<u>EGF316 - Advanced Structural Analysis</u>
<u>8. Fatigue</u>

8.1 Introduction

The field of Fatigue and Fracture Mechanics is primarily concerned with the ***initiation*** and ***propagation*** of a crack or cracks in a material until a point is reached when the component or structure can no longer sustain the level of applied loading. It is the study of cracks in structures, and the resistance of a material to fracture which is commonly known as its *'toughness'*.

In our analysis so far, loads (forces, moments, torques etc.) have been assumed to be constant. The behaviour of engineering materials under static loading conditions is well understood. However, in practice most engineering components are subjected to loading that varies with time. Such cyclic loading can be systematic or random.

- Wind and wave loading
- Reversed loading on a drive shaft
- Repeated bending of a crankshaft
- Pressure pulsations in pipework (water hammer)

It has been discovered that many components subjected to a cyclic load, fail at stresses well below the tensile strength, and sometimes even below the elastic limit of the material. These failures generally occur after a large number of stress cycles and are hence known as *fatigue failures*.

In the case of fatigue failures, a component subjected to a cyclic load is susceptible to cracks being initiated at a point where there is a tensile stress concentration due to:

- A change in section (fillets, holes, grooves, threads, projections…)
- A material defect
- Surface roughness
- Impact
- Surface treatments

No structure is completely free from defects and even on a microscopic scale, these defects act as stress concentrations which initiate the growth of cracks, as determined by Leonardo Da Vinci, back in the 17th century. He conducted experiments to study on the fracture strength of iron wires. He used different lengths of wire with the same diameter. He determined that the strength of the iron wires varied inversely with the wire length. He concluded that the shorter the wire, the less likely it was to contain any defect therefore the stronger it was.

Fracture of components due to fatigue is the most common cause of service failure, particularly in shafts, axles, aircraft wings etc. where cyclic stresses are present.

There are two main categories for the failure of structures:

- Negligence during design, construction or operation of the structure
- Application of a new design or new material, which produces an unexpected, and undesirable, result

The first category can be offset by existing procedures, which for a number of reasons – human error, ignorance, wilful misconduct – have not been adhered to. The latter category is more difficult to negate. Whenever a new material or design is introduced, they will require an enormous amount of testing before being put into service, but even this diligence cannot predict and prevent all problems.

"..... 90% of all failures are brought about by fatigue and 90% of all fatigue failures are brought about due to lack of attention to design detail"

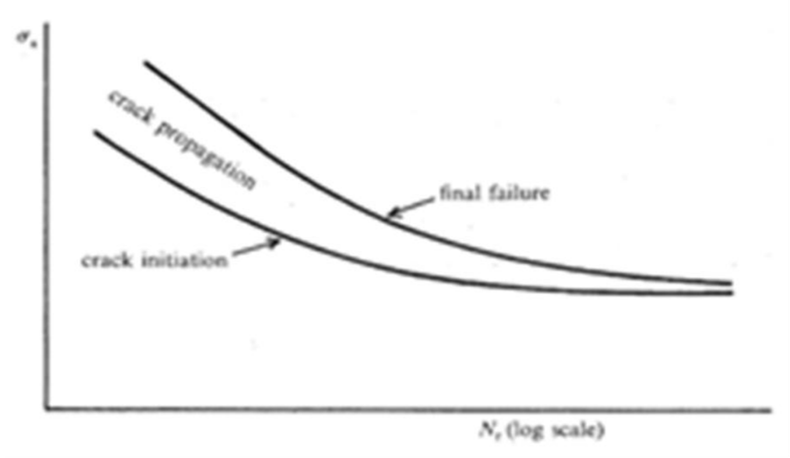
Due to the repeated loading and unloading the crack propagates through the component, until a critical crack size is reached, when the component fails catastrophically and completely. A sudden and total failure is very dangerous, so wherever possible fatigue must be considered at the design stage. This will allow us to either:

