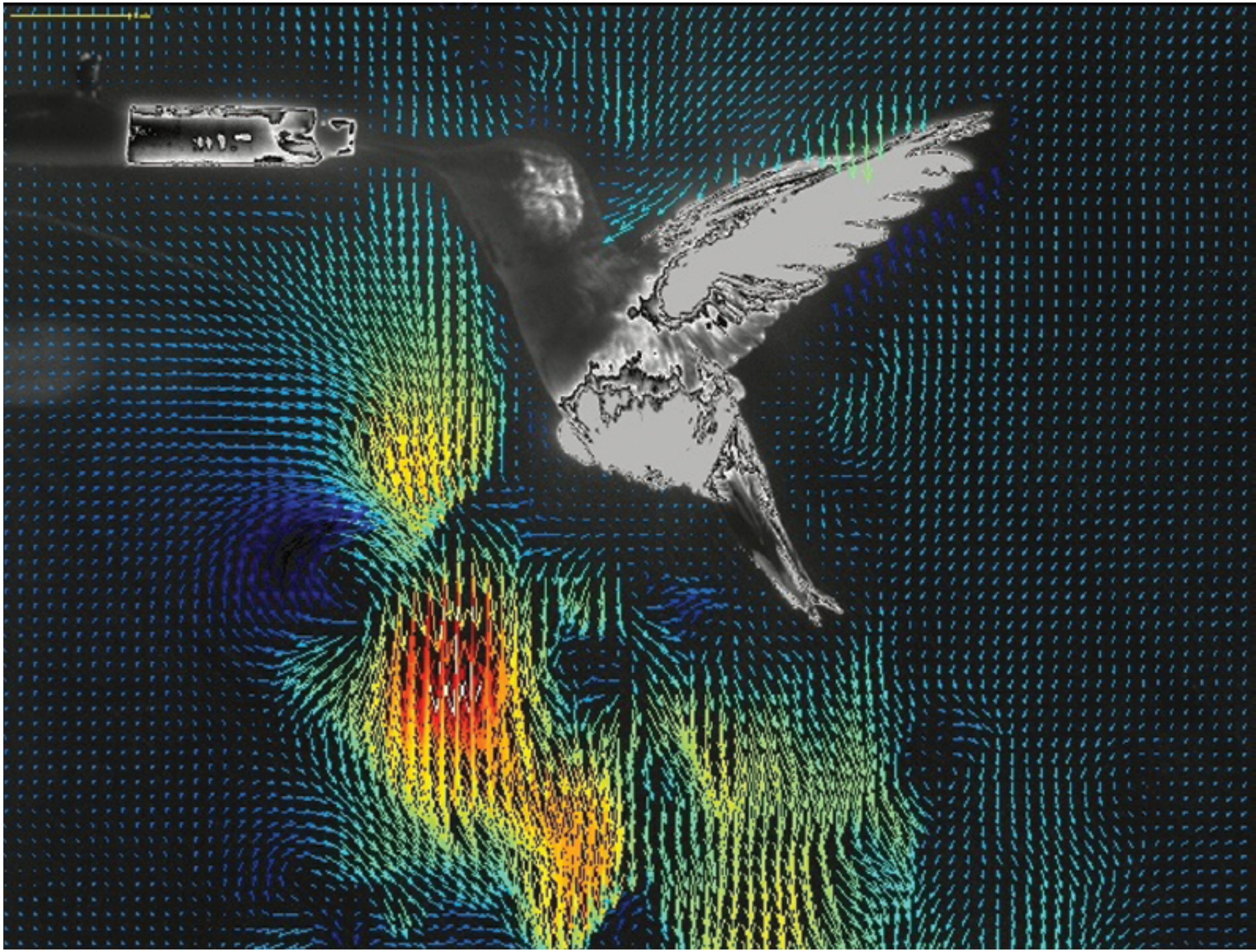
Lecture-9
Flow-rate and velocity measurement



Instantaneous PIV velocity vectors superimposed on a hummingbird in hover. Color scale is from low velocity (blue) to high velocity (red).

