Lecture session-UNIT 2

Topic: canonical form, Multilevel

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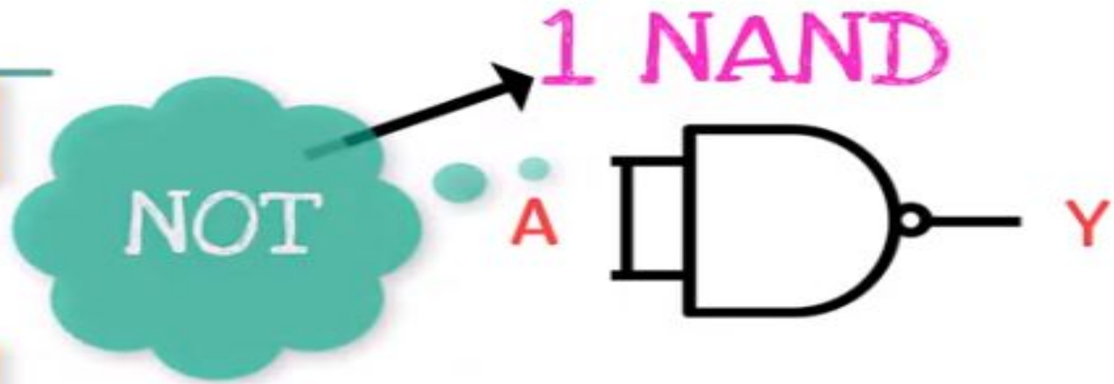
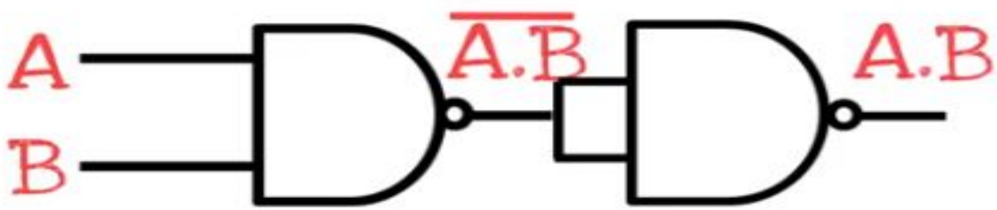
CHENNAI-119

Constructing NOT gate using NAND gate

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

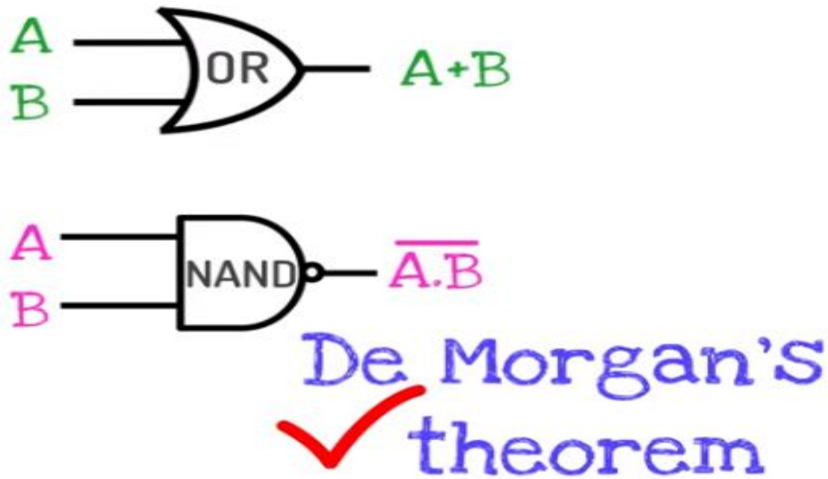


Constructing AND gate using NAND gate

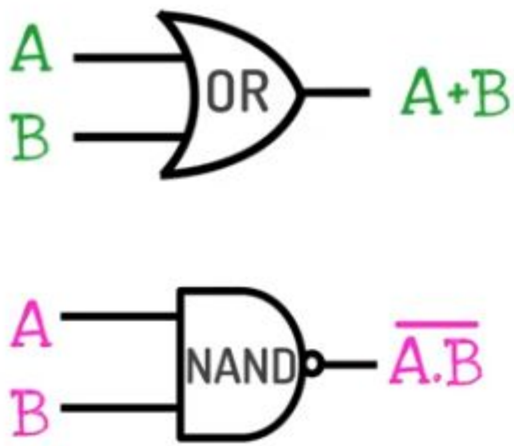


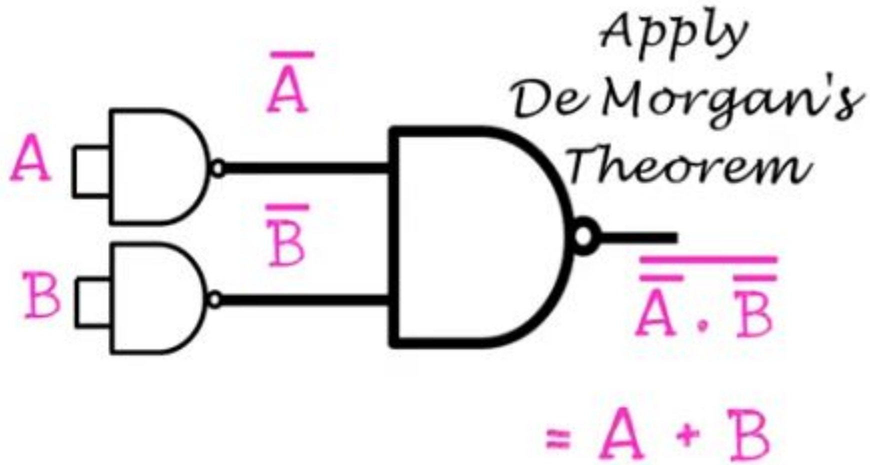
Using 2 NAND gates → AND

Constructing OR gate using NAND gate



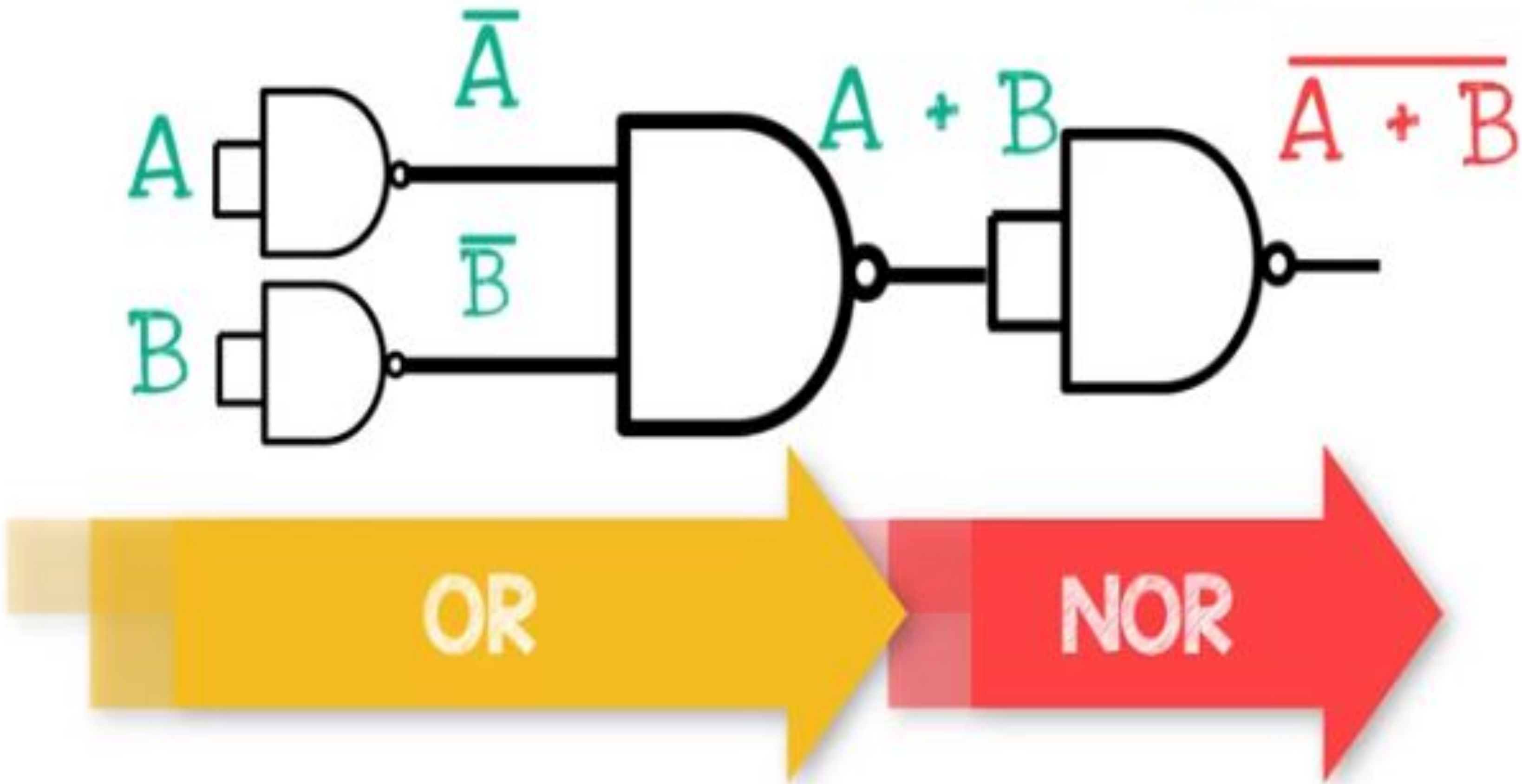
CONNECT OR gate with AND gate??



