Swansea University
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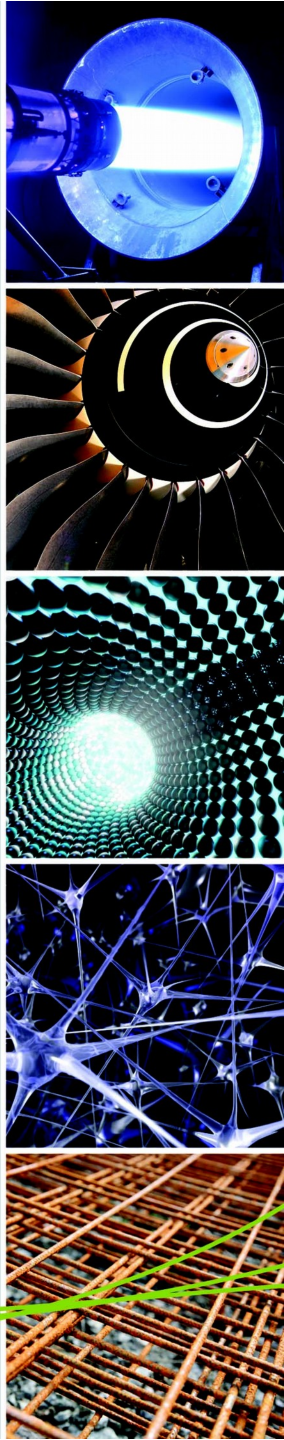
Advanced Structural Analysis
EGF316

Stress and Strain relationships



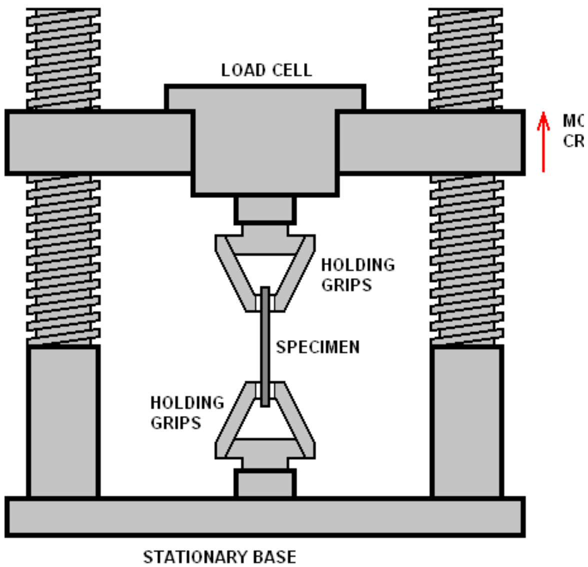
Lecture Content

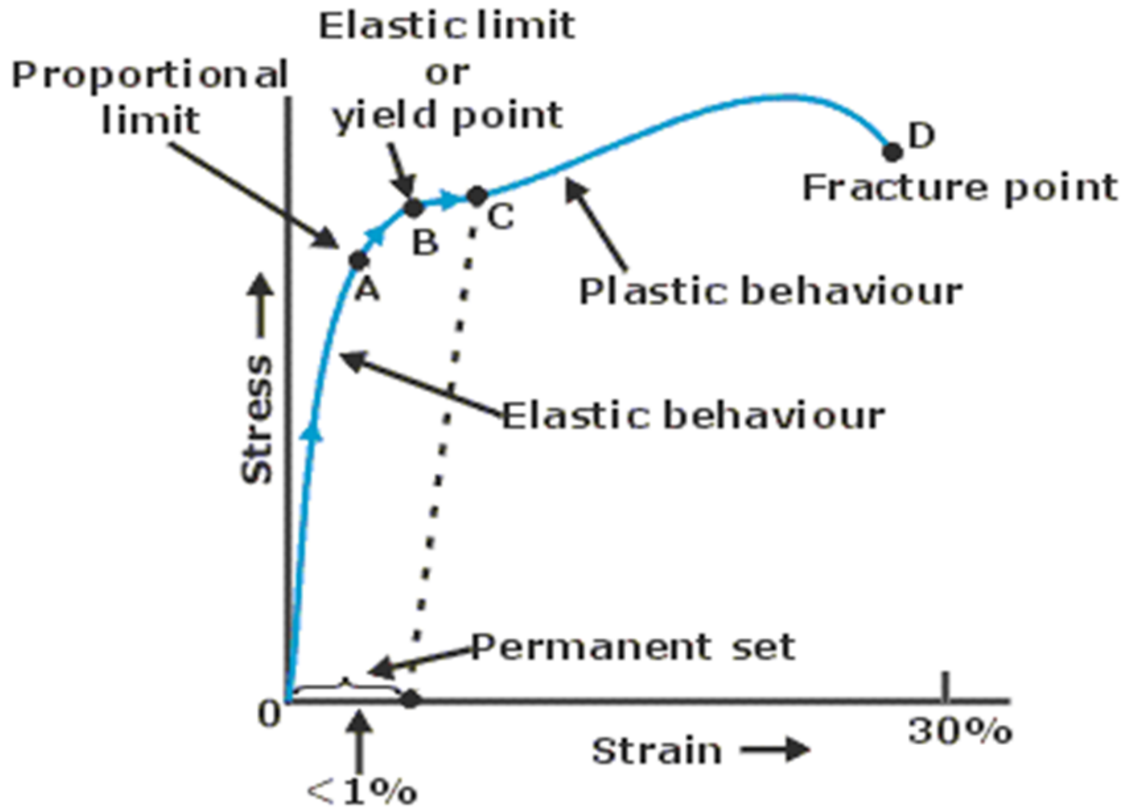
- Stress-strain diagram
- Hooke's law
- Young's modulus (E)
- Poisson's ratio (v)
- Plane stress
- Plane strain

