External Flow

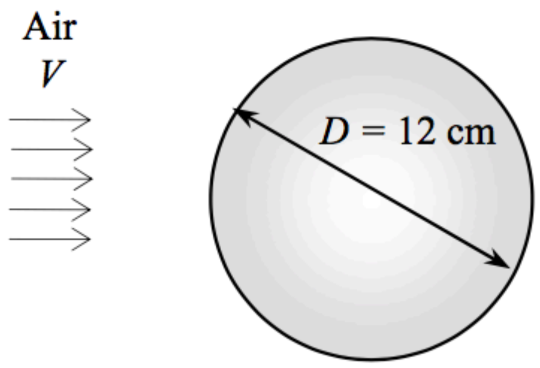
Ex. 1 During a high Reynolds number experiment, the total drag force acting on a spherical body of diameter $D = 12$ cm subjected to airflow at 1 atm and 5°C is measured to be 5.2 N. The pressure drag acting on the body is calculated by integrating the pressure distribution (measured by the use of pressure sensors throughout the surface) to be 4.9 N. Determine the friction drag coefficient of the sphere. *Answer:* 0.0115

Assumptions **1** The flow of air is steady and incompressible. **2** The surface of the sphere is smooth. **3** The flow over the sphere is turbulent (to be verified).

Properties The density and kinematic viscosity of air at 1 atm and 5°C are $\rho = 1.269$ kg/m^3 and $\nu = 1.382\times10^{-5}$ m^2/s. The drag coefficient of sphere in turbulent flow is $C_D = 0.2$, and its frontal area is $A = \pi D^2/4$ (Table 11-2).

Analysis The total drag force is the sum of the friction and pressure drag forces. Therefore,

$$F_{D,\text{friction}} = F_D - F_{D,\text{pressure}} = 5.2 - 4.9 = 0.3\ \text{N}$$



where $F_D = C_D A \dfrac{\rho V^2}{2}$ and $F_{D,\text{friction}} = C_{D,\text{friction}} A \dfrac{\rho V^2}{2}$

Taking the ratio of the two relations above gives

$$C_{D,\text{friction}} = \frac{F_{D,\text{friction}}}{F_D} C_D = \frac{0.3\ \text{N}}{5.2\ \text{N}}(0.2) = \mathbf{0.0115}$$

Now we need to verify that the flow is turbulent. This is done by calculating the flow velocity from the drag force relation, and then the Reynolds number:

$$F_D = C_D A \frac{\rho V^2}{2} \quad \rightarrow \quad V = \sqrt{\frac{2F_D}{\rho C_D A}} = \sqrt{\frac{2(5.2\ \text{N})}{(1.269\ \text{kg/m}^3)(0.2)[\pi(0.12\ \text{m})^2/4]}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)} = 60.2\ \text{m/s}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(60.2\ \text{m/s})(0.12\ \text{m})}{1.382\times10^{-5}\ \text{m}^2/s} = 5.23\times10^5$$

which is greater than 2×10^5. Therefore, the flow is turbulent as assumed.

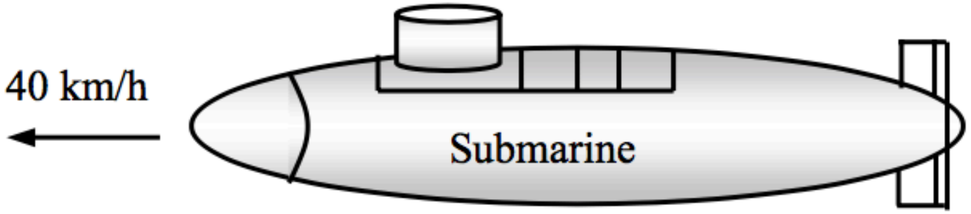
Discussion Note that knowing the flow regime is important in the solution of this problem since the total drag coefficient for a sphere is 0.5 in laminar flow and 0.2 in turbulent flow.

Ex. 2 A submarine can be treated as an ellipsoid with a diameter of 5 m and a length of 25 m. Determine the power required for this submarine to cruise horizontally and steadily at 40 km/h in seawater whose density is 1025 kg/m^3. Also determine the power required to tow this submarine in air whose density is 1.30 kg/m^3. Assume the flow is turbulent in both cases.

Solution A submarine is treated as an ellipsoid at a specified length and diameter. The powers required for this submarine to cruise horizontally in seawater and to tow it in air are to be determined.

