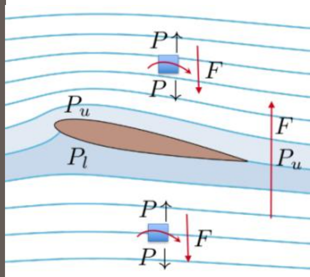
Swansea University
Prifysgol Abertawe

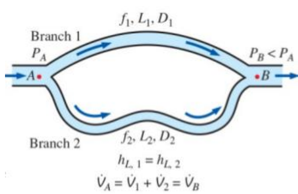
$$\mathrm{Re} = \frac{\rho V L}{\mu}$$

$$\mathrm{St} = \frac{f L}{V}$$

EGF320 - Fluid Flow

Week 4 – Fluid Kinematics





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Fluid Kinematics

Chapter 4

in

Fluid Mechanics: Fundamentals and Applications (Third edition)

Yunus A. Cengel and John M. Cimbala.

Chapter 4 – sections 4.1 to 4.9

Chapter 7 – sections 7.1 to 7.5

in

Fluid Mechanics (Fifth edition)

J. F. Douglas, J. M. Gasiorek, J. A. Swaffield, L. B. Jack

Fluid Flow - Basics

Uniform flow: velocity at a given instant is the same in magnitude and direction at every point in the domain.

Non-uniform flow: velocity changes from point to point.

Steady flow: velocity at a point does not change with time.

Unsteady flow: velocity at a point changes with time.

Steady uniform flow

Steady non-uniform flow

Unsteady uniform flow

Unsteady non-uniform flow

Study of Fluids in motion

Fluid Dynamics:

study of forces and their effect on the motion of the fluid.

Fluid Kinematics:

study of motion of the fluid without reference to the forces.

Pressure field:	$P = P(x, y, z, t)$
Velocity field:	$\vec{V} = \vec{V}(x, y, z, t)$
Acceleration field:	$\vec{a} = \vec{a}(x, y, z, t)$

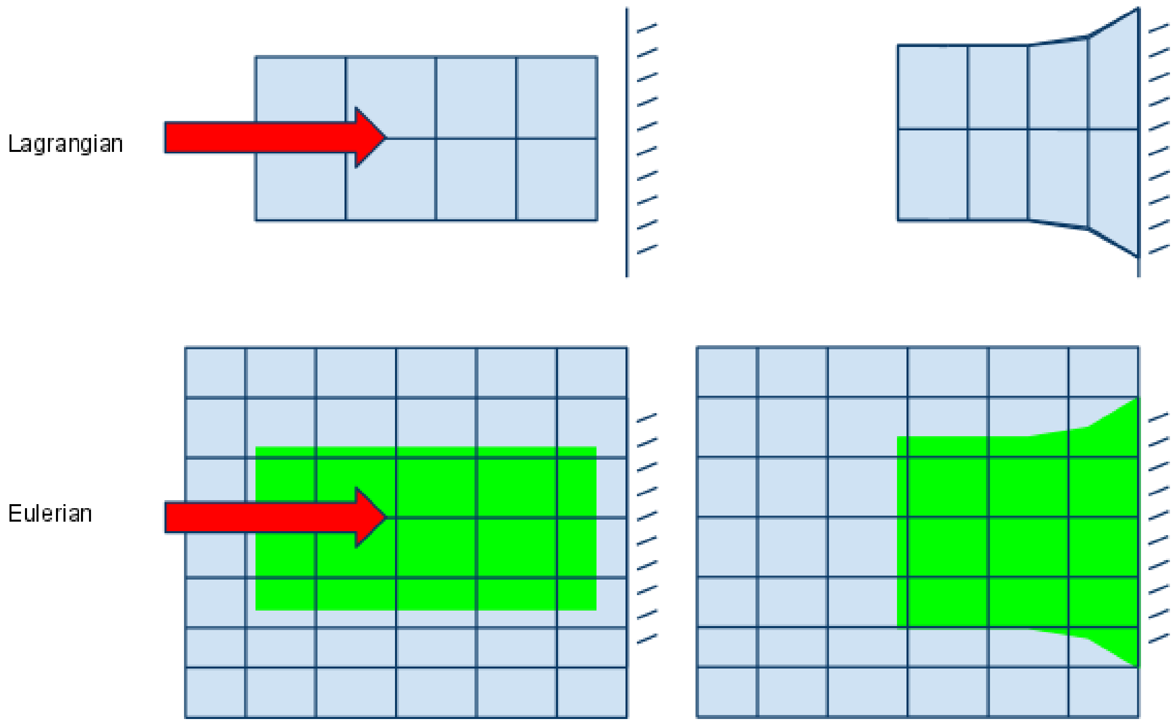
Collectively, these (and other) field variables define the **flow field**. The velocity field can be expanded in Cartesian coordinates as

$$\vec{V} = (u, v, w) = u(x, y, z, t)\vec{i} + v(x, y, z, t)\vec{j} + w(x, y, z, t)\vec{k}$$

Description of motion and Reference frames

Lagrangian description (or Lagrangian frame of reference):
to follow an individual fluid particle as it moves through space and time

Eulerian description (or Eulerian frame of reference):
To look at fluid motion by focussing on a specific location in space



Eulerian description

- A more common method for the study of fluid motion.
- In the Eulerian description of fluid flow, a finite volume called a flow domain or control volume is defined, through which fluid flows in and out.
- Instead of tracking individual fluid particles, we define field variables, functions of space and time, within the control volume.
- The field variable at a particular location at a particular time is the value of the variable for whichever fluid particle happens to occupy that location at that time.
- Experimental measurements are generally more suited to the Eulerian description.

Motion of a fluid particle

- Every fluid particle will obey the laws of mechanics in the same way as a particle in a solid body.
- Newton's second law:

Force = Mass X Acceleration

In any body of flowing fluid, the velocity at a given instant will generally vary from point to point over any specified region, and if the flow is unsteady the velocity at each point may vary with time. In this field of flow, at any given time, a particle at point A will have a different velocity from that of a particle at point B. The velocities at A and B may also change with time. Thus the change of velocity δv, which occurs when a particle moves from A to B through a distance δs in time δt, is given by

Total change of velocity	=	Difference of velocity between A and B at the given instant	+	Change of velocity at B occurring in time δt.

$$dv = \frac{\partial v}{\partial s}\delta s + \frac{\partial v}{\partial t}\delta t$$

Acceleration of a fluid particle

$$\text{Acceleration, a} = \frac{Dv}{Dt} = v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

The derivative D/D*t is known as the substantive derivative.*

The total acceleration, known as the *substantive acceleration, is composed of two parts*:

1.the *convective acceleration v(∂v/∂s)*

due to the movement of the particle from one point to another point at which the velocity at the given instant is different

2.the *local or temporal acceleration (∂v/∂t)*

due to the change of velocity at every point with time

Acceleration of a fluid particle

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_y = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Material derivative:

$$\frac{D}{Dt} = \frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{V} \cdot \vec{\nabla})$$

Material acceleration:

$$\vec{a}(x, y, z, t) = \frac{D\vec{V}}{Dt} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V}$$

Example 1:

EXAMPLE 4–1 **A Steady Two-Dimensional Velocity Field**

A steady, incompressible, two-dimensional velocity field is given by

$$\vec{V} = (u, v) = (0.5 + 0.8x)\,\vec{i} + (1.5 - 0.8y)\,\vec{j} \qquad (1)$$

where the *x*- and *y*-coordinates are in meters and the magnitude of velocity is in m/s. A **stagnation point** is defined as *a point in the flow field where the velocity is zero.* (*a*) Determine if there are any stagnation points in this flow field and, if so, where? (*b*) Sketch velocity vectors at several locations in the domain between $x = -2$ m to 2 m and $y = 0$ m to 5 m; qualitatively describe the flow field.

