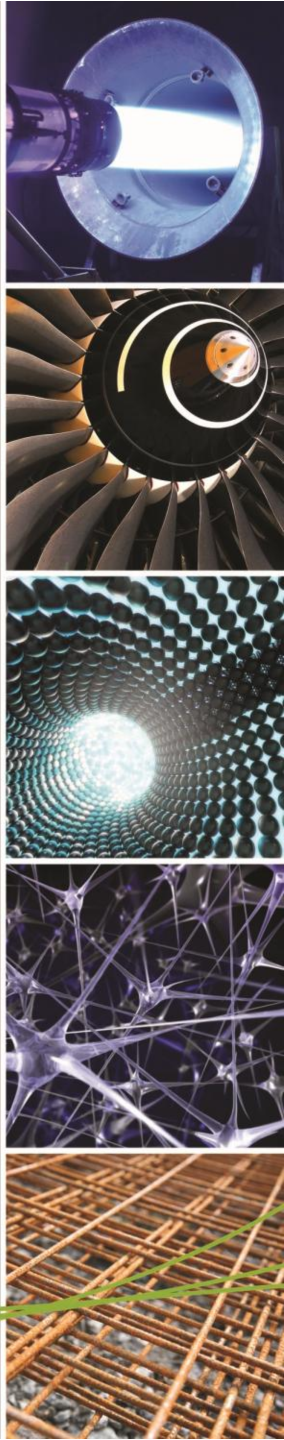

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Advanced Structural Analysis
EGF316

Theories of Failure and Stress Concentrations

Lecture Content

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- Failure mechanisms
- Theories of Failure
- Safety factors
- Stress concentrations

Failure mechanisms

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1. Wear and tear
2. Fatigue – cyclic loading
3. Corrosion
4. Creep
5. Buckling
6. Friction, etc..

Failure of material is usually classified into
1.) Ductile failure (yielding)
2.) Brittle failure (fracture)

A material can fail in brittle or ductile manner or both, depending upon the conditions.

Ductile Vs Brittle Fracture

Ductile Vs Brittle Fracture

	Ductile	Brittle
Fracture Stress	Greater than yield strength	Lower than yield strength
Energy Absorption	High	Low
Nature of Fracture	Necking, rough fracture surface, linking up of cavities	No necking, shiny granular surface, cleavage or intergranular
Type of Material	Metals	Ceramics, glasses
Crack Propagation	Slow	Fast
Nature of Failure	Plastic deformation warning, less catastrophic	Little deformation, more catastrophic

Theories of Failure

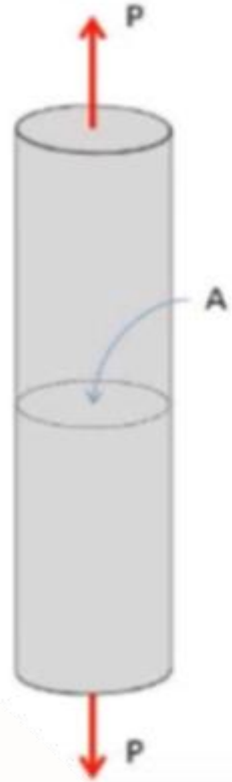
A “theory of failure” is a theory for predicting the conditions under which the (solid) materials fail under the action of external loads.

This, of course, is while ignoring the damage due to any causes internal to the material, for example, phase change.

Theories of Failure

1) Developing a “theory of failure” is difficult because determining the type of the predominant stresses – tensile, compressive, shear – is difficult.
2) Also, whether the predominant type of failure for the material is ductile or brittle is a significant factor → yield stress or ultimate stress

