Lecture-8
Piping networks and pump selection

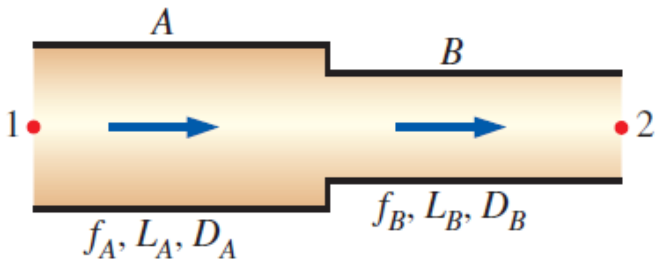
A piping network in an industrial facility.

Basic principles

- Conservation of mass throughout the system must be satisfied. This is done by requiring the total flow into a junction to be equal to the total flow out of the junction for all junctions in the system.

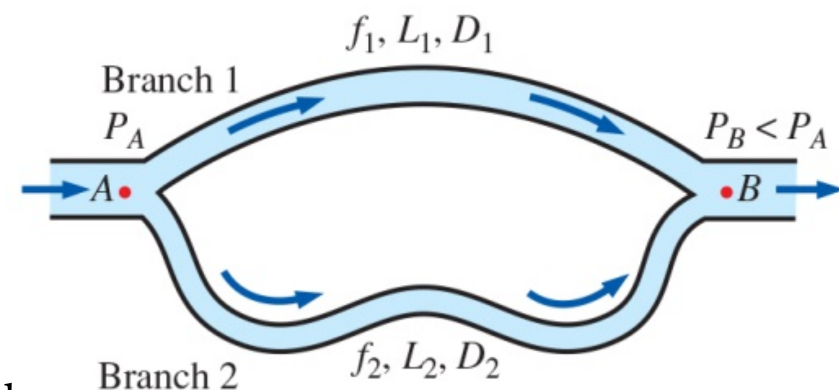
- Pressure drop (and thus head loss) between two junctions must be the same for all paths between the two junctions. This is because pressure is a point function and it cannot have two values at a specified point. In practice this rule is used by requiring that the algebraic sum of head losses in a loop (for all loops) be equal to zero.

Networks in series and parallel



$$\dot{V}_A = \dot{V}_B$$
$$h_{L,\,1\text{-}2} = h_{L,\,A} + h_{L,\,B}$$

For pipes *in series*, the flow rate is the same in each pipe, and the total head loss is the sum of the head losses in individual pipes.



$$h_{L,\,1} = h_{L,\,2}$$
$$\dot{V}_A = \dot{V}_1 + \dot{V}_2 = \dot{V}_B$$

For pipes *in parallel*, the head loss is the same in each pipe, and the total flow rate is the sum of the flow rates in individual pipes.

The relative flow rates in parallel pipes are established from the requirement that the head loss in each pipe be the same.

$$h_{L,\,1} = h_{L,\,2} \quad \rightarrow \quad f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g}$$

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2} \quad \text{and} \quad \frac{\dot{V}_1}{\dot{V}_2} = \frac{A_{c,\,1}V_1}{A_{c,\,2}V_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2}$$

The flow rate in one of the parallel branches is proportional to its diameter to the power 5/2 and is inversely proportional to the square root of its length and friction factor.

Piping systems with pumps and turbines

Energy equation

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},e} + h_L$$

$h_{\text{pump},u}$: useful head added by the pump

$h_{\text{turbine},e}$: extracted head by the turbine

$$\dot{W}_{\text{pump,shaft}} = \frac{\rho \dot{\mathcal{V}} g h_{\text{pump},u}}{\eta_{\text{pump}}}$$

