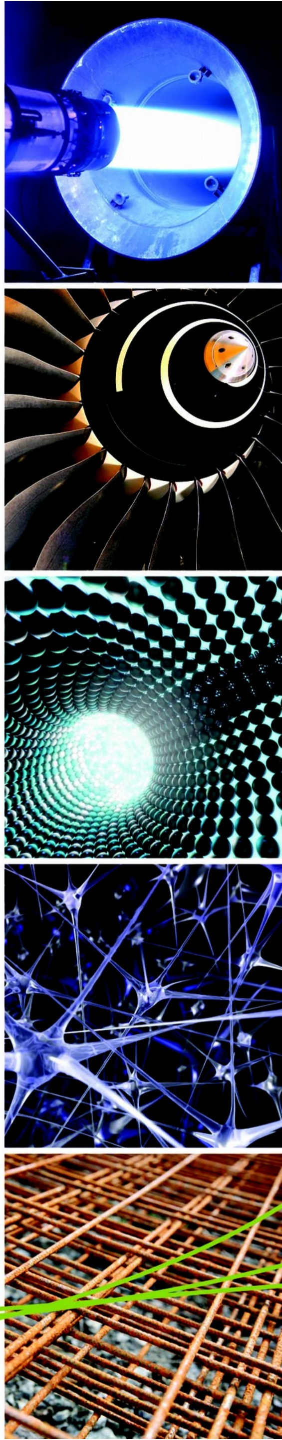
# Advanced Structural Analysis

# EGF316

## Stress and Strain

## The Basics



# Lecture Content



Fundamentals of Stress and Strain

- Stress
- Properties of stress
- Strain
- Properties of strain
- Plane stress
- Plane strain
- Transformation of stresses and strains
- Principal planes and principal stresses
- Maximum shear stress
- Principal strains

# Stress Analysis

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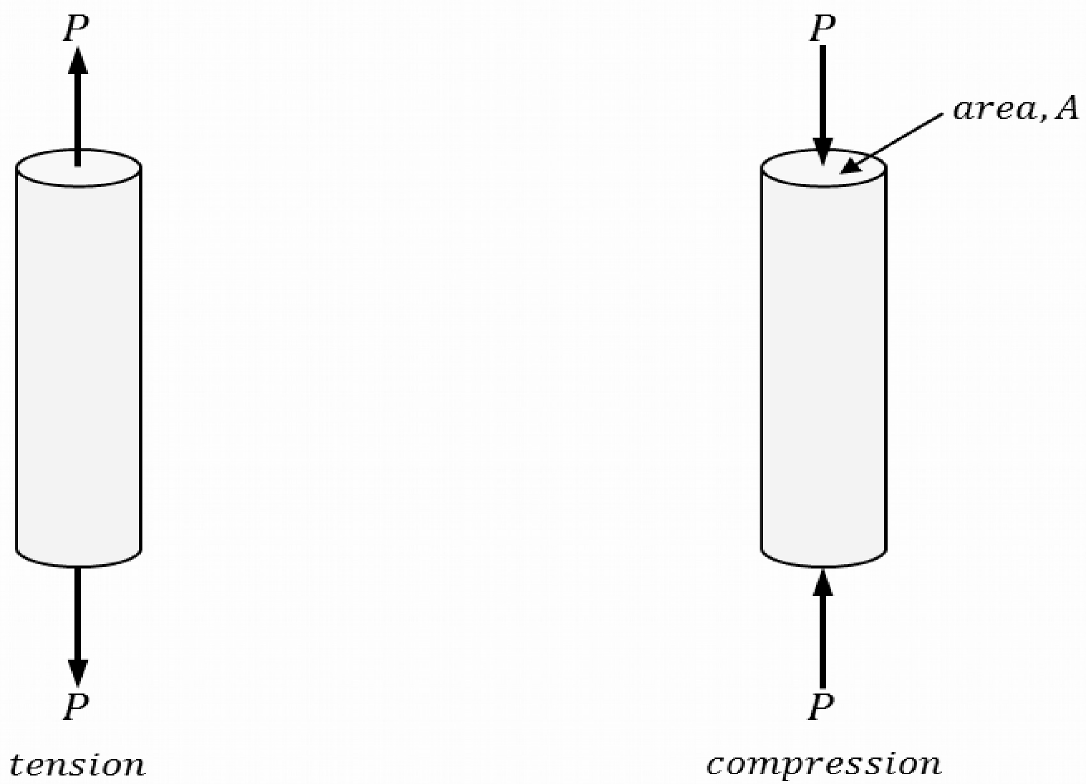
- Importance
  - Load bearing capacity of components
  - Stress concentration - Critical geometric features
  - Points of failure
- Techniques
  - Analytical
  - Empirical formulae
  - Experimental
  - Numerical, typically Finite Element Analysis

