

Assignment-1

Part-A

- 1) $P \rightarrow Q$ (a)
- 2) $(P \wedge T) \vee (Q \wedge F)$ (b)
- 3) $(P \rightarrow Q) \wedge (R \rightarrow Q) \Leftrightarrow (P \vee R) \rightarrow Q$ (d)
- 4) $P \vee (\neg P \wedge Q)$

P	Q	$\neg P$	$\neg P \wedge Q$	$P \vee (\neg P \wedge Q)$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F

The PDNF of given statement $(P \wedge Q)$

$$\vee (P \wedge \neg Q) \vee (\neg P \wedge Q) \text{ (b)}$$

- 5) $(\forall x) (m(x) \rightarrow w(x))$ (a)

Part-B

6)

Step	Statement	Reason
①	$p \rightarrow q$	Rule P
②	$q \rightarrow r$	Rule P
③	$p \rightarrow r$	Rule ①, ① and ② chain rule
④	p	Rule P
⑤	r	Rule T, ③ and ④ modus rule

7)

$$(1) (\neg p \vee q) \rightarrow r$$

$$(2) r \rightarrow (\neg p \vee q)$$

8)

$$(1) p \Leftrightarrow (p \wedge q) \vee (p \wedge \neg q) = \text{let the statement be } S$$

p	q	$p \wedge q$	$\neg q$	$p \wedge \neg q$	$(p \wedge q) \vee (p \wedge \neg q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	F	F	F
F	F	F	T	F	F

(2) let $S \text{ be } (P \vee Q) \wedge (P \vee \neg Q)$

P	Q	$P \vee Q$	$\neg Q$	$P \vee \neg Q$	$(P \vee Q) \wedge (P \vee \neg Q)$
T	T	T	F	T	T
T	F	T	T	T	T
F	T	T	F	F	F
F	F	F	T	T	F

9) $(P \rightarrow (Q \rightarrow R)) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$ let the Statement be S

P	Q	R	$Q \rightarrow R$	$P \rightarrow (Q \rightarrow R)$	$P \rightarrow Q$	$P \rightarrow R$	$(P \rightarrow Q) \rightarrow (P \rightarrow R)$	S
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	F	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

\therefore The given Statement S is a tautology

$$\begin{aligned}
 (10) \quad & (P \vee Q \vee \delta) \wedge (P \vee T \vee \neg Q) \wedge (P \vee \neg T \vee \gamma) \\
 \Rightarrow & (P \vee Q \vee \delta) \wedge (T \vee \neg Q) \wedge (D \vee F \vee \delta) \\
 \Rightarrow & (P \vee Q \vee \delta) \wedge T \wedge (P \vee \delta) \\
 \Rightarrow & (P \vee Q \vee \delta) \wedge (P \vee \delta) \\
 \Rightarrow & (P \vee \delta \vee Q) \wedge (P \vee \delta) \quad (\text{Absorption law}) \\
 & \quad (A \vee B) \wedge B \Rightarrow A \\
 \Rightarrow & P \vee \delta
 \end{aligned}$$

Part - C

$$\begin{aligned}
 (11) \quad & \text{Let } S: P \rightarrow [(P \rightarrow Q) \wedge \neg (R \vee \neg P)] \\
 \Rightarrow & P \rightarrow [(P \rightarrow Q) \wedge (Q \wedge P)] \\
 \Rightarrow & \neg P \vee [(P \rightarrow Q) \wedge (P \wedge Q)] \\
 \Rightarrow & \neg P \vee [(P \wedge Q) \wedge (\neg P \vee Q)] \\
 \Rightarrow & \neg P \vee [(P \wedge Q \wedge \sim P) \vee (P \wedge Q \wedge Q)] \\
 \Rightarrow & \neg P \vee [F \wedge Q \vee (P \wedge Q)] \\
 \Rightarrow & \neg P \vee [F \vee (P \wedge Q)] \\
 \Rightarrow & \neg P \vee (P \wedge Q) \\
 \Rightarrow & \neg P \wedge (Q \vee \neg Q) \vee (P \wedge Q) \\
 \Rightarrow & \text{The PDNF of given } S \Rightarrow (P \wedge \neg Q)
 \end{aligned}$$