Theories of Failure in 1D

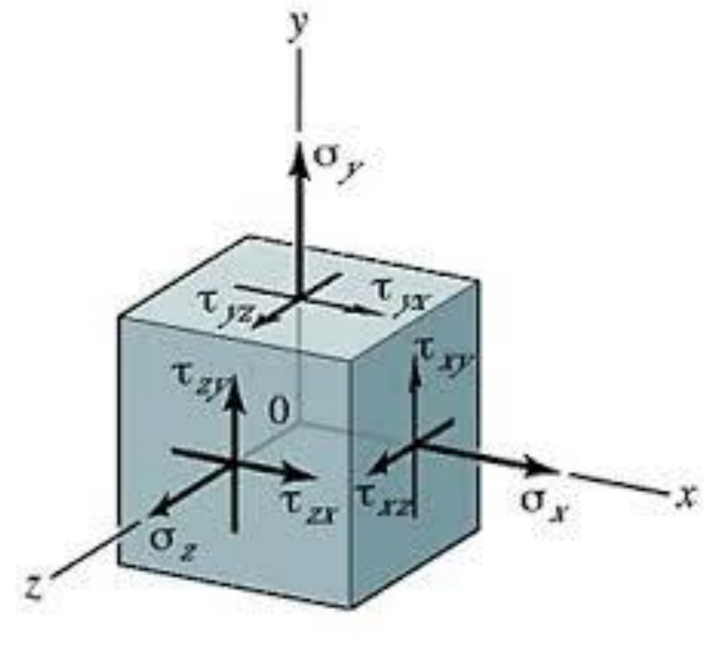


$$normal\ stress, \qquad \sigma_1 = \frac{P}{A}$$

Criterion:

$$\boxed{\sigma_1 \leq \sigma_{yield}}$$

Theories of Failure in 3D



State of stress in 3D is complicated. So, we need better theories.

Theories of Failure

A good failure theory requires:

- Observation of a large number of test results in different load conditions (empirical knowledge)
- A theory explaining the microscopic mechanisms involved which must be consistent with the observations

Theories of Failure

a) Rankine or Maximum Principal stress theory
b) Saint – venant or Maximum Principal strain theory
c) Tresca or Maximum shear stress theory
d) Von Mises & Hencky or Shear strain energy theory
e) Haigh or Total strain energy per unit volume theory

a) Mohr-Coulomb failure theory – cohesive-frictional solids
b) Drucker-Prager failure theory – pressure dependent solids
c) Cam-Clay failure theory - soils

Maximum principal stress theory:
Failure will occur when the maximum principal stress exceeds the material's yield strength.

$$\max\left(\sigma_{1,}\sigma_{2,}\sigma_3\right) \leq \sigma_{yield}$$

Maximum principal strain theory:
Failure will occur when the maximum principal strain exceeds the strain at the yield point.

$$\max\left(\varepsilon_{1,}\varepsilon_{2,}\varepsilon_3\right) \leq \varepsilon_{yield}$$

Tresca or Maximum Shear Stress Theory:

This theory considers that failure (yielding) will occur when the maximum shear stress in the complex stress state becomes equal to the material's limiting shear strength in a simple tensile test.

$$\sigma_1 - \sigma_3 \leq \sigma_{yield}$$

Note: σ1 and σ3 are the maximum and minimum principal stresses.

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Von Mises or Maximum Shear Strain Energy Theory:

This theory states that failure will occur when the maximum shear energy component in the complex stress system is equal to that at the yield point in a simple tensile test.

$$\sigma_{eq} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}$$

$$\sigma_{eq} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{xx} - \sigma_{zz})^2 + 6({\tau_{xy}}^2 + {\tau_{yz}}^2 + {\tau_{xz}}^2)}$$

$$\sigma_{eq} \leq \sigma_{yield}$$

Example 1:

If $\sigma_1 = 200MPa$ and $\sigma_2 = 100MPa$, calculate the limiting value of σ_3 to avoid yielding in accordance with Tresca and Von Mises. The yield stress $\sigma_y = 300MPa$.

Safety Factors

Tresca:

$$\sigma_1 - \sigma_3 \leq \frac{\sigma_y}{SF}$$

Von Mises:

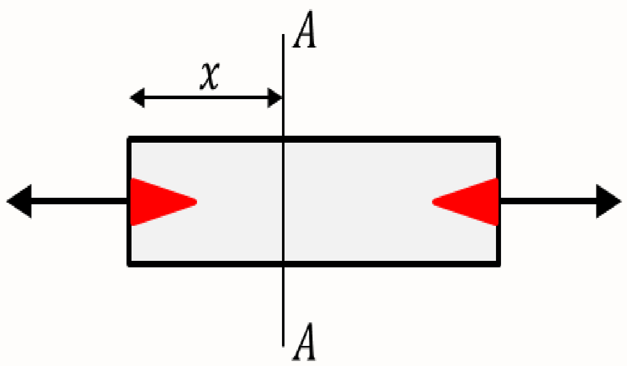
$$\sigma_{eq} \leq \frac{\sigma_{yield}}{SF}$$

Example 2:

A thin mild steel tube has a mean diameter of 100mm and a thickness of 3mm. Determine the maximum torque that the tube can transmit according to the Tresca and von Mises criteria and using a Safety Factor of 2.25.

