AC CIRCUITS

- Alternating current is an electric current which periodically reverses direction and changes its magnitude continuously with time in contrast to direct current which flows only in one direction.
- A DC quantity is one which has a constant magnitude irrespective of time, but an alternating quantity (AC) is one which has a varying magnitude and angle with respect to time.

SOME IMPORTANT TERMS

1. Wave form

A wave form is the graph in which the instantaneous value of any quantity is plotted against time.

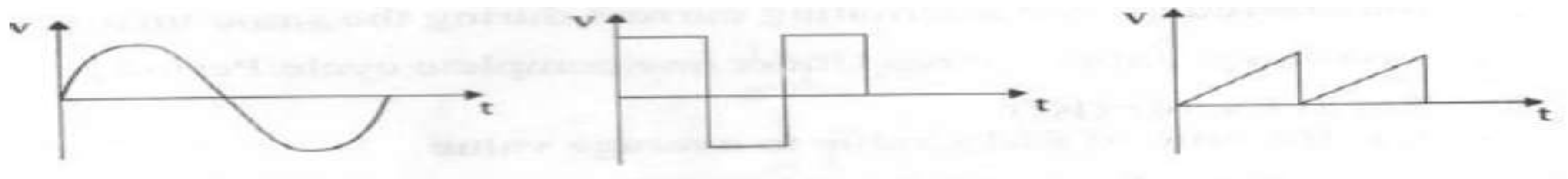


Figure 1(a) Sinusoidal waveform (b) Rectangular waveform (c) sawtooth waveform

2. Alternating Waveform

This is wave which reverses its direction at regularly recurring interval.

3. Cycle

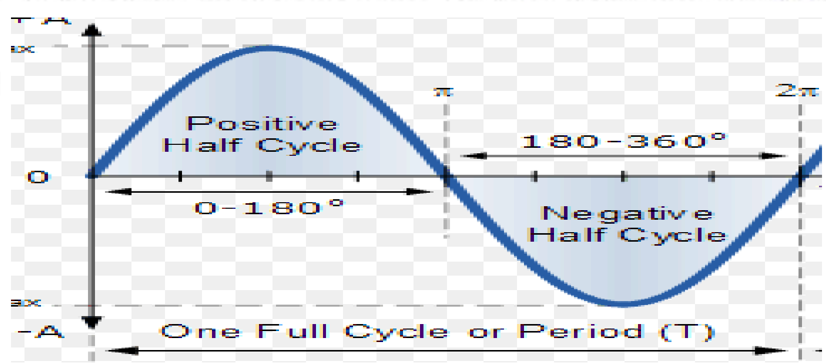


Figure 2

It is a set of positive and negative portion of waveforms.

4. **Time Period**

The time required for an alternating quantity to complete one cycle is called the time period and it denoted by T.

5. **Frequency**

The number of cycles per second is called frequency and is denoted by F.It is measured in cycles/second (cps) (or) Hertz

$$f=1/t$$

6. **Amplitude**

The maximum value of an alternating quantity in a cycle is called amplitude. It is also known as peak value.

7. **R.M.S value [Root Mean Square]**

The steady current when flowing through a given resistor for a given time produces the same amount of heat as produced by an alternating current when flowing through the same resistor for the same time is called R.M.S value of the alternating current.

$$\text{RMS Value} = \sqrt{\begin{matrix}\text{Area Under the square curve for} \\ \text{one complete cycle/Period}\end{matrix}}$$

8. **Average Value of AC**

The average value of an alternating current is defined as the DC current which transfers across any circuit the same change as is transferred by that alternating current during the same time.

Average Value = Area Under one complete cycle/Period.

9. **Form Factor (K_f)**

It is the ratio of RMS value to average value

Form Factor = RMS value/Average Value

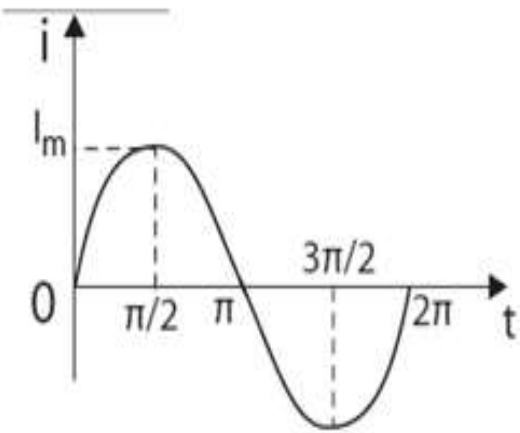
10. **Peak Factor (K_a)**

It is the ratio of Peak (or) maximum value to RMS value.

Peak Factor Ka=Peak Value/RMS value

ANALYTICAL METHOD FOR SINUSOIDAL CURRENT (OR) VOLTAGE

Full Wave Rectifier



Root Mean Square value

$$i = I_m \sin \omega t$$

$$I_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} i^2 d\theta \text{ [since it is symmetrical]}$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \sin^2 d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\pi$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Average Value

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi}\int_0^{\pi} \sin\theta \, d\theta$$

$$= -\frac{I_m}{\pi}[\cos\theta]_0^{\pi}$$

$$= -\frac{I_m}{\pi}[\cos\pi - \cos 0]$$

$$= -\frac{I_m}{\pi}(-1-1)$$

$$= \frac{2I_m}{\pi}$$

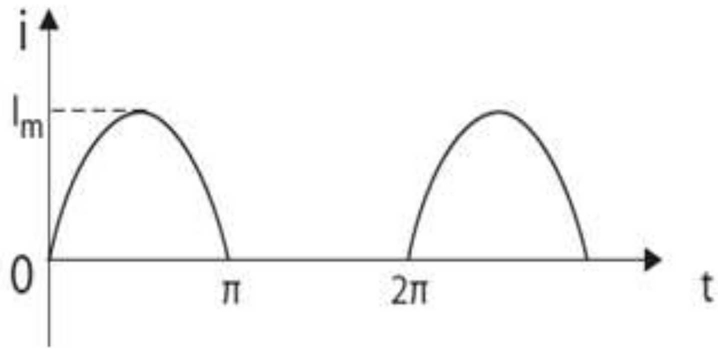
Form Factor

$$\text{Form Factor} = \frac{RMS}{Avg} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = 1.11$$

Peak Factor

$$\text{Peak Factor} = \frac{MAX}{RMS} = \frac{I_m}{RMS} = \frac{I_m}{\left(\frac{I_m}{\sqrt{2}}\right)} = 1.414$$

Half wave rectifier



RMS value

$I = I_m \sin\theta \qquad 0 < \theta < \pi$

$I = 0 \qquad \pi < \theta \leq 2\pi$

$$I_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$$

$$= \frac{1}{2\pi}\int_0^{\pi} i^2 d\theta + \int_{\pi}^{2\pi} i^2 d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i^2 d\theta + 0\right]$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2 d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{4\pi}\pi$$

$$I_{RMS} = \frac{I_m}{2}$$

Average Value

$$I_{av} = \frac{1}{2\pi}\int_0^{\pi} i\,d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i\,d\theta + 0\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{2\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= -\frac{I_m}{2\pi}[\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{2\pi}[\cos\pi - \cos 0]$$

$$= -\frac{I_m}{2\pi}(-1-1)$$

$$= \frac{2I_m}{2\pi} = \frac{I_m}{\pi}$$

- **Form Factor**

$$Form\ Factor = \frac{RMS}{Avg} = \frac{I_m}{2} / \frac{I_m}{\pi} = 1.57$$

- **Peak Factor**

$$Peak\ Factor = \frac{MAX}{RMS} = \frac{I_m}{RMS} / \frac{I_m}{\frac{I_m}{2}} = 2$$

Problem The equation of an alternating current is given by

$i = 40\sin 314\, t$

Determine

(i) Max value of current

(ii) Average value of current

(iii) RMS value of current

(iv) Frequency and angular frequency

(v) Form Factor

(vi) Peak Factor

Solution:

$$i = 40\sin 314\, t$$

We know that $i = I_m \sin \omega t$

So $I_m = 40$

$\omega = 314 \; rad/\sec$

(i) Maximum value of current = 40A

(ii) Average value of current

$$I_{Avg} = \frac{2I_m}{\pi} = \frac{2\times 40}{\pi} = 25.464A$$

(iii) RMS value of current

$$I_{Rms} = \frac{I_m}{\sqrt{2}} = \frac{40}{\sqrt{2}} = 28.28 \; Amp$$

(iv) Frequency $f = \dfrac{\omega}{2\pi} = \dfrac{314}{2\pi} \approx 50\,Hz$

(v) From Factor $= \dfrac{RMS}{Avg} = \dfrac{28.28}{25.46} = 1.11$

(vi) Peak Factor $= \dfrac{\text{Peak}}{\text{RMS}} = \dfrac{40}{28.28} = 1.414$

Phasor Representation of Sinusoidal varying alternating quantities

Phase

The phase is nothing but a fraction of time period that has elapsed from reference or zero position.