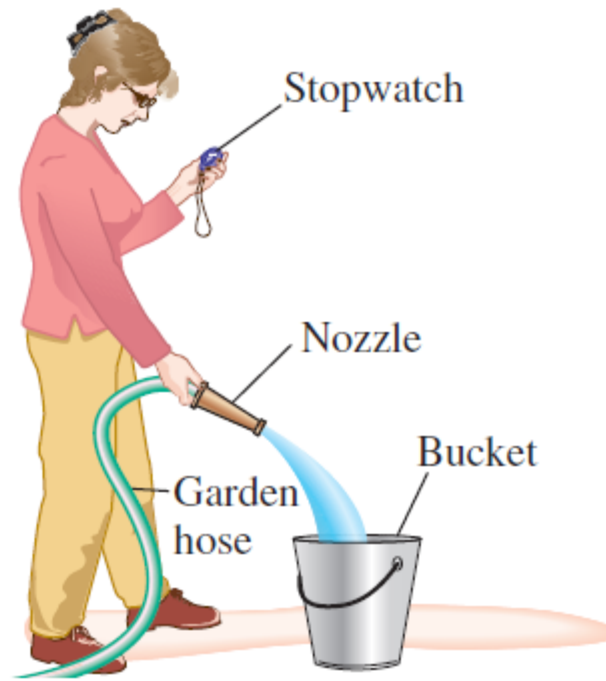
Velocity and flow-rate

Incompressible flows

$$\dot{V} = VA_c$$

Measuring the flow rate is usually done by measuring flow velocity, and many flowmeters are
simply velocimeters used for the purpose of metering flow.

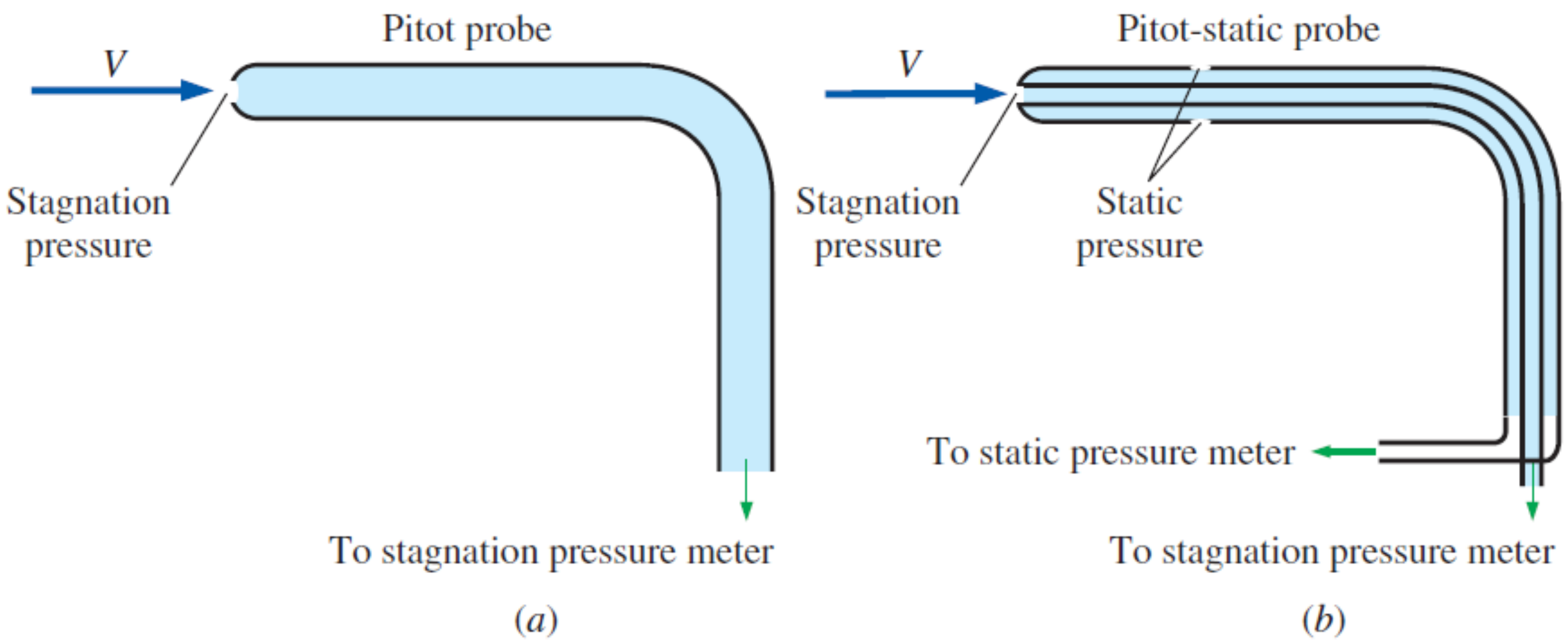
A primitive (but fairly accurate) way of measuring the flow rate of water through a garden hose involves collecting water in a bucket and recording the collection time.



