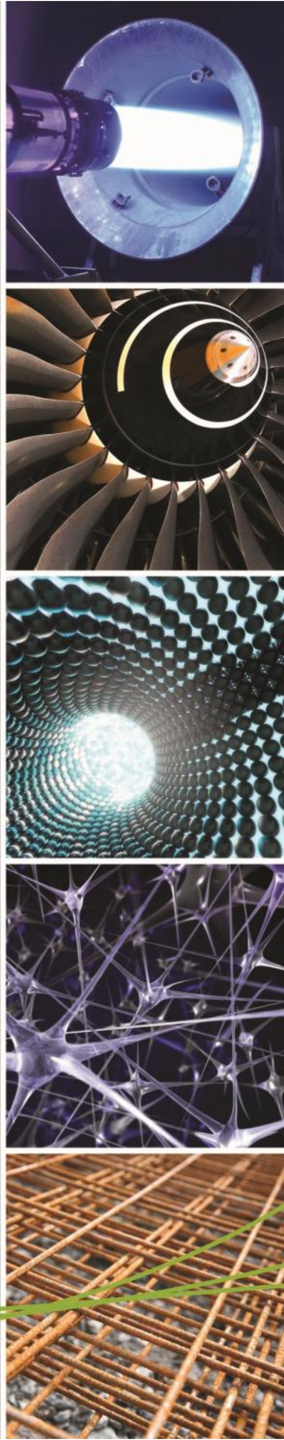
Advanced Structural Analysis
EGF316

Thin and Thick Cylinders



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Lecture Content

- Thin cylinders under pressure
  - Hoop stress
  - Longitudinal stress
  - Strains and changes in dimension

- Thick cylinders under pressure
  - Lamé’s theory
  - Longitudinal stress
  - Maximum shear stress
  - Strains and changes in dimension