In Phase

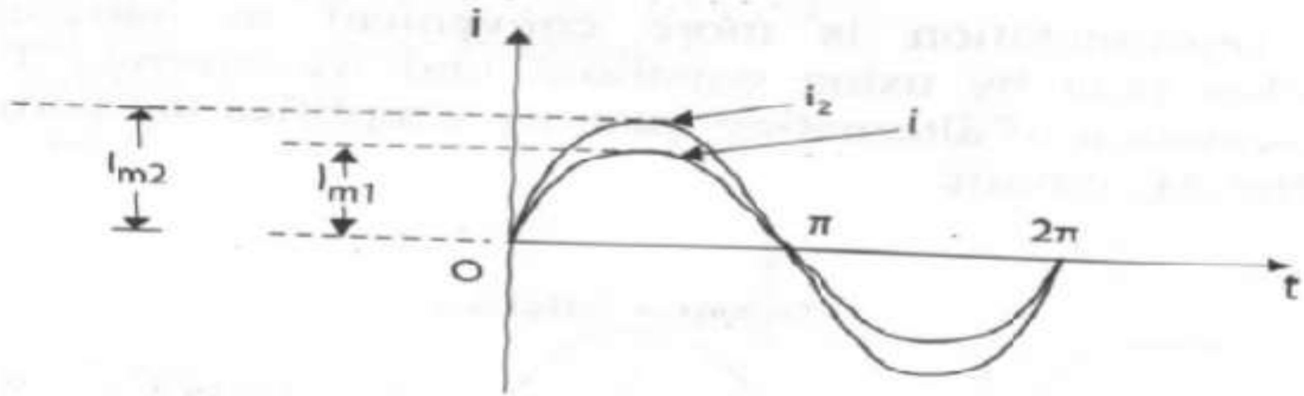
Two alternating quantities are said to be in phase, if they reach their zero value and maximum value at the same time.

Consider two alternating quantities represented by the equation

$i_1 = I_{m1} \sin\theta$

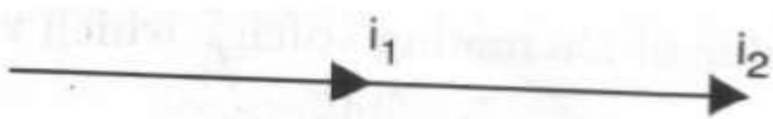
$i_2 = I_{m2} \sin\theta$

can be represented graphically as shown in Figure



Graphical representation of sinusoidal current

From above Figure it is clear that both i_1 and i_2 reaches their zero and their maximum value at the same time even though both have different maximum values. It is referred as both currents are in phase meaning that no phase difference is between the two quantities. It can also be represented as vector as shown in Fig .



Vector diagram

Out of Phase

Two alternating quantities are said to be out of phase if they do not reach their zero and maximum value at the same time. The Phase differences between these two quantities are represented in terms of '**lag**' and '**lead**' and it is measured in radians or in electrical degrees.

Lag

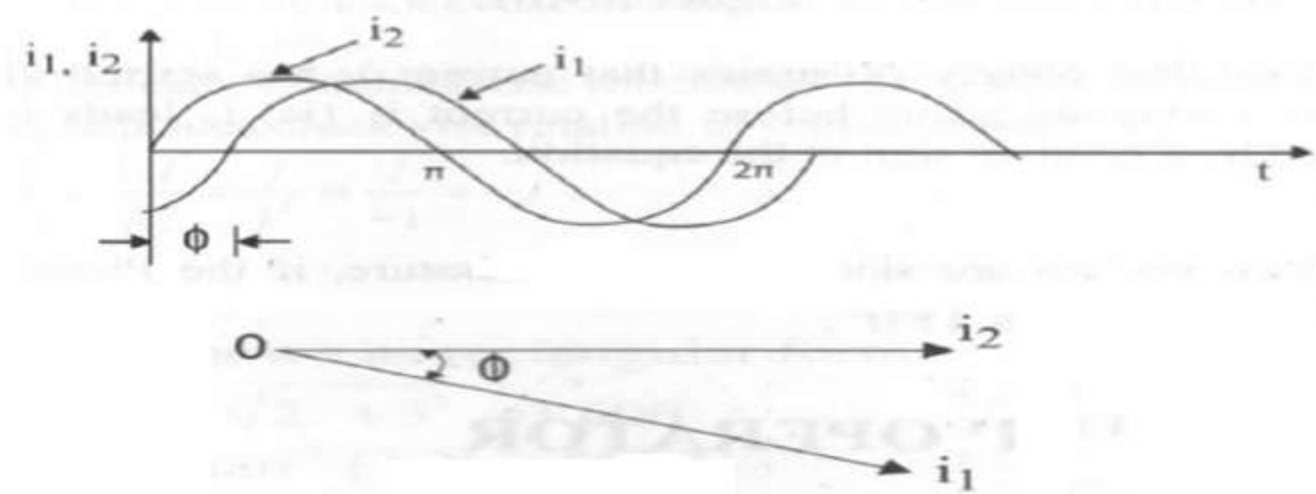
Lagging alternating quantity is one which reaches its maximum value and zero value later than that of the other alternating quantity.

Consider two alternating quantities represented by the equation:

$i_1 = I_{m1} \sin(\omega t - \Phi)$

$i_2 = I_{m2} \sin(\omega t)$

These equations can be represented graphically and in vector form as shown in Figure



Lead

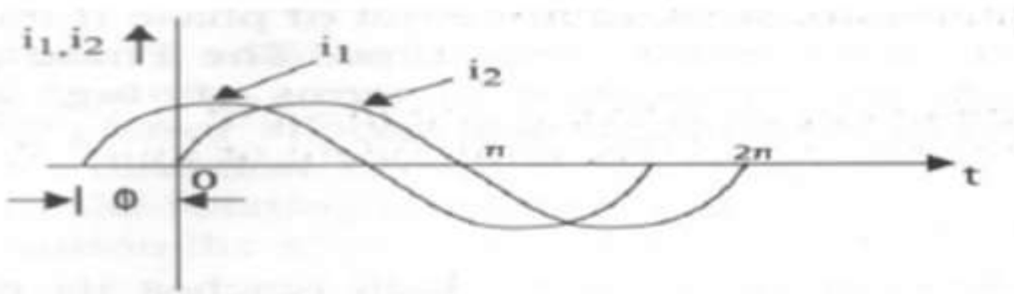
Leading alternating quantity is one which reaches its maximum value and zero value earlier than that of the other alternating quantity.