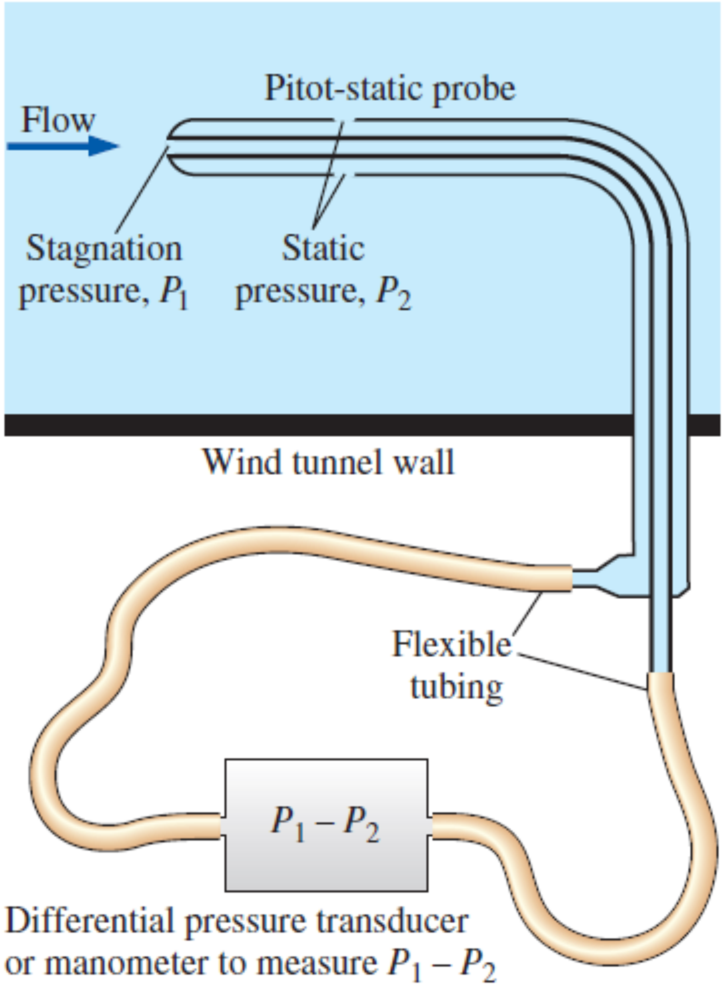
Pitot probes and Pitot-static probes

Pitot probes (also called *Pitot tubes*) and Pitot-static probes are widely used for flow speed measurement.

A Pitot probe is just a tube with a pressure tap at the stagnation point that measures stagnation pressure, while a Pitot-static probe has both a stagnation pressure tap and several circumferential static pressure taps and it measures both stagnation and static pressures



(*a*) A Pitot probe measures stagnation pressure at the nose of the probe, while (*b*) a Pitot-static probe measures both stagnation pressure and static pressure, from which the flow speed is calculated.

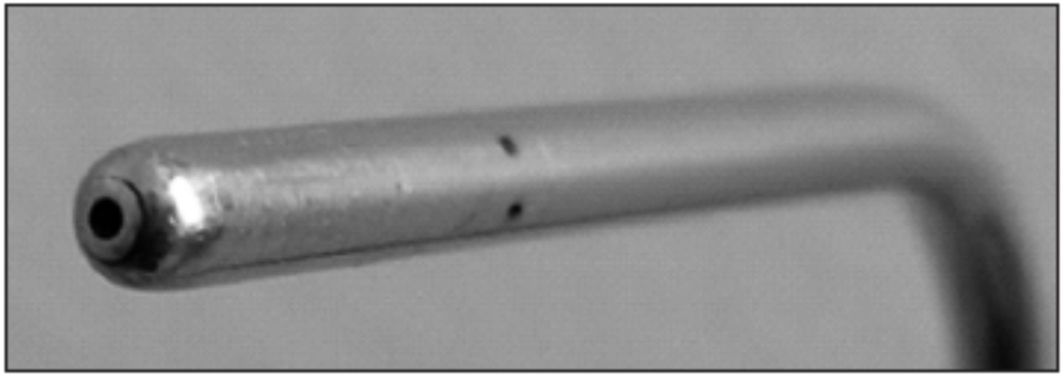


Measuring flow velocity with a Pitotstatic probe. (A manometer may be used in place of the differential pressure transducer.)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Pitot formula:

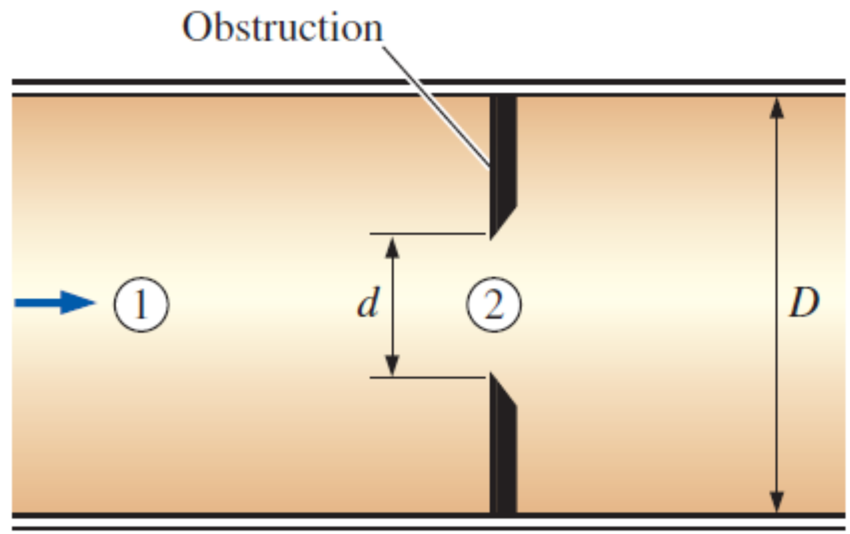
$$V = \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$