Assumptions **1** The submarine can be treated as an ellipsoid. **2** The flow is turbulent. **3** The drag of the towing rope is negligible. **4** The motion of submarine is steady and horizontal.

Properties The drag coefficient for an ellipsoid with $L/D = 25/5 = 5$ is $C_D = 0.1$ in turbulent flow (Table 11-2). The density of sea water is given to be 1025 kg/m^3. The density of air is given to be 1.30 kg/m^3.



Analysis Noting that 1 m/s = 3.6 km/h, the velocity of the submarine is equivalent to V = 40/3.6 = 11.11 m/s. The frontal area of an ellipsoid is $A = \pi D^2/4$. Then the drag force acting on the submarine becomes

In water: $$F_D = C_D A \frac{\rho V^2}{2} = (0.1)[\pi(5\text{ m})^2/4]\frac{(1025\text{ kg/m}^3)(11.11\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 124.2\text{ kN}$$

In air: $$F_D = C_D A \frac{\rho V^2}{2} = (0.1)[\pi(5\text{ m})^2/4]\frac{(1.30\text{ kg/m}^3)(11.11\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 0.1575\text{ kN}$$

Noting that power is force times velocity, the power needed to overcome this drag force is

In water: $$\dot{W}_{\text{drag}} = F_D V = (124.2\text{ kN})(11.11\text{ m/s})\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{1380\text{ kW}}$$

In air: $$\dot{W}_{\text{drag}} = F_D V = (0.1575\text{ kN})(11.11\text{ m/s})\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{1.75\text{ kW}}$$

Therefore, the power required for this submarine to cruise horizontally in seawater is 1380 kW and the power required to tow this submarine in air at the same velocity is 1.75 kW.

Discussion Note that the power required to move the submarine in water is about 800 times the power required to move it in air. This is due to the higher density of water compared to air (sea water is about 800 times denser than air).

Ex. 3 Bill gets a job delivering pizzas. The pizza company makes him mount a sign on the roof of his car. The frontal area of the sign is $A = 0.0569\ \text{m}^2$, and he estimates the drag coefficient to be $C_D = 0.94$ at nearly all air speeds. Estimate how much additional money it costs Bill per year in fuel to drive with the sign on his roof compared to without the sign. Use the following additional information: He drives about 16,000 km per year at an average speed of 72 km/h. The overall car efficiency is 0.332, $\rho_{\text{fuel}} = 804\ \text{kg/m}^3$, and the heating value of the fuel is 45,700 kJ/kg. The fuel costs \$0.925 per liter. Use standard air properties. Be careful with unit conversions.

Solution We are to estimate how much money is wasted by driving with a pizza sign on a car roof.

Properties $\rho_{\text{fuel}} = 804$ kg/m^3, $\text{HV}_{\text{fuel}} = 45{,}700$ kJ/kg, $\rho_{\text{air}} = 1.184$ kg/m^3, $\mu_{\text{air}} = 1.849 \times 10^{-5}$ kg/m·s

Analysis First some conversions: $V = 72$ km/h $= 20$ m/s and the total distance traveled in one year $= L = 16{,}000$ km $= 1.60 \times 10^7$ m. The additional drag force due to the sign is

$$F_D = \tfrac{1}{2}\rho_{\text{air}} V^2 C_D A$$

where A is the frontal area. The work required to overcome this additional drag is force times distance. So, letting L be the total distance driven in a year,

$$\text{Work}_{\text{drag}} = F_D L = \tfrac{1}{2}\rho_{\text{air}} V^2 C_D A L$$

The energy required to perform this work is much greater than this due to overall efficiency of the car engine, transmission, etc. Thus,

$$E_{\text{required}} = \frac{\text{Work}_{\text{drag}}}{\eta_{\text{overall}}} = \frac{\tfrac{1}{2}\rho_{\text{air}} V^2 C_D A L}{\eta_{\text{overall}}}$$

But the required energy is also equal to the heating value of the fuel HV times the mass of fuel required. In terms of required fuel *volume*, volume = mass/density. Thus,

$$\boxed{V_{\text{fuel required}} = \frac{m_{\text{fuel required}}}{\rho_{\text{fuel}}} = \frac{E_{\text{required}}/\text{HV}}{\rho_{\text{fuel}}} = \frac{\tfrac{1}{2}\rho_{\text{air}} V^2 C_D A L}{\rho_{\text{fuel}}\eta_{\text{overall}}\text{HV}}}$$

