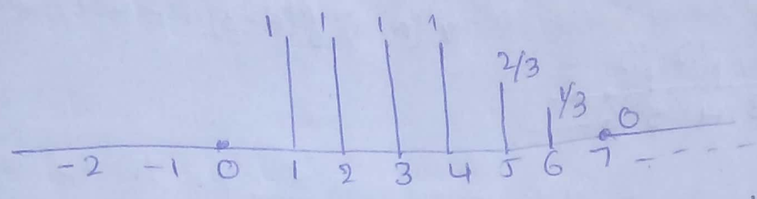
$x(n) \rightarrow$ delay by 4 samples

$x(n-4)$





c) Sketch the signal $x(-n+4)$



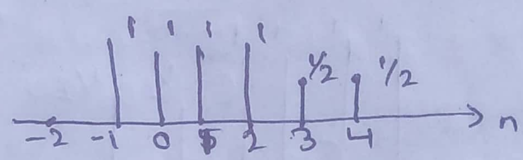
d) compare the result in part (b) & (C) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

By comparing results in parts (b) & (C) we can say that to get $x(-n+k)$ from $x(n)$ first we need to fold $x(n)$ which results in $x(-n)$ and then we need shift by k samples to right if $k>0$ (or) to left if $k<0$ results in $x(-n+k)$.

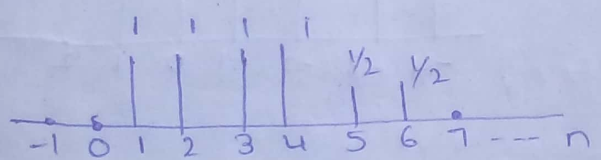
e) can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$?

Yes. $x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n-1) + u(n) - u(n-4)$.

2) A discrete time signal $x(n)$ is shown in the figure. sketch and label carefully each of the following signals.



a) $x(n-2)$

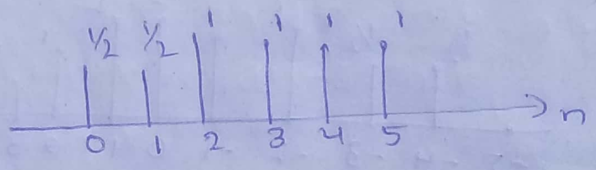


b) $x(4-n)$

$-1 \le 4 \le n \le 4$

$-5 \le -n \le 0$

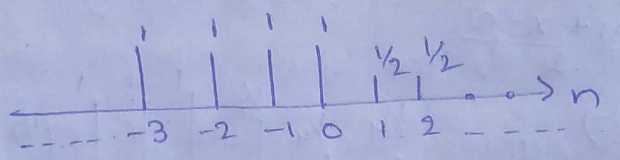
$5 \ge n \ge 0$



c) $x(n+2)$

$-1 \le n+2 \le 4$

$-3 \le n \le 2$

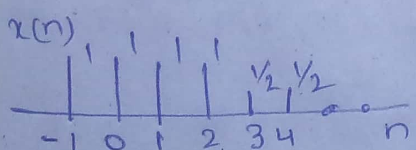


d) $x(n)\, u(2-n)$

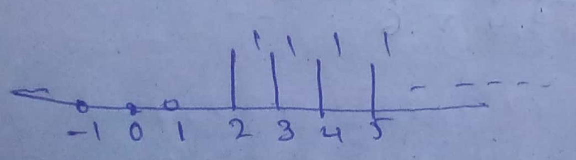
$u(2-n) \Rightarrow 1;\ 2-n \ge 0$

$1,\ -n \ge 2$

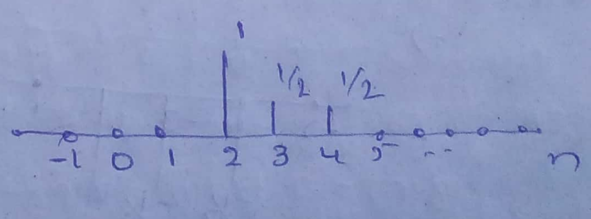
$n > 2$



$x(n)\, u(2-n)$

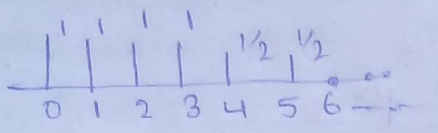


$\Rightarrow$



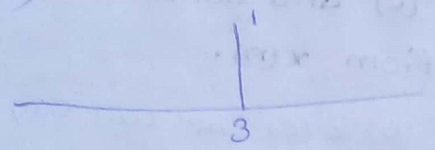
e) $x(n-1)\,\delta(n-3)$

$x(n-1)$

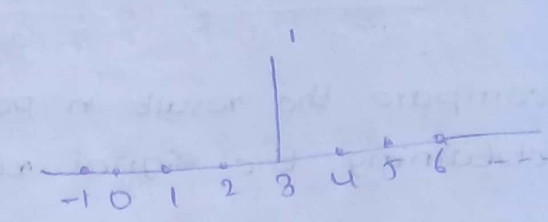


$x(n-1)\,\delta(n-3)$

$\delta(n-3)$



$\Rightarrow$



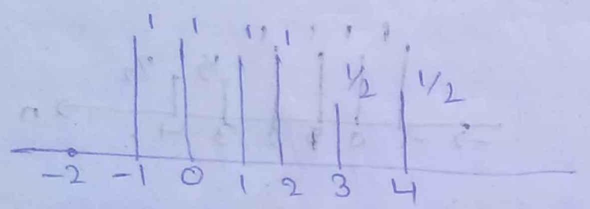
f) $x(n^2) \Rightarrow x(n) = \{x(-2), x(-1), x(0), x(1), x(2), x(3), x(4), \dots\}$

$x(n^2) = \{\dots x(4), x(1), x(0), x(1), x(4), x(9), x(16), \dots\}$

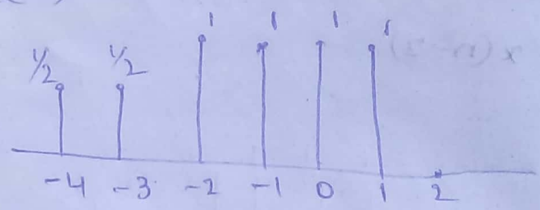
$= \{\dots \frac{1}{2}, 1, 1, 1, \frac{1}{2}, 0, 0, \dots\}$

g) even part $x_e(n) = \dfrac{x(n) + x(-n)}{2}$

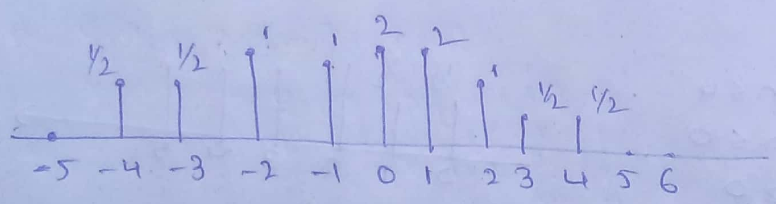
$x(n)$



$x(-n)$

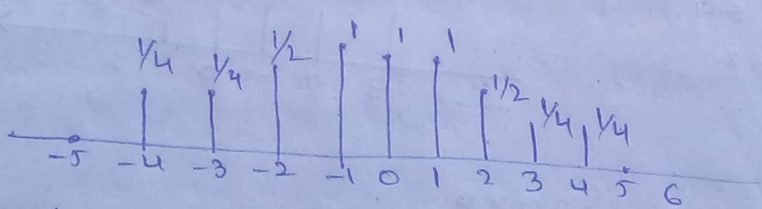


$x(n) + x(-n)$



$\dfrac{x(n) + x(-n)}{2}$

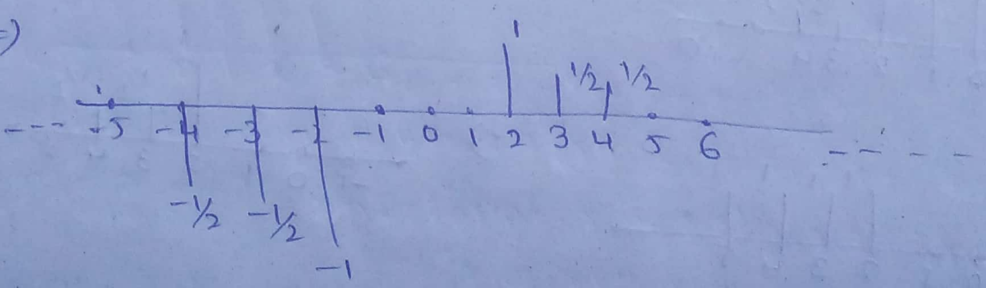
$\Rightarrow$



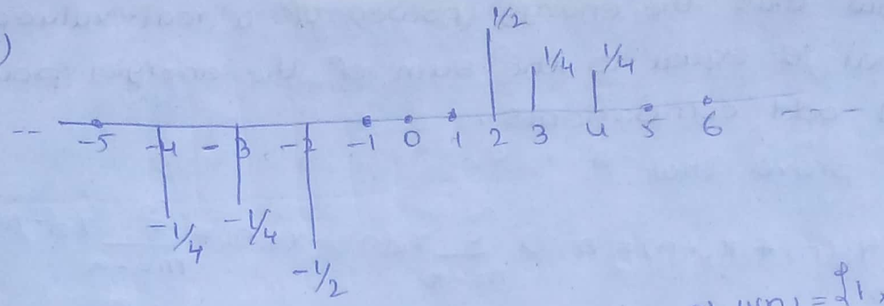
odd part $x_o(n) = \dfrac{x(n) - x(-n)}{2}$

$x(n) - x(-n)$

$\Rightarrow$



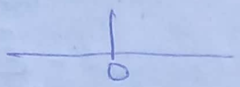
$\frac{x(n)-x(-n)}{2} \Rightarrow$



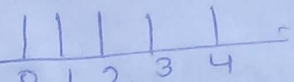
3) a) show that $\delta(n) = u(n) - u(n-1)$

as $u(n) = \{1, n \geq 0$

we know $\delta(n) =$

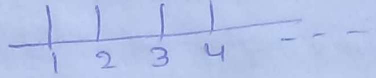


$u(n) =$



$u(n) = u(n-1)$

$u(n-1) =$

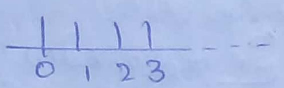


$\Rightarrow$

$\therefore \delta(n) = u(n) - u(n-1)$

b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

$u(n) =$



$\Rightarrow \sum_{k=-\infty}^{n} \delta(k) = u(n) = \begin{cases} 0; & n < 0 \\ 1; & n \geq 0 \end{cases}$

$\sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$

4) Show that any signal can be decomposed into an even & an odd component. Is the decomposition, unique? Illustrate at your component using the signal.

$x(n) = \{2, 3, 4, 5, 6\}$

$x_e(n) = \frac{x(n)+x(-n)}{2}$

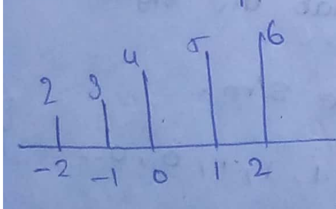
$x_o(n) = \frac{x(n)-x(-n)}{2}$

$x_e(n) = x_e(-n)$

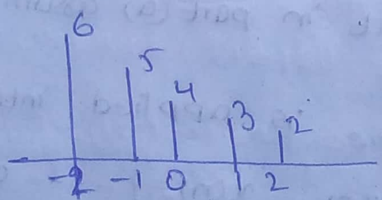
$x_o(n) = -x_o(n)$

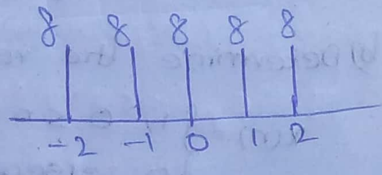
$\Rightarrow x(n) = x_e(n) + x_o(n)$

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

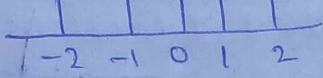


$x(-n)$



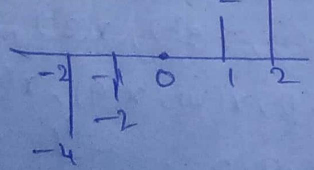


$x_e(n) = \frac{x_e(n)+x(-n)}{2} \Rightarrow$

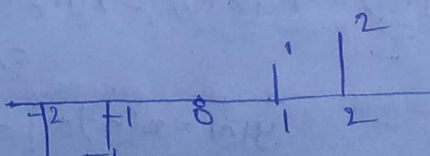


$x(n) - x(-n)$

$\Rightarrow$



$x_o(n) = \frac{x(n) - x(-n)}{2}$



5) Show that the energy (power) of a real-valued energy (power) signal is equal to the sum of the energies (power) of its even and odd components.