Assume a constant shear stress (thin tube) and a yield stress of 230 MPa.

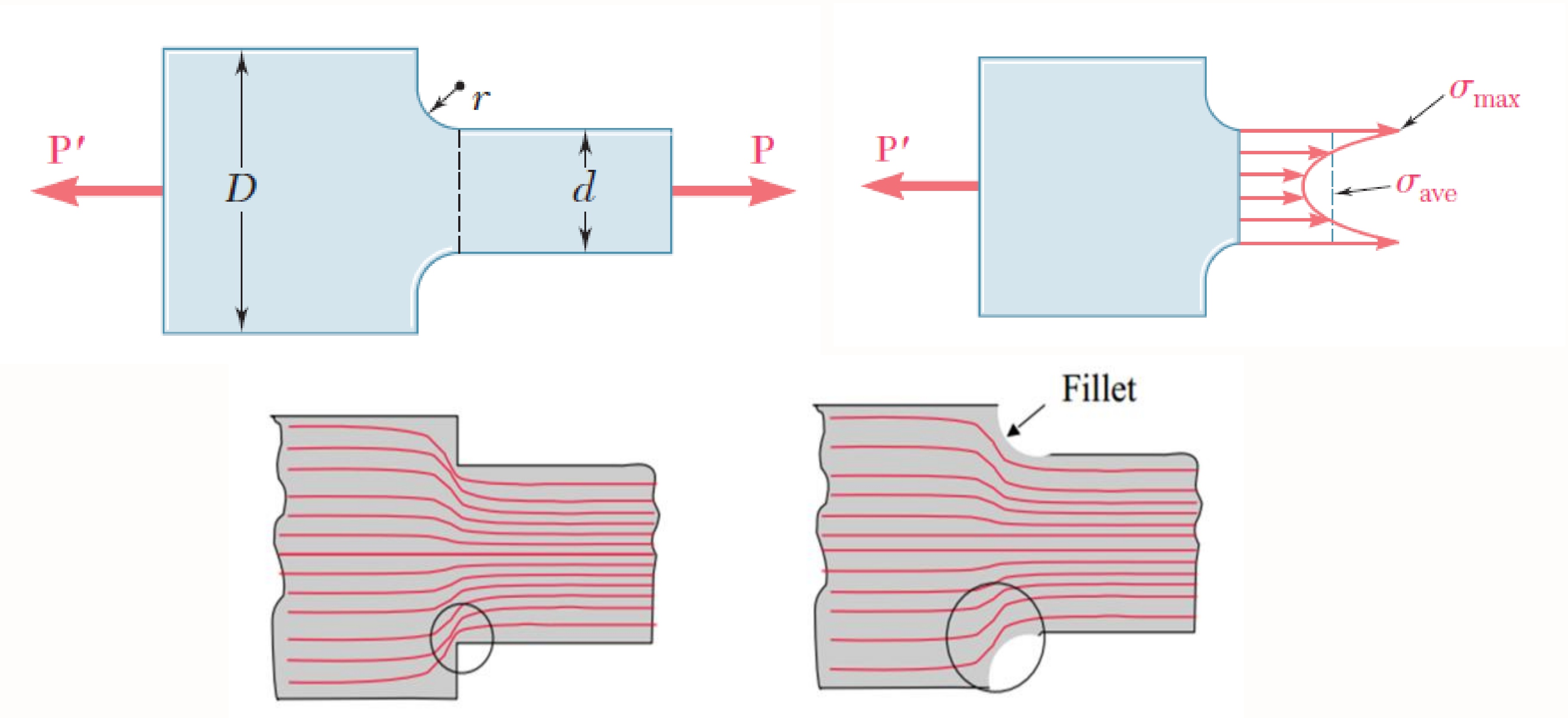
Stress Concentrations
concentration of stress



