

EGF316 - Advanced Structural Analysis

9. Linear Elastic Fracture Mechanics

9.1 Introduction

The initiation of cracks is very difficult to detect considering that almost all components contain some kind of cracks or crack-like flaws. The focus is therefore placed on *preventing*, *predicting* and *controlling* their propagation. The theory of fracture mechanics assumes the pre-existence of cracks and develops criteria for the catastrophic growth of these cracks. The designer must then ensure that no such criteria can be met in the structure.

As engineers we wish to provide quantitative answers to the following questions:

- How and where does a crack start?
- How will the crack grow and develop?
- What is the residual strength as a function of crack size?
- What crack size can be tolerated under service load?
- How long does it take for a crack to grow from a certain initial crack size to a maximum permissible size?
- What is the service life of a structure when a certain pre-existing flaw size is assumed to exist?
- During the period available for crack detection how often should the structure be inspected for cracks?
- How and when will failure occur?

If a crack is present in a structure it has the effect of reducing its strength. The structure may then fail at a load lower than the given safe or design working load. The failure is sudden and catastrophic and hence very dangerous. There have been a number of such notable failures.

9.1.1 Boston Molasses Tank failure

On 15th January 1919, a relatively new molasses tank in Boston measuring 27m in diameter and 15m high cracked spectacularly without any warning signs. About 7.5 million litres of molasses rushed through the streets of Boston at a speed of about 35 mph killing 21 people and injuring more than 100.

The steel tank was painted the same brown colour as the molasses themselves hindering the visual identification of any leaks. There had been a drastic change in air temperature over the preceding days from -17°C to 5°C . Subsequent studies proved that the tank had failed due to thermal shocks which led to higher stress levels than the tank could tolerate. The tank was not adequately designed to withstand the pressure created by the expanding molasses. It was also found that the tank had a low factor of safety and was not subject to sufficient testing prior to use.



9.1.2 Liberty Ships

World War II saw the construction of some of the first ships using welding of panels as opposed to traditional riveting. The Liberty Ships boasted an all-welded hull which saved time and weight compared to the previously employed riveting process.

The edges of riveted panels would act as natural crack inhibitors, but with welded hulls, cracks were able to propagate across panels over long distances. The welds were not heat-treated and this resulted in a heat-affected zone, which was brittle in comparison to the rest of the steel plates. Once the ships set sail a number suffered catastrophic failure and broke in two. The stresses that caused fracture to occur were quite low. The temperature of the North Atlantic was such that the ductile-brittle transition temperature was achieved and particularly affected the welded areas causing a brittle fracture. Out of the approximately 2,700 Liberty ships built, 145 broke in two and 700 suffered serious failures.

Further investigations concluded that the welds had been carried out by unskilled workers and contained crack-like flaws and the fractures originated from local stress concentration at the square hatch corners on the deck. Impact tests carried out on the steel used to manufacture the ships revealed that the grade of steel used was of poor toughness, contributing to the brittle fracture.

To reduce subsequent failures, design changes were made such as rounding and strengthening the square hatch corners and introducing riveted crack arresters at various locations.

These and other notable failures led to emergence of the field of fracture mechanics. The object of fracture mechanics is to provide quantitative answers to specific problems concerning cracks in structures.



9.2 Crack Detection

Structures that are susceptible to cracks should be examined on a regular basis to determine if cracks are present. There are many non-destructive testing (NDT) techniques, which can be employed to determine if cracks are present in a structure. These techniques include:

- Visual inspection
- Penetrative dyes
- Crack opening gauges
- Eddy current testing
- Ultrasonic testing
- X-Ray
- Acoustic emissions testing
- Magnetic particle testing

Once a crack has been found in a component the engineer has to decide what action is required.

- Should the component be replaced? This may be an expensive option due to the cost of the component, getting it fitted and down time.
- Can the component continue to be used? In some cases the component can still have a useful life. It may be possible to determine the load, which will cause complete failure and ensure that the component is not subjected to a load of that magnitude. The component would then be inspected on a regular basis to check on the development of the crack. In some cases it may be possible to use a technique, which will prevent the crack from growing. The simplest method is to drill a hole at the crack tip. This is not always possible and in some cases the hole may be a stress raiser in its own right.