Close-up of a Pitot-static probe, showing the stagnation pressure hole and two of the five static circumferential pressure holes.

Obstruction flowmeters

Flowmeters based on this principle are called obstruction flowmeters and are widely used to measure flow rates of gases and liquids.



Flow through a constriction in a pipe.

Mass balance: $\dot{V} = A_1 V_1 = A_2 V_2 \quad \rightarrow \quad V_1 = (A_2/A_1)V_2 = (d/D)^2 V_2$

Bernoulli equation ($z_1 = z_2$): $$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

Obstruction (with no loss): $$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \qquad \beta = d/D$$

$$\dot{V} = A_2 V_2 = (\pi d^2/4) V_2$$

Accounting for losses

The losses can be accounted for by incorporating a correction factor called the discharge coefficient C_d whose value (which is less than 1) is determined experimentally.

Obstruction flowmeters: $$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$A_0 = A_2 = \pi d^2/4 \quad \beta = d/D$$

The value of C_d depends on both b and the Reynolds number, and charts and curve-fit correlations for C_d are available for various types of obstruction meters.

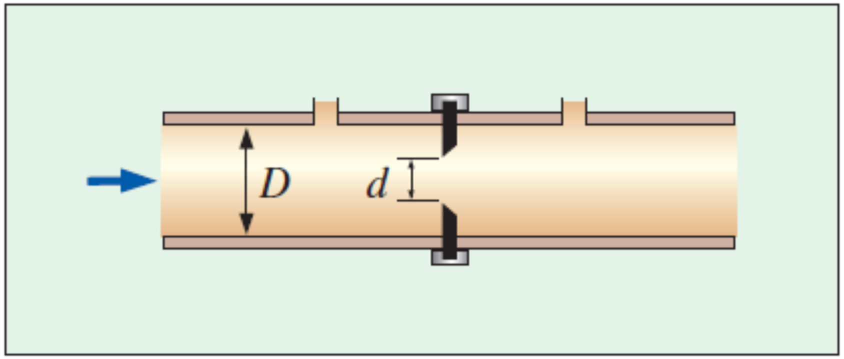
Orifice meters: $$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\text{Re}^{0.75}}$$

Nozzle meters: $$C_d = 0.9975 - \frac{6.53\beta^{0.5}}{\text{Re}^{0.5}}$$

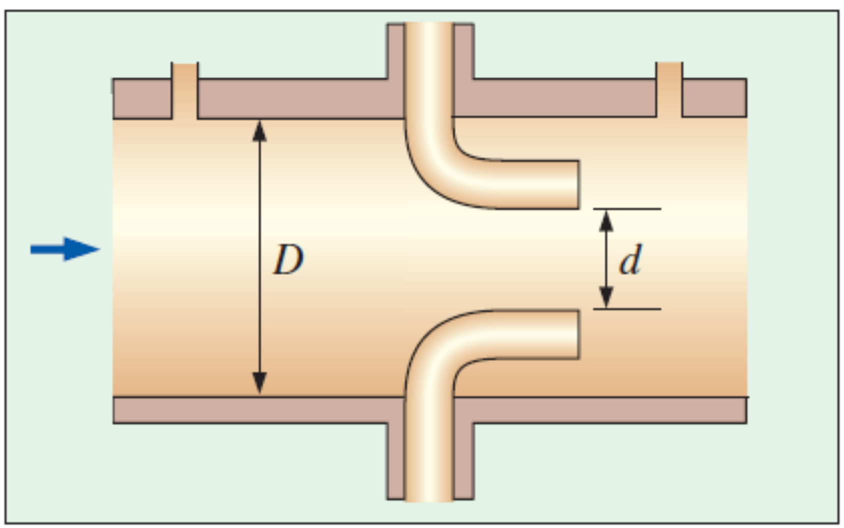
$$0.25 < \beta < 0.75 \text{ and } 10^4 < \text{Re} < 10^7$$

For flows with high Reynolds numbers (Re > 30,000), the value of C_d can be taken to be 0.96 for flow nozzles and 0.61 for orifices.