Shaft's mechanical energy to useful head

$$\eta_{\text{motor}} = \frac{\dot{W}_{\text{shaft}}}{\dot{W}_{\text{elect}}}$$

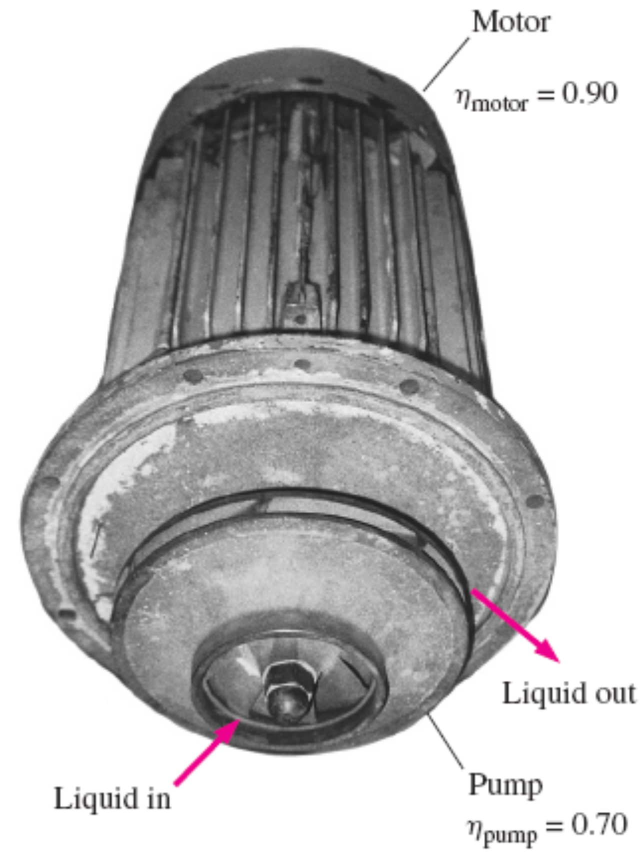
Electric energy to shaft's mechanical energy

Combining

$$\dot{W}_{\text{elect}} = \frac{\rho \dot{\mathcal{V}} g h_{\text{pump},u}}{\eta_{\text{pump-motor}}}$$

Electric energy to useful head

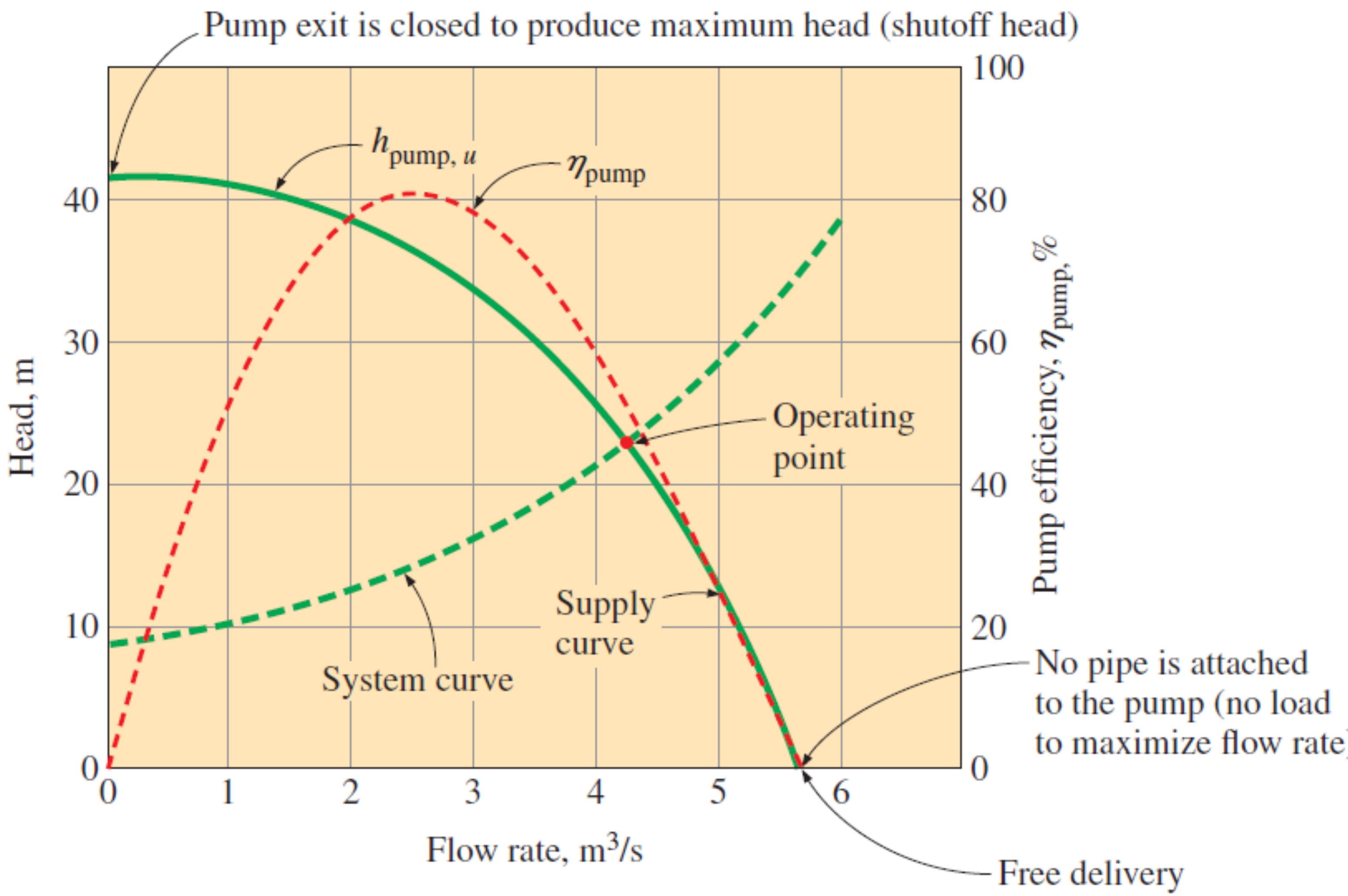
$$\eta_{\text{pump-motor}} = \eta_{\text{pump}} \eta_{\text{motor}}$$