First prove that

$$\sum_{n=-\infty}^{\infty} x_e(n) + x_o(n) = 0 \Rightarrow \sum_{n=-\infty}^{\infty} x_e(n) + x_o(n) = \sum_{m=-\infty}^{\infty} x_e(-m)\, x_o(-m)$$

$$= -\sum_{m=-\infty}^{\infty} x_e(m)\, x_o(m)$$

$$= -\sum_{m=-\infty}^{\infty} x_e(n)\, x_o(n)$$

$$= \sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$$

$$= 0.$$

Energies (powers)

$$\Rightarrow \sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} [x_e(n) + x_o(n)]^2$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + x_o^2(n) + 2x_e(n)\, x_o(n)$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + 2\sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$$

$$= E_e + E_o + 0$$

$$E = E_e + E_o$$

6) consider the s/m $y(n) = T[x(n)] = x(n^2)$

a) Determine if the s/m is time invariant.

Given $y(n) = T[x(n)] = x(n^2)$

$$x(n-k) \rightarrow y_1(n) = x[(n-k)^2]$$

$$= x(n^2 + k^2 - 2nk)$$

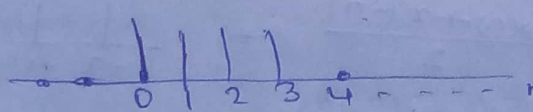
$$x(n-k) \neq y(n-k)$$

So the given s/m is time variant.

b) Determine the result in part (a) assume that the signal

$$x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$ is applied into the s/m.

1) sketch the signal $x(n)$ $\quad x(n) = \{ \ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0, 0 \ldots \}$



2) Det & sketch the signal $y(n) = T[x(n)]$

$$y(n) = T[x(n)] = x(n^2) \Rightarrow \{x(0), x(1), x(2^2), x(3^2), x(4^2) \ldots\}$$

$$\Rightarrow \{x(0), x(1), x(4), x(9), x(16), \ldots\}$$

$$y(n) = x(n^2) = \{\ldots 1, 1, 0, 0, 0, \ldots\}$$

0 1 2 3 4

3) Sketch the signal $y_2'(n) = y(n-2)$

$y(n-2) = \{ \ldots 0, 0, 1, 1, 0, 0, 0, 0, \ldots \}$
$\uparrow$

4) Sketch the signal $x_2(n) = x(n-2)$

$x(n-2) = \{ \ldots 0, 0, 1, 1, 1, 1, 0, \ldots \}$
$\uparrow$
0 1 2 3 4 5 6

5) Det and sketch the signal $y_2(n) = T\{x_2(n)\}$

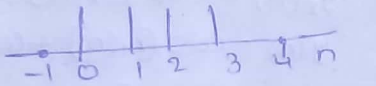
$y_2(n) = T[x_2(n-2)] = \{x(0), x(1), x(2), x(3), x(4), x(5), x(6)\}$
$= \{ \ldots 0, 1, 0, 0, 0, 1, 0, \ldots \}$
$\uparrow$

6) compare the signal $y_2(n)$ and $y(n-2)$. what is your conclusion?

$y_2(n) \neq y(n-2) \Rightarrow$ system is time variant.

c) Repeat part (b) for the system $y(n) = x(n) - x(n-1)$. can you use this statement about the time invariance of the s/m.

① $x(n) =$

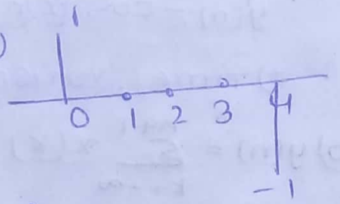


$= \{1, 1, 1, 1\}$
$\uparrow$

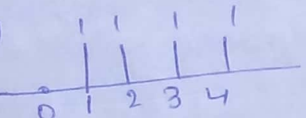
② $y(n) = x(n) - x(n-1)$

$x(n)$

$y(n) \Rightarrow x(n) - x(n-1) \Rightarrow$

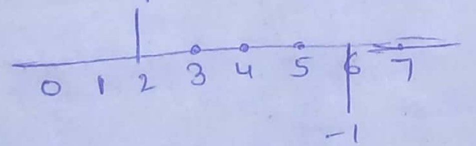


$x(n-1)$



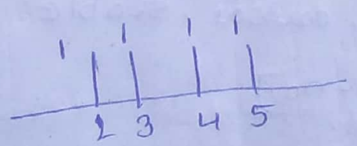
$\Rightarrow \{0, 1, 0, 0, 0, -1\}$
$\uparrow$

③ $y(n-2) \Rightarrow$



$\Rightarrow \{0, 0, 1, 0, 0, 0, -1\}$

④ $x(n-2) \Rightarrow$



$\Rightarrow \{0, 0, 1, 1, 1, 1\}$

⑤ $y_2(n) = \{0, 0, 1, 0, 0, 0, -1\}$

⑥ $y_2(n) = y(n-2) \Rightarrow$ s/m is time invariant.

d) Repeat parts (b) and (c) for the s/m $y(n) = T[x(n)] = n\,x(n)$.

$n \rightarrow$ integer value from 0 –

1) $y(n) = n\,x(n)$

$x(n) = \{ \ldots 0, 1, 1, 1, 1, 0, \ldots \}$
$\uparrow$

2) $y(n) = \{ \ldots 0, 1, 2, 3, 4, \ldots \}$
$\uparrow$

3) $y(n-2) = \{ \ldots 0, 0, 0, 1, 2, 3, 4, \ldots \}$
$\uparrow$

4) $x(n-2) = \{ \ldots 0, 0, 0, 1, 1, 1, 1 \ldots \}$
$\uparrow$

5) $y_2(n) = T[x(n-2)] = \{ \ldots 0, 0, 2, 3, 4, 5, \ldots \}$

6) $y_2(n) \neq y(n-2) \Rightarrow$ s/m is time variant.

7) 1) static or dynamic

2) Linear or Non linear

3) Time invariant or varying

4) causal or non causal

5) stable or unstable

Examine the following s/m with respect to the properties above

a) $y(n) = \cos[x(n)]$

i) static (only present i/p)

iv) only present i/p $\rightarrow$ causal

v) stable

iii) $y(n) = \cos[x(n-n_0)]$

$y'(n) = \cos[x(n-n_0)]$

$\Rightarrow$ Time variant

ii) $y_1(n) = \cos[x_1(n)]$

$y_2(n) = \cos[x_2(n)]$

$y(n) = \cos[x_1(n)] + \cos[x_2(n)]$

$y'(n) = \cos[x_1(n) + x_2(n)]$

$\Rightarrow$ Non-linear.

b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

Dynamic (depends on future values)

Linear, time invariant, non causal (also depends on future values).

unstable.

c) $y(n) = x(n) \cos[\omega_0(n)]$

static, linear, Time variant, causal, stable

$y(n) = x(n-n_0) \cos[\omega_0(n-n_0)]$

$y'(n) = x(n-n_0) \cos \omega_0 n.$

d) $y(n) = x(-n+2)$

Dynamic

$\Downarrow$

at $n=0 \Rightarrow y(0) = x(2)$

future value

$y_1(n) = x_1(-n+2) + x_2(-n+2)$

$y_2(n) = (x + x_2(-n+2)) \Rightarrow x_1(-n+2) + x_2(-n+2).$

Linear.

Non causal, Stable, Time invariant.

e) $y(n) = \text{Trun}\,[x(n)]$

static, non-linear, time invariant, causal, stable.

f) $y(n) = \text{Round}\,[x(n)]$

static, non-linear, time invariant, causal, stable.

g) $y(n) = |x(n)|$

static, non-linear, time invariant, causal, stable

h) static, linear, time invariant, causal, stable.

i) Dynamic, linear, Time variant, Non causal, stable

j) Dynamic, linear, Time variant, Non causal, stable

k) static, linear, time invariant, Non causal, stable.

l) Dynamic, linear, Time variant, causal, stable

m) static, non-linear, Time invariant, causal, stable.

n) $x(n) = x_a(nT)$

static, linear, time variant, non-causal, stable.

9) a) $x(n) = x(n+N) \quad \forall\, n \geq 0$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$y(n+N) = \sum_{k=-\infty}^{n+N} h(k)\, x(n+N-k)$$

$$y(n+N) = \sum_{k=n+1}^{n+N} h(k)\, x(n-k) + \sum_{k=-\infty}^{n} h(k)\, x(n-k)$$

$$y(n+N) = y(n) + \sum_{k=n+1}^{n+N} h(k)\, x(n-k)$$

For BIBO S/m $\quad \lim_{n\to\infty} |h(n)| = 0$

$$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k)\, x(n-k) = 0$$

$$\lim_{n\to\infty} y(n+N) = y(N)$$

$$\therefore\; y(N) = y(n+N).$$

b) If $x(n)$ is bounded & tends to a constant, the o/p will also tend to a constant.

$x(n) = x_0(n) + a\,u(n) \qquad x_0(n) \to$ bounded with $\lim_{n\to\infty} x_0(n) = 0$

$$\Rightarrow y(n) = a\sum_{k=0}^{\infty} h(k)\, u(n-k) + \sum_{k=0}^{\infty} h(k)\, x_0(n-k) = a\sum_{k=0}^{n} h(k) + y_0(n)$$

$\Rightarrow \sum_n x_o^2(n) < \infty \Rightarrow \sum_n y_o^2(n) < \infty$

hence, $\lim_{n\to\infty} |y_o(n)| = 0 \Rightarrow a \sum_{k=0}^{n} h(k) = \text{constant}$.

c) If $x(n)$ is a energy signal, the o/p $y(n)$ will also be an energy signal.

$$y(n) = \sum_k h(k)\, x(n-k)$$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty} \left[\sum_k h(k)\, x(n-k)\right]^2 = \sum_k \sum_l h(k) h(l) \sum_n x(n-k)\, x(n-l)$$

but $\sum_n x(n-k)\, x(n-l) \le \sum_n x^2(n)\, |h(l)|$

For BIBO stable S/m $\sum_k |h(k)| \le M$

Hence $E_y \le m^2 E_x$, so that $E_y < 0$ if $E_x < 0$.

10)

Sol:- As this is a time-invariant system.

$y_2(n)$ should have only 3 elements and

$y_3(n)$ should have 4 elements.

So, it is non-linear.