Stress-strain relations

- Stress = f1(strain)
- Strain = f2(stress)
- Varies from simple linear mapping to quite complicated models depending upon the material under consideration and also the strain regime (small or large/finite strains).
- Also, the relationships vary depending upon the Stress and Strain measures/definitions used.
- Tensile testing using Universal Testing Machine

Universal Testing Machine (UTM)

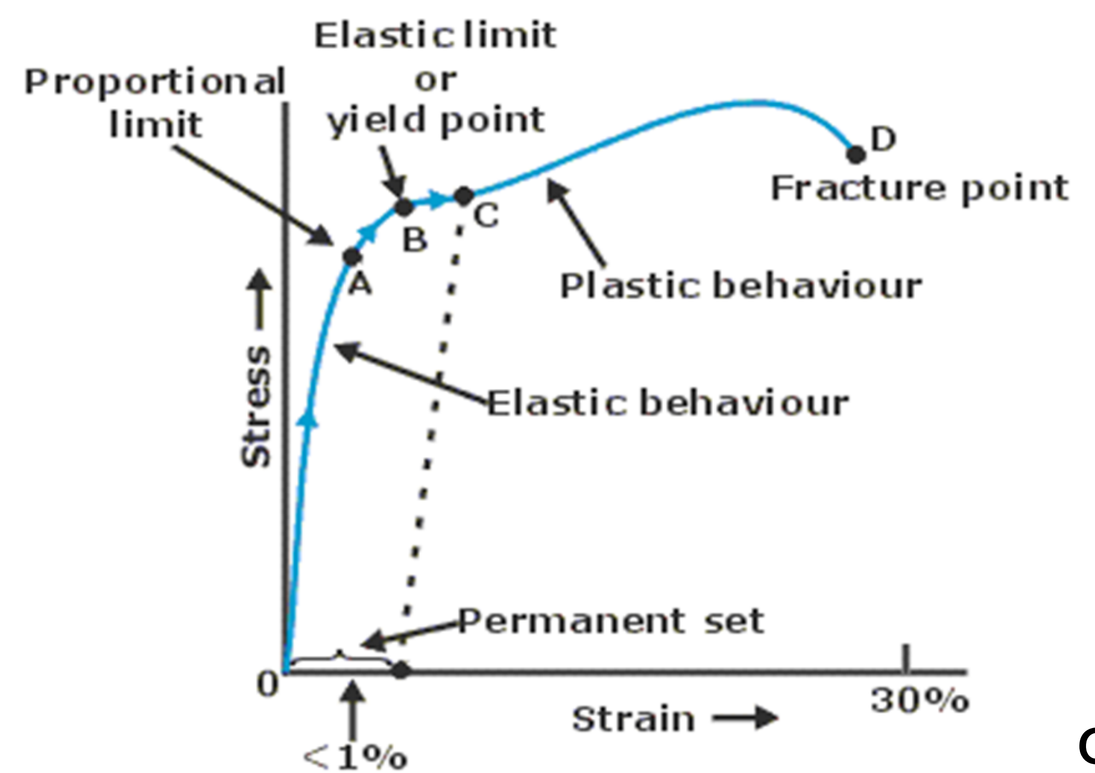


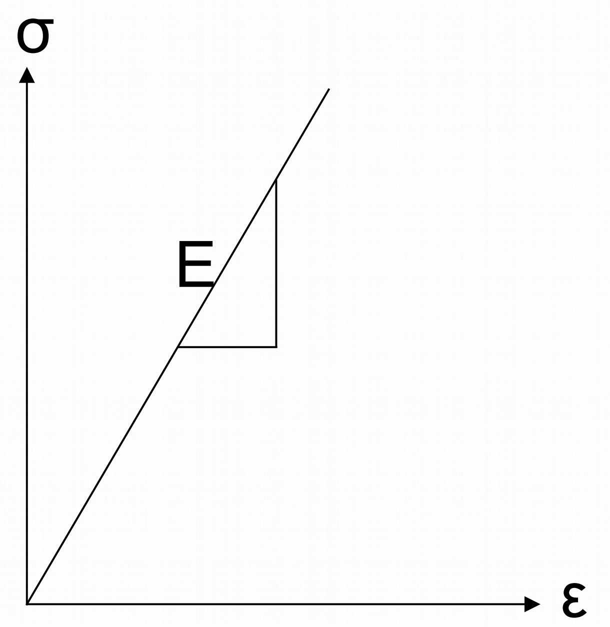