$\eta_{\text{pump–motor}} = \eta_{\text{pump}}\eta_{\text{motor}}$
$= 0.70 \times 0.90 = 0.63$

Demand and supply

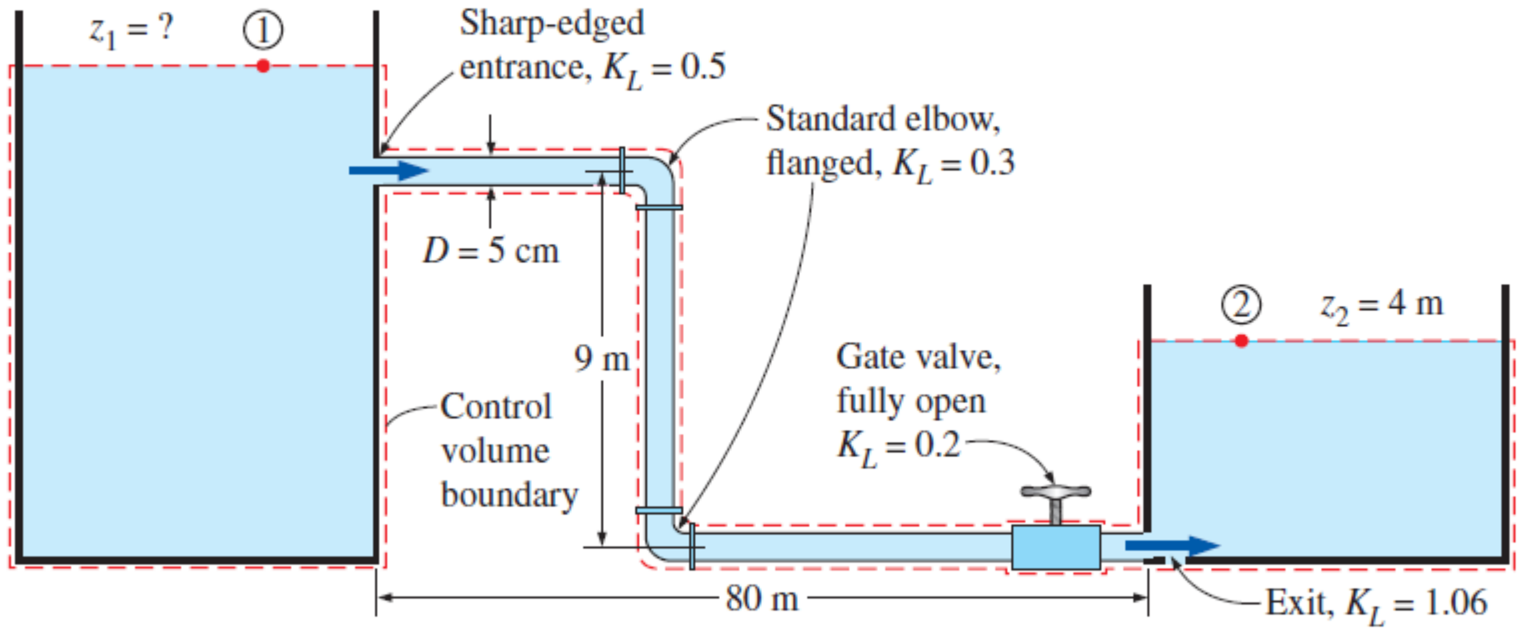
- The head loss of a piping system increases with the flow rate. A plot of required useful pump head $h_{pump, u}$ as a function of flow rate is called the system (or demand) curve.
- flow rate of a pump increases as the required head decreases (supply curve).