The above is our answer in variable form. Finally, we plug in the given values and properties to obtain the numerical answer,

$$V_{\text{fuel required}} = \frac{0.5(1.184\ \text{kg/m}^3)(20\ \text{m/s})^2(0.94)(0.0569\ \text{m}^2)(1.60\times10^7\ \text{m})}{(804\ \text{kg/m}^3)(0.332)(45{,}700\ \text{kJ/kg})}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

which yields $V_{\text{fuel required}} = 0.01661$ m^3, which is equivalent to 16.61 liters per year. At \$0.925 per liter, the total cost is about \$15.36 per year, or rounding to two significant digits, **the total cost is about \$15 per year**.

Ex. 4 The drag coefficient of a vehicle increases when its windows are rolled down or its sunroof is opened. A sports car has a frontal area of 1.7 m^2 and a drag coefficient of 0.32 when the windows and sunroof are closed. The drag coefficient increases to 0.41 when the sunroof is open. Determine the additional power consumption of the car when the sunroof is opened at (*a*) 55 km/h and (*b*) 110 km/h. Take the density of air to be 1.2 kg/m^3.

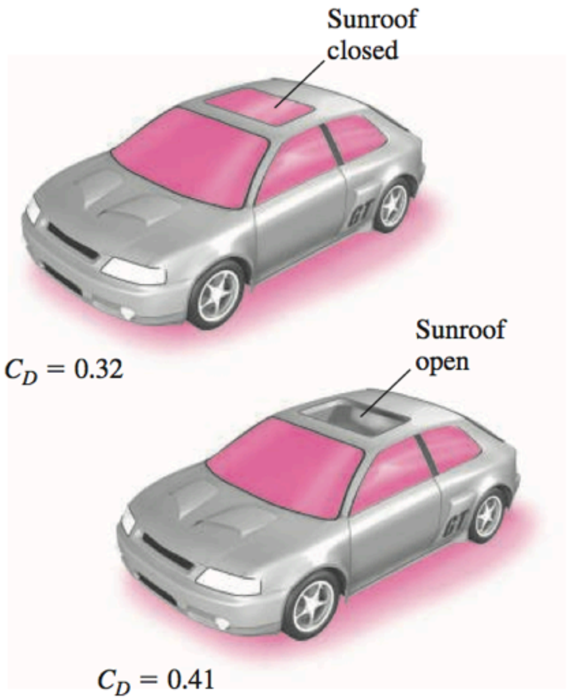
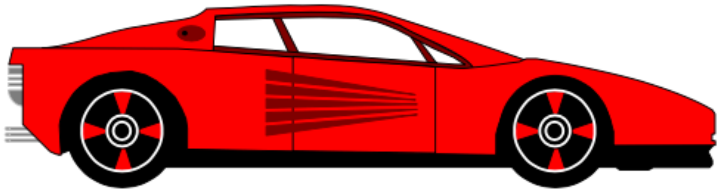


FIGURE P11–41

Solution The drag coefficient of a sports car increases when the sunroof is open, and it requires more power to overcome aerodynamic drag. The additional power consumption of the car when the sunroof is opened is to be determined at two different velocities.

Assumptions **1** The car moves steadily at a constant velocity on a straight path. **2** The effect of velocity on the drag coefficient is negligible.



Properties The density of air is given to be 1.2 kg/m^3. The drag coefficient of the car is given to be $C_D = 0.32$ when the sunroof is closed, and $C_D = 0.41$ when it is open.

Analysis (*a*) Noting that 1 km/h = 3.6 m/s and that power is force times velocity, the drag force acting on the car and the power needed to overcome it at 55 km/h are:

$$\textit{Closed sunroof:}\quad F_{D1} = 0.32(1.7\text{ m}^2)\frac{(1.2\text{ kg/m}^3)(55/3.6\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 76.2\text{ N}$$

$$\dot{W}_{\text{drag1}} = F_{D1}V = (76.2\text{ N})(55/3.6\text{ m/s})\left(\frac{1\text{ kW}}{1000\text{ N}\cdot\text{m/s}}\right) = 1.16\text{ kW}$$

$$\textit{Open sunroof:}\quad F_{D2} = 0.41(1.7\text{ m}^2)\frac{(1.2\text{ kg/m}^3)(55/3.6\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 97.6\text{ N}$$

$$\dot{W}_{\text{drag2}} = F_{D2}V = (97.6\text{ N})(55/3.6\text{ m/s})\left(\frac{1\text{ kW}}{1000\text{ N}\cdot\text{m/s}}\right) = 1.49\text{ kW}$$

