UNIT II

AC CIRCUITS

2.1 INTRODUCTION

We have seen so far about the analysis of DC circuit. A DC quantity is one which has a constant magnitude irrespective of time. But an alternating quantity is one which has a varying magnitude and angle with respect to time. Since it is time varying in nature, at any time it can be represented in three ways 1) By its effective value 2) By its average value and 3) By its peak value.

Some important terms

1. Wave form
   A wave form is a graph in which the instantaneous value of any quantity is plotted against time.

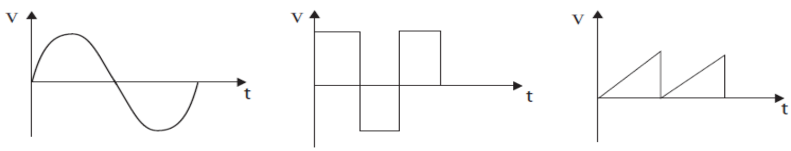

Fig 2.1(a-c)

2. Alternating Waveform
   This is wave which reverses its direction at regularly recurring interval.
3. Cycle

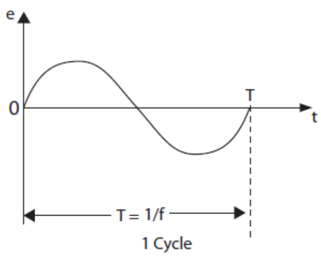

Figure 2.2

It is a set of positive and negative portion of waveforms.

4. Time Period
   The time required for an alternating quantity, to complete one cycle is called the time period and is denoted by T.
5. Frequency
   The number of cycles per second is called frequency and is denoted by f. It is measured in cycles/second (cps) (or) Hertz
   $$f = 1/T$$
6. Amplitude
   The maximum value of an alternating quantity in a cycle is called amplitude. It is also known as peak value.
7. R.M.S value [Root Mean Square]
   The steady current when flowing through a given resistor for a given time produces the same amount of heat as produced by an alternating current when flowing through the same resistor for the same time is called R.M.S value of the alternating current.
   $$RMS\ Value = \sqrt{\begin{matrix}Area\ Under\ the\ square\ curve\ for \\ one\ complete\ cycle / Period\end{matrix}}$$
8. Average Value of AC
   The average value of an alternating current is defined as the DC current which transfers across any circuit the same change as is transferred by that alternating current during the same time.

   Average Value = Area Under one complete cycle/Period.

9. Form Factor (Kf)
   It is the ratio of RMS value to average value

   Form Factor = RMS value/Average Value

10. Peak Factor (Ka)
    It is the ratio of Peak (or) maximum value to RMS value.

    Peak Factor Ka=Peak Value/RMS value

2.2 Analytical method to obtain the RMS, Average value, Form Factor and Peak factor for sinusoidal current (or) voltage

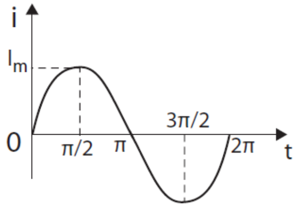

Figure 2.3

$$i = I_m \sin \omega t \;\; ; \omega t = \theta$$

Mean square of AC $I_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$

$$= \frac{1}{\pi}\int_0^{\pi} i^2 d\theta \text{ [since it is symmetrical]}$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \sin^2 d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\pi$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Average Value:

$$I_{av} = \int_0^{\pi} \frac{i d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, \mathrm{d}\theta$$

$$= \frac{I_m}{\pi}\int_0^{\pi} \sin\theta \, \mathrm{d}\theta$$

$$= -\frac{\mathrm{I_m}}{\pi}[\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{\pi}[\cos\pi - \cos 0]$$

$$= \frac{I_m}{\pi}(-1-1)$$

$$= -\frac{2I_m}{\pi}$$

$$Form\ Factor = \frac{RMS}{Avg} = \frac{\mathrm{I_m}/\sqrt{2}}{2\mathrm{I_m}/\pi} = 1.11$$

$$Peak\ Factor = \frac{MAX}{RMS} = \frac{I_m}{RMS} = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = 1.414$$

2.2.1 Expression for RMS, Average, Form Factor, Peak factor for Half wave rectifier

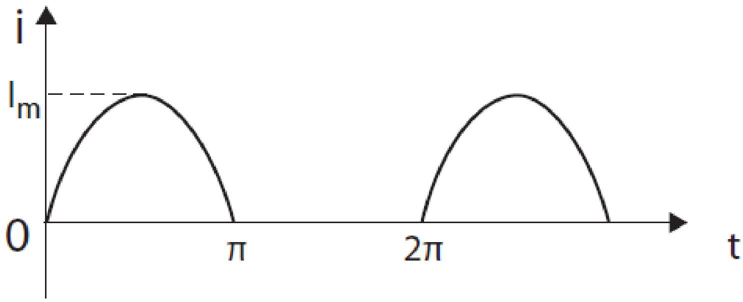

Figure 2.4

1) RMS value

$i = I_m \sin\theta \;;\quad 0 < \theta < \pi$

$i = 0 \;;\quad \pi < \theta \leq 2\pi$

Mean square of AC $I_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$

$$= \frac{1}{2\pi}\int_0^{\pi} i^2 d\theta + \int_{\pi}^{2\pi} i^2 d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i^2 d\theta + 0\right]$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2 d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{4\pi}\pi$$

$$I_{RMS} = \frac{I_m}{2}$$

Average Value:

$$I_{av} = \int_0^{\pi} \frac{i d\theta}{2\pi}$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i d\theta + 0\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta d\theta$$

$$= \frac{I_m}{2\pi}\int_0^{\pi} I_m \sin\theta d\theta$$

$$= \frac{I_m}{2\pi}\left[\cos\theta\right]_0^{\pi}$$

$$= \frac{I_m}{2\pi}\left[\cos\pi - \cos 0\right]$$

$$= -\frac{I_m}{2\pi}\left(-1-1\right)$$

$$= \frac{2I_m}{2\pi} = \frac{I_m}{\pi}$$

$$Form\,Factor = \frac{RMS}{Avg} = \frac{I_m}{2} / \frac{I_m}{\pi} = 1.57$$

$$Peak\ Factor = \frac{MAX}{RMS} = \frac{I_m}{RMS} / \frac{I_m}{\frac{I_m}{2}} = 2$$

Examples:

2.1) The equation of an alternating current is given by

i = 40sin 314 t

Determine

(i) Max value of current
(ii) Average value of current
(iii) RMS value of current
(iv) Frequency and angular frequency
(v) Form Factor
(vi) Peak Factor

Solution:

i = 40sin 314 t

We know that i = I_m sin ωt

S_o $I_m = 40$
ω = 314 $rad/\sec$
(i) Maximum value of current = 40A
(ii) Average value of current

$$I_{Avg} = \frac{2I_m}{\pi} = \frac{2\times 40}{\pi} = 25.464A$$

(iii) RMS value of current

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{40}{\sqrt{2}} = 28.28\ Amp$$

(iv) Frequency $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} \approx 50\,Hz$

(v) $Form\ Factor \frac{RMS}{Avg} = \frac{28.28}{25.46} = 1.11$

(vi) $Peak\ Factor = \frac{\max}{RMS} = \frac{40}{28.28} = 1.414$

2.2) what is the equation of a 50Hz voltage sin wave having an rms value of 50 volt

Solution:

f = 50Hz

V_{rms} = 50V

v = V_m sinωt

ω = 2πf = 2π×50=314 rad/sec

$V_m = V_{rms}\sqrt{2} = 50\times\sqrt{2} = 70.7$ volt

$\therefore v = 70.7\sin 314t$

2.3 PHASOR REPRESENTATION OF SINUSOIDAL VARYING ALTERNATING QUANTITIES

The Phasor representation is more convenient in handling sinusoidal quantities rather than by using equations and waveforms. This vector or Phasor representation of alternating quantity simplifies the complexity of the problems in the AC circuit.

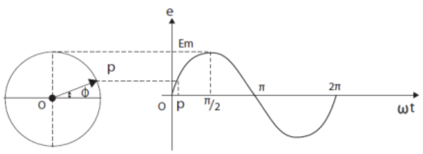

Figure 2.5

$\overline{OP}$ =E_m

E_m – the maximum value of alternating voltage which varies sinusoidally

Any alternating sinusoidal quantity (Voltage or Current) can be represented by a rotating Phasor, if it satisfies the following conditions.