- Avoid fatigue failure or,
- Predict when failure is likely to occur

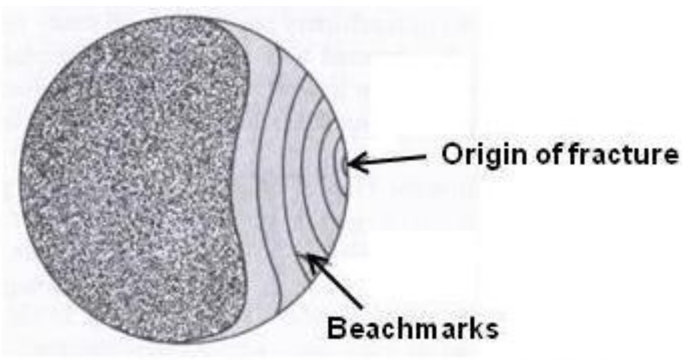
8.2 Failure Surface:

As previously mentioned, there are two key stages in fatigue failures:

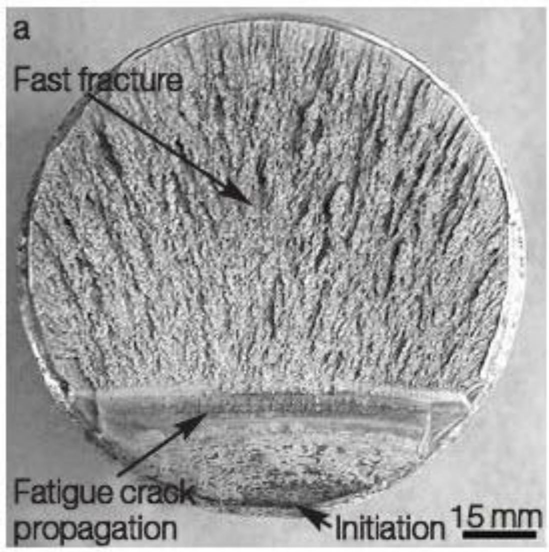
- Crack initiation (fatigue)
- Crack propagation (fracture mechanics) leading to sudden failure



This is reflected in the failure surface which consists of two main areas. The area over which the crack has propagated looks relatively smooth macroscopically, although under a microscope characteristic fatigue striations can be seen. These radiate out from the point of crack initiation. The second area is where the sudden failure has occurred.



