Numerical modelling for Fluid-structure interaction

EGEM07 – Fluid-structure interaction

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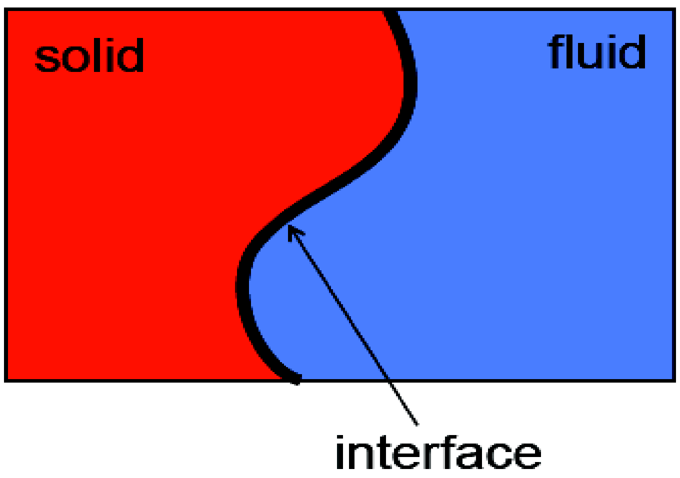
Introduction to FSI



- Interactions of fluid and solid
- A multi-physics phenomenon
- Abundant in nature
  - Almost every life form
- Occurs in many areas of engineering
  - **Aerospace:** Aircraft, parachutes, rockets
  - **Civil:** Bridges, dams, cable/roof structures
  - **Mechanical:** Automobiles, turbines, pumps
  - **Naval:** Ships, off-shore structures, submarines

Governing equations

Fluid: $\rho^f \frac{D\mathbf{v}^f}{Dt} + \nabla \cdot \sigma^f = \mathbf{b}^f$ (Eulerian)

Solid: $\rho^s \frac{\partial^2 \mathbf{d}^s}{\partial t^2} + \nabla \cdot \sigma^s = \mathbf{b}^s$ (Lagrangian)

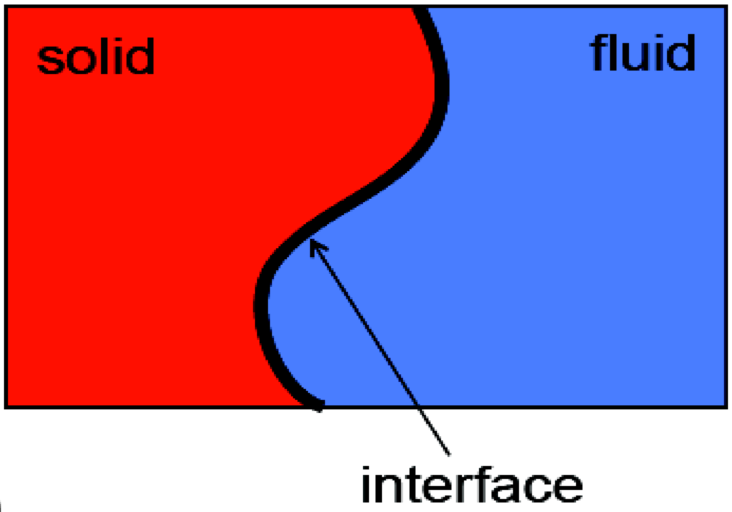
Interface:

Kinematic condition:

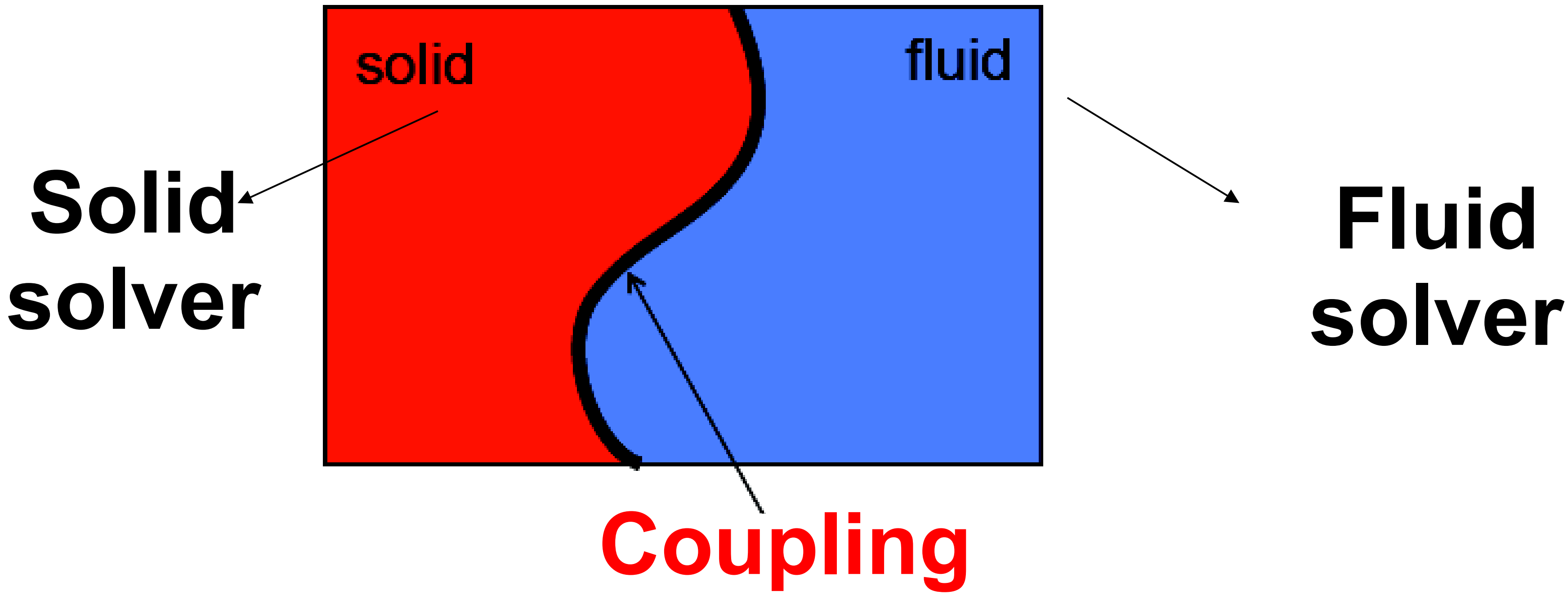
$$\mathbf{v}^f = \mathbf{v}^s$$

Equilibrium condition:

$$\sigma^f \cdot \mathbf{n}^f + \sigma^s \cdot \mathbf{n}^s = 0$$

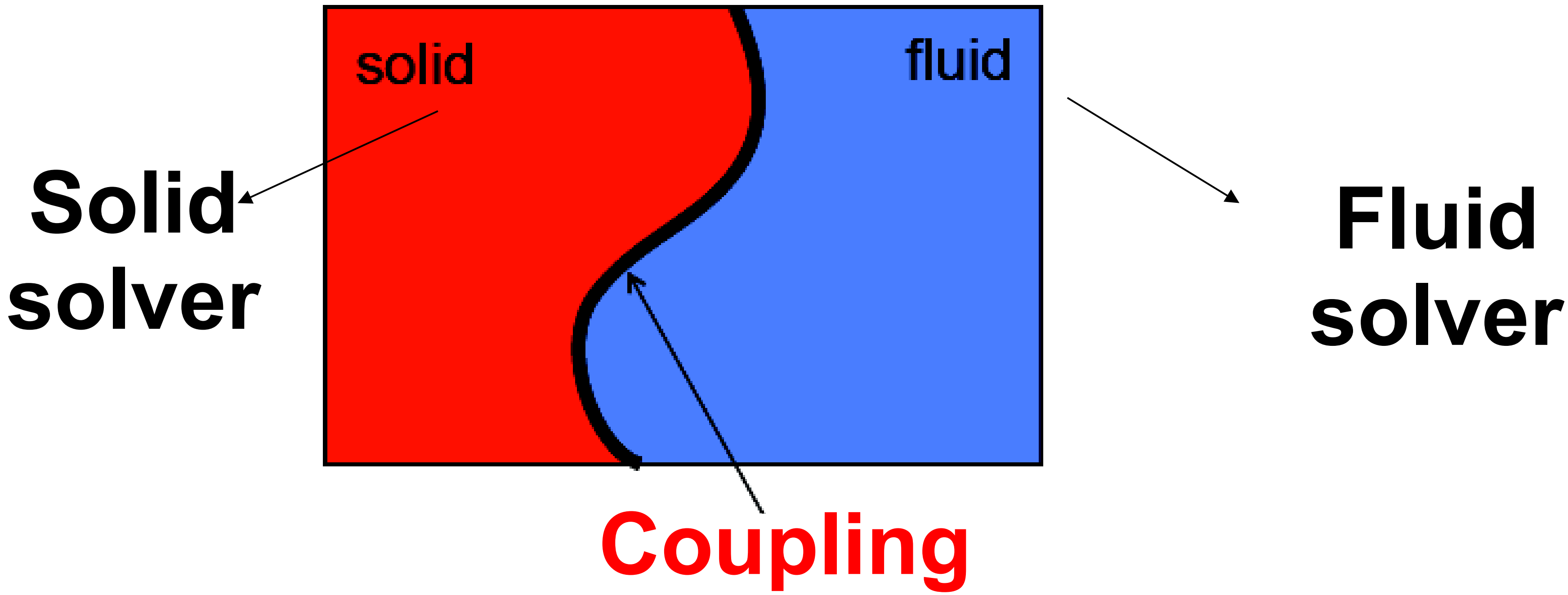


Aspects of numerical modelling



Can we solve all the FSI problems if we use the best available solvers for fluid and solid sub-problems?

