

Lecture-5

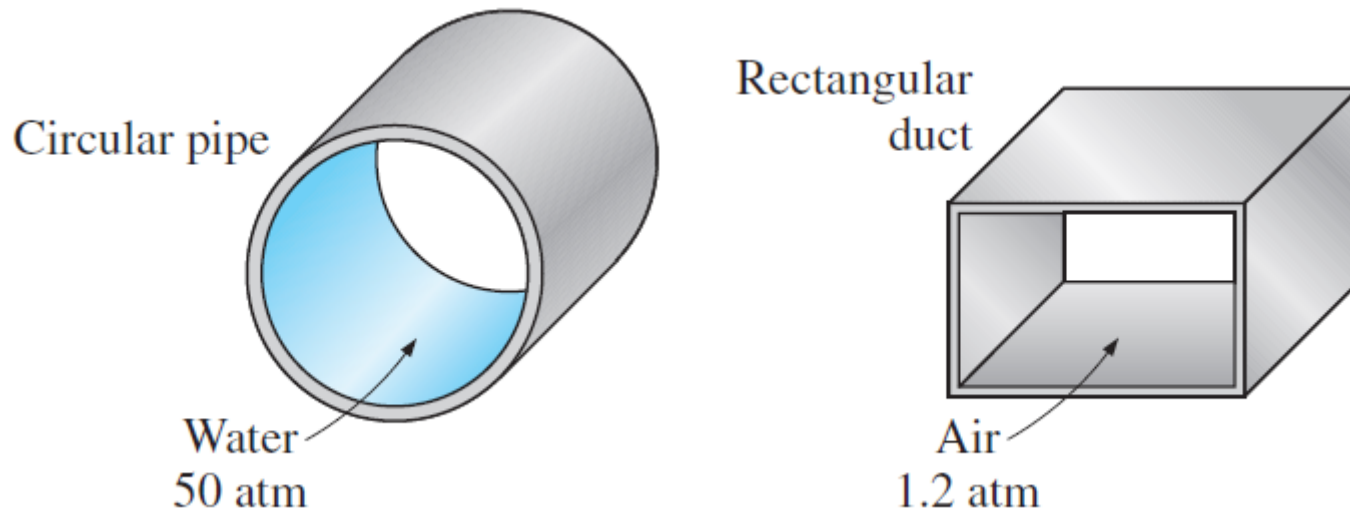
Introduction: Laminar and Turbulent flows

Internal flows through pipes, elbows, tees, valves, etc., as in this oil refinery, are found in nearly every industry.



Introduction

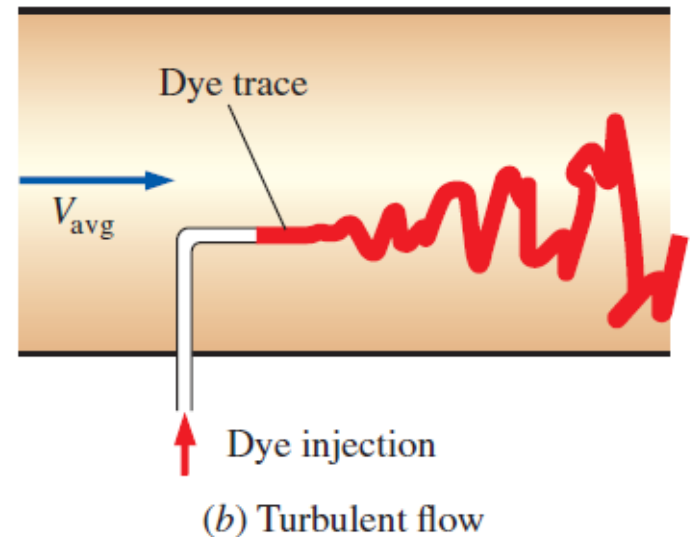
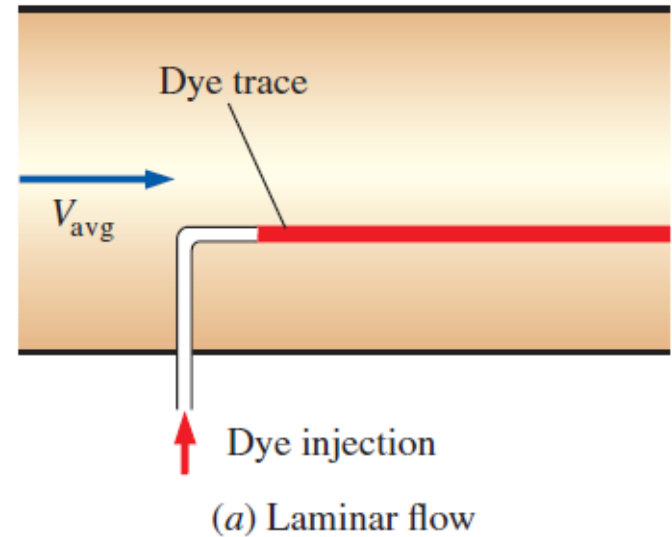
- Liquid or gas flow through pipes or ducts is commonly used in heating and cooling applications and fluid distribution networks.
- The fluid in such applications is usually forced to flow by a fan or pump through a flow section.
- We pay particular attention to friction, which is directly related to the pressure drop and head loss during flow through pipes and ducts.
- The pressure drop is then used to determine the pumping power requirement.



Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot.

Laminar and Turbulent flow

- Laminar flow
 - Smooth streamlines and highly ordered motion
 - Flow in parallel layers
 - Viscous forces are dominant.
- Turbulent flow
 - Irregular velocity fluctuations and highly disordered motion.
 - High mixing.
 - Inertial forces are dominant.





Reynolds number

$$\text{Re} = \frac{\text{Inertial Forces}}{\text{Viscous Forces}} = \frac{\rho V_{\text{avg}} D_h}{\mu} = \frac{V_{\text{avg}} D_h}{\nu}$$

- Small Reynolds numbers
 - Viscous forces dominate
 - Laminar flow
- Large Reynolds numbers
 - Inertial forces dominate
 - Turbulent flow
- Medium Reynolds numbers
 - Transitional flow.
 - Flow switched between laminar and turbulent flow seemingly randomly
- Re_{cr} , critical Reynolds number after which flow is turbulent.
 - Re_{cr} different for different geometries and flow conditions.
- For flow in circular pipe

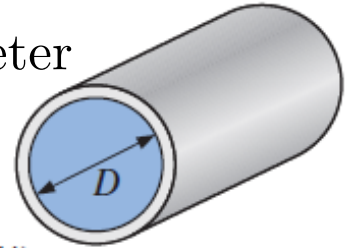
Laminar flow	$\text{Re} < 2300$
Transitional flow	$2300 \lesssim \text{Re} < \lesssim 10,000$
Turbulent flow	$\text{Re} \gtrsim 10,000$

hydraulic diameter: $D_h = \frac{4A_c}{p}$

A_c : cross-section of flow

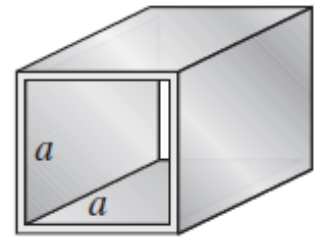
p : Wetted perimeter

Circular tube:



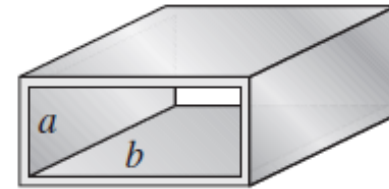
$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct:



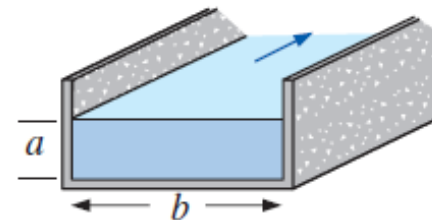
$$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct:



$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

Channel:



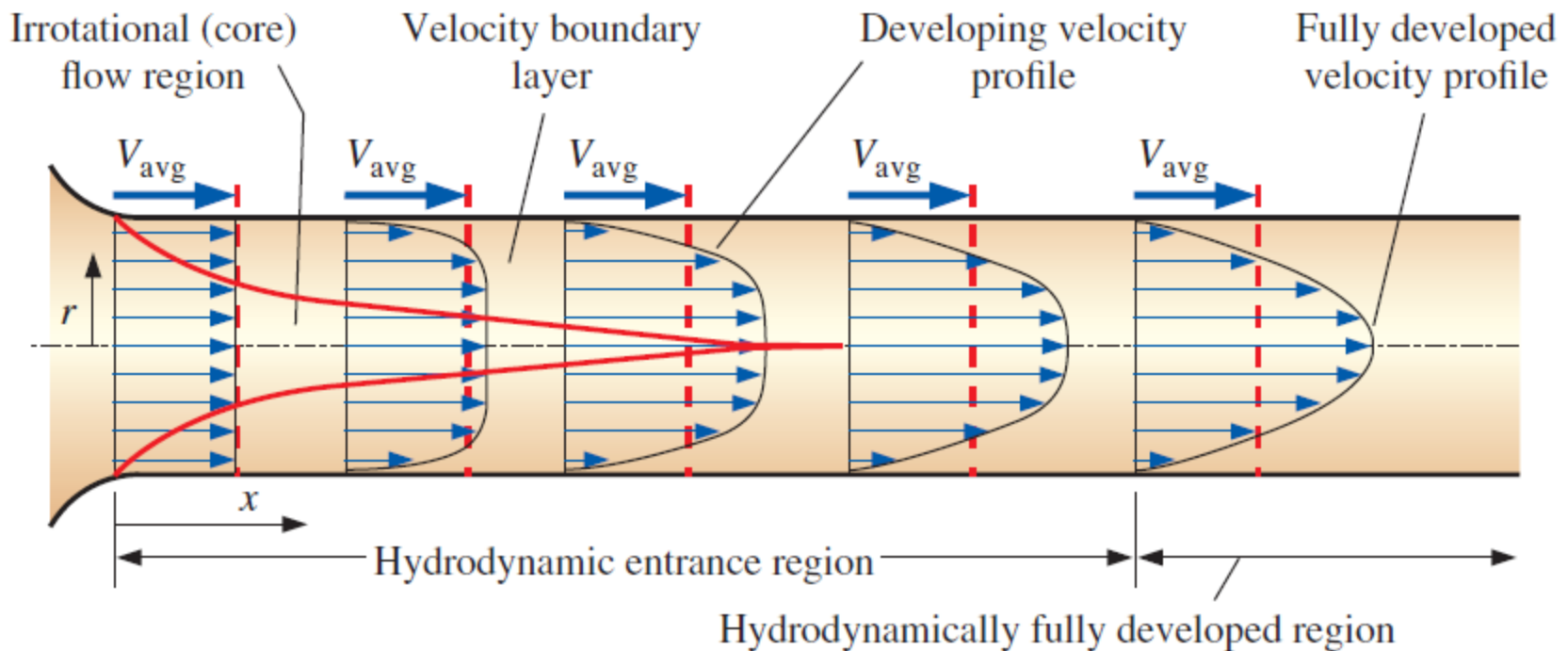
$$D_h = \frac{4ab}{2a+b}$$

Entrance region

Velocity boundary layer: The region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt.

Boundary layer region: The viscous effects and the velocity changes are significant.

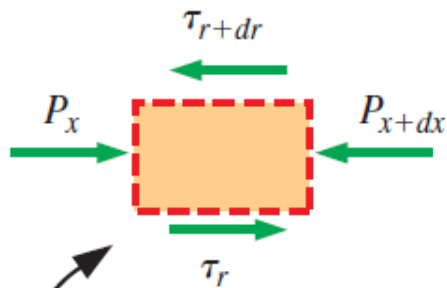
Irrotational (core) flow region: The frictional effects are negligible and the velocity remains essentially constant in the radial direction.



The development of the velocity boundary layer in a pipe. The developed average velocity profile is parabolic in laminar flow, but somewhat flatter or fuller in turbulent flow.