1. The magnitude of rotating phasor should be equal to the maximum value of the quantity.
2. The rotating phasor should start initially at zero and then move in anticlockwise direction. (Positive direction)
3. The speed of the rotating phasor should be in such a way that during its one revolution the alternating quantity completes one cycle.

Phase

The phase is nothing but a fraction of time period that has elapsed from reference or zero position.

In Phase

Two alternating quantities are said to be in phase, if they reach their zero value and maximum value at the same time.

Consider two alternating quantities represented by the equation

$i_1 = Im_1 \sin\theta$
$i_2 = Im_2 \sin\theta$

can be represented graphically as shown in Fig 2.6(a).

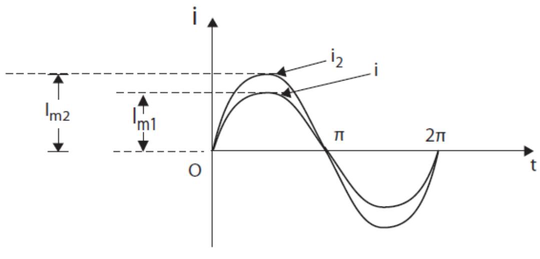

Figure 2.6(a) Graphical representation of sinusoidal current

From Fig 2.6(a), it is clear that both i_1 and i_2 reaches their zero and their maximum value at the same time even though both have different maximum values. It is referred as both currents are in phase meaning that no phase difference is between the two quantities. It can also be represented as vector as shown in Fig 2.6(b).

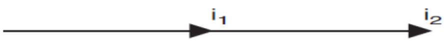

Figure 2.6(b) Vector diagram

Out of Phase

Two alternating quantities are said to be out of phase if they do not reach their zero and maximum value at the same time. The Phase differences between these two quantities are represented in terms of '**lag**' and '**lead**' and it is measured in radians or in electrical degrees.

Lag

Lagging alternating quantity is one which reaches its maximum value and zero value later than that of the other alternating quantity.

Consider two alternating quantities represented by the equation:

$i_1 = Im_1 \sin(\omega t - \Phi)$
$i_2 = Im_2 \sin(\omega t)$

These equations can be represented graphically and in vector form as shown in Fig 2.7(a) and Fig 2.7(b) respectively.

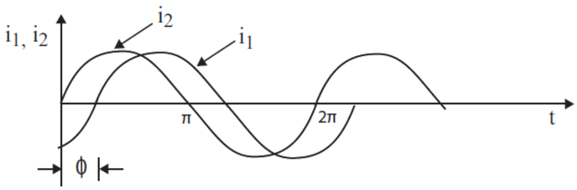

Figure 2.7a

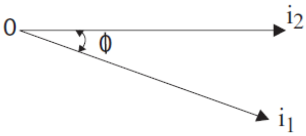

Figure 2.7b

It is clear from the Fig 2.7(a), the current i_1 reaches its maximum value and its zero value with a phase difference of 'Φ' electrical degrees or radians after current i_2. (ie) i_1 lags i_2 and it is represented by a minus sign in the equation.

Lead

Leading alternating quantity is one which reaches its maximum value and zero value earlier than that of the other alternating quantity.

Consider two alternating quantities represented by the equation:
$i_1 = Im_1 \sin(\omega t + \Phi)$
$i_2 = Im_2 \sin(\omega t)$

These equations can be represented graphically and in vector form as shown in Fig 2.8(a) and Fig 2.8(b) respectively.

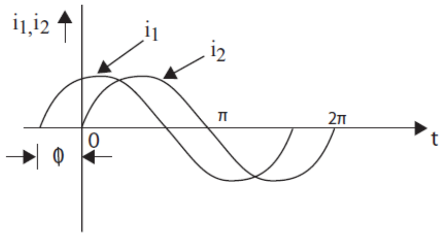

Figure 2.8(a)

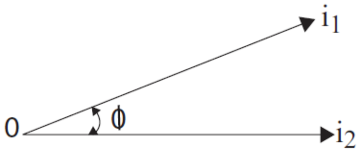

Figure 2.8(b)

The Fig 2.8(a) clearly illustrates that current i_1 has started already and reaches its maximum value before the current i_2. (ie) i_1 leads i_2 and it is represented by a positive sign in the equation.

Note:

1. Two vectors are said to be in quadrature, if the Phase difference between them is 90°.
2. Two vectors are said to be in anti phase, if the phase difference between them is 180°.

2.4 REVIEW OF 'J' OPERATOR

A vector quantity has both magnitude and direction. A vector' A' is represented in two axis plane as shown in Fig 3.10

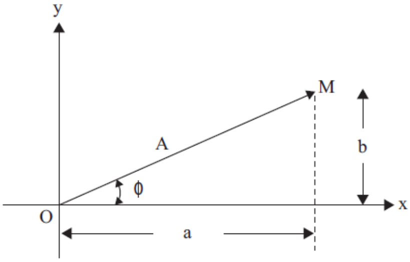

Figure 2.9

In Fig 2.9, OM represents vector A
Φ represents the phase angle of vector A

A = a + jb
a – Horizontal component or active component or in phase component
b – Vertical component or reactive component or quadrature component

The magnitude of vector $'A' = \sqrt{a^2 + b^2}$
Phase angle of Vector 'A' = $\alpha = \tan^{-1}$ (b/a)
Features of j – Operator

1. $j = \sqrt{-1}$
   It indicates anticlockwise rotation of Vector through 90°.
2. $j^2 = j \cdot j = -1$
   It indicates anticlockwise rotation of vector through 180°.
3. $j^3 = j \cdot j \cdot j = -j$
   It indicates anticlockwise rotation of vector through 270°.
4. $j^4 = j \cdot j \cdot j \cdot j = 1$
   It indicates anticlockwise rotation of vector through 360°.
5. –j indicates clockwise rotation of vector through 90°.
6. $\frac{1}{j} = \frac{1.j}{j.j} = \frac{j}{j^2} = \frac{j}{-1} = -j$

A vector can be written both in polar form and in rectangular form.
A = 2 + j3

This representation is known as rectangular form.
Magnitude of A = |A| = $\sqrt{2^2 + 3^2}$ =3.606
Phase angle of A = $\alpha = \tan^{-1}(3/2) = 56°.31$
$A = |A| \angle \alpha°$
$A = 3.606 \angle 56°.31$

This representation is known as polar form.

Note:

1. Addition and Subtraction can be easily done in rectangular form.
2. Multiplication and division can be easily done in polar form.

Examples:

2.3) A= 2 + j3; B = 4 + j5.

Add Vector A and Vector B and determine the magnitude and Phase angle of resultant vector.

Solution:

A + B = 2 + j3 + 4 + j5 = 6 + j8

$\therefore$ Magnitude = | A + B | = $\sqrt{6^2+8^2}$ = 10.0

Phase angle = α = $\tan^{-1}$ (B /A) = $\tan^{-1}$ (8/6) = 53°.13

2.4) A= 2 + j5; B = 4 – j2.

Subtract Vector A and Vector B and determine the magnitude and Phase angle of resultant vector.

Solution:

A – B = 2 + j5 – (4 – j2) = 2 + j5 – 4 + j2 = -2 + j7

$\therefore$ Magnitude = | A – B | = $\sqrt{-2^2+7^2}$ = 7.280

Phase angle = α = $\tan^{-1}$ (B /A) = $\tan^{-1}$ (7/-2) = -74°.055

2.5) A= 2 + j3; B = 4 – j5.

Perform A x B and determine the magnitude and Phase angle of resultant vector.

Solution:

A= 2 + j3

|A| = $\sqrt{2^2+3^2}$ =3.606

α = $\tan^{-1}$ (3/2) = 56°.310

A= 3.606 $\angle$ 56°.310

B = 4 – j5

|B| = $\sqrt{4^2+-5^2}$ =6.403

α = $\tan^{-1}$ (-5/4) = -51°.340

B= 6.403$\angle$ -51°.340

A X B = 3.606 $\angle$ 56°.310 X 6.403$\angle$ -51°.340

= 3.606 X 6.403$\angle$ (56°.310 + (-51°.340))

= 23.089 $\angle$4°.970

2.6) A= 4 – j2; B = 2 + j3.