Some typical examples

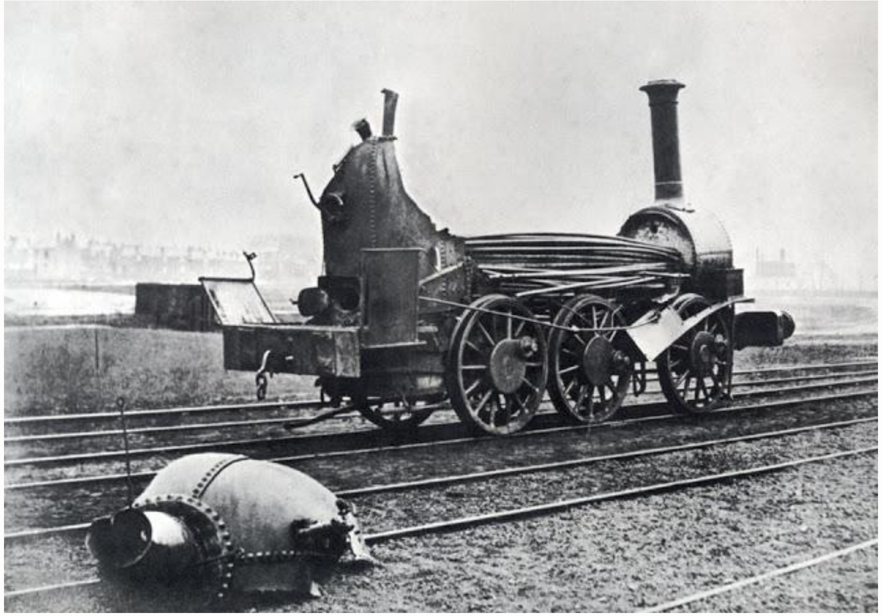
Credits: www.davi.com

Credits: www.istockphoto.com

Credits: www.weholite.com

Some failures in history

Credits: www.pveng.com