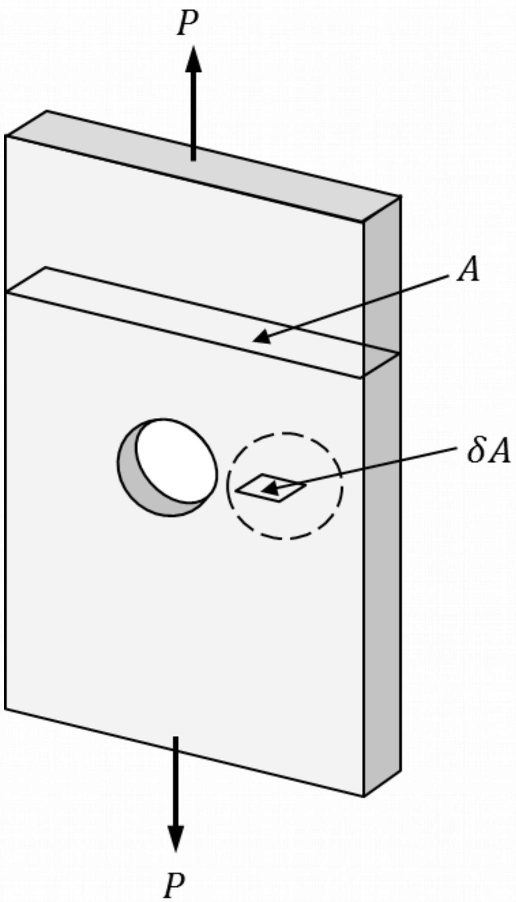
# Direct Stress or Cauchy Stress

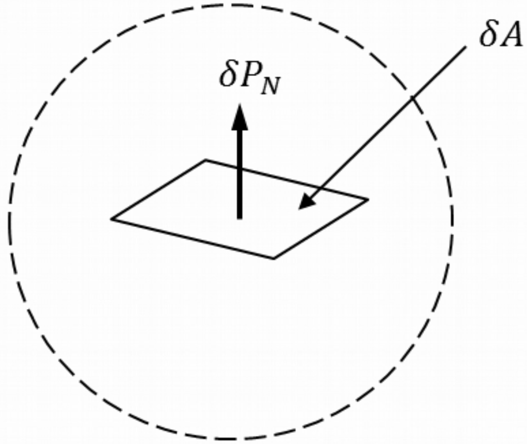
- Stress normal to the material cross-section