Perform $\frac{A}{B}$ and determine the magnitude and Phase angle of resultant vector.

Solution:

A= 4 – j2

|A| = $\sqrt{4^2+-2^2}$ = 4.472

α = $\tan^{-1}$ (-2/4) = -26°.565

A= 4.472 $\angle$ -26°.565

B = 2 + j3

$|B| = \sqrt{2^2 + 3^2} = 3.606$

$\alpha = \tan^{-1}(3/2) = 56°.310$

$B = 3.606\angle 56°.310$

$$\frac{A}{B} = \frac{4.472\angle{-26^o}.565}{3.606\angle 56^o.310} = \frac{4.472}{3.606}\angle\ {-26^o}.565 - 56^o.310 = 1.240\ \angle{-82.875}$$

2.5 ANALYSIS OF AC CIRCUIT

The response of an electric circuit for a sinusoidal excitation can be studied by passing an alternating current through the basic circuit elements like resistor (R), inductor (L) and capacitor (C).

2.5.1 Pure Resistive Circuit:

In the purely resistive circuit, a resistor (R) is connected across an alternating voltage source as shown in Fig.2.10

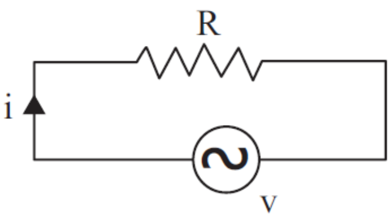

Figure 2.10

Let the instantaneous voltage applied across the resistance (R) be

$$V = V_m \sin\omega t$$

From Ohms law,

$$v = i\,R$$

$$I = \frac{v}{R} = \frac{V_m \sin\omega t}{R}$$

$$\because I_m = \frac{V_m}{R}$$

$$= I_m \sin\omega t$$

where,

$V_m \rightarrow$ Maximum value of voltage (V)
$I_m \rightarrow$ Maximum value of current (A)
$\omega \rightarrow$ Angular frequency (rad/sec)
$t \rightarrow$ Time period (sec)

Phasor Representation:

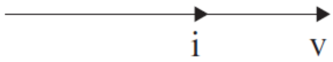

Figure 2.11

Comparing equations, we find that applied voltage and the resulting current are **inphase** with each other. Therefore in a purely resistive circuit there is no phase difference between voltage and current i.e., phase angle is zero (Φ=0).

If voltage is taken as reference, the phasor diagram for purely resistive circuit is shown in Fig.2.11

Waveform Representation:

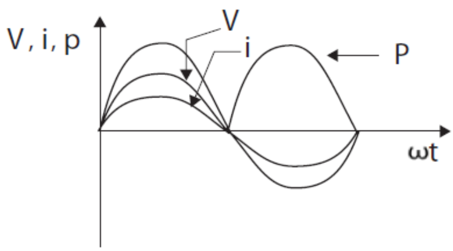

Figure 2.12

The waveform for applied voltage and the resulting current and power were shown in Fig.2.12. Since the current and voltage are inphase the waveforms reach their maximum and minimum values at the same instant.

Impedance:
In an AC circuit, impedance is the ratio of the maximum value of voltage to the maximum value of current.

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{V_m/R} = R$$

$$\therefore Z = R$$

Power:

(i) Instantaneous power:
It is defined as the product of instantaneous voltage and instantaneous current.

p= v i

$= V_m \sin\omega t \, I_m \sin\omega t = V_m I_m \sin^2\omega t$

$[\because \omega t = \theta]$

$p = V_m I_m \sin^2\theta$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin^2\theta d\theta$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{V_m I_m}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{V_m I_m}{2\pi}\left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right]$$

$$= \frac{V_m I_m}{2\pi}[\pi] = \frac{V_m I_m}{2}$$

$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{RMS} I_{RMS} = V.I$$

$$P = V\,I$$

Power Factor:

It is defined as the cosine of the phase angle between voltage and current.

$$\cos\phi = \cos 0 = 1(unity)$$

Problems:

2.7) A voltage of 240 sin 377t is applied to a 6Ω resistor. Find the instantaneous current, phase angle, impedance, instantaneous power, average power and power factor.

Solution:

Given: $v = 240 \sin 377t$

$V_m = 240$ V

$\omega = 377$ rad/sec

$R = 6\Omega$

Instantaneous current:

$$= \frac{V_m \sin\omega t}{R}$$

$$= \frac{240}{6}\sin 377t$$

$$= 40\sin 377tA$$

I. Phase angle:

$\phi = 0$

II. Impedance:

Z = R = 6Ω

III. Instantaneous power:

IV. $$p = V_m\ I_m\ \sin^2\omega t$$

$$= 240.40.\sin^2 377t$$

$$= 9600\sin^2 377t$$

V. Average power:

$$P = \frac{V_m I_m}{2} = 4800\text{watts}$$

VI. Power factor:

cosΦ = cos0 = 1

2.8) A voltage e = 200sinωt when applied to a resistor is found to give a power 100 watts. Find the value of resistance and the equation of current.

Solution:

Given: e = 200sinωt

$V_m = 200$

P = 100w

Average power, $P = \frac{V_m I_m}{2}$

$$100 = \frac{200 I_m}{2}$$

$I_m = 1$ A

Also, $V_m = I_m.R$

R = 200Ω

Instantaneous current, $I = I_m \sin\omega t = 1.\sin\omega t$ A

2.9) A voltage e = 250sinωt when applied to a resistor is found to give a power of 100W. Find the value of R and write the equation for current. State whether the value of R varies when the frequency is changed.

Solution:

Given: e = 250sinωt

$V_m = 250$
P = 100W

I. $P = \frac{V_m I_m}{2}$

$100 = \frac{250 I_m}{2}$

$I_m = 0.8$ A

II. $I_m = \frac{V_m}{R}$

$R = 312.5\Omega$

III. $I = 0.8\sin\omega t$

The resistance is independent of frequency, so the variation of frequency will not affect the resistance of the resistor.

2.5.2 Pure Inductive Circuit:

In this circuit, an alternating voltage is applied across a pure inductor (L) is shown in Fig. 2.13.

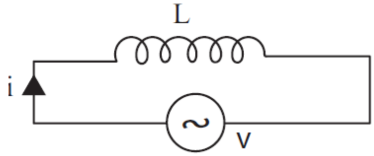

Figure 2.13

Let the instantaneous voltage applied across the inductance (L) be

$$v = V_m \sin\omega t$$

We know that the self induced emf always opposes the applied voltage.

$$V = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int v dt = \frac{1}{L}\int V_m \sin\omega t dt$$

$$= \frac{V_m}{\omega L}(-\cos\omega t) = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\left[\because I_m = \frac{V_m}{\omega L}\right]$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Phasor representation:

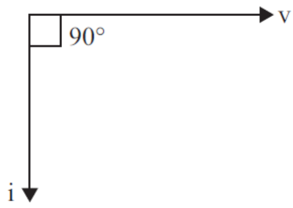

Figure 2.14

Comparing equations, the applied voltage and the resulting current are 90^0 out-of phase. Therefore in a purely inductive circuit there is a phase difference of 90^0 ie., phase angle is 90^0 ($\Phi = 90^0$). Clearly, the current **lags** behind the applied voltage.

Waveform representation:

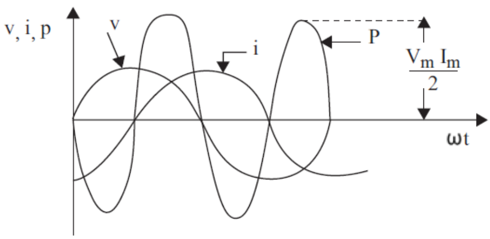

Figure 2.15

The waveform for applied voltage and the resulting current and the power were shown in Fig.2.15. The current waveform is lagging behind the voltage waveform by 90^0.