Stress-strain graph for ductile material

Tensile Testing – ductile Vs brittle materials

Linear Elasticity - Hooke's law

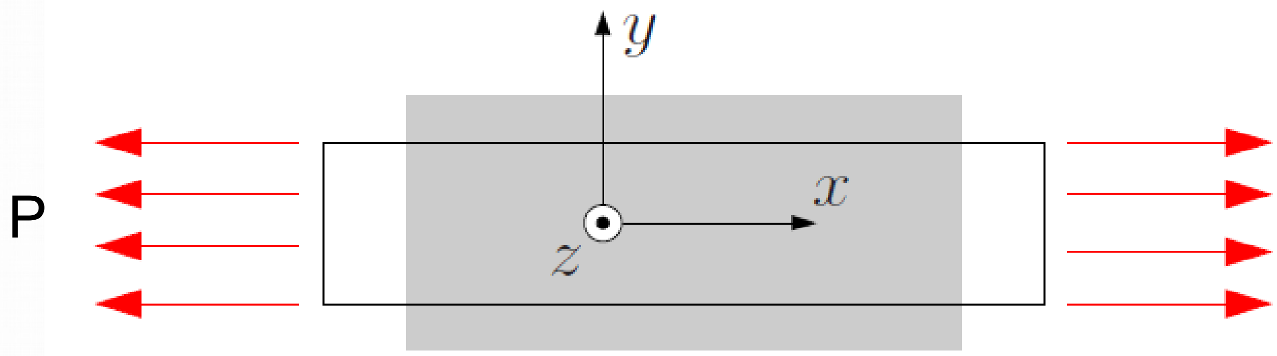




$\sigma = E\,\varepsilon \rightarrow$ Hooke's law

E = Young's modulus

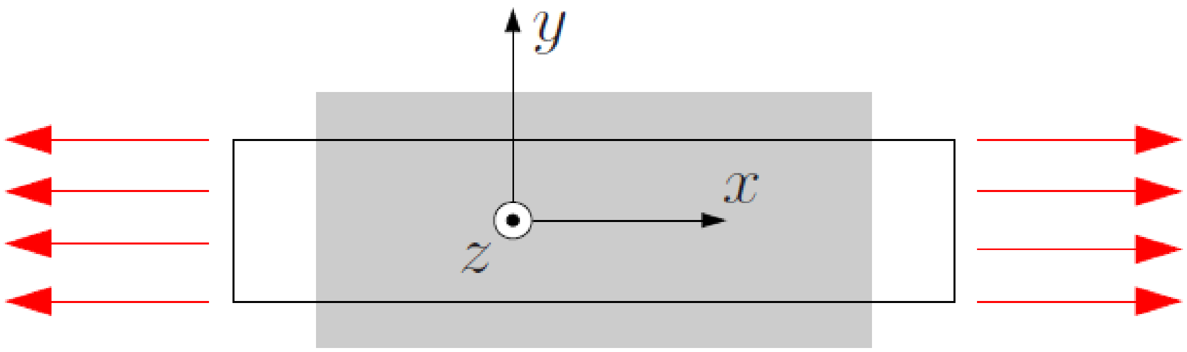
Linear Elasticity – uniform bar



$$\boxed{\sigma = \frac{P}{A}} \qquad \boxed{\varepsilon = \frac{\delta L}{L}}$$

$$E = constant = \frac{stress}{strain} = \frac{\sigma}{\varepsilon} = \frac{P}{A} \Big/ \frac{\delta L}{L} = \frac{PL}{A\delta L}$$