Analysis (*a*) Since $\vec{V}$ is a vector, *all* its components must equal zero in order for $\vec{V}$ itself to be zero. Using Eq. 4–4 and setting Eq. 1 equal to zero,

Stagnation point:

$$u = 0.5 + 0.8x = 0 \quad \rightarrow \quad x = -0.625 \text{ m}$$
$$v = 1.5 - 0.8y = 0 \quad \rightarrow \quad y = 1.875 \text{ m}$$

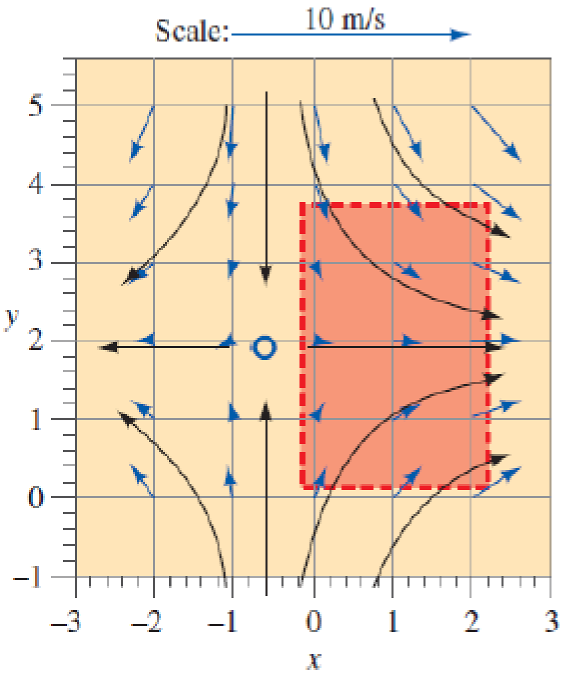
Yes. There is one stagnation point located at **$x = -0.625$ m, $y = 1.875$ m.**

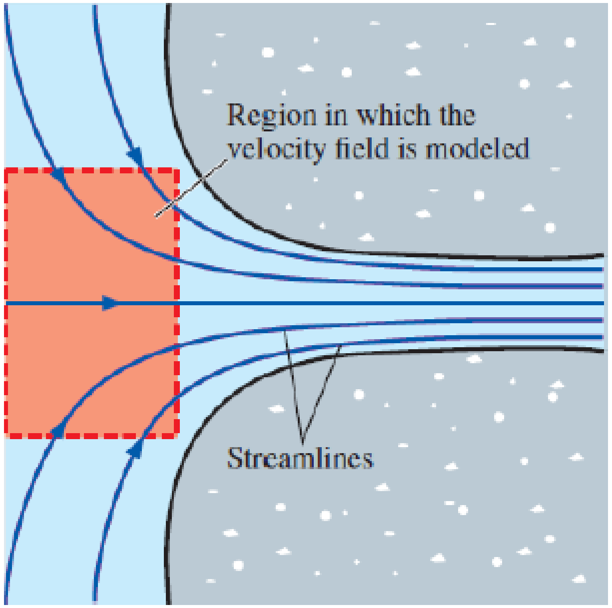
(*b*) The *x*- and *y*-components of velocity are calculated from Eq. 1 for several (*x*, *y*) locations in the specified range. For example, at the point ($x = 2$ m, $y = 3$ m), $u = 2.10$ m/s and $v = -0.900$ m/s. The magnitude of velocity (the *speed*) at that point is 2.28 m/s. At this and at an array of other locations, the velocity vector is constructed from its two components, the results of which are shown in Fig. 4–4. The flow can be described as stagnation point flow in which flow enters from the top and bottom and spreads out to the right and left about a horizontal line of symmetry at $y = 1.875$ m. The stagnation point of part (*a*) is indicated by the blue circle in Fig. 4–4.

If we look only at the shaded portion of Fig. 4–4, this flow field models a converging, accelerating flow from the left to the right. Such a flow might be encountered, for example, near the submerged bell mouth inlet of a hydroelectric dam (Fig. 4–5). The useful portion of the given velocity field may be thought of as a first-order approximation of the shaded portion of the physical flow field of Fig. 4–5.

Example 1 (contd):

$$\vec{V} = (u, v) = (0.5 + 0.8x)\,\vec{i} + (1.5 - 0.8y)\,\vec{j}$$





Flow field near the bell mouth inlet of a hydroelectric dam; a portion of the velocity field may be used as a first-order approximation of this physical flow field.

Velocity vectors for the velocity field. The scale is shown by the top arrow, and the solid black curves represent the approximate shapes of some streamlines, based on the calculated velocity vectors. The stagnation point is indicated by the blue circle. The shaded region represents a portion of the flow field that can approximate flow into an inlet.

Example 1 (contd):

Consider the steady, incompressible, two-dimensional velocity field of Example 4–1. (*a*) Calculate the material acceleration at the point ($x = 2$ m, $y = 3$ m). (*b*) Sketch the material acceleration vectors at the same array of x- and y-values as in Example 4–1.

Analysis (*a*) Using the velocity field of Eq. 1 of Example 4–1 and the equation for material acceleration components in Cartesian coordinates (Eq. 4–11), we write expressions for the two nonzero components of the acceleration vector:

$$a_x = \frac{\partial u}{\partial t} + \overbrace{u\frac{\partial u}{\partial x}} + \overbrace{v\frac{\partial u}{\partial y}} + w\frac{\partial u}{\partial z}$$