Impedance (Z):

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{V_m/\omega L} = \omega L$$

$Z = X_L$ [Impedance is equal to inductive reactance]

Power:

(i)Instantaneous power:

$$p = v\,i$$

$$= V_m \sin\omega t\ I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= V_m I_m \sin\omega t\ (-\cos\omega t)$$

$$= -V_m I_m \sin\omega t \cos\omega t = -V_m I_m \sin\theta \cos\theta$$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = -\frac{1}{\pi}\int_0^{\pi} V_m I_m \sin\theta \cos\theta\, d\theta$$

$$= -\frac{V_m I_m}{\pi}\int_0^{\pi} \frac{\sin 2\theta}{2} d\theta$$

$$[\because \sin 2\theta = 2\sin\theta\cos\theta]$$

$$= -\frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{V_m I_m}{4\pi}[\cos 2\pi - \cos 0]$$

$$= \frac{V_m I_m}{4\pi}[1-1] = 0$$

Thus, a pure inductor does not consume any real power. It is also clear from Fig. that the average demand of power from the supply for a complete cycle is zero. It is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of instantaneous power is $\left(\frac{V_m I_m}{2}\right)$.

Power Factor:

In a pure inductor the phase angle between the current and the voltage is 90^0 (lags).

$$\Phi = 90^0;\ \cos\Phi = \cos 90^0 = 0$$

Thus the power factor of a pure inductive circuit is zero lagging.

Problems:

2.10) A coil of wire which may be considered as a pure inductance of 0.225H connected to a 120V, 50Hz source. Calculate (i) Inductive reactance (ii) Current (iii) Maximum power delivered to the inductor (iv) Average power and (v) write the equations of the voltage and current.

Solution:

Given: $L = 0.225$ H

$V_{RMS} = V = 120$ V

$f = 50$Hz

I. Inductive reactance, $XL = 2\pi fL = 2\pi \times 50 \times 0.225 = 70.68\Omega$

II. Instantaneous current, $i = -I_m \cos\omega t$

$\because I_m = \frac{V_m}{\omega L}$ and $V_{RMS} = \frac{V_m}{\sqrt{2}}$, calculate I_m and V_m

$V_m = V_{RMS}\sqrt{2} = 169.71V$

$I_m = \frac{V_m}{\omega L} = \frac{169.71}{70.68} = 2.4A$

Maximum power, $P_m = \frac{V_m I_m}{2} = 203.74$ W

III. Average power, P=0

IV. Instantaneous voltage, $v = V_m \sin\omega t = 169.71 \sin 344t$ volts

Instantaneous current, $i = -2.4 \cos\omega t$ A

2.11) A pure inductance, L = 0.01H takes a current, 10 cos 1500t. Calculate (i) inductive reactance, (ii) the equation of voltage across it and (iii) at what frequency will the inductive reactance be equal to 40Ω.

Solution:

Given: $L = 0.01$ H

$I = 10\cos 1500t$

$I_m = 10A$

$\omega = 1500$ rad/sec

I. Inductive reactance, $X_L = \omega L = 1500 \times 0.01 = 15\Omega$

II. The voltage across the inductor, $e = L\frac{di}{dt}$

$$= 0.01\frac{d(10\cos 1500t)}{dt} = 0.01 \times 10[-\sin 1500t.1500]$$

$$= -150 \sin 1500t \text{ V}$$

III. $X_L = 40\Omega$; $2\pi fL = 40$

$$f = \frac{40}{2\pi \times 0.01} = 637\text{Hz}$$

2.12) In the circuit, source voltage is $v = 200\sin\left(314t + \frac{\pi}{6}\right)$ and the current is $i = 20\sin\left(314t - \frac{\pi}{3}\right)$ Find (i) frequency (ii) Maximum values of voltage and current (iii) RMS value of voltage and current (iv) Average values of both (v) Draw the phasor diagram (vi) circuit element and its values

Solution:

Given: $V_m = 200V$
$I_m = 20A$
$\omega = 314$ rad/sec

I. $\omega = 2\pi f$
f = 50Hz

II. $V_m = 200V$ and $I_m = 20A$

III. $V_{RMS} = \dfrac{V_m}{\sqrt{2}} = 141.42V$

$I_{RMS} = \dfrac{I_m}{\sqrt{2}} = 14.142A$

IV. For a sinusoidal wave, Average value of current, $I_{av} = \dfrac{2I_m}{\pi} = 12.732A$

Average value of voltage, $V_{av} = \dfrac{2V_m}{\pi} = 127.32A$

V. Phasor diagram

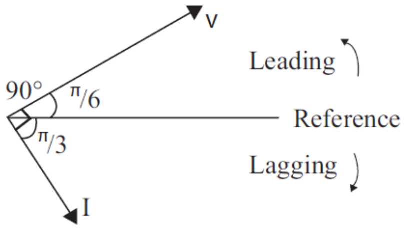

Figure 2.16

VI. From the phasor diagram, it is clear that I lags V by some angle (90^0). So the circuit is purely inductive.

$$I_m = \frac{V_m}{\omega L}$$

$$L = \frac{200}{314 \times 20} = 31.85mH$$

2.5.3 Pure Capacitive Circuit:

In this circuit, an alternating voltage is applied across a pure capacitor(C) is shown in Fig.2.17

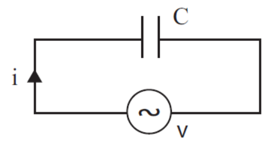

Figure 2.17

Let the instantaneous voltage applied across the inductance (L) be

$$v = V_m \sin\omega t$$

Let at any instant i be the current and Q be the charge on the plates.

So, charge on capacitor, Q = C.v

$$= C.\, V_m \sin\omega t$$

$$\text{Current}, i = \frac{dQ}{dt}$$

$$i = \frac{d}{dt}\left(CV_m \sin\omega t\right) = \omega CV_m \cos\omega t$$

$$= \omega CV_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\left[\because I_m = \omega CV_m\right]$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

From the above equations , we find that there is a phase difference of 90^0 between the voltage and current in a pure capacitor.

Phasor representation:

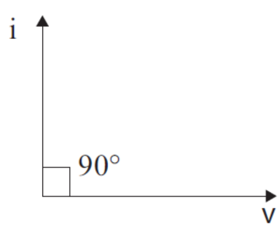

Figure 2.18

In the phasor representation, the current leads the voltage by an angle of 90^0.

Waveform representation:

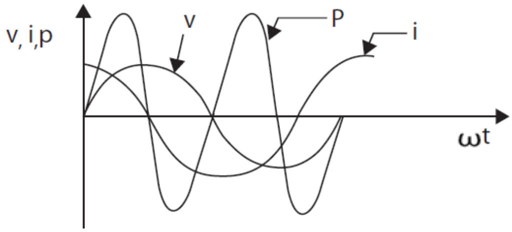

Figure 2.19

The current waveform is ahead of the voltage waveform by an angle of 90^0.

Impedance (Z):

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

$Z = X_C$ [Impedance is equal to capacitive reactance]

Power:

(i)Instantaneous power:

$$p = v\, i$$

$$= V_m \sin\omega t\; I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin\omega t\,(\cos \omega t)$$

$$= V_m I_m \sin\theta \cos \theta$$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin\theta \cos\theta d\theta$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \frac{\sin 2\theta}{2} d\theta$$

$$[\because \sin 2\theta = 2\sin\theta\cos\theta]$$

$$= \frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{V_m I_m}{4\pi}[-\cos 2\pi + \cos 0]$$

$$= \frac{V_m I_m}{4\pi}[-1+1] = 0$$

Thus, a pure capacitor does not consume any real power. It is also clear from Fig. that the average demand of power from the supply for a complete cycle is zero. Again, it is seen that power wave is a sine wave of frequency double that of the voltage and current. The maximum value of instantaneous power is $\left(\frac{V_m I_m}{2}\right)$.

Power Factor:

In a pure capacitor, the phase angle between the current and the voltage is 90^0 (leads).

$$\Phi = 90^0;\ \cos \Phi = \cos 90^0 = 0$$

Thus the power factor of a pure inductive circuit is zero leading.

Problems:

2.13) A 135μF capacitor has a 150V, 50Hz supply. Calculate (i) capacitive reactance (ii) equation of the current (iii) Instantaneous power (iv) Average power (v) RMS current (vi) Maximum power delivered to the capacitor.