Therefore, the additional power required for this car when the sunroof is open is

$$\dot{W}_{\text{extra}} = \dot{W}_{\text{drag2}} - \dot{W}_{\text{drag2}} = 1.49 - 1.16 = \mathbf{0.33\text{ kW}}\quad\text{(at 55 km/h)}$$

(*b*) We now repeat the calculations for 110 km/h:

Closed sunroof: $F_{D1} = 0.32(1.7\text{ m}^2)\dfrac{(1.2\text{ kg/m}^3)(110/3.6\text{ m/s})^2}{2}\left(\dfrac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 304.7\text{ N}$

$$\dot{W}_{\text{drag1}} = F_{D1}V = (304.7\text{ N})(110/3.6\text{ m/s})\left(\frac{1\text{ kW}}{1000\text{ N}\cdot\text{m/s}}\right) = 9.31\text{ kW}$$

Open sunroof: $F_{D2} = 0.41(1.7\text{ m}^2)\dfrac{(1.2\text{ kg/m}^3)(110/3.6\text{ m/s})^2}{2}\left(\dfrac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 390.5\text{ N}$

$$\dot{W}_{\text{drag2}} = F_{D2}V = (390.5\text{ N})(110/3.6\text{ m/s})\left(\frac{1\text{ kW}}{1000\text{ N}\cdot\text{m/s}}\right) = 11.93\text{ kW}$$

Therefore, the additional power required for this car when the sunroof is open is

$$\dot{W}_{\text{extra}} = \dot{W}_{\text{drag2}} - \dot{W}_{\text{drag2}} = 11.93 - 9.31 = \mathbf{2.62\text{ kW}} \quad \text{(at 110 km/h)}$$

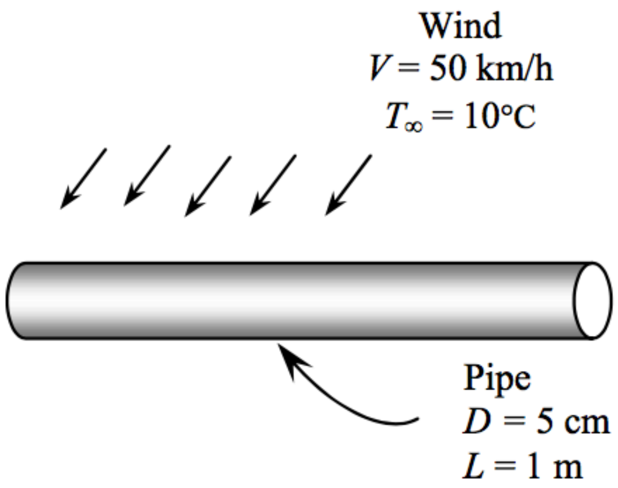
Discussion Note that the additional drag caused by open sunroof is 0.33 kW at 55 km/h, and 2.62 kW at 110 km/h, which is an increase of 8 folds when the velocity is doubled. This is expected since the power consumption to overcome drag is proportional to the cube of velocity.

Ex. 5 A long 5-cm-diameter steam pipe passes through some area open to the wind. Determine the drag force acting on the pipe per unit of its length when the air is at 1 atm and 10°C and the wind is blowing across the pipe at a speed of 50 km/h.

Solution A pipe is exposed to high winds. The drag force exerted on the pipe by the winds is to be determined.

Assumptions **1** The outer surface of the pipe is smooth so that Fig. 11-34 can be used to determine the drag coefficient. **2** Air flow in the wind is steady and incompressible. **3** The turbulence in the wind is not considered. **4**The direction of wind is normal to the pipe.

Properties The density and kinematic viscosity of air at 1 atm and 10°C are $\rho = 1.246\ \text{kg/m}^3$ and $\nu = 1.426\times10^{-5}\ \text{m}^2/\text{s}$.