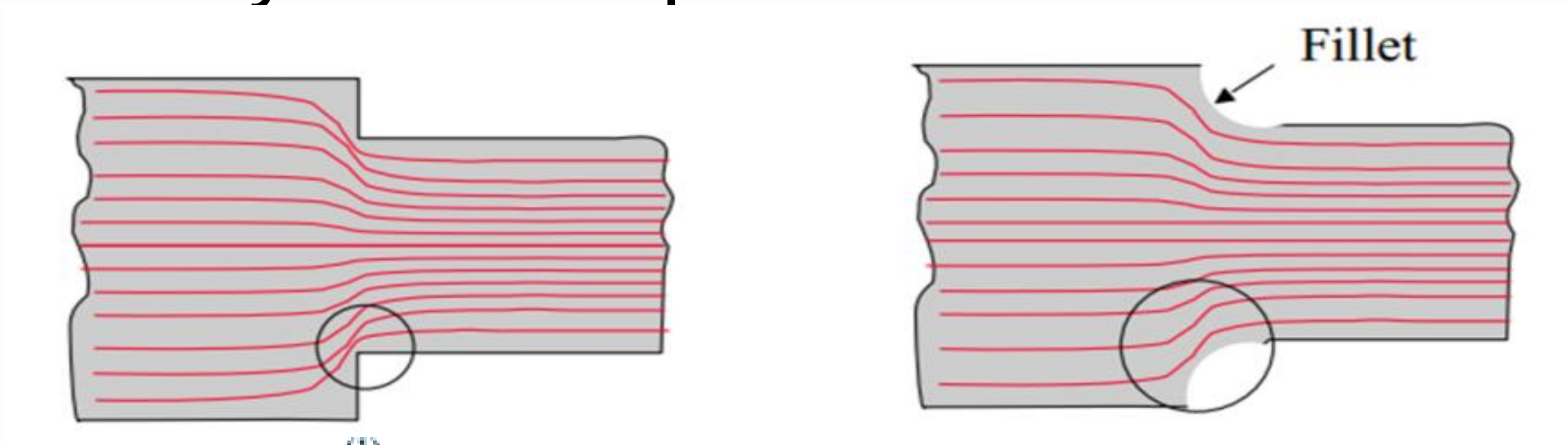
(a) concentrated load (W)

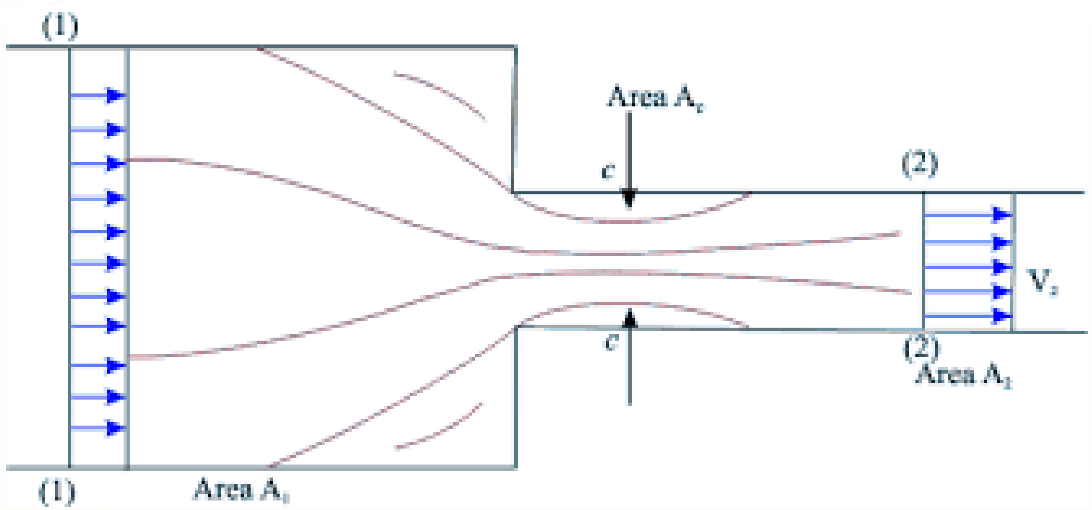
(b) distributed load (or pressure)