Poisson's Ratio



$$\nu = -\frac{\text{lateral strain}}{\text{longitudinal strain}}$$

The minus sign compensates for the fact that the lateral and axial strains, in general, have opposite signs.

$$\nu = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = -\frac{\varepsilon_{zz}}{\varepsilon_{xx}}$$

Hooke's law in shear

$$\tau = G\gamma$$

G is called the **shear modulus** or **the modulus of rigidity**

$$G = \frac{E}{2(1+\nu)}$$

Types of materials

- Homogeneous
  - Material properties are same everywhere
- Nonhomogeneous
  - Material properties depend upon the location
- Isotropic
  - Material properties are same in every direction
- Anisotropic
  - Material properties are different in different directions

We focus on Linear, Homogeneous and Isotropic materials.

Direct-Stress-Strain Relationship

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\sigma_{yy} - \nu\sigma_{zz}\right]$$
$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\sigma_{xx} - \nu\sigma_{zz}\right]$$
$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\sigma_{xx} - \nu\sigma_{yy}\right]$$

$$\sigma_{xx} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{xx} + \nu\varepsilon_{yy} + \nu\varepsilon_{zz}\right]$$
$$\sigma_{yy} = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\varepsilon_{xx} + (1-\nu)\varepsilon_{yy} + \nu\varepsilon_{zz}\right]$$
$$\sigma_{zz} = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\varepsilon_{xx} + \nu\varepsilon_{yy} + (1-\nu)\varepsilon_{zz}\right]$$

Shear Stress-Strain Relationship

3D state of stress:

$$\gamma_{xy} = \frac{\tau_{xy}}{G}, \qquad \gamma_{yz} = \frac{\tau_{yz}}{G}, \qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

$$\gamma_{xy} = \frac{2(1+v)\tau_{xy}}{E}, \qquad \gamma_{yz} = \frac{2(1+v)\tau_{yz}}{E}, \qquad \gamma_{zx} = \frac{2(1+v)\tau_{zx}}{E}$$

2D stress state - Plane Stress

Stresses in the plane normal to the plane of deformation are assumed to be zero.

$$\boxed{\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0}$$

Also $$\boxed{\gamma_{xz} = \gamma_{yz} = 0}$$

$$\begin{aligned}
\varepsilon_{xx} &= \frac{1}{E}\left[\sigma_{xx} - \nu\sigma_{yy}\right] \\
\varepsilon_{yy} &= \frac{1}{E}\left[\sigma_{yy} - \nu\sigma_{xx}\right] \\
\varepsilon_{zz} &= \frac{1}{E}\left[-\nu\sigma_{xx} - \nu\sigma_{yy}\right]
\end{aligned}$$

$$\begin{aligned}
\sigma_{xx} &= \frac{E}{(1-\nu^2)}\left[\varepsilon_{xx} + \nu\varepsilon_{yy}\right] \\
\sigma_{yy} &= \frac{E}{(1-\nu^2)}\left[\nu\varepsilon_{xx} + \varepsilon_{yy}\right]
\end{aligned}$$

2D stress state - Plane Strain

Strains in the plane normal to the plane of deformation are assumed to be zero.

$$\boxed{\varepsilon_{zz} = \gamma_{xz} = \gamma_{yz} = 0}$$

Also $\boxed{\tau_{xz} = \tau_{yz} = 0}$

$$\sigma_{xx} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{xx} + \nu\varepsilon_{yy}\right]$$

$$\sigma_{yy} = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\varepsilon_{xx} + (1-\nu)\varepsilon_{yy}\right]$$

$$\sigma_{zz} = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\varepsilon_{xx} + \nu\varepsilon_{yy}\right]$$

Find expressions for the strains in terms of stresses.

Plane Stress Example:

The principal strains at a point on a loaded thin plate are found to be 320×10^{-6} and 200×10^{-6}. What are the principal stresses if the modulus of elasticity is 200GPa and the Poisson's ratio is 0.3?

Plane Strain Example:

A strain gauge rosette is attached to a thin flat steel plate. When loaded in-plane, the following strain values are measured:

$$\varepsilon_A = 500 \times 10^{-6}$$
$$\varepsilon_B = -400 \times 10^{-6}$$
$$\varepsilon_C = -200 \times 10^{-6}$$

Gauge A lies along the x axis of the structure and Gauge B and C are at $+45^{\circ}$ and -45° to Gauge A, respectively. Determine:

i) The in-plane strains and stresses

ii) The in-plane principal strains

iii) The principal stresses and the maximum shear stress

For steel, assume $E = 200GPa, v = 0.3 \text{ and } \sigma_{yield} = 300MPa$.