$$= 0 + (0.5 + 0.8x)(0.8) + (1.5 - 0.8y)(0) + 0 = (0.4 + 0.64x)\ \text{m/s}^2$$

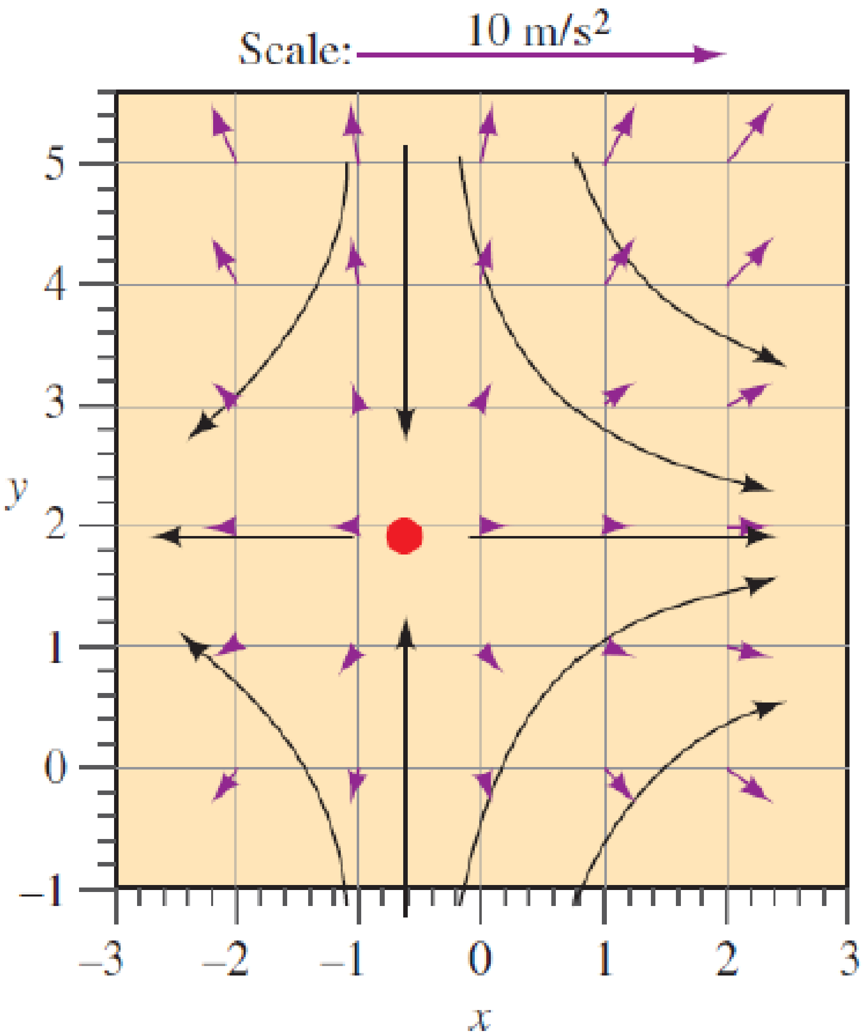
and

$$a_y = \frac{\partial v}{\partial t} + \overbrace{u\frac{\partial v}{\partial x}} + \overbrace{v\frac{\partial v}{\partial y}} + w\frac{\partial v}{\partial z}$$

$$= 0 + (0.5 + 0.8x)(0) + (1.5 - 0.8y)(-0.8) + 0 = (-1.2 + 0.64y)\ \text{m/s}^2$$

At the point ($x = 2$ m, $y = 3$ m), $a_x = 1.68\ \text{m/s}^2$ and $a_y = 0.720\ \text{m/s}^2$.

Example 1 (contd):



Acceleration vectors for the velocity field of Examples 4–1 and 4–3. The scale is shown by the top arrow, and the solid black curves represent the approximate shapes of some streamlines, based on the calculated velocity vectors. The stagnation point is indicated by the red circle.

FUNDAMENTALS OF FLOW VISUALIZATION

- Streamlines and stream tubes
- Pathlines
- Streaklines
- Timelines

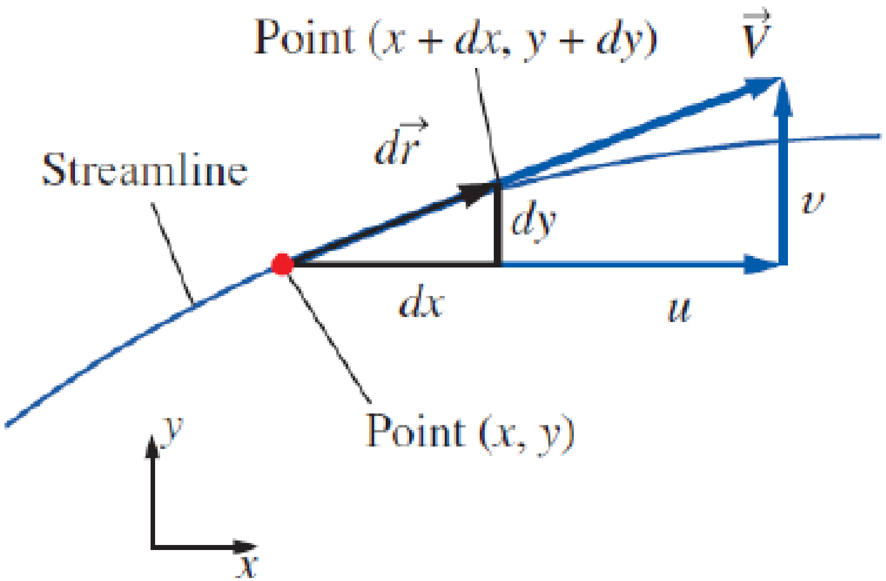
Streamlines

Streamline: A curve that is everywhere tangent to the instantaneous local velocity vector.

Streamlines are useful as indicators of the instantaneous direction of fluid motion throughout the flow field.

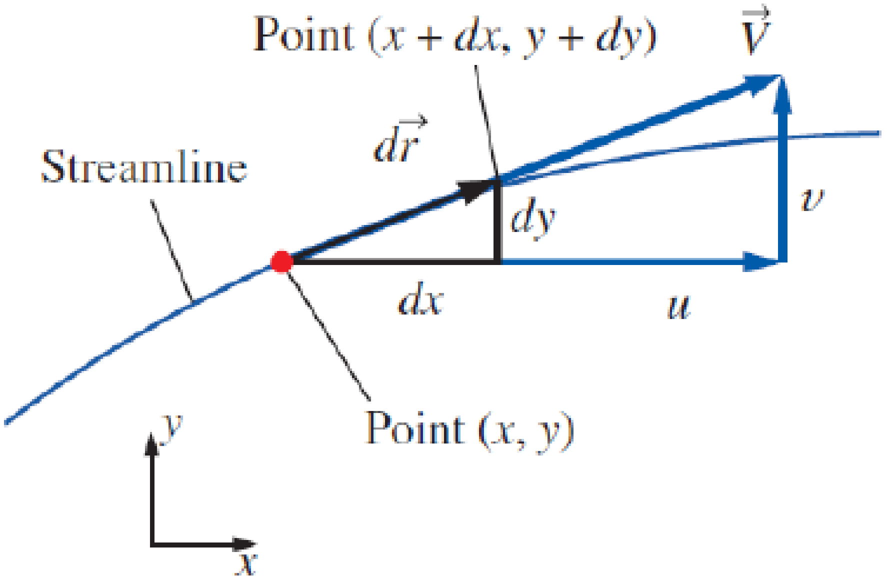
For example, regions of recirculating flow and separation of a fluid off of a solid wall are easily identified by the streamline pattern.

Streamlines cannot be directly observed experimentally except in steady flow fields.



For two-dimensional flow in the xy-plane, arc length $d\vec{r} = (dx, dy)$ along a *streamline* is everywhere tangent to the local instantaneous velocity vector $\vec{V} = (u, v)$.

Streamlines



$$Equation\ for\ a\ streamline, \frac{dr}{V} = \frac{dx}{u} + \frac{dy}{v} + \frac{dz}{w}$$

$$Streamline\ in\ the\ xy\ plane, \left(\frac{dy}{dx}\right) = \frac{v}{u}$$

Example 1 (contd):

For the steady, incompressible, two-dimensional velocity field of Example 4–1, plot several streamlines in the right half of the flow ($x > 0$) and compare to the velocity vectors plotted in Fig. 4–4.