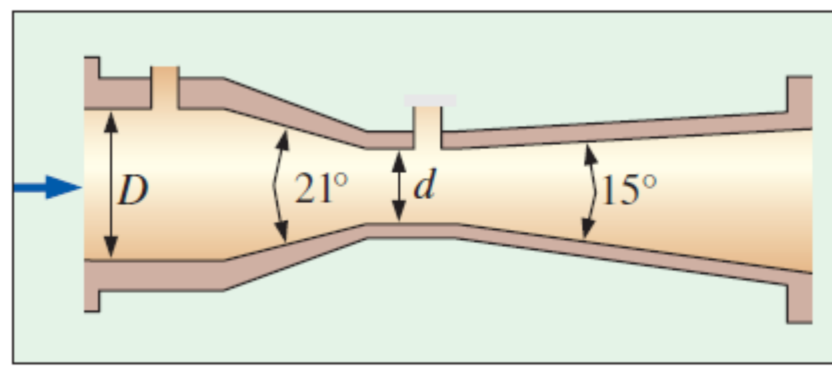
Common obstruction meters



(a) Orifice meter



(b) Flow nozzle

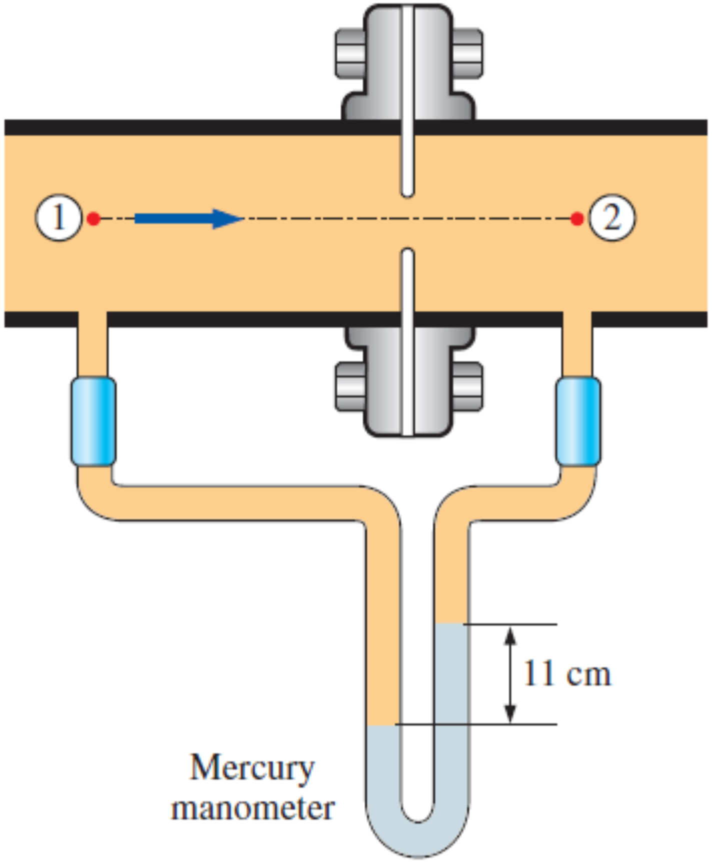


(c) Venturi meter

Example

EXAMPLE 8–10 Measuring Flow Rate with an Orifice Meter

The flow rate of methanol at 20°C ($\rho = 788.4\ \text{kg/m}^3$ and $\mu = 5.857 \times 10^{-4}\ \text{kg/m·s}$) through a 4-cm-diameter pipe is to be measured with a 3-cm-diameter orifice meter equipped with a mercury manometer across the orifice plate, as shown in Fig. 8–62. If the differential height of the manometer is 11 cm, determine the flow rate of methanol through the pipe and the average flow velocity.



Example (continued)

First guess:
Cd = 0.61

Properties The density and dynamic viscosity of methanol are given to be $\rho = 788.4 \text{ kg/m}^3$ and $\mu = 5.857 \times 10^{-4}$ kg/m·s, respectively. We take the density of mercury to be 13,600 kg/m^3.

Analysis The diameter ratio and the throat area of the orifice are

$$\beta = \frac{d}{D} = \frac{3}{4} = 0.75$$

$$A_0 = \frac{\pi d^2}{4} = \frac{\pi(0.03 \text{ m})^2}{4} = 7.069 \times 10^{-4} \text{ m}^2$$

The pressure drop across the orifice plate is

$$\Delta P = P_1 - P_2 = (\rho_{\text{Hg}} - \rho_{\text{met}})gh$$

Then the flow rate relation for obstruction meters becomes

$$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} = A_0 C_d \sqrt{\frac{2(\rho_{\text{Hg}} - \rho_{\text{met}})gh}{\rho_{\text{met}}(1 - \beta^4)}} = A_0 C_d \sqrt{\frac{2(\rho_{\text{Hg}}/\rho_{\text{met}} - 1)gh}{1 - \beta^4}}$$