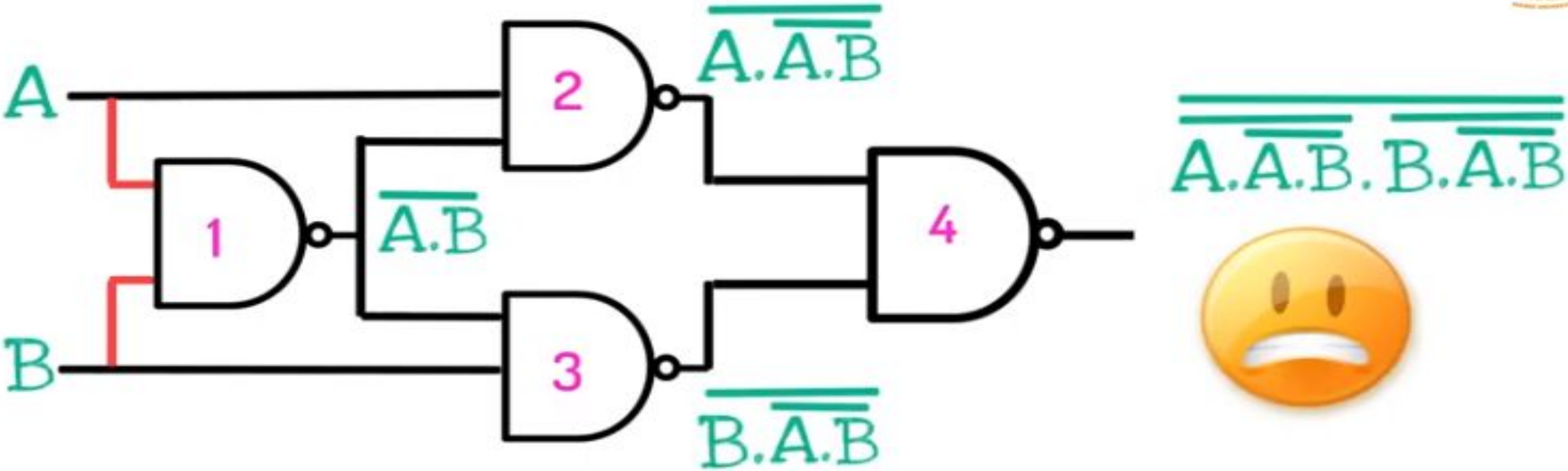


Constructing NOR gate using NAND gate

4 NAND gates



Constructing XOR gate using NAND gate



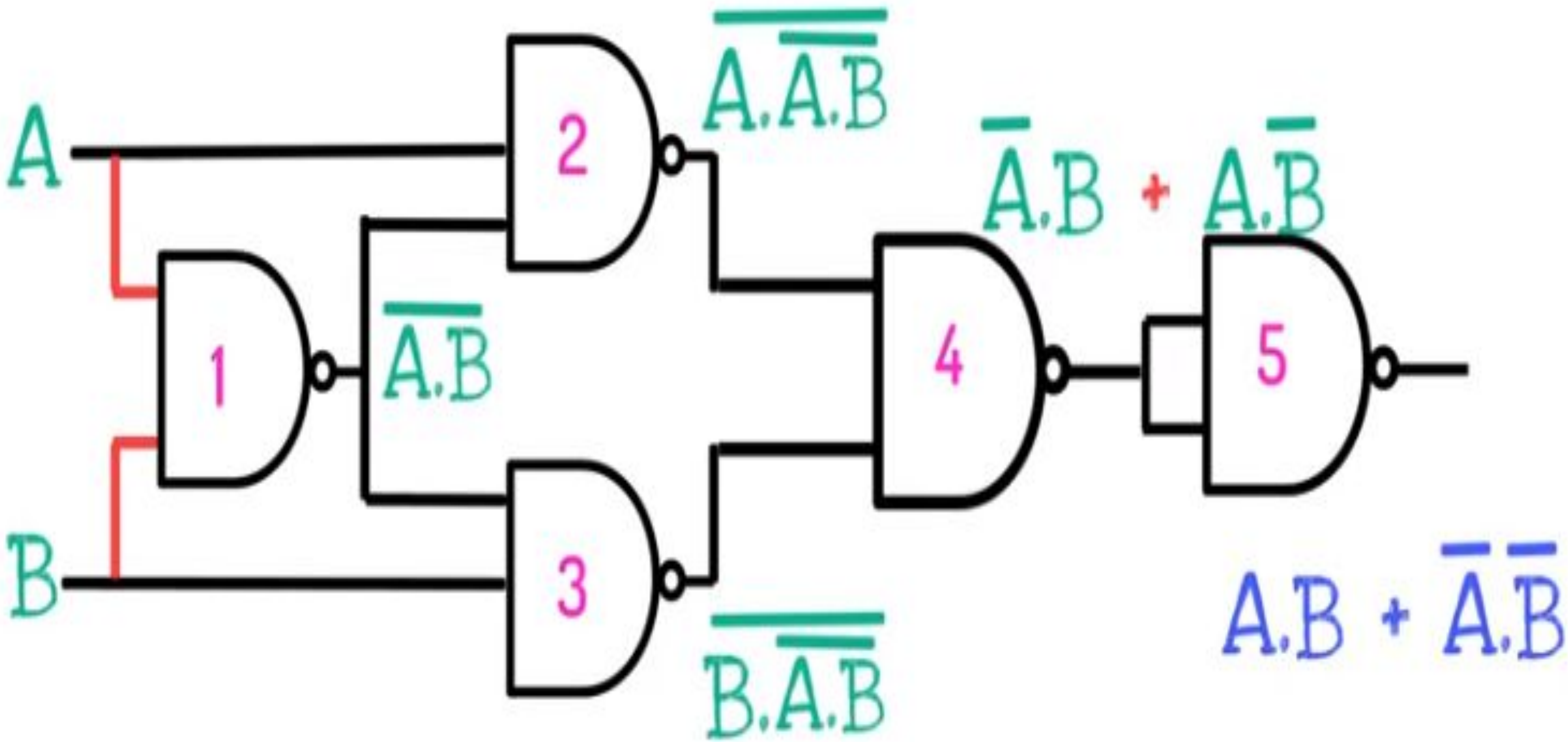
$$\overline{\overline{A.\overline{A.B}}.\overline{B.\overline{A.B}}} = A.\overline{A.B}+B.\overline{A.B}$$