SOLUTION An analytical expression for streamlines is to be generated and plotted in the upper-right quadrant.

Assumptions **1** The flow is steady and incompressible. **2** The flow is two-dimensional, implying no z-component of velocity and no variation of u or v with z.

Analysis Equation 4–16 is applicable here; thus, along a streamline,

$$\frac{dy}{dx} = \frac{v}{u} = \frac{1.5 - 0.8y}{0.5 + 0.8x}$$

We solve this differential equation by separation of variables:

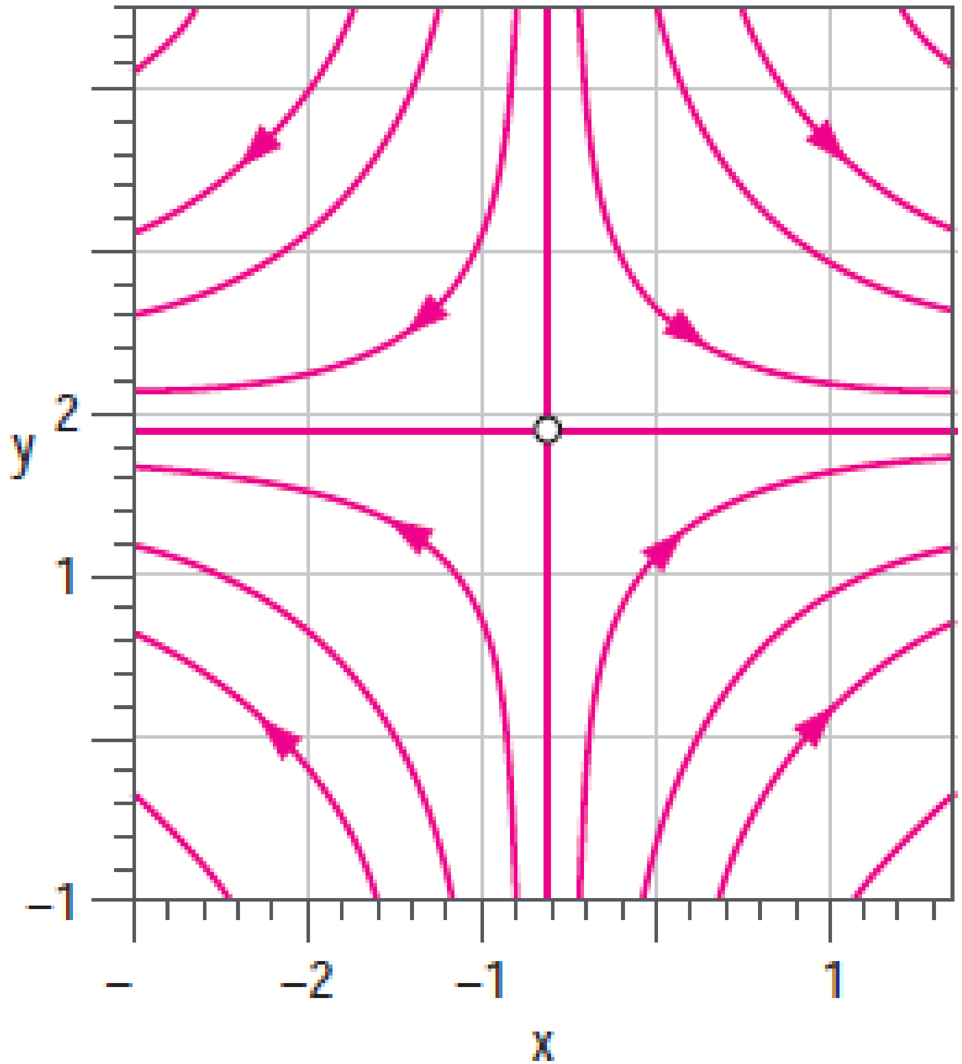
$$\frac{dy}{1.5 - 0.8y} = \frac{dx}{0.5 + 0.8x} \quad \rightarrow \quad \int \frac{dy}{1.5 - 0.8y} = \int \frac{dx}{0.5 + 0.8x}$$

After some algebra, we solve for y as a function of x along a streamline,

$$y = \frac{C}{0.8(0.5 + 0.8x)} + 1.875$$

where C is a constant of integration that can be set to various values in order to plot the streamlines. Several streamlines of the given flow field are shown

Example 1 (contd):

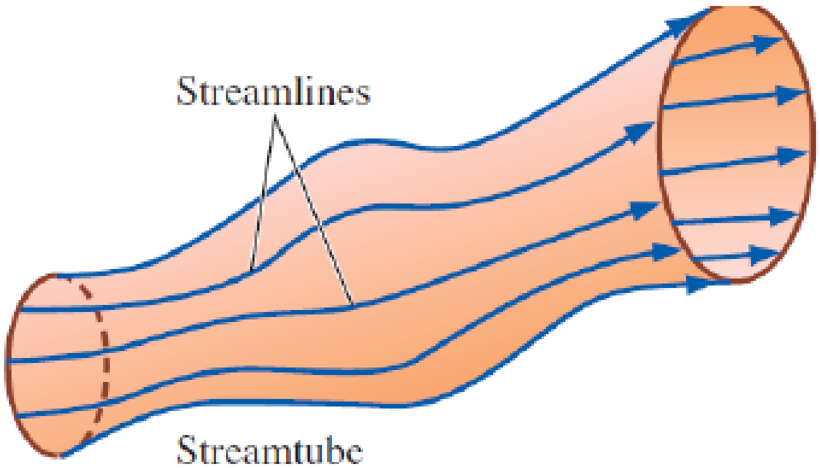


Streamtubes

A **streamtube** consists of a bundle of streamlines much like a communications cable consists of a bundle of fiber-optic cables.

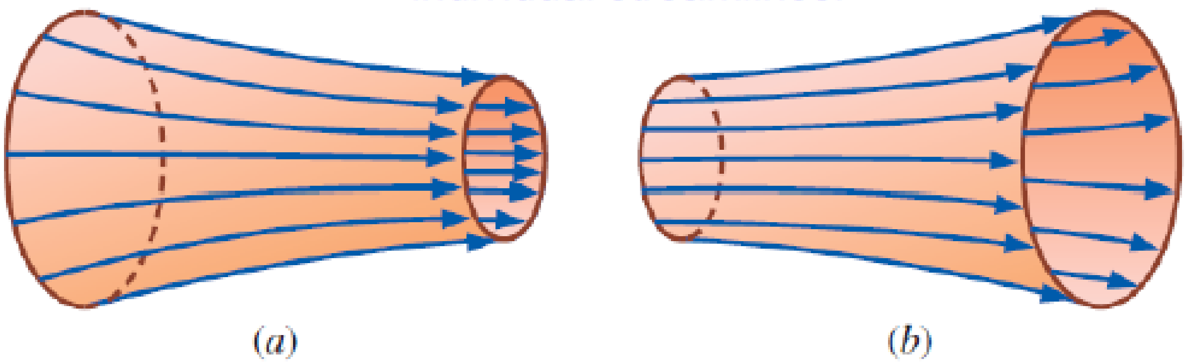
Since streamlines are everywhere parallel to the local velocity, fluid cannot cross a streamline by definition.