$$\boxed{\sigma = \frac{P}{A}}$$

# Direct Stress

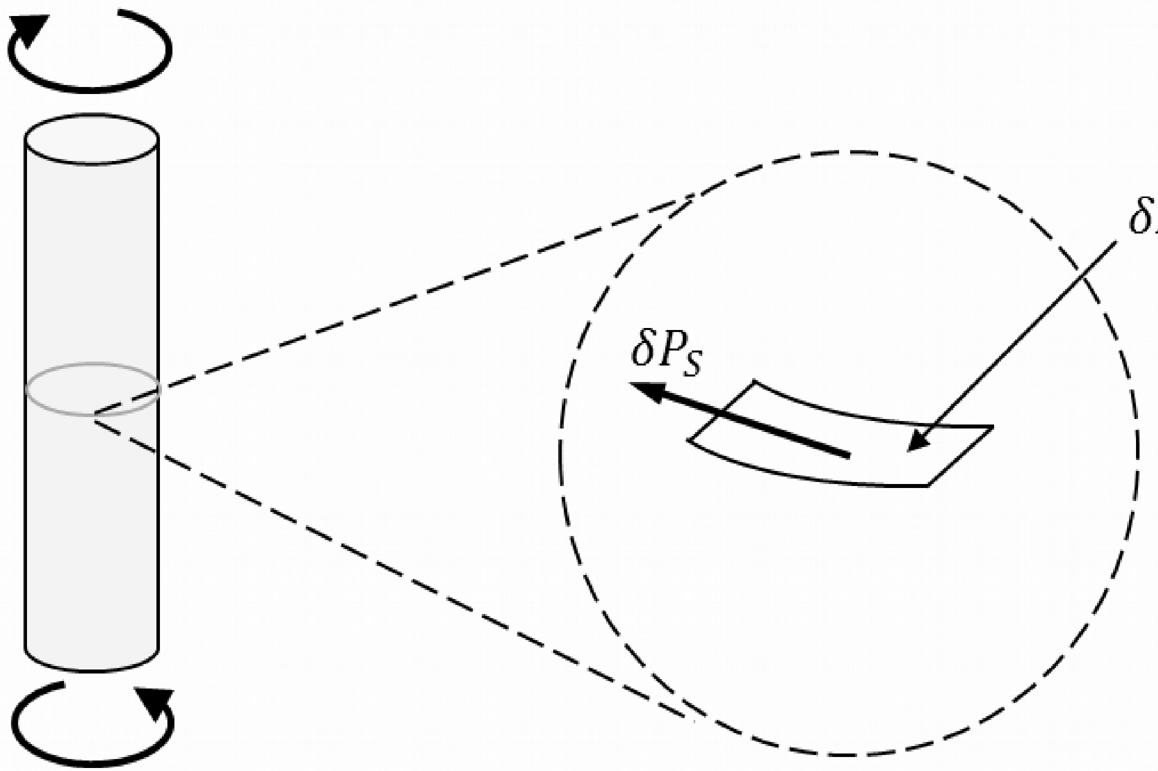



$$\sigma = \lim_{\delta A \to 0} \frac{\delta P_N}{\delta A}$$

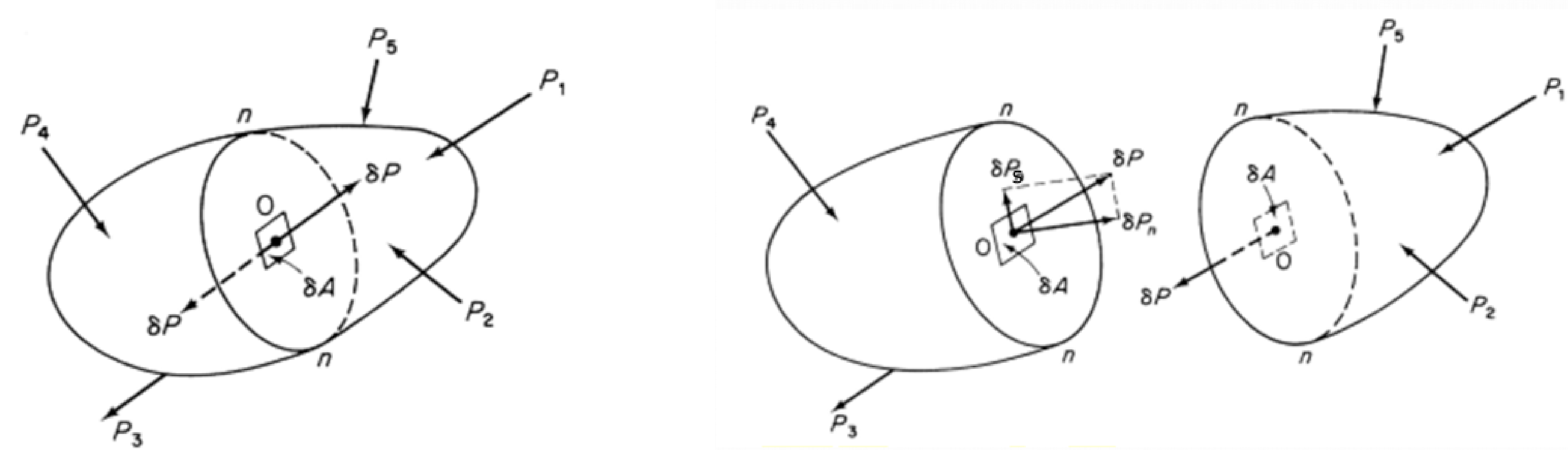
# Shear Stress

- Stress coplanar with the material cross-section



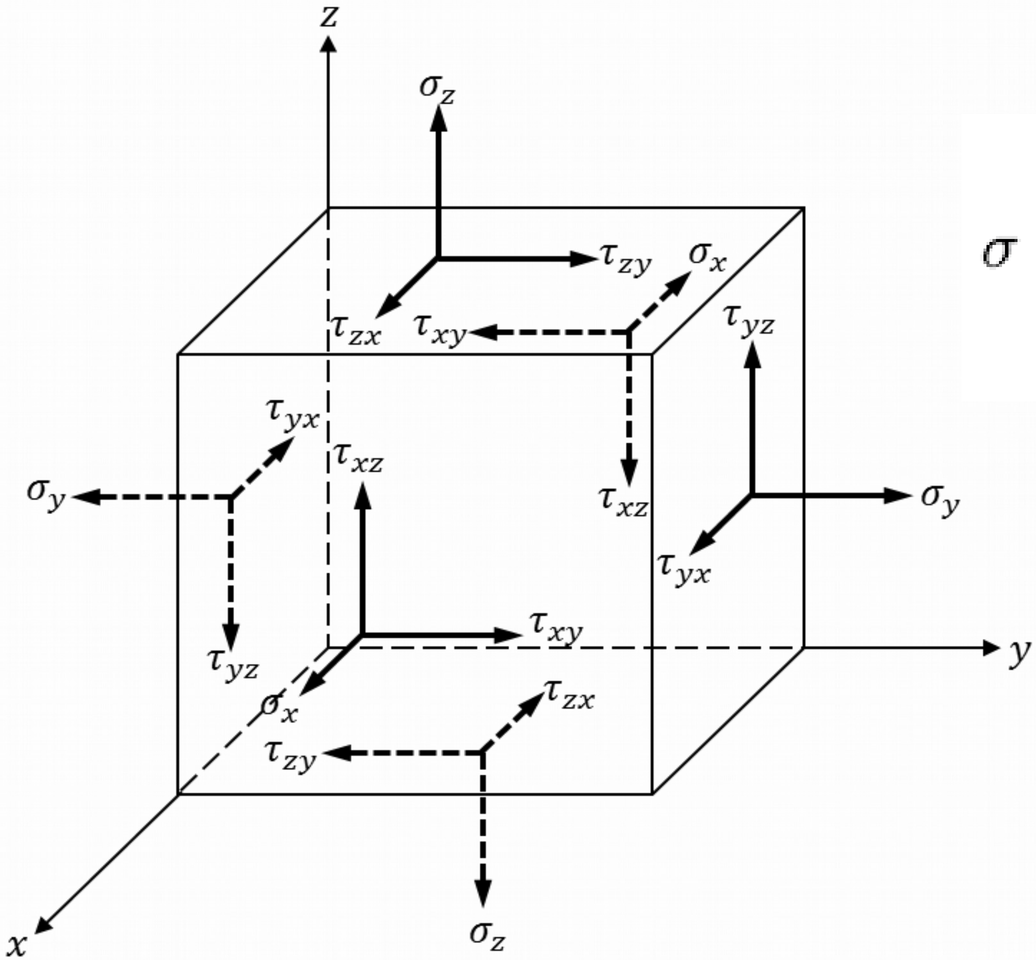

$$\tau = \lim_{\delta A \to 0} \frac{\delta P_S}{\delta A}$$

# Generic state of stress



# Local Equilibrium






$$\sigma = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix}$$

# Properties of Stress

Stress is classed as a *tensor* quantity:

$$\sigma_{ij} \qquad or \qquad \tau_{ij}$$

Where:

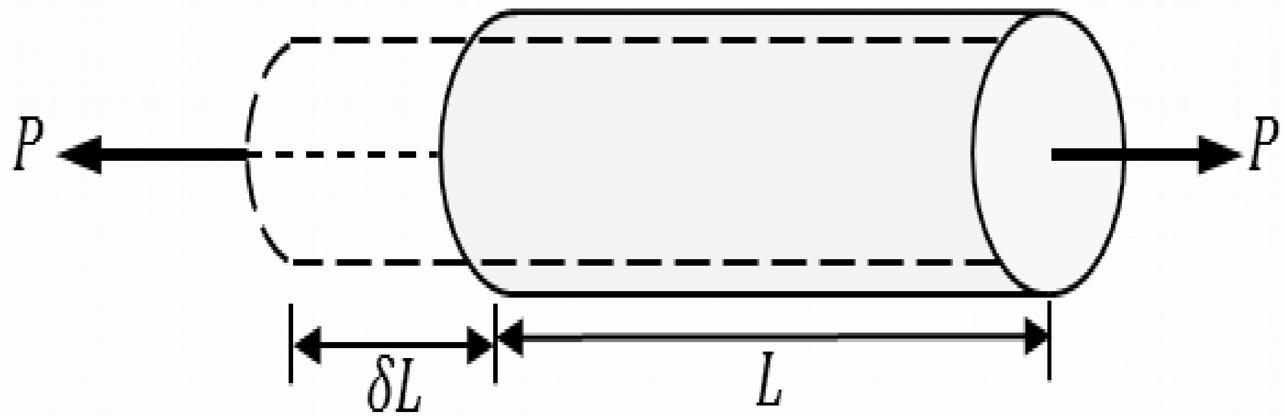
$$i = \text{plane (plane of constant } i\text{)} \;\; \text{face on which stesss acts}$$
$$j = \text{direction}$$

