Lecture session- UNIT -2

Topic: Binary logic functions and Boolean Algebra

By

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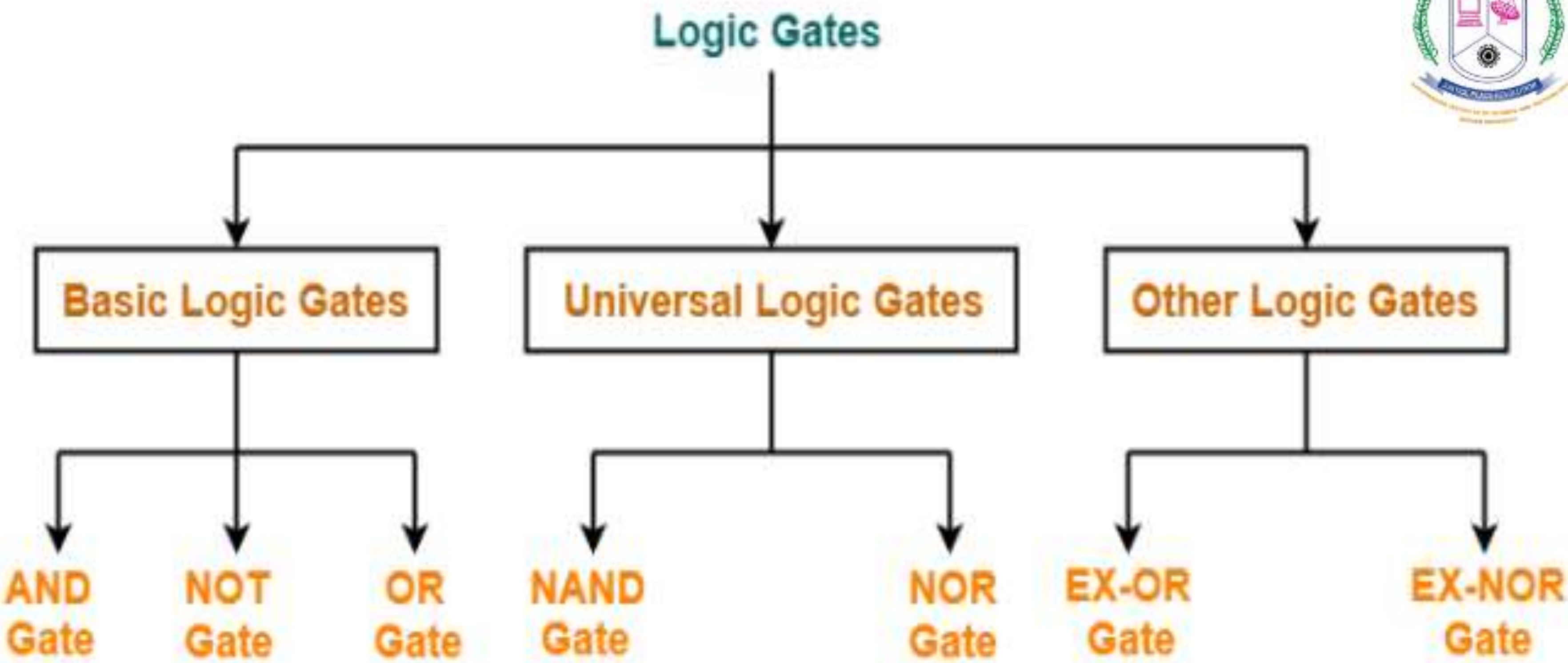
CHENNAI-119



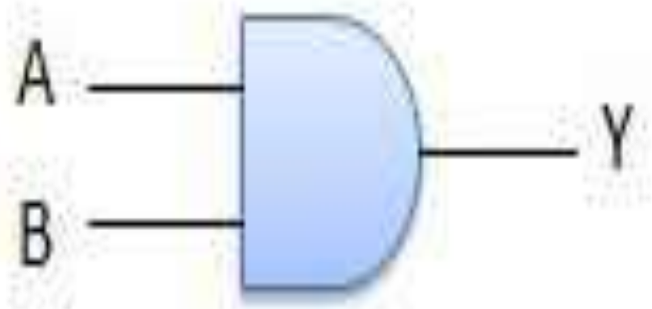
Types of Logic Gates

BINARY LOGIC GATES

AND Gate

A circuit which performs an AND operation is shown in figure. It has n input (n >= 2) and one output.