11)

Sol:- since $x_1(n) + x_2(n) = \delta(n)$

& S/m is linear, the impulse response of the system is

$$y_1(n) + y_2(n) = \{0, \underset{\uparrow}{3}, -1, 2, 1\}$$

If S/m was time invariant the response of $x_3(n)$ would be $\{\underset{\uparrow}{3}, 2, 1, 3, 1\}$

12) a)

Sol:- Any linear combination of signal in the form of

$x_i(n)$; $i = 1, 2, 3, \dots N$

because if we take $i = 1, 3$

$y_1(n) = x_1(n)$

$y_3(n) = x_3(n)$ $\Rightarrow y(n) = y_1(n) + y_3(n) = x_1(n) + x_3(n)$

$y(n) = x_1(n) + x_3(n)$

linear

b)

Sol:- Any $x_i(n-k)$ where k is any integer, $i = 1, 2, \dots N$

1^{st} replace $n = n - n_0 \Rightarrow x_i(n - n_0 - k)$

$x(n)$ by $x(n-n_0) \Rightarrow x_i(n-k-n_0)$

Time invariant

13)

Sol: A s/m to be BIBO stable only when bounded o/p produce bounded i/p.

$$y(n) = \sum_k h(k)\, x(n-k)$$

$$|y(n)| = \sum_k |h(k)|\, |x(n-k)|$$

$$= \sum_k |x(n-k)| \le m_n \text{ [some constant]}$$

so $|y(n)| = m_n \sum_k |h(k)|$

$|y(n)| < \infty \;\forall n$, if & only if $\sum_k |h(k)| < \infty$

so $\sum_{n=-\infty}^{\infty} |y(n)|$

$\Rightarrow$ A s/m to be BIBO stable only when bounded i/p produce bounded o/p.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \;;\; n \le n-k$$

$$|y(n)| = \sum_{k=-\infty}^{\infty} |h(k)|\,|x(n-k)| \;;\; k \ge 0$$

as $\sum_{k=-\infty}^{\infty} |x(n-k)| \le m_n$ for some constant.

$$|y(n)| = m_n \sum_{k=\infty}^{\infty} |h(k)| \;;\; n \le n-k \;;\; k \ge 0$$

$|y(n)|$ is $\le \infty$ if & only if $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$

so $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

14) a)

Sol: If a s/m is causal, o/p depends only on the present & past i/ps as $x(n) = 0 \;\forall n < n_0$ then $y(n)$ also becomes zero $\forall n < n_0$.

b)

Sol: $y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

For infinite impulse response

$h(n) = 0,\; n < 0$ and $n \ge m$

so $y(n)$ reduses to $y(n) = \sum_{k\ge 0}^{n-1} h(k)\, x(n-k)$

$y(n) = \sum$

15) a)

Sol:- For $a=1$, $\sum_{n=M}^{N} a^n = N-M+1$

for $a \neq 1$, $\sum_{n=m}^{N} a^n = \frac{a^m - a^{N+1}}{1-a}$

$(1-a)^N \sum_{n=m}^{N} a^n = a^m + a^{m+1} - a^{m+1} + \cdots a^N - a^N - a^{N+1}$

$= a^m - a^{N+1}$

b)

Sol:- For $M=0$, $|a|<1$ and $N \to \infty$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad |a|<1$$

16) a)

Sol:- $y(n) = \sum_k h(k)\, x(n-k)$

$\sum_n y(n) = \sum_n \sum_k h(k)\, x(n-k)$

$\sum_n y(n) = \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k)$

$\sum_n y(n) = \left(\sum_k h(k)\right)\left(\sum_n x(n)\right)$

b) i)

Sol:- $y(n) = \{1, 3, 7, 7, 7, 6, 4\}$

$\sum_n y(n) = 35$; $\sum_n x(n) = 7$, $\sum_n h(n) = 5$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$35 = 7 \times 5$

$35 = 35$

By Tabular Method

h(n) \ x(n)	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

ii)

Sol:- $x(n) = \{1, 2, -1\}$, $h(n) = \{1, 2, -1\}$

$y(n) = x(n) * h(n)$

$y(n) = \{1, 4, 2, -4, 1\}$

$\sum_n y(n) = 4$; $\sum_n x(n) = 2$, $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$4 = 2 \times 2$

$4 = 4$

h(n) \ x(n)	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

iii)

Sol:- $y(n) = \{0, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -2, 0, -\frac{5}{2}, 2\}$

$x(n)$ / $h(n)$	0	+1	-2	-3	-4
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$\frac{3}{2}$	-2
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$\frac{3}{2}$	-2
1	0	1	-2	3	-4
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$\frac{3}{2}$	-2

$\sum_n y(n) = 5$, $\sum_n x(n) = -2$, $\sum_n h(n) = \frac{5}{2}$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$-5 = -5$

iv)

Sol:- $y(n) = \{1, 2, 3, 4, 5\}$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$15 = \underline{15}$

v)

Sol:- $y(n) = \{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$

$x(n)$ / $h(n)$	$\overset{\downarrow}{1}$	-2	3
$\rightarrow 0$	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

$\sum_n y(n) = 8$; $\sum_n x(n) = 2$; $\sum_n h(n) = 4$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$\underline{8 = 8}$

vi)

Sol:- $y(n) = \{\underset{\uparrow}{0}\ 0, 1, -1, 2, 2, 1, 3\}$

$x(n)$ / $h(n)$	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

$\sum_n y(n) = 8$; $\sum_n x(n) = 4$, $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$8 = 4 \times 2$

$\underline{8 = 8}$

vii)

Sol:- $y(n) = \{\underset{\uparrow}{0}, 1, 4, -4, -5, -1, 3\}$

$x(n)$ / $h(n)$	$\overset{\downarrow}{0}$	1	4	-3
$\rightarrow 1$	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

$\sum_n y(n) = -2$; $\sum_n x(n) = -2$; $\sum_n h(n) = 1$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

viii)

Sol: $y(n) = \{ \underset{\uparrow}{1}, 2, 4, 3, 2 \}$

$\sum_n y(n) = 12$; $\sum_n x(n) = 4$; $\sum_n h(n) = 3$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$12 = 4 \times 3$

$12 = 12$

h(n) \ x(n)	↓ 1	1	2
→ 1	1	1	2
1	1	1	2
1	1	1	2

ix)

Sol: $y(n) = \{ \underset{\uparrow}{1}, -1, -5, 2, 3, -5, 1, 4 \}$

$\sum_n y(n) = 0$; $\sum_n x(n) = 4$; $\sum_n h(n) = 0$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$0 = 4 \times 0$

$0 = 0$

h(n) \ x(n)	1	1	↓ 0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
→ -4	-4	-4	0	-4	-4

x)

Sol: $y(n) = \{1, 4, 4, 4, 10, 4, 4, 4, 1\}$

$\sum_n y(n) = 36$, $\sum_n x(n) = 6$, $\sum_n h(n) = 6$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$36 = 6 \times 6$

$36 = 36$

h(n) \ x(n)	1	2	↓ 0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
→ 0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

xi)

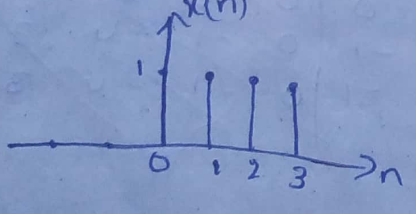
Sol: $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$

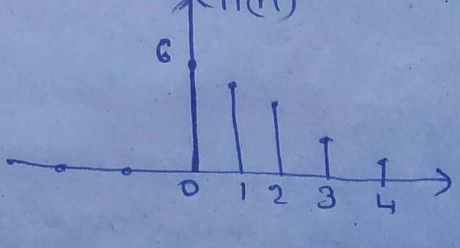
$y(n) = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n u(n)\right]$

$\sum_n y(n) = \frac{8}{3}$; $\sum_n h(n) = \frac{4}{3}$; $\sum_n x(n) = 2$.

17] compute and plot convolutions $x(n) * h(n)$ and $h(n) * x(n)$ for the pairs of signals shown below.

a]





$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$

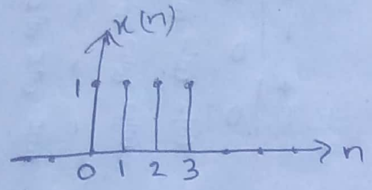
$h(n) = \{\underset{\uparrow}{6}, 5, 4, 3, 2, 1, 0\}$

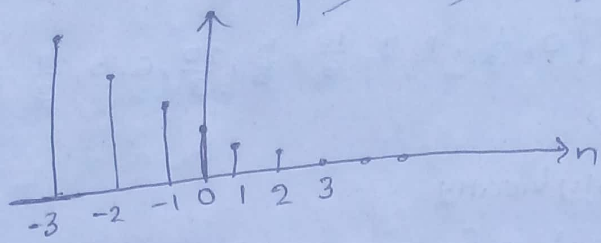
$y(n) = x(n) * h(n)$

$y(n) = \{\underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1\}$

h(n) \ x(n)	1	1	1
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1

b)





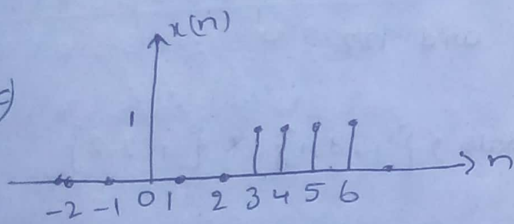
$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$

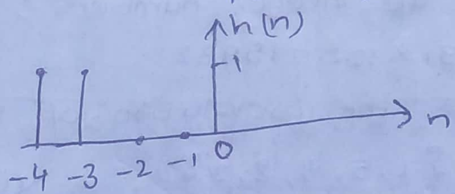
$h(n) = \{6, 5, 4, \underset{\uparrow}{3}, 2, 1, 0\}$

$y(n) = \{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\}$

h(n) \ x(n)	1	1	1
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1

c)





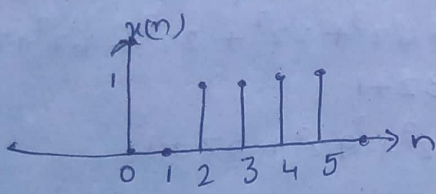
$x(n) = \{\underset{\uparrow}{0}, 0, 0, 1, 1, 1, 1\}$

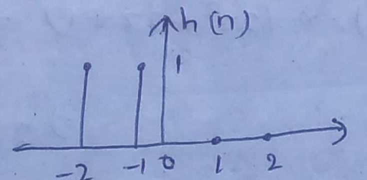
$h(n) = \{1, 1, 0, 0, \underset{\uparrow}{0}\}$

$y(n) = \{0, 0, 0, 1, 2, 2, 2, 1, 0, 0, 0\}$

h(n) \ x(n)	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

d)





$x(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$

$h(n) = \{1, 1, \underset{\uparrow}{0}, 0, 0\}$

$y(n) = \{0, 0, \underset{\uparrow}{1}, 2, 2, 2, 1, 0, 0, 0\}$

h(n) \ x(n)	0	0	1	1	1	1
1	0	0	1	1	1	1
1	0	0	1	1	1	1
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

18] Determine and sketch the convolution $y(n)$ of the signals

$$x(n) = \begin{cases} \frac{1}{3}n, & 0 \le n \le 6 \\ 0, & \text{elsewhere} \end{cases} \qquad h(n) = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere.} \end{cases}$$

a) graphically.

$x(n) = \{0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\}$

$h(n) = \{1, 1, 1, 1, 1\}$

$y(n) = \{0, \frac{1}{3}, 1, 2, \frac{10}{3}, \frac{5}{8}, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$

$h(n) \backslash x(n)$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
→ 1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2

b) Analytically

$x(n) = \frac{1}{3}n\,[u(n) - u(n-7)]$

$h(n) = u(n+2) - u(n-3)$

$y(n) = \frac{1}{3}n\,[u(n) - u(n-7)] * u(n+2) - u(n-3)$

$= \frac{1}{3}n\,[u(n) * u(n+2)] - \frac{1}{3}n\,[u(n) * u(n-3)] - \frac{1}{3}n\,[u(n-7) * u(n+2)]$
$+ \frac{1}{3}n\,[u(n-7) * u(n-3)].$

20] consider the following three operations

a] Multiply the integer numbers ; 131 and 122

$131 \times 122 = 15982.$

b] compute the convolution of signals ; $\{1, 3, 1\} * \{1, 2, 2\}$.

$y(n) = \{1, 5, 9, 8, 2\}$

$h(n) \backslash x(n)$	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

c) Multiply the polynomials:-

$1 + 3z + z^2$ and $1 + 2z + 2z^2$

$(z^2 + 3z + 1) \times (2z^2 + 2z + 1) \Rightarrow 2z^4 + 6z^3 + 2z^2 + 2z^3 + 6z^2 + 2z + z^2 + 3z + 1$

$\Rightarrow 2z^4 + 8z^3 + 9z^2 + 5z + 1.$

d] Repeat part (a) for the numbers 1.31 and 12.2

$1.31 \times 12.2 = 15.982$

e) comment on your results.

These are different ways to perform convolution.

