UNIT – 1

TOPIC: NUMBER SYSTEM AND ITS CONVERSION

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Number Systems

Number System is a basis for counting various items

- Decimal Number System (0 – 9)
- Binary Number System (0, 1)
- Octal Number System (0 – 7)
- Hexa Decimal Number System (0 – 9, A – F)

Number – Base – Digits

Name	Radix / Base	Digits
Binary	2	0,1
Octal	8	0,1,2,3,4,5,6,7
Decimal	10	0,1,2,3,4,5,6,7,8,9
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Any Number in the form
Eg. N= 1456.37 = $A_3A_2A_1A_0.A_{-1}A_{-2}$

$(1*10^3)+(4*10^2)+(5*10^1)+(6*10^0)+(3*10^{-1})+(7*10^{-2}) =$

$(A_3*r^3)+(A_2*r^2)+(A_1*r^1)+(A_0*r^0)+(A_{-1}*r^{-1})+(A_{-2}*r^{-2})$

A- Digit, r- Radix or Base

Numbers With Different Base

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
0	0000	00	0
1	0001	01	1
2	0010	02	2
3	0011	03	3
4	0100	04	4
5	0101	05	5
6	0110	06	6
7	0111	07	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Conversion of Any Number System into Decimal Number

Number in Decimal

$N = (A_{n-1}*r^{n-1})+\ldots (A_3*r^3)+(A_2*r^2)+(A_1*r^1)+(A_0*r^0)+(A_{-1}*r^{-1})+(A_{-2}*r^{-2})+ \ldots)+(A_{-m}*r^{-m})$

Example

$$2586_{10} = (2 \times 10^3) + (5 \times 10^2) + (8 \times 10^1) + (6 \times 10^0)$$
$$= 2000 + 500 + 80 + 6$$

Convert into Decimal

$(1\,1\,0\,1\,.\,1)_2$ n=4, m=1

$$N = (1*2^3) + (1*2^2) + (0*2^1) + (1*2^0) + (1*2^{-1})$$
$$= 8 + 4 + 0 + 1 + .5 = 13.5_{10}$$

$(475.25)_8$ n=3, m=2

$$N = (4*8^2) + (7*8^1) + (5*8^0) + (2*8^{-1}) + (5*8^{-2})$$
$$= 256 + 56 + 5 + 0.25 + 0.078125$$
$$= 317.32813_{10}$$

Conversion of Any Number System into Decimal Number Cont...

Number in Decimal

$N = (A_{n-1}*r^{n-1})+\ldots (A_3*r^3)+(A_2*r^2)+(A_1*r^1)+(A_0*r^0)+(A_{-1}*r^{-1})+(A_{-2}*r^{-2})+ \ldots)+(A_{-m}*r^{-m})$

Convert into Decimal

$(9B2.1A)_H$ n=3, m=2

$$N= (9*16^2) +(B*16^1) +(2*16^0) +(1*16^{-1}) + (A*16^{-2})$$

$$= (9*16^2) +(11*16^1) +(2*16^0) +(1*16^{-1}) + (10*16^{-2})$$

$$= 2304 + 176 + 2 + 0.0625 + 0.039 = 2482.1_{10}$$

$(3102.12)_4$ n=4, m=2

$$N= (3*4^3) + (1*4^2) +(0*4^1) +(2*4^0) +(1*4^{-1}) + (2*4^{-2})$$

$$= 192 + 16 + 0 + 2 + 0.25 + 0.125 = 210.375_{10}$$

Conversion of Decimal Number into Any Radix Number

It is performed in 2 Steps

1. Conversion of Integer Part – successive division method
2. Conversion of Fractional Part – successive multiplication method