In general:

$$\text{for direct stresses } i = j$$
$$\text{for shear stresses } i \neq j$$

Stress is defined at every infinitesimally small point

# Direct Strain



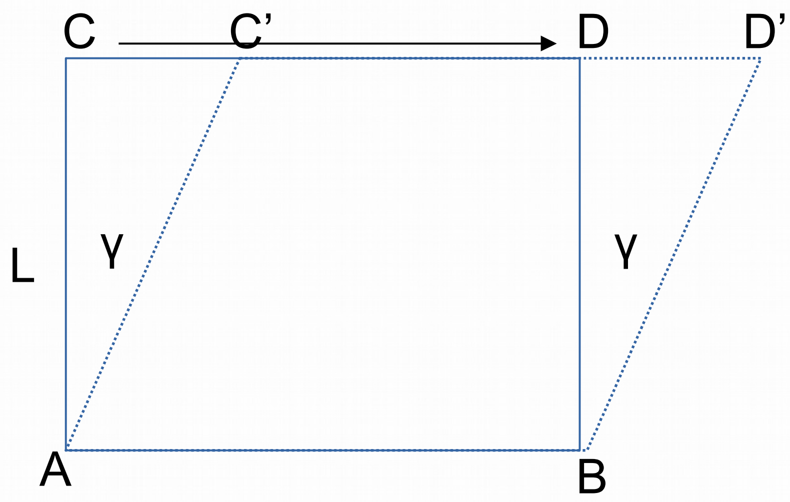

$$\text{Cauchy/Engineering strain} = \frac{\text{change in length}}{\text{original length}}$$

$$\boxed{\varepsilon = \frac{\delta L}{L}}$$

$$\text{Logarithmic/True strain} = \ln\left(\frac{L + \delta L}{L}\right)$$

# Shear Strain

– Change in the angle




$$\tan(\gamma) = \frac{CC'}{AC}$$

$$\text{For small } \gamma,\ \tan(\gamma) \sim= \gamma = \frac{CC'}{AC}$$

$$\frac{shear\ stress}{shear\ strain} = \frac{\tau}{\gamma} = G$$

# Strain tensor



Infinitesimal strain tensor:

$$\varepsilon_{ij} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{bmatrix}$$