Logic diagram



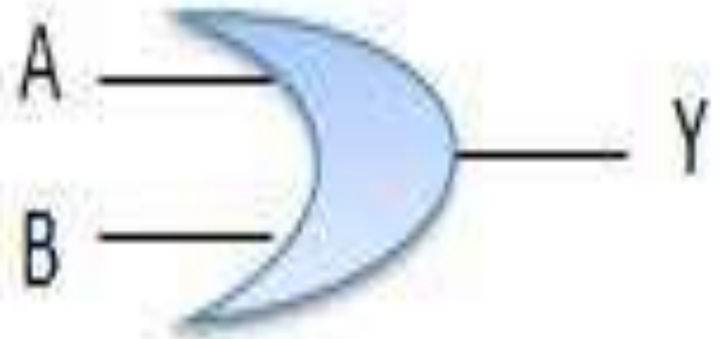
Truth Table

Inputs		Output
A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate

A circuit which performs an OR operation is shown in figure. It has n input (n >= 2) and one output.

Logic diagram



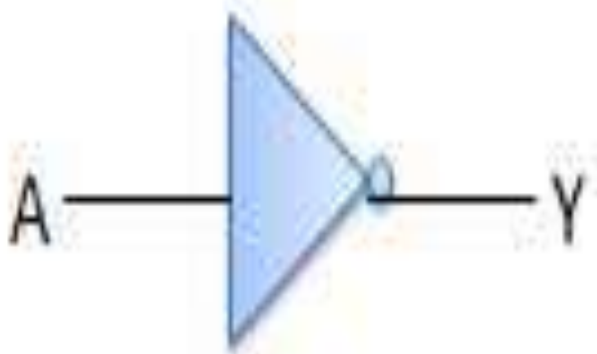
Truth Table

Inputs		Output
A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1

NOT Gate

NOT gate is also known as **Inverter**. It has one input A and one output Y.

Logic diagram



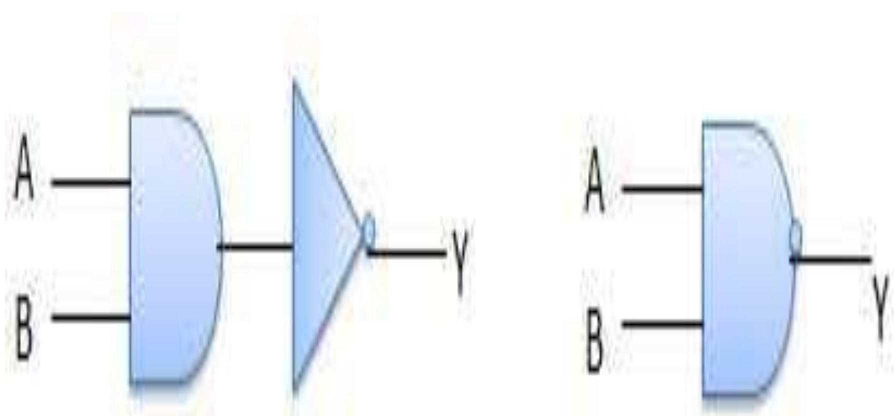
Truth Table

Inputs	Output
A	B
0	1
1	0

NAND Gate

A NOT-AND operation is known as NAND operation. It has n input (n >= 2) and one output.