21] compute the convolution $y(n) * h(n)$ of the following pairs of the signals.

a] $x(n) = a^n u(n)$, $h(n) = b^n u(n)$ when $a \neq b$ & when $a = b$.

$$y(n) = x(n) * h(n)$$
$$= a^n u(n) * b^n u(n)$$
$$= [a^n * b^n]\, u(n)$$
$$y(n) = \sum_{k=0}^{n} a^k u(k)\, b^{n-k} u(n-k)$$
$$= b^n \sum_{k=0}^{n} a^k u(k)\, b^{-k}$$
$$= b^n \sum_{k=0}^{n} \cancel{a^k} (ab)^{-k}$$

if $a \neq b$ then $y(n) = \dfrac{b^{n+1} - a^{n+1}}{b-a}\, u(n)$

if $a = b \Rightarrow b^n (n+1)\, u(n)$

b] $x(n) = \begin{cases} 1 ; & n = -2, 0, 1 \\ 2 ; & n = -1 \\ 0 ; & \text{elsewhere} \end{cases}$ $h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

$x(n) = \{1, 2, 1, 1\}$ $h(n) = \{\underset{\uparrow}{1}, -1, 0, 0, 1, 1\}$

$y(n) = \{1, 1, \underset{\uparrow}{-1}, 0, 0, 3, 3, 2, 1\}$

h(n) \ x(n)	1	2	1	1
1	1	2	1	1
-1	-1	-2	-1	-1
0	0	0	0	0
0	0	0	0	0
1	1	2	1	1
1	1	2	1	1

c] $x(n) = u(n+1) - u(n-4) - \delta(n-5)$

$h(n) = [u(n+2) - u(n-3)]\ 3(\delta(n))$

d] $x(n) = u(n) - u(n-5)$;

$h(n) = u(n-2) - u(n+8) + u(n-11) - u(n-17)$

g] $x(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$

$h(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$y(n) = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, 1\}$

h(n) \ x(n)	1	1	1	1	1	0	-1
1	1	1	1	1	1	0	-1
2	2	2	2	2	2	0	-2
3	3	3	3	3	3	0	-3
2	2	2	2	2	2	0	2
1	1	1	1	1	1	0	-1

d) $x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$h'(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1, 1\}$

$h(n) = h'(n) + h'(n-9)$

$y(n) = y'(n) + y'(n-9)$, where

$y'(n) = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

h(n) \ x(n)	1	1	1	1	1
0	0	0	0	0	0
0	0	0	0	0	0
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1

22) $x(n) = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

$h_1(n) = \{1, 1\}$ $\qquad h_4(n) = \{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$

$h_2(n) = \{1, 2, 1\}$ $\qquad h_5(n) = \{\frac{1}{4}, \frac{-1}{2}, \frac{1}{4}\}$

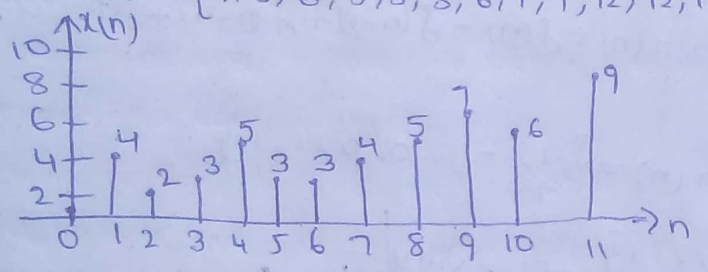
$h_3(n) = \{\frac{1}{2}, \frac{1}{2}\}$

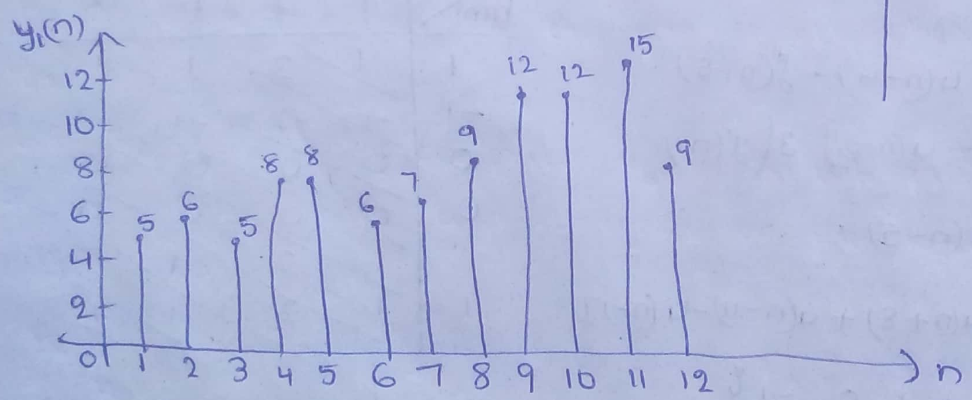
Sketch $x(n)$, $y_1(n)$, $y_2(n)$ on one graph and $x(n)$, $y_3(n)$, $y_4(n)$, $y_5(n)$ on another graph.

Sol: $y_1(n) = x(n) * h_1(n)$

$y_1(n) = \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 12, 15, 9\}$

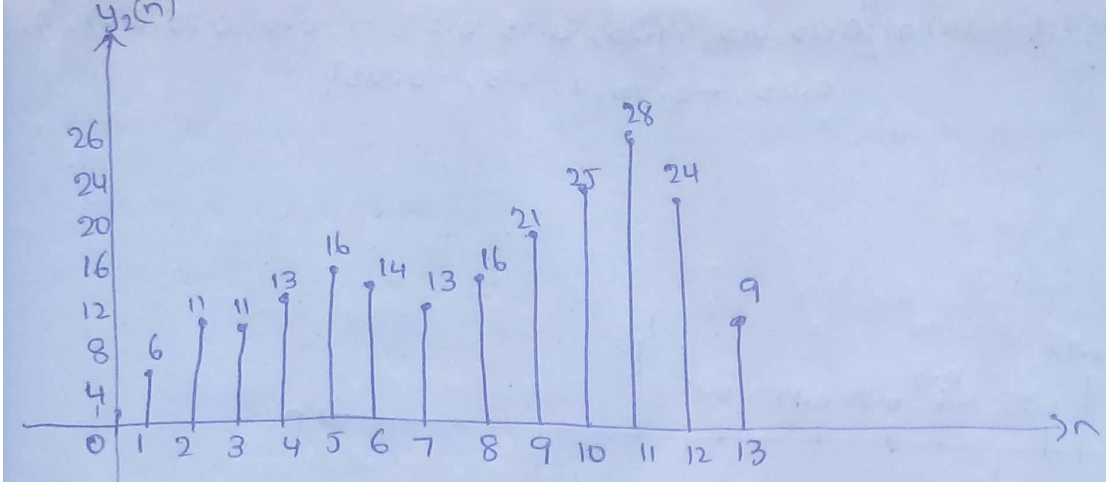
h(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9





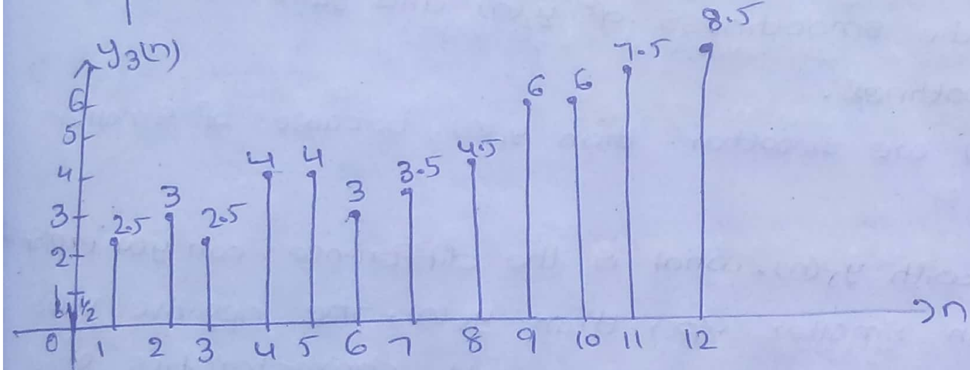
$y_2(n) = x(n) * h_2(n) \Rightarrow \{1, 6, 11, 11, 13, 16, 14, 13, 16, 21, 25, 28, 24, 9\}$

h(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
2	2	8	4	6	10	6	6	8	10	14	12	18
1	1	4	2	3	5	3	3	4	5	7	6	9



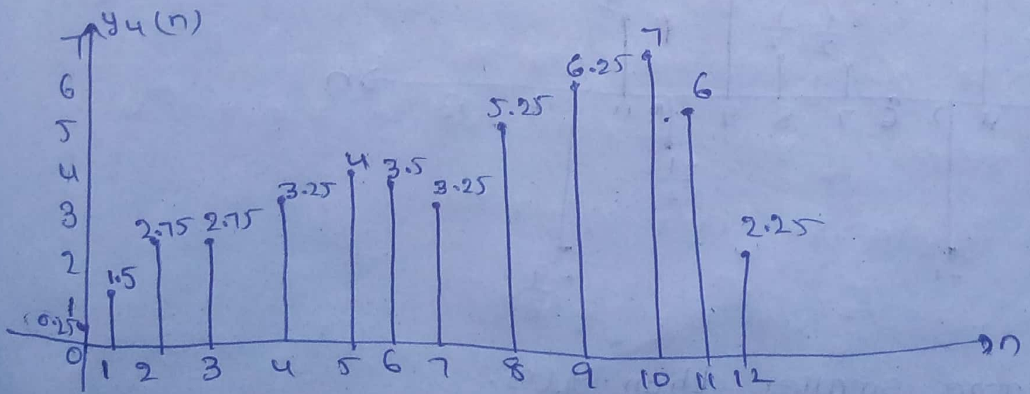
$\Rightarrow y_3(n) = x(n) * h_3(n) \Rightarrow \left\{ \frac{1}{2}, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6, 7.5, \frac{9}{2} \right\}$

$h(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{2}$	$\frac{1}{2}$	2	1	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$	$\frac{7}{2}$	3	$\frac{9}{2}$
$\frac{1}{2}$	$\frac{1}{2}$	2	1	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$	$\frac{7}{2}$	3	$\frac{9}{2}$

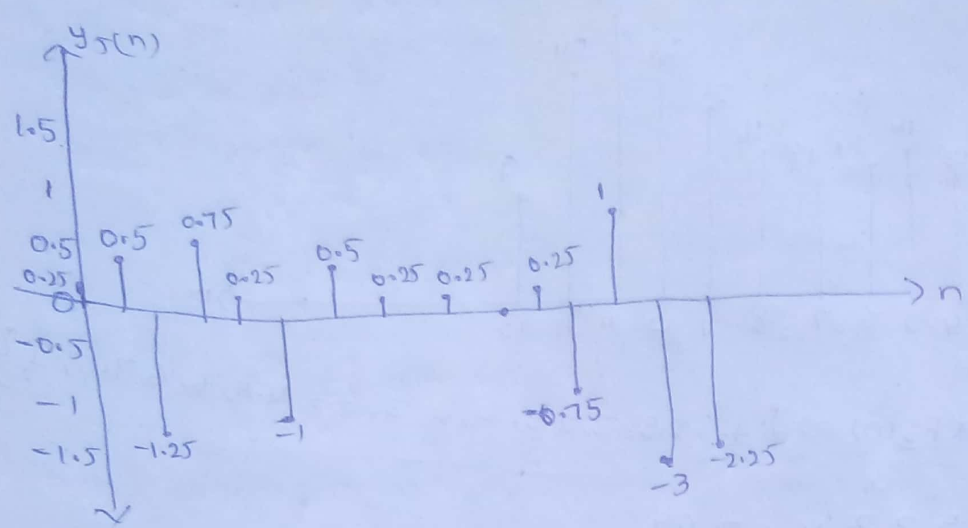


$\Rightarrow y_4(n) = x(n) * h_4(n) \Rightarrow \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 5.25, 6.25, 7, 6, 2.25\}.$

$h(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{3}{2}$	$\frac{9}{4}$
$\frac{1}{2}$	$\frac{1}{2}$	2	1	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$	$\frac{7}{2}$	3	$\frac{9}{2}$
$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{3}{2}$	$\frac{9}{4}$



$\Rightarrow y_5(n) = x(n) * h_5(n) \Rightarrow \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0.25, 0, 0.25, -0.75, 1, -3, -2.25\}$



b) what is the difference b/w $y_1(n)$ and $y_2(n)$ and b/w $y_3(n)$ & $y_4(n)$.

$y_3(n) = \frac{1}{2} y_1(n)$; $h_3(n) = \frac{1}{2} h_1(n)$

$y_4(n) = \frac{1}{4} y_2(n)$; $h_4(n) = \frac{1}{4} h_2(n)$

c) comment on the smoothness of $y_2(n)$ and $y_4(n)$. which factors affect the smoothness.

$y_2(n)$ and $y_4(n)$ are smoother than $y_1(n)$ because of smaller scalar factor.