Fluid within a streamtube must remain there and cannot cross the boundary of the streamtube.



A *streamtube* consists of a bundle of individual streamlines.

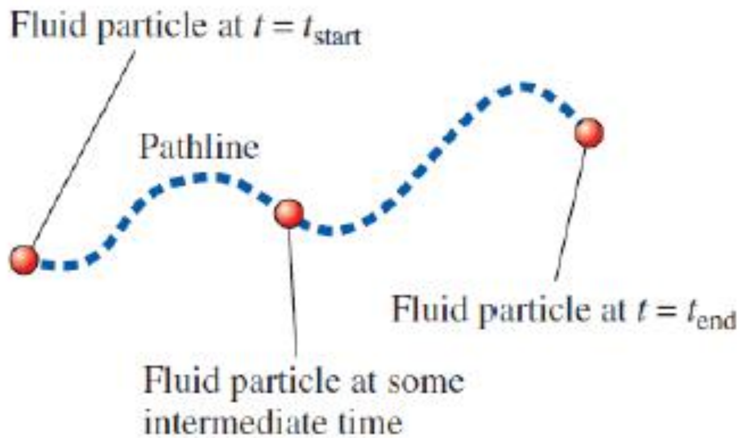
Both streamlines and streamtubes are instantaneous quantities, defined at a particular instant in time according to the velocity field at that instant.



In an incompressible flow field, a streamtube (*a*) decreases in diameter as the flow accelerates or converges and (*b*) increases in diameter as the flow decelerates or diverges.

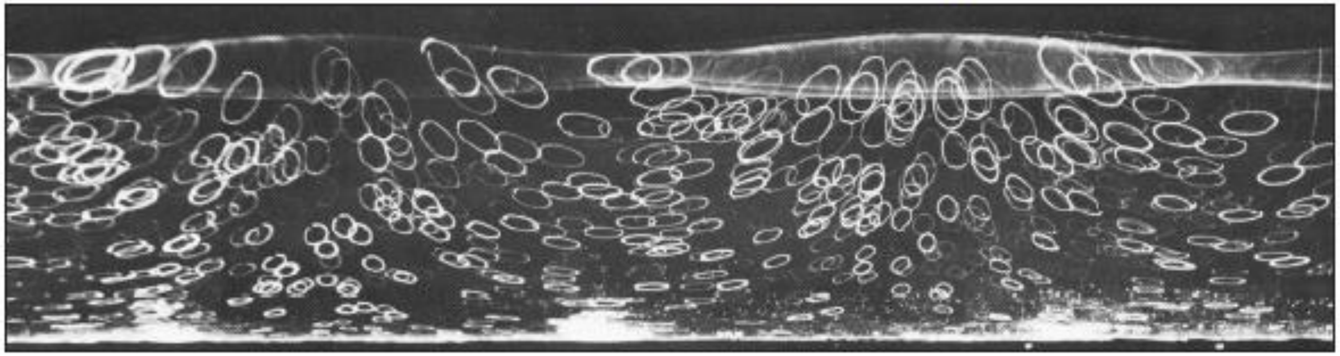
Pathlines

- **Pathline:** The actual path traveled by an individual fluid particle over some time period.
- A pathline is a Lagrangian concept in that we simply follow the path of an individual fluid particle as it moves around in the flow field.
- Thus, a pathline is the same as the fluid particle's material position vector ($x_{particle}(t)$, $y_{particle}(t)$, $z_{particle}(t)$) traced out over some finite time interval.



A *pathline* is formed by following the actual path of a fluid particle.

Pathlines produced by white tracer particles suspended in water and captured by time-exposure photography; as waves pass horizontally, each particle moves in an elliptical path during one wave period.

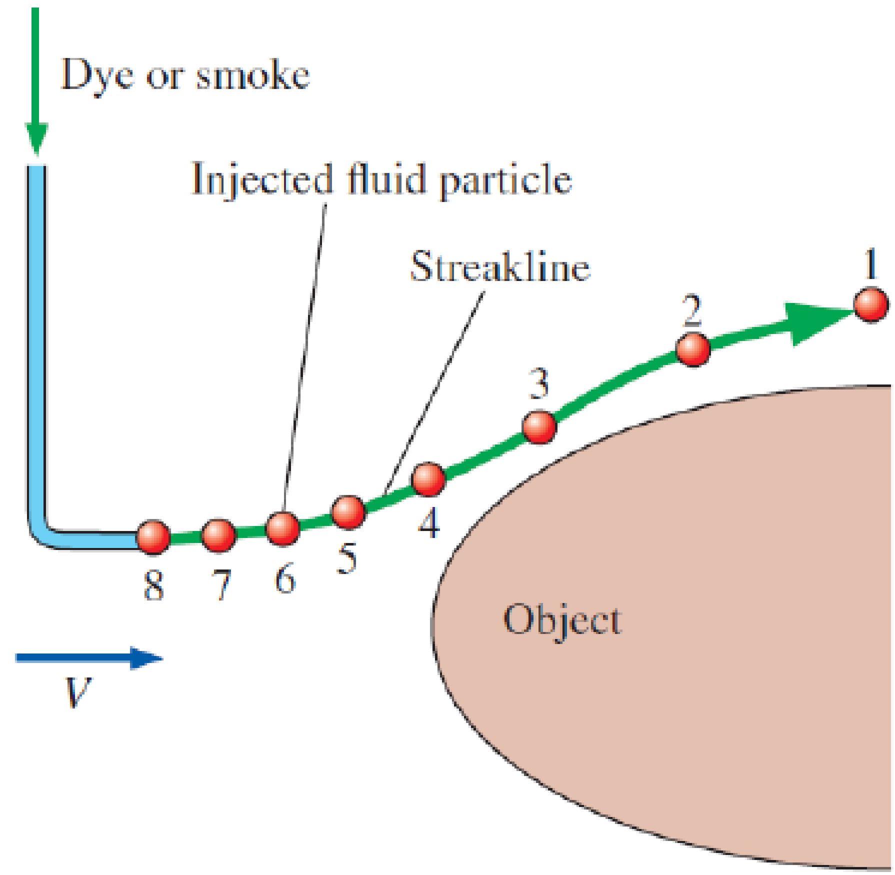


Streaklines

Streakline: The locus of fluid particles that have passed sequentially through a prescribed point in the flow.

Streaklines are the most common flow pattern generated in a physical experiment.

If you insert a small tube into a flow and introduce a continuous stream of tracer fluid (dye in a water flow or smoke in an air flow), the observed pattern is a streakline.



A *streakline* is formed by continuous introduction of dye or smoke from a point in the flow. Labeled tracer particles (1 through 8) were introduced sequentially.