Solution:

Given: $V_{RMS} = V = 150V$

$C = 135\mu F$

$f = 50Hz$

I. $X_C = \frac{1}{\omega C} = 23.58\Omega$

II. $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad \because I_m = \omega C V_m$ and $V_{RMS} = \frac{V_m}{\sqrt{2}}$

$V_m = 150 X \sqrt{2} = 212.13V$

$I_m = 314X135X10^{-6}X212.13 = 8.99A$

$i = 8.99 \sin\left(314t + \frac{\pi}{2}\right) A$

III. $p = V_m I_m \sin\omega t (\cos \omega t) = 212.13X8.99 \sin 314t.\cos 314t$

$= 66642.6 \sin 314t.\cos 314t = 66642.6 \frac{\sin 628t}{2}$

$[\because \sin 2\theta = 2 \sin\theta \cos\theta]$

$= 33321.3 \sin 628t\ W$

IV. Average power, $P = 0$

V. $I_{RMS} = \frac{I_m}{\sqrt{2}} = 6.36A$

VI. $P_m = \frac{V_m I_m}{2} = 953.52\ W$

2.14) A voltage of 100V is applied to a capacitor of 12μF. The current is 0.5 A. What must be the frequency of supply

Solution:

Given: V_{RMS}= V= 100V
C = 12μF
I = 0.5A

I. Find V_m and I_m

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

V_m=100X $\sqrt{2}$ =141.42V

$$I_{RMS} = \frac{I_m}{\sqrt{2}}$$

I_m= 0.5X $\sqrt{2}$ = 0.707A

II. $I_m = \omega CV_m = 2\pi fCV_m$

f = 66.3Hz

2.5.4 RL Series Circuit

Let us consider a circuit is which a pure resistance R and a purly inductive coil of inductance L are connected in series as shown in diagram.

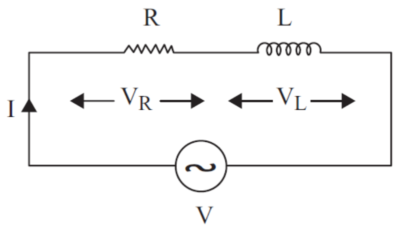

Figure 2.20

Let V = V_m Sin ωt be the applied voltage.
i = Circuit current at any constant.
I = Effective Value of Circuit Current.
V_R= Potential difference across inductor.
V_L= Potential difference across inductor.
F= Frequency of applied voltage.

The same current I flows through R and L hence I is taken as reference vector.

Voltage across resistor V_R= IR in phase with I
Voltage with inductor V_L= IX_L leading I by 90°

The phasor diagram of RL series circuit is shown below.

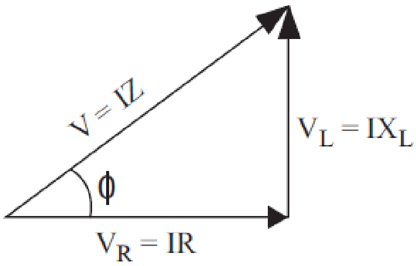

Figure 2.21

At any constant, applied voltage

$$V = V_R + V_L$$
$$V = IR + jIX_L$$
$$V = I\,(R + jx_L)$$
$$\frac{V}{I} = R + jx_L$$
$$= z \text{ impedance of circuit}$$
$$Z = R + j\,x_L$$
$$|z| = \sqrt{R^2 + X_L^2}$$

From phasor disgram,

$$\tan\phi = \frac{x_L}{R}$$
$$\phi = \tan^{-1}\left(\frac{x_L}{R}\right)$$

ϕ is called the phasor angle and it is the angle between V and I, its value lies between 0 to 90^0.

So impedence $Z = R + jX_L$

$$= |Z| < \phi$$

The current and voltage waveform of series RL Circuit is shown below.

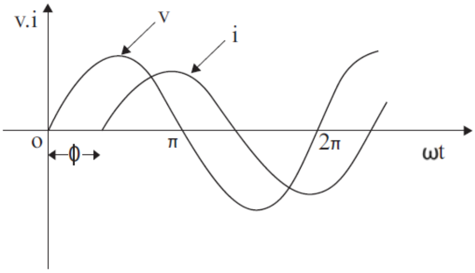

Figure 2.22

$V = V_m \sin\omega t$

$I = I_m \sin(\omega t - \phi)$

The current I lags behind the applied voltage V by an angle ϕ.

From phasor diagram,

Power factor $\cos\phi = \frac{R}{Z}$

Actual Power P = VI cos ϕ – Current component is phase with voltage

Reactive or Quadrature Power

Q = VI sinϕ – Current component is quadrature with voltage

Complex or Apparent Power

S = VI – Product of voltage and current

S = P + jQ

Problem

2.15) A series RL Circuit has

$$i(t) = 5\ sin\left(314t + \frac{2\pi}{3}\right) \text{ and } V(t) = 20\ sin\left(314t + \frac{5\pi}{3}\right)$$

Determine (a) the impedence of the circuit

(b) the values of R_1L and power factor

(c) average power of the circuit

Solution:

$$i(t) = 5 \sin(314t + \frac{2\pi}{3})$$

$$V(t) = 20 \sin(314t + \frac{5\pi}{3})$$

Phase angle of current $\theta_i = \frac{2\pi}{3} = \frac{2\times180}{3} = 120°$

Phase angle of voltage $\theta_v = \frac{5\pi}{3} = \frac{5\times180}{3} = 150°$

Phase angle between voltage and current $\theta = \theta_v \sim \theta_i$

$$= 150 - 120$$

$$\theta = 30^0$$

Power factor = cos θ

= cos 30

= 0.866 (lagging)

Impedence of the circuit $Z = \frac{V_m}{I_m}$

$$= \frac{20}{5}$$

$$Z = 4\Omega$$

(i) But cos $\phi = \frac{R}{Z}$

$$0.866 = \frac{R}{4}$$

$$\therefore R = 4 \times 0.866$$

$$R = 3.46\Omega$$

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$X_L = \sqrt{Z^2 + R^2}$$

$$= \sqrt{(4)^2 - (3.46)^2}$$

$$X_L = 2\Omega$$

$$\omega L = 2\Omega$$

$$L = \frac{2}{\omega}$$

$$= \frac{2}{3/4}$$

$$L = 6.37 \times 10^{-3} \text{ H}$$

(ii) Average power = VI cos ϕ

$$= \frac{20}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} (0.866)$$

= 43.3 watts

2.16) A coil having a resistance of 6Ω and an inductance of 0.03 H is connected across a 100V, 50Hz supply, Calculate.

(i) The current
(ii) The phase angle between the current and the voltage
(iii) Power factor
(iv) Power

Solution:

R = 6Ω
L = 0.03 H
$X_L = 2\pi fL$
$X_L = 2\pi \times 50 \times 0.03$
$X_L = 9.42\Omega$

$$|Z| = \sqrt{(R)^2 + (X_L)^2}$$

$$= \sqrt{(6)^2 + (9.42)^2}$$

$$|Z| = 11.17\Omega$$

(i) $I = \dfrac{V}{Z} = \dfrac{100}{11.17} = 8.95$ amps

(ii) $\phi = tan^{-1}\left(\dfrac{X_L}{R}\right)$

$= tan^{-1}\left(\dfrac{9.42}{6}\right)$

Φ = 57.5 (lagging)

(iii) Power factor = cos ϕ
= cos 57.5
= 0.537 (lagging)

(iv) Power = Average power
= VI cos Φ
= 100 ×8.95×0.537

Power = 480.6 Watts

2.17) A 10Ω resistor and a 20 mH inductor are connected is series across a 250V, 60 Hz supply. Find the impedence of the circuit, Voltage across the resistor, voltage across the inductor, apparent power, active power and reactive power.