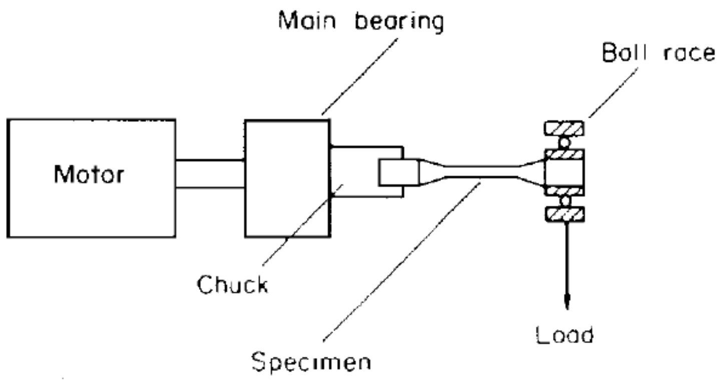
Fatigue Fracture with Beachmarks



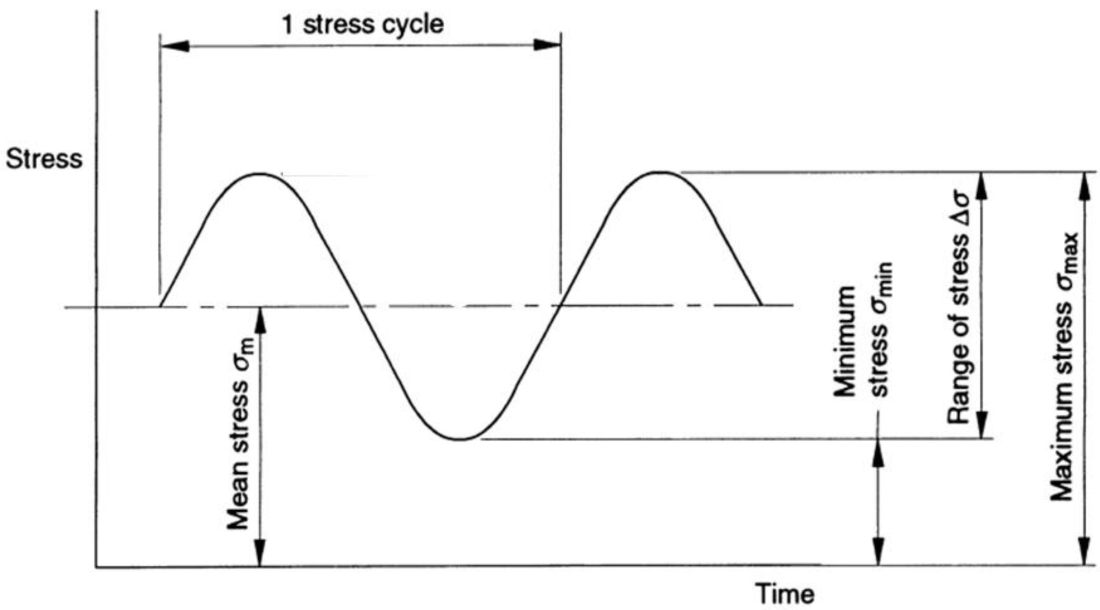
8.3 Fatigue testing:

Fatigue tests are often carried out on a Wohler machine under rotating-bending conditions with a zero mean stress.

The specimen is supported as a cantilever type beam. The top surface is in tension and the bottom surface in compression. As the specimen rotates, the surfaces switch places so each segment of the surface is continuously moving between tension and compression. This produces a stress-cycle curve.



Another common type of fatigue test is using a 'push-pull' machine such as a Haigh machine, which differs from the Wohler test in that the tensile mean stress is positive (not zero). Let us define some common terms to describe fatigue. Consider the below plot:



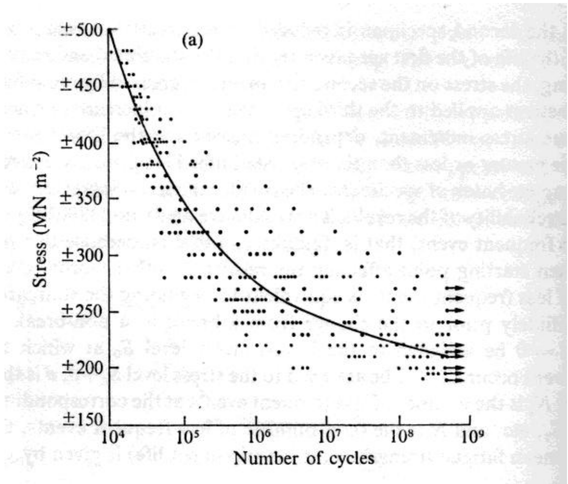
For stress cycles that are not symmetrical about the time axis, the mean stress is defined as the average of the maximum and minimum peak stresses:

$$\text{Mean stress}, \sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad (8.1)$$

The stress range is the difference between these peak values:

$$\text{Stress range}, \Delta\sigma = \sigma_{max} - \sigma_{min} \qquad (8.2)$$

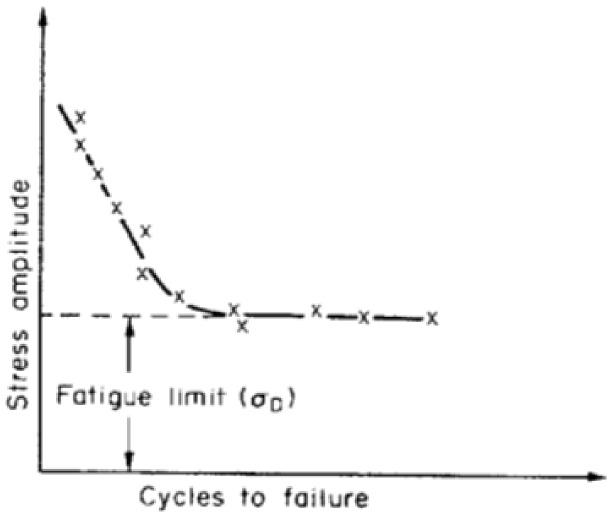
Due to the nature of the fatigue, a series of tests are carried out for a given load to determine the number of cycles to failure. The results can vary considerably, even in a well conducted test. The test is repeated for a series of different loads. The fatigue data is presented as an *S-N* curve – this is stress amplitude against the corresponding number of cycles to failure. The plot below shows typical test results for a fatigue test:



From the S-N curve we can also determine the *fatigue limit* or the *endurance limit*.