Laminar flow in pipes

- We consider **steady, laminar, incompressible flow** of a fluid with constant properties in the **fully developed** region of a straight circular pipe.
- In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile $u(r)$ remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component in the direction normal to the pipe axis is everywhere zero. There is no acceleration since the flow is steady and fully developed.



$$(2\pi r dr P)_x - (2\pi r dr P)_{x+dx} + (2\pi r dx \tau)_r - (2\pi r dx \tau)_{r+dr} = 0$$

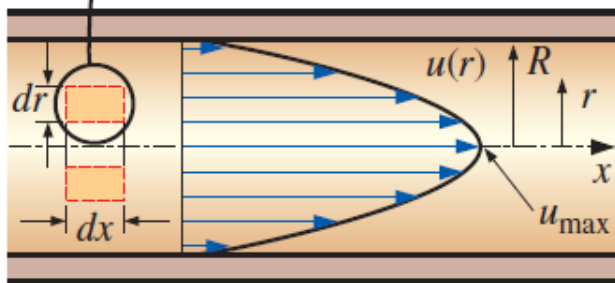
$$r \frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

$$r \frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0$$

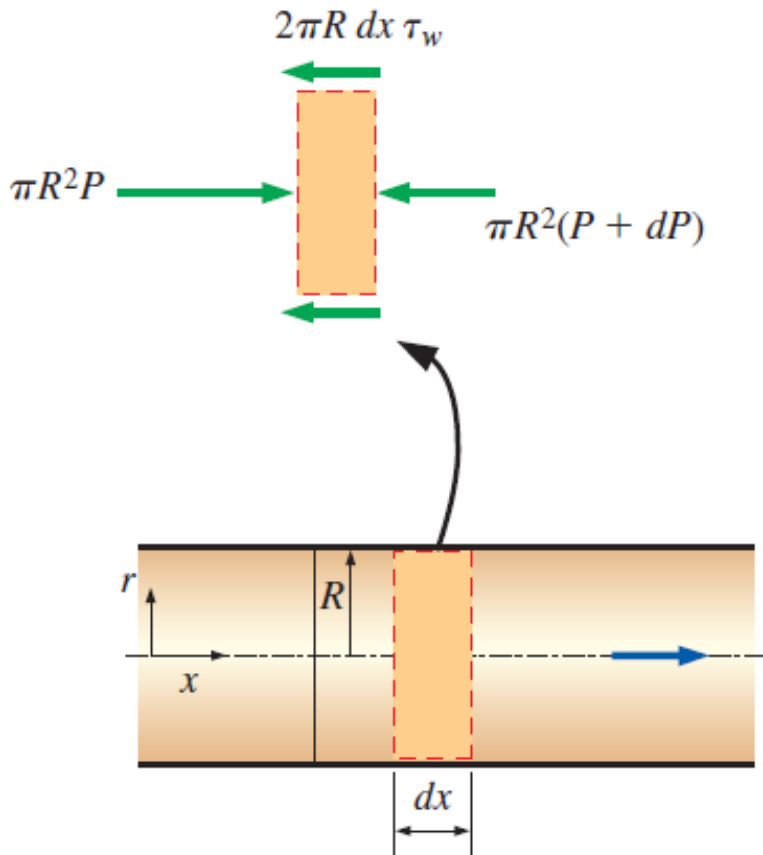
$$\tau = -\mu(du/dr) \qquad \frac{\mu}{r} \frac{d}{dr} \left(r \frac{du}{dr} \right) = \frac{dP}{dx}$$

Since LHS is a function of r and RHS a function of x , and the equality must be satisfied for all values of x and r , we can conclude that both LHS and RHS are equal to a constant

$$\frac{\mu}{r} \frac{d}{dr} \left(r \frac{du}{dr} \right) = \text{constant} = \frac{dP}{dx}$$



Laminar flow in pipes



Force balance:

$$\pi R^2 P - \pi R^2 (P + dP) - 2\pi R dx \tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