Substituting, the flow rate is determined to be

$$\dot{V} = (7.069 \times 10^{-4} \text{ m}^2)(0.61)\sqrt{\frac{2(13{,}600/788.4 - 1)(9.81 \text{ m/s}^2)(0.11 \text{ m})}{1 - 0.75^4}}$$
$$= 3.09 \times 10^{-3} \text{ m}^3/\text{s}$$

which is equivalent to 3.09 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{3.09 \times 10^{-3} \text{ m}^3/\text{s}}{\pi(0.04 \text{ m})^2/4} = 2.46 \text{ m/s}$$

Example (continued)

Substituting $\beta = 0.75$ and $\text{Re} = 1.32 \times 10^5$ into the orifice discharge coefficient relation

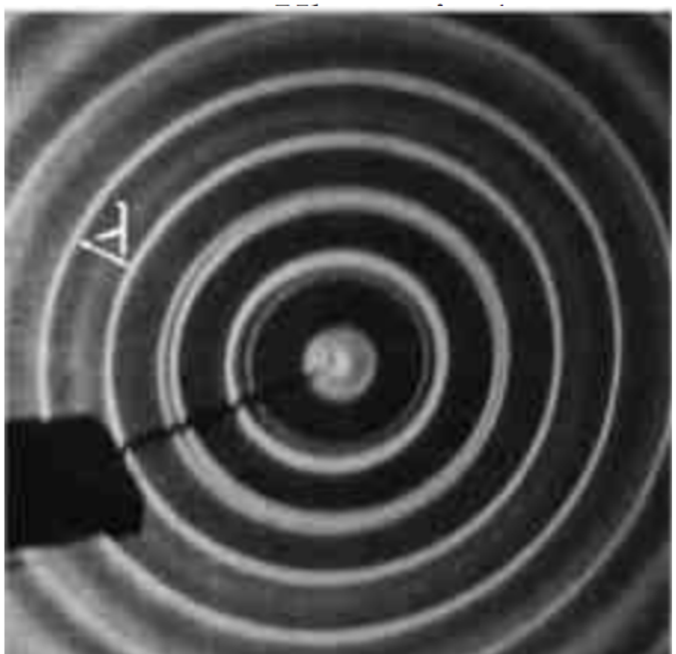
$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\text{Re}^{0.75}}$$

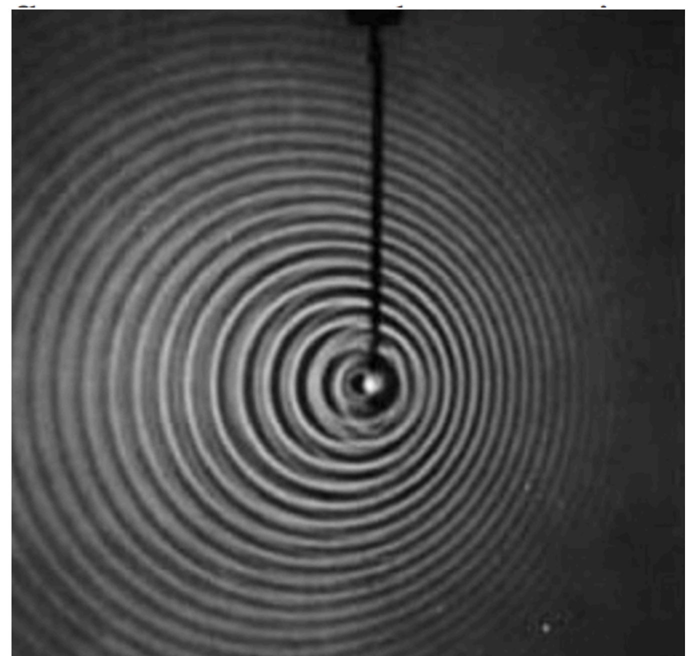
gives $C_d = 0.601$, which differs from the original guessed value of 0.61. Using this refined value of C_d, the flow rate becomes 3.04 L/s, which differs from our original result by 1.6 percent. After a couple iterations, the final converged flow rate is **3.04 L/s,** and the average velocity is **2.42 m/s** (to three significant digits).

Discussion If the problem is solved using an equation solver such as EES, then it can be formulated using the curve-fit formula for C_d (which depends on the Reynolds number), and all equations can be solved simultaneously by letting the equation solver perform the iterations as necessary.