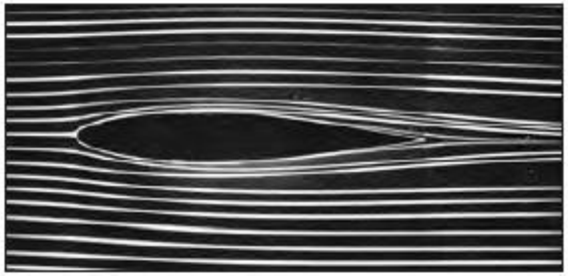


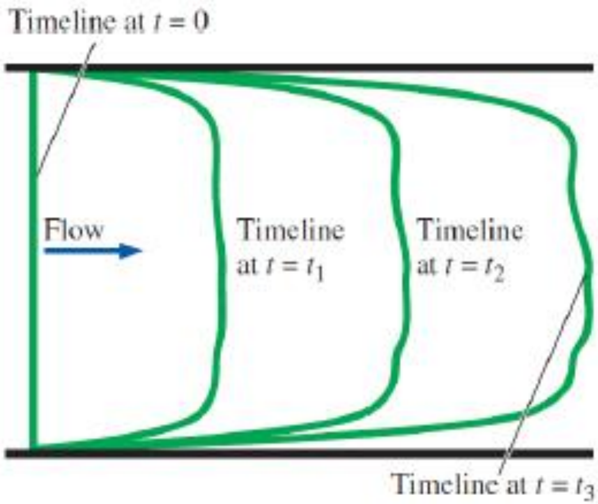
FIGURE 4–24
Streaklines produced by colored fluid introduced upstream since the flow is steady, these streaklines are the same as streamlines and pathlines.
Courtesy NERA. Photograph by Werl .

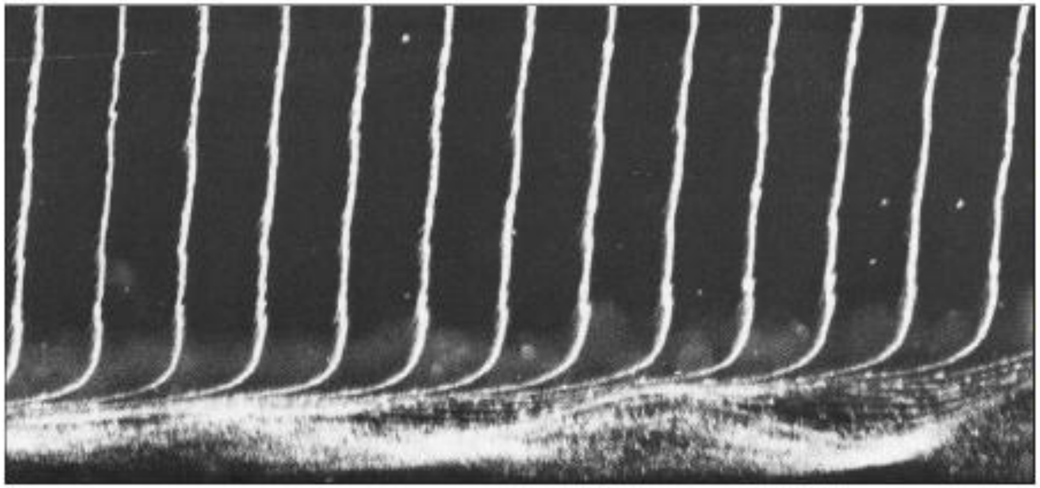
Streaklines are often confused with streamlines or pathlines. While the three flow patterns are identical in steady flow, they can be quite different in unsteady flow. The main difference is that a streamline represents an instantaneous flow pattern at a given instant in time, while a streakline and a pathline are flow patterns that have some age and thus a time history associated with them. A streakline is an instantaneous snapshot of a time-integrated flow pattern. A pathline, on the other hand, is the time-exposed flow path of an individual particle over some time period.

Timelines

Timeline: A set of adjacent fluid particles that were marked at the same (earlier) instant in time.

Timelines are particularly useful in situations where the uniformity of a flow (or lack thereof) is to be examined.



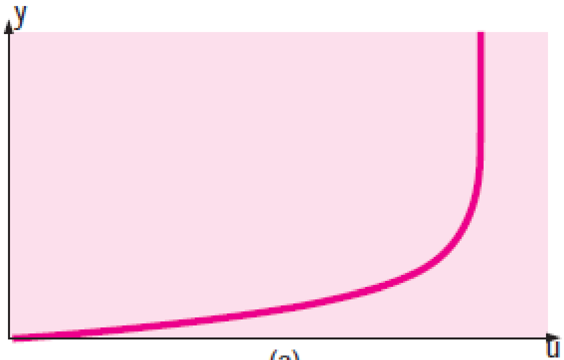


Timelines are formed by marking a line of fluid particles, and then watching that line move (and deform) through the flow field; timelines are shown at $t = 0$, t_1, t_2, and t_3.

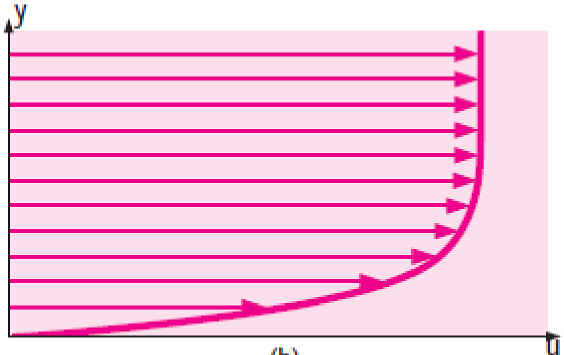
Timelines produced by a hydrogen bubble wire are used to visualize the boundary layer velocity profile shape. Flow is from left to right, and the hydrogen bubble wire is located to the left of the field of view. Bubbles near the wall reveal a flow instability that leads to turbulence.

Plots of fluid flow data

Profile plots

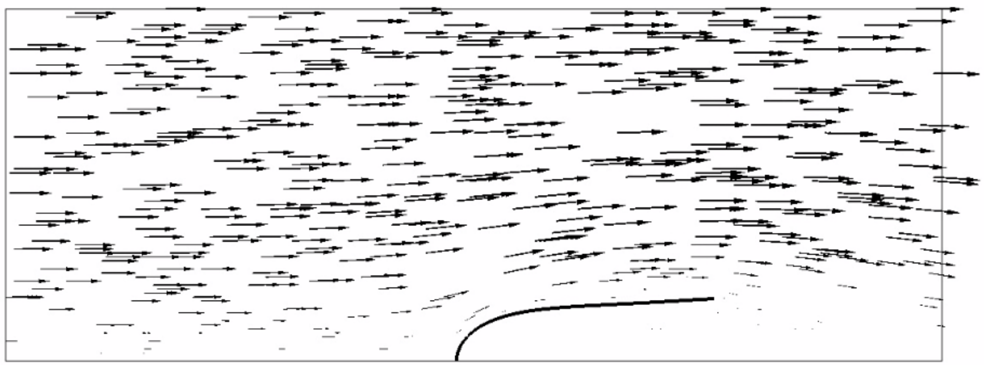


(a)