Geometric Discontinuities



Similarity with flow problems

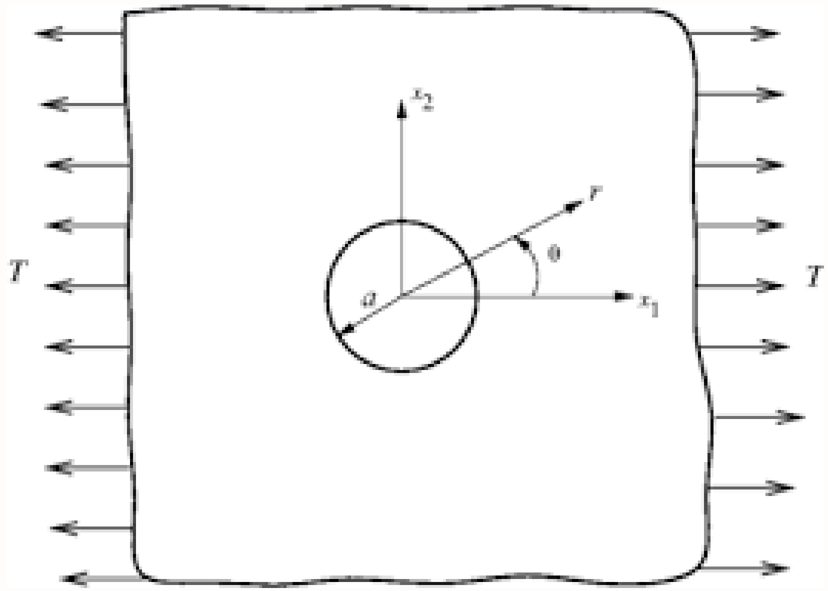




How to calculate concentrated stresses?

1) Analytical methods
2) Photoelastic methods - laboratory experiments
3) Empirical data – Stress concentration factors
4) Virtual methods – numerical techniques
    - Finite Element Analysis

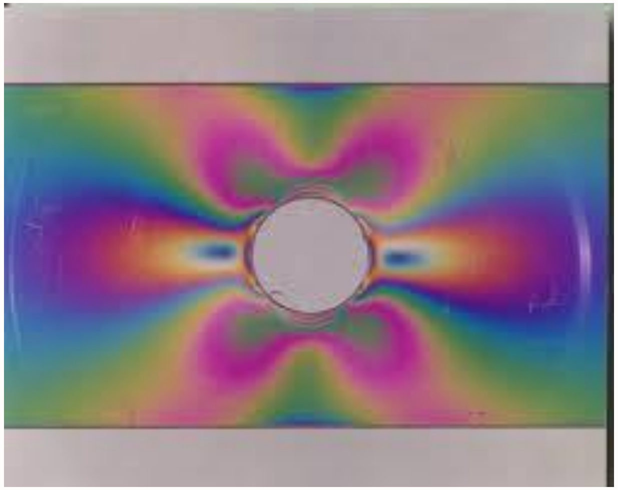
1.) Analytical methods



$$\sigma = f(r, \theta, T)$$

- ❖ Limited to very simple geometries

2.) Photoelastic methods



- ❖ Widely used experimental technique before the invention of numerical methods
- ❖ Limited to small components