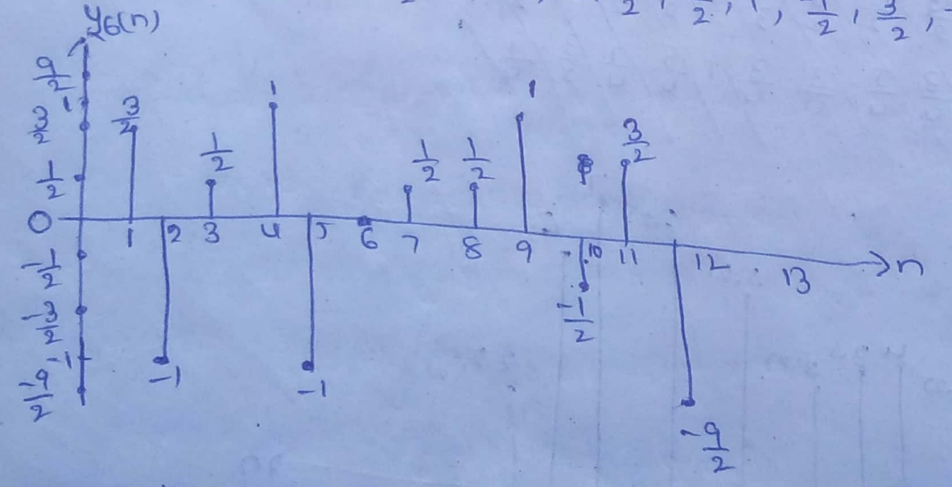
d) compare $y_4(n)$ with $y_5(n)$. what is the difference can you explain it?

$y_4(n)$ results in smaller o/p, than $y_5(n)$. The negative value of $h_5(0)$ is responsible for the non-smooth characteristics of $y_5(n)$.

e) Let $h_6(n) = \{\frac{1}{2}, \frac{-1}{2}\}$ compute $y_6(n)$. sketch $x(n)$, $y_2(n)$ and $y_6(n)$ on the same figure and comment on the results.

$y_6(n) = x(n) * h_6(n)$.

$y_6(n) = \{\frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, \frac{-1}{2}, \frac{3}{2}, -\frac{9}{2}\}$



$y_2(n)$ is more smaller than $y_6(n)$.

23] $\delta(n) = h(n) * u(n)$ & i/p $x(n)$.

Sol: we can express $\delta(n) = u(n) - u(n-1)$

$h(n) = h(n) * \delta(n)$

$= h(n) * [u(n) - u(n-1)]$

$= h(n) * u(n) - h(n) * u(n-1)$

$= \delta(n) - \delta(n-1)$

then $y(n) = h(n) * x(n)$

$= [\delta(n) - \delta(n-1)] * x(n)$

$= \delta(n) * x(n) - \delta(n-1) * x(n)$

24] $y(n) = ny(n-1) + x(n)$, $n \geq 0$. check if system is LTI & stable.

Sol:- $y(n) = ny(n-1) + x(n)$, $n \geq 0$

$$\left.\begin{aligned} y_1(n) &= ny_1(n-1) + x_1(n) \\ y_2(n) &= ny_2(n-1) + x_2(n) \end{aligned}\right\} \oplus \Rightarrow y(n) = ny_1(n-1) + x_1(n) + ny_2(n-1) + x_2(n).$$

$y(n) = ny(n-1) + x(n)$

$x(n) = ax_1(n) + bx_2(n)$

$y(n) = ay_1(n) + by_2(n)$.

Hence the s/m is linear.

$\Rightarrow y(n-1) = (n-1)y(n-2) + x(n-1)$

delayed $\Rightarrow y(n-1) = ny(n-2) + x(n-1)$

so the s/m is TI.

$\Rightarrow$ If $x(n) = u(n)$, then $|x(n)| \leq 1$, for this bounded i/p, o/p is $y(0) = 0$, $y(1) = 2$, $y(2) = 5$, - - - - -, unbounded. so s/m is unstable.

25] consider the signal $\delta(n) = a^n u(n)$, $0 < a < 1$

a] show that any sequence $x(n)$ can be decomposed as

$x(n) = \sum_{n=-\infty}^{\infty} c_K r(n-k)$ and express c_K in terms of $x(n)$.

Sol: $\delta(n) = r(n) - ar(n-1)$

$\delta(n-K) = r(n-k) - ar(n-k-1)$

$x(n) = \sum_{K=-\infty}^{\infty} x(k) \delta(n-k)$

$= \sum_{K=-\infty}^{\infty} x(k) [r(n-k) - ar(n-k-1)]$

$x(n) = \sum_{K=-\infty}^{\infty} x(k) r(n-k) - a \sum_{K=-\infty}^{\infty} x(k) r(n-k-1)$

$x(n) = \sum_{K=-\infty}^{\infty} x(k) r(n-k) - a \sum_{K=-\infty}^{\infty} x(k-1) r(n-k)$

$$= \sum_{K=-\infty}^{\infty} [x(K) - a x(K-1)] \gamma(n-K)$$

thus $\quad C_K = x(K) - a x(K-1)$

b] $y(n) = \tau[x(n)]$

$$= \tau\left[\sum_{K=-\infty}^{\infty} C_K \gamma(n-K)\right]$$

$$= \sum_{K=-\infty}^{\infty} C_K \tau[\gamma(n-K)]$$

$$= \sum_{K=-\infty}^{\infty} C_K g(n-K)$$

c) $h(n) = \tau[\delta(n)]$

$$h(n) = \tau[\gamma(n) - a\gamma(n-1)]$$

$$= g(n) - a g(n-1)$$

26] Determine the zero- i/p resistance of the s/m described by the second-order differential equation.

$$x(n) - 3y(n-1) - 4y(n-2) = 0$$

with $x(n) = 0$

$$-3y(n-1) - 4y(n-2) = 0 \qquad [\div(-3)]$$

$$y(n-1) + \frac{4}{3}y(n-2) = 0$$

at $n=0$

$$y(-1) = -\frac{4}{3}y(-2)$$

at $n=1$

$$y(0) = -\frac{4}{3}y(-1) = \left(-\frac{4}{3}\right)^2 y(-2)$$

$$y(1) = \left(-\frac{4}{3}\right)^3 y(-2)$$

$$\vdots$$

$$y(K) = \left(-\frac{4}{3}\right)^{K+2} y(-2)$$

zero i/p ~~resistance~~ response.

27] $y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$ when $x(n) = 2^n u(n)$.

sol'n $y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$

$$x(n) = y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2)$$

characteristic equation is

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0; \qquad \lambda = \frac{1}{2}, \frac{1}{3}$$

so $y_h(n) = c_1 \left(\frac{1}{2}\right)^n + c_2 \left(\frac{1}{3}\right)^n$

$x(n) = 2^n u(n)$

$y_p(n) = k(2^n)\, u(n)$

so $k(2^n)u(n) - k\left(\frac{5}{6}\right)(2^{n-1})u(n-1) + k\left(\frac{1}{6}\right)(2^{n-2})u(n-2) = 2^n u(n)$

for $n=2$, $4k - \frac{5k}{3} + \frac{k}{6} = 4$

$$k = \frac{8}{5}$$

Total solution is

$$y_p(n) + y_n(n) = y(n)$$

$$y(n) = \frac{8}{5}(2^n)u(n) + c_1\left(\frac{1}{2}\right)^n u(n) + c_2\left(\frac{1}{3}\right)^n u(n)$$

Assume $y(-2) = y(-1) = 0$ so $y(0) = 1$

then $y(1) = \frac{5}{6}y(0) + 2 = \frac{17}{6}$

so $\frac{8}{5} + c_1 + c_2 = 1$

$$c_1 + c_2 = \frac{3}{5} \rightarrow (1)$$

$$\frac{16}{5} + \frac{1}{2}c_1 + \frac{1}{3}c_2 = \frac{17}{6}$$

$$3c_1 + 2c_2 = \frac{-11}{5} \rightarrow (2)$$

By solving 1 & 2

$$c_1 = -1, \quad c_2 = \frac{2}{5}.$$

So the total solution is

$$y(n) = \left[\frac{8}{5}(2^n) - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right]u(n).$$

28)

Soln at $y(-1) = 1$

The given equation is $y(n) = (-a)^{n+1} + \frac{(1-(-a)^{n+1})}{1+a}$

$$y(n) = y_{zi}(n) + y_{zs}(n)$$

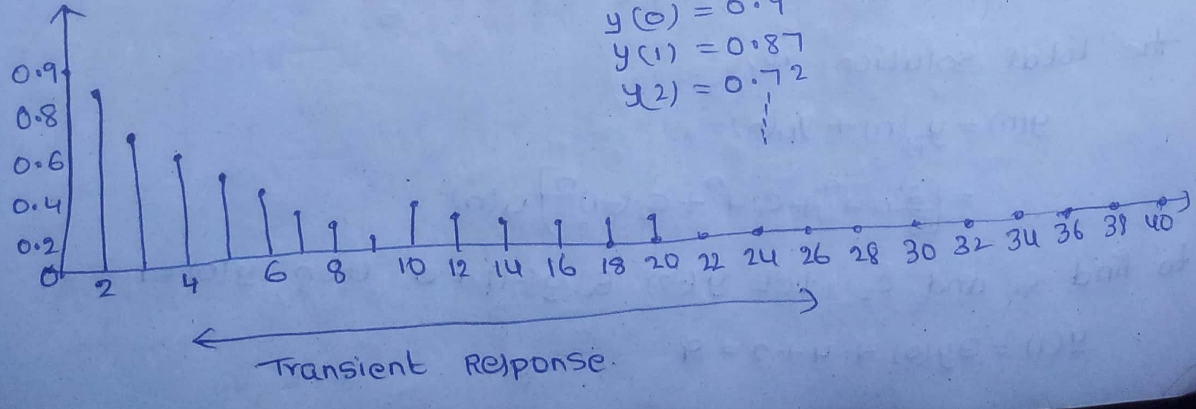
= Transient + ~~statistical~~ steady state

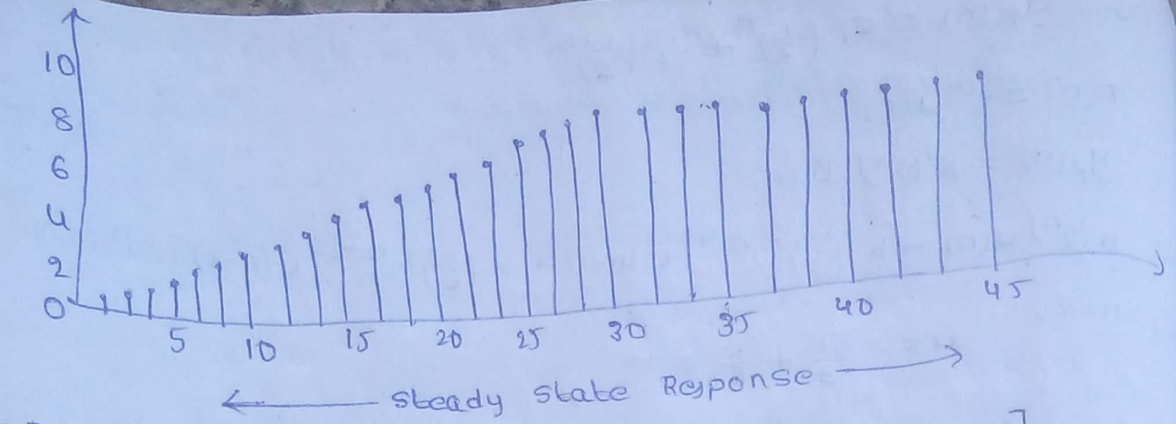
$y(0) = 0.9$

$y(1) = 0.87$

$y(2) = 0.72$

⋮





29] $h_1(n) = a^n [u(n) - u(n-N)]$ & $h_2(n) = [u(n) - u(n-m)]$

Sol:- $h(n) = h_1(n) * h_2(n)$

$$= \sum_{k=-\infty}^{\infty} a^k [u(k) u(k-N)] [u(n-k) - u(n-k-m)]$$

$$= \sum_{k=-\infty}^{\infty} a^k u(k) u(n-k) - \sum_{k=-\infty}^{\infty} a^k u(k) u(n-k-m) -$$

$$\sum_{k=-\infty}^{\infty} a^k u(k-N) u(n-k) + \sum_{k=-\infty}^{\infty} a^k u(k-N) u(n-k-m)$$

$$= \left(\sum_{k=0}^{n} a^k - \sum_{k=0}^{n-m} a^k\right) - \left(\sum_{k=N}^{n} a^k - \sum_{k=N}^{n-m} a^k\right)$$

$h(n) = 0$

30] $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ to i/p $x(n) = 4^n u(n)$

Sol:- $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

characteristic equation is

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda = 4, -1$$

so $y_h(n) = c_1 4^n + c_2 (-1)^n$

$x(n) = 4^n u(n)$

$y_p(n) = k n 4^n u(n)$

$$kn 4^n u(n) - 3k(n-1) 4^{n-1} u(n-1) - 4k(n-2) 4^{n-2} u(n-2) = 4^n u(n) + 2(4)^{n-1} u(n-1)$$

for $n=2$, $k(32-12) = 4^2 + 8 = 24 \rightarrow k = \frac{6}{5}$

the total solution is,

$$y(n) = y_p(n) + y_h(n)$$

$$= \left[\frac{6}{5} n 4^n + c_1 4^n + c_2 (-1)^n\right] u(n)$$

to find c_1 and c_2, Let $y(-2)$ & $y(-1) = 0$ then $y(0) = 1$

$y(1) = 3y(0) + 4 + 2 = 9$

$c_2 + c_1 = 1 \rightarrow ①$

$\frac{24}{5} + 4c_1 - c_2 = 9 \Rightarrow 4c_1 - c_2 = \frac{21}{5} \rightarrow ②$

from ① & ②

$c_1 = \frac{26}{25}$ & $c_2 = \frac{-1}{25}$

so $y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25}(-1)^n\right] u(n)$.