9.3 Damage Tolerant Design

There are two different approaches to designing structures to be tolerant to defects. These are Safe Life Design and Fail Safe Design.

9.3.1 Safe Life Design

The structure is designed to operate safely within its expected life. It is designed to withstand its calculated working stress for a set period of time.

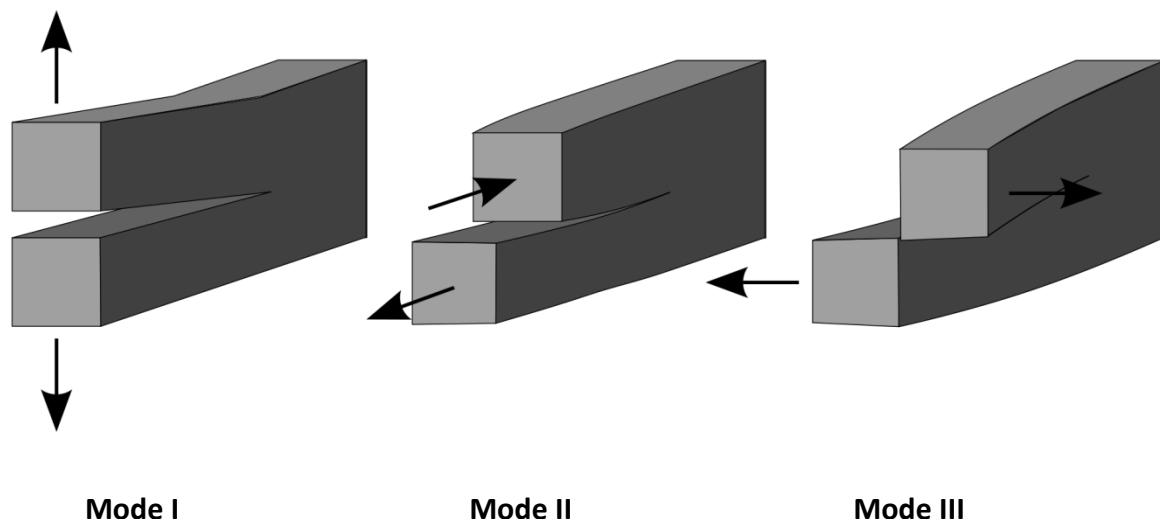
9.3.2 Fail Safe Design

The structure is designed such that it can operate safely without any in-service catastrophic failures. This is achieved by one three means:

- Multiple load paths are set up such that if one member were to fail, the remaining members are able to continue to operate safely for a period of time
- Acknowledging the presence of a crack, the focus is on stopping it from propagating and delaying or eliminating the risk of the crack re-propagating. Cracks can be 'healed', patched or a hole can be drilled (skiing gondola lifts)
- A leak before break philosophy is widely used, especially in the nuclear industry. If a crack is present, it is expected that a leak will be seen in advance of any subsequent disastrous failure.

9.4 Modes of Failure

A sharp crack in a stressed body can propagate in a combination of three opening modes as shown below:



Mode I represents opening due to a purely tensile load. The displacement of the crack surfaces move perpendicular to the plane of the crack.

Mode II represents opening due to in plane shear. In the sliding mode the crack surfaces move normal to the leading edge of the crack front while remaining in the crack plane.

Mode III represents opening due to out-of-plane shear. The crack surface in this case moves parallel to the leading edge of the crack front while remaining in the crack plane.

The most commonly seen failures are due to cracks propagating predominantly in Mode I, and we will concentrate on this mode in our analysis. Materials are thus generally characterised by their resistance to fracture in this mode. The subsequent theories that we will examine in this modules will be based upon the assumption of Mode I failure.

9.5 The Field of Fracture Mechanics

The field of fracture mechanics uses two progressive theories to attempt to characterize a material given that it contains internal flaws. These are:

- Linear Elastic Fracture Mechanics (LEFM)
- Elastic Plastic Fracture Mechanics (EPFM)

We will only consider LEFM in this module.