$$= A.(\overline{A}+\overline{B})+B.(\overline{A}+\overline{B})$$

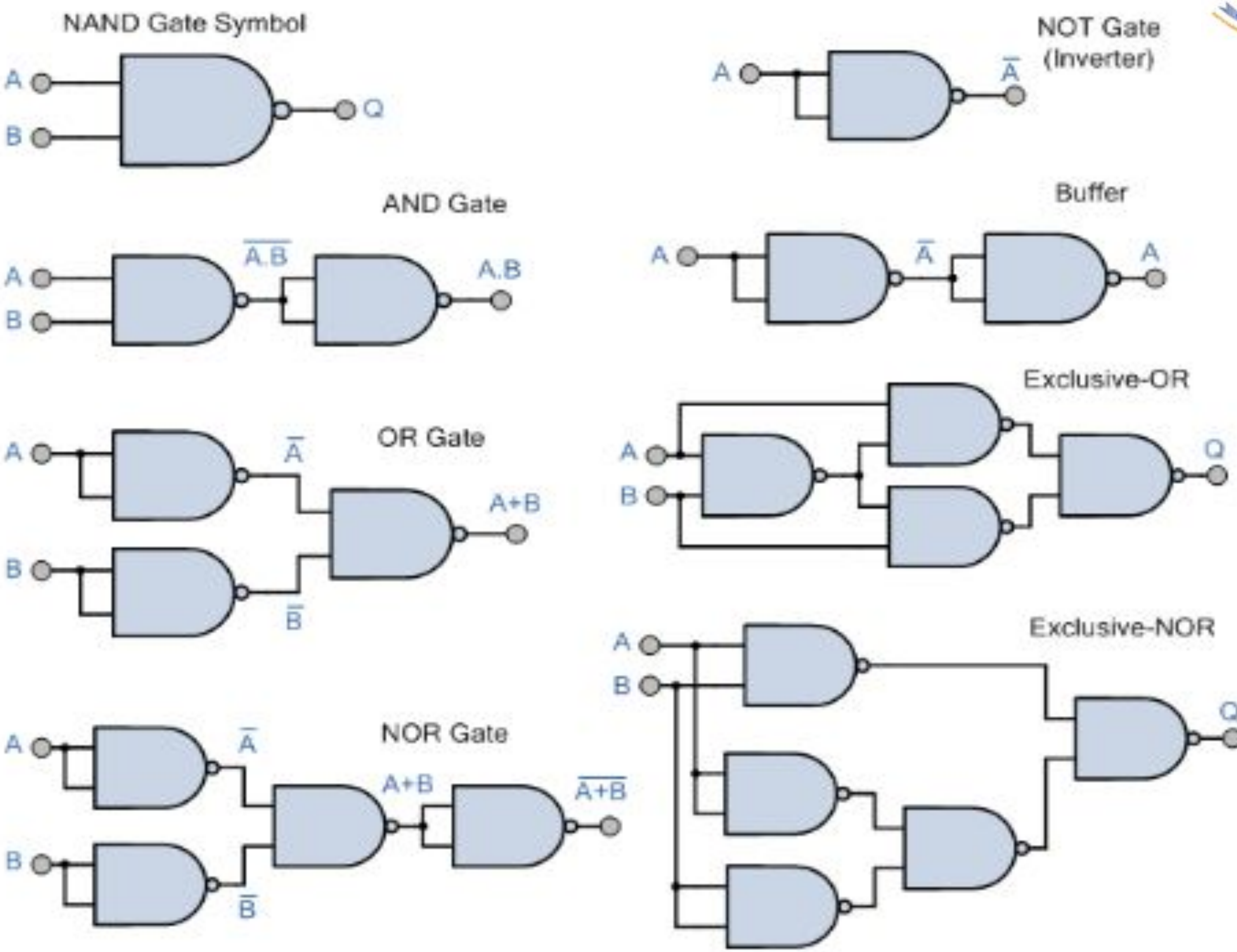
$$= A.\overline{A} + A.\overline{B} + B.\overline{A} + B.\overline{B}$$

$$= \overline{A}.B + A.\overline{B}$$

Constructing XNOR gate using NAND gate

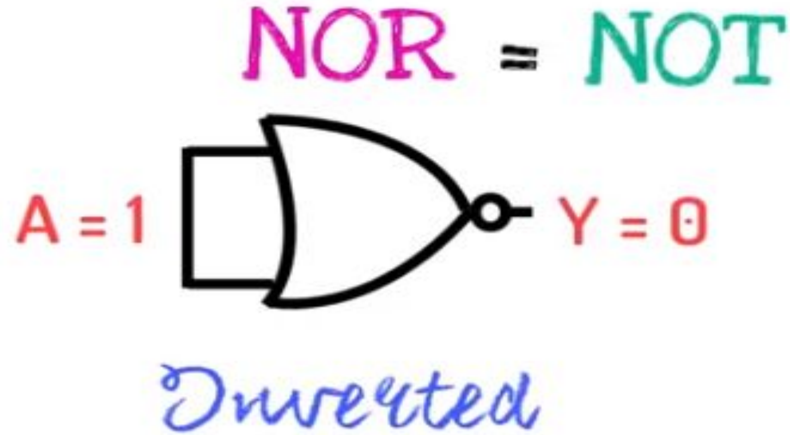


Logic Gates using only NAND Gates

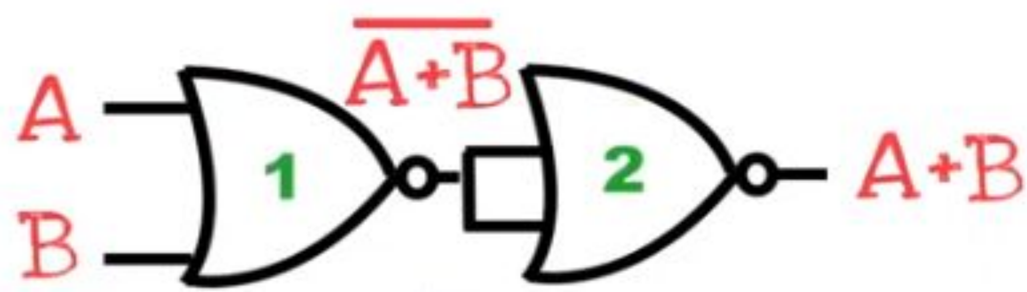


Constructing NOT gate using NOR gate

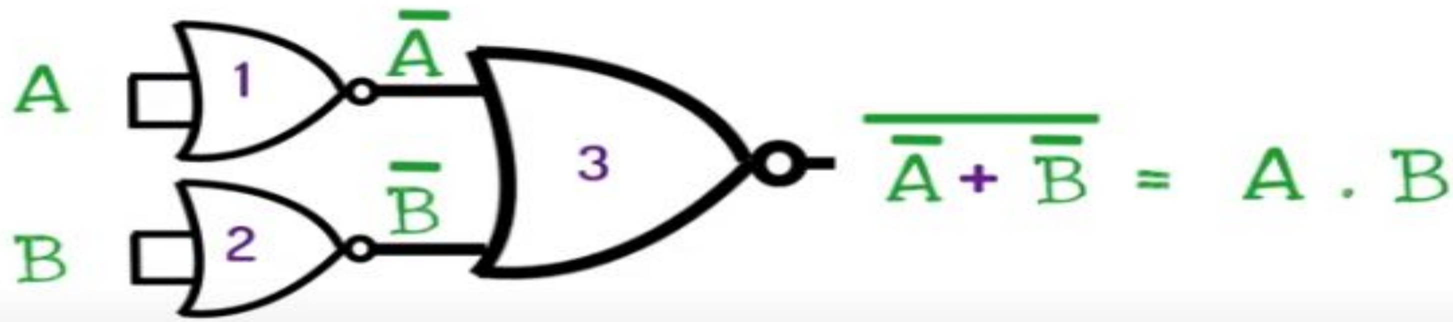
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0