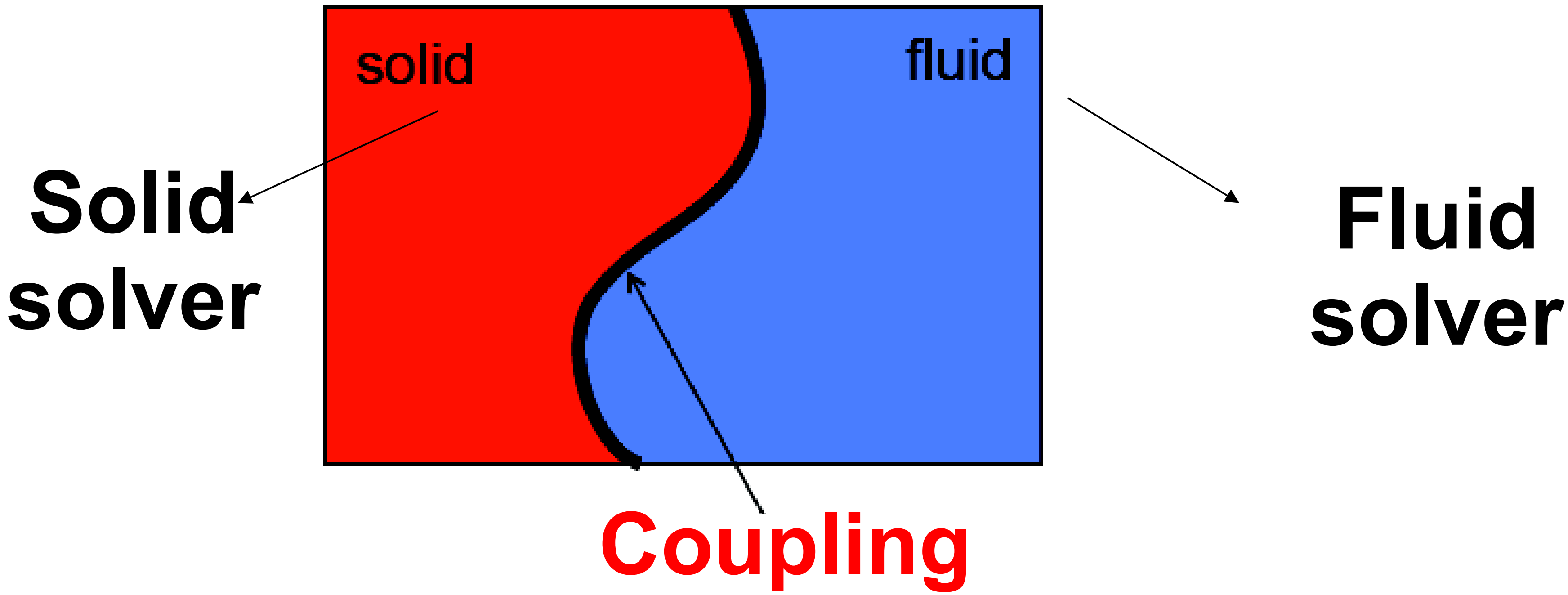
Aspects of numerical modelling



Can we solve all the FSI problems if we use the best available solvers for fluid and solid sub-problems?

No. But, why?

Aspects of numerical modelling



Can we solve all the FSI problems if we use the best available solvers for fluid and solid sub-problems?

No. But, why? The devil is at the interface.

Caution!

If someone tells you that his/her scheme/tool can solve a FSI problem without actually looking at the problem, then it is highly likely that **he/she is lying**.

Important properties of numerical schemes for FSI

(1) Existence

- Does the tool have FSI capability?

(2) Robustness

- For a reasonable time step, does the scheme work without crashing?

(3) Accuracy

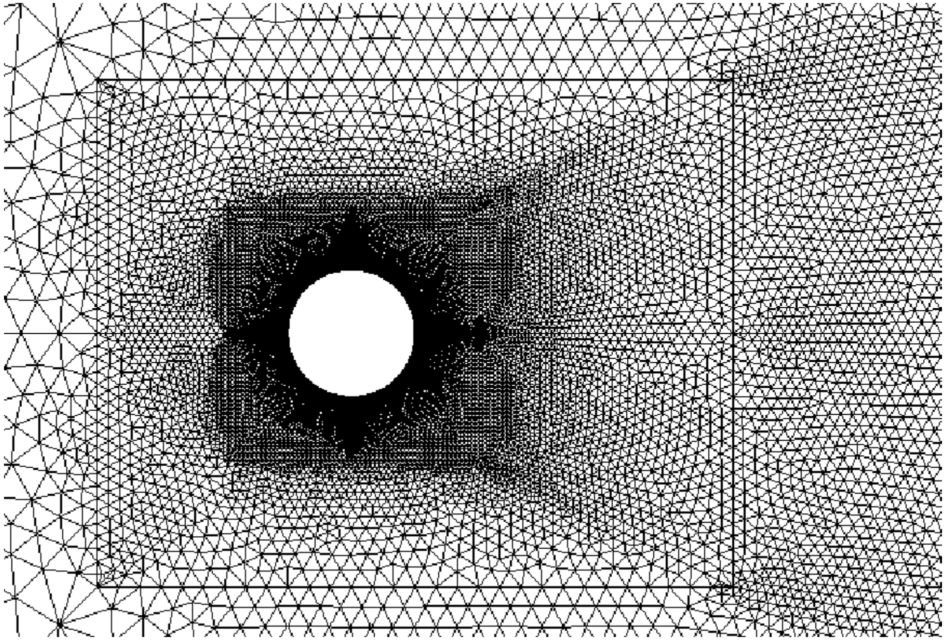
- How accurate is the solution?

(4) Efficiency

- What is the amount of time required?

Body-fitted meshes

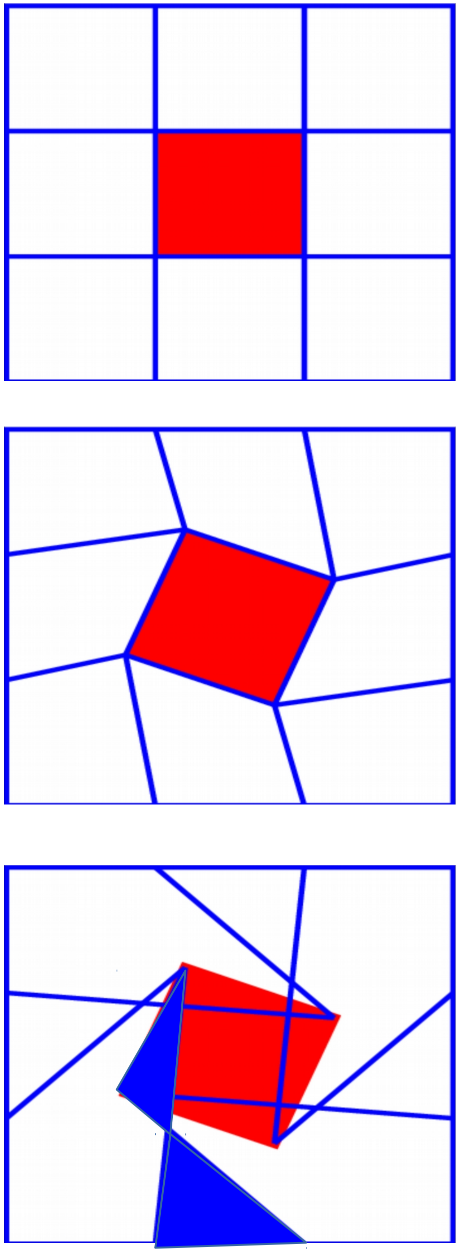
- Meshes aligned with the solid boundary
- Finite Element or Finite Volume schemes for the fluid problem



How to deal with moving solids?

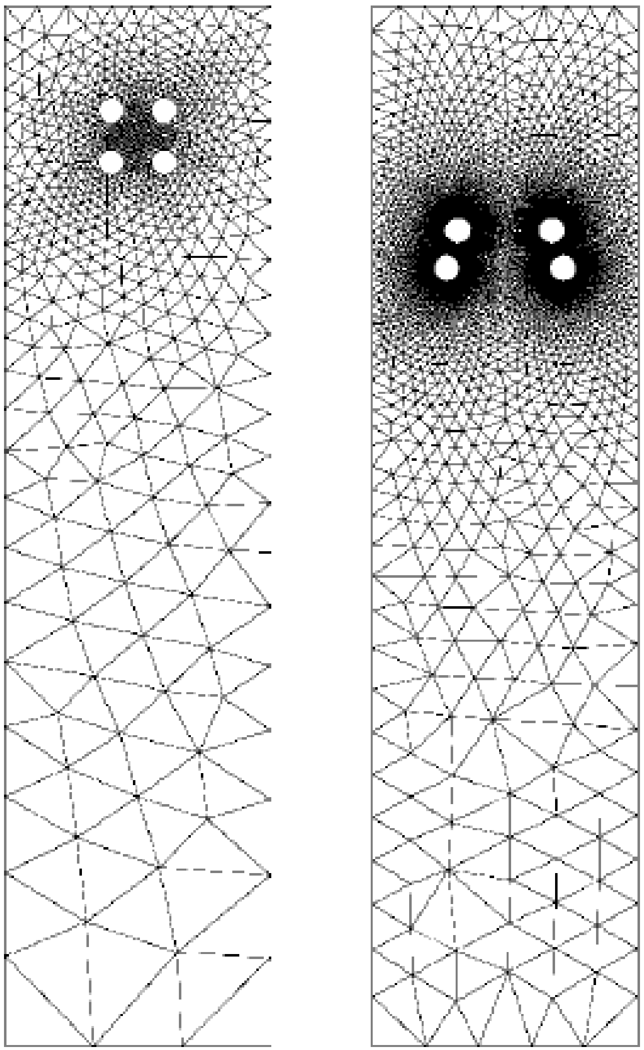
Body-fitted meshes

- When the solid moves
  - ➔ Surrounding fluid mesh also moves
  - ➔ Arbitrary Lagrangian-Eulerian (ALE) formulation for the fluid
  - ➔ For small displacements
    - ▪ mesh deformation schemes
  - ➔ For large displacements
    - ▪ re-meshing techniques

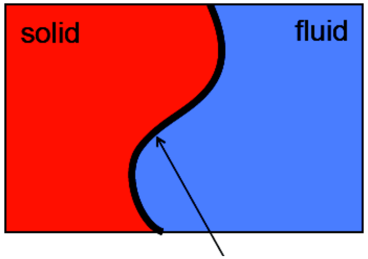


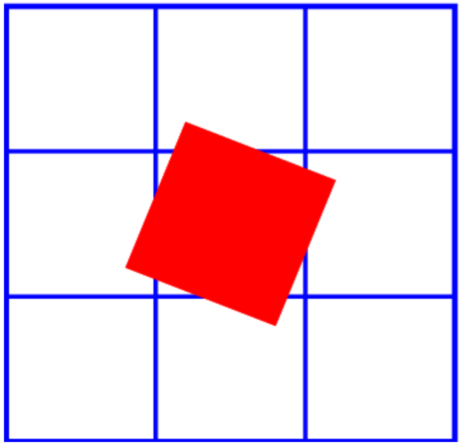
Body-fitted meshes

- Advantages
  - Efficient and accurate for simple problems
  - Well established
  - Available in commercial software
- Disadvantages
  - Mesh generation is cumbersome
  - Require sophisticated re-meshing algorithms
  - Complicated and inefficient in 3D
  - Difficulty in capturing topological changes