Logic diagram



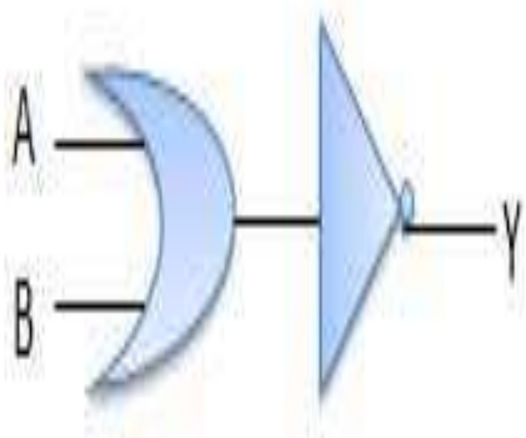
Truth Table

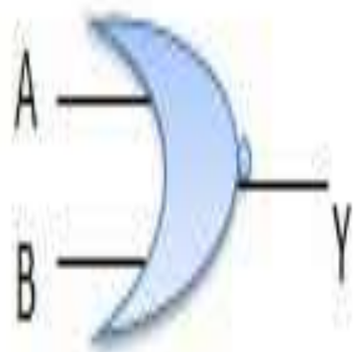
Inputs		Output
A	B	$\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

NOR Gate

A NOT-OR operation is known as NOR operation. It has n input (n >= 2) and one output.

Logic diagram





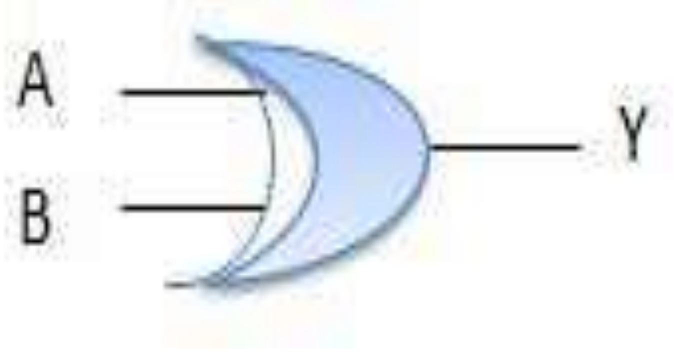
Inputs		Output
A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

XOR Gate

XOR or Ex-OR gate is a special type of gate. It can be used in the half adder, full adder and subtractor. The exclusive-OR gate is abbreviated as EX-OR gate or sometime as X-OR gate. It has n input (n >= 2) and one output.

Truth Table

Logic diagram

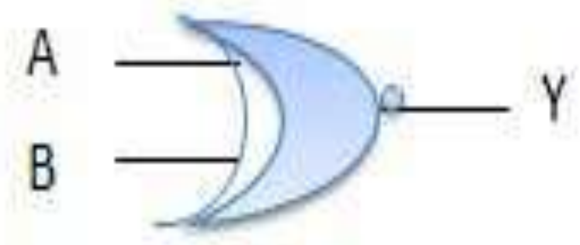


Inputs		Output
A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

XNOR Gate

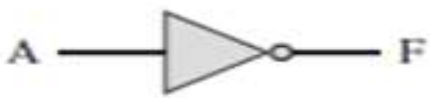
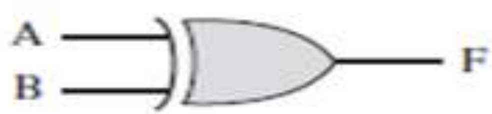
XNOR gate is a special type of gate. It can be used in the half adder, full adder and subtractor. The exclusive-NOR gate is abbreviated as EX-NOR gate or sometime as X-NOR gate. It has n input (n >= 2) and one output.