Credits: www.wikipedia.org

Stresses in cylinders

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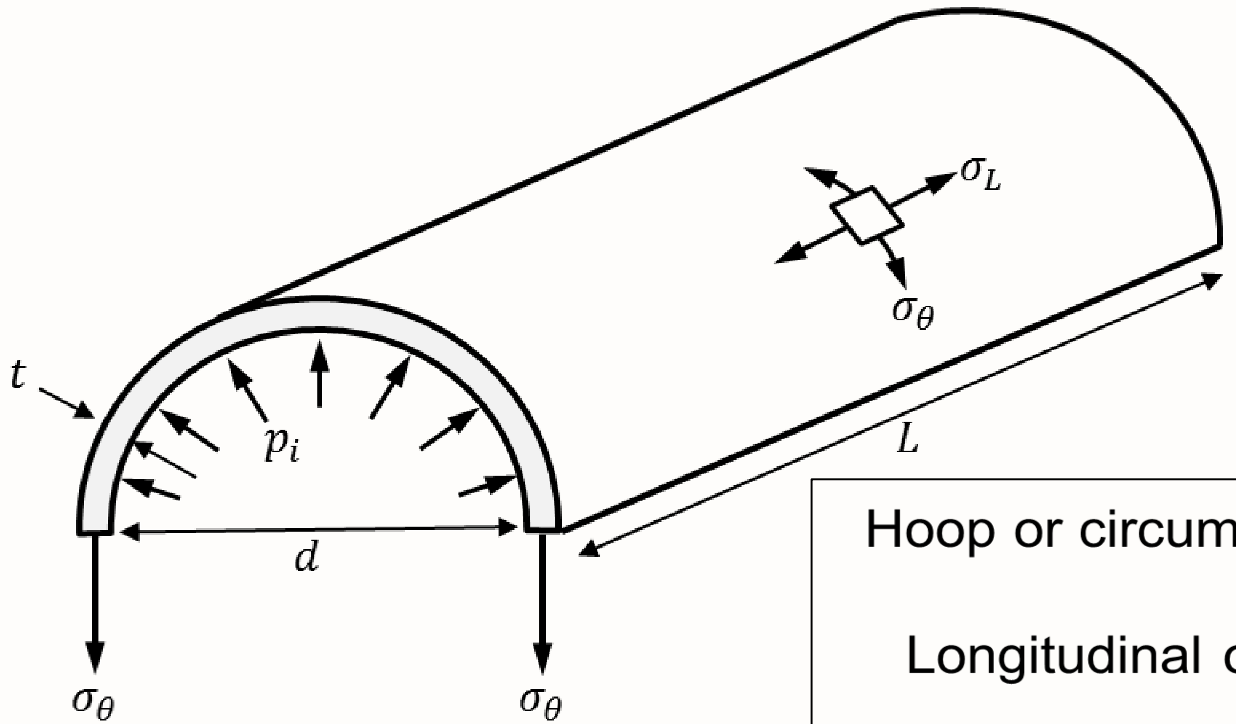
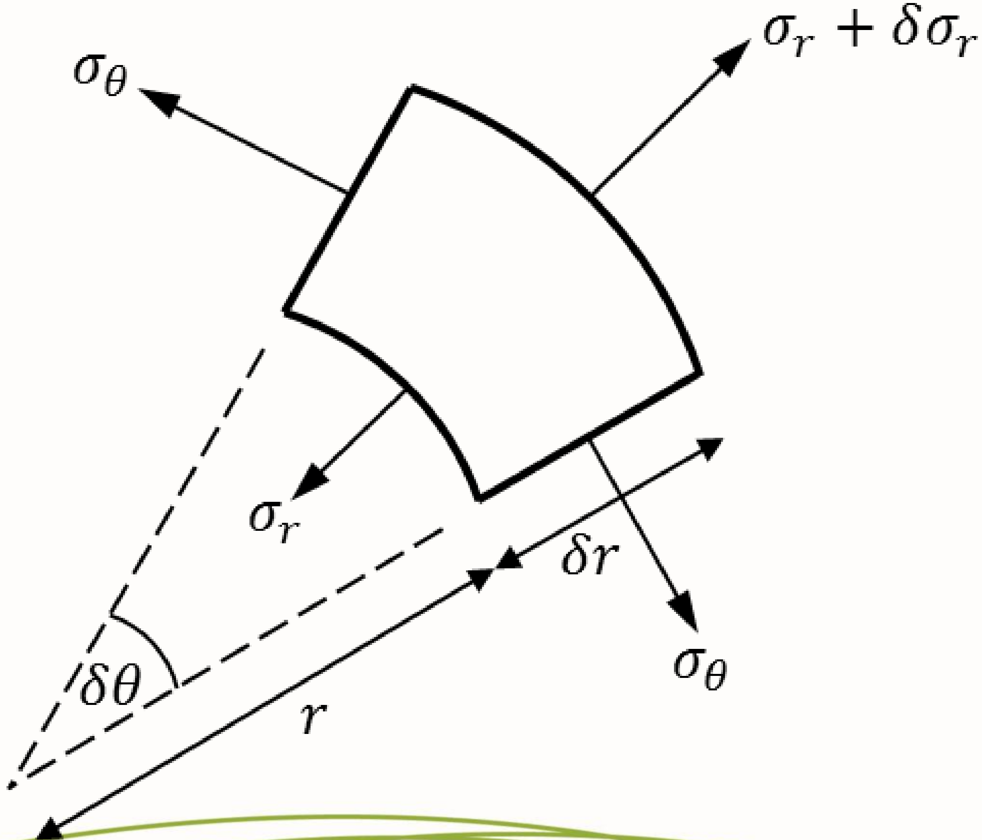