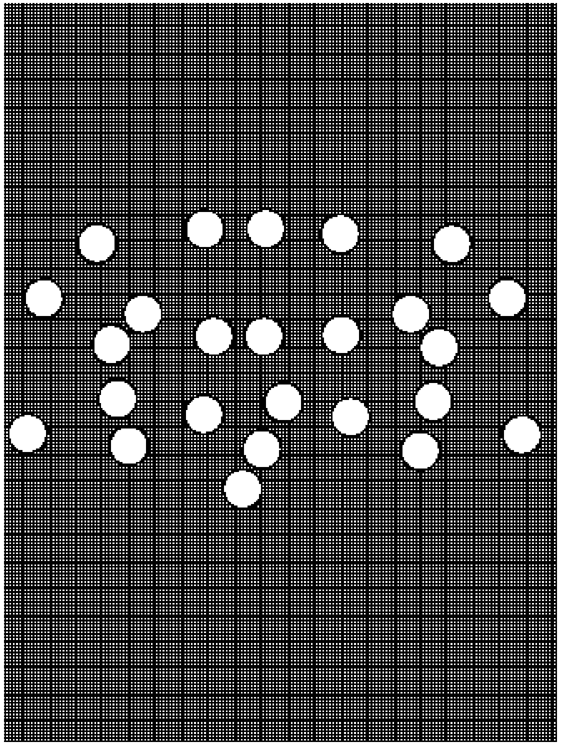


Unfitted/immersed methods





- Solids immersed/embedded on fixed grids
- Advantages
  - No need for body-fitted meshes
  - No need for re-meshing
  - Ideal for multi-phase flows, fracture
  - Complex FSI problems can be solved
- Disadvantages
  - Needs to develop a fluid solver
  - Majority of the schemes are only 1st order accurate in time
  - Very limited availability in commercial software



Integration in time

- Only implicit schemes are considered
- **Fluid:**
  - 1st order - Backward Euler
  - 2nd order – Crank-Nicolson/Trapezoidal, Generalised-alpha, BDF2
- **Solid:**
  - 1st order - Backward Euler
  - 2nd order - Crank-Nicolson/Trapezoidal, Generalised-alpha

✔ Spatial discretisation

✔ Temporal discretisation

Coupling strategies

Monolithic Vs Staggered

Governing equations

Fluid: $$\rho^f \frac{D\mathbf{v}^f}{Dt} + \nabla \cdot \sigma^f = \mathbf{b}^f$$ (Eulerian)

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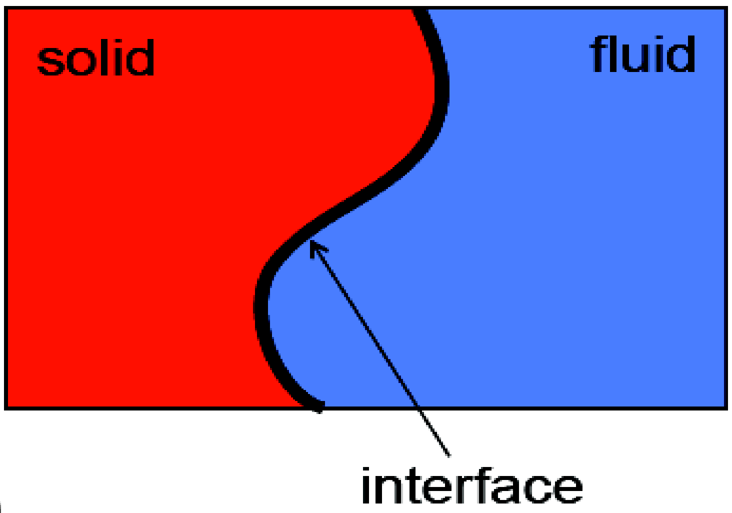
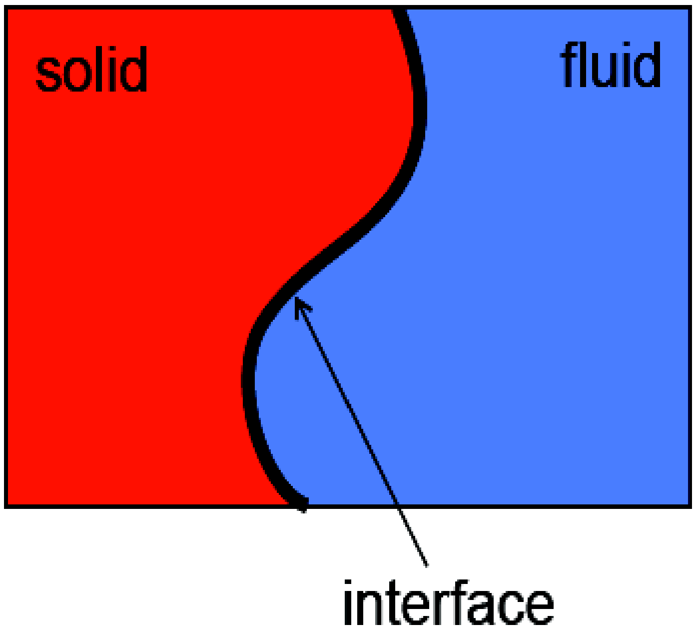
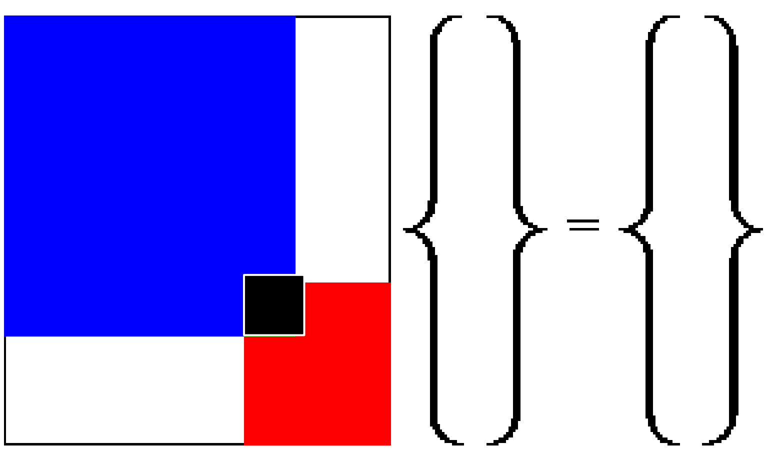


Coupling

- Data transfer between fluid and solid
- Types of techniques
  - → Dirichlet-Neumann (body-fitted, unfitted)
  - → Robin-Robin (body-fitted, unfitted)
  - → Body-force (standard Immersed methods)
- We consider Dirichlet-Neumann
  - → The most intuitive and physical
  - → Velocity boundary condition on the Fluid
  - → Force boundary condition on the Solid

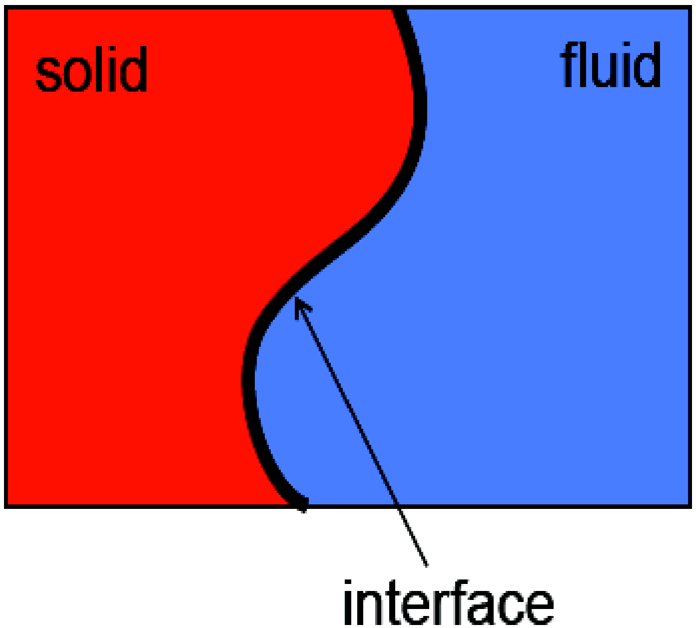
Monolithic schemes

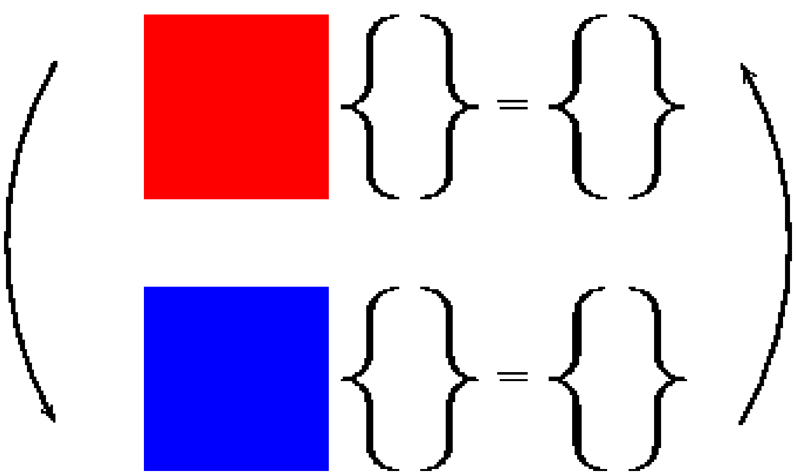




- Fixed-point or Newton-Raphson
- <u>Advantages</u>
  - ➔ No added-mass instabilities
  - ➔ 2^{nd} order accuracy in time is possible
- <u>Disadvantages</u>
  - ➔ Need to develop customised solvers
  - ➔ Computationally expensive
  - ➔ Difficult to linearise
  - ➔ Convergence issues

Staggered schemes





- Solve solid and fluid separately
- <u>Advantages</u>
  - ➔ Computationally appealing
  - ➔ Existing solvers can be used
- <u>Disadvantages</u>
  - ➔ Added-mass instabilities
  - ➔ Difficult to get 2nd order accurate schemes for FSI with flexible structures in the presence of significant added-mass
  - ➔ Efficiency and accuracy decrease with the increase in added mas

Summary of FSI schemes

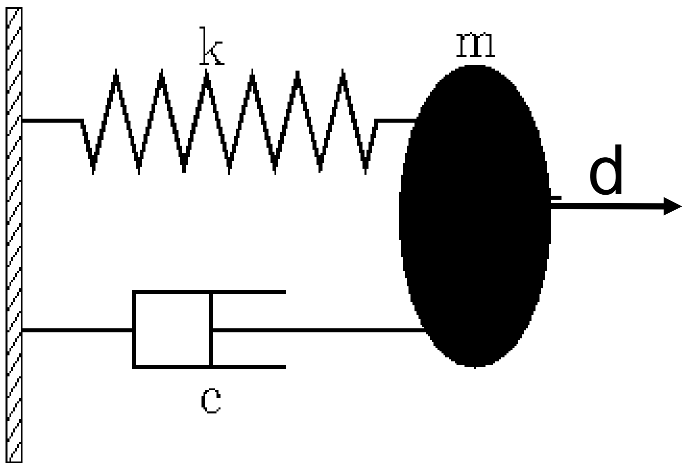
	Monolithic	**Staggered**
Body-fitted	✓ Commerical software ✓ No added-mass issue ✘ Expensive	✓ Efficient ✓ Easiest of all ✘ Added-mass issues
Unfitted	✓ No added-mass issue ✘ Complicated ✘ Expensive	✓ Efficient ✓ Relatively easy ✓ Many applications ✘ Added-mass issue

What is added mass issue?