Logic diagram



Truth Table

Inputs		Output
A	B	$A \odot B$
0	0	1
0	1	0
1	0	0
1	1	1

Name	Graphical Symbol	Algebraic Function	Truth Table
AND		$F = A \bullet B$ or $F = AB$	A B | F 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR		$F = A + B$	A B | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
NOT		$F = \overline{A}$ or $F = A'$	A | F 0 | 1 1 | 0
NAND		$F = \overline{AB}$	A B | F 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR		$F = \overline{A + B}$	A B | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0
XOR		$F = A \oplus B$	A B | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0

BOOLEAN ALGEBRA

Boolean Algebra is used to analyze and simplify the digital (logic) circuits. It uses only the binary numbers i.e. 0 and 1. It is also called as **Binary Algebra** or **logical Algebra**.

Rule in Boolean Algebra

Following are the important rules used in Boolean algebra.

- •Variable used can have only two values. Binary 1 for HIGH and Binary 0 for LOW.
- •Complement of a variable is represented by an over bar (-). Thus, complement of variable B is represented as . Thus if B = 0 then = 1 and B = 1 then = 0.
- •OR ing of the variables is represented by a plus (+) sign between them. For example OR ing of A, B, C is represented as A + B + C.
- •Logical ANDing of the two or more variable is represented by writing a dot between them such as A.B.C. Sometime the dot may be omitted like ABC.

Boolean Laws

There are six types of Boolean Laws.

Commutative law

Any binary operation which satisfies the following expression is referred to as commutative operation.

law states that changing the sequence of the variables does not have any effect on the output of a logic circuit.

$$(i)\ A.B = B.A \qquad (ii)\ A + B = B + A$$

Associative law

This law states that the order in which the logic operations are performed is irrelevant as their effect is the same.

$$(i)\ (A.B).C = A.(B.C) \qquad (ii)\ (A + B) + C = A + (B + C)$$

Distributive law

Distributive law states the following condition.

$$A.(B + C) = A.B + A.C$$

AND law

These laws use the AND operation. Therefore they are called as **AND** laws.

(i) $A.0 = 0$ (ii) $A.1 = A$

(iii) $A.A = A$ (iv) $A.\overline{A} = 0$

OR law

These laws use the OR operation. Therefore they are called as **OR** laws.

(i) $A + 0 = A$ (ii) $A + 1 = 1$

(iii) $A + A = A$ (iv) $A + \overline{A} = 1$

INVERSION law

This law uses the NOT operation. The inversion law states that double inversion of a variable results in the original variable itself.

$$\overline{\overline{A}} = A$$

OR RULES (ADDITION)

1. 0 + 0 = 0
2. 0 + 1 = 1
3. 1 + 0 = 1
4. 1 + 1 = 1
5. A + 0 = A
6. A + 1 = 1
7. A + A (bar) =1

AND RULES (MULTIPLICATION)

1. 0.0 = 0
2. 0.1 = 0
3. 1.0 = 0
4. 1.1 = 1
5. A.1 = A
6. A.0 = 0
7. A.A = A
8. A.A (bar) = 0

REDUNDANT LITERAL RULE:

$$A + \overline{A}B = A + B$$

$$Similarly,$$

$$A(\overline{A} + B) = AB$$

Inputs			Output
A	B	$\bar{A}B$	$A + \bar{A}B$
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	1

Inputs		Output
A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

Basic Rules of Boolean Algebra

1. $A + 0 = A$	7. $A \cdot A = A$
2. $A + 1 = 1$	8. $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	9. $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	10. $A + AB = A$
5. $A + A = A$	11. $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	12. $(A + B)(A + C) = A + BC$

DeMorgan's Theorem

$\overline{(AB)} = (\overline{A} + \overline{B})$

$\overline{(A + B)} = (\overline{A}\ \overline{B})$

Name	AND form	OR form
Identity law	$1A = A$	$0 + A = A$
Null law	$0A = 0$	$1 + A = 1$
Idempotent law	$AA = A$	$A + A = A$
Inverse law	$A\bar{A} = 0$	$A + \bar{A} = 1$
Commutative law	$AB = BA$	$A + B = B + A$
Associative law	$(AB)C = A(BC)$	$(A + B) + C = A + (B + C)$
Distributive law	$A + BC = (A + B)(A + C)$	$A(B + C) = AB + AC$
Absorption law	$A(A + B) = A$	$A + AB = A$
De Morgan's law	$\overline{AB} = \bar{A} + \bar{B}$	$\overline{A + B} = \bar{A}\bar{B}$