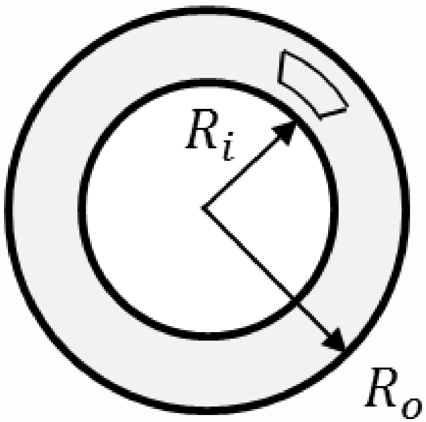
Hoop or circumferential stress, σ_θ

Longitudinal or axial stress, σ_L

Radial stress, σ_r

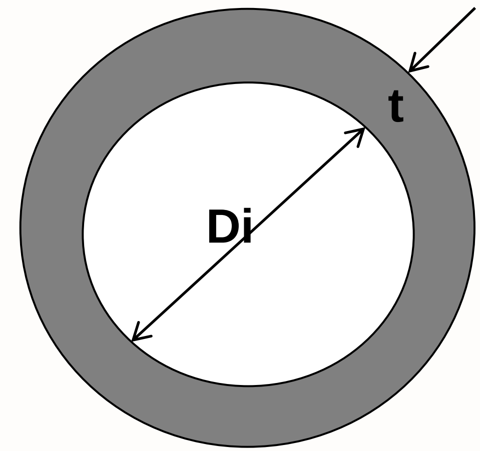
Stresses in cylinders
– cylindrical coordinates