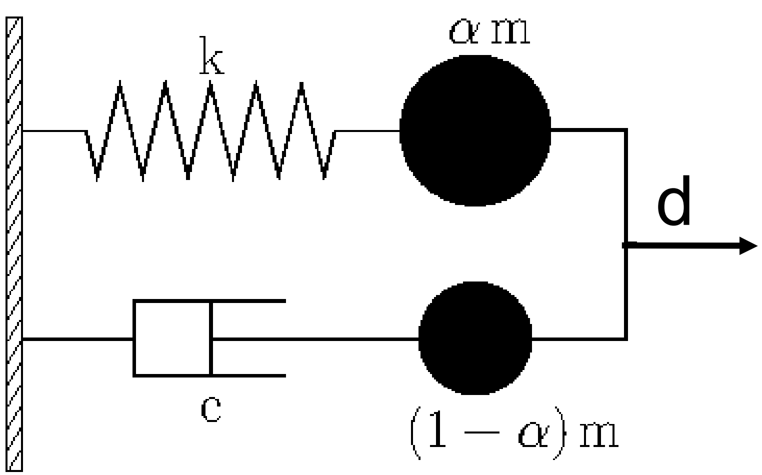
Instability arising when

1) The density of the solid is close to or less than that of the fluid
   - Blood flow through arteries
2) When the structure is very thin
   - Shell structures
3) When the structure is highly flexible
   - Roof membranes, parachutes

A model problem for FSI



$$m\,\ddot{d} + c\,\dot{d} + k\,d = 0$$



$$\begin{aligned} \alpha\ \ddot{d} + \omega^2\, d &= f^s \\ (1-\alpha)\dot{u} + 2\,\xi\,\omega\,u &= f^f \\ \dot{d} &= u \\ f^s + f^f &= 0 \end{aligned}$$

Dettmer, W. G. and Peric, D. *A new staggered scheme for fluid-structure interaction*, IJNME, 93, 1-22, 2013.

A stabilised immersed framework for FSI

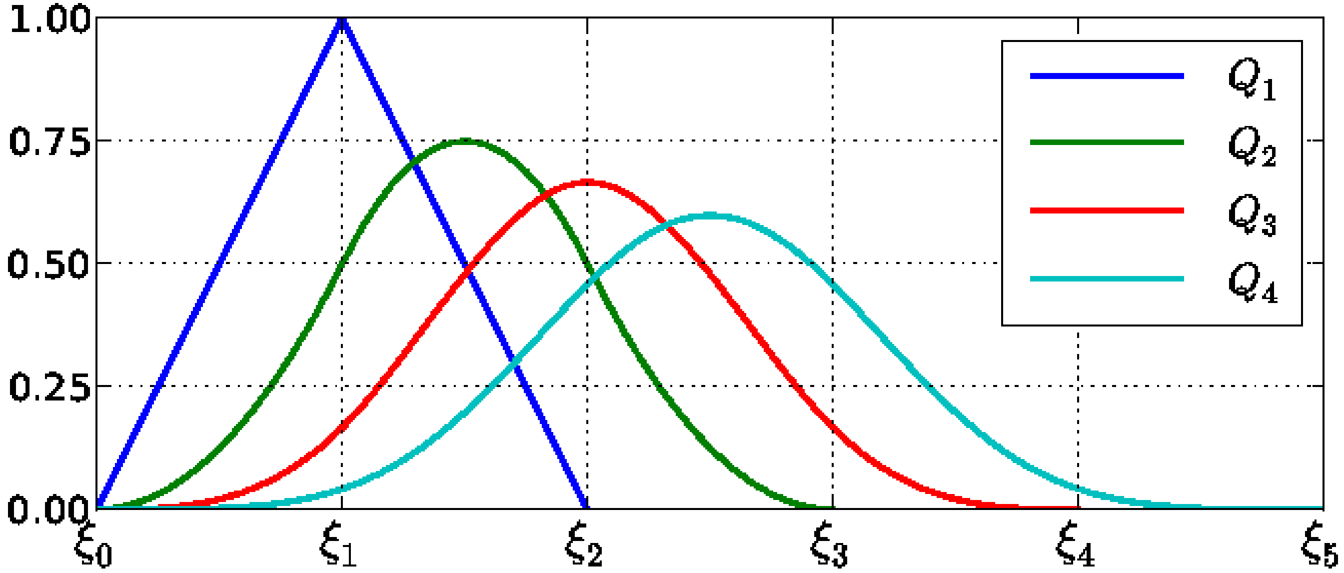
- Combines the state-of-the-art
- Hierarchical b-splines
- SUPG/PSPG stabilisation for the fluid
- Ghost-penalty stabilisation for cut-cells
- Solid-Solid contact
- Staggered solution schemes
- Wide variety of applications

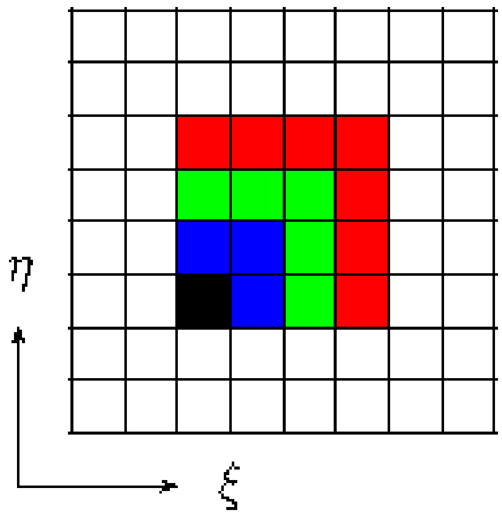
B-Splines and hierarchical refinement - spatial discretisations for unfitted meshes

B-Splines

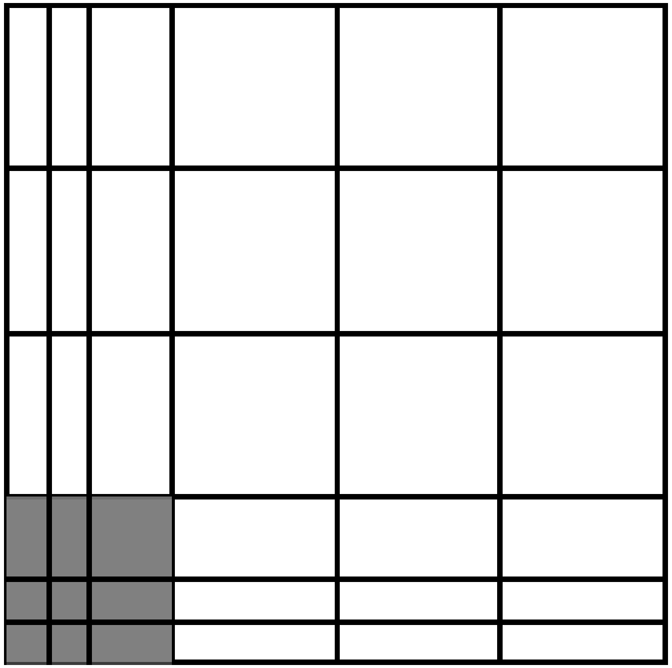
$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leq \xi \leq \xi_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$





Hierarchical B-Splines



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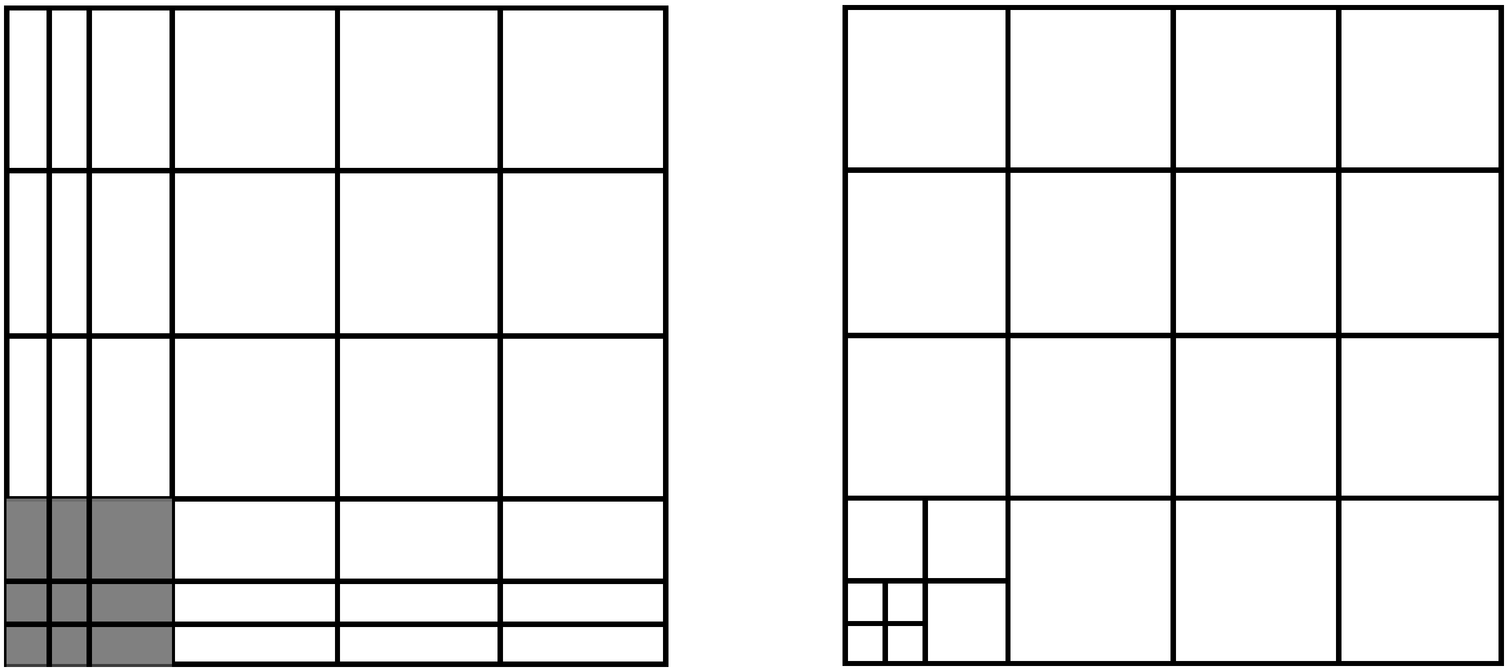

