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Lecture session –UNIT 2

Topic: Karnaugh map

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CHENNAI-119

Karnaugh Map

We have simplified the Boolean functions using Boolean postulates and theorems. It is a time consuming process and we have to re-write the simplified expressions after each step.

To overcome this difficulty,

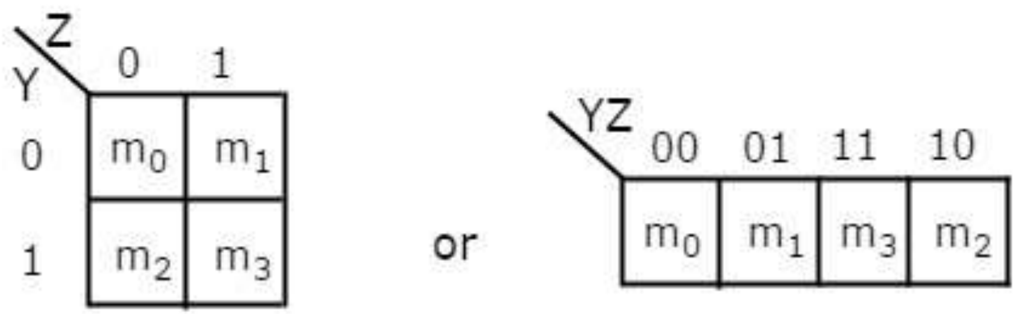
Karnaugh introduced a method for simplification of Boolean functions in an easy way. This method is known as Karnaugh map method or K-map method. It is a graphical method, which consists of 2^n cells for ‘n’ variables. The adjacent cells are differed only in single bit position.

K-Maps for 2 to 5 Variables

K-Map method is most suitable for minimizing Boolean functions of 2 variables to 5 variables. Now, let us discuss about the K-Maps for 2 to 5 variables one by one.

2 Variable K-Map

The number of cells in 2 variable K-map is four, since the number of variables is two. The following figure shows **2 variable K-Map**.



There is only one possibility of grouping 4 adjacent min terms.

•The possible combinations of grouping 2 adjacent min terms are $\{(m_0, m_1), (m_2, m_3), (m_0, m_2)$ and $(m_1, m_3)\}$.

3 Variable K-Map

The number of cells in 3 variable K-map is eight, since the number of variables is three. The following figure shows **3 variable K-Map**.

X \ YZ	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

- There is only one possibility of grouping 8 adjacent min terms.
- The possible combinations of grouping 4 adjacent min terms are $\{(m_0, m_1, m_3, m_2), (m_4, m_5, m_7, m_6), (m_0, m_1, m_4, m_5), (m_1, m_3, m_5, m_7), (m_3, m_2, m_7, m_6)$ and $(m_2, m_0, m_6, m_4)\}$.
- The possible combinations of grouping 2 adjacent min terms are $\{(m_0, m_1), (m_1, m_3), (m_3, m_2), (m_2, m_0), (m_4, m_5), (m_5, m_7), (m_7, m_6), (m_6, m_4), (m_0, m_4), (m_1, m_5), (m_3, m_7)$ and $(m_2, m_6)\}$.
- If x=0, then 3 variable K-map becomes 2 variable K-map.

4 Variable K-Map

The number of cells in 4 variable K-map is sixteen, since the number of variables is four. The following figure shows **4 variable K-Map**.

WX \ YZ	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

- There is only one possibility of grouping 16 adjacent min terms.
- Let R_1, R_2, R_3 and R_4 represents the min terms of first row, second row, third row and fourth row respectively. Similarly, C_1, C_2, C_3 and C_4 represents the min terms of first column, second column, third column and fourth column respectively. The possible combinations of grouping 8 adjacent min terms are $\{(R_1, R_2), (R_2, R_3), (R_3, R_4), (R_4, R_1), (C_1, C_2), (C_2, C_3), (C_3, C_4), (C_4, C_1)\}$.
- If w=0, then 4 variable K-map becomes 3 variable K-map.

5 Variable K-Map

The number of cells in 5 variable K-map is thirty-two, since the number of variables is 5. The following figure shows **5 variable K-Map**.