31] $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

sol: characteristic equation :- $\lambda^2 - 3\lambda - 4 = 0$

$\lambda = -4, 1$

$y_h(n) = c_1 4^n + c_2 (-1)^n$

$x(n) = \delta(n)$

$y(0) = 1$ and $y(1) - 3y(0) = 2$

$y(1) = 5$

so $c_1 + c_2 = 1 \rightarrow ①$

$4c_1 - c_2 = 5 \rightarrow ②$

from ① & ② $c_1 = \frac{6}{5}$ & $c_2 = \frac{-1}{5}$

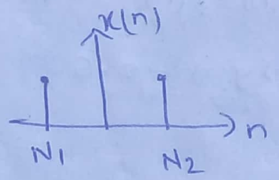
$\therefore h(n) = \left[\frac{6}{5} 4^n - \frac{1}{3}(-1)^n\right] u(n)$.

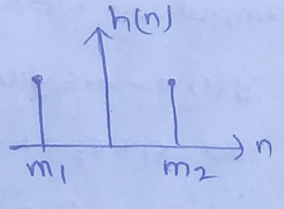
32]

a] Determine the range $L_1 \leq n \leq L_2$ of their convolution, in terms of N_1, N_2, M_1 and M_2.

$L_1 = N_1 + M_1$

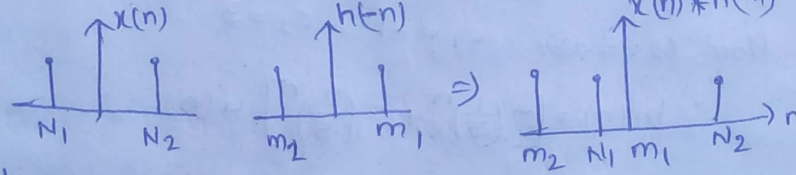
$L_2 = N_2 + M_2$





b] partial overlap from left

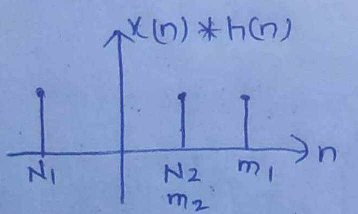
$\rightarrow x(n) * h(n) \Rightarrow$



Low $N_1 + m_1$ & high $m_2 + N_1 - 1$

If fully overlap then $N_1 + m_2$ (Low) & high $N_2 + m_1$.

Partial overlap from right

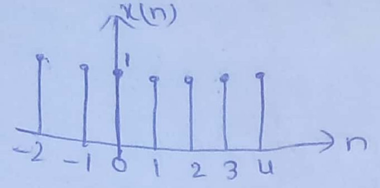


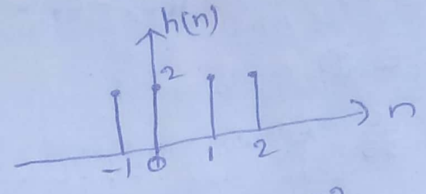
Low ⇒ $N_2 + m_1 + 1$

high ⇒ $N_2 + m_2$

If fully overlaped high $N_2 + m_1$; Low $= N_1 + m_2$.

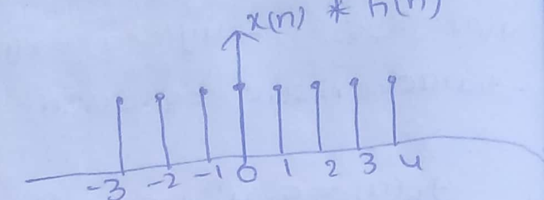
c] $x(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\}$ $\qquad h(n) = \{2, \underset{\uparrow}{2}, 2, 2\}$ 6)





$m_1 = -1, \; m_2 = 2.$

$N_1 = -2, \; N_2 = 4$



Partial overlap from left

Low $N_1 + m_1 = -3$

high $m_2 + N_1 - 1 = 2 - 2 - 1 = -1$

full overlap $n = 0, \; n = 3$

partial right; $n = 4, \; n = 6, \; L_2 = 6.$

33]

a] $y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

Sol: $x(n) = y(n) - 0.6y(n-1) - 0.08y(n-2)$

characteristic equation

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$$\lambda = \frac{1}{2}, \frac{2}{5}$$

$$y_h(n) = c_1\left(\frac{1}{5}\right)^n + c_2\left(\frac{2}{5}\right)^n$$

Impulse response $x(n) = \delta(n)$ with $y(0) = 1$

$y(1) - 0.6y(0) = 0 \Rightarrow y(1) = 0.6$

so $c_1 + c_2 = 1 \rightarrow ①$

$\frac{1}{5}c_1 + \frac{2}{5}c_2 = 0.6 \rightarrow ②$

from ① & ② $\quad c_1 = -1, \; c_2 = 3.$

$$\therefore h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right]u(n)$$

step response $x(n) = u(n)$

$$s(n) = \sum_{k=0}^{n} y\,h(n-k), \quad n \geq 0$$

$$= \sum_{k=0}^{n}\left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= 2\left(\frac{2}{5}\right)^n \sum_{k=0}^{n}\left(\frac{5}{2}\right)^k - \left(\frac{1}{5}\right)^n \sum_{k=0}^{n}(5)^k$$

$$= \left[2\left(\frac{2}{5}\right)^n\left(\left(\frac{5}{2}\right)^{n+1} - 1\right) - \left(\frac{1}{5}\right)^k\left(5^{n+1} - 1\right)\right]u(n).$$

b) $y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2)$

$2x(n) - x(n-2) = y(n) - 0.7y(n-1) + 0.1y(n-2)$

characteristic equation

$\lambda^2 - 0.7\lambda + 0.1 = 0$

$\lambda = \frac{1}{2}, \frac{1}{5}$

$y_h(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{5}\right)^n$

Impulse Response $x(n) = \delta(n)$, $y(0) = 2$

$y(1) - 0.7y(0) = 0 \Rightarrow y(1) = 1.4$

$C_1 + C_2 = 2$

$\frac{1}{2}C_1 + \frac{1}{5}C_2 = \frac{7}{5} \rightarrow (1)$

$C_1 + \frac{2}{5}C_2 = \frac{14}{5} \rightarrow (2)$

solving (1) & (2)

$C_1 = \frac{10}{3}$, $C_2 = \frac{-4}{3}$

so $h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n)$

Step Response $s(n) = \sum_{k=0}^{n} h(n-k)$

$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$

$= \frac{10}{3}\left(\frac{1}{2}\right)^n\sum_{k=0}^{n}2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n\sum_{k=0}^{n}5^k$

$= \frac{10}{3}\left[\frac{1}{2}^n(2^{n+1}-1)u(n)\right] - \frac{4}{3}\left[\frac{1}{5}^n(5^{n+1}-1)\right]u(n)$

34] $h(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \le n \le 4 \\ 0, & \text{else where} \end{cases}$

$y(n) = \{\underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1; 0\}$

Sol:- $h(n) = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots\}$

$y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0, \dots\}$.

$y(0) = x(0)h(0)$

$y(0) = x(0)\cdot 1 \Rightarrow x(0) = 1$

$y(1) = x(1) + h(1)x(0)$

$2 = x(1) + \frac{1}{2}(1) \Rightarrow \cancel{x(0)}\ x(1) = \frac{3}{2}$

$y(2) = x(2) + h(2)x(1) + h(1)x(0)$.

$2.5 = x(2) + \frac{1}{4}\left(\frac{3}{2}\right) + \frac{1}{2}(1)$

So

$x(n) = \{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2}, \ldots\}$.

35]

a] Express the overall impulse response in terms of $h_1(n), h_2(n)$, $h_3(n)$ & $h_4(n)$.

Sol:-

$h(n) = h_1(n) * [h_2(n) - \{h_3(n) * h_4(n)\}]$

b] Determine $h(n)$ when $h_1(n) = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{2}\}$

$h_2(n) = h_3(n) = (n+1)u(n)$

$h_4(n) = \delta(n-2)$.

Sol:-

$h_3(n) * h_4(n) = (n+1)u(n) * \delta(n-2)$

$= (n+1)u(n-2) = (n+1)u(n-2)$

$h_2(n) - [h_3(n) * h_4(n)] = (n+1)u(n) - (n+1)u(n-2)$

$= 2u(n) - \delta(n)$.

$h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$

$h(n) = \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right] * [2u(n) - \delta(n)]$

$= \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$.

c] Determine the response of the s/m in part (b) if

$x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$.

$x(n) = \{1, 0, \underset{\uparrow}{0}, 3, 0, -4\}$

36]

Sol:- $s(n) = u(n) * h(n)$

$s(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$

$= \sum_{k=0}^{\infty} h(n-k)$

$= \sum_{k=0}^{\infty} a^{n-k}$

$= \frac{a^{n+k} - 1}{a-1}$; $n \geq 0$.

For $x(n) = u(n+5) - u(n-10)$ then

$s(n+5) - s(n-10) = \frac{a^{n+6} - 1}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10)$

from given figure $y(n) = x(n) * h(n) - x(n) * h(n-2)$

$$y(n) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10) - \frac{a^{n+4}-1}{a-1} u(n+3) + \frac{a^{n+1}-1}{a-1} u(n-12)$$

37] compute & sketch step response of the s/m.

$$y(n) = \frac{1}{M} \sum_{k=0}^{m-1} x(n-k)$$

$$h(n) = \left[\frac{u(n) - u(n-m)}{m}\right]$$

$$s(n) = \sum_{k=-\infty}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k) = \begin{cases} \frac{n+1}{m}, & n < m \\ 1, & n \geq m. \end{cases}$$

38] $h(n) = \begin{cases} a^n, & n \geq 0 \\ 0, & \text{otherwise.} \end{cases}$

Sol:- $$\sum_{k=-\infty}^{\infty} |h(n)| = \sum_{n=0,\ n=\text{even}}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n} = \frac{1}{1-|a|^2}$$

stable if $|a| < 1$

39] $h(n) = a^n u(n)$ to i/p signal $x(n) = u(n) - u(n-10)$.

Sol:- $h(n) = a^n u(n)$

$$y_1(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} a^{n-k}$$

$$= a^m \sum_{k=0}^{n} a^{-k}$$

$$= \frac{1-a^{n+1}}{1-a} u(n)$$

$$y(n) = y_1(n) - y_1(n-10)$$

$$= \frac{1}{1-a}\left[(1-a^{n+1})\, u(n) - (1-a^{n-9})\, u(n-10)\right]$$

40] $x(n) = \begin{cases} 1, & 0 \leq n \leq 10 \\ 0, & \text{otherwise} \end{cases}$

Sol:- From 36th problem with $a = \frac{1}{2}$

$$y(n) = 2\left[1 - \left(\tfrac{1}{2}\right)^{n+1}\right] u(n) - 2\left[1\left(\tfrac{1}{2}\right)^{n-9}\right] u(n-10).$$

41]

a] $x(n) = 2^n u(n)$.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k 2^{n-k}$$

$$= 2^n \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k$$

$$= 2^n \left(1 - \left(\frac{1}{4}\right)^{n+1}\right)\left(\frac{4}{3}\right)$$

$$= \frac{2}{3}\left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right] u(n).$$

b] $x(n) = u(-n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{\infty} h(k)$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = 2, \quad n<0$$

$$y(n) = \sum_{k=n}^{\infty} h(k)$$

$$= \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)^k$$

$$= 2 - \left(\frac{1-\left(\frac{1}{2}\right)^n}{\frac{1}{2}}\right)$$

$$= 2\left(\frac{1}{2}\right)^n, \quad n \geq 0$$

42]

a] what is the impulse response, $h_c(n)$ of the overall S/m.