Cracks produce very high level of stresses which lead to the development of a plastic zone at the tip of the crack. LEFM only allows for a highly localised plastic zone near the crack tip. LEFM is mostly used for low toughness materials where cracks occur with minimal damage – elastic deformation. These materials tend to be brittle with high yield strength.

At room temperature high strength steel, precipitation hardened aluminium, polymers below glass transition temperature, ceramics and ceramic composites are examples of materials for which LEFM remains valid.

9.6 Linear Elastic Fracture Mechanics (LEFM) Approaches

LEFM allows an analytical approach to fracture by relating the stress distribution in the vicinity of the crack to other parameters such as the nominal applied stress and the geometry and orientation of the crack. It allows us to represent material fracture properties in terms of a single parameter.

There are two basic approaches employed in fracture analysis.

Energy Balance Approach

This approach is based on the release of elastic strain energy produced by crack extension. Approach states that a crack grows as a result of the available energy. The energy available

for crack growth needs to exceed the energy required for the generation of a new crack surfaces for the crack to propagate.

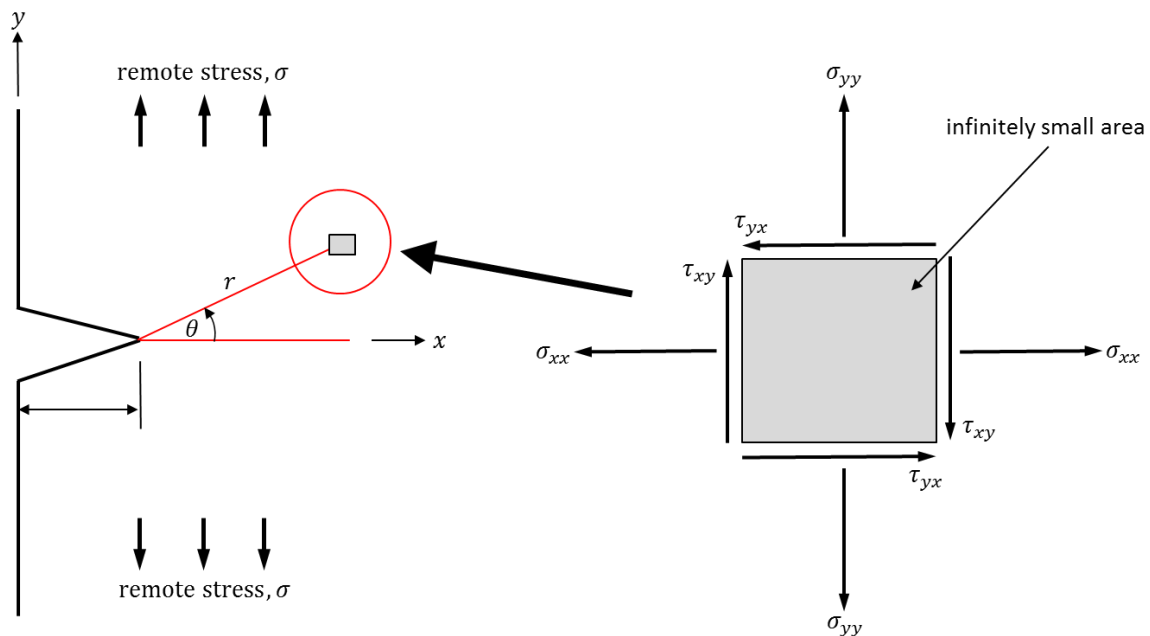
Approach Based on Stress State at Crack tip

- a) Stress function approach considers elastic stresses only (Westergard, 1939)*
- b) Irwin's (1964) estimation of crack tip plasticity
- c) Dugdale's (1960) model of crack tip plasticity

*we will focus on this approach

9.6.1 Stress function approach (Westergard, 1939)

Westergard considered the stress state in the vicinity of a crack tip. He let the origin of x and y -axis lie at the crack centre with x aligned with the crack length.