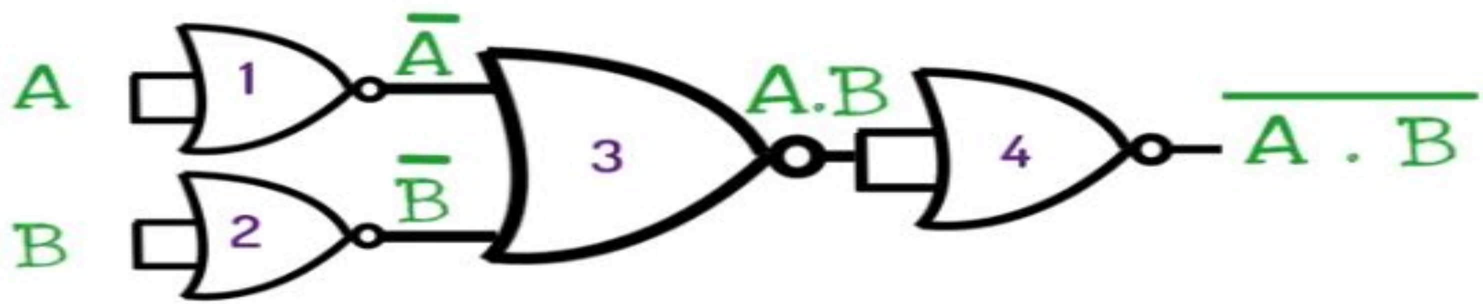
Constructing OR gate using NOR gate



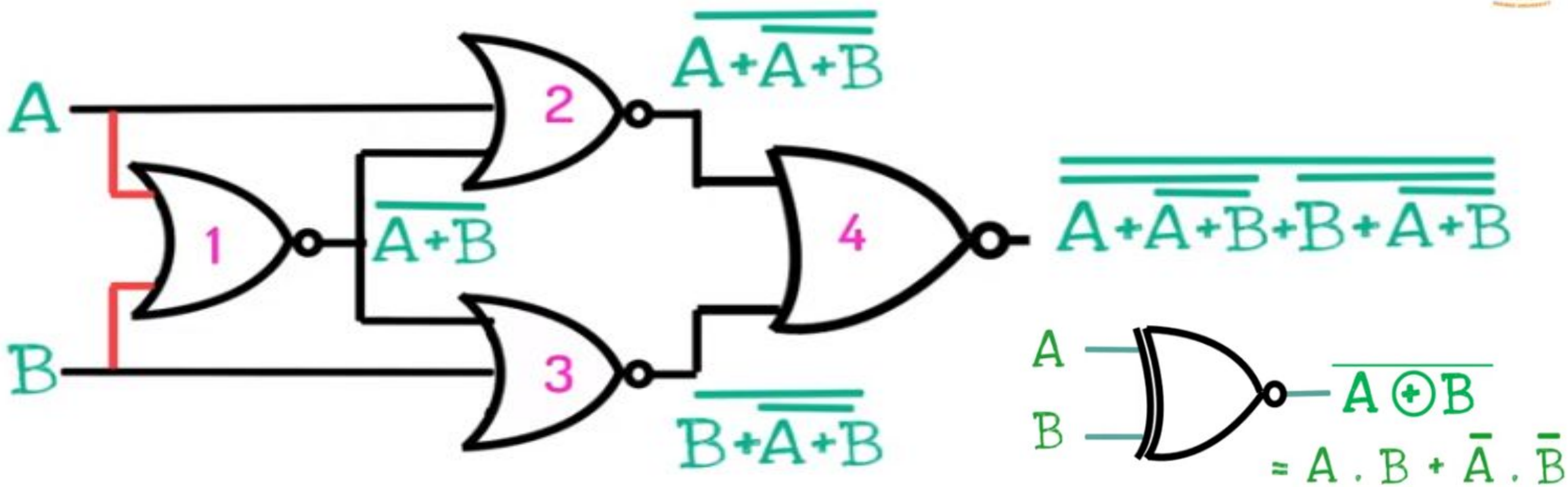
Constructing AND gate using NOR gate



Constructing NAND gate using NOR gate



Constructing XNOR gate using NOR gate



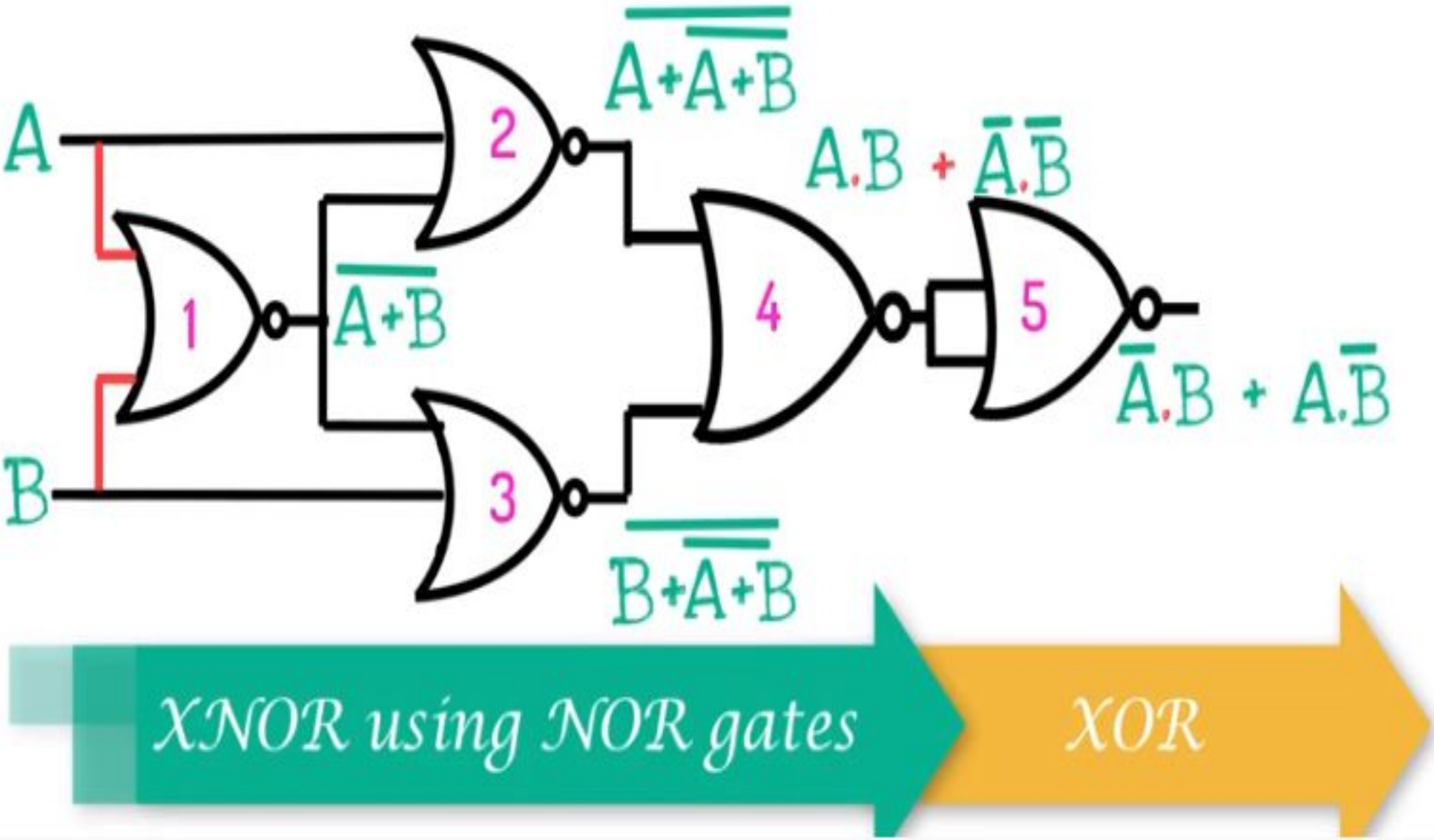
$$\overline{\overline{A+\overline{A+B}}+\overline{B+\overline{A+B}}} = A+\overline{A+B} \cdot B+\overline{A+B}$$