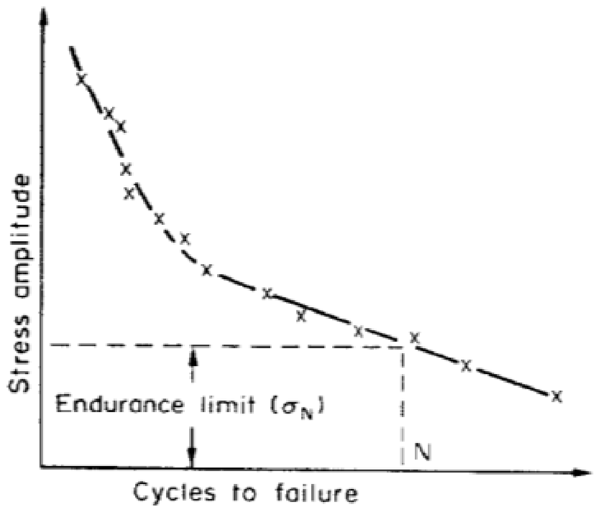
8.4 Fatigue Limit:

The *fatigue limit*, S_L or σ_e or S_e, is the stress condition below which a material may endure an infinite number of cycles prior to failure. Ferrous metal specimens often produce asymptotic *S-N* curves (infinite life, no fatigue failure will occur).



8.5 Endurance Limit:

For most other materials, the curve continues to fall. An *Endurance Limit,* S_E*,* is used for a specified life, normally between 10^9 and 10^{10} cycles. The endurance limit (or fatigue strength) is the stress condition under which a specimen would have a fatigue life of N cycles as shown.



8.6 Fatigue Categorisation:

8.6.1 Low cycle fatigue (LCF):

- *N* is < 10,000 (10^4) cycles
- Significant plasticity
- Dominated by plastic strain (complex)

8.6.2 High cycle fatigue (HCF):

- *N* is > 10,000 (10^4) cycles
- Elastic loading
- Dominated by elastic strain

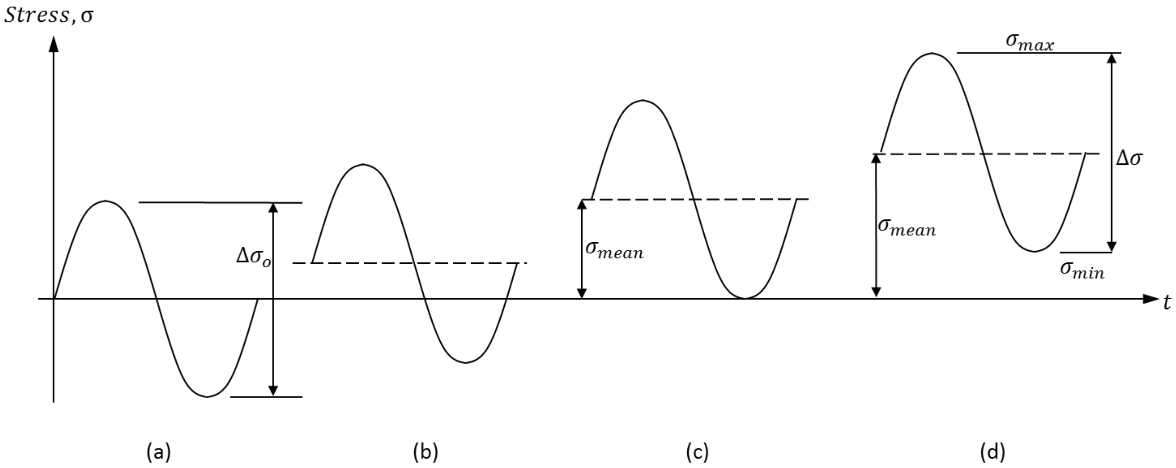
We will focus on High Cycle Fatigue. As S increases, the number of cycles to failure, N, reduces. The effects of stress concentrations are not considered at this stage.