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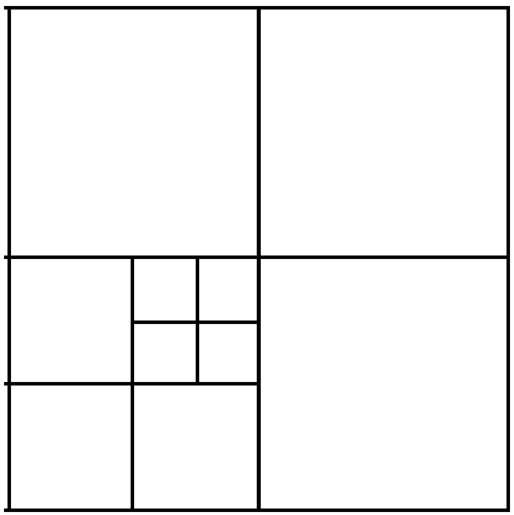
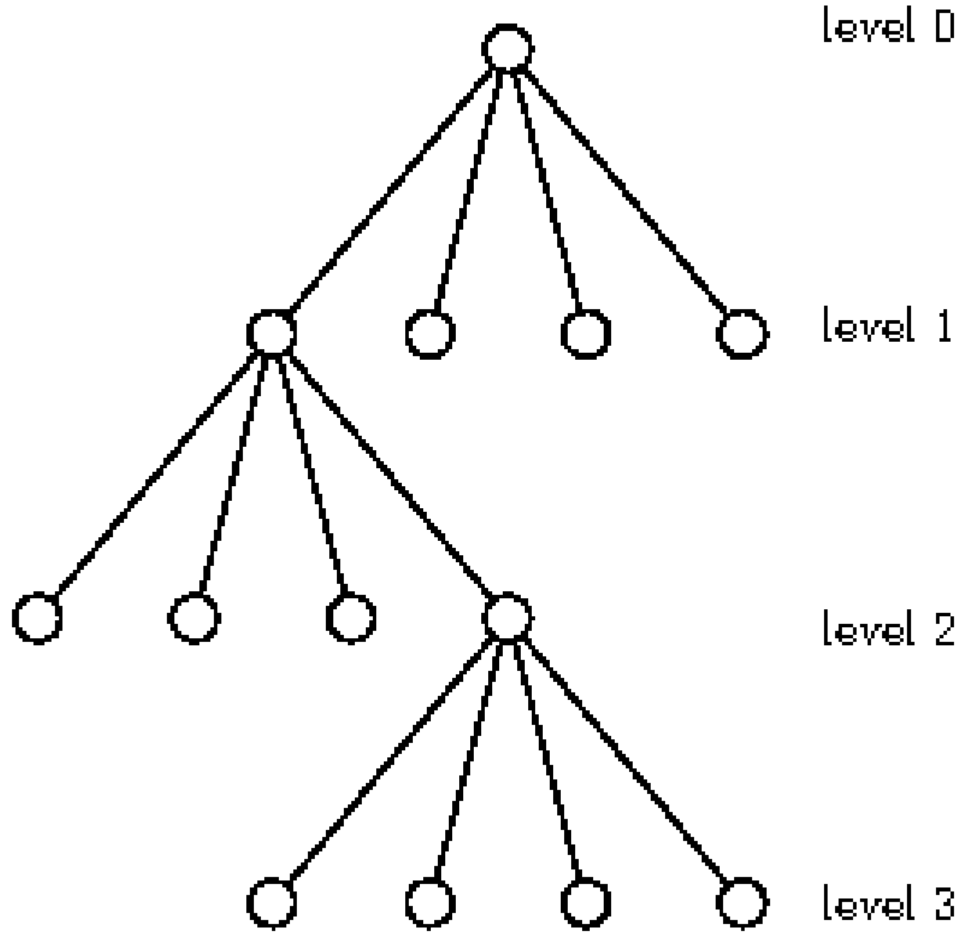
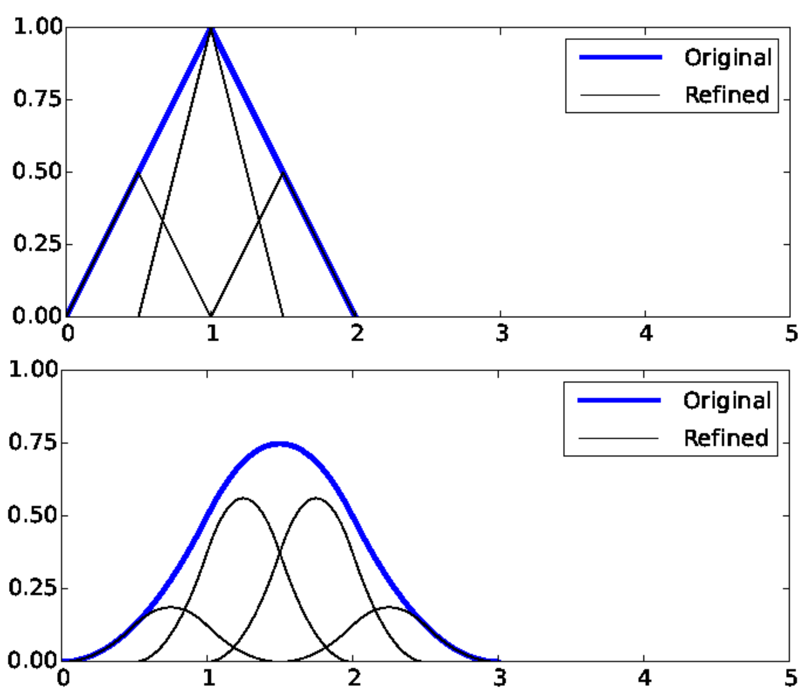
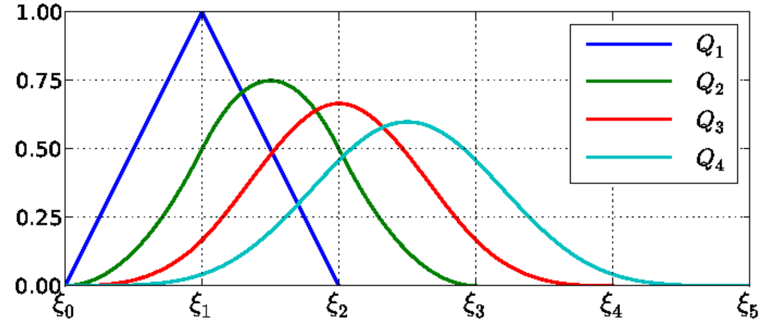