Sol:- $h_c(n) = h_1(n) * h_2(n) * h_3(n)$

$$= [\delta(n) - \delta(n-1)] * u(n) * h(n)$$

$$= [u(n) - u(n-1)] * h(n)$$

$$= \delta(n) * h(n) = h(n).$$

b] NO.

43)

a] $x(n)\,\delta(n-n_0) = x(n_0)$. thus only the value of $x(n)$ at $n=n_0$ is of interest.

$x(n) * \delta(n-n_0) = x(n-n_0)$. thus, we obtained shifted version of $x(n)$ sequence.

b]

Sol:- $y(n) = \sum_{k=-\infty}^{\infty} h(k)\,x(n-k)$

$= h(n) * x(n)$

Linearity:- $x_1(n) \to y_1(n) = h(n) * x_1(n)$

$x_2(n) \to y_2(n) = h(n) * x_2(n)$

$= \alpha\, h(n) * x_1(n) + \beta h(n) * x_2(n)$

$= \alpha\, y_1(n) + \beta y_2(n)$.

Time invariance:-

$x(n) \to y_1(n) = h(n) * x(n)$

$x(n-n_0) \to y_1(n) = h(n) * x(n-n_0)$

$= \sum_{k} h(k)\, x(n-n_0-k)$

$= y(n-n_0)$.

c] $h(n) = \delta(n-n_0)$

45]

Sol:- $y(n) = \frac{-1}{2} y(n-1) + x(n) + 2x(n-2)$

at $y(-2) = \frac{-1}{2} y(-3) + x(-2) + 2x(-4) = 1$

$y(-1) = \frac{-1}{2} y(-2) + x(-1) + 2x(-3) = \frac{3}{2}$.

$y(0) = \frac{-1}{2} y(-1) + 2x(-2) + x(0) = \frac{17}{4}$

$y(1) = \frac{-1}{2} y(0) + x(1) + 2x(-1) = \frac{47}{8}$.

47]

a] $x(n) = \{\underset{\uparrow}{1}, 0, 0, \ldots\}$

$y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

$y(0) = x(0) = 1$

$y(1) = \frac{1}{2} y(0) + x(1) + x(0) = \frac{3}{2}$.

$y(2) = \frac{1}{2} y(1) + x(2) + x(1) = \frac{3}{4}$. thus we obtain

$y(n) = \{1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, \ldots\}$.

b] $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

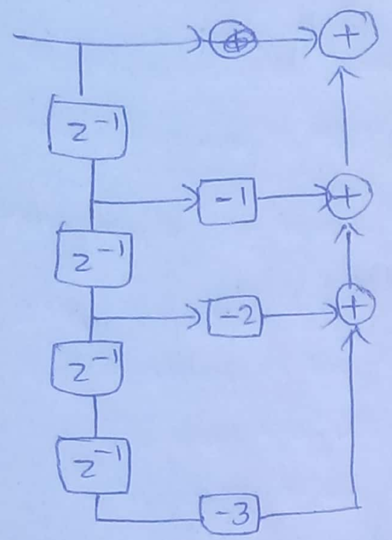
c] as in part (a) we obtain

$$y(n) = \left\{1, \frac{5}{2}, \frac{13}{4}, \frac{29}{8}, \frac{61}{16}, \ldots\right\},$$

d] $y(n) = u(n) * h(n)$

$$= \sum_k u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k)$$



$y(0) = h(0) = 1$

$y(1) = h(0) + h(1) = \frac{5}{2}$

$y(2) = h(0) + h(1) + h(2) = \frac{13}{4}$ etc.

e] from part (a), $h(n) = 0$ for $n<0 \Rightarrow$ the s/m is causal.

$$\sum_{n=0}^{\infty} |h(n)| = 1 + \frac{3}{2}\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right) = 4 \Rightarrow \text{s/m is stable.}$$

48]

a] $y(n) = ay(n-1) + bx(n)$

$h(n) = ba^n u(n)$

$$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1$$

$b = 1-a$.

b] $s(n) = \sum_{k=0}^{n} h(n-k)$

$$= b\left[\frac{1-a^{n+1}}{1-a}\right] u(n)$$

$$s(\infty) = \frac{b}{1-a} = 1$$

$b = 1-a$.

c] $b = 1-a$ in both the cases.

49]

a] $y(n) = 0.8y(n-1) + 2x(n) + 3x(n-1)$

$y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)$

the characteristic equation is

$\lambda - 0.8 = 0$

$\lambda = 0.8$

$y_h(n) = c(0.8)^n$

Let us first consider the response of the s/m.

$$y(n) - 0.8y(n-1) = x(n)$$

to $x(n) = \delta(n)$. Since $y(0) = 1$, it follows that $c = 1$. Then, the impulse response of the original s/m is

$$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$$
$$= \cancel{28}\; 2\delta(n) + 4.6(0.8)^{n-1} u(n-1).$$

b) The inverse s/m is characterized by the difference equation

$$x(n) = -1.5x(n-1) + \frac{1}{2}y(n) - 0.4y(n-1).$$

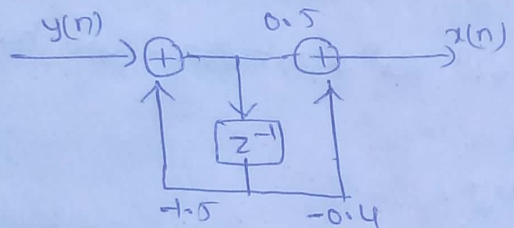
50)

a) $y(n) = 0.9y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$

$y(n) - 0.9y(n-1) = x(n) + 2x(n-1) + 3x(n-2)$.

For $x(n) = \delta(n)$, we have

$y(0) = 1$, $y(1) = 2.9$, $y(2) = 5.61$, $y(3) = 5.049$, $y(4) = 4.544$, $y(5) = 4.090$.



b) $s(0) = y(0) = 1$

$s(1) = y(0) + y(1) = 3.91$

$s(2) = y(0) + y(1) + y(2) = 9.51$

$s(3) = y(0) + y(1) + y(2) + y(3) = 14.56$

$s(4) = \sum_{0}^{4} y(n) = 19.10$

$s(5) = \sum_{0}^{5} y(n) = 23.19$.

c) $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2)$

$= \delta(n) + 2.9\delta(n-1) + 5.61(0.9)^{n-2} u(n-2)$

51)

a) $y(n) = \frac{1}{3}x(n) + \frac{1}{3}x(n-3) + y(n-1)$

for $x(n) = \delta(n)$, we have

$h(n) = \{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \ldots\}$

b) $y(n) = \frac{1}{2}y(n-1) + \frac{1}{8}y(n-2) + \frac{1}{2}x(n-2)$

with $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$, we obtain

$h(n) = \{0, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{16}, \frac{1}{8}, \frac{11}{128}, \frac{15}{256}, \frac{41}{1024}, \ldots\}$

c) $y(n) = 1.4y(n-1) - 0.48y(n-2) + x(n)$

with $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$, we obtain

$h(n) = \{1, 1.4, 1.48, 1.4, 1.2496, 1.0774, 0.9086, \ldots\}$

d) All three systems are IIR.

e) $y(n) = 1.4y(n-1) - 0.48y(n-2) + x(n)$

the characteristic equation is

$\lambda^2 - 1.4\lambda + 0.48 = 0$ hence

$\lambda = 0.8, 0.6.$ and

$y_h(n) = c_1(0.8)^n + c_2(0.6)^n$ for $x(n) = \delta(n)$. we have,

$c_1 + c_2 = 1$ and

$0.8c_1 + 0.6c_2 = 1.4$

$c_1 = 4, \quad c_2 = -3$

$h(n) = [4(0.8)^n - 3(0.6)^n]u(n).$

52)

a) $h_1(n) = c_0\delta(n) + c_1\delta(n-1) + c_2\delta(n-2)$

$h_2(n) = b_2\delta(n) + b_1\delta(n-1) + b_0\delta(n-2).$

$h_3(n) = a_0\delta(n) + (a_1 + a_0 + a_2)\delta(n-1) + a_1a_2\delta(n-2).$

b) the only question is whether

$h_3(n) \stackrel{?}{=} h_2(n) = h_1(n)$

Let $a_0 = c_0$

$a_1 + a_2c_0 = c_1, \Rightarrow a_1 + a_2c_0 - c_1 = 0$

$a_2a_1 = c_2, \Rightarrow \frac{c_2}{a_2} = a_1$

$\Rightarrow \frac{c_2}{a_2} + a_2c_0 - c_1 = 0$

$\Rightarrow c_0a_2^2 - c_1a_2 + c_2 = 0.$

For $c_0 \neq 0$, the quadratic has a real solution if and only if $c_1^2 - 4c_0c_2 \geq 0$.

53]

a] $y(n) = \frac{1}{2}y(n) + x(n) + x(n-1)$

For $y(n) - \frac{1}{2}y(n-1) = x(n) + x(n-1)$ $\qquad x(n) = \delta(n)$

$$h(n) = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1).$$

b] $h_1(n) * [\delta(n) + \delta(n-1)] = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1).$

54)

a) $x_1(n) = \{\underset{\uparrow}{1}, 2, 4\}$ $\qquad h(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

convolution : $y_1(n) = \{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\}$

correlation : $r_1(n) = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

b) $x_2(n) = \{\underset{\uparrow}{0}, 1, -2, 3, -4\}$ $\qquad h_2(n) = \{\frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}\}$

convolution : $y_2(n) = \{\frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, \frac{-5}{2}, -2\}$

correlation : $r_2(n) = \{\frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, \frac{-5}{2}, -2\}$

Note $y_2(n) = r_2(n)$, $\because h_2(-n) = h_2(n)$ (C).

c) $x_3(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $\qquad h_3(n) = \{\underset{\uparrow}{4}, 3, 2, 1\}$

convolution : $y_3(n) = \{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\}$

Correlation : $r_3(n) = \{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\}$.

d] $x_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $\qquad h_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

convolution : $y_4(n) = \{\underset{\uparrow}{1}, 4, 10, 20, 25, 24, 16\}$.

correlation : $r_4(n) = \{4, 11, 20, \underset{\uparrow}{30}, 20, 11, 4\}$

Note that $h_3(-n) = h_4(n+3)$

hence $r_3(n) = y_4(n+3)$

$h_4(-n) = h_3(n+3)$

$r_4(n) = y_3(n+3)$.

55]

Sol: length of $h(n) = 2$

$h(n) = \{h_0, h_1\}$

$h_0 = 1$

$3h_0 + h_1 = 4$ $\Rightarrow \boxed{h_0 = 1,\ h_1 = 1}$

56]

Sol:- (2.5.6) $y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$

(2.5.9) $w(n) = -\sum_{k=1}^{N} a_k w(n-k) + x(n)$

(2.5.10) $y(n) = \sum_{k=0}^{M} b_k w(n-k)$

From 2.5.9 we obtain $x(n) = w(n) + \sum_{k=1}^{N} a_k w(n-k)$ → (A)

by substituting (2.5.10) for $y(n)$ and (A) into (2.5.6).

we obtain L.H.S = R.H.S.

57]

Sol:- $y(-1) = y(-2) = 0$

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

The characteristic equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$\lambda = 2, 2.$ Hence

$$y_h(n) = c_1 2^n + c_2 n 2^n.$$

The partial solution is

$$y_p(n) = k(-1)^n u(n).$$

Substituting this solution is into the difference equation

we obtain,

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n).$$

For $n=2$, $k(1+4+4) = 2 \Rightarrow k = \frac{2}{9}$. The total solution is

$$y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

from the intial conditions, we obtain $y(0)=1, y(1)=2.$

$$c_1 + \frac{2}{9} = 1$$

$$\Rightarrow c_1 = \frac{7}{9}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2$$

$$\Rightarrow c_2 = \frac{1}{3}.$$

58]

Sol:- From problem (57)

$$h(n) = \left[c_1 2^n + c_1 n 2^n\right] u(n)$$

with $y(0)=1$, $y(1)=3$, we have

$C_1 = 1$; $2C_1 + 3C_2 = 1$

$\cancel{C_2 = \frac{1}{2}}$ $\cancel{2+2C_2 = 1}$ $3C_2 = \cancel{\frac{8}{2}} - 2$

$1 + C_2 = \frac{1}{\cancel{2}}$

$C_2 = \frac{1}{2}$

thus $h(n) = \left[2^n + \frac{1}{2} n 2^n\right] u(n)$.