$$= (A+ A.\bar{B} \;.(B+\bar{A}.\bar{B})$$

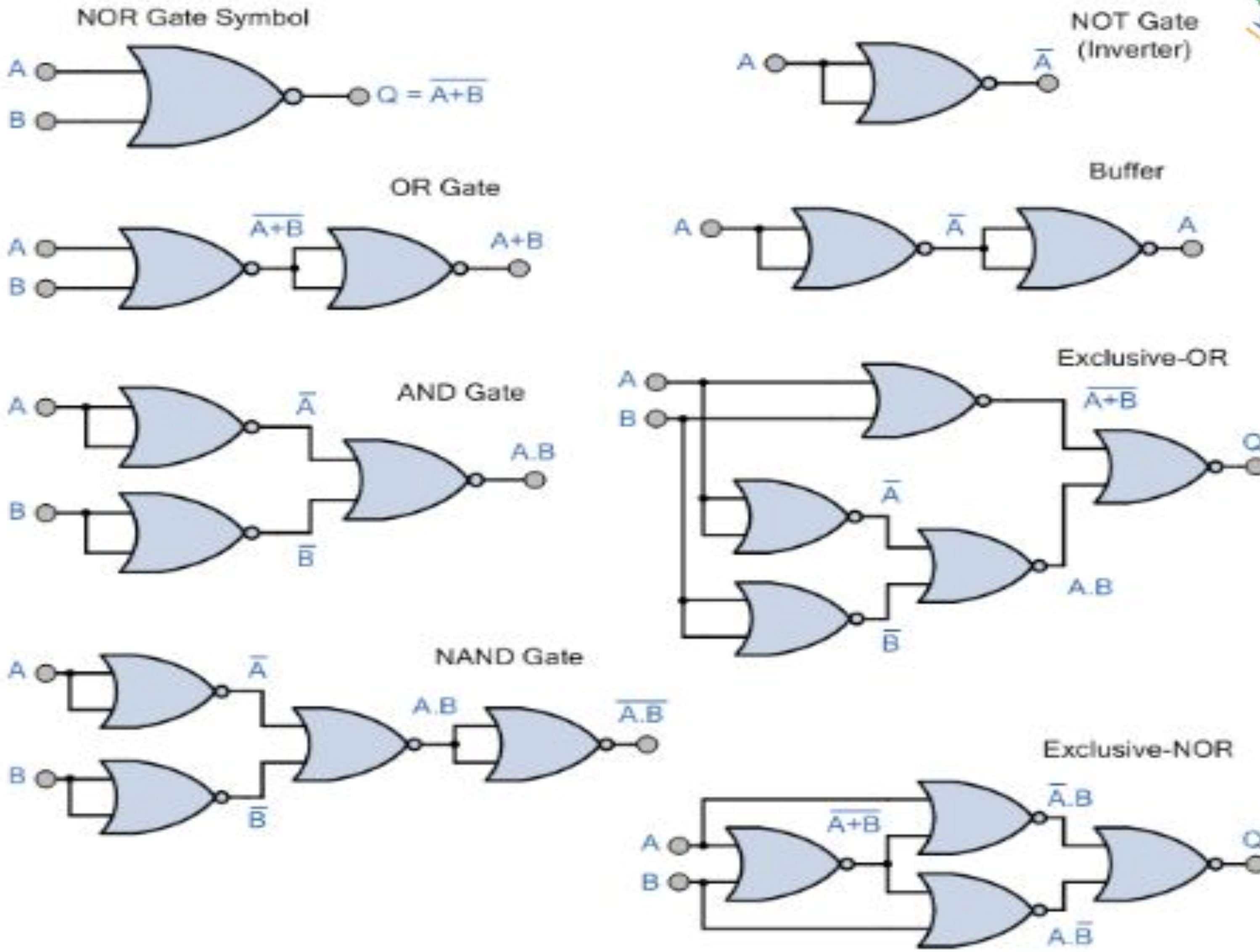
$$= A.B + A.\bar{A}.\bar{B} + \bar{A}.\bar{B}.B + \bar{A}.\bar{B}.\bar{A}.\bar{B}$$

$$= A.B + \bar{A}.\bar{B}$$

Constructing XOR gate using NOR gate

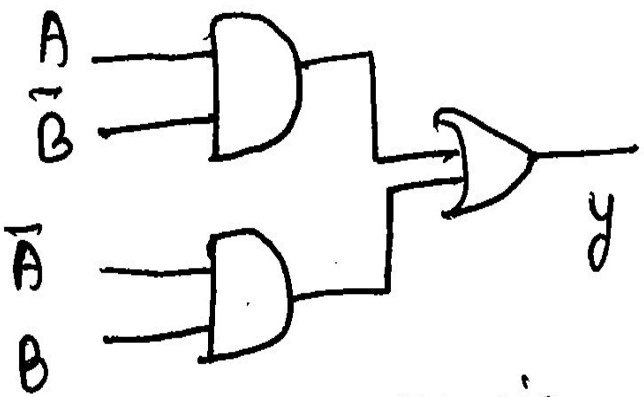


Logic Gates using only NOR Gates

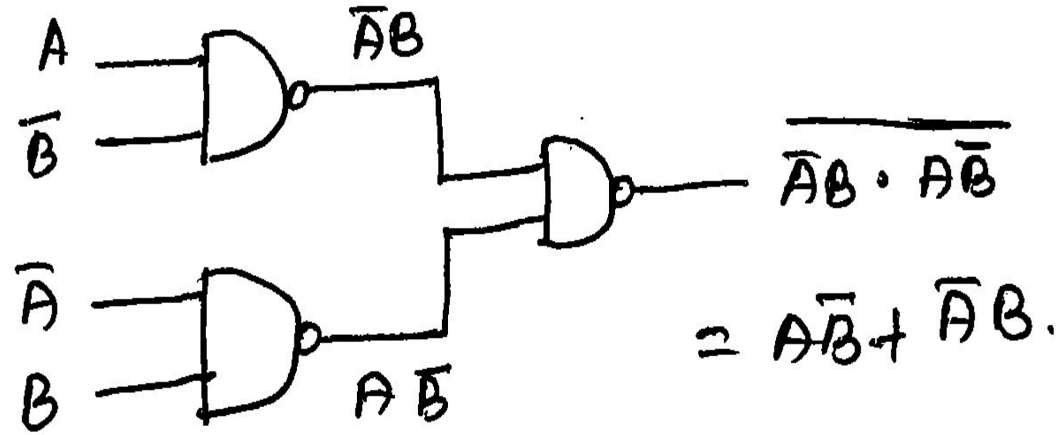


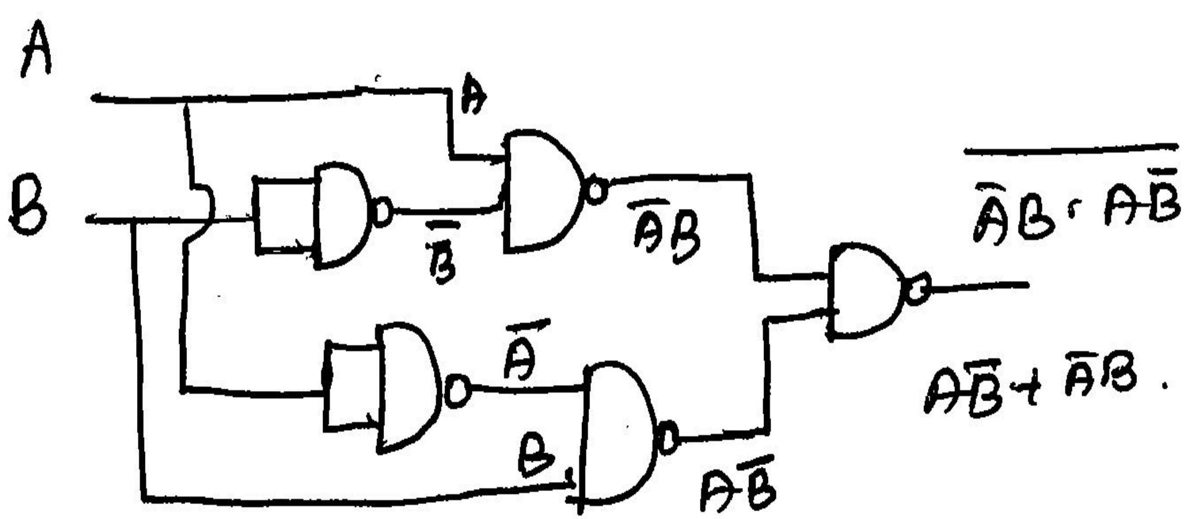
Implement EX-OR gate using only NAND gates.