Integrating $\frac{\mu}{r} \frac{d}{dr} \left(r \frac{du}{dr} \right) = \frac{dP}{dx}$

gives $u(r) = \frac{r^2}{4\mu} \left(\frac{dP}{dx} \right) + C_1 \ln r + C_2$

and with boundary conditions

$$\frac{\partial u}{\partial r} = 0 \text{ at } r = 0$$

$$u = 0 \text{ at } r = R$$

we obtain the velocity profile

$$u(r) = -\frac{R^2}{4\mu} \left(\frac{dP}{dx} \right) \left(1 - \frac{r^2}{R^2} \right)$$

and average velocity is

$$V_{\text{avg}} = \frac{2}{R^2} \int_0^R u(r) r dr = -\frac{R^2}{8\mu} \left(\frac{dP}{dx} \right)$$

therefore

$$u(r) = 2V_{\text{avg}} \left(1 - \frac{r^2}{R^2} \right)$$

$$u_{\text{max}} = 2V_{\text{avg}}$$

Pressure drop and head loss

A quantity of interest in the analysis of pipe flow is the *pressure drop* ΔP since it is directly related to the power requirements of the fan or pump to maintain flow. We note that $dP/dx = \text{constant}$, and integrating from $x = x_1$ where the pressure is P_1 to $x = x_1 + L$ where the pressure is P_2 gives

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} \quad \text{Laminar flow:} \quad \Delta P = P_1 - P_2 = \frac{8\mu L V_{\text{avg}}}{R^2} = \frac{32\mu L V_{\text{avg}}}{D^2}$$

A pressure drop due to viscous effects represents an irreversible pressure loss, and it is called **pressure loss** ΔP_L .

$$\text{Pressure loss: } \Delta P_L = f \frac{L}{D} \frac{\rho V_{\text{avg}}^2}{2} \quad \text{head loss: } h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V_{\text{avg}}^2}{2g}$$

f : friction factor

head loss h_L represents the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe

These two equations are valid for all types (laminar and turbulent) of fully developed internal flows (circular, non-circular, smooth/rough surfaces); the friction factor will change from one scenario to other.

For circular laminar flow

$$f = \frac{64\mu}{\rho D V_{\text{avg}}} = \frac{64}{\text{Re}}$$

Power of pump and Poiseuille's law

Work required by the pump

$$\dot{W}_{\text{pump},L} = \dot{V}\Delta P_L = \dot{V}\rho gh_L = \dot{m}gh_L$$

\mathcal{V} : volume

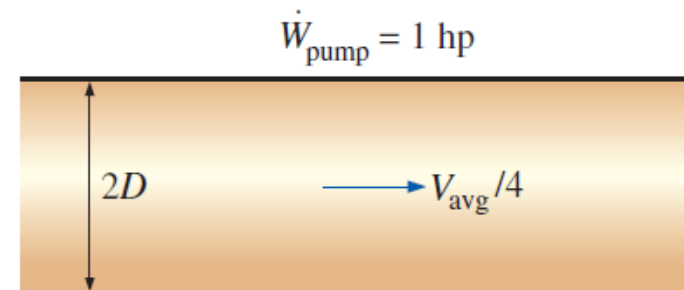
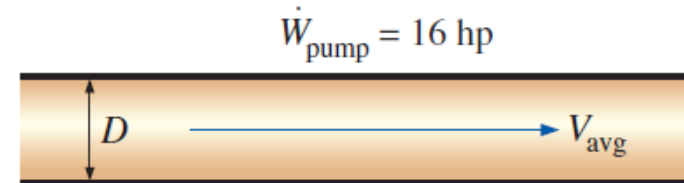
$\dot{\mathcal{V}}$: flow-rate, Volume/sec, rate of change of volume, dV/dt

Poiseuille's law

$$\dot{\mathcal{V}} = V_{\text{avg}}A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P\pi D^4}{128\mu L}$$

For a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the diameter of the pipe.

The pumping power requirement for a laminar flow piping system can be reduced by a factor of 16 by doubling the pipe diameter.



Energy equation

The pressure drop ΔP equals the pressure loss ΔP_L in the case of a horizontal pipe, but this is not the case for inclined pipes or pipes with variable cross-sectional area.