Analysis Noting that $D = 0.05$ m and 1 m/s = 3.6 km/h, the Reynolds number for flow over the pipe is

$$\text{Re} = \frac{VD}{\nu} = \frac{(50/3.6\ \text{m/s})(0.05\ \text{m})}{1.426\times10^{-5}\ \text{m}^2/\text{s}} = 0.4870\times10^5$$

The drag coefficient corresponding to this value is, from Fig. 11-34, C_D = 1.0. Also, the frontal area for flow past a cylinder is $A = LD$. Then the drag force becomes

See lecture notes for C_D curve

$$F_D = C_D A\frac{\rho V^2}{2} = 1.0(1\times0.05\ \text{m}^2)\frac{(1.246\ \text{kg/m}^3)(50/3.6\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{6.01\ N}\ \text{(per m length)}$$

Discussion Note that the drag force acting on a unit length of the pipe is equivalent to the weight of 0.6 kg mass. The total drag force acting on the entire pipe can be obtained by multiplying the value obtained by the pipe length. It should be kept in mind that wind turbulence may reduce the drag coefficients by inducing turbulence and delaying flow separation.

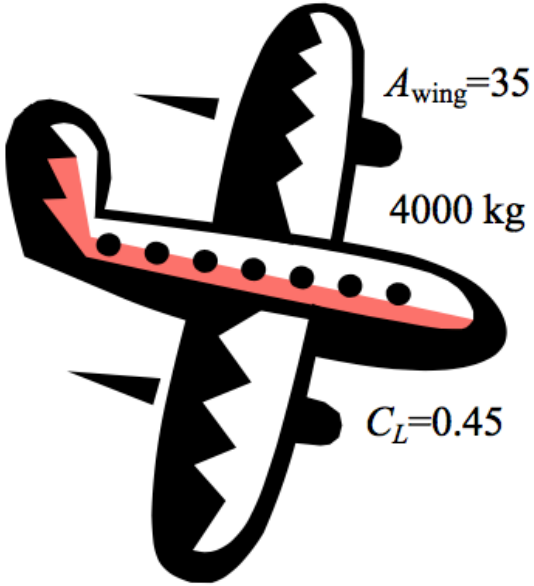
Ex. 6 A small aircraft has a wing area of 35 m^2, a lift coefficient of 0.45 at takeoff settings, and a total mass of 4000 kg. Determine (*a*) the takeoff speed of this aircraft at sea level at standard atmospheric conditions, (*b*) the wing loading, and (*c*) the required power to maintain a constant cruising speed of 300 km/h for a cruising drag coefficient of 0.035.

Solution The wing area, lift coefficient at takeoff settings, the cruising drag coefficient, and total mass of a small aircraft are given. The takeoff speed, the wing loading, and the required power to maintain a constant cruising speed are to be determined.

Assumptions **1** Standard atmospheric conditions exist. **2** The drag and lift produced by parts of the plane other than the wings are not considered.

Properties The density of standard air at sea level is ρ= 1.225 kg/m^3.

Analysis (*a*) An aircraft will takeoff when lift equals the total weight. Therefore,



$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho V^2 A \quad \rightarrow \quad V = \sqrt{\frac{2W}{\rho C_L A}}$$

Substituting, the takeoff speed is determined to be

$$V_{\text{takeoff}} = \sqrt{\frac{2mg}{\rho C_{L,\text{takeoff}} A}} = \sqrt{\frac{2(4000\text{ kg})(9.81\text{ m/s}^2)}{(1.225\text{ kg/m}^3)(0.45)(35\text{ m}^2)}}$$
$$= 63.8\text{ m/s} = \mathbf{230\ km/h}$$

(*b*) Wing loading is the average lift per unit planform area, which is equivalent to the ratio of the lift to the planform area of the wings since the lift generated during steady cruising is equal to the weight of the aircraft. Therefore,

$$F_{\text{loading}} = \frac{F_L}{A} = \frac{W}{A} = \frac{(4000\text{ kg})(9.81\text{ m/s}^2)}{35\text{ m}^2} = \mathbf{1121\ N/m^2}$$

(*c*) When the aircraft is cruising steadily at a constant altitude, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force, which is

$$F_D = C_D A \frac{\rho V^2}{2} = (0.035)(35\text{ m}^2)\frac{(1.225\text{ kg/m}^3)(300/3.6\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 5.211\text{ kN}$$

Noting that power is force times velocity, the propulsive power required to overcome this drag is equal to the thrust times the cruising velocity,

$$\text{Power} = \text{Thrust} \times \text{Velocity} = F_D V = (5.211\text{ kN})(300/3.6\text{ m/s})\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{434\ kW}$$

Therefore, the engines must supply 434 kW of propulsive power to overcome the drag during cruising.