Boolean Algebra Theorems

★ Duality

- The *dual* of a Boolean algebraic expression is obtained by interchanging the AND and the OR operators and replacing the 1's by 0's and the 0's by 1's.
- $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$
- $x + (y \cdot z) = (x + y) \cdot (x + z)$

Applied to a valid equation produces a valid equation

★ Theorem 1

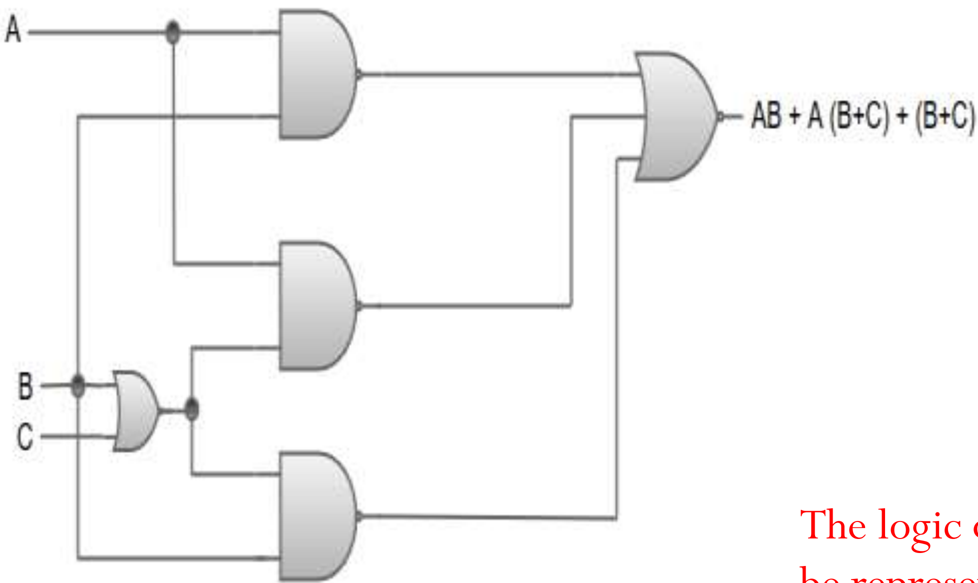
- $x \cdot x = x$ $\qquad$ $x + x = x$

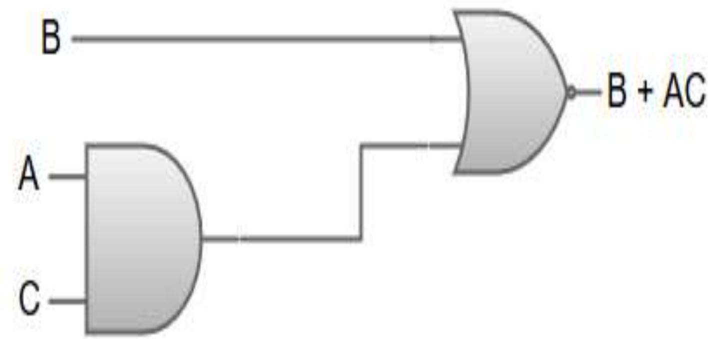
★ Theorem 2

- $x \cdot 0 = 0$ $\qquad$ $x + 1 = 1$

Let us consider an example of a Boolean function:

AB+A (B+C) + B (B+C)





The logic diagram for Boolean function B + AC can be represented as:

AB + A (B+C) + B (B+C)