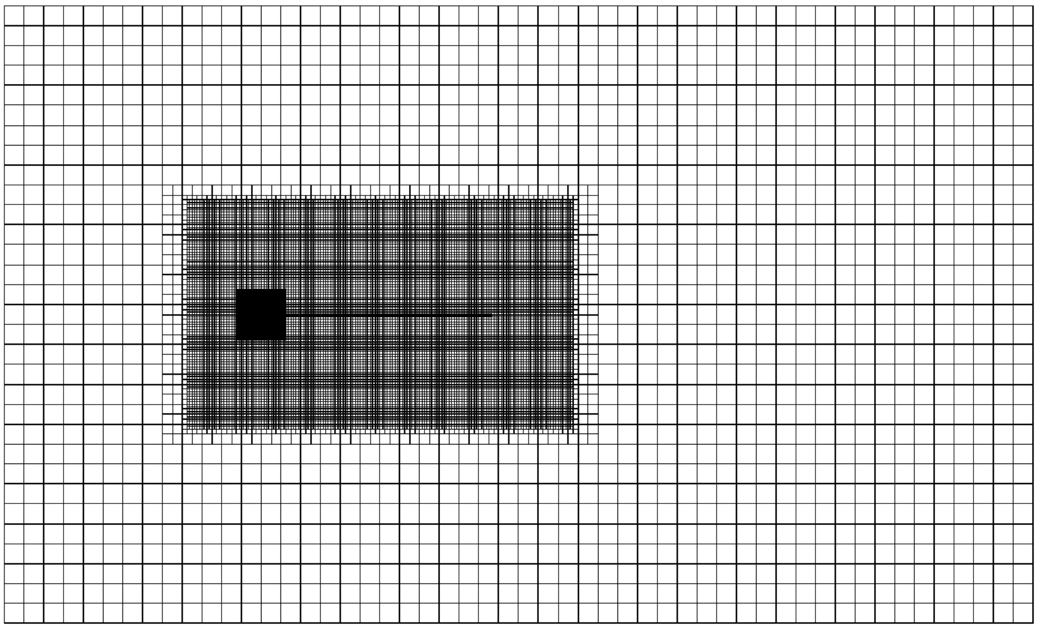


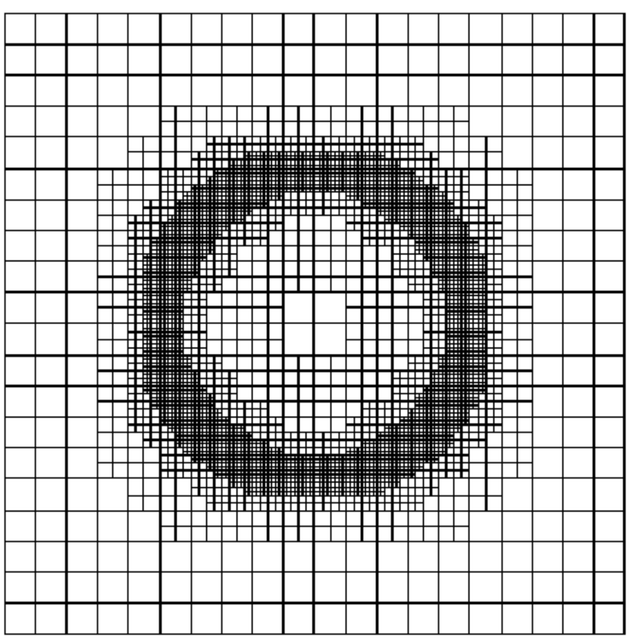
B-Splines

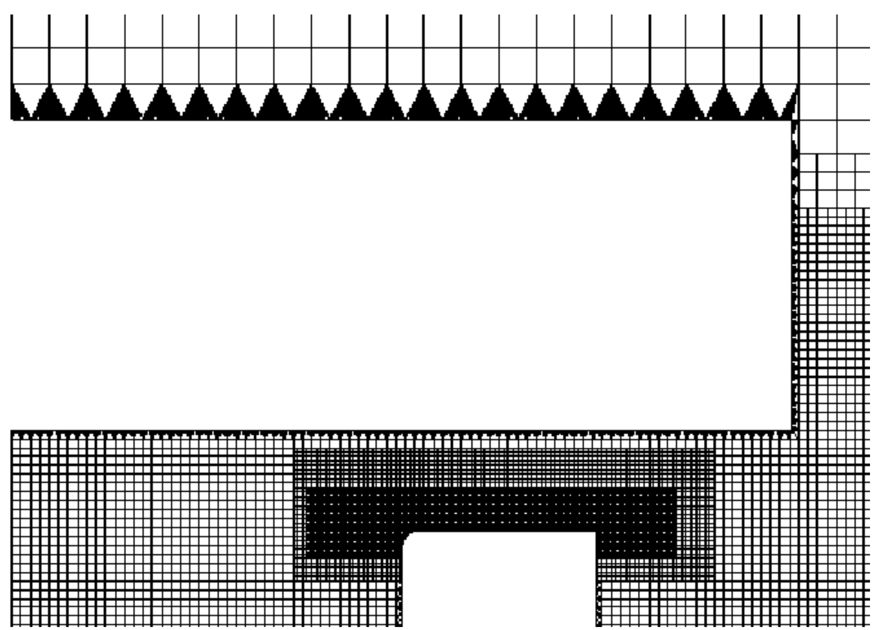
- Nice mathematical properties
  - Tensor product nature
  - Partition of unity
  - Higher-order continuities across element boundaries
- Always positive
- No hanging nodes
- Ease of localised refinements
- Efficient programming techniques and data structures

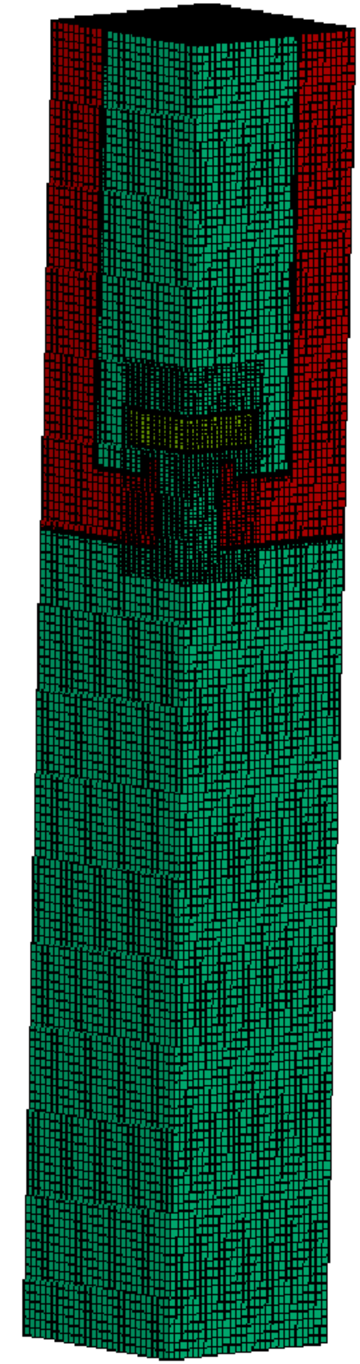


Sample meshes









Formulation

Incompressible Navier-Stokes

$$\rho^f \frac{\partial \mathbf{v}^f}{\partial t} + \rho^f (\mathbf{v}^f \cdot \nabla)\mathbf{v}^f - \mu^f \Delta \mathbf{v}^f + \nabla p = \mathbf{g}^f \quad \text{in} \quad \Omega^f$$

$$\nabla \cdot \mathbf{v}^f = 0 \quad \text{in} \quad \Omega^f$$

$$\mathbf{v}^f = \mathbf{v}^s \quad \text{on} \quad \Gamma_D^f$$

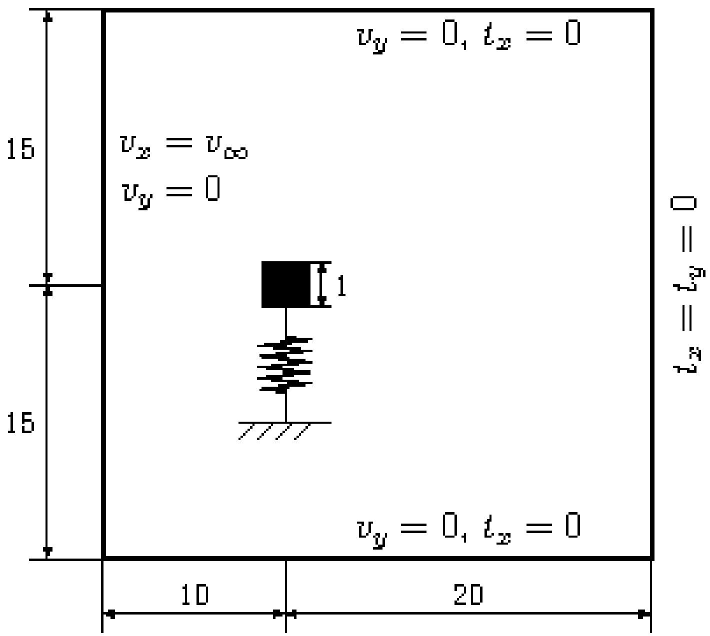
$$\sigma^f \cdot \mathbf{n}^f = \mathbf{t}^f \quad \text{on} \quad \Gamma_N^f$$

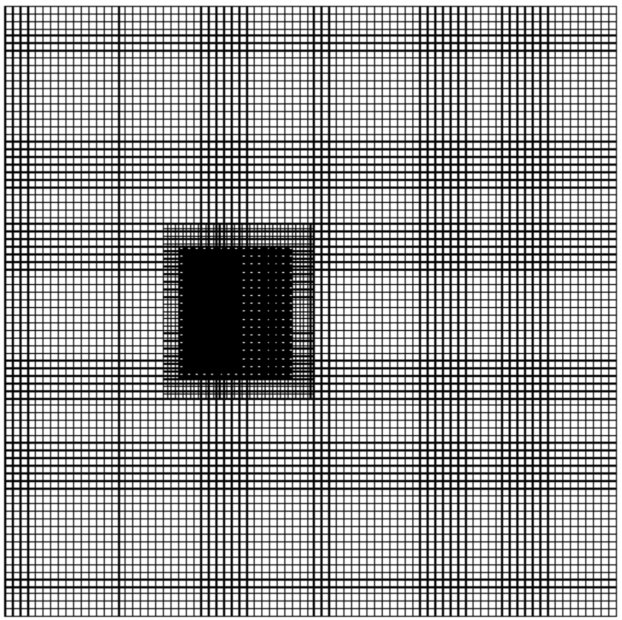
Variational formulation

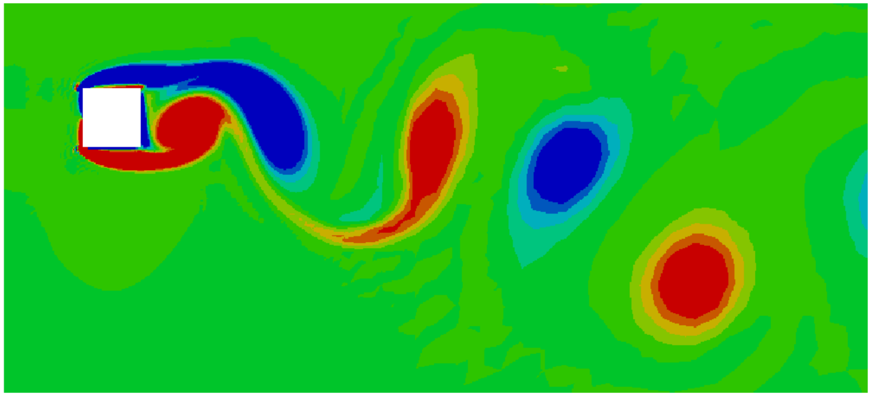
$$B^f_{\mathsf{Gal}}(\{\mathbf{w}^f, q\}, \{\mathbf{v}^f, p\}) + B^f_{\mathsf{Stab}}(\{\mathbf{w}^f, q\}, \{\mathbf{v}^f, p\}) + B^f_{\mathsf{Nitsche}}(\{\mathbf{w}^f, q\}, \{\mathbf{v}^f, p\}) + B^f_{\mathsf{GP}}(\{\mathbf{w}^f, q\}, \{\mathbf{v}^f, p\}) = F^f_{\mathsf{Gal}}(\{\mathbf{w}^f, q\})$$

Time integration:
Backward Euler (O(dt)) and Generalised-alpha (O(dt^2))