# Plane Stress

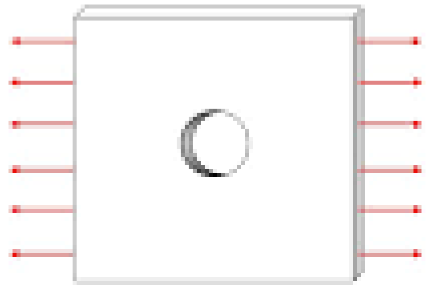


Many structural components are made from thin sheets. Stresses acting across the thickness of such sheets are very small and can be neglected to simplify analysis

If $z$-axis is in the direction of the thickness, the stress state reduces to a 2D stress state in the $xy$-plane

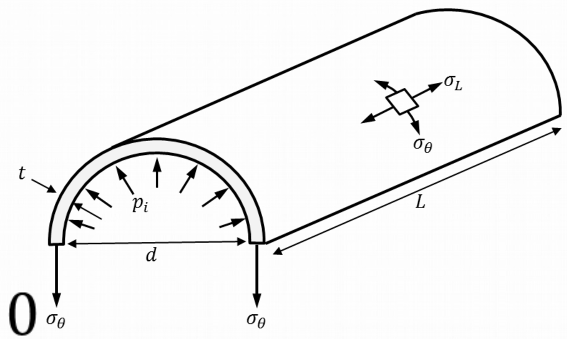
Plane stress assumption:

$$\sigma_z = \tau_{yz} = \tau_{zx} = 0$$



# Plane Strain

Plane strain describes situations where the dimension of the structure in one direction (z-direction) is very large in comparison with the dimensions of the structure in the other two directions.




Plane strain assumption:

$$\varepsilon_{zz} = \gamma_{yz} = \gamma_{zx} = 0$$

As for principal stresses, principal strains occur on planes of zero shear strain.