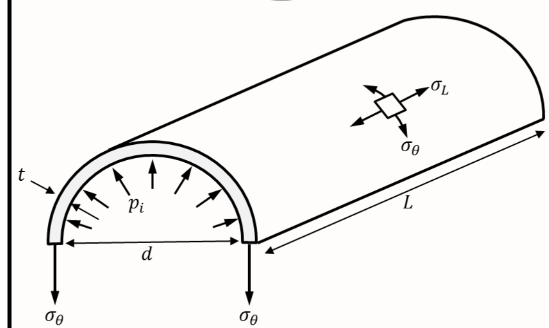




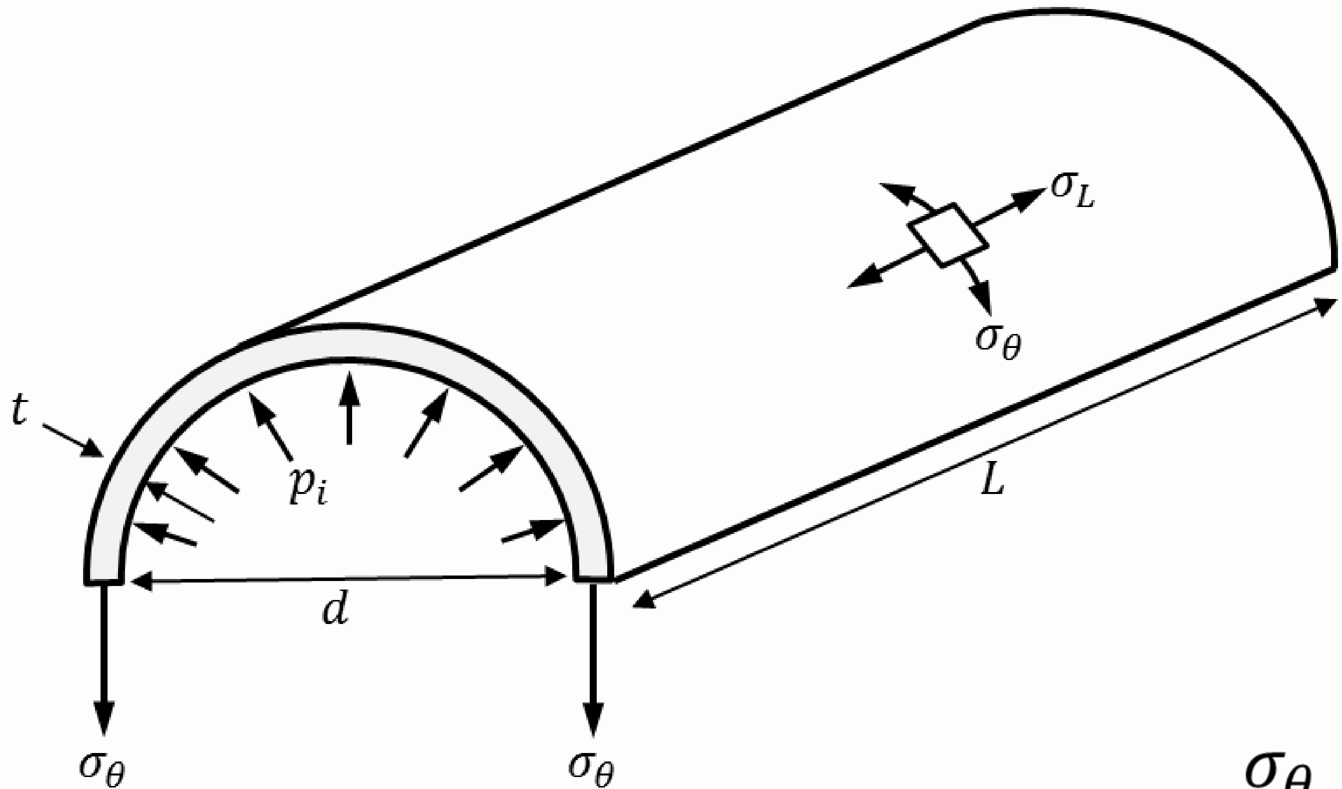
Thin Vs Thick Cylinders

Thin Cylinders	Thick Cylinders
$\frac{D_i}{t} > 20$	$\frac{D_i}{t} < 20$
$\sigma_L, \sigma_r, \sigma_\theta$ all are constant	σ_L is constant
	σ_r, σ_θ vary with radius
σ_r is negligible	