Example-1

EXAMPLE 8–8 **Gravity-Driven Water Flow in a Pipe**

Water at 10°C flows from a large reservoir to a smaller one through a 5-cm-diameter cast iron piping system, as shown in Fig. 8–51. Determine the elevation z_1 for a flow rate of 6 L/s.



Properties The density and dynamic viscosity of water at 10°C are ρ = 999.7 kg/m³ and μ = 1.307 × 10^{-3} kg/m·s. The roughness of cast iron pipe is ε = 0.00026 m (Table 8–2).

Example-1 continued

Analysis The piping system involves 89 m of piping, a sharp-edged entrance ($K_L = 0.5$), two standard flanged elbows ($K_L = 0.3$ each), a fully open gate valve ($K_L = 0.2$), and a submerged exit ($K_L = 1.06$). We choose points 1 and 2 at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocities at both points are nearly zero ($V_1 \approx V_2 \approx 0$), the energy equation for a control volume between these two points simplifies to

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \frac{V_1^2}{2g}^{\nearrow 0} + z_1 = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \frac{V_2^2}{2g}^{\nearrow 0} + z_2 + h_L \quad \rightarrow \quad z_1 = z_2 + h_L$$

where

$$h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.006\ \text{m}^3/\text{s}}{\pi(0.05\ \text{m})^2/4} = 3.06\ \text{m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(999.7\ \text{kg/m}^3)(3.06\ \text{m/s})(0.05\ \text{m})}{1.307 \times 10^{-3}\ \text{kg/m·s}} = 117{,}000$$

The flow is turbulent since Re > 4000. Noting that $\varepsilon/D = 0.00026/0.05 = 0.0052$, the friction factor is determined from the Colebrook equation (or the Moody chart),

Example-1 continued

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{0.0052}{3.7} + \frac{2.51}{117{,}000\sqrt{f}}\right)$$

It gives $f = 0.0315$. The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{ entrance}} + 2K_{L,\text{ elbow}} + K_{L,\text{ valve}} + K_{L,\text{ exit}}$$
$$= 0.5 + 2 \times 0.3 + 0.2 + 1.06 = 2.36$$

Then the total head loss and the elevation of the source become

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} = \left(0.0315\,\frac{89\text{ m}}{0.05\text{ m}} + 2.36\right)\frac{(3.06\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 27.9\text{ m}$$