Successive Division for Integer Part conversion

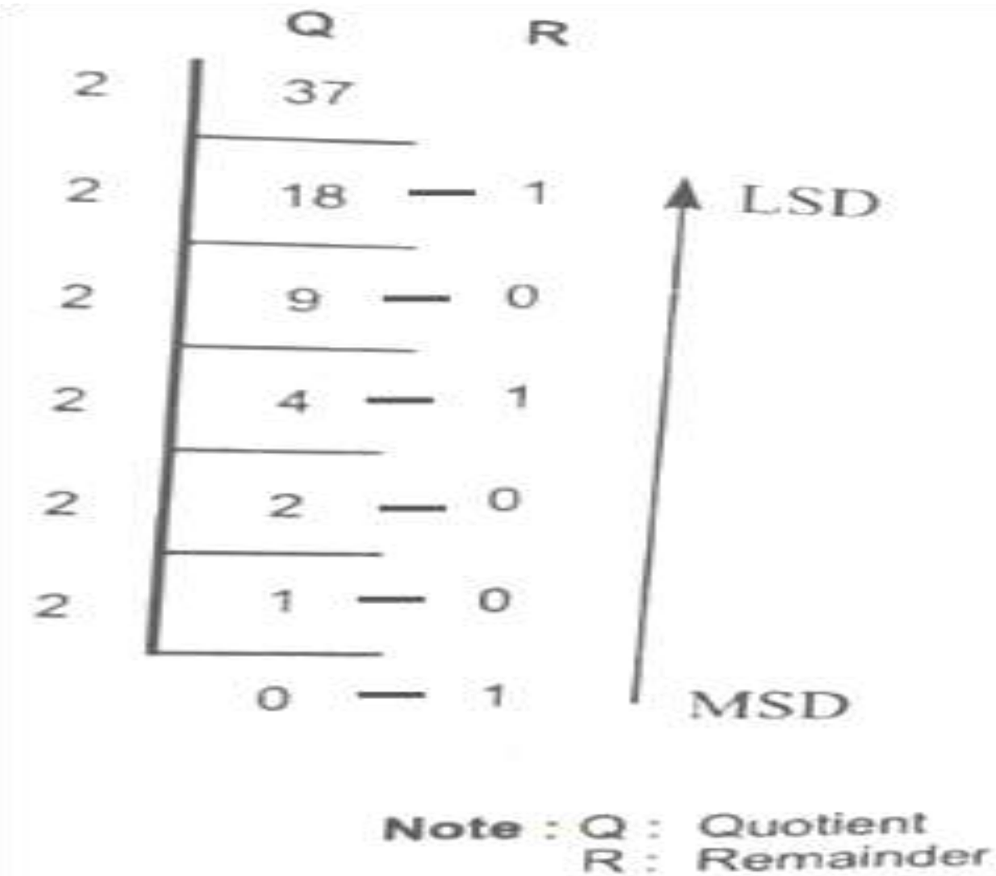
In this method we repeatedly divide the integer part of the decimal number by r (the new radix) until quotient is zero. The remainder of each division becomes the numeral in the new radix. The remainders are taken in the reverse order to form a new radix number. This means that the first remainder is the least significant digit (LSD) and the last remainder is the most significant digit (MSD) in the new radix number. This procedure is illustrated in following examples.

Successive multiplication for Fractional Part conversion

Conversion of fractional decimal numbers to another radix number is accomplished using a successive multiplication method. In this method, the number to be converted is multiplied by the radix of the new number, producing a product that has an integer part and a fractional part. The integer part (carry) of the product becomes a numeral in the new radix number. The fractional part is again multiplied by the radix and this process is repeated until fractional part reaches 0 or until the new radix number is carried out to sufficient digits. The integer part (carry) of each product is read downward to represent the new radix number. This is illustrated in following examples.

Perform the following Conversions

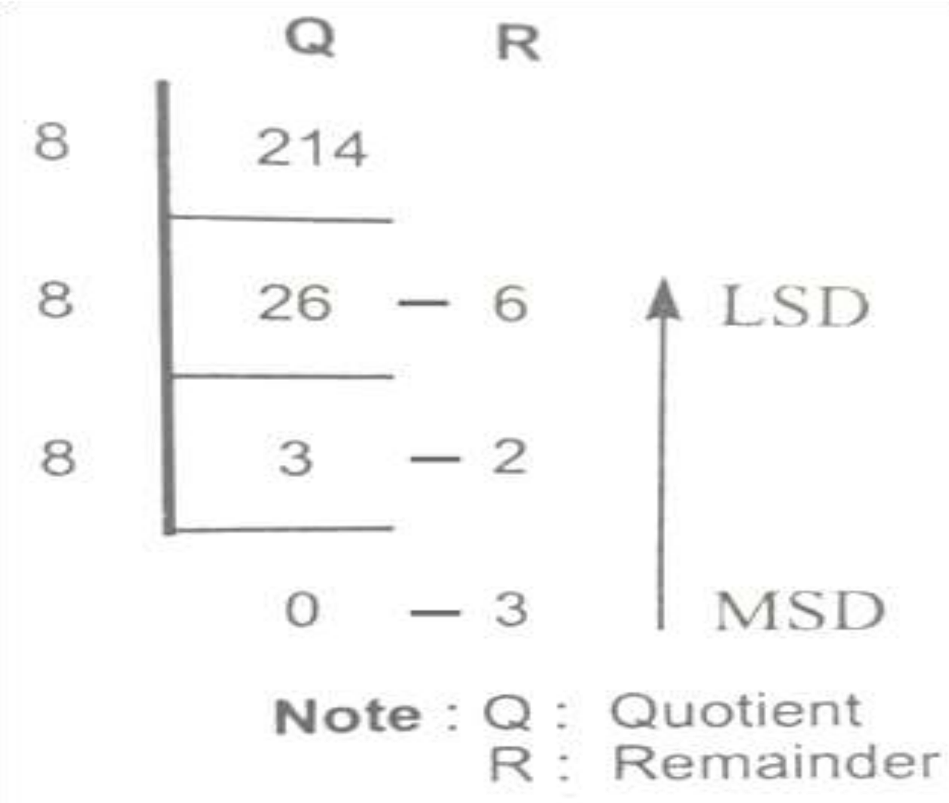
Convert the decimal number 37 into its binary equivalent