AB + AB + AC + BB + BC {Distributive law; A (B+C) = AB+AC, B (B+C) = BB+BC}

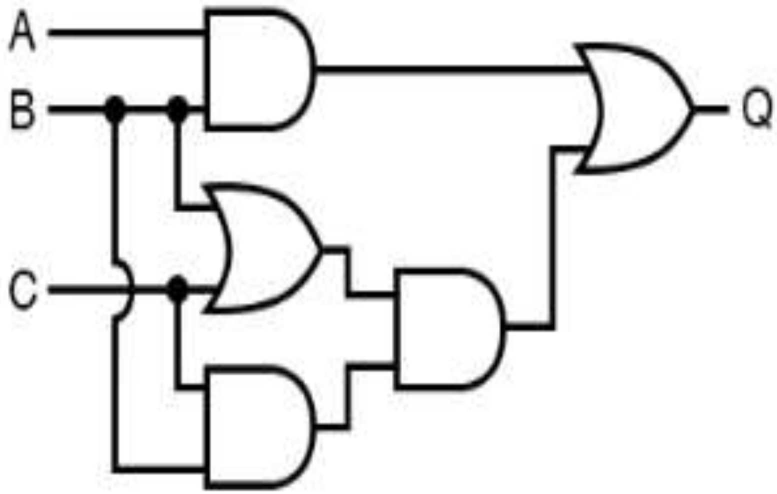
AB + AB + AC + B + BC {Idempotent law; BB = B}

AB + AC + B + BC {Idempotent law; AB+AB = AB}

AB + AC +B {Absorption law; B+BC = B}

B + AC {Absorption law; AB+B = B}

EXAMPLE 2



$AB + BC(B + C)$

Distributing terms

$AB + BBC + BCC$

Applying identity $AA = A$ to 2nd and 3rd terms

$AB + BC + BC$

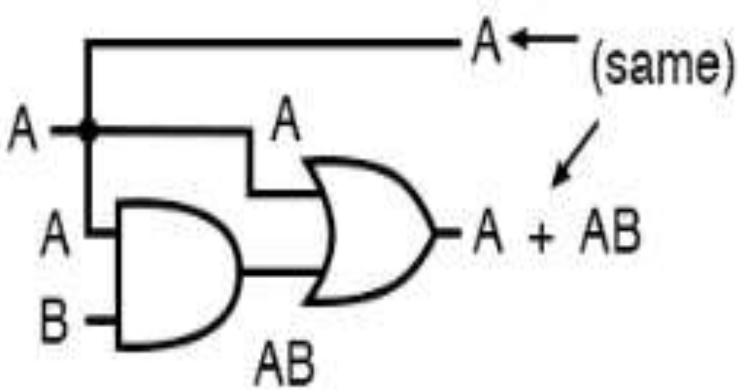
Applying identity $A + A = A$ to 2nd and 3rd terms

$AB + BC$

Factoring B out of terms

$B(A + C)$

EXAMPLE 3



$A + AB$

Factoring A out of both terms

$A(1 + B)$

Applying identity $A + 1 = 1$

$A(1)$

Applying identity $1A = A$

A

Example 4

- $(A + B)(A + C) = A + BC$
- This rule can be proved as follows:
- $(A + B)(A + C) = AA + AC + AB + BC$ (Distributive law)

