Swansea University
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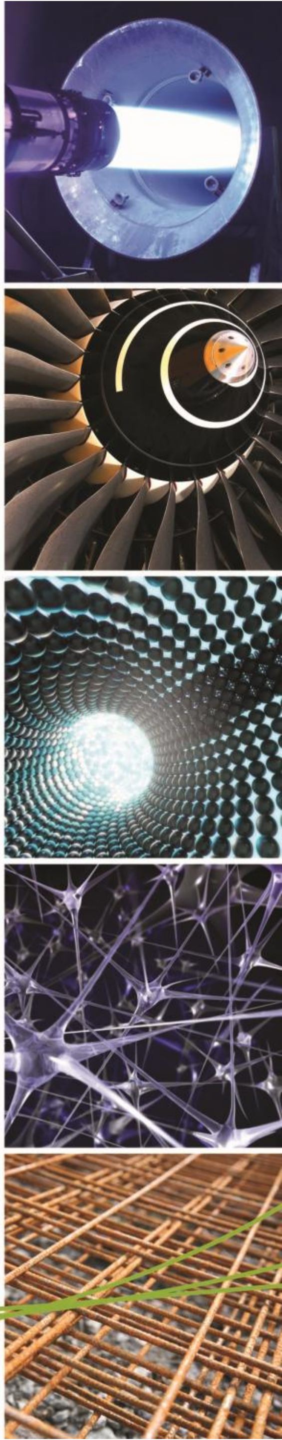
# Advanced Structural Analysis

# EGF316

## Linear Elastic Fracture Mechanics



# Lecture Content



- Introduction to LEFM
- Crack Detection
- Damage Tolerant Design
- Modes of Failure
- Fracture Mechanics
- LEFM Approaches
- LEFM and Fatigue

# Introduction

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Crack initiation is very difficult to detect. So, the focus is therefore on *preventing*, *predicting* and *controlling* their propagation.

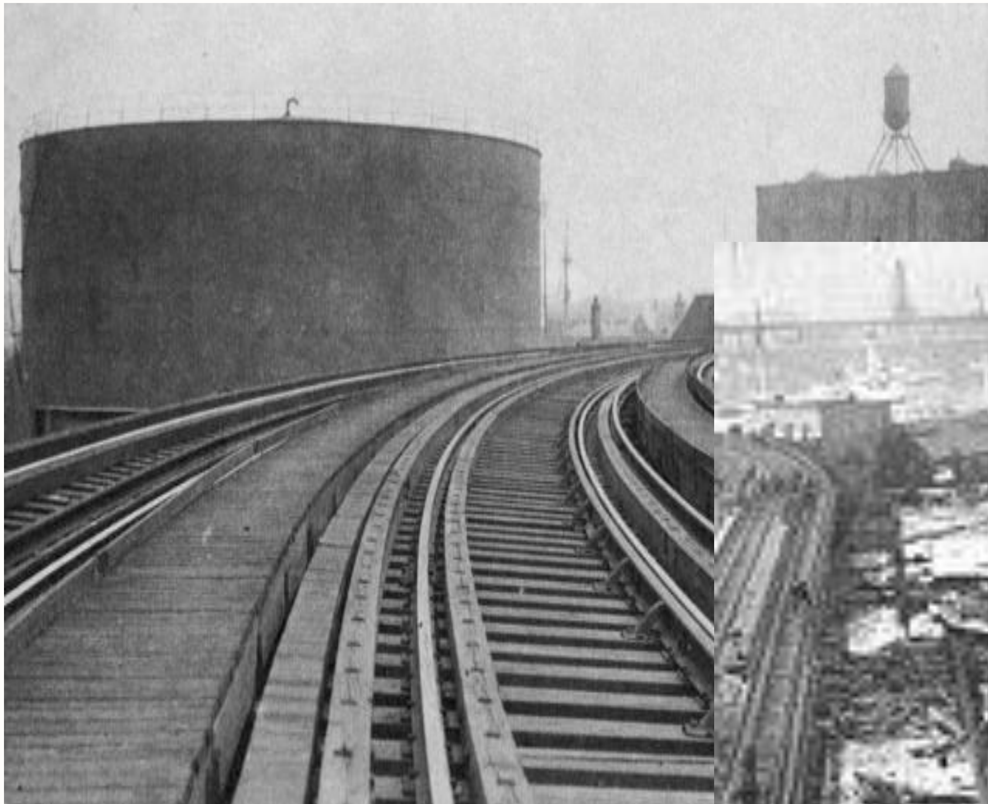
The theory of fracture mechanics assumes the pre-existence of cracks and develops criteria for the catastrophic growth of these cracks.