$37_{10} = (1\ 0\ 0\ 1\ 0\ 1)_2$

Perform the following Conversions

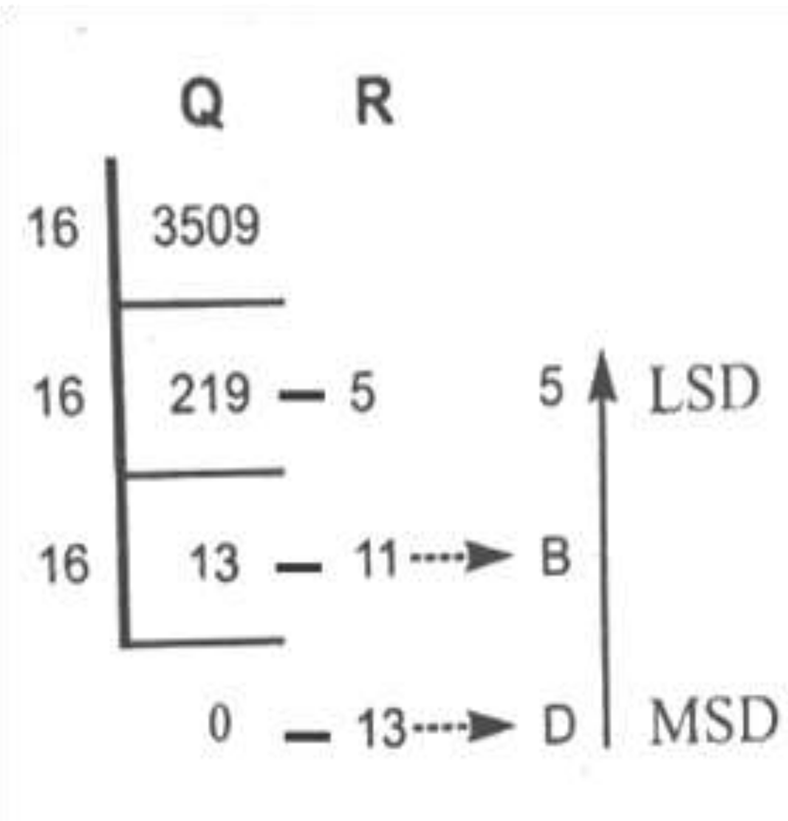
Convert the decimal number 214 into its octal equivalent