# Transformation of Stresses



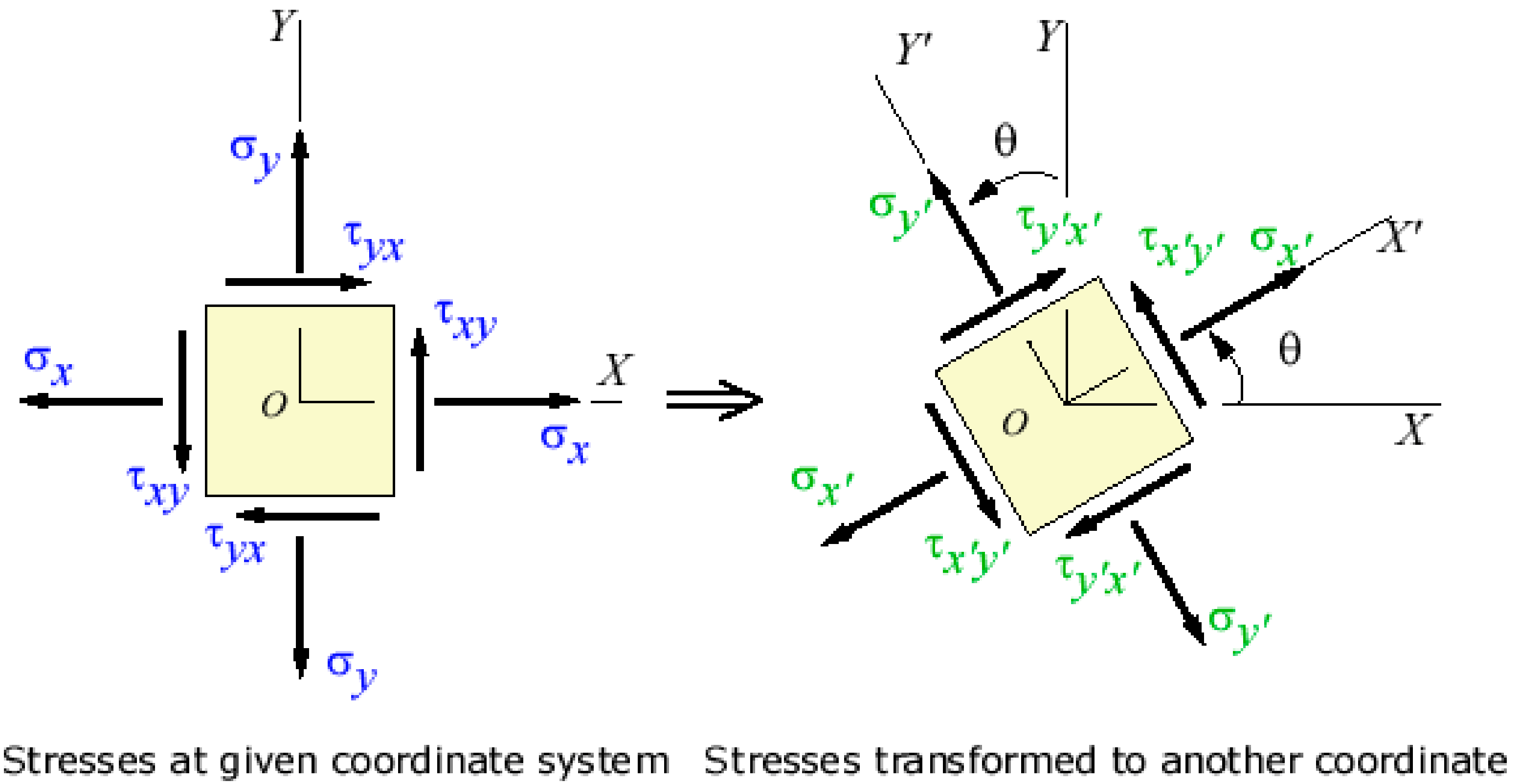


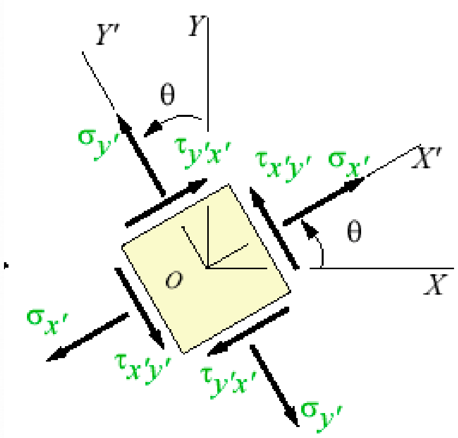

Image source: www.efunda.com

# Transformation of Stresses

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$




For plane stress: $\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$

# Principal Stresses

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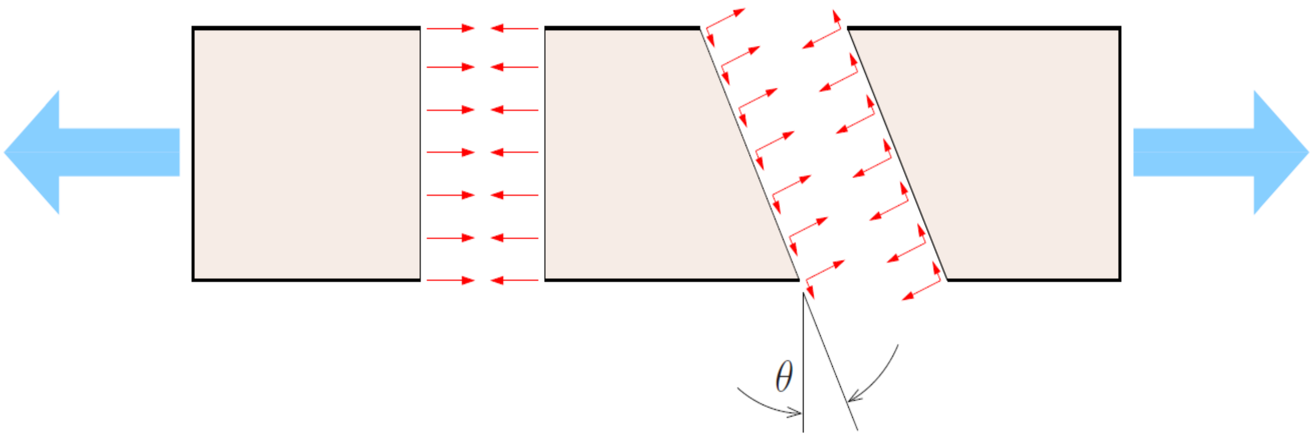
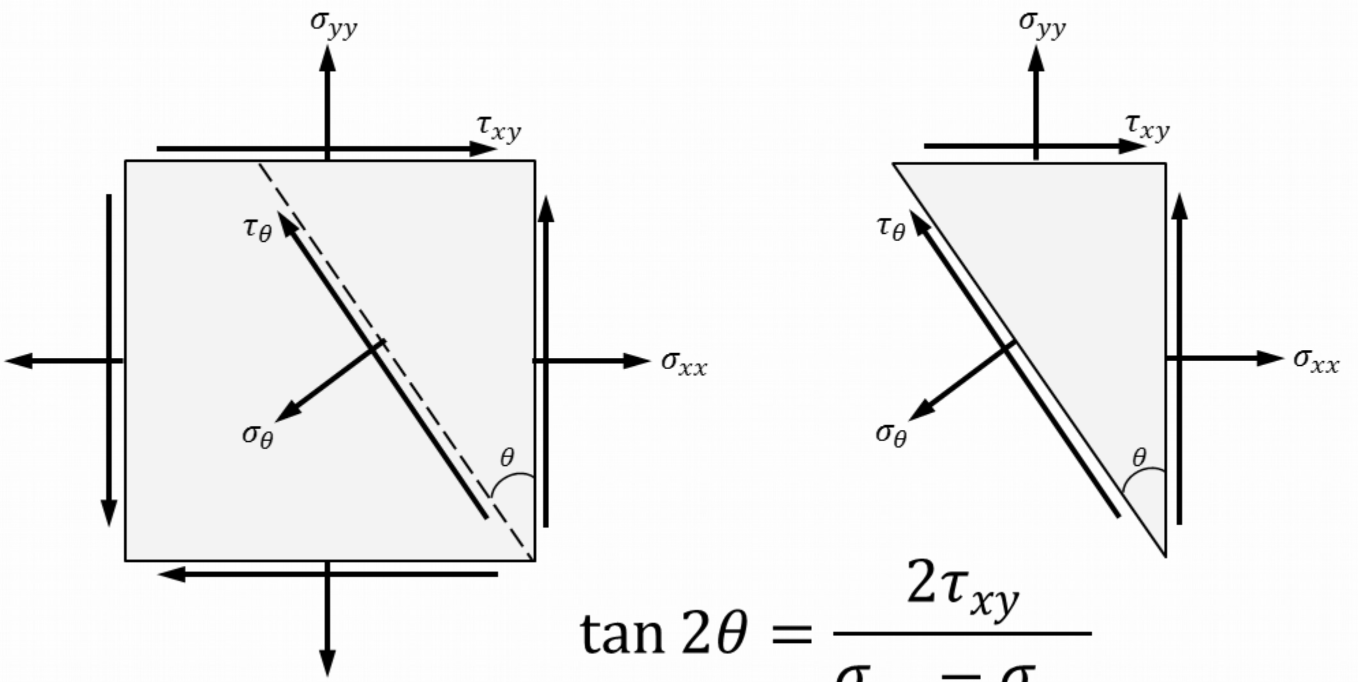
For each possible stress state, there is one specific orientation of the coordinate axes, such that the shear stresses disappear and the direct stresses adopt maximum and minimum values