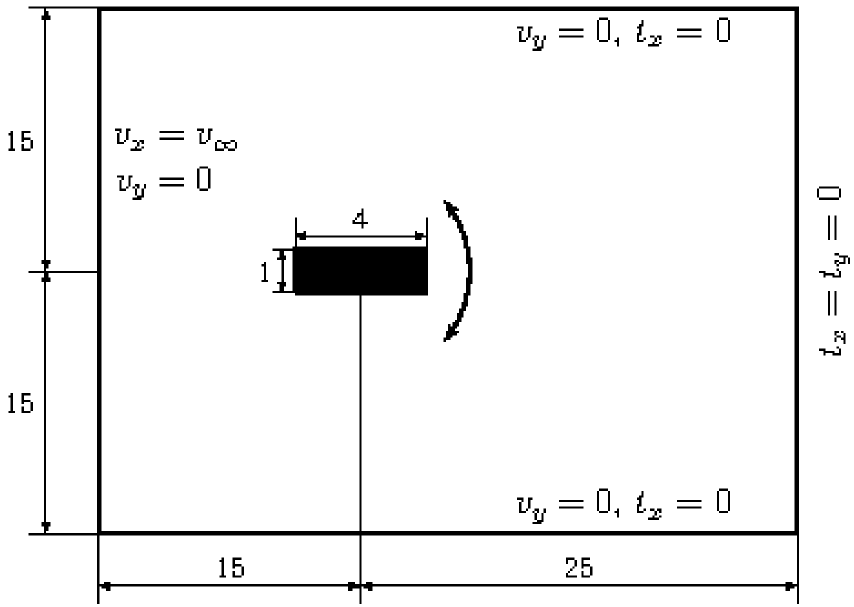
Transverse Galloping

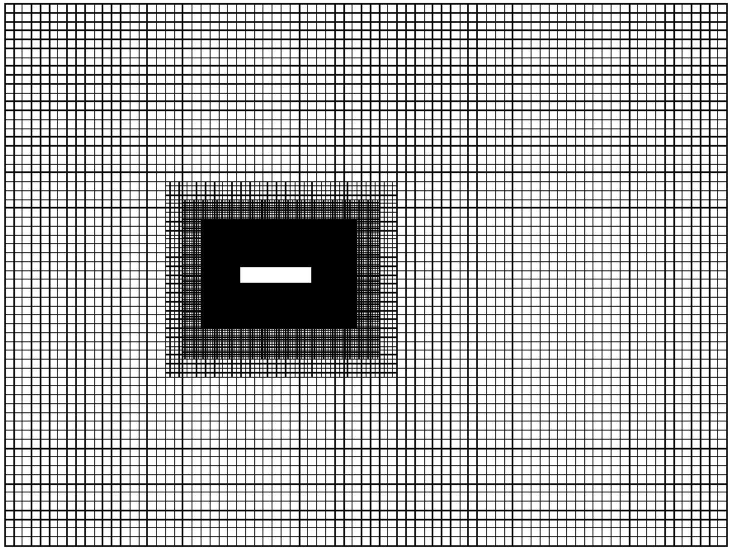


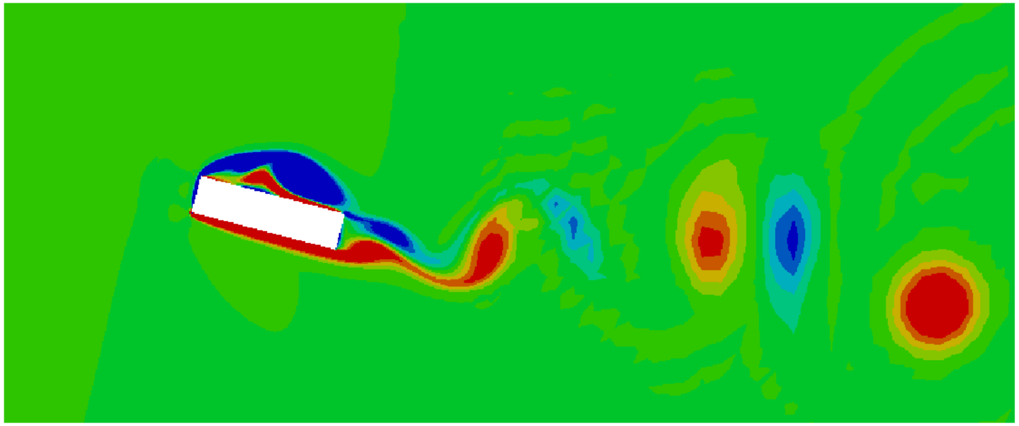




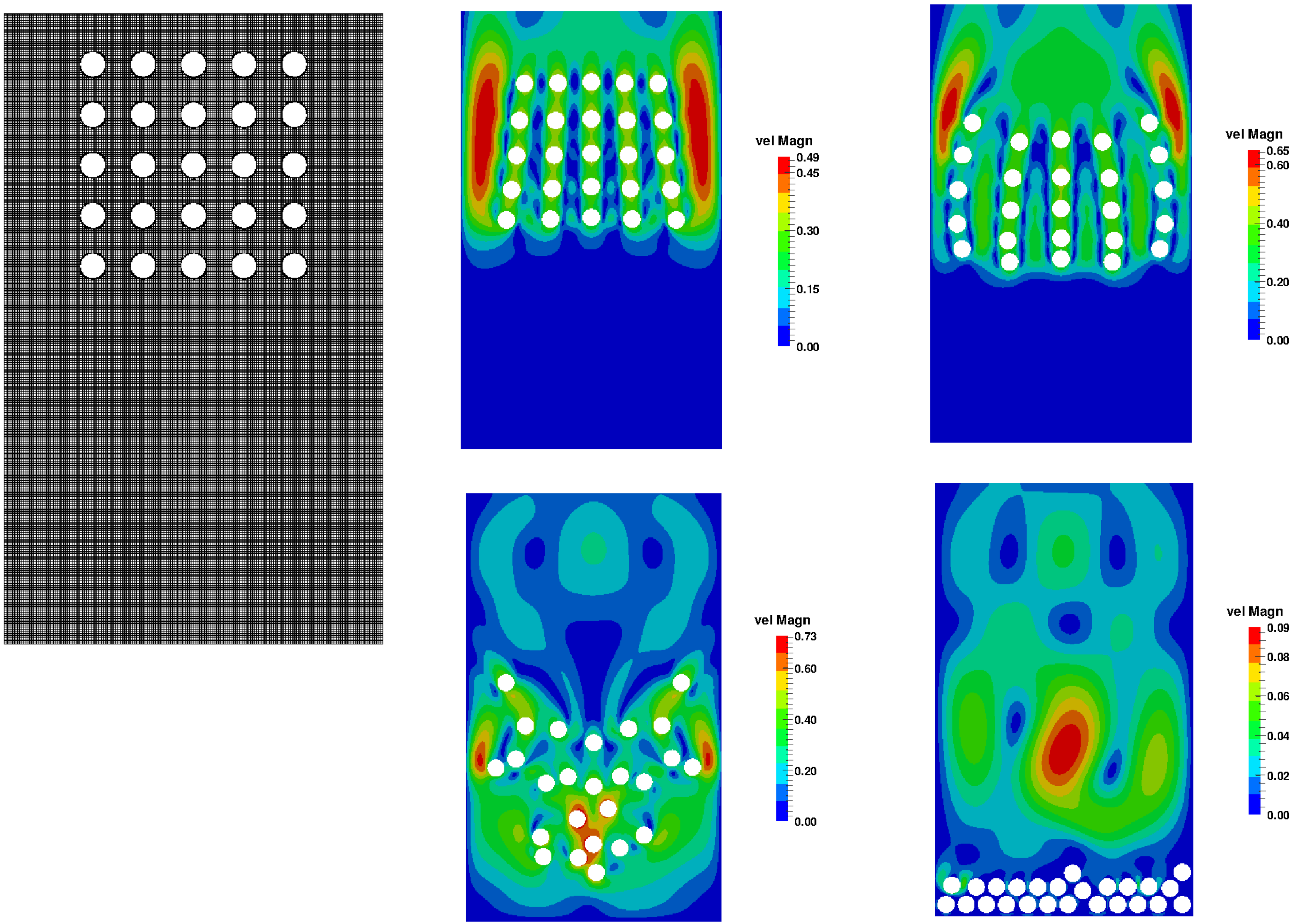
Rotational Galloping



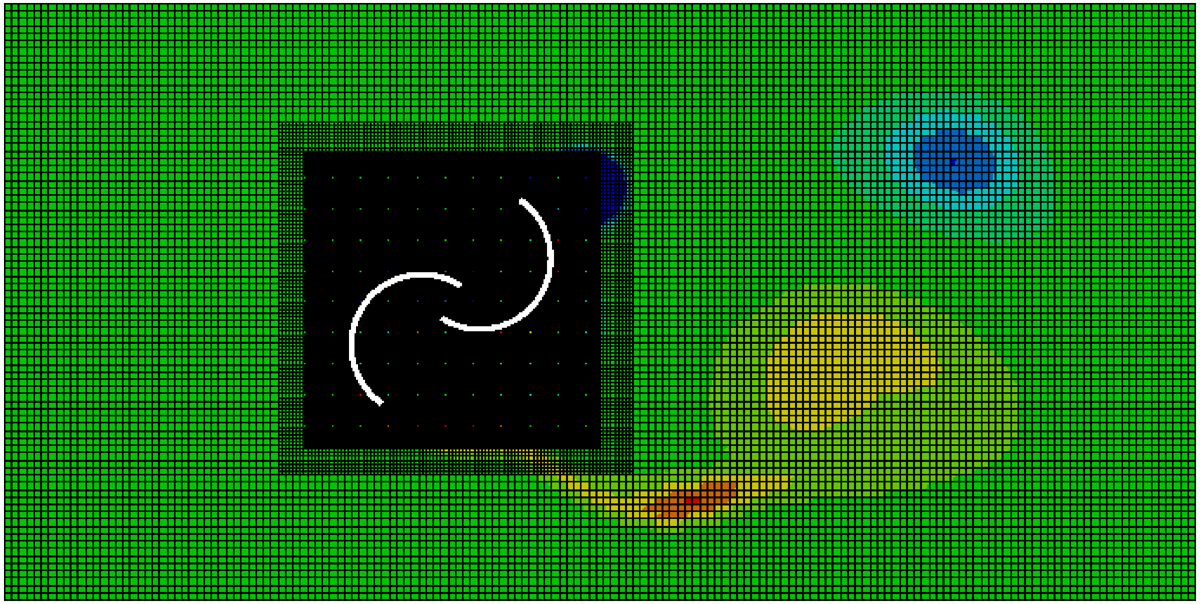


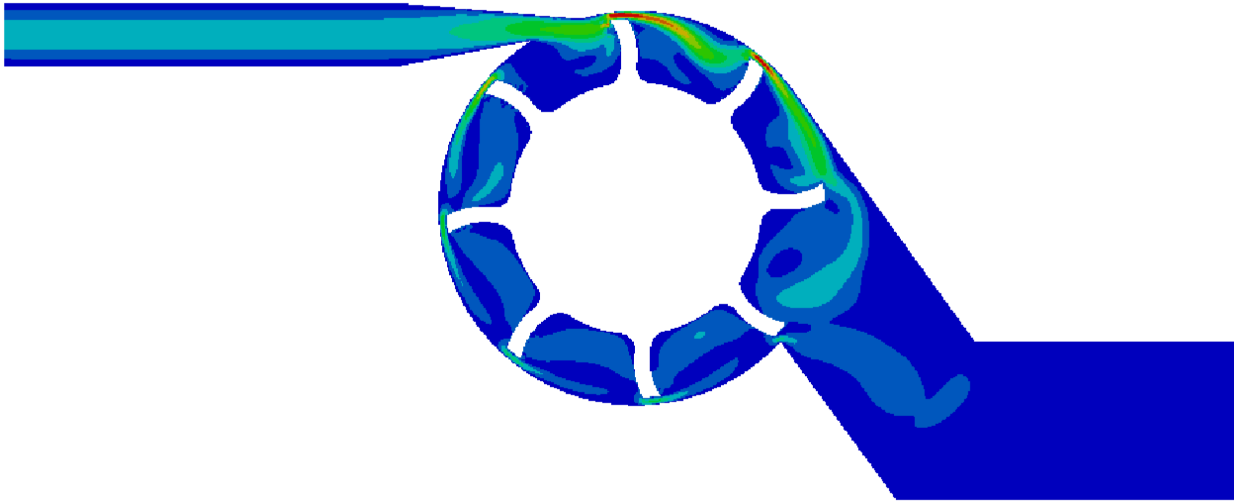


Sedimentation of multiple particles

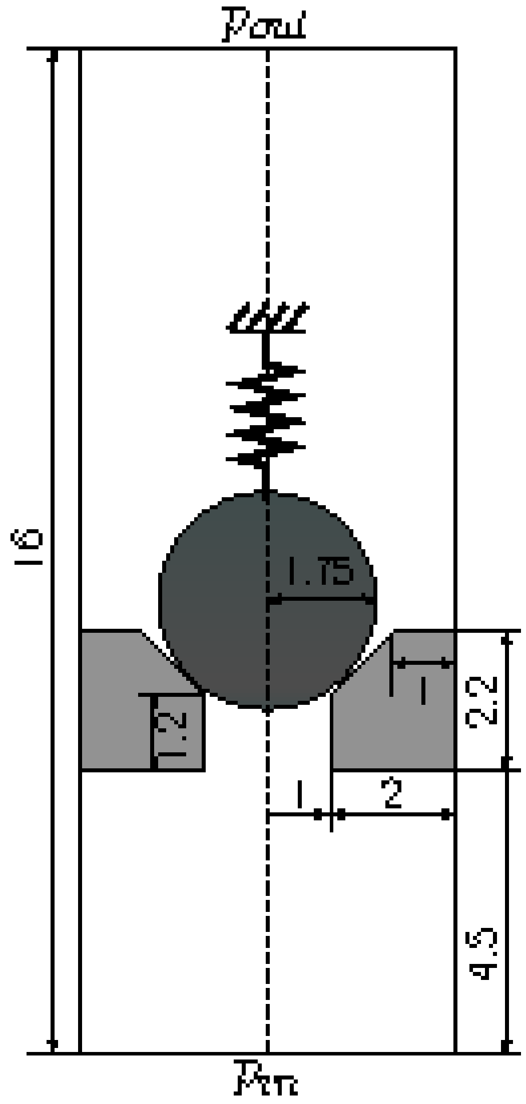


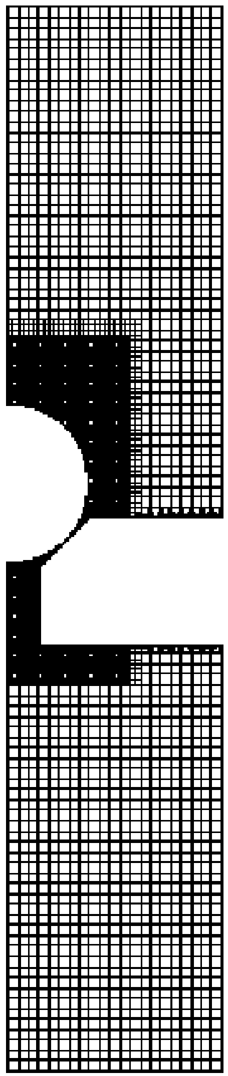
Model turbines

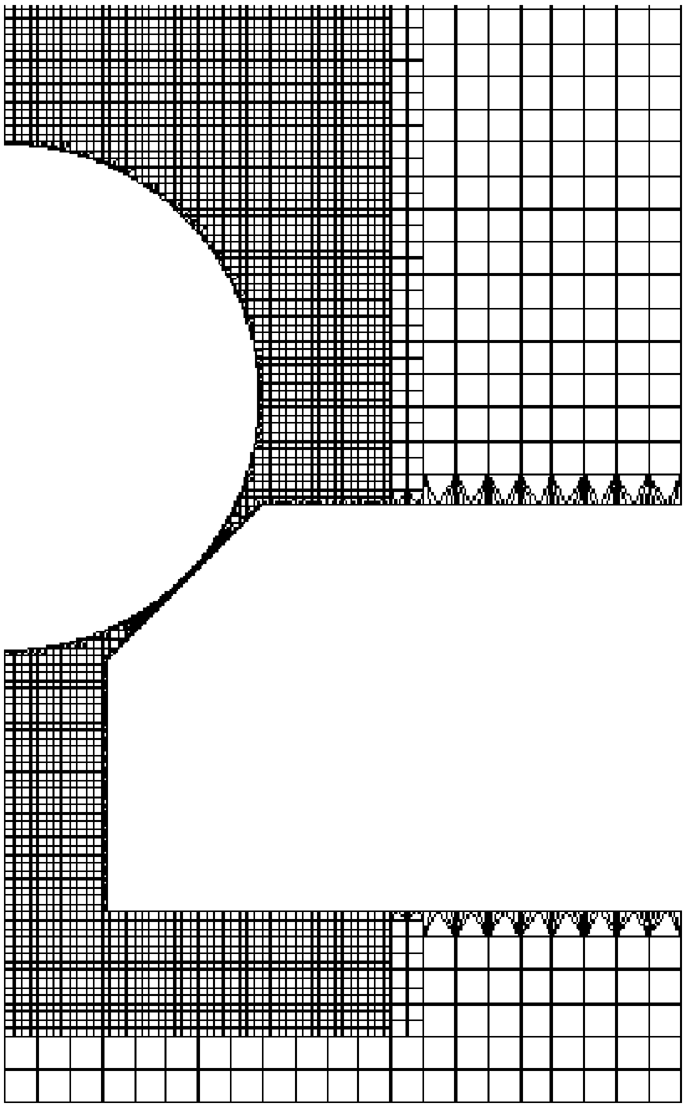




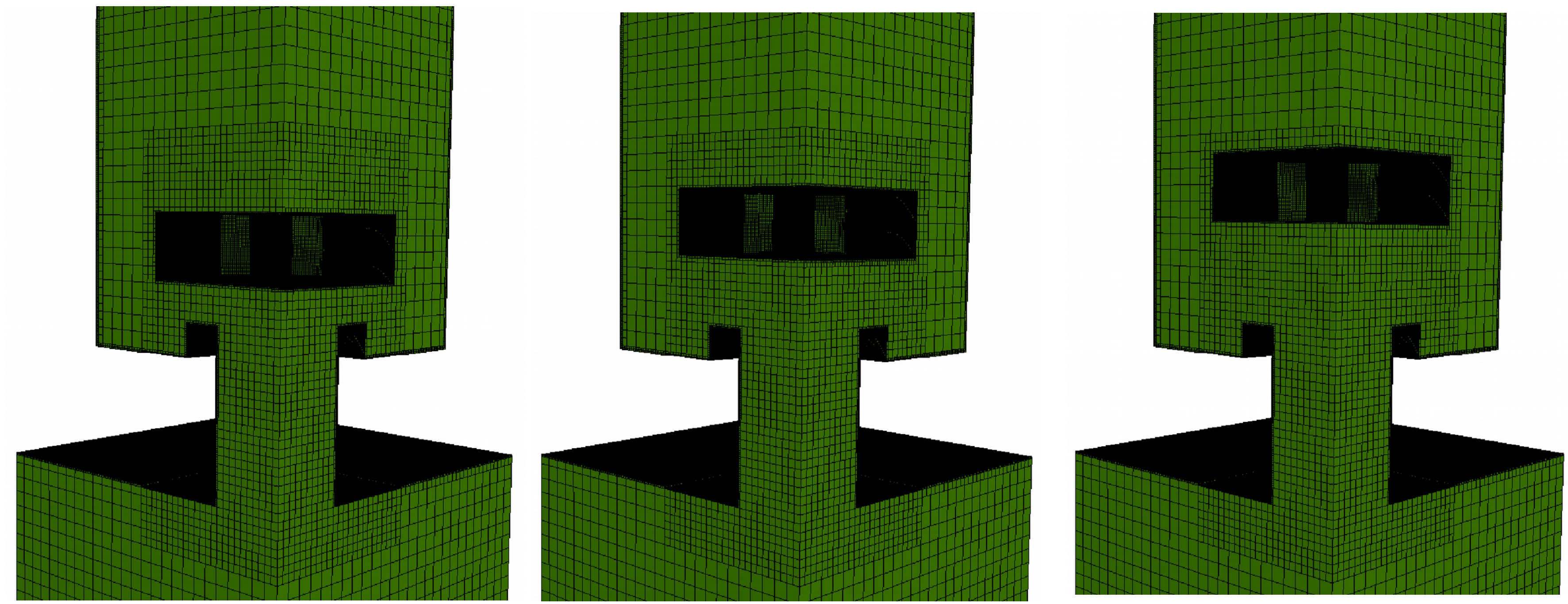
Ball check valve







Relief valve in 3D



References

(1) W. G. Dettmer and D. Perić. *A new staggered scheme for fluid-structure interaction*, IJNME, 93, 1-22, 2013.

(2) W. G. Dettmer, C. Kadapa, D. Perić, *A stabilised immersed boundary method on hierarchical b-spline grids,* CMAME, Vol. 311, pp. 415-437, 2016.

(3) C. Kadapa, W. G. Dettmer, D. Perić, *A stabilised immersed boundary method on hierarchical b-spline grids for fluid-rigid body interaction with solid-solid contact,* CMAME, Vol. 318, pp. 242-269, 2017.

(4) Y. Bazilevs, K. Takizawa, T. E. Tezduyar, *Computational Fluid-Structure Interaction: Methods and Applications*, Wiley, 2013.