Solution:

R = 10Ω
L = 20 mH = 20×10^{-3}H
$X_L = 2\pi fL$
$= 2\pi\times60\times20\times10^{-3}$
$X_L = 7.54\Omega$

(i) $|Z| = \sqrt{R + (X_L)^2} = \sqrt{(10)^2 + (7.54)^2} = 12.5\Omega$

(ii) $I = \dfrac{V}{Z} = \dfrac{250}{12.5} = 20\,amps$

V_R = IR = 20×10=200 volts

(iii) $V_L = I\,X_L$ = 20×7.54=150.8 volts

(iv) Apparent power S = VI
= 250×20
S= 5000VA

$$\cos\phi = \frac{R}{Z} = \frac{10}{12.5} = 0.8 \ \text{(lagging)}$$

Active power = VI cos ϕ

=250×20×0.8

P = 4000 Watts

$$\sin\phi = \sqrt{1-\cos^2\Phi} \ = \sqrt{1-(0.8)^2} \ = 0.6$$

Reactive Power Q = VI sin ϕ

= 250×20×0.6

Q= 3000 KVAR

2.18) Two impedances (5+j7)Ω and (10-j7)Ω are connected in series across a 200V supply. Calculate the current, power factor and power.

Solution:

$Z_1 = 5 + j7$

$Z_2 = 10 - j7$

V = 200 volts

$Z_{Total} = Z_1 + Z_2$

$= 5 + j7 + 10 - j7$

$Z_{Total} = 15 < 0^{\cdot}$

$\therefore \ \phi = 0^{\cdot}$

Taking V as referenve,

$V = 200 < 0^{\circ\cdot}$ Volts

(i) $I = \frac{V}{Z} = \frac{200\angle 0^\circ}{15\angle 0^\circ} = 13.33\angle 0^\circ \, amps$

(ii) ϕ = 0

PF = cos ϕ = cos 0 = 1

(iii) Power = VI cos ϕ

= 200 ×13.33×1

Power = 2666 watts

2.5.5 RC Series Circuit

Let us consider the circuit shown in diagram in which a pure resistance R and a pure capacitance C are connected in series.

Figure 3.24

Let

$V = V_m \sin\omega t$ be the applied voltage.
I = Circuit current of any instant
I = Effective value of circuit current
V_R = Potential Difference across Resistor
V_c = Potential Difference across Capacitor
f = Frequency of applied voltage
The same Current I flows through R and C
Voltage across R = V_R = IR in phase with I
Voltage across C = $V_c = IX_c$ lagging I by 90^0
Applied voltage $V = IR - jIX_c$

$$= I(R - jx_c)$$

$$\frac{V}{I} = R - jX_c = Z$$

Z – Impedence of circuit

$$|Z| = \sqrt{R^2 + X_c^2}$$

Phasor Diagram of RC series circuit is,

Figure 3.25

From Triangle

$$\tan\phi = \frac{X_c}{R} = \frac{1/\omega c}{R} = \frac{1}{\omega c R}$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega c R}\right)$$

ϕ is called Phase angle and it is angle between V and I. Its value lies between 0 and -90^o.

The current and voltage waveform of series RC Circuit is,

Figure 3.26

$V = V_m \sin \omega t$

$I = I_m \sin (\omega t - \phi)$

The current I leads the applied voltage V by an angle ϕ.

From Phasor Diagram,

Power factor $\cos\phi = \frac{R}{Z}$

Actual or real power $P = VI \cos\phi$

Reactive or Quardrature power $Q = VI \sin\phi$

Complex or Apparent Power $S = P + jQ$

$= VI$

Figure 3.27

PROBLEMS

3.20 A capacitor having a capacitarce of 10 μF is connected in series with a non-inductive resistor of 120Ω across 100V, 50HZ calculate the current, power and the Phase Difference between current and supply voltage.

(Non-inductive Resistor means a Pureresistor)

Solution:

C = 10 μF
R = 120Ω
V = 100V
F = 50Hz

$$X_c = \frac{1}{2\pi fc} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}}$$

$$= 318\,\Omega$$

$$|Z| = \sqrt{R^2 + X_c^{\ 2}}$$

$$= 340\,\Omega$$

(a) $$|I| = \frac{|V|}{|Z|}$$

$$= \frac{100}{340}$$

$$= 0.294\, amps$$

(b) $$Phase Difference\, \phi = \tan^{-1}\left(\frac{X_c}{R}\right)$$

$$= \tan^{-1}\left(\frac{318}{120}\right)$$

$$\phi = 69.3^\circ \quad (Leading)$$

$$\cos\phi = \cos(69.3)^\circ$$

$$= 0.353 \quad (Leading)$$

$$Power = |V||I|\cos\phi$$

$$= 100 \times 0.294 \times 0.353$$

$$= 10.38\, Watts$$

3.21 The Resistor R in series with capacitance C is connected to a 50HZ, 240V supply. Find the value of C so that R absorbs 300 watts at 100 volts. Find also the maximum charge and the maximum stored energy in capacitance.

Solution:

V = 240 volt
F = 50Hz

Power absorbed by R = 300 watts
Voltage across R = 100 volts

$$|V|^2 = |V_R|^2 + |V_C|^2$$
$$|V_C| = \sqrt{|V|^2 - |V_R|^2}$$
$$= \sqrt{(240)^2 - (100)^2}$$
$$|V_C| = 218.2\, volts$$

For Resistor, Power absorbed = 300 volts

$$|I|^2 R = |V_R||I| = 300$$
$$|I| = \frac{300}{|V_R|} = \frac{300}{100} = 3 amps$$
$$|X_C| = \frac{V_C}{|I|} \quad (Apply\, ohm's\, law\, for\, C)$$
$$= \frac{218.2}{3} = 72.73\Omega$$
$$\frac{1}{2\pi fc} = 72.73$$
$$C = \frac{1}{2\pi \times 50 \times 72.73} = 43.77 \times 10^{-6} F$$
$$C = 43.77 \mu F$$

Maximum charge = Q_m = C × maximum V_c
Maximum stared energy = 1/2 (C × maximum V_c^2)
Maximum $V_c = \sqrt{2}$ × Rms value of V_c
= $\sqrt{2}$ × 218.2 = 308.6 volts

Now

Maximum charge = Q_m = 43.77 x 10^{-6} x 308.6
= 0.0135 Coulomb
Maximum energy stored
= ½ (43.77 x 10^{-6}) $(308.6)^2$
= 2.08 joules.

3.22 A Capacitor and Resistor are connected in series to an A.C. supply of 60 volts, 50HZ. The current is 2A and the power dissipated in the Resistor is 80 watts. Calculate (a) the impedance (b) Resistance (c) capacitance (d) Power factor.

Solution

$$|V| = 60 volts$$

$$F = 50\,Hz$$

$$|I| = 2\,amps$$

Power Dissipated = P = 80 watts

(a) $|Z| = \frac{|V|}{|I|} = \frac{60}{2} = 30\Omega$

(b) $As\,P = I^2R$

$$R = \frac{P}{I^2} = \frac{80}{4}$$

$$= 20\Omega$$

(c) $Since, |Z|^2 = R^2 + X_c{}^2$

$$X_c = \sqrt{(z)^2 - R^2}$$

$$= \sqrt{30^2 - 20^2} = 22.36\Omega$$

$$\frac{1}{2\pi\,fc} = 22.36$$

$$c = \frac{1}{2\pi\,f\,(22.36)}$$

$$= \frac{1}{2\pi \times 50 \times 22.36}$$

= 142 × 10^{-6} F

C = 142 µF

(or) Power factor = $\cos\phi = \frac{R}{|Z|}$

$$= = \frac{20}{30}$$

= 0.67(Leading)

It is capacitive circuit.

3.23 A metal filament lamp, Rated at 750 watts, 100V is to be connected in series with a capacitor across a 230V, 60Hz supply. Calculate (i) The capacitance required (ii) The power factor

Solution

Rating of the metal filament W =750watts

V_R = 100 volts

W = I^2R = V_RI

$$I = \frac{W}{V_R} = \frac{750}{100} = 7.5\,amps$$

It is like RC Series Circuit

So

$$V^2 = V_R^2 + V_C^2$$

$$V_C = \sqrt{V^2 - V_R^2}$$

$$= \sqrt{(230)^2 - (100)^2}$$

$$= 207\,volts$$

Applying Ohm's Law for C

$$|X_C| = \frac{|V_C|}{|I|} = \frac{207}{7.5}$$

$$= 27.6\Omega$$

$$\frac{1}{2\pi fc} = 27.6$$

$$c = \frac{1}{2\pi \times f \times 27.6} = \frac{1}{2\pi \times 60 \times 27.6}$$

$$= 96.19\,\mu F$$

$$Power\ \ factor = \cos\phi = \frac{R}{|Z|}$$

$$|Z| = \frac{|V|}{|I|} = \frac{230}{7.5} = 30.7\Omega$$

$$R = \frac{W}{I^2} = \frac{750}{(7.5)^2}$$

$$= 13.33\Omega$$

$$Powerfactor = \cos\phi = \frac{R}{Z}$$

$$\cos\phi = \frac{13.33}{30.7}$$

$$= 0.434\text{(Leading)}$$

3.5.6 RLC series circuit

Let v = RMS value of the voltage applied to series combination
I = RMS value of the current flowing
V_R = voltage across R
V_L = voltage across L
V_C = voltage across C

Figure 3.28

A circuit consisting of pure R, pure L and pure C connected in series is known as RLC series circuit.