$214_{10} = (326)_8$

Perform the following Conversions

Convert the decimal number 3509 into its hexadecimal equivalent

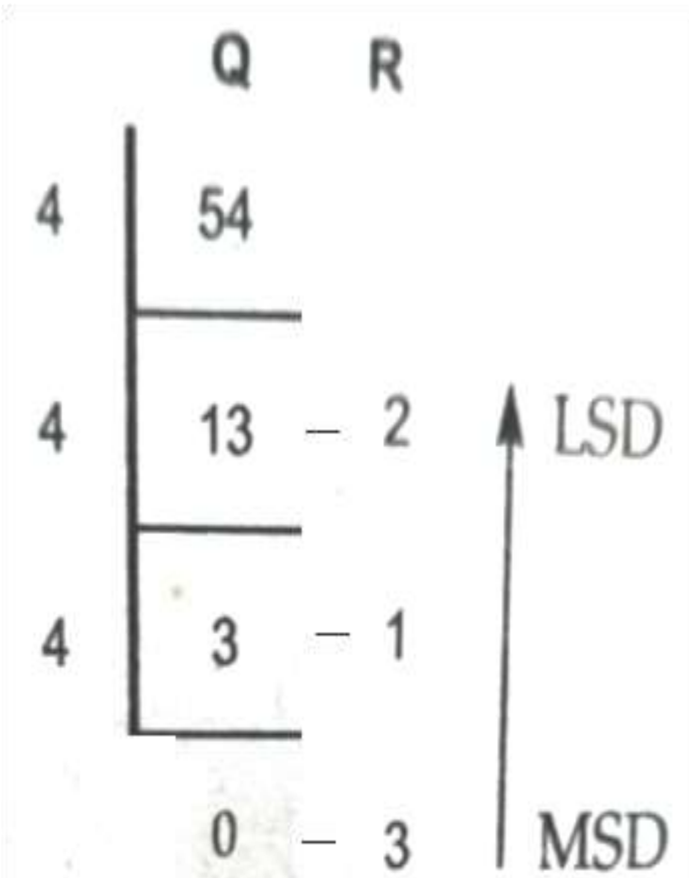


$3509_{10} = (\text{DB5})_{16}$

$3509_{10} = (\text{DB5})_{H}$

Perform the following Conversions

Convert the decimal number 54 into radix 4



$54_{10} = (312)_4$

Perform the following Conversions

Convert the decimal number .8125 into its binary equivalent

Fraction		Radix		Result		Recorded Carries		
0.8125	×	2	=	1.625	=	0.625	with a carry of 1	MSD
0.625	×	2	=	1.25	=	0.25	with a carry of 1	
0.25	×	2	=	0.5	=	0.5	with a carry of 0	
0.5	×	2	=	1.0	=	0.0	with a carry of 1	LSD

$.8125_{10} = (.1101)_2$

Perform the following Conversions

Convert the decimal number .45 into its Octal equivalent

Fraction		Radix		Result		Recorded Carries			
0.45	×	8	=	3.6	=	0.6	with a carry of	3	MSD
0.6	×	8	=	4.8	=	0.8	with a carry of	4	
0.8	×	8	=	6.4	=	0.4	with a carry of	6	
0.4	×	8	=	3.2	=	0.2	with a carry of	3	
0.2	×	8	=	1.6	=	0.6	with a carry of	1	LSD

$.45_{10} = (.34631)_8$

Perform the following Conversions

Convert the decimal number .64 into its Hex equivalent

Fraction	Radix		Result				
0.64	× 16	= 10.24	= 0.24	with a carry of	10	= A	MSD
0.24	× 16	= 3.84	= 0.84	with a carry of	3	= 3	
0.84	× 16	= 13.44	= 0.44	with a carry of	13	= D	
0.44	× 16	= 7.04	= 0.04	with a carry of	7	= 7	LSD

$.64_{10} = (.A3D7)_{16}$

Perform the following Conversions

Convert the decimal number 37.8125 into its binary equivalent

	Q	R	
2	37		
2	18	1	LSD
2	9	0	
2	4	1	
2	2	0	
2	1	0	
	0	1	MSD

Note : Q : Quotient
R : Remainder

Fraction	Radix		Result			Recorded Carries	
0.8125	× 2	=	1.625	=	0.625	with a carry of 1	MSD
0.625	× 2	=	1.25	=	0.25	with a carry of 1	
0.25	× 2	=	0.5	=	0.5	with a carry of 0	
0.5	× 2	=	1.0	=	0.0	with a carry of 1	LSD

$37_{10} = (1\ 0\ 0\ 1\ 0\ 1.\ 1\ 1\ 0\ 1)_2$

Perform the following Conversions

Convert the decimal number 214.45 into its octal equivalent

	Q	R	
8	214		
8	26	— 6	LSD
8	3	— 2	
	0	— 3	MSD

Note : Q : Quotient
R : Remainder

$$214.45_{10} = (326.34631)_8$$

Fraction		Radix		Result		Recorded Carries		
0.45	×	8	=	3.6	=	0.6 with a carry of	3	MSD
0.6	×	8	=	4.8	=	0.8 with a carry of	4	
0.8	×	8	=	6.4	=	0.4 with a carry of	6	
0.4	×	8	=	3.2	=	0.2 with a carry of	3	
0.2	×	8	=	1.6	=	0.6 with a carry of	1	LSD