59]

Sol:- $u(n-k) = \begin{cases} 1, & n \geq k \\ 0, & \text{otherwise} \end{cases}$

$$x(n) = x(n) * \delta(n)$$
$$= x(n) * [u(n) - u(n-1)]$$
$$= [x(n) - x(n-1)] * u(n)$$
$$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, u(n-k)$$

60]

Sol:- Let $h(n)$ be the impulse response of the s/m.

$$\delta(k) = \sum_{m=-\infty}^{k} h(m)$$
$$h(k) = \delta(k) - \delta(k-1)$$
$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$
$$= \sum_{k=-\infty}^{\infty} [\delta(k) - \delta(k-1)]\, x(n-k)$$

61]

Sol:- $x(n) = \begin{cases} 1, & n_0 - N \leq n \leq n_0 + N \\ 0, & \text{otherwise.} \end{cases}$

$y(n) = \begin{cases} 1, & -N \leq n \leq N \\ 0, & \text{otherwise.} \end{cases}$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

The range of non-zero values of $r_{xx}(l)$ is determined by

$$n_0 - N \leq n \leq n_0 + N$$
$$n_0 - N \leq n - l \leq n_0 + N$$

which implies

$$-2N \leq l \leq 2N$$

for a given shift l, the number of terms in the summation for which both $x(n)$ and non-zero is $2N+1-|l|$ and the value each term is 1. Hence,

$$r_{xx}(l) = \begin{cases} 2N+1-|l| & , -2N \le l \le 2N \\ 0 & , \text{otherwise} \end{cases}$$

for $r_{xy}(l)$ we have

$$r_{xy}(l) = \begin{cases} 2N+1-|l-n_0| & , n_0-2N \le l \le n_0+2N \\ 0 & , \text{otherwise.} \end{cases}$$

62]

a] $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\,x(n-l)$

$r_{xx}(-3) = x(0)\,x(3) = 1$

$r_{xx}(-2) = x(0)x(2) + x(1) + x(3) = 3$

$r_{xx}(-1) = x(0)\,x(1) + x(1)\,x(2) + x(2)\,x(3) = 5.$

$r_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7$

also $r_{xx}(-l) = r_{xx}(l)$

$\therefore r_{xx}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$

b] $r_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)\,y(n-l).$

we obtain $r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

we obtain $y(n) = x(-n+3)$ which is equivalent to reversi the sequence $x(n)$. This has not changed the value..

63]

Sol: $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\,x(n-l).$

$$= \begin{cases} 2N+1-|l| & , -2N \le l \le 2N \\ 0 & , \text{otherwise.} \end{cases}$$

$r_{xx}(0) = 2N+1.$

$\therefore$ the normalized autocorrelation is

$$\rho_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1-|l|), & -2N \le l \le 2N \\ 0 & , \text{otherwise} \end{cases}$$

64]

a] $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$

$= \sum_{n=-\infty}^{\infty} [\delta(n) + r_1 \delta(n-k_1) + r_2 \delta(n-k_2)] * [\delta(n-l) + r_1 \delta(n-l-k_1) + r_2 \delta(n-l-k_2)]$

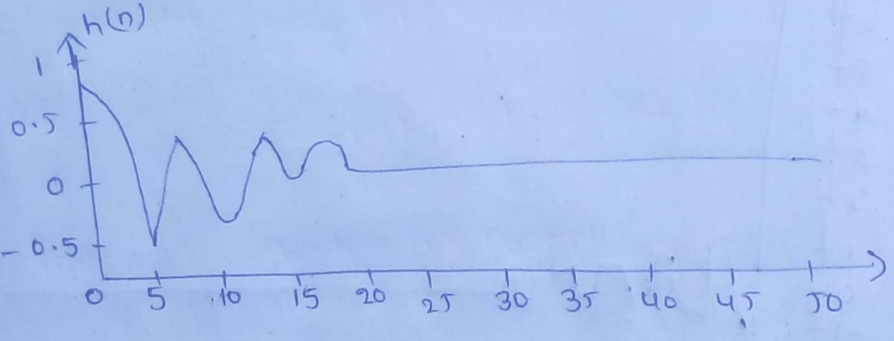
$= (1 + r_1^2 + r_2^2)\, r_{\delta\delta}(l) + r_1 [r_{\delta\delta}(l+k_1) + r_{\delta\delta}(l-k_1)] + r_2 [r_{\delta\delta}(l+k_2) + r_{ss}(l-k_2)] + r_1 r_2 [r_{ss}(l+k_1-k_2) + r_{ss}(l+k_2-k_1)]$.

b] $r_{\delta\delta}(l)$ has peaks at $l = 0, \pm k_1, \pm k_2$ and $\pm(k_1+k_2)$. suppose that $k_1 < k_2$. Then, we can determine r_1 and k_1. The problem is to determine r_2 and k_2 from the other peaks.

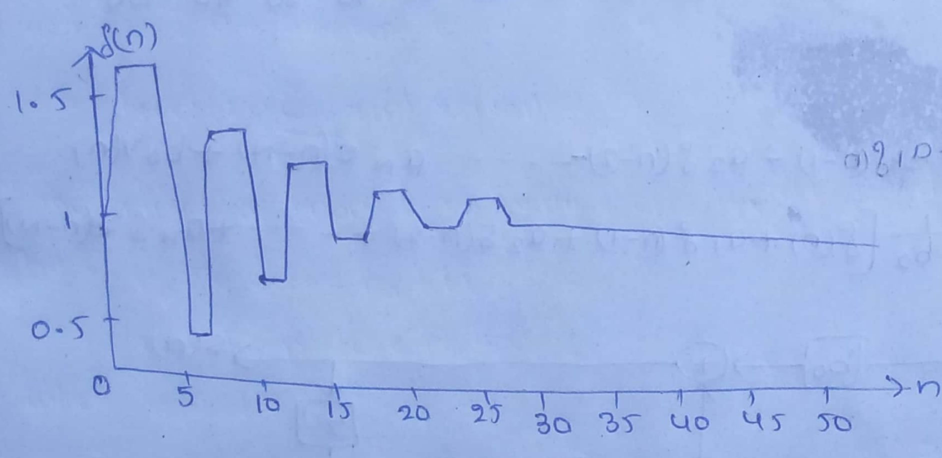
c] If $r_2 = 0$, the peaks occur at $l = 0$, and $l = \pm k_1$. That is easy to obtain r_1 and ~~to~~ k_1.

66]

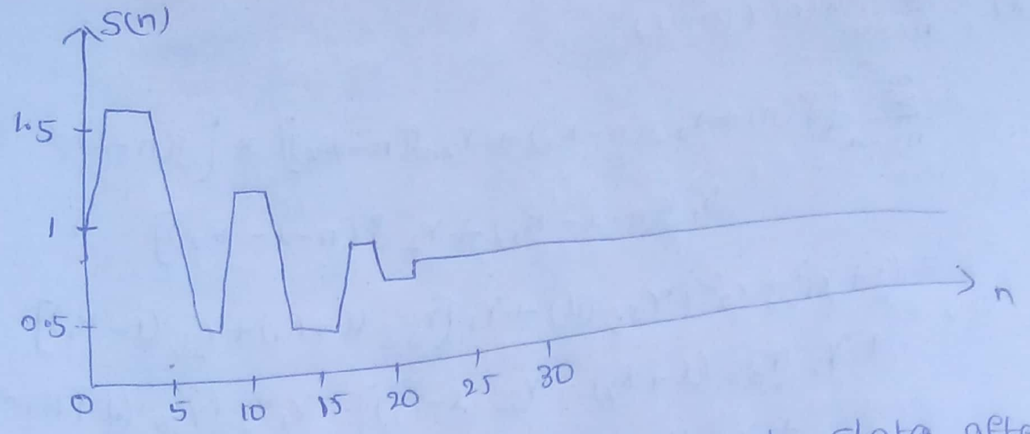
a]



b]

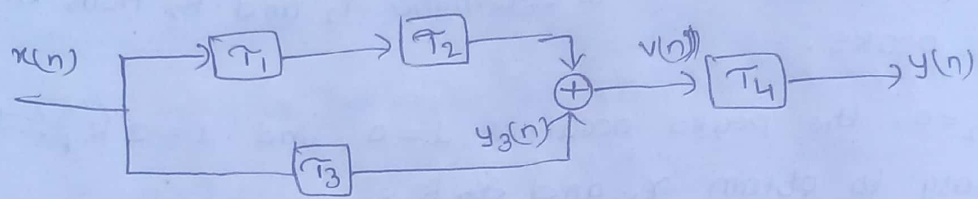


c]



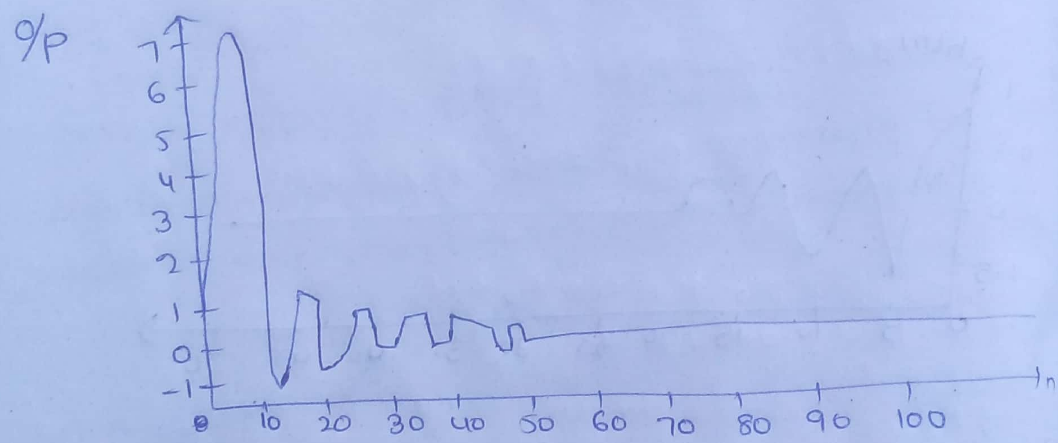
d] c, b are similar except c have steady state after n=20, where b have nearly at n=30.

67] Plot h(n) for $0 \le n \le 99$



Sol:

O/P

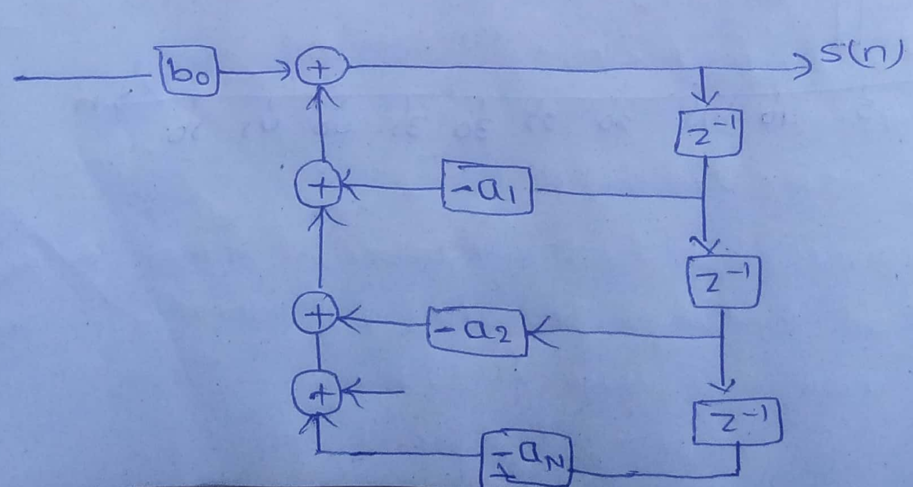


44]

a] $s(n) = -a_1 s(n-1) - a_2 s(n-2) - \dots - a_N s(n-N) + b_0 v(n)$

b] $v(n) = \frac{1}{b_0}\left[s(n) + a_1 s(n-1) + a_2 s(n-2) + \dots + a_N s(n-N)\right]$

a]



b)