Phasor diagram

Take I as reference
$V_R = I \times R$
$V_L = I \times X_L$
$V_C = I \times X_C$

Assume $X_L > X_C$

Then $V_L > V_C$

Figure 3.29

The above figure shows the phasor diagram for the combined circuit.
From the voltage triangle

$$|V|^2 = |V_R|^2 + (|V_L| - |V_C|)^2$$
$$= |IR|^2 + (|IX_L| - |IX_C|)^2$$
$$= |I|^2 + [R^2 + (X_L - X_C)^2]$$
$$|V| = |I|\sqrt{R^2 + (X_L - X_C)^2}$$
$$|Z| = \frac{|V|}{|I|}$$
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{R^2 + X^2} \qquad \because X = (X_L - X_C)$$

Three cases of Z

Case 1 If $X_L > X_C$
The circuit behaves like RL circuit. Current lags behind voltage. So power factor is lagging.

Case 2 If $X_L < X_C$
The circuit behaves like RC circuit current leads applied voltage power factor is leading.

Case 3 When $X_L = X_C$, the circuit behaves like pure resistance. Current is in phase with the applied voltage power factor is unity.
Impedance triangle

Figure 3.30

For $X_L > X_C$ For $X_L > X_C$.

1. If applied voltage
   $V = V_m \sin \omega t$ and ϕ is phase angle then 'i' is given by
   1) $i = I_m \sin (\omega t - \theta)$, for $X_L < X_C$
   2) $i = I_m \sin (\omega t + \theta)$, for $X_L > X_C$
   3) $i = I_m \sin \omega t$ for $X_L = X_C$

2. Impedance for RLC series circuit in complex form (or) rectangular form is given by

$$Z = R + j\,(X_L - X_C)$$

Problems

3.24 In a RLC series circuit, the applied voltage is 5V. Drops across the resistance and inductance are 3V and 1V respectively. Calculate the voltage across the capacitor. Draw the phaser diagram.

$V = 5V \qquad V_R = 3V \qquad V_L = 1V$

$V^2 = V_R^{\,2} + (V_L - V_C)^2$

$(V_L - V_C)^2 = V^2 - V_R^{\,2}$

$= 25 - 9 = 16$

$V_L - V_C = \pm 4$

$V_C = V_L \pm 4 = 1+4$

$V_C = 5V$

3.25 A coil of resistance 10Ω and in inductance of 0.1H is connected in series with a capacitance of 150μF across a 200v, 50HZ supply. Calculate

a) the inductive reactance of the coil.
b) the capacitive reactance
c) the reactance
d) current
e) power factor

$R = 10\Omega$

$L = 0.1\ H$

$C = 150\ \mu F \qquad = 150 \times 10^{-6}\ F$

$V = 200V \qquad f = 50\ Hz$

a) $X_L = 2\pi fL = 2\pi\,(50)\,0.1$

$= 31.4\ \Omega$

b) $$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi(50)(150\times10^{-6})}$$

$= 21.2\Omega$

c) the reactance $X = X_L - X_C$

$= 31.4 - 21.2$

$= 10.2\ \Omega$ (Inductive)

d) $$|Z| = \sqrt{R^2 + X^2}$$

$$= \sqrt{10^2 + (10.2)^2}$$

$$= 14.28\Omega\ (Inductive)$$

$$I = \frac{|V|}{|Z|} = \frac{200}{14.28} = 14 amps$$

e) $P.F = \cos\phi = \frac{R}{|Z|} = \frac{100}{14.28}$

$= 0.7$ (lagging) (I lags behind V)

3.5.7 Parallel AC circuit

When the impedance and connected in parallel and the combination is excited by AC source it is called parallel AC circuit.

Consider the parallel circuit shown in figure.

$$X_{C1} = \frac{1}{2\pi fc_1} = \frac{1}{\omega c_1}$$

$$X_{C2} = 2\pi fL_2 = \omega L_2$$

Impedance $|Z_1| = \sqrt{R_1^2 + X_{C1}^2}$

$$\phi_1 = \tan^{-1}\left(\frac{X_{C1}}{R_1}\right)$$

$$|Z_2| = \sqrt{R_2^2 + X^2_{L2}}$$

$$\phi_2 = \tan^{-1}\left(\frac{X_{L2}}{R_2}\right)$$

Conductance = g
Susceptance = b
Admittance = y

Branch 1

Conductance $g_1 = \frac{R_1}{|Z_1|^2}$

$$b_1 = \frac{X_{C1}}{|Z_1|^2} \ (positive)$$

$$|Y_1| = \sqrt{g_1^2 + b_1^2}$$

Branch 2

$$g_2 = \frac{R_2}{|Z_2|^2}$$

$$b_2 = \frac{X_{C2}}{|Z_2|^2} \text{ (Negative)}$$

$$|Y_2| = \sqrt{g_2^2 + b_2^2}$$

Total conductance $G = g_1 + g_2$
Total Suceptance $B = b_1 - b_2$

Total admittance $|Y| = \sqrt{G^2 + B^2}$

Branch current $|I_1| = |V||Y_1|$

$|I_2| = |V||Y_2|$

$|I| = |V||Y|$

Phase angle $= tan^{-1}\left(\frac{B}{G}\right)$ lag if B-negative

Power factor $\cos\phi = \frac{|G|}{|Y|}$

Problems:

3.26 Two impedances of parallel circuit can be represented by (20 + j15) and (10 – j60) Ω. If the supply frequency is 50 Hz, find the resistance, inductance or capacitance of each circuit.

$Z_1 = 20 + j15\ \Omega$
$Z_2 = 10 - j60\ \Omega$
$F = 50$ Hz
$Z_1 = R_1 + jX_L$
$Z_2 = R_2 - jX_C$

J term positive for in inductive
J term negative for capacitive.

For circuit 1, $R_1 = 20\Omega$

$X_1 = X_L = 2\pi fL = 2\pi\ (50)\ (L)$

$X_L = 15$

$2\pi\ (50)\ L = 15$

$$L = \frac{15}{2\pi(50)}$$

L = 48 mH

For circuit 2

$Z_2 = 10 - j60$
$R_2 = 10$
$X_2 = X_C = 60\ \Omega$

ie, $\frac{1}{2\pi fC} = 60$

$C = \frac{1}{2\pi(50)60}$

C = 53 μF.