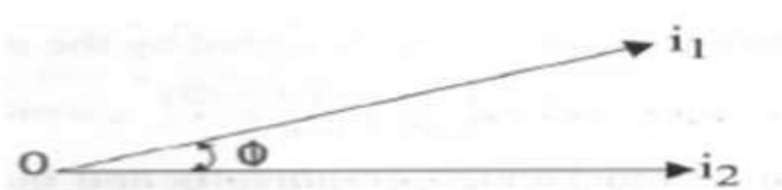
Consider two alternating quantities represented by the equation:

$i_1 = I_{m1} \sin(\omega t + \Phi)$

$i_2 = I_{m2} \sin(\omega t)$

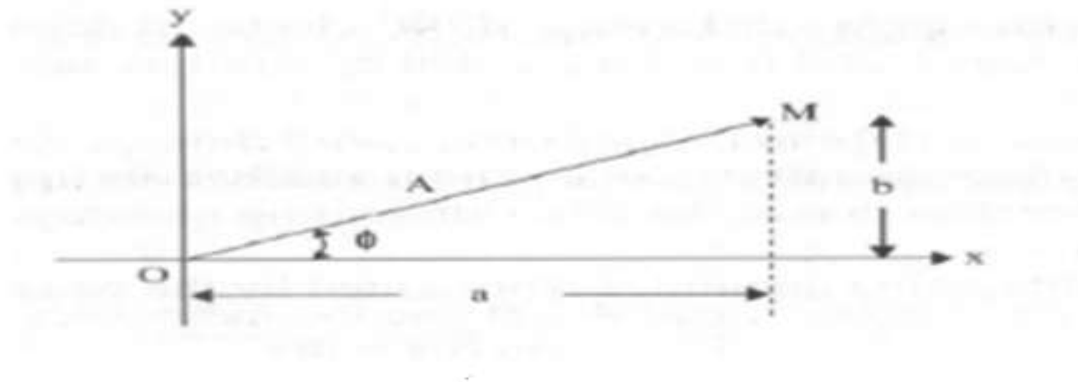
These equations can be represented graphically and in vector form as shown in Figure





Review of 'j' Operator

A vector quantity has both magnitude and direction. A vector' A' is represented in two axis plane as shown in Figure



In Figure OM represents vector A

Φ represents the phase angle of vector A

A = a + jb

a – Horizontal component or active component or in phase component

b – Vertical component or reactive component or quadrature component

The magnitude of vector 'A' = $\sqrt{a^2 + b^2}$

Phase angle of Vector 'A' = α = $\tan^{-1}$ (b/a)

$A = a + jb$

Features of j – Operator

1. $j = \sqrt{-1}$
   It indicates anticlockwise rotation of vector through 90°.
2. $j^2 = j \cdot j = -1$
   It indicates anticlockwise rotation of vector through 180°.
3. $j^3 = j \cdot j \cdot j = -j$
   It indicates anticlockwise rotation of vector through 270°.
4. $j^4 = j \cdot j \cdot j \cdot j = 1$
   It indicates anticlockwise rotation of vector through 360°.
5. $-j$ indicates clockwise rotation of vector through 90°.
6. $\frac{1}{j} = \frac{1.j}{j.j} = \frac{j}{j^2} = \frac{j}{-1} = -j$

:or can be written both in polar form and in rectangular form.

A = 2 + j3

This representation is known as rectangular form.

Magnitude of A = |A| = $\sqrt{2^2+3^2}$ =3.606

Phase angle of A = α = tan-1 (3/2) = 56°.31

A=|A| $\angle\alpha°$

A=3.606$\angle$56°.31

This representation is known as polar form.

Note:

1. Addition and Subtraction can be easily done in rectangular form.
2. Multiplication and division can be easily done in polar form.

Examples:

1. $A = 2 + j3$; $B = 4 + j5$.

Add Vector A and Vector B and determine the magnitude and Phase angle of resultant vector.

Solution:

$A + B = 2 + j3 + 4 + j5 = 6 + j8$

$\therefore$ Magnitude $= |A + B| = \sqrt{6^2 + 8^2} = 10.0$

Phase angle $= \alpha = \tan^{-1}(B/A) = \tan^{-1}(8/6) = 53°.13$

2. *A= 2 + j3 ; B = 4 – j5.*

Perform A x B and determine the magnitude and Phase angle of resultant vector.

Solution:

A= 2 + j3

$|A| = \sqrt{2^2 + 3^2}$ =3.606

α = tan^{-1} (3/2) = 56°.310

A= 3.606 ∠ 56°.310

B = 4 – j5

$|B| = \sqrt{4^2 + (-5)^2}$ =6.403

α = tan^{-1} (-5/4) = -51°.340

B= 6.403∠ -51°.340

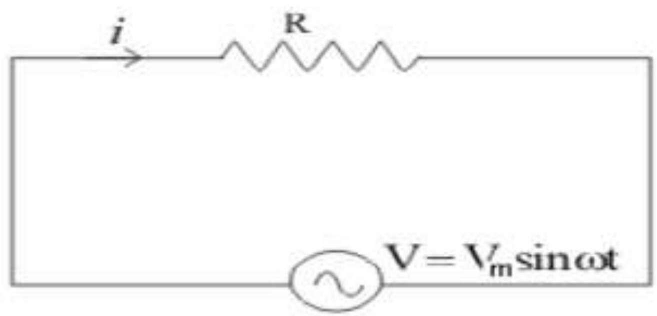
A X B = 3.606 ∠ 56°.310 X 6.403∠ -51°.340

= 3.606 X 6.403∠ (56°.310 + (-51°.340))

= 23.089 ∠4°.970

SINUSOIDAL EXCITATION APPLIED TO PURE RESISTANCE

Consider the circuit, in which a resistor of value R ohms is connected across an alternating voltage source.



Let the applied voltage across the resistance be $V=V_m \sin \omega t$

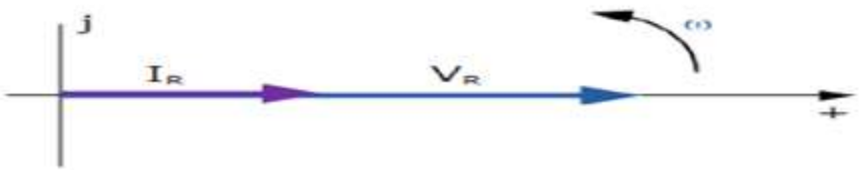
The resulting current has instantaneous value I by ohm's law $V = iR$

$$i = \frac{V}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$$

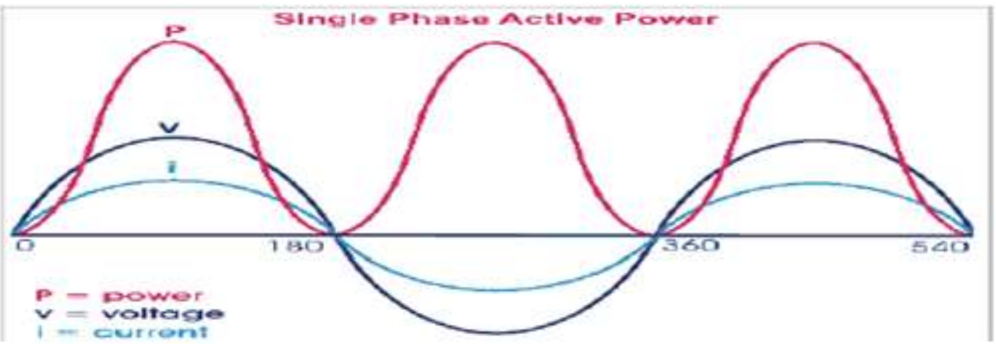
Where, $I_m = \frac{V_m}{R} = Peak\ value\ of\ the\ circuit\ current$

Comparing the voltage and the current we find voltage and the current are in phase with each other.

Phasor representation: In pure resistive circuit, no phase difference between the voltge and current (φ=0).



Waveform representation: Since the current and voltage are in phase, the waveforms reach their maximum and minimum values at the same instant.