$= A + AC + AB + BC$ $\quad (AA = A)$

$= A(1 + C) + AB + BC$ $\quad (1 + C = 1)$

$= A.1 + AB + BC$

$= A(1 + B) + BC$ $\quad (1 + B = 1)$

$= A.1 + BC$ $\quad (A.1 = A)$

$= A + BC$

Example 5

$$\overline{A}\,\overline{B} + AB + \overline{A}B$$

$$\overline{A}\,\overline{B} + B(A + \overline{A})$$

$$\overline{A}\,\overline{B} + B \cdot 1$$

$$\overline{A}\,\overline{B} + B$$

$$B + \overline{A}\,\overline{B}$$

$$(B + \overline{A})(B + \overline{B})$$

$$(B + \overline{A}) \cdot 1$$

$$B + \overline{A}$$

$$\overline{A} + B$$

Example 6

$$ABC + \overline{A}B + AB\overline{C}$$

$$B(AC + \overline{A} + A\overline{C})$$

$$B(A[C + \overline{C}] + \overline{A})$$

$$B(A \cdot 1 + \overline{A})$$

$$B(A + \overline{A})$$

$$B \cdot 1$$

$$B$$

Example 7

$$AB + A(CD + C\overline{D})$$

$$A(B + [CD + C\overline{D}])$$

$$A(B + C[D + \overline{D}])$$

$$A(B + C\cdot 1)$$

$$A(B + C)$$

Example 8

$$\overline{A} + AB + A\overline{C} + A\overline{B}\overline{C}$$

$$\overline{A} + AB + A\overline{C}(1 + \overline{B})$$

$$\overline{A} + AB + A\overline{C}\cdot 1$$

$$\overline{A} + AB + A\overline{C}$$

$$\overline{A} + A(B + \overline{C})$$

$$(\overline{A} + A)(\overline{A} + [B + \overline{C}])$$

$$1\cdot(\overline{A} + [B + \overline{C}])$$

$$\overline{A} + B + \overline{C}$$

Example 9

$$A + \bar{A}B = A + B$$

$A + \bar{A}B$

$\downarrow$ Applying the previous rule to expand A term

$A + AB = A$

$A + AB + \bar{A}B$

$\downarrow$ Factoring B out of 2nd and 3rd terms

$A + B(A + \bar{A})$

$\downarrow$ Applying identity $A + \bar{A} = 1$

$A + B(1)$

$\downarrow$ Applying identity $1A = A$

$A + B$

10. $(B\bar{C} + \bar{A}D)\,(A\bar{B} + C\bar{D})$

$$B\bar{C}A\bar{B} + B\bar{C}C\bar{D} + \bar{A}DA\bar{B} + \bar{A}DC\bar{D}$$

$$(B\bar{B})\bar{C}A + (\bar{C}C)B\bar{D} + (\bar{A}A)D\bar{B} + (D\bar{D})\bar{A}C$$

$$0\cdot\bar{C}A + 0\cdot B\bar{D} + 0\cdot D\bar{B} + 0\cdot\bar{A}C$$

$$0$$

simplify $F = (A+B)(A+\bar{C}) + \bar{A}\bar{B} + \bar{A}\bar{C}$.

$$= A\cdot A + A\cdot\bar{C} + A\cdot B + B\cdot\bar{C} + \bar{A}\bar{B} + \bar{A}\bar{C}.$$

$$= A + \bar{C}(A+\bar{A}) + A\cdot B + B\bar{C} + \bar{A}\bar{B}.$$

$$= A + \bar{C} + AB + B\bar{C} + \bar{A}\bar{B}$$

$$= A(1+B) + \bar{C}(1+B) + \bar{A}\bar{B}$$

$$= A + \bar{C} + \bar{A}\bar{B} \qquad \because A + \bar{A}\bar{B} = A + \bar{B}$$

$$= A + \bar{C} + \bar{B}$$

Reduce the expression.

$$F = (\overline{A + B\bar{C}})(A\bar{B} + ABC).$$

Demorganic $\overline{A+B\bar{C}} = (\bar{A}\cdot\overline{B\bar{C}})(A\bar{B}+ABC).$

$$= (\bar{A}BC)(A\bar{B} + ABC)$$

$$= \bar{A}AB\bar{B}C + A\bar{A}B\cdot B\cdot C\cdot C.$$

$$= 0 + 0$$

$$= 0$$

Redue the expression.

$$F = A + B[AC + (B+\bar{C})D]$$

$$= A + B[AC + BD + \bar{C}D]$$

$$= A + ABC + B\cdot B\cdot D + B\bar{C}D$$

$$= A(1+BC) + BD + B\bar{C}D$$

$$= A + BD(1+\bar{C}).$$

$$= A + BD.$$

THANK YOU
ANY QUERIES?