Principal stresses occur on planes of zero shear stress

# Principal Stresses




$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}$$

$$\sigma_{max,min} = \sigma_{1,2} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}{}^2}$$

# Principal Stresses

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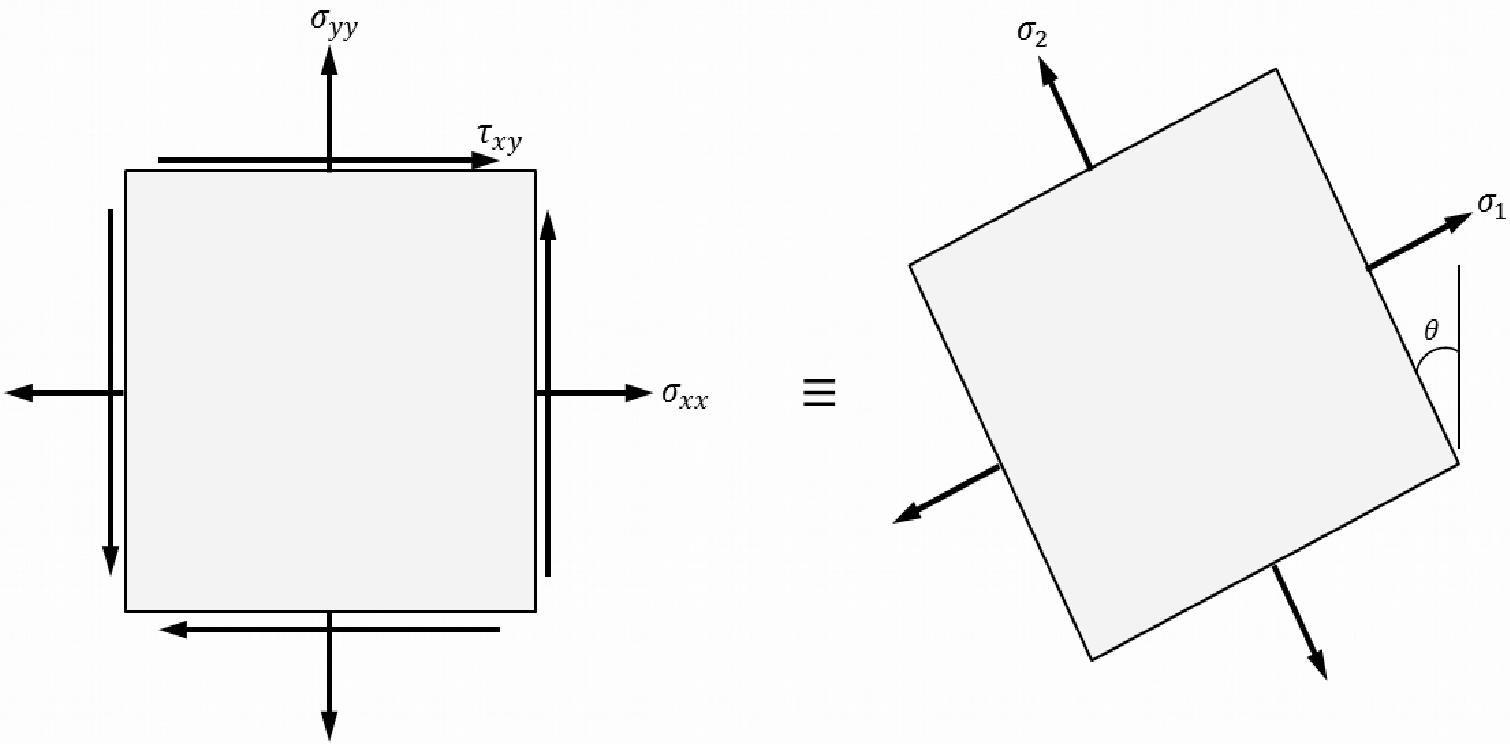