The designer must then ensure that no such criteria can be met in the structure.

# What are the key questions that engineers need to consider in fracture mechanics?

# Boston Molasses Tank





# Liberty Ships



# Crack Detection



Structures that are susceptible to cracks should be examined on a regular basis to determine if cracks are present.

Non-destructive testing (NDT) techniques

1) Visual inspection
2) Penetrative dyes
3) Crack opening gauges
4) Eddy current testing
5) Ultrasonic testing
6) X-Ray
7) Acoustic emissions testing
8) Magnetic particle testing

# Damage Tolerant Design



Safe Life Design:

Products are designed to survive a specific design life

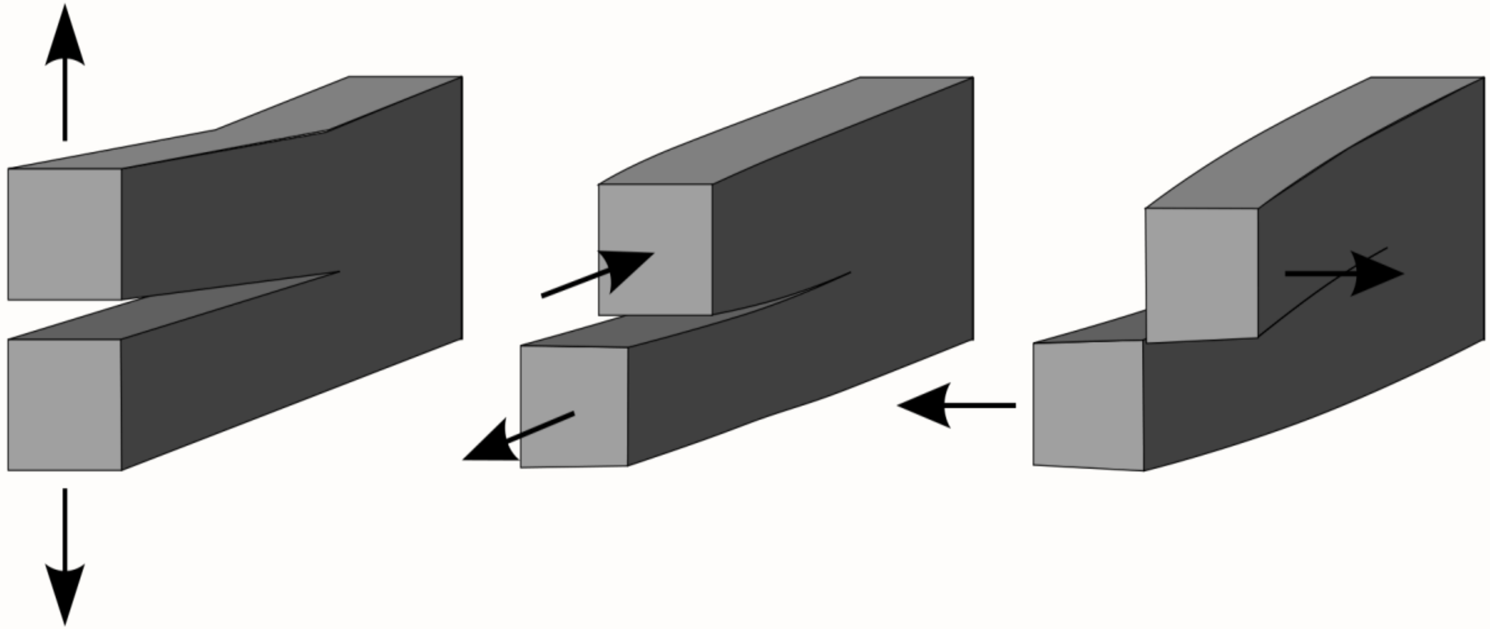
Fail Safe Design:

In the event failure, the structure inherently responds in a way that will cause no or minimal harm.

- Multiple load paths

# Modes of Failure





**Mode I** **Mode II** **Mode III**

**Mode I** is the predominant mode of failure due to crack.

# Fracture Mechanics - theories



Linear Elastic Fracture Mechanics (LEFM)

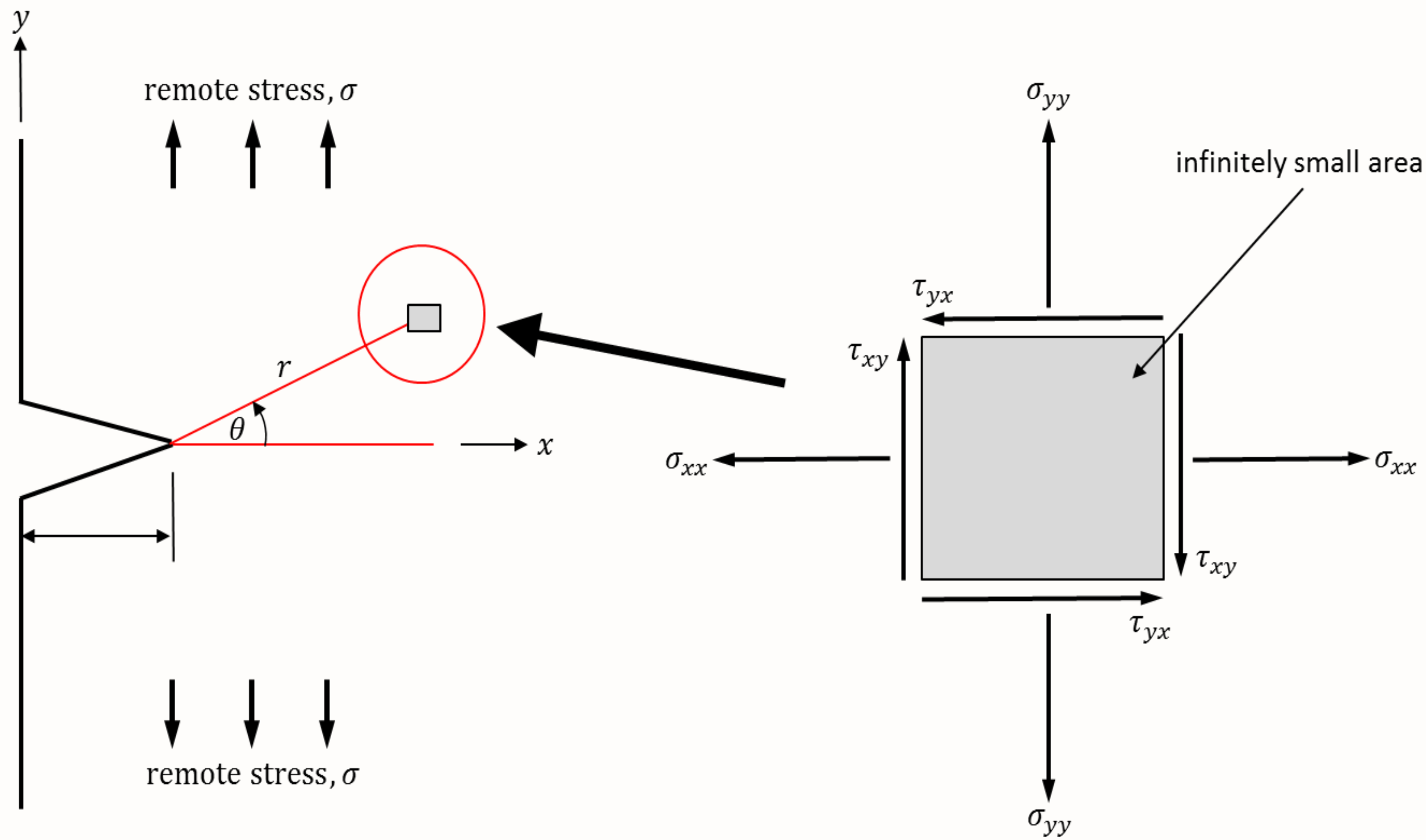
Elastic Plastic Fracture Mechanics (EPFM)