Ultrasonic flowmeters

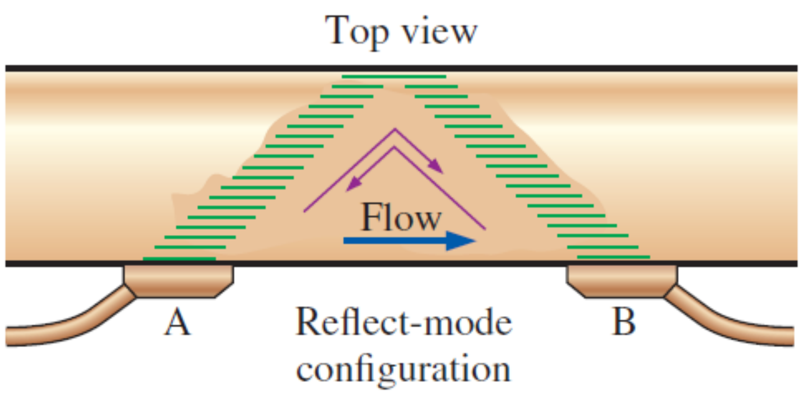
It is a common observation that when a stone is dropped into calm water, the waves that are generated spread out as concentric circles uniformly in all directions. But when a stone is thrown into flowing water such as a river, the waves move much faster in the flow direction (the wave and flow velocities are added since they are in the same direction) compared to the waves moving in the upstream direction (the wave and flow velocities are subtracted since they are in opposite directions). As a result, the waves appear spread out downstream while they appear tightly packed upstream. The difference between the number of waves in the upstream and downstream parts of the flow per unit length is proportional to the flow velocity, and this suggests that flow velocity can be measured by comparing the propagation of waves in the forward and backward directions with respect to the flow. **Ultrasonic flowmeters** operate on this principle, using sound waves in the ultrasonic range (beyond human hearing ability, typically at a frequency of 1 MHz).





Transit time and frequency-shift (Doppler) ultrasonic flow meters

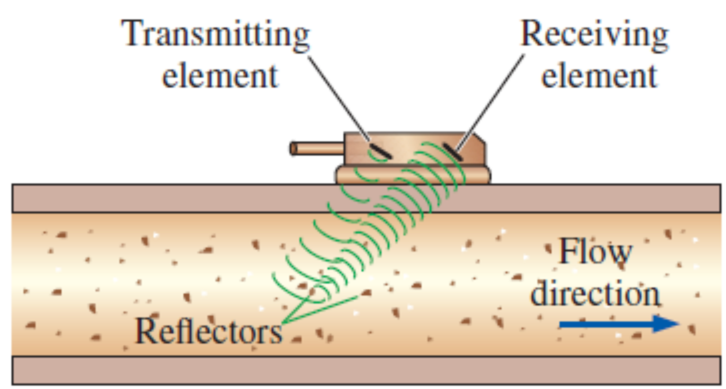
Transit-time: Two transducers that alternately transmit and receive ultrasonic waves, one in the direction of flow and the other in the opposite direction. The travel time for each direction can be measured accurately, and the difference in the travel time is calculated.



$$V = KL\,\Delta t$$

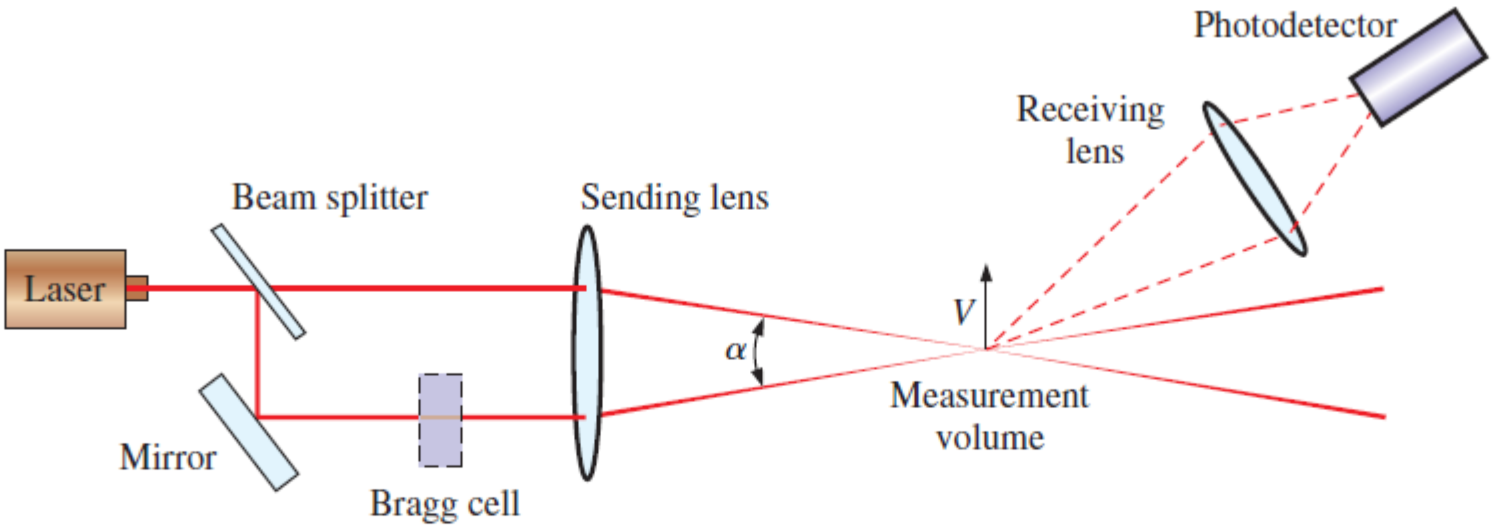
L is the distance between the transducers and K is a constant