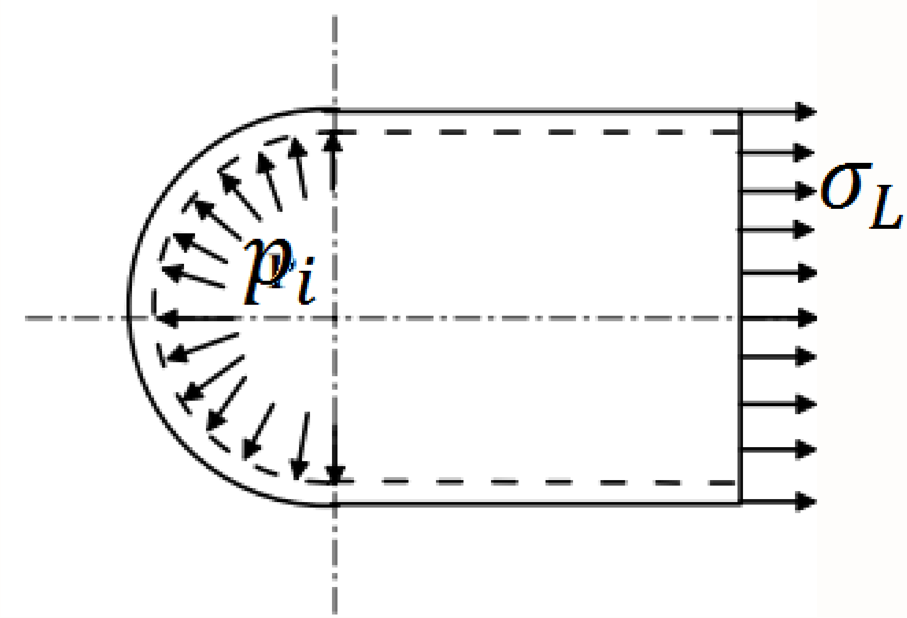


Thin Cylinder – hoop stresses



$$\sigma_\theta = \frac{p_i d}{2t}$$

Thin Cylinder - longitudinal stress



$$\sigma_L = \frac{p_i d}{4t}$$

Changes in Dimensions

Change in length:

$$\delta L = \frac{p_i d}{4tE}(1 - 2v)L$$

Change in diameter:

$$\delta d = \frac{p_i d^2}{4tE}(2 - v)$$

Change in volume:

$$\delta V = \frac{p_i d}{4tE}(5 - 4v)V$$

Example 1:

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A thin cylinder 60mm internal diameter, 225mm long with walls 2.7mm thick is subjected to an internal pressure of $6MN/m^2$. Calculate:

- The hoop stress
- The longitudinal stress
- The change in length
- The change in diameter

You may assume that $E = 200GN/m^2$ and $v = 0.3$

Example 2:

A 1m long thin cylinder has an internal diameter of 200mm with a wall thickness of 3mm. If it found to undergo a change to its internal volume of $9 \times 10^{-6} m^3$ when subject to an internal pressure p. You may assume that $E = 210 GN/m^2$ and $v = 0.3$.

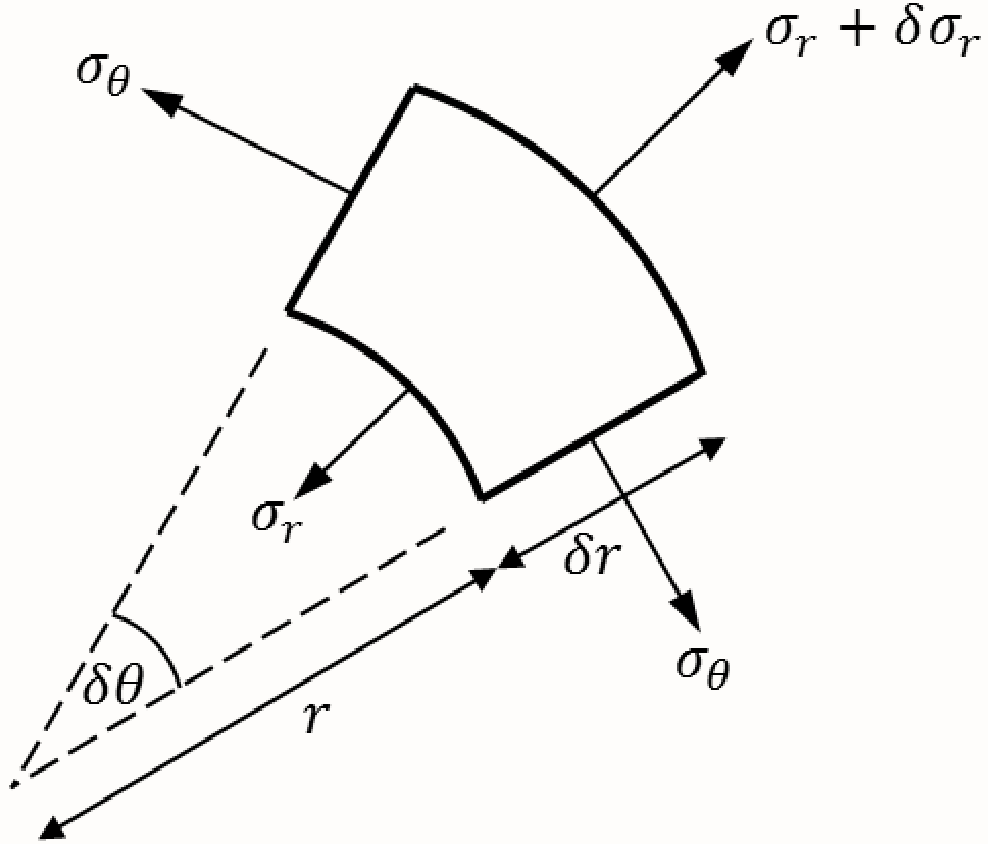
Calculate the hoop and longitudinal stresses.