$$y = \bar{A}B + A\bar{B}$$

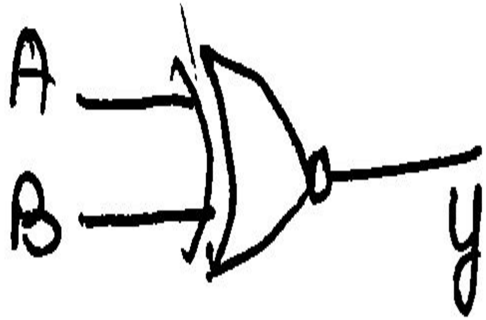


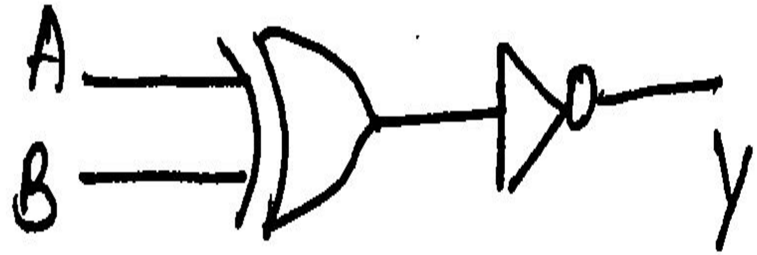
AND-OR Logic

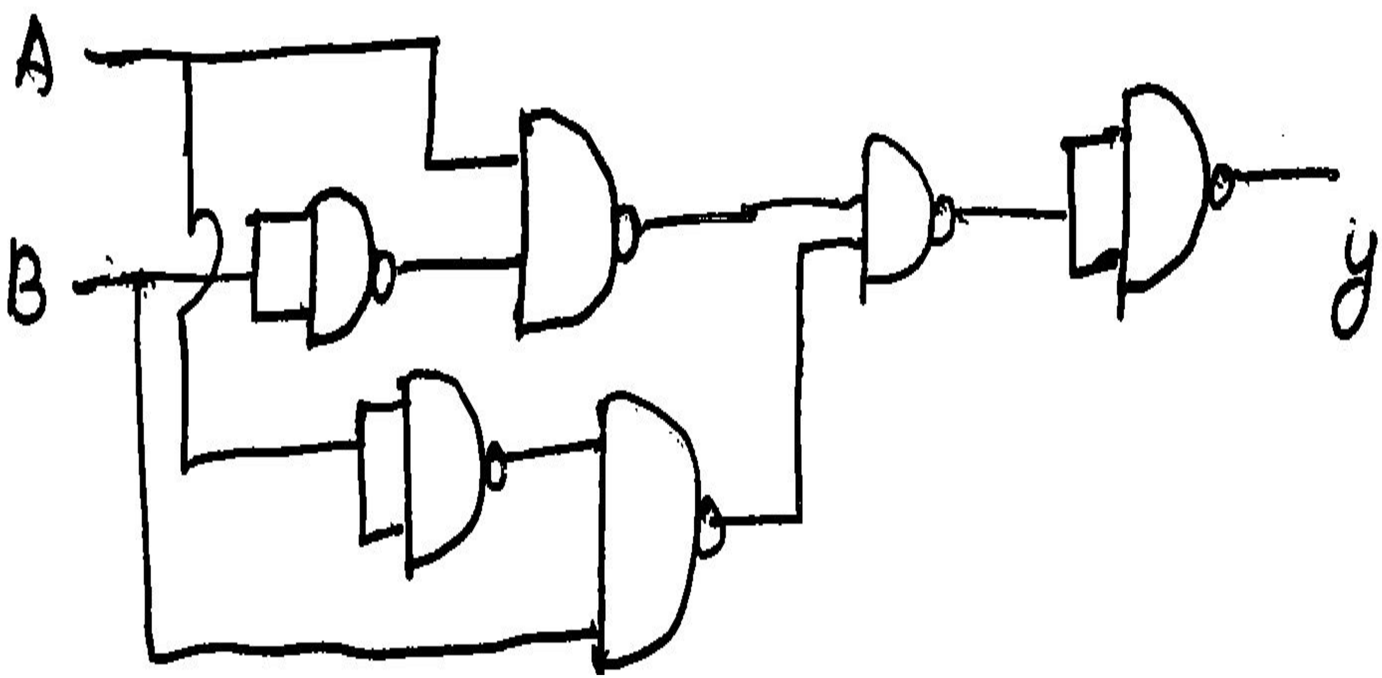




Implement EX-NOR gate using only NAND gate.







Implement EX-NOR gate using only NOR - gate.

$\overline{\overline{(\bar{A}+B)}+\bar{C}}$

$\overline{\overline{(\bar{A}+B)}}\cdot\overline{\bar{C}}$

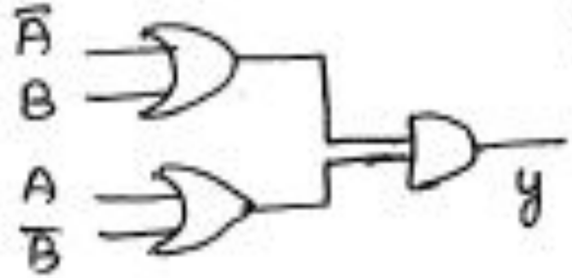
$(\bar{A}+B)\cdot C$

$$y = AB + \bar{A}\bar{B}$$

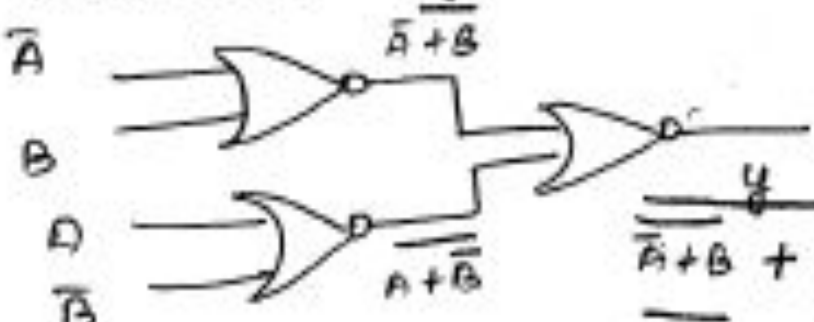
$$= \overline{A\bar{B} + \bar{A}B}$$

$$= \overline{A\bar{B}}\cdot\overline{\bar{A}B}$$

$$= (\bar{A}+B)\cdot(A+\bar{B})$$



NOR - NOR Logic



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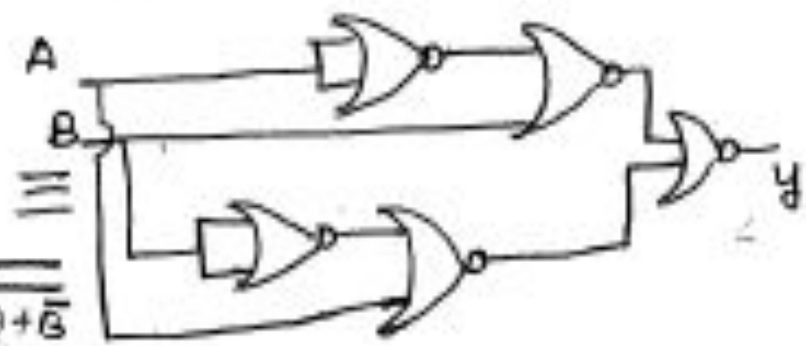