V=0

WX \ YZ	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

V=1

WX \ YZ	00	01	11	10
00	m_{16}	m_{17}	m_{19}	m_{18}
01	m_{20}	m_{21}	m_{23}	m_{22}
11	m_{28}	m_{29}	m_{31}	m_{30}
10	m_{24}	m_{25}	m_{27}	m_{26}

- There is only one possibility of grouping 32 adjacent min terms.
- There are two possibilities of grouping 16 adjacent min terms. i.e., grouping of min terms from m_0 to m_{15} and m_{16} to m_{31}.
- If v=0, then 5 variable K-map becomes 4 variable K-map.

In the above all K-maps, we used exclusively the min terms notation. Simi you can use exclusively the Max terms notation.

Re-arranging the Truth Table

- A two-variable function has four possible minterms. We can re-arrange these minterms into a Karnaugh map

x	y	minterm
0	0	x'y'
0	1	x'y
1	0	xy'
1	1	xy

X \ Y	0	1
0	x'y'	x'y
1	xy'	xy

- Now we can easily see which minterms contain common literals
  - Minterms on the left and right sides contain y' and y respectively
  - Minterms in the top and bottom rows contain x' and x respectively

X \ Y	0	1
0	x'y'	x'y
1	xy'	xy

	y'	y
x'	x'y'	x'y
x	xy'	xy

Karnaugh Map Simplifications

- Imagine a two-variable sum of minterms:

$$x'y' + x'y$$

- Both of these minterms appear in the top row of a Karnaugh map, which means that they both contain the literal x'

		y
	$x'y'$	$x'y$
x	xy'	xy

- What happens if you simplify this expression using Boolean algebra?

$$\begin{aligned} x'y' + x'y &= x'(y' + y) && [\text{ Distributive }] \\ &= x' \bullet 1 && [\ y + y' = 1\ ] \\ &= x' && [\ x \bullet 1 = x\ ] \end{aligned}$$

More Two-Variable Examples

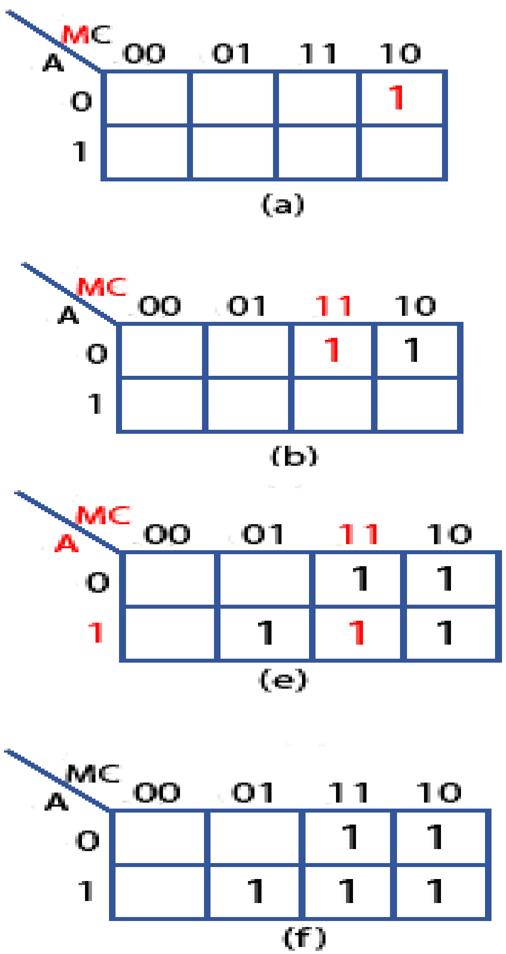
- Another example expression is $x'y + xy$
  - Both minterms appear in the right side, where y is uncomplemented
  - Thus, we can reduce $x'y + xy$ to just y

		y
	$x'y'$	$x'y$
X	xy'	xy

- How about $x'y' + x'y + xy$?
  - We have $x'y' + x'y$ in the top row, corresponding to x'
  - There's also $x'y + xy$ in the right side, corresponding to y
  - This whole expression can be reduced to $x' + y$

		y
	$x'y'$	$x'y$
X	xy'	xy

The Karnaugh map can be populated with data from either a truth table or a Boolean equation.