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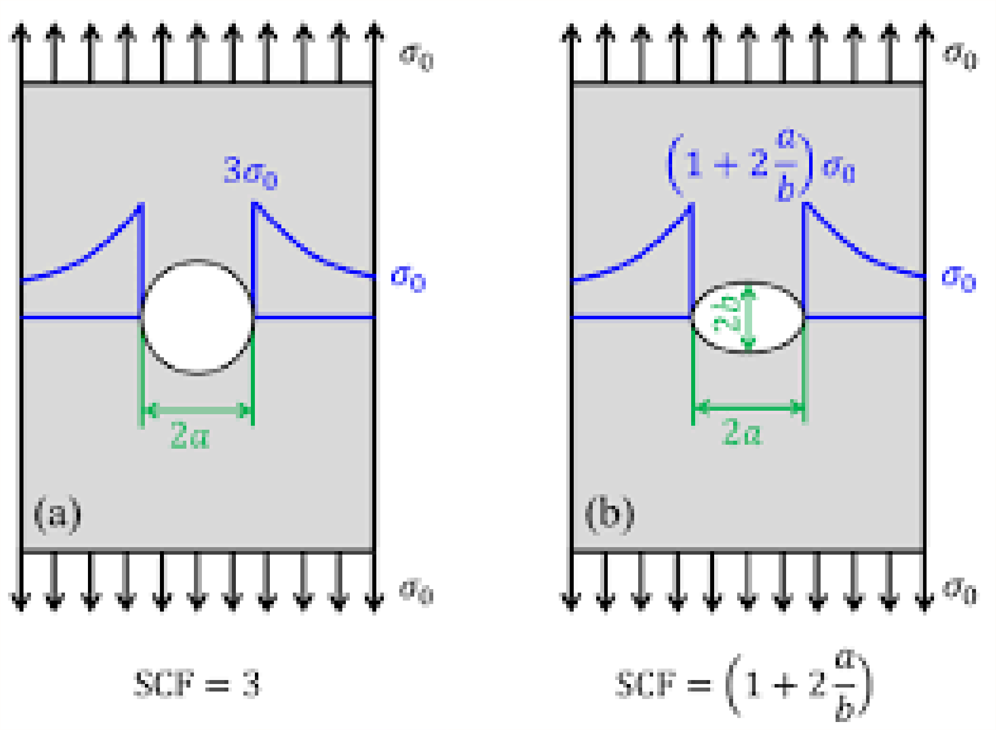
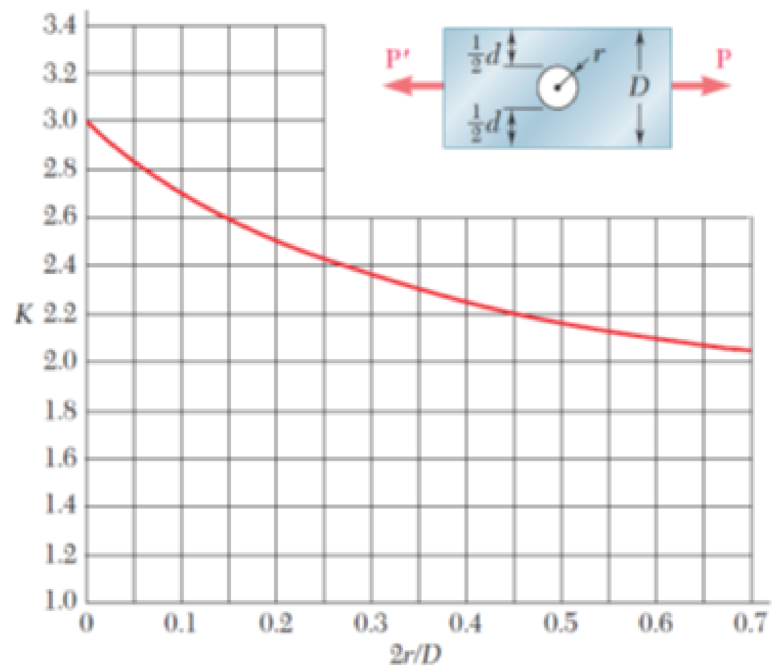
3.) Empirical data - stress concentration factors

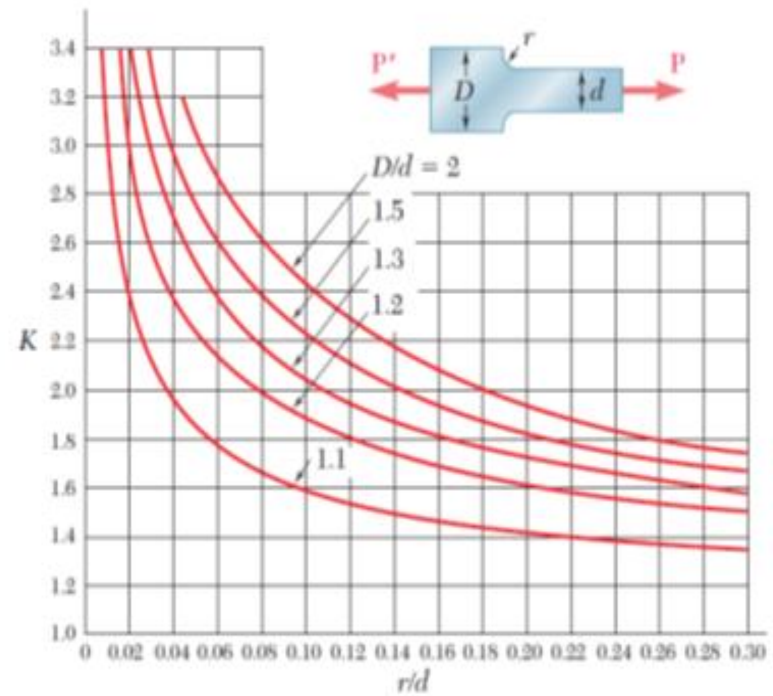
❖ Help in predicting concentrated stresses