He considered an infinitely small area located at a distance r and an angle θ from the crack tip (where $r \ll a$) in an infinite plate and derived expressions for the Cartesian stress components.

For a point close to the crack tip on the crack x -axis ($\theta = 0$), he demonstrated that:

$$\tau_{xy} = 0 \quad \text{and} \quad \sigma_{xx} = \sigma_{yy} = \sigma \sqrt{\frac{a}{2r}} \quad (9.1)$$

where,

σ is the remote (applied) stress

a is the crack half length*

r is the distance from the crack tip

σ_{yy} is the *opening stress* and governs crack stability

*a crack which is not on the edge of a plate (e.g. crack in an infinite plate) has a length of $2a$.

Irwin furthered this work and discovered that the opening stress had a common form and for $\theta = 0$, this became:

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \quad (9.2)$$

where K_I is the Mode I stress intensity factor and distinguishes one plate from another. As a general rule:

$$K_I = Q\sigma\sqrt{a\pi} \quad (9.3)$$

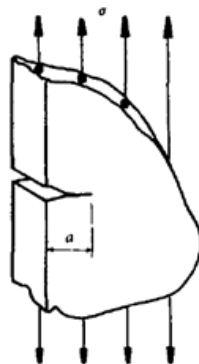
where Q is either a constant for infinite plates, or a function of finite plate geometry and type of crack.

- For an infinite plate with a central through-crack

$$Q = 1 \quad (9.4)$$

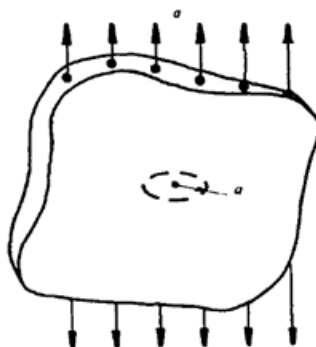
- For a semi-infinite plate containing an edge crack:

$$Q = 1.12 \quad (9.5)$$



- For an infinite plate containing an embedded circular crack or a semi-circular surface crack, each of radius a , lying in a plane normal to σ :

$$Q = \frac{2}{\pi} \quad (9.6)$$



Failure will occur when K_I reached a critical value, K_{IC} , known as the *Fracture Toughness* of the material. This has units of $\text{MNm}^{3/2} = \text{MPa}\sqrt{\text{m}}$ or $\text{Nmm}^{3/2}$.

Fixed stress:

If σ is fixed, then failure will occur when $a = a_{crit}$:

$$K_{IC} = Q\sigma\sqrt{\pi a_{crit}} \quad \text{or rearranging} \quad a_{crit} = \frac{1}{\pi} \left(\frac{K_{IC}}{Q\sigma} \right)^2 \quad (9.7)$$

Fixed crack length:

If a is fixed, then failure will occur when $\sigma = \sigma_{crit}$:

$$K_{IC} = Q\sigma_{crit}\sqrt{\pi a} \quad \text{or rearranging} \quad \sigma_{crit} = \frac{K_{IC}}{Q\sqrt{\pi a}} \quad (9.8)$$

Example 1:

An infinite steel plate contains a crack 5mm long. Calculate the maximum allowable design stress that could be applied around the boundary if $K_{IC} = 105 \text{ MN/m}^{3/2}$

If $\sigma_{yield} = 1500 \text{ MPa}$, comment on the type of failure that would occur if this design stress is exceeded.

So, a is fixed and failure will occur when $\sigma = \sigma_{crit}$:

$$a = \frac{0.005}{2} = 0.0025 \text{ m}$$

$$Q = 1$$

$$\sigma_{crit} = \frac{K_{IC}}{Q\sqrt{\pi a}} = \frac{105}{1\sqrt{\pi 0.0025}} = 1184.8 \text{ MPa}$$

Considering that:

$$\sigma_{crit} < \sigma_{yield}$$

We can deduce that this will be a brittle failure.