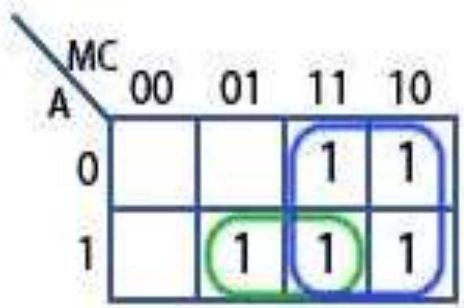


Table 2.4.1

A	M	C	X	Boolean
0	0	0	0	
0	0	1	0	
0	1	0	1	M
0	1	1	1	M•C
1	0	0	0	
1	0	1	1	A•C
1	1	0	1	A•M
1	1	1	1	A•M•C

$$X = M + A\bullet C$$

Karnaugh Map Rules

1. Groups should contain as many '1' cells (i.e. cells containing a logic 1) as possible and no blank cells.

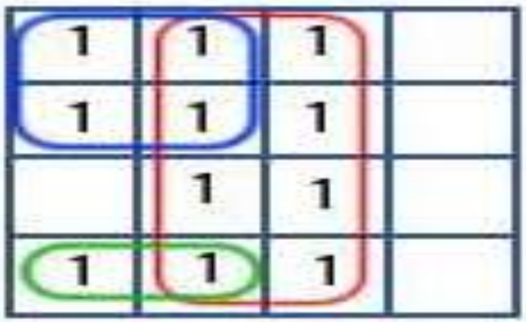
2. Groups can only contain 1, 2, 4, 8, 16 or 32... etc. cells (powers of 2).

3. A '1' cell can only be grouped with adjacent '1' cells that are immediately above, below, left or right of that cell; no diagonal grouping.

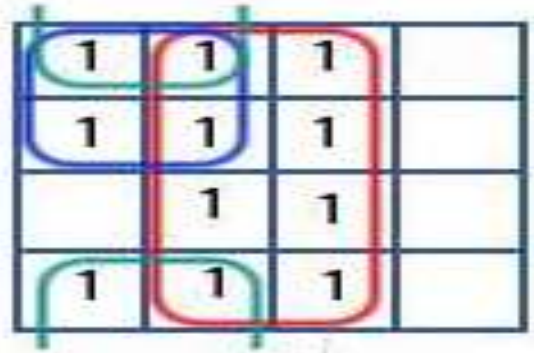
4. Groups of '1' cells can overlap. This helps make smaller groups as large as possible, which is an advantage in finding the simplest solution.

5. The top/bottom and left/right edges of the map are considered to be continuous, as shown in Fig. 2.4.3, so larger groups can be made by grouping cells across the top and bottom or left and right edges of the map.

6. There should be as few groups as possible.



(a)



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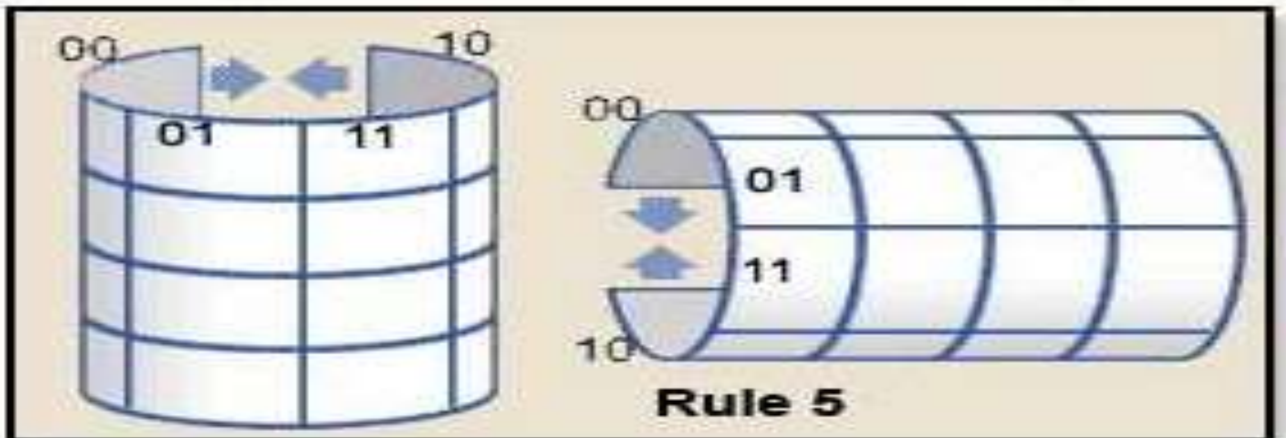