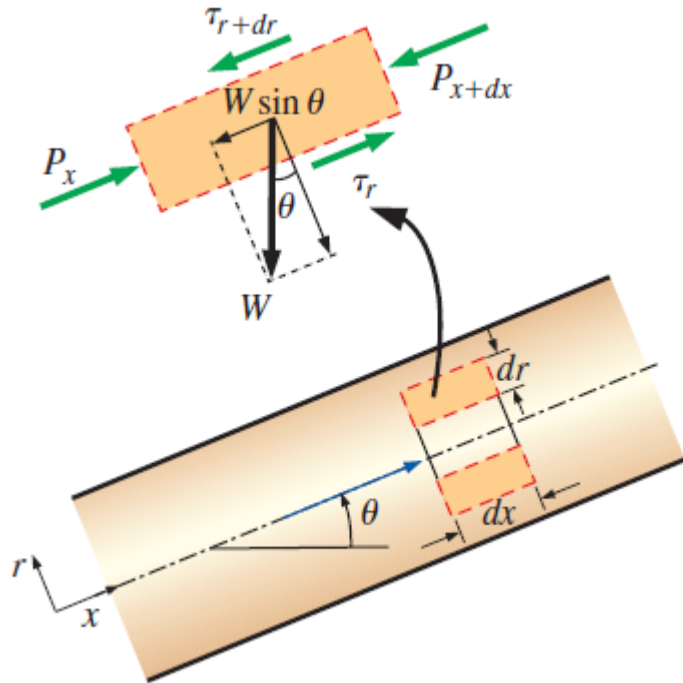
This can be demonstrated by writing the energy equation for steady, incompressible one-dimensional flow in terms of heads as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine}, e} + h_L$$

$$P_1 - P_2 = \rho(\alpha_2 V_2^2 - \alpha_1 V_1^2)/2 + \rho g[(z_2 - z_1) + h_{\text{turbine}, e} - h_{\text{pump}, u} + h_L]$$

Therefore, the pressure drop $\Delta P = P_1 - P_2$ and pressure loss $\Delta P_L = \rho g h_L$ for a given flow section are equivalent if (1) the flow section is horizontal so that there are no hydrostatic or gravity effects ($z_1 = z_2$), (2) the flow section does not involve any work devices such as a pump or a turbine since they change the fluid pressure ($h_{\text{pump}, u} = h_{\text{turbine}, e} = 0$), (3) the cross-sectional area of the flow section is constant and thus the average flow velocity is constant ($V_1 = V_2$), and (4) the velocity profiles at sections 1 and 2 are the same shape ($\alpha_1 = \alpha_2$).

Gravity



Free-body diagram of a ring-shaped differential fluid element of radius r , thickness dr , and length dx oriented coaxially with an inclined pipe in fully developed laminar flow.

$$W_x = W \sin \theta = \rho g V_{\text{element}} \sin \theta = \rho g (2\pi r dr dx) \sin \theta$$

$$(2\pi r dr P)_x - (2\pi r dr P)_{x+dx} + (2\pi r dx \tau)_r - (2\pi r dx \tau)_{r+dr} - \rho g (2\pi r dr dx) \sin \theta = 0$$

$$\frac{\mu}{r} \frac{d}{dr} \left(r \frac{du}{dr} \right) = \frac{dP}{dx} + \rho g \sin \theta$$

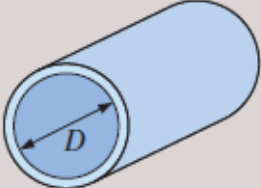
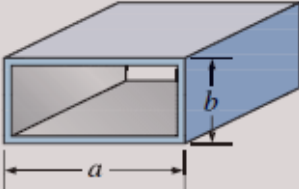
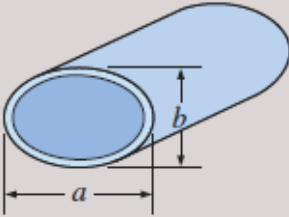
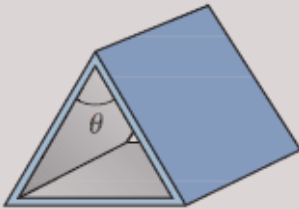
$$u(r) = -\frac{R^2}{4\mu} \left(\frac{dP}{dx} + \rho g \sin \theta \right) \left(1 - \frac{r^2}{R^2} \right)$$

$$V_{\text{avg}} = \frac{(\Delta P - \rho g L \sin \theta) D^2}{32\mu L} \quad \dot{V} = \frac{(\Delta P - \rho g L \sin \theta) \pi D^4}{128\mu L}$$