9.7 Application of LEFM to Fatigue

For a constant stress range, $\Delta\sigma$:

$$K_I = Q\sigma\sqrt{\pi a} \quad \text{becomes} \quad \Delta K_I = Q\Delta\sigma\sqrt{\pi a} \quad (9.9)$$

where $\Delta\sigma = \sigma_{max} - \sigma_{min}$.

9.8 Fatigue Crack Growth

The development of a crack can be broken down into three main stages:

Stage I involves the initiation of a crack at a defect of stress concentration and its subsequent growth along a favourable path within the microstructure.

Stage II crack growth describes when the crack has become sufficiently large that the microstructure has a reduced effect on the crack direction, and it will generally propagate in a direction normal to the maximum principle stress direction.

Stage III sees the crack continue to grow in this direction until its size approaches the critical crack size as determined by the Stress Intensity, K_{IC} . At this point the crack growth will accelerate until a point where K_{IC} is exceeded and failure occurs.

Paris (1964) carried out an extensive range of experimental testing that allowed graphical representation of the three stages of growth of a crack in terms of the crack growth per cycle (da/dN) and the amplitude of the stress intensity factor. The crack initiation of Stage I can be detected using devices such as crack opening gauges. However, as this stage is very short compared with Stage II for analysis purposes it is generally ignored.

Similarly, Stage III constitutes a very small portion of the components life and this too is generally ignored in the determination of the fatigue life of the component. It would not be advisable to include Stage III in any analysis as the point of fast fracture is imminent and the component should be removed from service prior to this to avoid a catastrophic failure

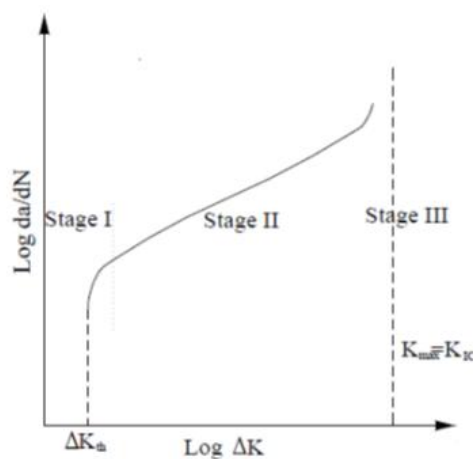


Figure: Crack growth rate due to a cyclic load of constant amplitude

The crack growth rate is described in terms of Stage II growth only and this takes the form:

$$\frac{da}{dN} = C(\Delta K)^n \quad (9.10)$$

where,

a is the crack size in metres

N the number of cycles to failure

ΔK is the change in the stress intensity factor in $\text{MN/m}^{3/2}$

C and n are material coefficients where it should be noted that n usually lies between 2 and 7, but values around 4 are generally found.

This is known as the *Paris-Erdogan Law*.

Substituting

$$\Delta K_I = Q\Delta\sigma\sqrt{\pi a}$$

into

$$\frac{da}{dN} = C(\Delta K)^n$$

and integrating gives,

$$N_f = \frac{1}{Q^n C (\Delta\sigma)^n \pi^{n/2}} \left[\frac{(a_{crit})^{1-n/2} - (a_i)^{1-n/2}}{1 - n/2} \right] \quad (9.11)$$

where,

N_f is the number of cycles to failure

a_{crit} is the critical crack size at maximum stress (in metres)

a_i is the initial crack size (in metres)

This applies for a constant cyclic stress and is used to predict the lifetime of a component if the stress amplitude remains constant and the maximum crack size is known. The life of the component may vary markedly if there are occasional overloads, environmental factors (e.g. corrosion), changes in stress concentration etc. The number of cycles to failure should be taken as a guide and regular checks should be made to monitor the actual crack growth.

Example 2:

Calculate the number of cycles to failure when a repeated stress range of 175MPa is applied to an infinite plate containing a 0.2mm long crack.