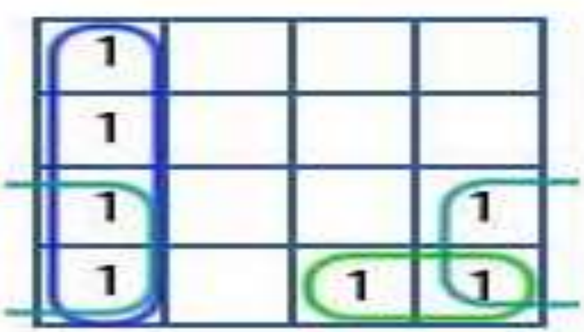

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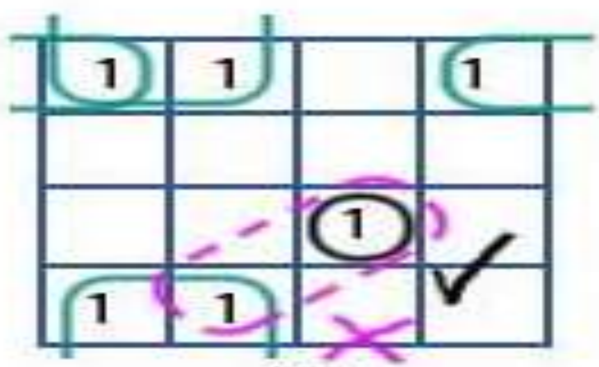


(d)

Simplification of SOP expression:

Example 1: Minimize the expression $y = A\bar{B}C + \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}C$

Solution: map plotted for the given variable.

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$ 0	1 (0)	1 (1)	1 (3)	0 (2)
A 1	1 (4)	1 (5)	0 (7)	0 (6)

$A\bar{B}C = 101$ $\quad A\bar{B}\bar{C} = 100$

$\bar{A}\bar{B}C = 001$ $\quad \bar{A}\bar{B}\bar{C} = 000$

$\bar{A}BC = 011$

$g_1 = \bar{B}$ $\quad g_2 = C\bar{A}$.

Ans: $y = C\bar{A} + \bar{B}$

Example 2: minimize the expression.

$$y = \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D}$$

AB \ CD	00	01	11	10
00	0	0	0	1
01	1	1	0	0
11	1	1	0	0
10	0	1	0	0

$\bar{A}B\bar{C}\bar{D} = 0100$

$\bar{A}B\bar{C}D = 0101$

$AB\bar{C}\bar{D} = 1100$

$AB\bar{C}D = 1101$

$A\bar{B}\bar{C}D = 1001$

$\bar{A}\bar{B}C\bar{D} = 0010$

$g_1 = B\bar{C}$ $\quad g_2 = A\bar{C}D$ $\quad g_3 = \bar{A}\bar{B}C\bar{D}$

Ans: $y = \bar{A}\bar{B}C\bar{D} + A\bar{C}D + B\bar{C}$

Example 3 :

Reduce the following four variables function to its minimum sum of products form.

$$y = \bar{A}\bar{B}C\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D} + A\bar{B}CD + A\bar{B}\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}\bar{D}.$$

Solution:

AB \ CD	00	01	11	10
00	1 ($_0$)	0 ($_1$)	1 ($_3$)	1 ($_2$)
01	0 ($_4$)	0 ($_5$)	0 ($_7$)	0 ($_6$)
11	1 ($_{12}$)	0 ($_{13}$)	0 ($_{15}$)	1 ($_{14}$)
10	1 ($_8$)	0 ($_9$)	1 ($_{11}$)	1 ($_{10}$)

$g_3 = \bar{B}\bar{D}$

$g_1 = A\bar{D}$ $\quad g_2 = \bar{B}C$.

$\bar{A}\bar{B}C\bar{D} = 0010$

$ABC\bar{D} = 1110$

$A\bar{B}C\bar{D} = 1010$

$A\bar{B}CD = 1011$

$A\bar{B}\bar{C}\bar{D} = 1000$

$AB\bar{C}\bar{D} = 1100$

$\bar{A}\bar{B}CD = 0011$

$\bar{A}\bar{B}\bar{C}\bar{D} = 0000$

Solution: $y = \bar{B}\bar{D} + A\bar{D} + \bar{B}C$.