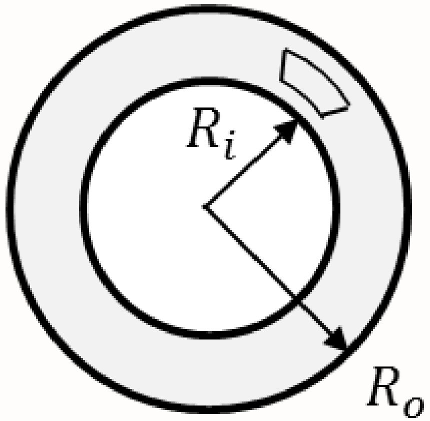
Thick Cylinders - Lamé's Theory

For the following analysis, we will assume:

- The material is *isotropic* and *homogeneous*
- Longitudinal stresses in the cylinder wall are constant
- The thick walled cylinder can be considered as a large number of thin cylinders, thickness δr
- The cylinder is subjected to uniform internal or external pressure (or both)

Lamé's Theory





Lamé's Theory

$$\sigma_r = A - \frac{B}{r^2}$$

$$\sigma_\theta = A + \frac{B}{r^2}$$

$$\sigma_L = \frac{end\ load}{cross\ sectional\ area}$$

Internal Pressure Only

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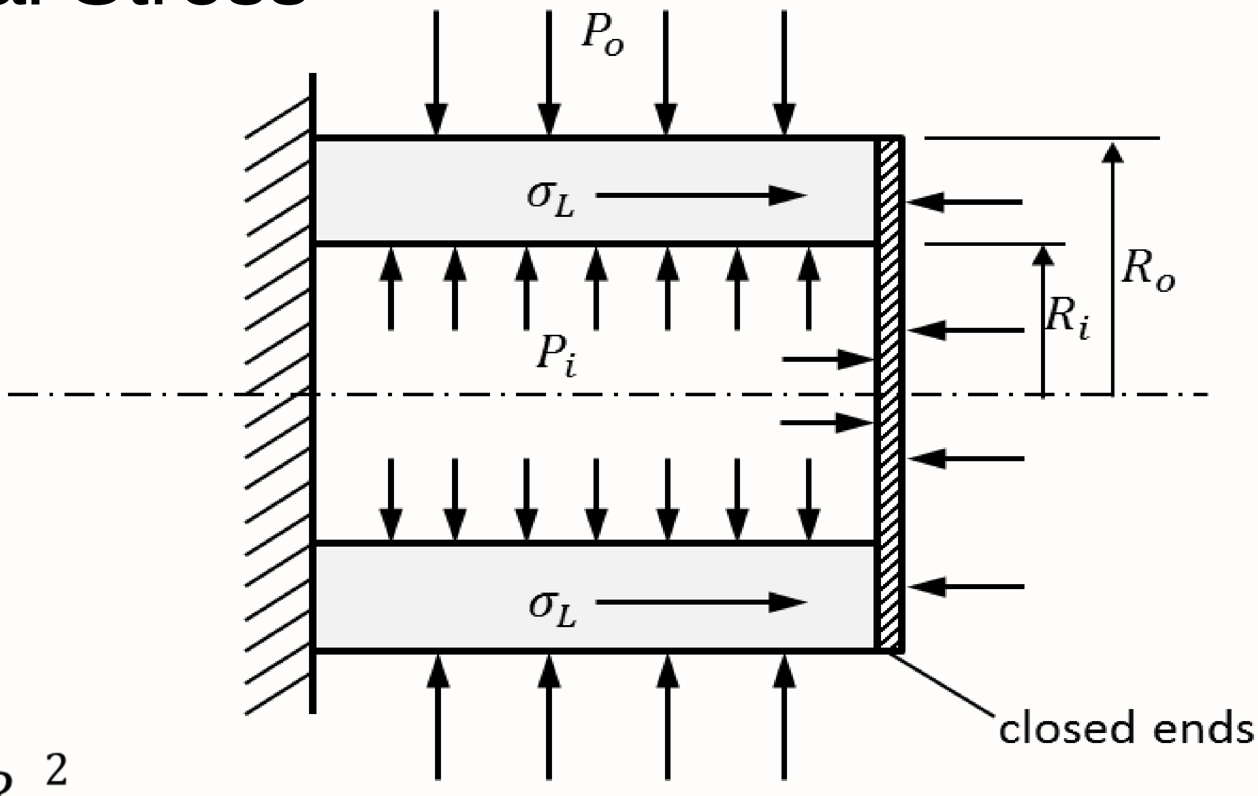
$$\sigma_r = \frac{p_i {R_i}^2}{({R_o}^2 - {R_i}^2)} \left(\frac{r^2 - {R_o}^2}{r^2} \right)$$

$$\sigma_\theta = A + \frac{B}{r^2} = \frac{p_i {R_i}^2}{({R_o}^2 - {R_i}^2)} \left(\frac{r^2 + {R_o}^2}{r^2} \right)$$

Longitudinal Stress



$$\sigma_L = \frac{P_i R_i^{\ 2} - P_o R_o^{\ 2}}{\left(R_o^{\ 2} - R_i^{\ 2}\right)} = \text{constant}$$

Maximum Shear Stress

$$\tau_{max} = \frac{\sigma_\theta - \sigma_r}{2}$$

$$\tau_{max} = \frac{B}{r^2}$$

Strains

$$\varepsilon_r = \frac{\sigma_r - v(\sigma_\theta + \sigma_L)}{E}$$

$$\varepsilon_\theta = \frac{\sigma_\theta - v(\sigma_r + \sigma_L)}{E}$$

$$\varepsilon_L = \frac{\sigma_L - v(\sigma_r + \sigma_\theta)}{E}$$

$$\text{diametral strain} = \varepsilon_\theta = \varepsilon_d = \varepsilon_r$$

Changes in Dimensions

Change in diameter:

$$\delta D = \frac{2r}{E}(\sigma_\theta - v\sigma_r - v\sigma_L)$$

Change in length:

$$\delta L = \frac{L}{E}(\sigma_L - v\sigma_r - v\sigma_\theta)$$

Change in volume:

$$\delta V = V(\varepsilon_L + 2\varepsilon_\theta)$$

Example 3:

A tube has 100mm inner diameter and the walls are 20mm thick. It is subjected to an internal pressure of 20MPa.

Calculate the maximum error in hoop stress at the surface if a thin tube criterion based on the inner diameter is used.

Example 4:

A thick steel pressure vessel, 200mm inside diameter and 300mm outside diameter, is subjected to an internal pressure of 30MPa and an external pressure of 10MPa.

Calculate the maximum hoop stress and the longitudinal stress in the material.

Calculate the change in internal volume per unit length of the pressure vessel under the conditions described above.

Assume $E = 200GPa$ and $v = 0.3$.

Class Test in next session – 30%

Content – Course material so far

Type – Open book

Duration – 1 hour