y

$\overline{\overline{\bar{A}+B} + \overline{A+\bar{B}}}$

$\overline{\overline{\bar{A}+B}}\cdot\overline{\overline{A+\bar{B}}}$

$(\bar{A}+B)\cdot(A+\bar{B})$

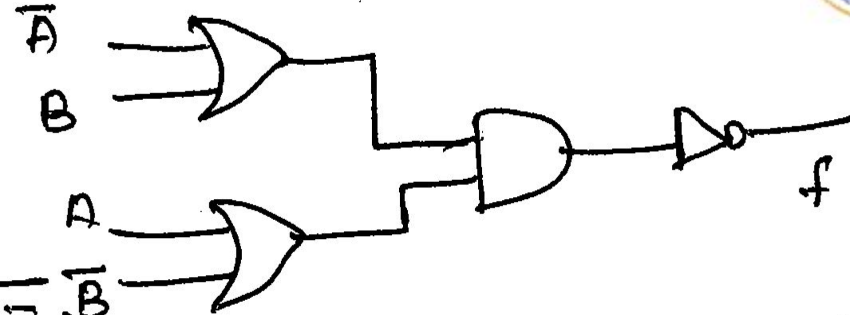
2. Implement Ex-OR using NOR only.

$$F = A\bar{B} + \bar{A}B.$$

$$\bar{F} = \overline{\overline{A\bar{B} + \bar{A}B}}$$

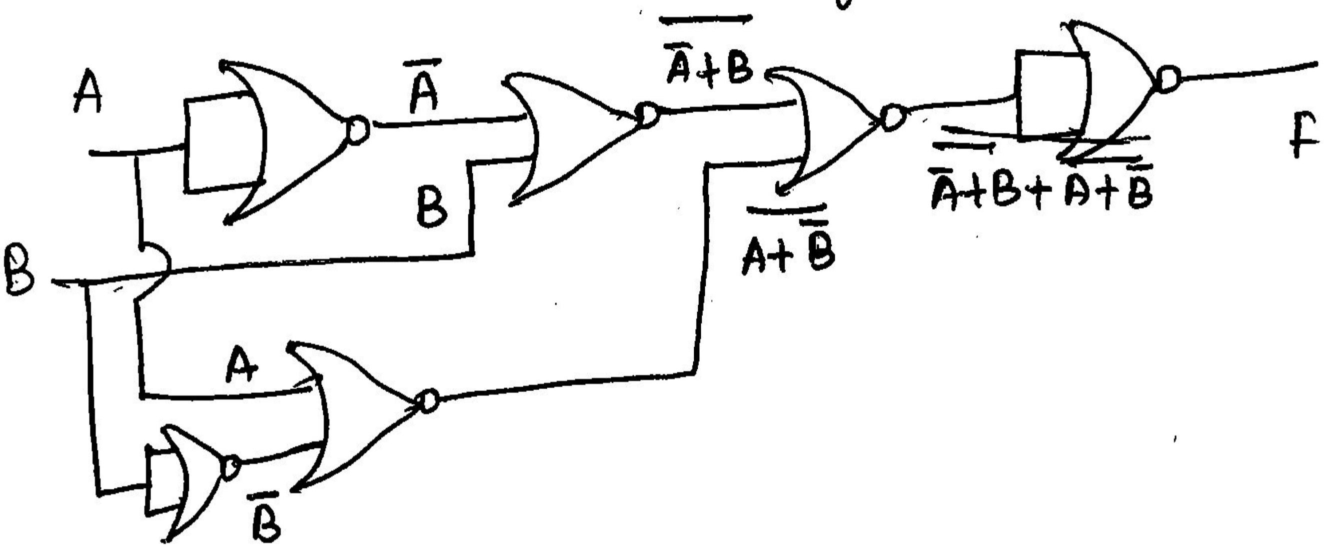
$$= \overline{\overline{A\bar{B}} \cdot \overline{\bar{A}B}}$$

$$= \overline{(\bar{A}+B) \cdot (A+\bar{B})}$$



OR - AND - NOT Logic

NOR - NOR logic



$$\overline{\overline{\overline{\bar{A}+B} + \overline{A+\bar{B}}}}$$

$$\overline{(\bar{A}+B)(A+\bar{B})}$$

Canonical and Standard Forms

We need to consider formal techniques for the simplification of Boolean functions.
Identical functions will have exactly the same canonical form.

- Minterms and Maxterms
- Sum-of-Minterms and Product-of- Maxterms
- Product and Sum terms
- Sum-of-Products (SOP) and Product-of-Sums (POS)

Definitions

Literal: A variable or its complement
Product term: literals connected by •
Sum term: literals connected by +
Minterm: a product term in which all the variables appear exactly once, either complemented or uncomplemented.
Maxterm: a sum term in which all the variables appear exactly once, either complemented or uncomplemented.
Canonical form: Boolean functions expressed as a sum of Minterms or product of Maxterms are said to be in canonical form.

Minterm

- Represents exactly one combination in the truth table.
- Denoted by m_j, where j is the decimal equivalent of the minterm's corresponding binary combination (b_j).
- A variable in m_j is complemented if its value in b_j is 0, otherwise is uncomplemented.

Example: Assume 3 variables (A, B, C), and j=3. Then, b_j = 011 and its corresponding minterm is denoted by $m_j = A'BC$

Maxterm

- Represents exactly one combination in the truth table.
- Denoted by M_j, where j is the decimal equivalent of the maxterm's corresponding binary combination (b_j).
- A variable in M_j is complemented if its value in b_j is 1, otherwise is uncomplemented.

Example: Assume 3 variables (A, B, C), and j=3. Then, b_j = 011 and its corresponding maxterm is denoted by $M_j = A+B'+C'$

Truth Table notation for Minterms and Maxterms

- Minterms and Maxterms are easy to denote using a truth table.

Example: Assume 3 variables x,y,z (order is fixed)

x	y	z	Minterm	Maxterm
0	0	0	$x'y'z' = m_0$	$x+y+z = M_0$
0	0	1	$x'y'z = m_1$	$x+y+z' = M_1$
0	1	0	$x'yz' = m_2$	$x+y'+z = M_2$
0	1	1	$x'yz = m_3$	$x+y'+z' = M_3$
1	0	0	$xy'z' = m_4$	$x'+y+z = M_4$
1	0	1	$xy'z = m_5$	$x'+y+z' = M_5$
1	1	0	$xyz' = m_6$	$x'+y'+z = M_6$
1	1	1	$xyz = m_7$	$x'+y'+z' = M_7$

- Every function F() has two canonical forms:
  - Canonical Sum-Of-Products (sum of minterms)
  - Canonical Product-Of-Sums (product of maxterms)

Canonical Sum-Of-Products:

The minterms included are those m_j such that F( ) = 1 in row j of the truth table for F( ).

Canonical Product-Of-Sums:

The maxterms included are those M_j such that F( ) = 0 in row j of the truth table for F( ).

Example

Consider a Truth table for $f_1(a,b,c)$ at right

The canonical sum-of-products form for f_1 is

$f_1(a,b,c) = m_1 + m_2 + m_4 + m_6$

$= a'b'c + a'bc' + ab'c' + abc'$

The canonical product-of-sums form for f_1 is

$f_1(a,b,c) = M_0 \bullet M_3 \bullet M_5 \bullet M_7$

$= (a+b+c)\bullet(a+b'+c')\bullet (a'+b+c')\bullet(a'+b'+c')$.