Alternatively, use the energy equation to achieve the same result

Friction factors

Friction factor for fully developed *laminar flow* in pipes of various cross sections ($D_h = 4A_c/\rho$ and $Re = V_{avg} D_h/\nu$)

Tube Geometry	a/b or θ°	Friction Factor f
<p>Circle</p> 	—	64.00/Re
<p>Rectangle</p> 	$\frac{a/b}{1}$ 1 2 3 4 6 8 ∞	56.92/Re 62.20/Re 68.36/Re 72.92/Re 78.80/Re 82.32/Re 96.00/Re
<p>Ellipse</p> 	$\frac{a/b}{1}$ 1 2 4 8 16	64.00/Re 67.28/Re 72.96/Re 76.60/Re 78.16/Re
<p>Isosceles triangle</p> 	$\frac{\theta}{10^\circ}$ 10° 30° 60° 90° 120°	50.80/Re 52.28/Re 53.32/Re 52.60/Re 50.96/Re

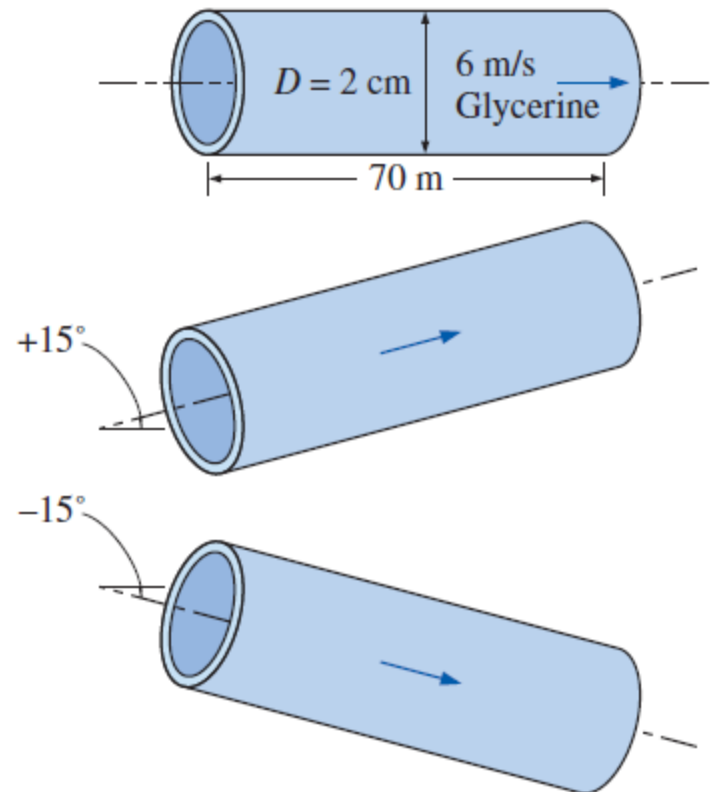
Pressure loss: $\Delta P_L = f \frac{L}{D} \frac{\rho V_{avg}^2}{2}$

head loss: $h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V_{avg}^2}{2g}$

Example

EXAMPLE 8–1 Laminar Flow in Horizontal and Inclined Pipes

Consider the fully developed flow of glycerin at 40°C through a 70-m-long, 4-cm-diameter, horizontal, circular pipe. If the flow velocity at the centerline is measured to be 6 m/s, determine the velocity profile and the pressure difference across this 70-m-long section of the pipe, and the useful pumping power required to maintain this flow. For the same useful pumping power input, determine the percent increase of the flow rate if the pipe is inclined 15° downward and the percent decrease if it is inclined 15° upward. The pump is located outside this pipe section.



Example (contd.)