Perform the following Conversions

Convert the decimal number 3509.64 into its hexadecimal equivalent

	Q	R		
16	3509			
16	219	5	5	LSD
16	13	11	B	
	0	13	D	MSD

$3509.64_{10} = (\text{DB5.A3D7})_{16}$

Fraction	Radix	Result					
0.64	× 16	= 10.24	= 0.24	with a carry of	10	= A	MSD
0.24	× 16	= 3.84	= 0.84	with a carry of	3	= 3	
0.84	× 16	= 13.44	= 0.44	with a carry of	13	= D	
0.44	× 16	= 7.04	= 0.04	with a carry of	7	= 7	LSD

Binary to octal

Starting at the binary point and working left, separate the bits into groups of **three** and replace each group with the corresponding **octal** digit.

$8 = 2^3$

$$10001011_2 = 010\ 001\ 011$$
$$= 2\ \ 1\ \ 3 \quad = 213_8$$

$$1\,1\,0\,1\,0\,0\,1.1\,0\,1\,0\,1_2 = 001\ 101\ 001\,.\,101\ 010$$
$$= 1\ \ 5\ \ 1\ .\ 5\ \ 2$$
$$= 151.52_8$$

Binary to Hexadecimal

Starting at the binary point and working left, separate the bits into groups of **four** and replace each group with the corresponding **hexadecimal** digit.

$16 = 2^4$

$$10001011_2 = 1000\ 1011 = 8\ B = 8B_{16}$$

$$1001\,1101\,0101\,.\,1001\,01_2 = 1001\ 1101\ 0101\,.\,1001\ 0100 = 9\ 13\ 5\,.\,9\ 4 = 9D5.94_{16} = 9D5.94_H$$

Octal to Binary

Replace each **octal** digit with the corresponding **3-bit** binary string.

$$213_8 = 010\ \ 001\ \ 011 = 10001011_2$$

$$751.62_8 = 111\ \ 101\ \ 001\ .\ 110\ \ 010$$
$$= 111101001.110010_2$$

Hexadecimal to Binary

Replace each **hexadecimal** digit with the corresponding **4-bit** binary string.

$$8B_{16} = 1000 \;\; 1011 = 10001011_2$$

$$A\,2\,5\,F\,.\,1\,C\,4_{16}$$

$$= 1010 \;\; 0010 \;\; 0101 \;\; 1111\,.\; 0001 \;\; 1100 \;\; 0100$$

$$= 1010001001011111.000111000100_2$$

Octal to Hexadecimal

This can be performed in 2 steps

- Octal to Binary conversion
- Binary to Hexadecimal conversion

4725.4_8

Step1: $4725.4_8 = 100111010101.100_2$

Step 2: 1001 1101 0101 . 1000 $= 9D5.8_{16}$

$4725.4_8 = 9D5.8_{16}$

720.12_8

Step1: $720.12_8 = 111010000.001010_2$

Step 2: 0001 1101 0000 . 0010 1000 $= 1D0.28_{16}$

$720.12_8 = 1D0.28_{16}$

Hexadecimal to Octal

This can be performed in 2 steps

- Hexadecimal to Binary conversion
- Binary to Octal conversion

$9D5.8_{16}$

Step 1: $9D5.8_{16} = 1001\ 1101\ 0101\ .\ 1000_2$

Step 2: $100\ 111\ 01\ 0\ 101\ .100_2 = 4725.4_8$

$9D5.8_{16} = 4725.4_8$

$1D0.28_{16}$

Step 1: $1D0.28_{16} = 0001\ 1101\ 0000\ .\ 0010\ 1000_2$

Step 2: $000\ 111\ 010\ 000.001\ 010_2 = 0720.12_8$

$1D0.28_{16} = 720.12_8$

Binary Arithmetic

- Binary Addition
- Binary Subtraction
- Binary Multiplication
- Binary Division

Note:

1 Nybble (or nibble) = 4 bits

1 Byte = 2 nibbles = 8 bits

2 Bytes = 16 Bits = 1 word

1 Kilobyte (KB) = 1024 bytes
1 Megabyte (MB) = 1024 kilobytes = 1,048,576 bytes
1 Gigabyte (GB) = 1024 megabytes = 1,073,741,824 bytes

