Swansea University
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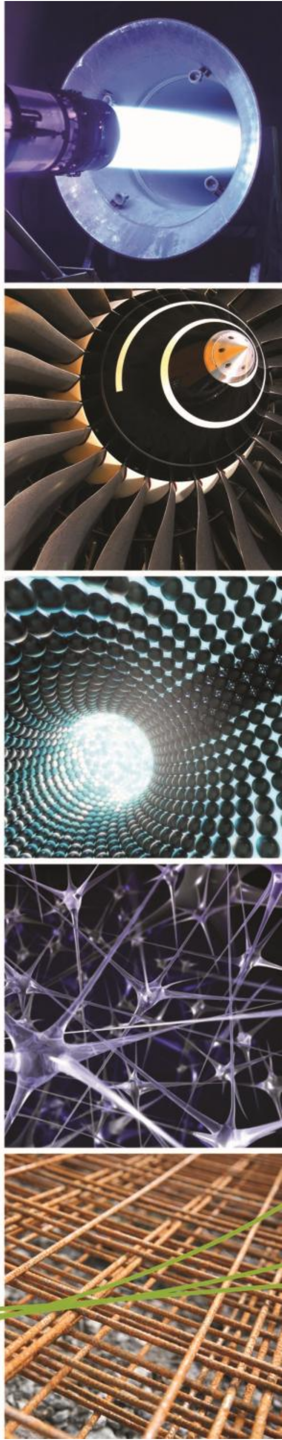
Advanced Structural Analysis