Discussion The power determined above is the power to overcome the drag that acts on the wings only, and does not include the drag that acts on the remaining parts of the aircraft (the fuselage, the tail, etc). Therefore, the total power required during cruising will be greater. The required rate of energy input can be determined by dividing the propulsive power by the propulsive efficiency.

Ex. 7 Consider an aircraft that takes off at 260 km/h when it is fully loaded. If the weight of the aircraft is increased by 10 percent as a result of overloading, determine the speed at which the overloaded aircraft will take off. *Answer: 273 km/h*

Solution The takeoff speed of an aircraft when it is fully loaded is given. The required takeoff speed when the weight of the aircraft is increased by 10% as a result of overloading is to be determined.

Assumptions **1** The atmospheric conditions (and thus the properties of air) remain the same. **2** The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane remains the same.

Analysis An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho V^2 A \quad \rightarrow \quad V = \sqrt{\frac{2W}{\rho C_L A}}$$

We note that the takeoff velocity is proportional to the square root of the weight of the aircraft. When the density, lift coefficient, and area remain constant, the ratio of the velocities of the overloaded and fully loaded aircraft becomes

$$\frac{V_2}{V_1} = \frac{\sqrt{2W_2 / \rho C_L A}}{\sqrt{2W_1 / \rho C_L A}} = \frac{\sqrt{W_2}}{\sqrt{W_1}} \quad \rightarrow \quad V_2 = V_1 \sqrt{\frac{W_2}{W_1}}$$

Substituting, the takeoff velocity of the overloaded aircraft is determined to be

$$V_2 = V_1 \sqrt{\frac{1.2 W_1}{W_1}} = (260 \text{ km/h})\sqrt{1.1} = \mathbf{273\ km/h}$$

Discussion A similar analysis can be performed for the effect of the variations in density, lift coefficient, and planform area on the takeoff velocity.

Ex. 7 A jumbo jet airplane has a mass of about 400,000 kg when fully loaded with over 400 passengers and takes off at a speed of 250 km/h. Determine the takeoff speed when the airplane has 100 empty seats. Assume each passenger with luggage is 140 kg and the wing and flap settings are maintained the same.

Solution The takeoff speed of an aircraft when it is fully loaded is given. The required takeoff speed when the aircraft has 100 empty seats is to be determined.

Assumptions **1** The atmospheric conditions (and thus the properties of air) remain the same. **2** The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane remains the same. **3** A passenger with luggage has an average mass of 140 kg.

Analysis An aircraft will takeoff when lift equals the total weight. Therefore,

$$W = F_L \quad \rightarrow \quad W = \tfrac{1}{2} C_L \rho V^2 A \quad \rightarrow \quad V = \sqrt{\frac{2W}{\rho C_L A}}$$

We note that the takeoff velocity is proportional to the square root of the weight of the aircraft. When the density, lift coefficient, and wing area remain constant, the ratio of the velocities of the under-loaded and fully loaded aircraft becomes

$$\frac{V_2}{V_1} = \frac{\sqrt{2W_2 / \rho C_L A}}{\sqrt{2W_1 / \rho C_L A}} = \frac{\sqrt{W_2}}{\sqrt{W_1}} = \frac{\sqrt{m_2 g}}{\sqrt{m_1 g}} = \frac{\sqrt{m_2}}{\sqrt{m_1}} \quad \rightarrow \quad V_2 = V_1 \sqrt{\frac{m_2}{m_1}}$$

where $m_2 = m_1 - m_{\text{unused capacity}} = 400{,}000\text{ kg} - (140\text{ kg/passanger}) \times (100\text{ passengers}) = 386{,}000\text{ kg}$

Substituting, the takeoff velocity of the overloaded aircraft is determined to be

$$V_2 = V_1 \sqrt{\frac{m_2}{m_1}} = (250\text{ km/h})\sqrt{\frac{386{,}000}{400{,}000}} = \mathbf{246\ km/h}$$

Discussion Note that the effect of empty seats on the takeoff velocity of the aircraft is small. This is because the most weight of the aircraft is due to its empty weight (the aircraft itself rather than the passengers and their luggage.)