Impedance:

$$Z = \frac{V}{I} = \frac{V_m \sin \omega t}{I_m \sin \omega t} = \frac{V_m}{I_m} = \frac{V_m}{V_m/_R} = R$$

Average Power:

$$P = Vi = V_m \sin \omega t \;\; I_m \sin \omega t = V_m I_m sin^2 \omega t$$

ωt=θ, $$P = V_m I_m sin^2 \omega t$$

Average power for one cycle $= \frac{V_m I_m}{\Pi} \int_0^{\Pi} sin^2\theta . d\theta = \frac{VmIm}{\pi} \int_0^{\pi} \frac{1-\cos 2\theta}{2} \, d\theta$

$$= \frac{VmIm}{2\pi} \left[0 - \frac{\sin 2\theta}{2} \right]_0^{\pi}$$

$$= \frac{VmIm}{2\pi} \left[\pi - \frac{\sin 2\pi}{2} - 0 + sin \frac{\theta}{2} \right]$$

$$= \frac{VmIm}{2\pi} . \pi = \frac{VmIm}{2} = \frac{Vm}{\sqrt{2}} . \frac{Im}{\sqrt{2}} = V . I$$

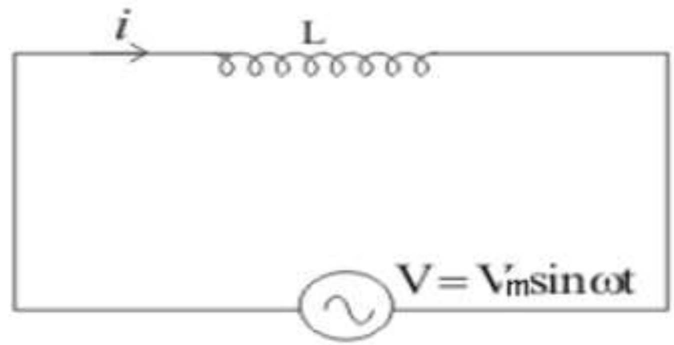
Average Power = VI watts

Power Factor: It is the cosine of the phase angle between voltage and current.

Cos Φ =cos 0= 1 (unity)

SINUSOIDAL EXCITATION APPLIED TO PURE INDUCTANCE

alternating voltage is applied across a pure inductor of self inductance L henry.



Let the applied voltage be $V = V_m \sin \omega t$

We know that the self induced emf always opposes the applied voltage. $V = L\frac{di}{dt}$

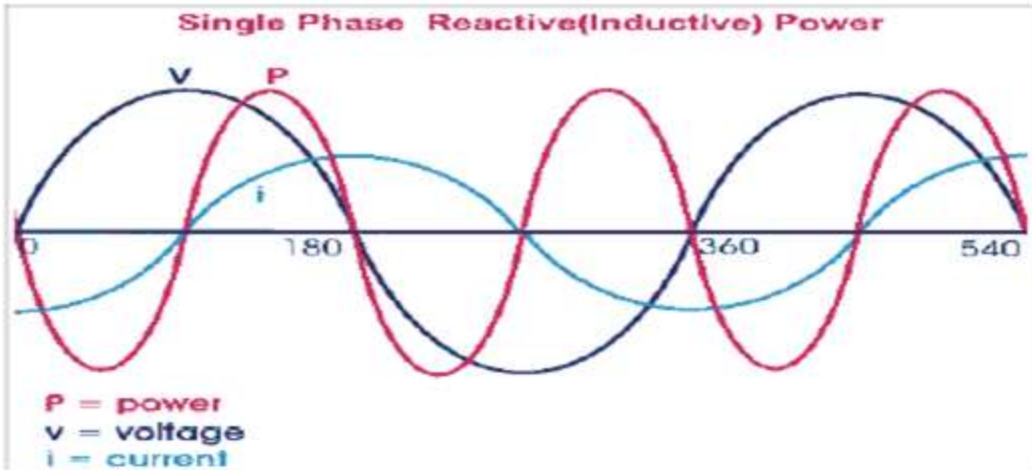
$$i = \frac{1}{L}\int V\,dt = \frac{1}{L}\int V_m \sin \omega t\,dt = \frac{V_m}{L}\left(\frac{-\cos \omega t}{\omega}\right) = \frac{-V_m}{L\omega}\cos \omega t$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

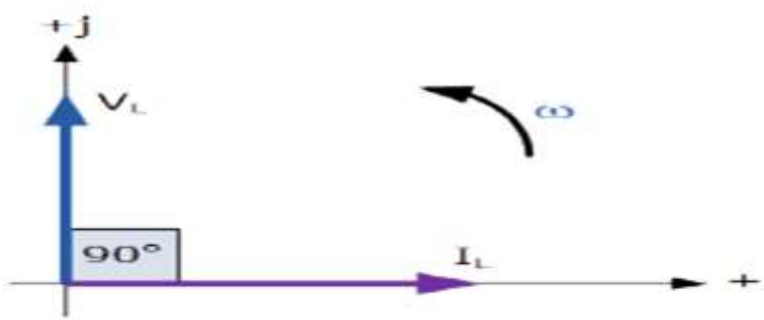
Here the current through the inductor lags the applied voltage by an angle 90°

<u>Waveform representation:</u> The current waveform is lagging behind the voltage waveform by 90°

Waveform representation: The current waveform is lagging behind the voltage waveform by 90°



Phasor representation: voltage is reference and current lags voltage by 90°



Impedence (Z):

$$Z = \frac{\text{Maximum value of v}}{\text{maximum value of I}} = \frac{Vm}{Im} = \frac{Vm}{Vm/L\omega}$$

$$Z = \omega L = inductive\ Reactance = X_l = \omega L = 2\pi f l$$

Power:

$$P = Vi = Vm \sin \omega t\ \ Im \sin\ (\omega t - \pi/2)$$

Average Power:

$$P = -\frac{1}{\pi}\int_0^{\pi} VmIm \sin\theta \cos\theta \, . d\theta = \frac{-1}{\pi}\int_0^{\pi} \frac{VmIm}{2} \sin 2\theta \, . d\theta$$

$$= \frac{VmIm}{2\pi}\left[\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{VmIm}{4\pi}(\cos 2\pi - \cos 0) = 0$$

Pure inductor does not consume any real power.

Power Factor: In the pure inductor the phase angle between the current and the voltage phasors is 90˚.

SINUSOIDAL EXCITATION APPLIED TO PURE CAPACITANCE

The capacitor of valve C farad is connected across an alternating voltage source. The voltage across the capacitance is $V = V_m \sin \omega t$.

The characteristic equation of the capacitor is $V = \frac{1}{c}\int i\,dt$

$$i = C\frac{dv}{dt} = C\frac{d}{dt}(Vm \sin \omega t)$$

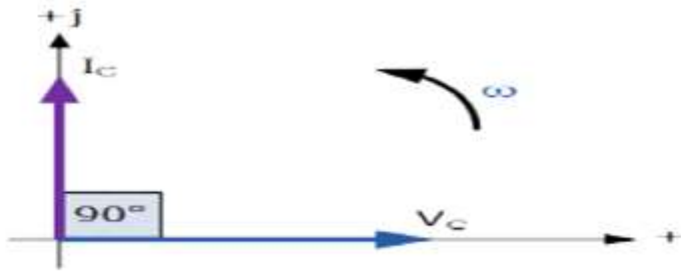
$$= \omega C\ Vm \cos \omega t \ = Im \cos \omega t = Im \cos \omega t$$

$$Im = \ \omega C\ .Vm$$

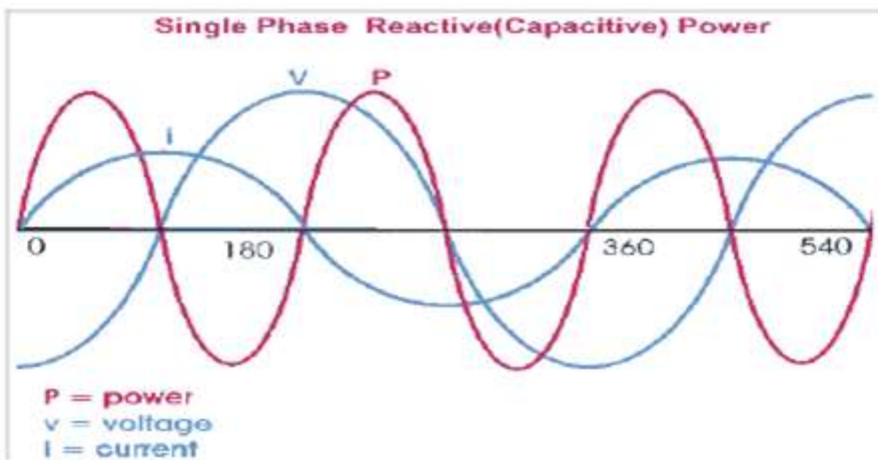
$$i = Im \sin(\omega t + 90^\circ)$$

We find that there is a phase difference of 90˚ between the voltage and the current in a pure capacitor. The current in a pure capacitor leads the applis voltage by an angle 90˚.