Doppler: The transducer transmits a sound wave at a fixed frequency through the pipe wall and into the flowing liquid. The waves reflected by impurities, such as suspended solid particles or entrained gas bubbles, are relayed to a receiving transducer. The change in the frequency of the reflected waves is proportional to the flow velocity



Laser Doppler Velocimetry

The operating principle of LDV is based on sending a highly coherent monochromatic (all waves are in phase and at the same wavelength) light beam toward the target, collecting the light reflected by small particles in the target area, determining the change in frequency of the reflected radia- tion due to the Doppler effect, and relating this frequency shift to the flow velocity of the fluid at the target area.

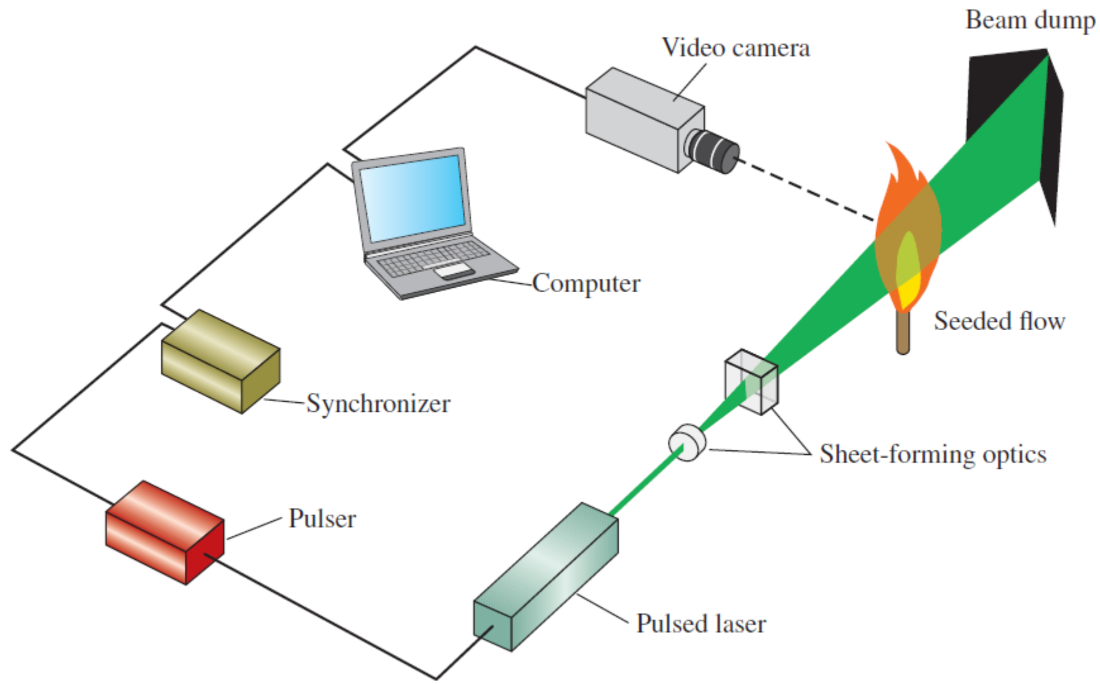


It can accurately measure velocity at a very small volume, and thus it can also be used to study the details of flow at a locality, including turbulent fluctuations, and it can be traversed through the entire flow field without intrusion

Particle image velocimetry

The entire instantaneous velocity profile at a cross section of pipe can be obtained with a single PIV measurement.

A PIV system can be viewed as a camera that can take a snapshot of velocity distribution at any desired plane in a flow.



The first step is to seed the flow with suitable particles in order to trace the fluid motion. Then a pulse of laser light sheet illuminates a thin slice of the flow field at the desired plane, and the positions of particles in that plane are determined by detecting the light scattered by particles on a digital video or photographic camera positioned at right angles to the light sheet (Fig. 8–76). After a very short time period, typically microseconds, the particles are illuminated again by a second pulse of laser light sheet, and their new positions are recorded. Using the information on these two superimposed camera images, the particle displacements are determined for all particles, and the magnitude of velocity of each particle in the plane of the laser light sheet is determined.

Instantaneous PIV velocity vectors superimposed on a hummingbird in hover. Color scale is from low velocity (blue) to high velocity (red).