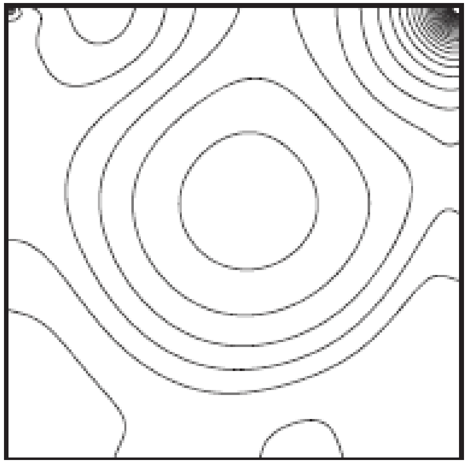


(b)

Vector plots



Contour plots



Strain rate and vorticity

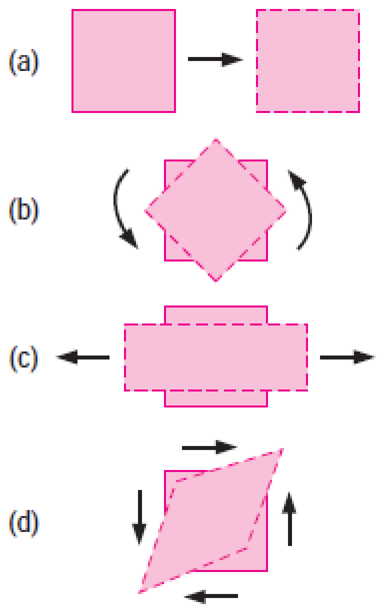


FIGURE 4–34
Fundamental types of fluid element motion or deformation: (a) translation, (b) rotation, (c) linear strain, and (d) shear strain.

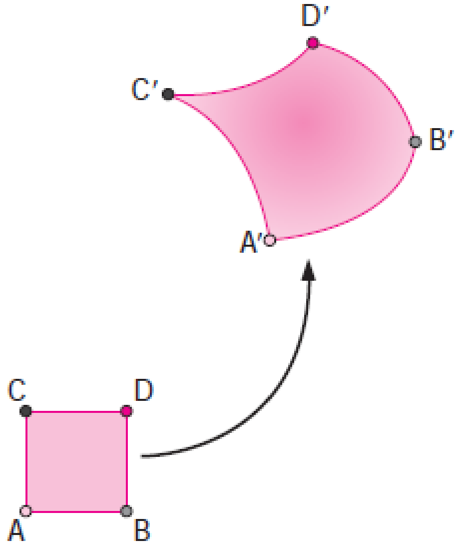


FIGURE 4–39
A fluid element illustrating translation, rotation, linear strain, shear strain, and volumetric strain.

Strain rate tensor in Cartesian coordinates:

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial u}{\partial x} & \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}\right) & \dfrac{\partial v}{\partial y} & \dfrac{1}{2}\left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x} + \dfrac{\partial u}{\partial z}\right) & \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y} + \dfrac{\partial v}{\partial z}\right) & \dfrac{\partial w}{\partial z} \end{pmatrix}$$

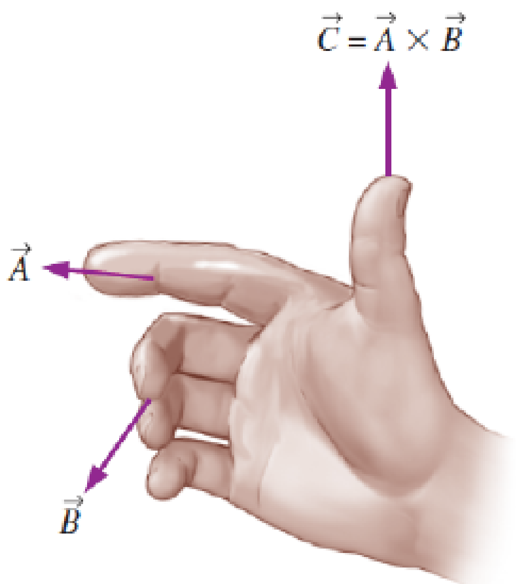
Strain rate and vorticity

Another kinematic property of great importance to the analysis of fluid flows is the **vorticity vector**, defined mathematically as the curl of the velocity vector

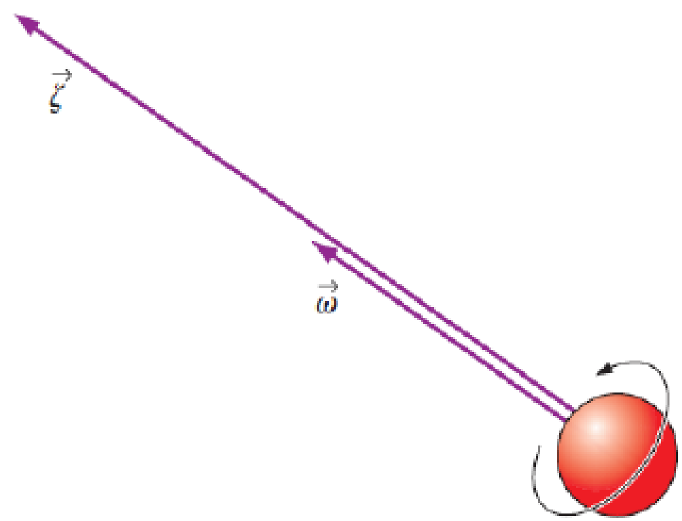
Vorticity vector: $$\vec{\zeta} = \vec{\nabla} \times \vec{V} = \text{curl}(\vec{V})$$

Rate of rotation vector: $$\vec{\omega} = \frac{1}{2}\vec{\nabla} \times \vec{V} = \frac{1}{2}\text{curl}(\vec{V}) = \frac{\vec{\zeta}}{2}$$

Vorticity is equal to twice the angular velocity of a fluid particle



The direction of a vector cross product is determined by the right-hand rule.



The *vorticity vector* is equal to twice the angular velocity vector of a rotating fluid particle.

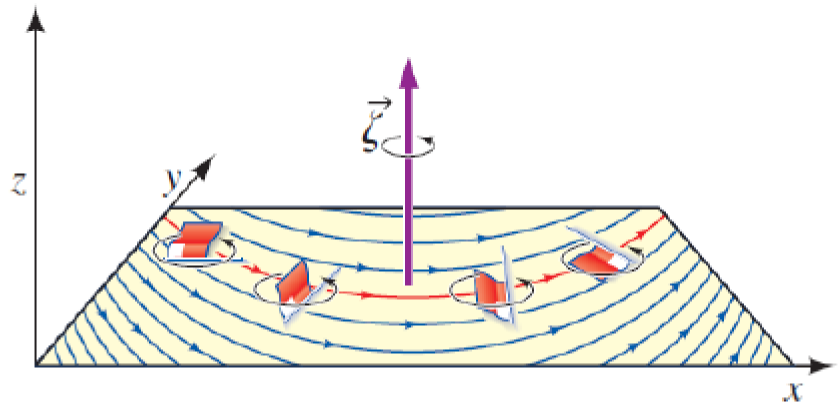
Vorticity

Vorticity vector in Cartesian coordinates:

$$\vec{\zeta} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

Two-dimensional flow in Cartesian coordinates:

$$\vec{\zeta} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

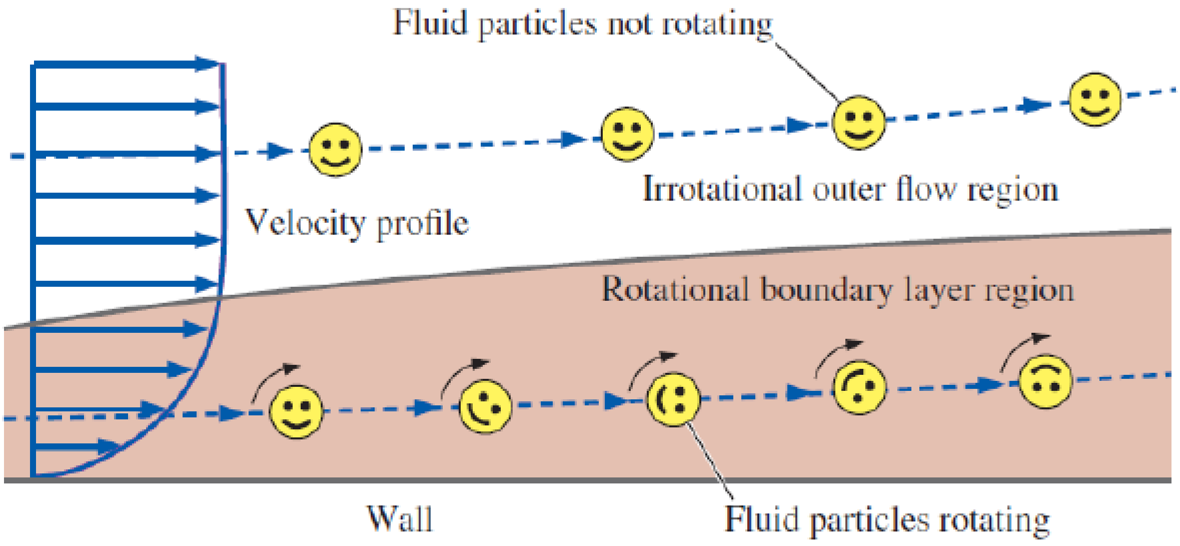


For a two-dimensional flow in the *xy*-plane, the vorticity vector always points in the *z*- or *z*-direction. In this illustration, the flag-shaped fluid particle rotates in the counterclockwise direction as it moves in the *xy*-plane; its vorticity points in the positive *z*-direction as shown.

Vorticity

- If the vorticity at a point in a flow field is nonzero, the fluid particle that happens to occupy that point in space is rotating; the flow in that region is called **rotational**.
- Likewise, if the vorticity in a region of the flow is zero (or negligibly small), fluid particles there are not rotating; the flow in that region is called **irrotational**.
- Physically, fluid particles in a rotational region of flow rotate end over end as they move along in the flow.

The difference between rotational and irrotational flow: fluid elements in a rotational region of the flow rotate, but those in an irrotational region of the flow do not.



Example

Consider the following steady, incompressible, two-dimensional velocity field:

$$\vec{V} = (u, v) = x^2\vec{i} + (-2xy - 1)\vec{j} \qquad (1)$$

Is this flow rotational or irrotational? Sketch some streamlines in the first quadrant and discuss.

SOLUTION We are to determine whether a flow with a given velocity field is rotational or irrotational, and we are to draw some streamlines in the first quadrant.

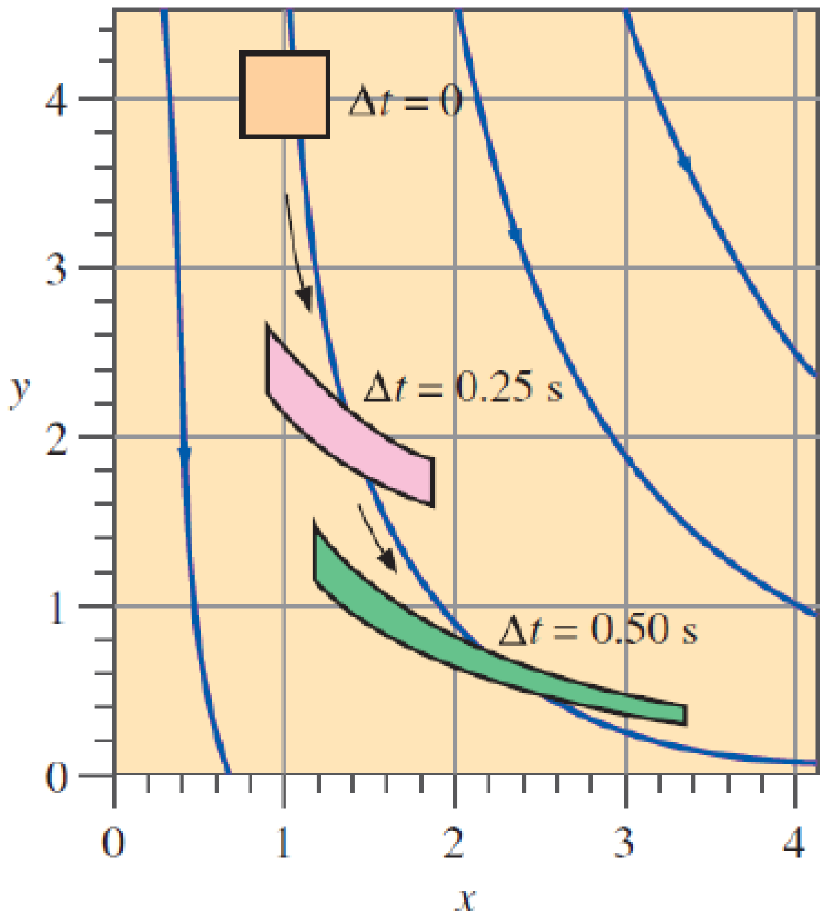
Analysis Since the flow is two-dimensional, Eq. 4–31 is valid. Thus,

Vorticity:
$$\vec{\zeta} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k} = (-2y - \quad)\vec{k} = -2y\vec{k} \qquad (2)$$

Since the vorticity is nonzero, this flow is **rotational.** In Fig. 4–47 we plot several streamlines of the flow in the first quadrant; we see that fluid moves downward and to the right. The translation and deformation of a fluid parcel is also shown: at $\Delta t = 0$, the fluid parcel is square, at $\Delta t = 0.25$ s, it has moved and deformed, and at $\Delta t = 0.50$ s, the parcel has moved farther and is further deformed. In particular, the right-most portion of the fluid parcel moves faster to the right and faster downward compared to the left-most portion, stretching the parcel in the x-direction and squashing it in the vertical direction. It is clear that there is also a net *clockwise* rotation of the fluid parcel, which agrees with the result of Eq. 2.

Discussion From Eq. 4–29, individual fluid particles rotate at an angular velocity equal to $\vec{\omega} = -y\vec{k}$, half of the vorticity vector. Since $\vec{\omega}$ is not constant, this flow is *not* solid-body rotation. Rather, $\vec{\omega}$ is a linear function of y. Further analysis reveals that this flow field is incompressible; the shaded areas representing the fluid parcel in Fig. 4–47 remain constant at all three instants in time.

Example (contd.)



Deformation of an initially square fluid parcel subjected to the velocity field of Example 4–8 for a time period of 0.25 s and 0.50 s. Several streamlines are also plotted in the first quadrant. It is clear that this flow is *rotational*.