Phasor representation: voltage is reference and current leads voltage by 90°.



Waveform Representation: The current waveform is ahead of the voltage waveform by an angle of 90˚.



npedence(Z):

$$Z = \frac{\text{Maximum value of voltage}}{\text{maximum value of current}} = \frac{Vm}{Im} = \frac{Vm}{\omega C\,.\,Vm} = \frac{1}{\omega C} = XC$$

Capacitive Reactance = $Xc = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

Power:

Instantaneous power P= Vi

$$P = Vm \sin\theta \;\; Im\cos\theta = VmIm\sin\theta\cos\theta$$

Average Power:

$$P = \frac{1}{\pi}\int_0^{\pi} VmIm\sin\theta\cos\theta\,.\,d\theta = \frac{1}{\pi}\int_0^{\pi}\frac{VmIm}{2}\sin 2\theta\,.\,d\theta$$

$$= \frac{VmIm}{2\pi}\left[\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{VmIm}{4\pi}(\cos 2\pi - \cos 0) = 0$$

Pure capacitor do not consume any real power.

Power factor: The phase angle between the voltage and current is 90˚ lead.

1: A voltage of 240 sin 377t is applied to a 6Ω resistor. Find the instantaneous current, phase angle, impedance, instantaneous power, average power and power factor.

Solution:

Given: $v = 240 \sin 377t$

$V_m = 240$ V

$\omega = 377$ rad/sec

$R = 6\Omega$

Instantaneous current:

$$= \frac{V_m \sin\omega t}{R}$$

$$= \frac{240}{6}\sin 377t$$

$$= 40\sin 377t A$$

I. Phase angle:

$\phi = 0$

II. Impedance:

$Z = R = 6\Omega$

III. Instantaneous power:

IV.

$$p = V_m\ I_m\ \sin^2\omega t$$

$$= 240.40.\sin^2 377t$$

$$= 9600\sin^2 377t$$

V. Average power:

$$P = \frac{V_m I_m}{2} = 4800\text{watts}$$

VI. Power factor:

$\cos\Phi = \cos 0 = 1$

2: A voltage e = 200sinωt when applied to a resistor is found to give a power 100 watts. Find the value of resistance and the equation of current.

Solution:

Given: $e = 200\sin\omega t$

$V_m = 200$

$P = 100w$

Average power, $P = \dfrac{V_m I_m}{2}$

$100 = \dfrac{200 I_m}{2}$

$I_m = 1\ A$

Also, $V_m = I_m.R$

$R = 200\Omega$

Instantaneous current, $I = I_m \sin\omega t = 1.\sin\omega t\ A$

3: *A coil of wire which may be considered as a pure inductance of 0.225H connected to a 120V, 50Hz source. Calculate (i) Inductive reactance (ii) Current (iii) Maximum power delivered to the inductor (iv) Average power and (v) write the equations of the voltage and current.*

Solution:

Given: L = 0.225 H

$V_{RMS} = V = 120$ V

f = 50Hz

I. Inductive reactance, $X_L = 2\pi fL = 2\pi \times 50 \times 0.225 = 70.68\Omega$

II. Instantaneous current, $i = -I_m \cos\omega t$

$\because I_m = \frac{V_m}{\omega L} and V_{RMS} = \frac{V_m}{\sqrt{2}}$, calculate I_m and V_m

$V_m = V_{RMS}\sqrt{2}$ =169.71V

$$I_m = \frac{V_m}{\omega L} = \frac{169.71}{70.68} = 2.4A$$

Maximum power, $P_m = \frac{V_m I_m}{2} = 203.74$ W

III. Average power, P=0

IV. Instantaneous voltage, v = Vm sinωt = 169.71 sin 314t V

Instantaneous current, i = -2.4 cos 314t A

4: *A 135μF capacitor has a 150V, 50Hz supply. Calculate (i) capacitive reactance (ii) equation of the current (iii) Instantaneous power (iv)Average power (v) RMS current (vi) Maximum power delivered to the capacitor.*

Solution:

Given: V_{RMS}= V= 150V

C = 135μF

f = 50Hz

I. $X_C = \frac{1}{\omega C} = 23.58\Omega$

II. $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \because I_m - \omega C V_m$ and $V_{RMS} - \frac{V_m}{\sqrt{2}}$

$V_m = 150 X \sqrt{2} = 212.13V$

$I_m = 314 X 135 X 10^{-6} X 212.13 = 8.99A$

$i = 8.99 \sin\left(314t + \frac{\pi}{2}\right)$ A

III. p= $V_m I_m$ sinωt (cos ωt)=212.13X8.99 sin314t.cos314t

$$= 66642.6 \sin 314t.\cos 314t = 66642.6 \frac{\sin 628t}{2} \qquad [\because \sin 2\theta = 2\sin\theta\cos\theta]$$

$$= 33321.3 \sin 628t \text{ W}$$

IV. Average power, P = 0

V. $I_{RMS} = \frac{I_m}{\sqrt{2}}$ =6.36A

VI. $P_m = \frac{V_m I_m}{2}$ = 953.52 W

5: *A voltage of 100V is applied to a capacitor of 12µF. The current is 0.5 A. What must be the frequency of supply*

Solution:

Given: $V_{RMS} = V = 100V$

$C = 12\mu F$

$I = 0.5A$

I. Find V_m and I_m

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$V_m = 100 X \sqrt{2} = 141.42V$

$$I_{RMS} = \frac{I_m}{\sqrt{2}}$$

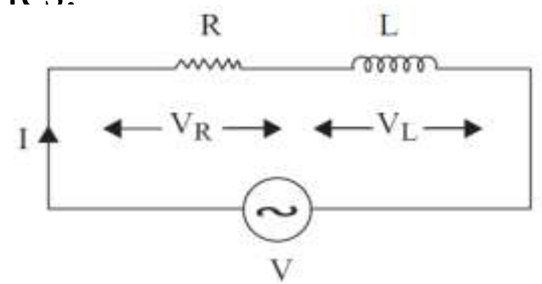
$I_m = 0.5 X \sqrt{2} = 0.707A$

II. $I_m = \omega C V_m = 2\pi f C V_m$

$f = 66.3Hz$

SINUSOIDAL EXCITATION APPLIED TO RL Series Circuit

Let us consider a circuit is which a pure resistance R and a purely inductive coil of inductance L are connected in series.



Let

$V=V_m$ Sinwt be the applied voltage.

I= Effective Value of Circuit Current.

V_R= Potential difference across inductor.

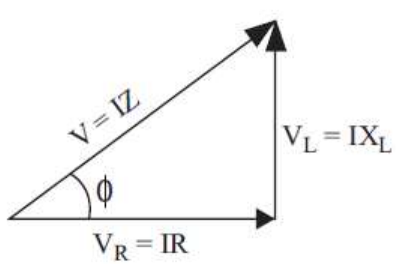
V_L= Potential difference across inductor. F= Frequency of applied voltage.

The same current I flows through R and L hence I is taken as reference vector.

Voltage across resistor V_R= IR in phase with I

Voltage with inductor V_L= IX_L leading I by 90°

At any constant, applied voltage



Phasor Diagram

At any instant, applied voltage

$$V = V_R + V_L$$
$$V = IR + jIX_L$$
$$V = I\,(R + jX_L)$$
$$\frac{V}{I} = R + jX_L$$
$$= z \text{ impedance of circuit}$$
$$Z = R + j\,X_L$$
$$|z| = \sqrt{R^2 + X_L^2}$$

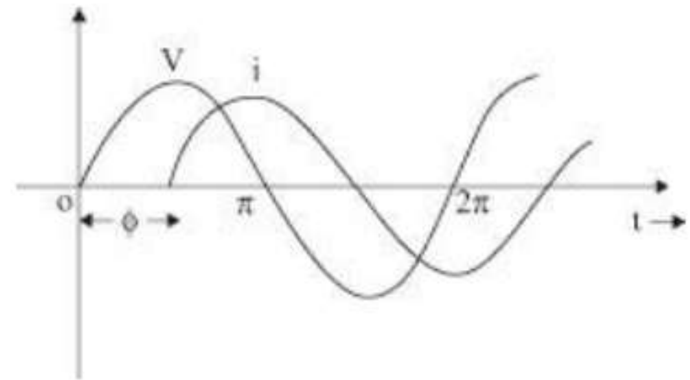
From phasor disgram,

$$\tan\phi = \frac{X_L}{R}$$

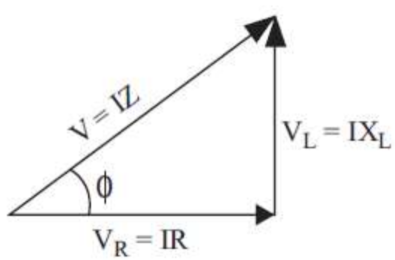
$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

ϕ is called the phasor angle and it is the angle between V and I, its v between 0 to 90^0.