Example 4:

Reduce the following function to its minimum sum of products form.

$$Y = \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D + ABCD + A\bar{B}CD$$

AB \ CD	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	1	1	1	0
10	0	0	1	0

$g_1 = \bar{A}\bar{C}D$

$g = \bar{A}BC$

$g_2 \; AB\bar{C}$

$g_3 = ACD$

$\bar{A}\bar{B}\bar{C}D = 0001 = 1$ $\quad AB\bar{C}D = 1101 = 13$

$\bar{A}B\bar{C}D = 0101 = 5$ $\quad ABCD = 1111 = 15$

$\bar{A}BCD = 0111 = 7$ $\quad A\bar{B}CD = 1011 = 11$

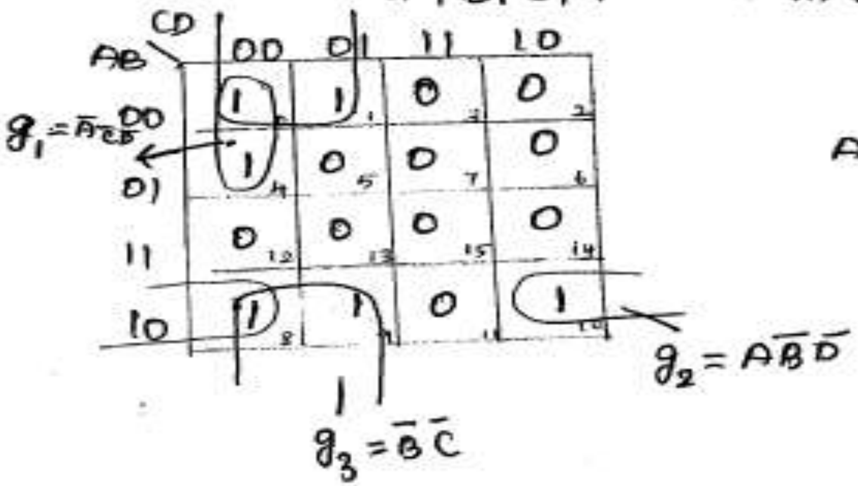
$\bar{A}BC\bar{D} = 0110 = 6$

$AB\bar{C}\bar{D} = 1100 = 12$

Answer: $\bar{A}BC + ACD + AB\bar{C} + \bar{A}\bar{C}D$

Reduce the following function using K-map techniques.

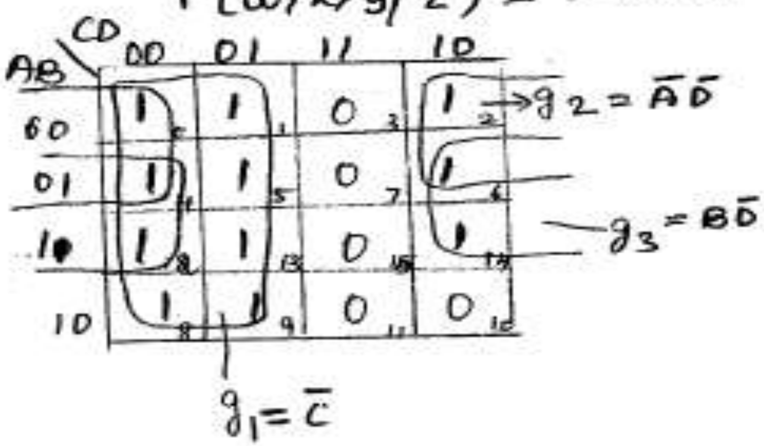
$F(A,B,C,D) = \Sigma m(0,1,4,8,9,10)$.



Ans: $F(A,B,C,D) = \bar{A}\bar{C}\bar{D} + \bar{B}\bar{C} + A\bar{B}\bar{D}$

Plot the following Boolean function on a karnaugh map and simplify it.

$F(w,x,y,z) = \Sigma(0,1,2,4,5,6,8,9,12,13,14)$.