Binary Addition

Rules of Binary Addition

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 0$, and carry 1 to the next more significant bit

Perform the binary addition

$11101.101_2 + 10010.001_2$

$$
\begin{array}{r}
1 \\
11101.101_2 + \\
10010.001_2 \\
\hline
101111.110
\end{array}
$$

Ans: 101111.110_2

$35.5 + 18.25$

$$
\begin{array}{rr}
 & 1 \\
35.5 = & 100011.10_2 + \\
18.25 = & 010010.01_2 \\
\hline
 & 110101.11
\end{array}
$$

Ans: 110101.11_2

Binary Subtraction

Rules of Binary Subtraction

0 - 0 = 0
0 - 1 = 1, and borrow 1 from the next more significant bit
1 - 0 = 1
1 - 1 = 0

Perform the binary subtraction

$11101.101_2 - 10010.001_2$

$$
\begin{array}{r}
1 \\
11101.101 - \\
10010.001 \\
\hline
01011.100
\end{array}
$$

Ans: 01011.100_2

$35.5 - 18.25$

$$
\begin{array}{rr}
 & 11 \\
35.5 = & 100011.10_2 - \\
18.25 = & 010010.01_2 \\
\hline
 & 010001.01
\end{array}
$$

Ans: 010001.01_2

Binary Multiplication

Rules of Binary Multiplication

$0 \times 0 = 0$

$0 \times 1 = 0$

$1 \times 0 = 0$

$1 \times 1 = 1$, and no carry or borrow bits

Perform the binary multiplication

$111_2 * 11_2$

$$
\begin{array}{rrrrrl}
 & & 1 & 1 & 1 & * \\
 & & & 1 & 1 & \\
\hline
 & 1 & 1 & & & \leftarrow \text{carry} \\
 & & 1 & 1 & 1 & \\
 & 1 & 1 & 1 & & \\
\hline
1 & 0 & 1 & 0 & 1_2 & : \text{Ans} \\
\hline
\end{array}
$$

Perform the binary multiplication

$1\,1\,0\,1_2 * 1\,0\,0\,1_2$

$1\,1\,0\,1_2 * 1\,0\,0\,1_2$

$$
\begin{array}{cccccccl}
 & & & 1 & 1 & 0 & 1 & * \\
 & & & 1 & 0 & 0 & 1 & \\
\hline
 & & 1 & & & & & \leftarrow \text{carry} \\
 & & & 1 & 1 & 0 & 1 & \\
 & & 0 & 0 & 0 & 0 & & \\
 & 0 & 0 & 0 & 0 & & & \\
1 & 1 & 0 & 1 & & & & \\
\hline
1 & 1 & 1 & 0 & 1 & 0 & 1_2 & : \text{Ans} \\
\hline
\end{array}
$$

Perform the binary multiplication

17.25 * 5.5

$= 10001.01_2 * 101.1_2$

```
        1 0 0 0 1 . 0 1 *
              1 0 1 . 1
      ---------------------
              1 <--- carry
        1 0 0 0 1 0 1
      1 0 0 0 1 0 1
    0 0 0 0 0 0 0
  1 0 0 0 1 0 1
  -------------------------
  1 0 1 0 1 1 0 . 1 1 1
  -------------------------
```

1010110.111_2 : Ans

Binary Division

Perform the binary division

$$111101_2 \div 101_2$$

```
       1100        Quotient
    --------
101 | 111101
      101
     -----
      0101
       101
      -----
        001        Remainder
```

$$111101_2 \div 101_2 = 1100\ (Q)$$

$$001\ (R)$$

Perform the binary division

$14 \div 4$

$= 1110_2 \div 100_2$

$$
\begin{array}{r|l}
 & \;\;1\;\;1\,.\,1 \\ \hline
1\;0\;0 & 1\;\;1\;\;1\;\;0 \\
 & 1\;\;0\;\;0 \\ \hline
 & \;\;\;\;1\;\;1\;\;0 \\
 & \;\;\;\;1\;\;0\;\;0 \\ \hline
 & \;\;\;\;\;\;\;\;1\;\;0\;\;0 \\
 & \;\;\;\;\;\;\;\;1\;\;0\;\;0 \\ \hline
 & \;\;\;\;\;\;\;\;0\;\;0\;\;0
\end{array}
$$

$1110_2 \div 100_2 = 11.1_2$