So impedence $Z = R + jX_L$
$= |Z| \angle \phi$



Waveform Representation



Phasor Diagram

$V = V\mathbf{m} \sin\omega t$
$I = I\mathbf{m} \sin(\omega t - \phi)$
The current I lags behind the applied voltage V by an angle ϕ.
From phasor diagram,
Power factor cos ø=R/Z
Actual Power P = VI cos ø = Current component is phase with voltage
Reactive or Quadrature Power
Q = VI sinϕ = Current component is quadrature with voltage
Complex or Apparent Power
S = VI = Product of voltage and current
S = P + jQ

6:A coil having a resistance of 6Ω and an inductance of 0.03 H is connected across a 100V, 50Hz supply, Calculate.

(i) The current

(ii) The phase angle between the current and the voltage

(iii) Power factor

(iv) Power

Solution:

$R = 6\Omega$

$L = 0.03\ H$

$X_L = 2\pi fL$

$X_L = 2\pi \times 50 \times 0.03$

$X_L = 9.42\Omega$

$$|Z| = \sqrt{(R)^2 + (X_L)^2}$$

$$= \sqrt{(6)^2 + (9.42)^2}$$

$$|Z| = 11.17\Omega$$

$$\text{(i) } I = \frac{V}{Z} = \frac{100}{11.17} = 8.95 \text{ amps}$$

$$\text{(ii) } \phi = tan^{-1}\left(\frac{X_L}{R}\right)$$

$$= tan^{-1}\left(\frac{9.42}{6}\right)$$

$\Phi = 57.5$ (lagging)

(iii) Power factor $= \cos \phi$

$= \cos 57.5$

$= 0.537$ (lagging)

(iv) Power = Average power

$= VI \cos \Phi$

$= 100 \times 8.95 \times 0.537$

Power = 480.6 Watts

7: 10Ω resistor and a 20 mH inductor are connected is series across a 250V, 60 Hz supply. Find the impedence of the circuit, Voltage across the resistor, voltage across the inductor, apparent power, active power and reactive power.

Solution:

R = 10Ω

L = 20 mH = 20×10-3H

XL = 2πfL

= 2π×60×20×10-3

XL = 7.54Ω

(i) $Z = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 7.54^2} = 12.5\Omega$

(ii) $I = \frac{V}{Z} = \frac{250}{12.5} = 20A$

$V_R = IR = 20\times10 = 200$ volts

(iii) $V_L = I X_L = 20\times7.54 = 150.8$ volts

(iv) Apparent power S = VI

= 250×20

S= 5000VA

$\cos\emptyset = \frac{R}{Z} = \frac{10}{12.5} = 0.8$(Lagging)

Active power = VI cos ϕ

=250×20×0.8

P = 4000 Watts

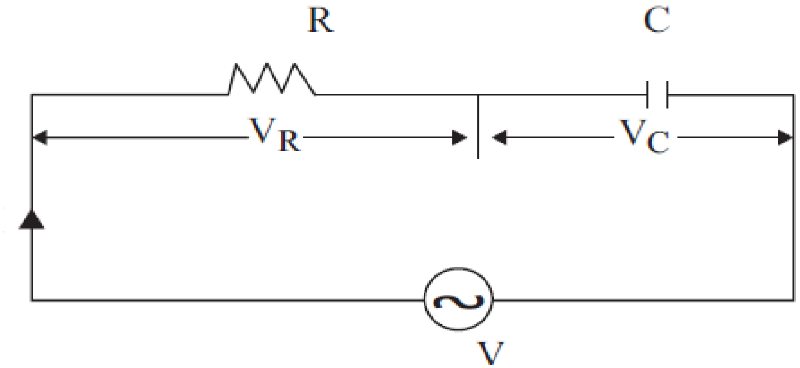
$\sin\phi = \sqrt{1-\cos^2\Phi} = \sqrt{1-(0.8)^2} = 0.6$

Reactive Power Q = VI sin ϕ

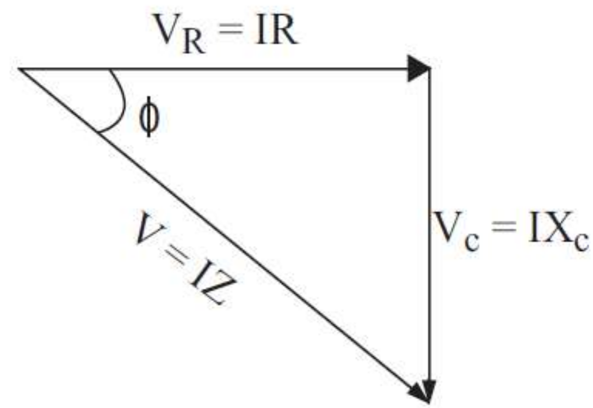
= 250×20×0.6

Q= 3000 KVAR

SINUSOIDAL EXCITATION APPLIED TO RC Series Circuit



Phasor Diagram of RC series circuit is,



Voltage across R = V_R = IR in phase with I

Voltage across C = $V_c = IX_c$ lagging I by 90^0

Applied voltage $V = IR - jIX_c$

$$= I(R - jx_c)$$

$$\frac{V}{I} = R - jX_c = Z$$

Z – Impedence of circuit

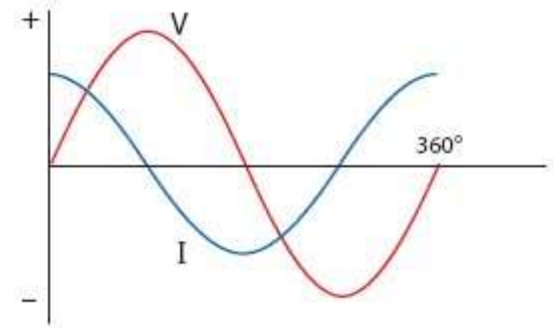
$$|Z| = \sqrt{R^2 + X_c^2}$$

$$\tan\phi = \frac{X_c}{R} = \frac{1/\omega c}{R} = \frac{1}{\omega c R}$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega c R}\right)$$

SINUSOIDAL EXCITATION APPLIED TO RC Series Circuit

ϕ is called Phase angle and it is angle between V and I. Its value lies between 0 and – 90°.



Waveform Representation

$V = V_m \sin \omega t$

$I = I_m \sin (\omega t + \phi)$

The current I leads the applied voltage V by an angle ϕ.

From Phasor Diagram,

Power factor $\cos\phi = \dfrac{R}{Z}$

Actual or real power $P = VI \cos\phi$

Reactive or Quardrature power $Q = VI \sin\phi$

Complex or Apparent Power $S = P + jQ$

$= VI$|

8:A capacitor having a capacitance of 10 μF is connected in series with a non-inductive resistor of 120Ω across 100V, 50HZ calculate the current, power and the Phase Difference between current and supply voltage.