- Observe that: $m_j = M_j'$

a	b	c	f_1
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Shorthand: ∑ and ∏

- $f_1(a,b,c) = \sum m(1,2,4,6)$, where ∑ indicates that this is a sum-of-products form, and $m(1,2,4,6)$ indicates that the minterms to be included are m_1, m_2, m_4, and m_6.
- $f_1(a,b,c) = \prod M(0,3,5,7)$, where ∏ indicates that this is a product-of-sums form, and $M(0,3,5,7)$ indicates that the maxterms to be included are M_0, M_3, M_5, and M_7.
- Since $m_j = M_j'$ for any *j*,
  $\sum m(1,2,4,6) = \prod M(0,3,5,7) = f_1(a,b,c)$
-

Conversion between Canonical Forms

- Replace ∑ with ∏ (or *vice versa*) and replace those *j*'s that appeared in the original form with those that do not.
  - Example:

$$
\begin{aligned}
f_1(a,b,c) &= a'b'c + a'bc' + ab'c' + abc' \\
&= m_1 + m_2 + m_4 + m_6 \\
&= \sum(1,2,4,6) \\
&= \prod(0,3,5,7) \\
&= (a+b+c)\bullet(a+b'+c')\bullet(a'+b+c')\bullet(a'+b'+c')
\end{aligned}
$$

Steps to Convert SOP to standard SOP form:

Step1: Find the missing literal in each product term if any.

Step2: AND each product term having missing literals with term form by ORing the literals and its Complement

Step3: Expand the terms by applying distributive law and reorder the literals in the product term.

Step4: Reduce the expression by omitting repeated Product terms if any. Because $A+A=A$.

Steps to Convert POS to standard POS.

Step 1: Find the missing literals in each sum term if any.

Step 2: OR each sum term having missing literal/s with terms form by ANDing the literal and its Complement.

Step 3: Expand the terms by applying distributive law and reorder the literals in the sum term.

Step 4: Reduce the expression by omitting repeated sum terms if any, Because $A \cdot A = A$.

Conversion of SOP from standard to canonical form

Example-1.

Express the Boolean function $F = A + B'C$ as a sum of minterms.

Solution: The function has three variables: A, B, and C. The first term A is missing two variables; therefore,

$$A = A(B + B') = AB + AB'$$

This function is still missing one variable, so

$$A = AB(C + C') + AB'(C + C')$$
$$= ABC + ABC' + AB'C + AB'C'$$

The second term $B'C$ is missing one variable; hence,

$$B'C = B'C(A + A') = AB'C + A'B'C$$

Combining all terms, we have

$$F = A + B'C$$
$$= ABC + ABC' + AB'C + AB'C' + A'B'C$$

But $AB'C$ appears twice, and according to theorem $(x + x = x)$, it is possible to remove one of those occurrences. Rearranging the minterms in ascending order, we finally obtain

$$F = A'B'C + AB'C + AB'C + ABC' + ABC$$
$$= m1 + m4 + m5 + m6 + m7$$

When a Boolean function is in its sum-of-minterms form, it is sometimes convenient to express the function in the following brief notation:

$$F(A, B, C) = \sum m\,(1, 4, 5, 6, 7)$$

Example-2.

Express the Boolean function $F = xy + x'z$ as a product of maxterms.

Solution: First, convert the function into OR terms by using the distributive law:

$$F = xy + x'z = (xy + x')(xy + z)$$
$$= (x + x')(y + x')(x + z)(y + z)$$
$$= (x'+ y)(x + z)(y + z)$$

The function has three variables: x, y, and z. Each OR term is missing one variable; therefore,

$$x'+ y = x' + y + zz' = (x' + y + z)(x' + y + z')$$
$$x + z = x + z + yy' = (x + y + z)(x + y' + z)$$
$$y + z = y + z + xx' = (x + y + z)(x' + y + z)$$

Combining all the terms and removing those which appear more than once, we finally obtain

$$F = (x + y + z)(x + y' + z)(x' + y + z)(x' + y + z)$$

F= M0M2M4M5

A convenient way to express this function is as

follows: $F(x, y, z) = \pi M(0, 2, 4, 5)$

The product symbol, π, denotes the ANDing of maxterms; the numbers are the indices of the maxterms of the function.

Convert the given expressions in standard POS form.

$$F(A,B,C) = (A+B)\cdot(B+C).$$

<u>Solution:</u>

A is missing.

$$= (A+B)\,(B+C)$$

C is missing

$$= (A+B)+C\cdot\bar{C}\cdot(B+C)+A\cdot\bar{A}$$

$$= \underline{(A+B+C)}\,(A+B+\bar{C})\,\underbrace{(A+B+C)}_{\text{repeated}}\,(\bar{A}+B+C)$$

$$= (A+B+C)\,(A+B+\bar{C})\,(\bar{A}+B+C).$$

Convert the given expression in standard SOP form

$$F(A,B,C) = A + ABC$$

$$F(A,B,C) = A\cdot(B+\bar{B})\cdot(C+\bar{C}) + ABC$$

Expand the term and re-order

$$F(A,B,C) = AB + A\bar{B}\cdot(C+\bar{C}) + ABC$$

$$= ABC + A\bar{B}\bar{C} + AB\bar{C} + A\bar{B}\bar{C} + ABC + A\bar{B}C$$

$$= ABC + AB\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$$

Obtain the canonical sum of product form of the following function:

$$F(A, B) = A + B$$

Solution.

The given function contains two variables A and B.

The variable B is missing from the first term of the expression and the variable A is missing from the second term of the expression.

Therefore, the first term is to be multiplied by $(B + B')$ and the second term is to be multiplied by $(A + A')$ as demonstrated below.

$F(A, B) = A + B = A.1 + B.1$

$= A(B + B') + B(A + A')$

$= AB + AB' + AB + A'B$

$= AB + AB' + A'B$ (as $AB + AB = AB$)

Hence the canonical sum of the product expression of the given function is

$$F(A, B) = AB + AB' + A'B.$$

Obtain the canonical sum of product form of the following function.

$$F(A, B, C) = A + BC$$

Solution.

Here neither the first term nor the second term is minterm.
The given function contains three variables A, B, and C.
The variables B and C are missing from the first term of the expression and the variable A is missing from the second term of the expression.
Therefore, the first term is to be multiplied by $(B + B')$ and $(C + C')$.
The second term is to be multiplied by $(A + A')$.
This is demonstrated below.

$$\begin{aligned} F(A, B, C) &= A + BC \\ &= A(B + B')(C + C') + BC(A + A') \\ &= (AB + AB')(C + C') + ABC + A'BC \\ &= ABC + AB'C + ABC' + AB'C' + ABC + A'BC \\ &= ABC + AB'C + ABC' + AB'C' + A'BC \text{ (as } ABC + ABC = ABC) \end{aligned}$$

Hence the canonical sum of the product expression of the given function is

$$\mathbf{F(A, B,C) = ABC + AB'C + ABC' + AB'C' + A'BC.}$$

Obtain the canonical product of the sum form of the following function.

$$F(A, B, C) = (A + B')(B + C)(A + C')$$

Solution.

In the above three-variable expression, C is missing from the first term, A is missing from the second term, and B is missing from the third term.

Therefore, CC' is to be added with first term, AA' is to be added with the second, and BB' is to be added with the third term.

This is shown below.

$F(A, B, C) = (A + B')(B + C)(A + C')$

$= (A + B' + 0)(B + C + 0)(A + C' + 0)$

$= (A + B' + CC')(B + C + AA')(A + C' + BB')$

$= (A + B' + C)(A + B' + C')(A + B + C)(A' + B + C)(A + B + C')(A + B' + C')$

[using the distributive property, as $X + YZ = (X + Y)(X + Z)$]

$= (A + B' + C)(A + B' + C')(A + B + C)(A' + B + C)(A + B + C')$ [as $(A + B' + C')(A + B' + C') = A + B' + C'$]

Hence the canonical product of the sum expression for the given function is

$$F(A, B, C) = (A + B' + C)(A + B' + C')(A + B + C)(A' + B + C)(A + B + C')$$

Thank you