Answer $= B\bar{D} + \bar{A}\bar{D} + \bar{C}$

$= x\bar{z} + \bar{w}\bar{z} + \bar{y}$

Incompletely specified functions (Don't care terms)

* In some logic circuits, certain input conditions never occur, therefore the corresponding output never appears.
* The output level is not defined, either high or low.
* indicated by 'x' or 'd' called don't care conditions or incompletely specified function.

Example:

$F(A,B,C,D) = \sum m(1,2,4,7,8) + d(10,11,12,13,14,15)$.

Minimization of incompletely specified functions.

* don't care condition, it is either '0' or '1'.

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	x
1	1	1	x

A \ BC	00	01	11	10
0	0	1	1	0
1	0	1	x	x

⇓

A \ BC	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$y = C$

Example: 1.

Find the reduced SOP form of the following function

$$F(A,B,C,D) = \sum m(1,3,7,11,15) + \sum d(0,2,4).$$

AB \ CD	00	01	11	10
00	X_{0}	1_{1}	1_{3}	X_{2}
01	X_{4}	$_{5}$	1_{7}	$_{6}$
11	$_{12}$	$_{13}$	1_{15}	$_{14}$
10	$_{8}$	$_{9}$	1_{11}	$_{10}$

AB \ CD	00	01	11	10
00	0	1	1	0
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

$g_1 = \bar{A}\bar{B} + CD.$

2. Reduce the following function using karnaugh's map

$$F(A,B,C) = \sum m(0,1,3,7) + \sum d(2,5).$$

A \ BC	00	01	11	10
0	1	1	1	X
1		X	1	

$\Rightarrow$

A \ BC	00	01	11	10
0	1	1	1	1
1		1	1	

$g_1 = \bar{A}$

$y = \bar{A} + C$

$g_2 = C$

3 Reduce the following function using Karnaugh map technique.

$$F(A,B,C,D) = \Sigma m(5,6,7,12,13) + \Sigma d(4,9,14,15).$$

AB \ CD	00	01	11	10
00	(0)	(1)	(3)	(2)
01	X (4)	1 (5)	1 (7)	1 (6)
11	1 (12)	1 (13)	X (15)	X (14)
10	(8)	X (9)	(11)	(10)

Ans. $f = B$.

AB \ CD	00	01	11	10
00				
01	1	1	1	1
11	1	1	1	1
10		0		

$g_1 = B$

4 Reduce the following function using Karnaugh map

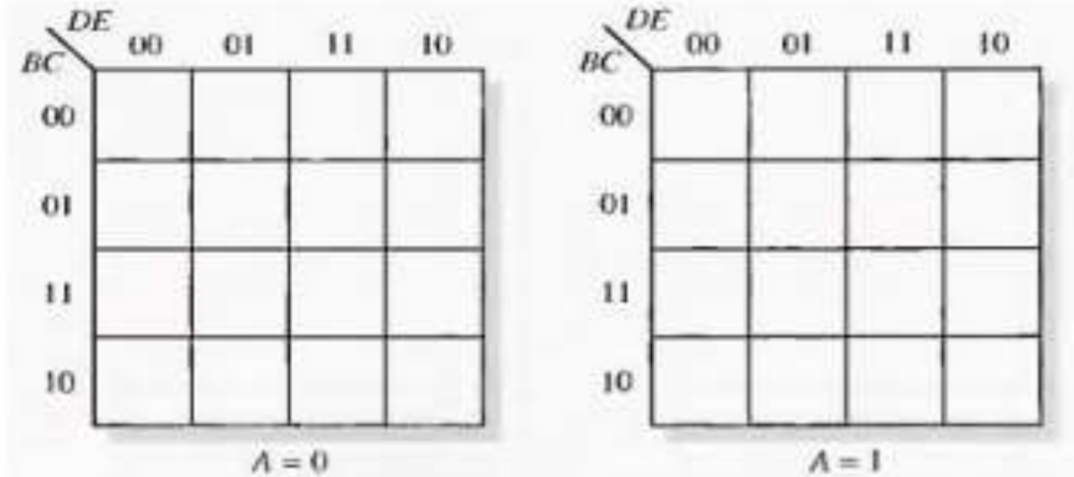
$$f(w,x,y,z) = \Sigma m(0,7,8,9,10,12) + \Sigma d(2,5,13).$$