# Maximum Shear Stress

$$tan2\theta = -\frac{(\sigma_{xx} - \sigma_{yy})}{2\tau_{xy}}$$

$$\tau_{max,min} = \pm\sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + {\tau_{xy}}^2}$$

# Example:

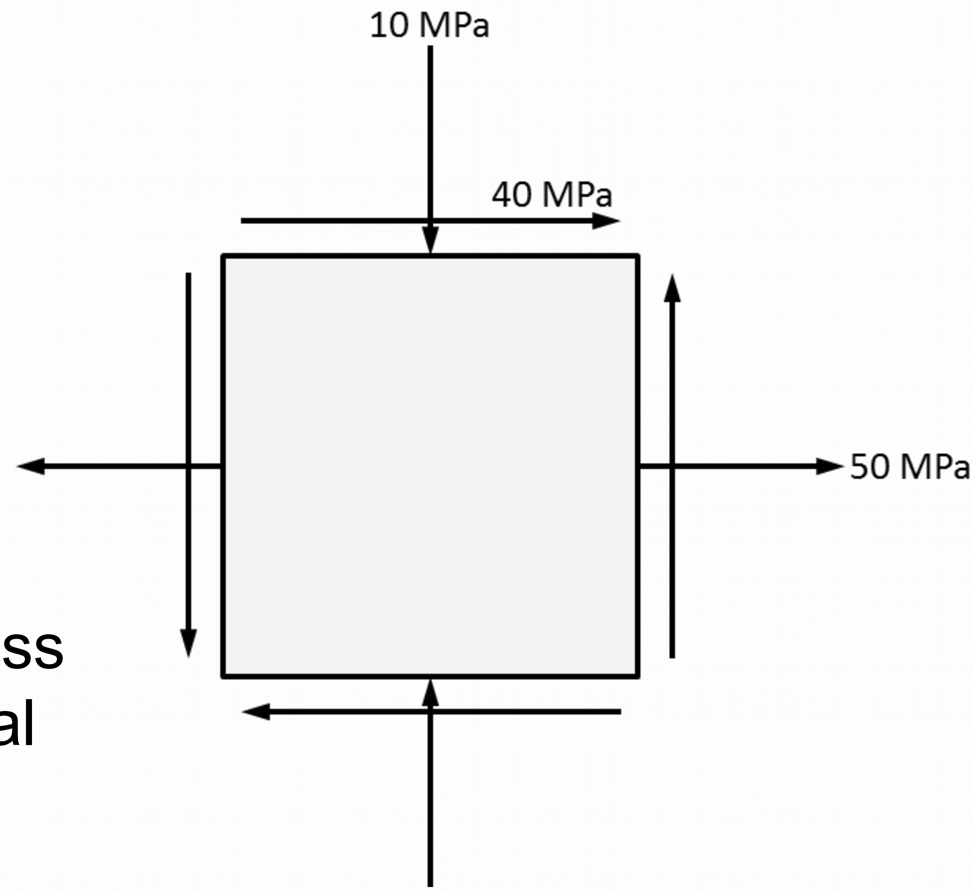
Determine:

(a) The principal planes

(b) The principal stresses

(c) The maximum shear stress and corresponding normal stress

(d) Draw the planes



# Transformation of Strains

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$$\varepsilon_\theta = \frac{1}{2}(\varepsilon_{xx} + \varepsilon_{yy}) + \frac{1}{2}(\varepsilon_{xx} - \varepsilon_{yy})\cos 2\theta + \gamma_{xy}\frac{1}{2}\sin 2\theta$$

$$\gamma_\theta = \frac{1}{2}\left(\varepsilon_{xx} - \varepsilon_{yy}\right)\sin 2\theta - \frac{1}{2}\gamma_{xy}\cos 2\theta$$

# Principal Strains

- Analogous to principal stresses.
- Principal strains occur on planes of zero shear strain and can be shown to be given by:

$$\varepsilon_{max,min} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} \pm \sqrt{\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$