# LEFM Approaches



LEFM allows an analytical approach to fracture by relating the stress distribution in the vicinity of the crack to other parameters such as the nominal applied stress and the geometry and orientation of the crack.

# Stress Function Approach



# Stress Function Approach



$$K_I = Q\sigma\sqrt{a\pi}$$

$K_I$ is the Mode I stress intensity factor
$\sigma$ is the remote (applied) stress
$a$ is the crack half length*
$Q$ is either a constant for infinite plates, or a function of finite plate geometry and type of crack

$K_I$ has units of MNm$^{3/2}$ = MPa$\sqrt{\mathrm{m}}$ or Nmm$^{3/2}$

# Stress Function Approach

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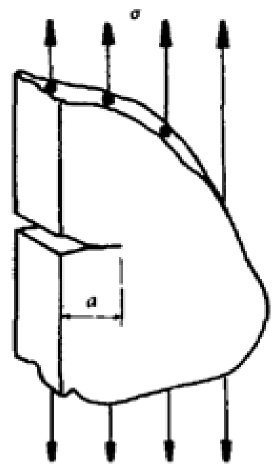
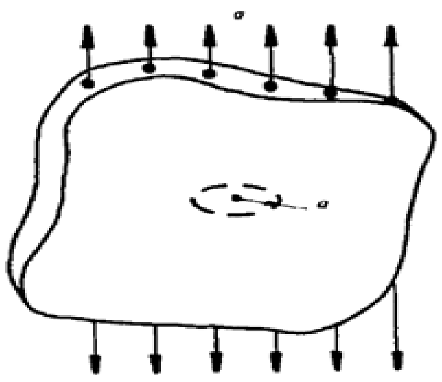
For an infinite plate with a central through-crack, $Q = 1$

For a semi-infinite plate containing an edge crack, $Q = 1.12$




For an infinite plate containing an embedded circular crack or a semi-circular surface crack, $Q = \frac{2}{\pi}$

# Stress Function Approach

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Failure will occur when $K_I$ reaches a critical value, $K_{IC}$, known as the *Fracture Toughness*

# Example 1:



An infinite steel plate contains a crack 5mm long.

Calculate the maximum allowable design stress that could be applied around the boundary if $K_{IC}$ = 105 $MN/m^{3/2}$

If $\sigma_{yield}$ = 1500MPa, comment on the type of failure that would occur if this design stress is exceeded.

# LEFM and Fatigue

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For a constant stress range $\Delta\sigma$:

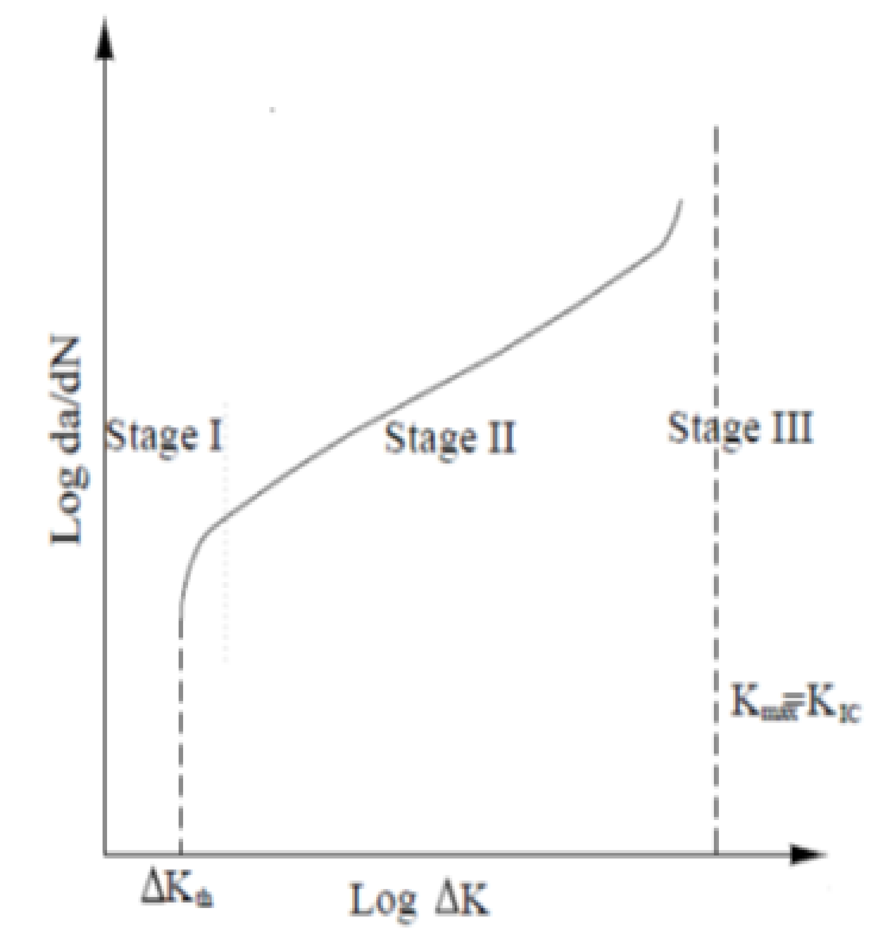
$$\Delta K_I = Q\Delta\sigma\sqrt{\pi a}$$

Where:

$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

# Fatigue Crack Growth






Stage I: initiation, growth along favorable path within microstructure

Stage II: growth, microstructure effect on crack direction reduced, propagates normal to maximum principal stress direction

Stage III: accelerated growth as crack approaches critical size, failure

# Fatigue Crack Growth

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Paris-Erdogan Law:

$$\frac{da}{dN} = C(\Delta K)^n$$

$a$ is the crack size in metres
$N$ the number of cycles to failure
$\Delta K$ is the change in the stress intensity
$C$ and $n$ are material coefficients

# Fatigue Crack Growth

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$$N_f = \frac{1}{Q^n C(\Delta\sigma)^n \pi^{n/2}} \left[ \frac{(a_{crit})^{1-n/2} - (a_o)^{1-n/2}}{1 - n/2} \right]$$

$N_f$ is the number of cycles to failure
$a_{crit}$ is the critical crack size at maximum stress (in metres)
$a_o$ is the initial crack size (in metres)

# Example 2:



Calculate the number of cycles to failure when a repeated stress range of 175MPa is applied to an infinite plate containing a 0.2mm long crack.

Assume:

$$K_{IC} = 54 MN/m^{3/2}$$

$$\frac{da}{dN} = 40 \times 10^{-12} (\Delta K)^4$$

# Example 3:

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Calculate the pressure that would cause failure of a thin cylinder of 7.5m inner diameter with 7.5mm thick walls when it contains a through-thickness crack 12.5mm long aligned with the longitudinal direction.

If this vessel is to be subjected to 4000 cycles of repeated internal pressure, determine the maximum cyclic pressure, based upon a safety factor of 2, given:

$$\frac{da}{dN} = 50 \times 10^{-12}(\Delta K)^4$$

$$K_{1C} = 60MN/m^{3/2}$$