AB \ CD	00	01	11	10
00	1			X
01		X	1	
11	1	X		
10	1	1		1

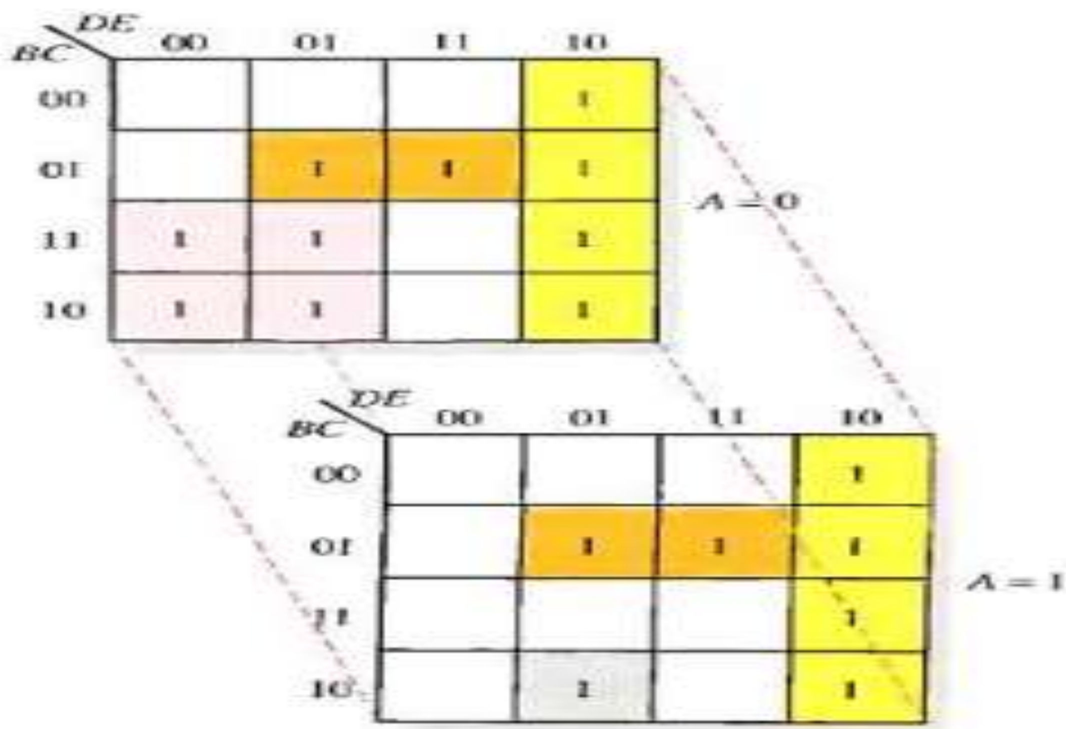
$g_3 = \bar{A}BD$

$g_1 = \bar{B}\bar{D}$ $g_2 = A\bar{C}$

$$f(w,x,y,z) = \bar{x}\bar{z} + w\bar{y} + \bar{w}xz$$



DE
BC
00 01 11 10
00
01
11
10
A = 0
A = 1

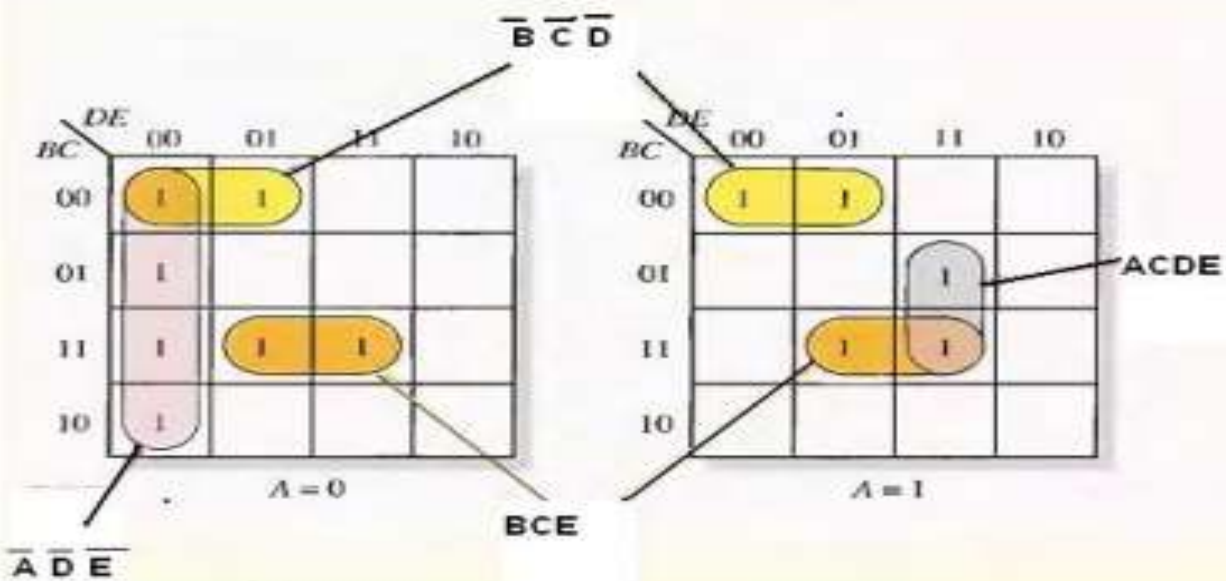


DE
BC
00 01 11 10
00
01
11
10
1
A = 0
A = 1

Example:

Use a Karnaugh map to minimize the following standard SOP 5-variable expression:

$$X = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\,\overline{E} + \overline{A}\,\overline{B}C\overline{D}\,\overline{E} + \overline{A}BC\overline{D}\,\overline{E} + \overline{A}BC\overline{D}E + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}E + \overline{A}B\overline{C}\,\overline{D}\,\overline{E}$$
$$+ \overline{A}BCDE + A\overline{B}\,\overline{C}\,\overline{D}\,\overline{E} + A\overline{B}\,\overline{C}\,\overline{D}E + ABC\overline{D}E + ABCDE + A\overline{B}CDE$$



$$X = \overline{A}\,\overline{D}\,\overline{E} + \overline{B}\,\overline{C}\,\overline{D} + BCE + ACDE$$

minimize the expression

$$y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+C)$$
$$(A+B+C).$$