Properties The density and dynamic viscosity of glycerin at 40°C are $\rho = 1252 \text{ kg/m}^3$ and $\mu = 0.3073 \text{ kg/m}\cdot\text{s}$, respectively.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is expressed as

$$u(r) = u_{\max} \left(1 - \frac{r^2}{R^2} \right)$$

Substituting, the velocity profile is determined to be

$$u(r) = (6 \text{ m/s}) \left(1 - \frac{r^2}{(0.02 \text{ m})^2} \right) = 6(1 - 2500r^2)$$

where u is in m/s and r is in m. The average velocity, the flow rate, and the Reynolds number are

$$V = V_{\text{avg}} = \frac{u_{\max}}{2} = \frac{6 \text{ m/s}}{2} = 3 \text{ m/s}$$

$$\dot{V} = V_{\text{avg}} A_c = V(\pi D^2/4) = (3 \text{ m/s})[\pi(0.04 \text{ m})^2/4] = 3.77 \times 10^{-3} \text{ m}^3/\text{s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(1252 \text{ kg/m}^3)(3 \text{ m/s})(0.04 \text{ m})}{0.3073 \text{ kg/m}\cdot\text{s}} = 488.9$$

which is less than 2300. Therefore, the flow is indeed laminar. Then the friction factor and the head loss become

$$f = \frac{64}{\text{Re}} = \frac{64}{488.9} = 0.1309$$

$$h_L = f \frac{LV^2}{D 2g} = 0.1309 \frac{(70 \text{ m})}{(0.04 \text{ m})} \frac{(3 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 105.1 \text{ m}$$

Example (contd.)

The energy balance for steady, incompressible one-dimensional flow is given by Eq. 8–28 as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L$$

For fully developed flow in a constant diameter pipe with no pumps or turbines, it reduces to

$$\Delta P = P_1 - P_2 = \rho g(z_2 - z_1 + h_L)$$

Then the pressure difference and the required useful pumping power for the horizontal case become

$$\begin{aligned}\Delta P &= \rho g(z_2 - z_1 + h_L) \\ &= (1252 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0 + 105.1 \text{ m}) \left(\frac{1 \text{ kPa}}{1000 \text{ kg/m} \cdot \text{s}^2} \right) \\ &= \mathbf{1291 \text{ kPa}} \\ \dot{W}_{\text{pump, u}} &= \dot{V} \Delta P = (3.77 \times 10^3 \text{ m}^3/\text{s})(1291 \text{ kPa}) \left(\frac{1 \text{ kW}}{\text{kPa} \cdot \text{m}^3/\text{s}} \right) = \mathbf{4.87 \text{ kW}}\end{aligned}$$

The elevation difference and the pressure difference for a pipe inclined upwards 15° is

$$\begin{aligned}\Delta z &= z_2 - z_1 = L \sin 15^\circ = (70 \text{ m}) \sin 15^\circ = 18.1 \text{ m} \\ \Delta P_{\text{upward}} &= (1252 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(18.1 \text{ m} + 105.1 \text{ m}) \left(\frac{1 \text{ kPa}}{1000 \text{ kg/m} \cdot \text{s}^2} \right) \\ &= \mathbf{1513 \text{ kPa}}\end{aligned}$$

Example (contd.)

Then the flow rate through the upward inclined pipe becomes

$$\dot{V}_{\text{upward}} = \frac{\dot{W}_{\text{pump}, u}}{\Delta P_{\text{upward}}} = \frac{4.87 \text{ kW}}{1513 \text{ kPa}} \left(\frac{1 \text{ kPa} \cdot \text{m}^3/\text{s}}{1 \text{ kW}} \right) = 3.22 \times 10^{-3} \text{ m}^3/\text{s}$$

which is a decrease of approximately 15%. Similarly when the pipe is inclined downwards, an increase in flow-rate will be observed

Discussion Note that the flow is driven by the combined effect of pumping power and gravity. As expected, gravity opposes uphill flow, enhances downhill flow, and has no effect on horizontal flow. Downhill flow can occur even in the absence of a pressure difference applied by a pump. For the case of $P_1 = P_2$ (i.e., no applied pressure difference), the pressure throughout the entire pipe would remain constant, and the fluid would flow through the pipe under the influence of gravity at a rate that depends on the angle of inclination, reaching its maximum value when the pipe is vertical. When solving pipe flow problems, it is always a good idea to calculate the Reynolds number to verify the flow regime—laminar or turbulent.