Assume:

$$K_{IC} = 54MN/m^{3/2}$$

$$\frac{da}{dN} = 40 \times 10^{-12} (\Delta K)^4$$

We know:

$$\frac{da}{dN} = C(\Delta K)^n$$

Therefore:

$$C = 40 \times 10^{-12}, \quad n = 4$$

And:

$$\Delta\sigma = \sigma_{max} - \sigma_{min} = 175 - 0 = 175MPa$$

Initial crack length:

$$a_i = \frac{0.2 \times 10^{-3}}{2} = 10^{-4}$$

To find the critical crack length:

$$a_{crit} = \frac{1}{\pi} \left(\frac{K_{IC}}{Q\Delta\sigma} \right)^2 = \frac{1}{\pi} \left(\frac{K_{IC}}{\Delta\sigma} \right)^2 \quad (Q = 1)$$

$$a_{crit} = \frac{1}{\pi} \left(\frac{54}{175} \right)^2 = 0.03m$$

Therefore:

$$N_f = \frac{1}{Q^n C (\Delta\sigma)^n \pi^{n/2}} \left[\frac{(a_{crit})^{1-n/2} - (a_i)^{1-n/2}}{1 - n/2} \right]$$

$$N_f = \frac{1}{1^4 (40 \times 10^{-12}) (175)^4 \pi^2} \left[\frac{(0.03)^{-1} - (10^{-4})^{-1}}{-1} \right]$$

$$N_f = 2.7008 \left[\frac{33.3 - 10000}{-1} \right] = 26,920 \text{ cycles}$$

Example 3:

Calculate the pressure that would cause failure of a thin cylinder of 7.5m inner diameter with 7.5mm thick walls when it contains a through-thickness crack 12.5mm long aligned with the longitudinal direction.

If this vessel is to be subjected to 4000 cycles of repeated internal pressure, determine the maximum cyclic pressure, based upon a safety factor of 2, given:

$$\frac{da}{dN} = 50 \times 10^{-12} (\Delta K)^4$$

$$K_{1C} = 60 \text{ MN}/\text{m}^{3/2}$$

(a) Assume infinite plate with central crack.

$$\sigma_{\theta} = \frac{pd}{2t} = \frac{p \times 7.5}{2 \times 7.5 \times 10^{-3}} = 500p$$

$$K_{Ic} = Q \sigma_c \sqrt{\pi a} = 1 \times \sigma_c \sqrt{\pi a} = \sigma_c \sqrt{\pi a}$$

$$\sigma_{crit} = \frac{K_{Ic}}{\sqrt{\pi a}} = \frac{60 \times 10^6}{\sqrt{\pi \times \frac{12.5}{2} \times 10^{-3}}} = 500p$$

$$p = 0.86 \text{ MPa}$$

(b) $\Delta \sigma = 500p$

$$a_{crit} = \frac{1}{\pi} \left(\frac{K_{Ic}}{Q \sigma} \right)^2 = \frac{1}{\pi} \left(\frac{60}{500p} \right)^2 = \frac{4.584 \times 10^{-3}}{p^2}$$

Using:

$$N_f = \frac{1}{Q^n C (\Delta \sigma)^n \pi^{n/2}} \left[\frac{(a_{crit})^{1-n/2} - (a_o)^{1-n/2}}{1 - n/2} \right]$$

$$4000 = \frac{1}{50 \times 10^{-12} (500p)^4 \pi^2} \left[\frac{\left(\frac{4.584 \times 10^{-3}}{p^2} \right)^{-1} - \left(\frac{12.5}{2} \times 10^{-3} \right)^{-1}}{-1} \right]$$

$$4000 = \frac{1}{50 \times 10^{-12} (500p)^4 \pi^2} \left(160 - \frac{p^2}{4.584 \times 10^{-3}} \right)$$

$$123370.1p^4 = 160 - 218.2p^2$$

$$123370.1p^4 + 218.2p^2 - 160 = 0$$

$$p^2 = \frac{-218.2 \pm \sqrt{218.2^2 - 4(123370.1)(-160)}}{2(123370.1)}$$

$$p^2 = \frac{-218.2 \pm 8888.4}{246740.2} = 0.0351$$

$$p = 0.1875 MPa$$

Therefore, the maximum cyclic pressure is:

$$\frac{0.1875}{SF} = \frac{0.1875}{2} = 0.094 MPa$$