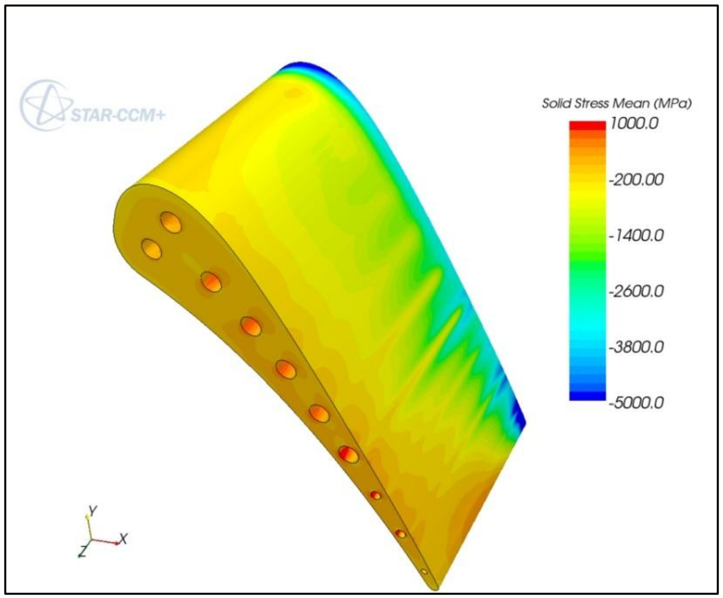
$$K_t = \frac{\sigma_{max}}{\sigma_{nom}}$$

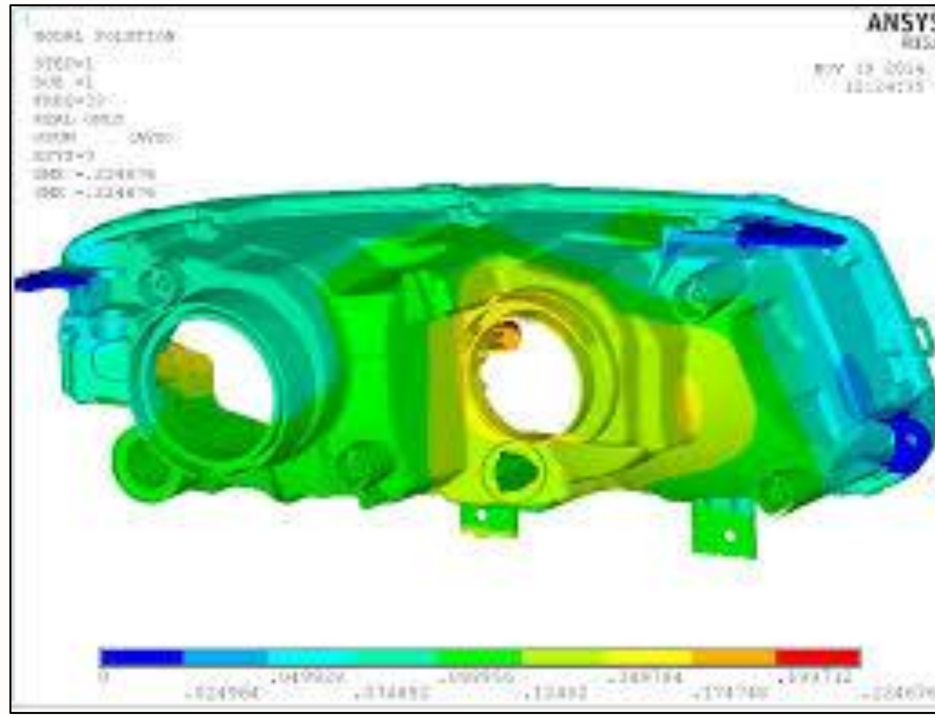
Stress concentration factors





4.) Numerical methods
Finite Element Analysis





Static Load Design

$$\sigma_{max} = K_t \sigma_{nom}$$

Elastic design model: all stresses must be elastic and the nominal stress must be such that the maximum stress does not exceed yield - a factor of safety is often employed.

Limited plasticity: local yielding is allowed in the region of the stress concentration and hence the nominal stress can be significantly closer to yield within some factor of safety.

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Fatigue Design

Fatigue stress concentration factor

$$K_f = \frac{maximum\ stress\ in\ notched\ specimen}{maximum\ stress\ in\ notch-free\ specimen}$$

$$K_f = q(K_t - 1) + 1$$

q is the notch sensitivity factor; a measure of how sensitive a material is to notches or geometric discontinuities.

Example 3:

Determine the stresses in the fillets of the component shown below due to an axial load of 100N. The thickness of the part is 1mm.