$$z_1 = z_2 + h_L = 4 + 27.9 = \mathbf{31.9\ m}$$

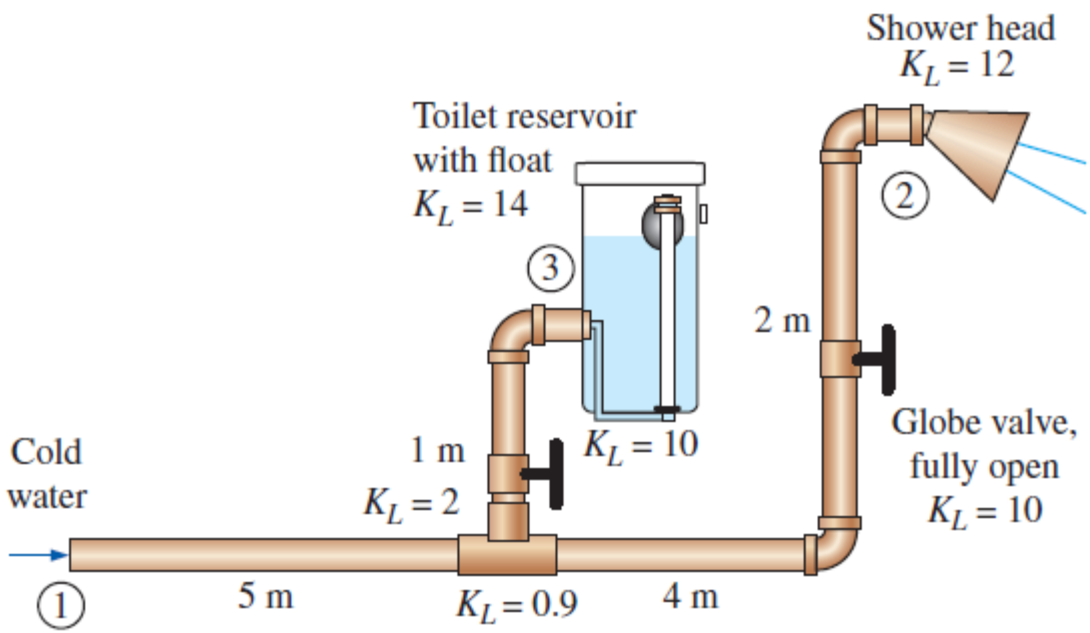
Therefore, the free surface of the first reservoir must be 31.9 m above the ground level to ensure water flow between the two reservoirs at the specified rate.

Discussion Note that $fL/D = 56.1$ in this case, which is about 24 times the total minor loss coefficient. Therefore, ignoring the sources of minor losses in this case would result in about 4 percent error. It can be shown that at the same flow rate, the total head loss would be 35.9 m (instead of 27.9 m) if the valve were three-fourths closed, and it would drop to 24.8 m if the pipe between the two reservoirs were straight at the ground level (thus eliminating the elbows and the vertical section of the pipe). The head loss could be reduced further (from 24.8 to 24.6 m) by rounding the entrance. The head loss can be reduced significantly (from 27.9 to 16.0 m) by replacing the cast iron pipes by smooth pipes such as those made of plastic.

Example-2

EXAMPLE 8–9 Effect of Flushing on Flow Rate from a Shower

The bathroom plumbing of a building consists of 1.5-cm-diameter copper pipes with threaded connectors, as shown in Fig. 8–52. (*a*) If the gage pressure at the inlet of the system is 200 kPa during a shower and the toilet reservoir is full (no flow in that branch), determine the flow rate of water through the shower head. (*b*) Determine the effect of flushing of the toilet on the flow rate through the shower head. Take the loss coefficients of the shower head and the reservoir to be 12 and 14, respectively.



Properties The properties of water at 20°C are $\rho = 998$ kg/m^3, $\mu = 1.002 \times 10^{-3}$ kg/m·s, and $\nu = \mu/\rho = 1.004 \times 10^{-6}$ m^2/s. The roughness of copper pipes is $\varepsilon = 1.5 \times 10^{-6}$ m.

Example-2 (continued)

Analysis This is a problem of the second type since it involves the determination of the flow rate for a specified pipe diameter and pressure drop. The solution involves an iterative approach since the flow rate (and thus the flow velocity) is not known.