High cycle fatigue is a common cause of failure. It encompasses elastic loading where the maximum stress is lower that the yield stress of the material. Factors affecting the number of cycles to failure, *N*, are:

- The applied stress range
- The mean stress value
- The change in stress amplitude
- The stress state
- The presence of any stress concentrations
- Temperature
- Corrosion

8.7 The Effect of the Mean Stress:

We will consider four types of cycle:



(a) Fully reversed cycle:

$$\sigma_{mean} = 0, \quad |\sigma_{max}| = |\sigma_{min}|$$

(b) Alternating cycle

$$\sigma_{max} \text{ is positive,} \quad \sigma_{min} \text{ is negative,} \quad \sigma_{mean} \neq 0 \quad (\text{either} + \text{ve or} - \text{ve})$$

(c) Repeated cycle

$$\sigma_{min} = 0, \quad \sigma_{max} \text{ is positive,} \quad \sigma_{mean} = \frac{\sigma_{max}}{2} = \frac{\Delta\sigma}{2}$$

(d) Fluctuating cycle

$$\sigma_{max}, \quad \sigma_{min}, \quad \text{and} \quad \sigma_{mean} \quad \text{are all positive or all negative}$$

It has been shown experimentally that as the mean stress in (b), (c) and (d) increases, the safe range of cyclic stress $\Delta\sigma$, defining the fatigue or endurance limit, decreases.

A number of investigations have been carried out looking at the quantitative effect of varying the tensile mean stress. A number of design rules to predict the effects of mean stress on the safe range of stress $\Delta\sigma_o$ under the reversed stress cycles have been developed.

The limiting conditions are then:

i) When $\sigma_{mean} = 0, \quad \Delta\sigma_o = 2\sigma_e$ for the reversed cycles
ii) When $\Delta\sigma = 0, \quad \sigma_{mean} = \sigma_u$ (or σ_y for a static tension test)

The following three failure criteria are widely popular.

Goodman:

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_u} = \frac{1}{FS} \qquad (8.3a)$$

or,

$$\sigma_a = \frac{\sigma_e}{FS}\left[1 - \left(\frac{FS * \sigma_m}{\sigma_u}\right)\right] \qquad (8.3b)$$

Soderberg:

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_y} = \frac{1}{FS} \qquad (8.4a)$$

or,

$$\sigma_a = \frac{\sigma_e}{FS}\left[1 - \left(\frac{FS * \sigma_m}{\sigma_y}\right)\right] \qquad (8.4b)$$

Gerber:

$$\frac{FS * \sigma_a}{\sigma_e} + \left(\frac{FS * \sigma_m}{\sigma_u}\right)^2 = 1 \qquad (8.5a)$$

where,

σ_u is the ultimate tensile strength of the material, and

σ_y is the yield strength of the material.

FS is the factor of safety.

When FS=1, the above equations simply to

Goodman:

$$\sigma_a = \sigma_e\left[1 - \left(\frac{\sigma_m}{\sigma_u}\right)\right] \qquad (8.3c)$$

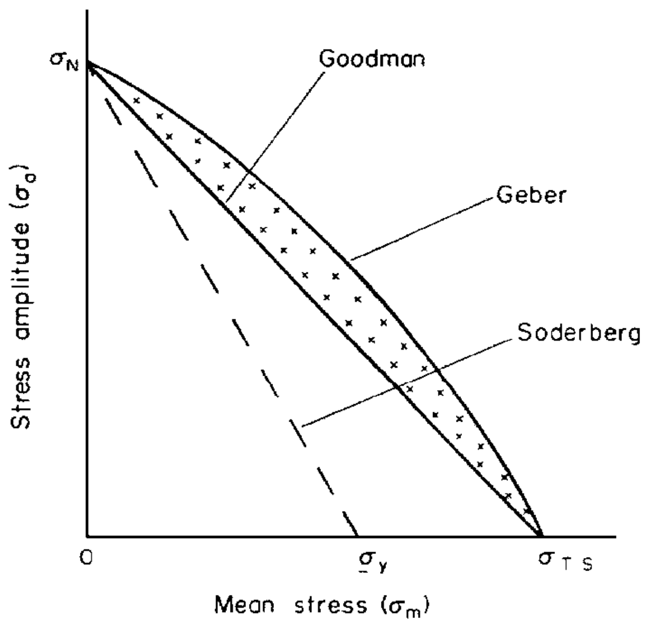
Soderberg:

$$\sigma_a = \sigma_e\left[1 - \left(\frac{\sigma_m}{\sigma_y}\right)\right] \qquad (8.4c)$$

Gerber:

$$\sigma_a = \sigma_e\left[1 - \left(\frac{\sigma_m}{\sigma_u}\right)^2\right] \qquad (8.5b)$$

The above equations can be represented graphically. In reality, it has been found that the majority of experimental test results lie within the envelope formed by the parabolic Gerber curve and the Goodman straight line. However, the Soderberg criterion is often used as it gives an additional margin of safety.



Example 1:

The ultimate tensile strength of a low carbon steel is 870MPa and the yield stress is 380MPa. The fatigue limit, σ_e is $\pm$230MPa:

i) Estimate the safe range of stress, $\Delta\sigma$ for a repeated cycle based on the Gerber and Goodman equations

ii) Estimate the safe range of stress, $\Delta\sigma$, the maximum and the minimum stress for a fluctuating tensile cycle with a mean stress of 185 MPa, using the Soderberg equation.

Assume a factor of safety of 1.

Solution:

i) For a repeated cycle, we have,

$$\sigma_m = \frac{\Delta\sigma}{2}$$

Goodman:

$$\sigma_a = \sigma_e\left[1 - \left(\frac{\sigma_m}{\sigma_u}\right)\right] = 230\left[1 - \left(\frac{\Delta\sigma}{2 \times 870}\right)\right]$$

$$\Delta\sigma = 460 - \frac{460\Delta\sigma}{1740}$$

$$\Delta\sigma + \frac{460\Delta\sigma}{1740} = \Delta\sigma\left(1 + \frac{460}{1740}\right) = \Delta\sigma\left(1 + \frac{130}{UTS}\right) = 1.26\Delta\sigma = 460$$

$$1.26\Delta\sigma = 460$$

$$\Delta\sigma = 363.8MPa$$

Gerber:

$$\sigma_a = \sigma_e\left[1-\left(\frac{\sigma_m}{\sigma_u}\right)^2\right] = 230\left[1-\left(\frac{\Delta\sigma}{2\times 870}\right)^2\right] = 230\left[1-\frac{\Delta\sigma^2}{1740^2}\right]$$

$$\Delta\sigma == 460-\frac{460\Delta\sigma^2}{1740^2} = 460-\frac{460\Delta\sigma^2}{3027600} = 460-\frac{23\Delta\sigma^2}{1513800}$$

$$151380\Delta\sigma == 69634800 - 23\Delta\sigma^2$$

$$23\Delta\sigma^2 + 151380\Delta\sigma - 69634800 = 0$$

$$\Delta\sigma = \frac{-151380 \pm \sqrt{151380^2 - 4(23)(-69634800)}}{2(23)} = 431.7MPa$$

ii) We have:

$$\sigma_m = 185MPa; \qquad \sigma_{yield} = 380MPa$$

Soderberg:

$$\sigma_a = \sigma_e\left[1-\left(\frac{\sigma_m}{\sigma_y}\right)\right] = 460\left[1-\left(\frac{185}{380}\right)\right] = 118\ MPa$$

$$\Delta\sigma = 2\sigma_a = 236.0MPa$$

Hence:

$$\sigma_{max} = \sigma_m + \sigma_a = 185 + 118 = 303MPa$$

$$\sigma_{min} = \sigma_m - \sigma_a = 185 - 118 = 67MPa$$

Example 2:

A steel shaft is subjected to a fluctuating axial load of +120kN and –20kN. The shaft diameter is 25mm. The reversed stress fatigue limit is 600MPa, the UTS is 400MPa and the yield strength is 200MPa. Determine the maximum allowable stress range and therefore the factor of safety for the shaft according to Goodman, Gerber and Soderberg.