C = 10 μF

R = 120Ω

V = 100V

F = 50Hz

$$X_c = \frac{1}{2\pi fc} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}}$$

$$= 318\,\Omega$$

$$|Z| = \sqrt{R^2 + X_c^{\,2}}$$

$$= 340\,\Omega$$

$$|I| = \frac{|V|}{|Z|}$$

$$= \frac{100}{340}$$

$$= 0.294\, amps$$

$$Phase Difference\, \phi = \tan^{-1}\left(\frac{X_c}{R}\right)$$

$$= \tan^{-1}\left(\frac{318}{120}\right)$$

$$\phi = 69.3^\circ \quad (Leading)$$

$$\cos\phi = \cos(69.3)^\circ$$

$$= 0.353 \quad (Leading)$$

$$Power = |V||I|\cos\phi$$

$$= 100 \times 0.294 \times 0.353$$

$$= 10.38\; Watts$$

9:A Capacitor and Resistor are connected in series to an A . C . Supply of 60volts,50 Hz. The current is 2A and the power dissipated in the Resistor is 80Watts. Calculate the Impedance, Resistance, Capacitance and Power factor

$|V|=60V \quad f=50Hz \quad |I|=2A$

Power Dissipated $=P=80W$

$$|Z| = \frac{|V}{|I|} = \frac{60}{2} = 30\ \Omega$$

$$P=I^2R \quad R = \frac{P}{I^2} = \frac{80}{4} = 20\Omega$$

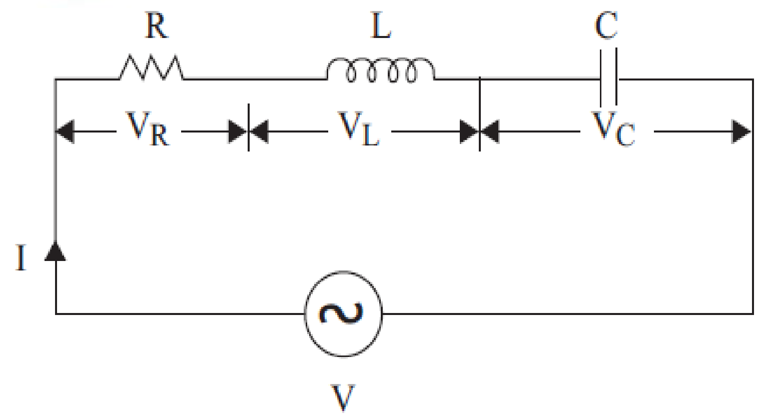
$$Z = \sqrt{R^2 + X_C^2}$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{30^2 - 20^2} = 22.36\Omega$$

$$X_C = \frac{1}{2\pi f_c} = 22.36\Omega \qquad C=\frac{1}{2\pi\times50\times22.36} \qquad C=142\times 10^{-6}\ F$$

$$\text{Power Factor} = \cos\Phi = \frac{R}{|Z|} = \frac{20}{30} = 0.67(\text{Leading})$$

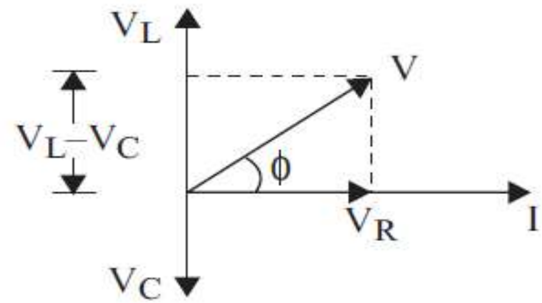
SINUSOIDAL EXCITATION APPLIED TO RLC Series Circuit



Assume Inductive reactance > Capacitive reactan

$X_L > X_C$

Then $V_L > V_C$



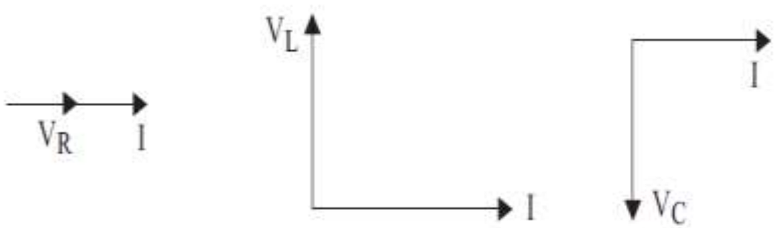
- **Phasor diagram**

  Take I as reference

  $V_R = I \times R$

  $V_L = I \times X_L$

  $V_C = I \times X_C$



For pure Resistance   For pure Inductance   For pure Capacitance

$$|V|^2 = |V_R|^2 + (|V_L| - |V_C|)^2$$

$$= |IR|^2 + (|IX_L| - |IX_C|)^2$$

$$= |I|^2 \, [R^2 + (X_L - X_C)^2]$$

$$|V| = |I| \sqrt{R^2 + (X_L - X_C)^2}$$

$$|Z| = \frac{|V|}{|I|}$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + X^2} \qquad \because X = (X_L - X_C)$$

RLC Series Circuit

- **Three cases of Z**

<u>Case 1</u> If $X_L > X_C$

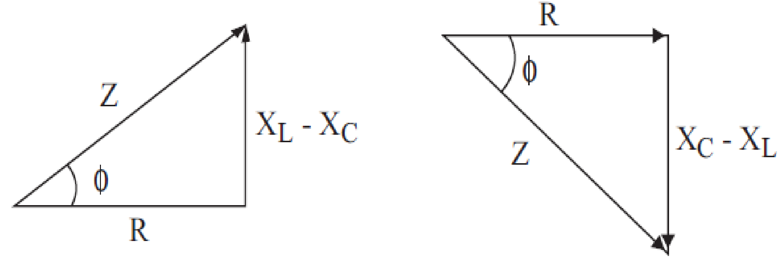
The circuit behaves like RL circuit. Current lags behind voltage. So powe factor is lagging.

<u>Case 2</u> If $X_L < X_C$

The circuit behaves like RC circuit Current leads applied voltage. So powe factor is leading.

<u>Case 3</u> When $X_L = X_C$, the circuit behaves like pure resistive circuit. Current is in phase with the applied voltage. So power factor is unity.

Impedance triangle



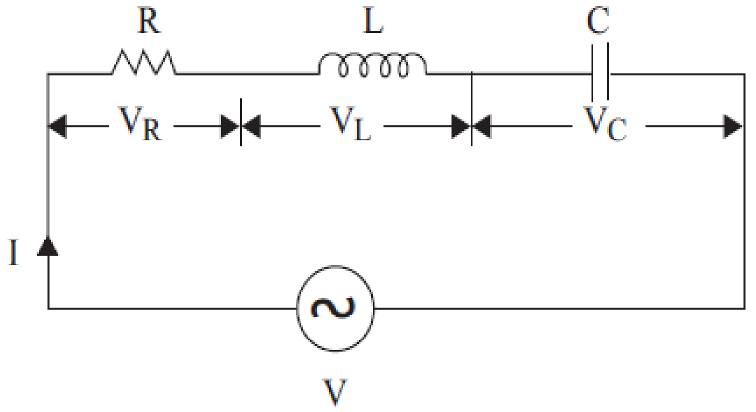
1. If applied voltage
   $v = V_m \sin \omega t$ and ϕ is phase angle then 'i' is g
   1) $i = I_m \sin(\omega t - \phi)$, for $X_L > X_C$
   2) $i = I_m \sin(\omega t + \phi)$, for $X_L < X_C$

   3) $i = I_m \sin \omega t$ for $X_L = X_C$
2. Impedance for RLC series circuit in complex form (or) rectangu form is given by

   $Z = R + j\,(X_L - X_C)$

10:In a RLC series circuit, the applied voltage is 5V. Drops across the resistance and inductance are 3V and 1V respectively. Calculate the voltage across the capacitor. Draw the phasor diagram.

$$V^2 = V_R^2 + (V_L - V_C)^2$$



$$(V_L - V_C)^2 = V^2 - V_R^2$$

$$= 25 - 9 = 16$$

$$V_L - V_C = \pm 4$$

$$V_C = V_L \pm 4 = 1 + 4$$

$$V_C = 5V$$

11:A coil of resistance 10Ω and in inductance of 0.1H is connected in series with a capacitance of 150μF across a 200v, 50HZ supply. Calculate

a) the inductive reactance of the coil.