(*a*) The piping system of the shower alone involves 11 m of piping, a tee with line flow ($K_L = 0.9$), two standard elbows ($K_L = 0.9$ each), a fully open globe valve ($K_L = 10$), and a shower head ($K_L = 12$). Therefore, $\sum K_L = 0.9 + 2 \times 0.9 + 10 + 12 = 24.7$. Noting that the shower head is open to the atmosphere, and the velocity heads are negligible, the energy equation for a control volume between points 1 and 2 simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine}, e} + h_L$$

$$\rightarrow \quad \frac{P_{1,\text{ gage}}}{\rho g} = (z_2 - z_1) + h_L$$

Therefore, the head loss is

$$h_L = \frac{200{,}000 \text{ N/m}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} - 2 \text{ m} = 18.4 \text{ m}$$

Also,

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} \quad \rightarrow \quad 18.4 = \left(f\frac{11 \text{ m}}{0.015 \text{ m}} + 24.7\right)\frac{V^2}{2(9.81 \text{ m/s}^2)}$$

Example-2 (continued)

since the diameter of the piping system is constant. Equations for the average velocity in the pipe, the Reynolds number, and the friction factor are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad \rightarrow \quad V = \frac{\dot{V}}{\pi(0.015\ \text{m})^2/4}$$

$$\text{Re} = \frac{VD}{\nu} \quad \rightarrow \quad \text{Re} = \frac{V(0.015\ \text{m})}{1.004 \times 10^{-6}\ \text{m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

This is a set of four equations with four unknowns, and solving them with an equation solver such as EES gives

$$\dot{V} = 0.00053\ \text{m}^3/\text{s},\ \ f = 0.0218,\ \ V = 2.98\ \text{m/s}, \quad \text{and} \quad \text{Re} = 44{,}550$$

Therefore, the flow rate of water through the shower head is **0.53 L/s.**

(*b*) When the toilet is flushed, the float moves and opens the valve. The discharged water starts to refill the reservoir, resulting in parallel flow after the tee connection. The head loss and minor loss coefficients for the shower branch were determined in (*a*) to be $h_{L,\,2} = 18.4$ m and $\sum K_{L,\,2} = 24.7$, respectively. The corresponding quantities for the reservoir branch can be determined similarly to be

Example-2 (continued)

$$h_{L,3} = \frac{200{,}000\ \text{N/m}^2}{(998\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} - 1\ \text{m} = 19.4\ \text{m}$$

$$\sum K_{L,3} = 2 + 10 + 0.9 + 14 = 26.9$$

The relevant equations in this case are

$$\dot{V}_1 = \dot{V}_2 + \dot{V}_3$$

$$h_{L,2} = f_1 \frac{5\ \text{m}}{0.015\ \text{m}} \frac{V_1^2}{2(9.81\ \text{m/s}^2)} + \left(f_2 \frac{6\ \text{m}}{0.015\ \text{m}} + 24.7\right) \frac{V_2^2}{2(9.81\ \text{m/s}^2)} = 18.4$$

$$h_{L,3} = f_1 \frac{5\ \text{m}}{0.015\ \text{m}} \frac{V_1^2}{2(9.81\ \text{m/s}^2)} + \left(f_3 \frac{1\ \text{m}}{0.015\ \text{m}} + 26.9\right) \frac{V_3^2}{2(9.81\ \text{m/s}^2)} = 19.4$$

$$V_1 = \frac{\dot{V}_1}{\pi(0.015\ \text{m})^2/4}, \quad V_2 = \frac{\dot{V}_2}{\pi(0.015\ \text{m})^2/4}, \quad V_3 = \frac{\dot{V}_3}{\pi(0.015\ \text{m})^2/4}$$

$$\text{Re}_1 = \frac{V_1(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}}, \quad \text{Re}_2 = \frac{V_2(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}}, \quad \text{Re}_3 = \frac{V_3(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right)$$

$$\frac{1}{\sqrt{f_3}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_3\sqrt{f_3}}\right)$$

Example-2 (continued)

Solving these 12 equations in 12 unknowns simultaneously using an equation solver, the flow rates are determined to be

$$\dot{V}_1 = 0.00090\ \text{m}^3/\text{s}, \quad \dot{V}_2 = 0.00042\ \text{m}^3/\text{s}, \quad \text{and} \quad \dot{V}_3 = 0.00048\ \text{m}^3/\text{s}$$

Therefore, the flushing of the toilet **reduces the flow rate of cold water through the shower by 21 percent** from 0.53 to 0.42 L/s, causing the shower water to suddenly get very hot (Fig. 8–53).