Solution:

Given data:

$$\sigma_e = 600\ MPa$$
$$\sigma_u = 400\ MPa$$
$$\sigma_y = 200\ MPa$$

We know that stress,

$$\sigma = \frac{load}{area}$$

Using the above equation, we can calculate the maximum and minimum stresses as,

$$\sigma_{max} = \frac{120 \times 10^3}{\pi \times 12.5^2} = 244.5MPa$$

$$\sigma_{min} = \frac{-20 \times 10^3}{\pi \times 12.5^2} = -40.7MPa$$

And the mean stress value is computed as,

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = 101.9MPa$$

$$Stress\ range, \qquad \Delta\sigma = \sigma_{max} - \sigma_{min} = 285.2MPa$$

$$Alternating\ Stress, \qquad \sigma_a = \frac{\Delta\sigma}{2} = 142.6\ MPa$$

Goodman:

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_u} = \frac{1}{FS}$$

$$\frac{142.6}{600} + \frac{101.9}{400} = \frac{1}{FS}$$

$$0.492416 = \frac{1}{FS}$$

$$FS = 2.03$$

Soderberg:

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_y} = \frac{1}{FS}$$

$$\frac{142.6}{600} + \frac{101.9}{200} = \frac{1}{FS}$$

$$0.74716 = \frac{1}{FS}$$

$$FS = 1.34$$

Gerber:

$$\frac{\sigma_a}{\sigma_e} FS + \left(\frac{\sigma_m}{\sigma_u}\right)^2 FS^2 = 1$$

$$\frac{142.6}{600} FS + \left(\frac{101.9}{400}\right)^2 FS^2 = 1$$

$$0.2376 * FS + 0.064 * FS^2 = 1$$

Solving this quadratic equation, we get two roots. Choose positive one, as FS cannot be negative. We get,

$$FS = 2.51$$

8.8 Cumulative Fatigue Damage

Many engineering components are subjected to varying levels of load at various frequencies for varying periods of time. The load on a car suspension, for example, depends on the road surface, the frequency depends on the speed at which the car is driven and the number of cycles also depends on the time it takes to complete the journey. Therefore fatigue life calculations based on constant stress amplitude cannot be applied.

How can we estimate the fatigue life under such conditions?

No complete solution to this problem has been found, due to the complexity of obtaining accurate data. However, some theorems have been developed and can be used as a guide to help predict the life of a component. These should NEVER be used to obtain absolute values unless they agree with experimental data.

8.8.1 Miner's Linear Damage Rule (1945)

In 1945, Miner made popular a rule that had been first proposed some years before, to allow consideration of varying stress amplitudes.

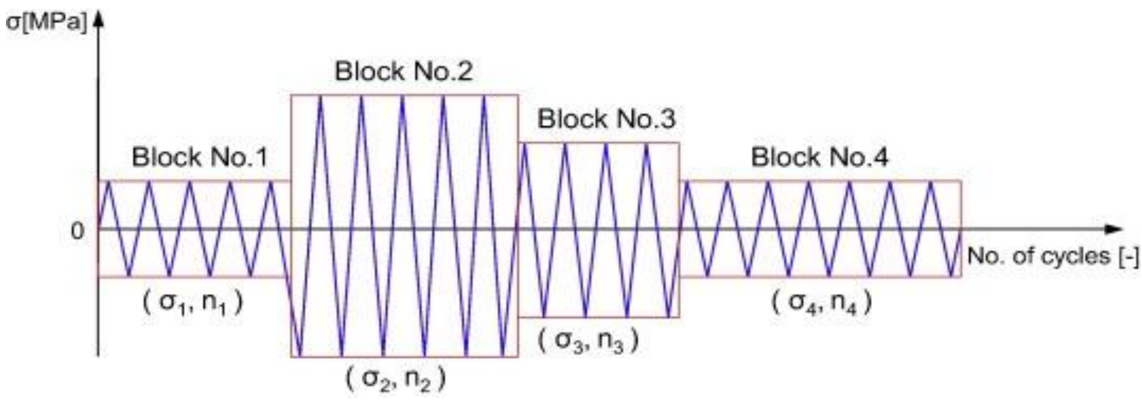
For a component subjected to a stress, σ_1 for n_1 cycles and then a stress σ_2 for n_2 cycles, etc. then Miner's rule states that:

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} \dots \dots \dots \dots \frac{n_i}{N_i} = C$$

* Can only be used in this form when the mean stress is zero

Where:

n is the number of applied cycles at a given level of stress
N is the maximum number of cycles for the applied stress
C is a constant in the range $0.7 \leq C \leq 2.2$



Usually, for design purposes, C is assumed to be 1. Unless stated otherwise, we will use $C = 1$ for the purpose of our analysis.

$$\sum \frac{n}{N} = 1 \qquad (8.6)$$

Although generally used this rule has some limitations. It does not allow for consideration of the order in which the stress reversals occur. In some circumstances, cycles of low stress followed by high stress cause more damage than would be predicted by the rule. It does not consider the effect of an overload or high stress which may result in a compressive residual stress that may retard crack growth. High stress followed by low stress may have less damage due to the presence of compressive residual stress

Example 3:

Cyclic bending stresses of 75, 60 and 40MPa are applied to a beam in a lifting machine. If the portion of time spent at these three levels is 30, 50 and 20% respectively, estimate the working life, in days, when the machine operates continuously at 10 cycles per day. Assume that the fatigue lives to be 10^3, 10^4 and 10^5 cycles at 75, 60 and 40MPa respectively.

Let the total number of cycles to failure be N_f.

$$\sum \frac{n}{N} = 1$$

$$\frac{0.3N_f}{1000} + \frac{0.5N_f}{10000} + \frac{0.2N_f}{100000} = 1$$

$$(30 + 5 + 0.2)N_f = 100000$$

$$N_f = \frac{100000}{35.2} = 2840.9 \text{ cylces} = 284 \text{ days}$$

Example 4:

A steel bracket used to support a rotating machine is subjected to variable stress amplitude high cycle fatigue loading. An analysis of the stress frequency response over a period of 2000 cycles shows that the variable stress amplitudes can be divided into six blocks over that period, as shown in the following table.

If it is assumed that the frequency response for each period of 2000 cycles follows the same form and using the constant stress amplitude versus number of cycles to failure data also given in Table, estimate the number of cycles to failure for the bracket, according to Miner's linear damage rule.

Stress	130MPa	120MPa	70MPa	100MPa	200MPa	150MPa
Duration	700 cycles	400 cycles	350 cycles	200 cycles	250 cycles	100 cycles

No of cycles to failure (based on constant stress amplitude)	10^8	10^9	10^9	10^7	10^8	10^6

Let the total number of cycles to failure be N_f:

$$\sum \frac{n}{N} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \frac{n_4}{N_4} + \frac{n_5}{N_5} + \frac{n_6}{N_6} = 1$$

$$\frac{\left({}^{700}/_{2000}\right)N_f}{10^8} + \frac{\left({}^{400}/_{2000}\right)N_f}{10^9} + \frac{\left({}^{350}/_{2000}\right)N_f}{10^9} + \frac{\left({}^{200}/_{2000}\right)N_f}{10^7} + \frac{\left({}^{250}/_{2000}\right)N_f}{10^8} + \frac{\left({}^{100}/_{2000}\right)N_f}{10^6} = 1$$

$$\frac{0.35N_f}{10^8} + \frac{0.2N_f}{10^9} + \frac{0.175N_f}{10^9} + \frac{0.1N_f}{10^7} + \frac{0.125N_f}{10^8} + \frac{0.05N_f}{10^6} = 1$$

$$(3.5 + 0.2 + 0.175 + 10 + 1.25 + 50)N_f = 10^9$$

$$N_f = 15355086 \text{ cycles}$$