EGF316

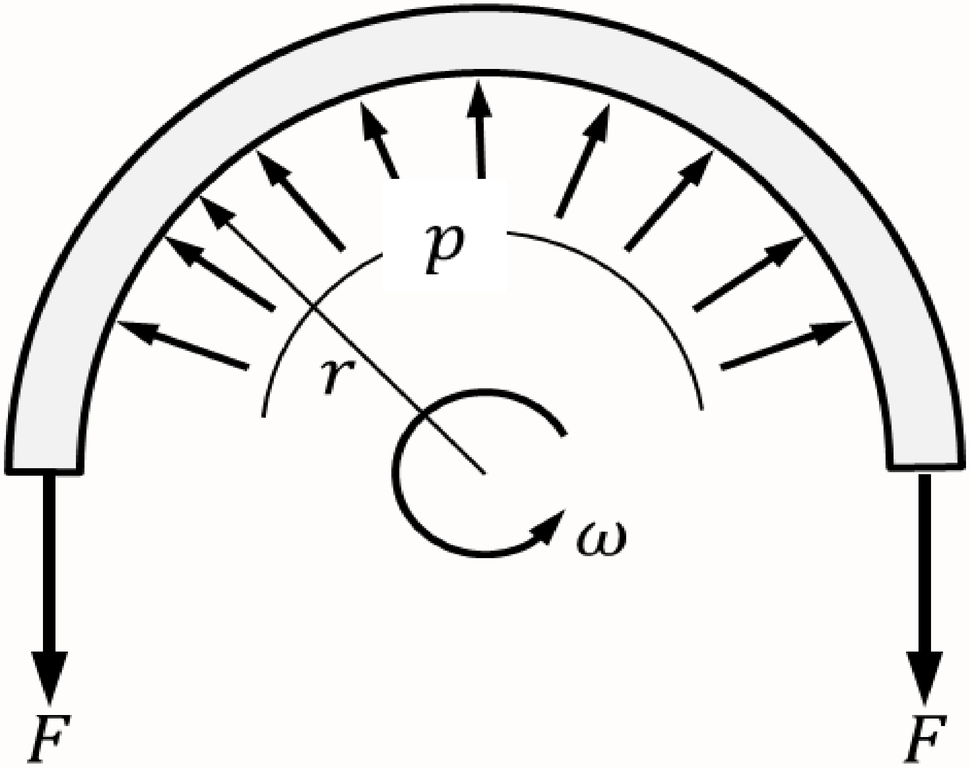
Rotating Discs



Lecture Content

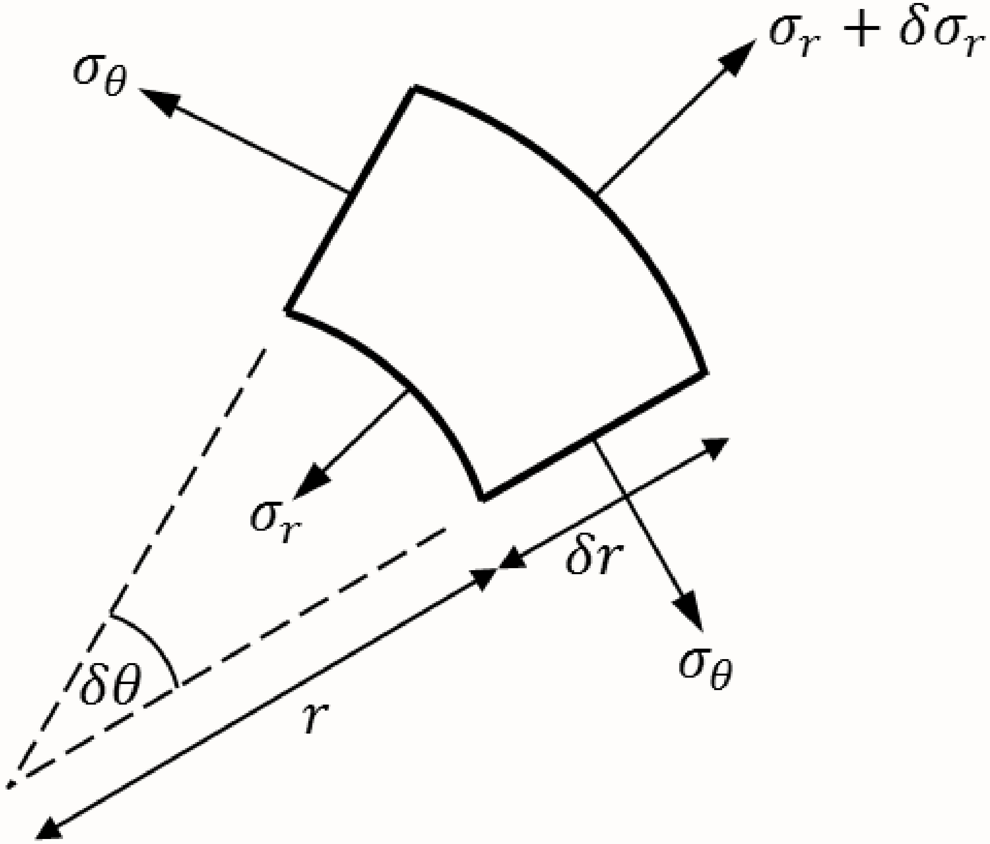
- Thin rotating rings or cylinder
- Rotating discs
- Solid circular disc of uniform thickness
- Circular disc of uniform thickness with a central hole
- Maximum stresses
- Effect of turbine blades on a rotating disc

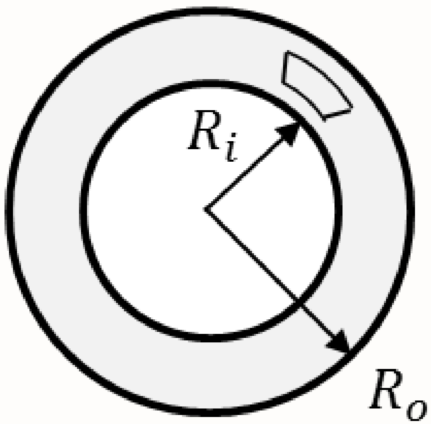
Thin Rotating Ring or Cylinder



$$\sigma_\theta = \rho\omega^2 r^2$$

Rotating Disc





Rotating Solid Disc

$$\sigma_r = A - \frac{B}{r^2} - \frac{(3+v)}{8}\rho\omega^2 r^2$$

$$\sigma_\theta = A + \frac{B}{r^2} - \frac{(1+3v)}{8}\rho\omega^2 r^2$$

Solid Circular Disc

$$\sigma_r = \frac{(3+v)}{8}\rho\omega^2\left(R_o^2 - r^2\right)$$

$$\sigma_\theta = \frac{\rho\omega^2}{8}\left[(3+v)R_o^2 - (1+3v)r^2\right]$$

Circular Disc with Central Hole

$$\sigma_r = (3+v)\frac{\rho\omega^2}{8}\left[{R_i}^2 + {R_o}^2 - \frac{{R_i}^2{R_o}^2}{r^2} - r^2\right]$$

$$\sigma_\theta = \frac{\rho\omega^2}{8}\left[(3+v)\left[{R_i}^2 + {R_o}^2 + \frac{{R_i}^2{R_o}^2}{r^2}\right] - (1+3v)r^2\right]$$

Example 1:

A steel ring of outer diameter 300mm and internal diameter 200mm is shrunk onto a solid steel shaft.

The interface is such that the radial pressure between the mating surfaces remains above 30MN/m^2 at all times whilst the assembly rotates.

The circumferential stress on the inside surface of the ring must not exceed 240MN/m^2

Determine the maximum speed at which the assembly can rotate.

You may assume that $\rho = 7500\mathrm{kg/m^3}, \nu = 0.3, E = 210\mathrm{GPa}$.

Maximum Hoop Stress

$$\sigma_\theta = \frac{\rho\omega^2}{8}\left[(3+v)\left[{R_i}^2 + {R_o}^2 + \frac{{R_i}^2{R_o}^2}{r^2}\right] - (1+3v)r^2\right]$$

Maximum when $r = R_i$:

$$\hat{\sigma}_\theta = \frac{\rho\omega^2}{4}\left[(3+v){R_o}^2 + (1-v){R_i}^2\right]$$

As $R_i \to 0$:

$$\hat{\sigma}_\theta = \frac{\rho\omega^2}{4}(3+v){R_o}^2$$

Comparison:

Solid disc:

$$\sigma_\theta = \frac{\rho\omega^2}{8}\left[(3+v)R_o^2 - (1+3v)r^2\right]$$

At centre of when $r = 0$:

$$\sigma_\theta = \frac{\rho\omega^2}{8}(3+v){R_o}^2$$

Disc with central hole:

$$\sigma_\theta = \frac{\rho\omega^2}{4}(3+v){R_o}^2$$

Maximum Radial Stress

$$\sigma_r = (3+v)\frac{\rho\omega^2}{8}\left[{R_i}^2 + {R_o}^2 - \frac{{R_i}^2{R_o}^2}{r^2} - r^2\right]$$

Max when:

$$\frac{d\sigma_r}{dr} = 0 \quad \rightarrow \quad r = \sqrt{R_i R_o}$$

$$\hat{\sigma}_r = (3+v)\frac{\rho\omega^2}{8}(R_o - R_i)^2$$