b) the capacitive reactance

c) the reactance

d) current

e) power factor

$R = 10\Omega$

$L = 0.1\ H$

$C = 150\ \mu F = 150 \times 10^{-6}\ F$

$V = 200V \quad f = 50\ Hz$

Solution

a) $X_L = 2\pi fL = 2\pi\ (50)\ 0.1$

$= 31.4\ \Omega$

b) $$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi(50)(150\times10^{-6})}$$

$= 21.2\Omega$

c) the reactance $X = X_L - X_C$

$= 31.4 - 21.2$

$= 10.2\ \Omega$ (Inductive)

$$|Z| = \sqrt{R^2 + X^2}$$

$$= \sqrt{10^2 + (10.2)^2}$$

$$= 14.28\Omega (Inductive)$$

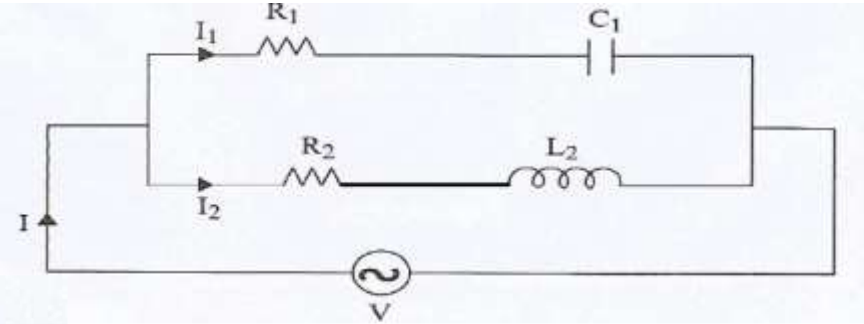
$$I = \frac{|V|}{|Z|} = \frac{200}{14.28} = 14 amps$$

e)

$$P.F = \cos\phi = \frac{R}{|Z|} = \frac{10}{14.28}$$

= 0.7 (lagging) (I lags behind V)

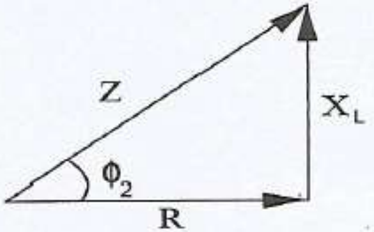
PARALLEL AC CIRCUITS



$$X_{C1} = \frac{1}{2\pi f c_1} = \frac{1}{\omega c_1}$$

$$X_{L2} = 2\pi f L_2 = \omega L_2$$

Impedance $|Z_1| = \sqrt{R_1^{\,2} + X_{C1}^{\,2}}$



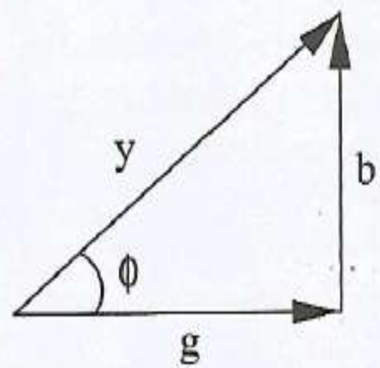
Impedance triangle

$$\phi_1 = \tan^{-1}\left(\frac{X_{C1}}{R_1}\right)$$

$$|Z_2| = \sqrt{R_2^{\,2} + X^2_{\;L2}}$$

$$\phi_2 = \tan^{-1}\left(\frac{X_{L2}}{R_2}\right)$$

Conductance = g
Susceptance = b
Admittance = y



Admittance triangle

Branch 1

Conductance $g_1 = \dfrac{R_1}{|Z_1|^2}$

$$b_1 = \frac{X_{C1}}{|Z_1|^2} \; (positive)$$

$$|Y_1| = \sqrt{g_1^{\,2} + b_1^{\,2}}$$

Branch 2

$$g_2 = \frac{R_2}{|Z_2|^2}$$

$$b_2 = \frac{X_{L_2}}{|Z_2|^2} \text{ (Negative)}$$

$$|Y_2| = \sqrt{g_2^{\,2} + b_2^{\,2}}$$

Total conductance $G = g_1 + g_2$

Total Suceptance $B = b_1 - b_2$

Total admittance $|Y| = \sqrt{G^2 + B^2}$

Branch current $|I_1| = |V||Y_1|$

$|I_2| = |V||Y_2|$

$|I| = |V||Y|$

Phase angle $\phi = tan^{-1}\left(\dfrac{B}{G}\right)$ lag if B-is negative

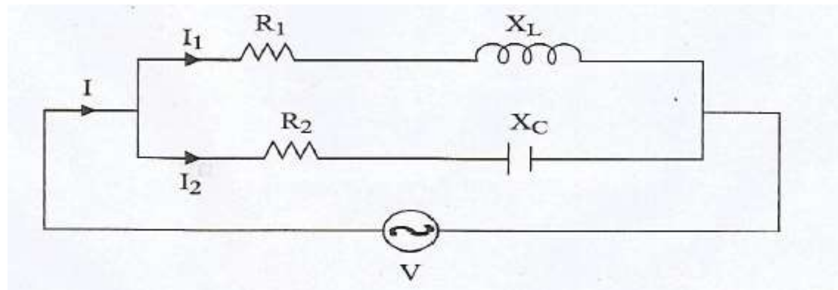
Power factor $\cos\phi = \dfrac{|G|}{|Y|}$

12:Two impedances of parallel circuit can be represented by (20 + j15) and (10 – j60) Ω. If the supply frequency is 50 Hz, find the resistance, inductance & capacitance of each circuit.

$Z_1 = 20 + j15\ \Omega$

$Z_2 = 10 - j60\ \Omega$

F = 50 Hz



Solution:

$Z_1 = R_1 + jX_L$

$Z_2 = R_2 - jX_C$

J term positive for in inductive

J term negative for capacitive.

For circuit 1, $R_1 = 20\Omega$

$X_1 = X_L = 2\pi fL = 2\pi\ (50)\ (L)$

$X_L = 15$

$2\pi\ (50)\ L = 15$

$$L = \frac{15}{2\pi(50)}$$

L = 48 mH

For circuit 2

$Z_2 = 10 - j60$

$R_2 = 10$

$X_2 = X_C = 60\ \Omega$

ie, $\frac{1}{2\pi fC} = 60$

$$C = \frac{1}{2\pi(50)60}$$

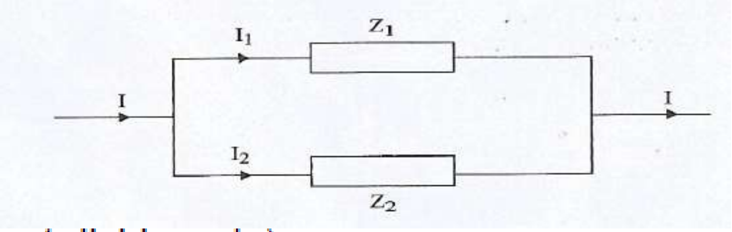
C = 53 μF.

13:Two circuits, the impedances of which are Z1 = (10 + j15) Ω and Z2 = (6 – j8) Ω are connected in parallel. If the total current supplied is 15A. What is the power taken by each branch

$$Z1 = (10 + j15)\Omega = 18.03\angle 56.3$$

$$Z2 = (6 - j8)\Omega = 10\angle -53.13$$

I =15 A



$$I_1 = I\frac{Z_2}{Z_1 + Z_2}$$ (Current divider rule)

$$= \frac{15\angle 0^0 \times 10\angle -53.13^0}{16 + j7}$$

($Z_1 + Z_2$ = 10 + j15 + 6–j8)

$$I_1 = \frac{150\angle -53.13^0}{17.46\angle 23.63}$$

$$I_1 = 8.6\angle -76.76\ A$$

By KCL $I_2 = I - I_1$

$$= 15\angle 0 - 8.6\angle -76.76$$

= 15 – (1.97 – j8.37)

= 15.5 – 32.7A

Power taken by branch 1

= power dissipated in resistance of branch 1

$$= |I_1|^2 R_1 = (8.6)^2 \times 10$$

=739.6 watts

Power taken by branch 2

$$= |I_2|^2 R_2$$

$$= (15.5)^2 \times 6$$

=1442 watts

THANK YOU