A \ B+C	00	01	11	10
0	0_{0}	0_{1}	0_{3}	$_{2}$
1	0_{4}	$_{5}$	0_{7}	$_{6}$

$A+B+\bar{C} = 001 \Rightarrow M_1$

$A+\bar{B}+\bar{C} = 011 \Rightarrow M_3$

$\bar{A}+\bar{B}+\bar{C} = 111 \Rightarrow M_7$

$\bar{A}+B+C = 100 \Rightarrow M_4$

$A+B+C = 000 \Rightarrow M_0$

$g_1 = B+C$ $\quad g_2 = \bar{B}+\bar{C}$

$g_3 = \bar{A}+\bar{C}$

Answer: $y = (B+C)(\bar{B}+\bar{C})$
$(A+\bar{C})$

Minimize the following expressions in POS form.

$$y = (\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{C}+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})(\bar{A}+B+C+D)$$
$$(A+\bar{B}+\bar{C}+D)(A+\bar{B}+\bar{C}+\bar{D})(A+B+C+D)(\bar{A}+\bar{B}+C+\bar{D})$$

A+B \ C+D	00	01	11	10
00	0			
01			0	0
11	0	0	0	0
10	0			

$\bar{A}+\bar{B}+C+D = 1100 = 12$

$\bar{A}+\bar{B}+\bar{C}+D = 1110 = 14$

$\bar{A}+\bar{B}+\bar{C}+\bar{D} = 1111 = 15$

$\bar{A}+B+C+D = 1000 = 8$

$A+\bar{B}+\bar{C}+D = 0110 = 6$

$A+\bar{B}+\bar{C}+\bar{D} = 0111 = 7$

$A+B+C+D = 0000 = 0$

$\bar{A}+\bar{B}+C+\bar{D} = 1101 = 13$

$g_1 = \bar{B}+\bar{C}$

$g_2 = \bar{A}+\bar{B}$

$g_3 = (B+C+D)$

Ans: $y = (B+C+D)(\bar{B}+\bar{C})(\bar{A}+\bar{B})$

THANK YOU