2.3.27 Two circuits, the impedances of which are $Z_1 = (10 + j15)\ \Omega$ and $Z_2 = (6 - j8)\ \Omega$ are connected in parallel. If the total current supplied is 15A. What is the power taken by each branch.

$$Z1 = (10 + j15)\Omega = 18.03\angle 56.3$$
$$Z2 = (6 - \mathrm{j}8)\Omega = 10\angle -53.13$$
$$I = 15\ A$$
$$I_1 = I\frac{Z_2}{Z_1 + Z_2} \qquad \text{(Current divider rule)}$$
$$= \frac{15\angle 0^0 \times 10\angle -53.13^0}{16 + j7}$$
$$(Z_1 + Z_2 = 10 + j15 + 6 - j8)$$
$$I_1 = \frac{150\angle -53.13^0}{17.46\angle 23.63}$$
$$I_1 = 8.6\angle -76.76\ A$$

By KCL $I_2 = I - I_1$

$$= 15\angle 0 - 8.6\angle -76.76$$
$$= 15 - (1.97 - j8.37)$$
$$= 15.5 - 32.7A$$

Power taken by branch 1

= power dissipated in resistance of branch 1

$$= |I_1|^2 R_1 = (8.6)^2 \times 10$$
$$= 739.6 \text{ watts}$$

Power taken by branch 2

$$= |I_2|^2 R_2$$
$$= (15.5)^2 \times 6$$
$$= 1442 \text{ watts}$$

3.28 A 100Ω resistance and 0.6H inductance are connected in parallel across a 230v 50 Hz supply. Find the line current, impedance, power dissipated and parameter of the equivalent series circuit.

$$Z_1 = R = 100\Omega$$
$$Z_2 = j\,X_L = j2\pi fL$$
$$= j\,(2\pi\times 50\times 0.6)$$
$$= j\,188.5\Omega$$
$$= 188.5\angle 90$$
$$Z_T = Z_1 * Z_2$$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{100\angle 0 \times 188.5\angle 90}{100 + j188.5}$$

$$= \frac{18850\angle 90}{213.4\angle 62}$$

$= 88.33 \ \angle 28$

$= 78 + j41.46 \Rightarrow R + jX_L$

$Total\ impedance \left|Z_T\right| = 88.33\Omega$

R = 78Ω, X_L = 41.46Ω

X_L = 2πfLeq

41.46 = 2π × 50 × Leq

$$Leq = \frac{41.46}{2\pi \times 50}$$

Leq = 132 mH

= 30 – j40 + 24 +j32

= 54 – j8

= 54.6 $\angle$ – 8.43 A

Comparing 'V' and 'I_T' current I_T lag voltage 'V'

∴ ϕ = 8.43° lag

Power factor = cos ϕ= cos 8.43

= 0.99 lag

True Power $= W = \left|V\right|\left|I\right|\cos\phi$

=200 × 54.6 × cos 8.43

= 10802 watts

= 10.802 KW

Apparent Power $= \left|V\right| I$

= 200× 54.6

= 10920 VA = 10.920 KVA

Reactive Power $= \left|V\right| I \sin\phi$

= 200 × 54.6 × sin 8.43

= 1601 VAR

= 1.601 KVAR

Let Z_{total} = Total impedance

$$Z_{Total} = \frac{V}{I_{total}} = \frac{200\angle 0^0}{54.6\angle -8.43}$$

= 3.663 $\angle$ 8.43

= 3.623 + j0.54

= R +j X_L

R = 3.623Ω X_L = 0.54 Ω

(or)

$$Z_{Total} = Z1*Z2$$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(2.4 + j3.2)(3 - j4)}{2.4 + j3.2 + 3 - j4}$$

$$= \frac{4\angle 53.13 \times 5\angle -53.13}{5.46\angle -8.43}$$

$$= \frac{20\angle 0^0}{5.46\angle -8.43}$$

$$= 3.663 \angle 8.43$$

$$= 3.623 + j\,0.54\ \Omega$$

3.6 THREE PHASE A.C. CIRCUITS

Three Phase Connection

We have seen above only about single phase systems. Generally generation transmission and distribution of electrical energy are of three phase in nature. Three phase system is a very common poly phase system. It could be viewed combination of three single phase system with a phase difference of 120° between every pair. Generation, transmission and distribution of three phase power is cheaper. Three phase system is more efficient compared to single phase system. Uniform torque production occurs in three phase system where as pulsating torque is produced in the case of single phase system. Because of these advantages the overall generation, transmission and distribution of electrical power is usually of three phase.

There are two possible connections in 3-phase system. One is star connection and the other one is delta or mesh connection. Each type of connection is governed by characteristics equations relating the currents and the voltages.

3.6.1 Star Connection

Here three similar ends of the three phase coils are joined together to form a common point. Such a point is called star point or the neutral point. The free ends of the three phase coils will be operating at specific potential with respect to the zero potential of star point.

It may also be noted that wires are drawn from the three free ends for connecting loads. We actually have here three phase four wire system and three phase three wire system.

Analysis

Let us analyze the relationship between currents and voltages. In a three phase circuit, the voltage across the individual coil is known as phase voltage and the voltage between two lines is called line voltage. Similarly the current flowing through the coil is called phase current and the current flowing through the line is called line current.

Notations Defined

E_R , E_Y , E_B	: Phase voltages of R, Y and B phases.
I_R , I_Y , I_B	: Phase currents
V_{RY}, V_{YB}, V_{BR}	: Line voltages
I_{L1}, I_{L2}, I_{L3}	: Line currents

Figure 3.32

A balanced system is one in which the currents in all phases are equal in magnitude and are displaced from one another by equal angles. Also the voltages in all the phases are equal in magnitude and are displaced from one another by equal angles. Thus,

$$E_R=E_Y=E_B=E_P \qquad V_{RY}=V_{YB}=V_{BR}=V_L$$
$$I_R=I_Y=I_B=I_P \qquad I_{L1}=I_{L2}=I_{L3}=I_L$$

Figure 3.33

Current Relationship:

Apply Kirchhoff's current law at nodes R_1, Y_1, B_1 We get

$$I_R = I_{LI} ; I_Y = I_{L1} ; I_B=I_{L3}$$

This means that in a balanced star connected system, phase current equals the line current

$I_P = I_L$

Phase current = Line current

Voltage relationship:

Let us apply Kirchhoff's voltage law to the loop consisting of voltages $E_{R;} V_{Ry}$ and E_y.

$$\vec{E}_R - \vec{E}_Y = \vec{V}_{RY}$$

Using law of parallelogram

$$\left|\vec{V}_{RY}\right| = V_{RY} = \sqrt{E_R^2 + E_R^2 + 2E_R E_Y \cos 60}$$

$$= \sqrt{E_P^2 + E_P^2 + 2E_P E_P \cos 60} == E_P\sqrt{3}$$

Similarly,

$$\vec{E}_Y - \vec{E}_B = \vec{V}_{YB} \ and\ \vec{E}_B - \vec{E}_R = \vec{V}_{BR}$$
$$V_{RY} = E_P\sqrt{3} \ and\ V_{BR} = E_P\sqrt{3}$$

Hence $V_L = \sqrt{3}\,E_P$

Line Voltage $= \sqrt{3}$ phase voltage

Power relationship:

Let cosϕ be the power factor of the system.

Power consumed in one phase=$E_P l_P \cos\phi$

Power consumed in three phase $= 3\left(\frac{V_L}{\sqrt{3}}\right) I_L cos\phi$

$$= \sqrt{3} V_L l_L cos\phi \ watts$$

Reactive power in one phase $= E_P l_P sin\phi$

Total Reactive power $= 3E_P l_P \sin\phi$

$$= \sqrt{3} V_L I_L \sin\phi \quad VAR$$

Apparent power per phase=$E_P I_P$

Total Apparent Power=$3E_P l_P = \sqrt{3} V_L I_L$ Volt

3.6.2 Delta Connection:

The dissimilar ends of the three phase coils are connected together to form a mesh. Wires are drawn from each junction for connecting load. We can connect only three phase loads as there is no fourth wire available.

Figure 3.33

Let us analyze the relationship between currents and voltages. The system is balanced one. Notation used in the star connection are used here.

E_R, E_Y, E_B :	Phase voltages of R, Y and B phases.
I_R, l_Y, I_B :	Phase currents
V_{RY}, V_{YB}, V_{BR}:	Line voltages
I_{L1}, I_{L2}, I_{L3} :	Line currents

Voltage relationship:
Let us apply Kirchhoff's voltage law to the loop consisting of voltages E_R,V_{RY}

We Have $E_R=V_{RY}$
Similarly $E_Y = V_{YB}$ *and* $E_B = V_{BR}$
Thus $E_P=V_L$
Phase voltage= line voltage

Current Relationship:
Apply Kirchhoff's current law at node A (i.e.) R_1 , B_2 We get

$$\vec{I}_R - \vec{I}_B = \vec{I}_{1,1}$$

Referring to the phasor diagram and applying the law of parallelogram,
We get

$$I_{L1} = \sqrt{I_R^2 + I_Y^2 + 2I_R I_Y \cos 60}$$
$$= \sqrt{I_P^2 + I_P^2 + 2I_P I_P \cos 60}$$

Similarly,

$$\vec{I}_Y - \vec{I}_R = \vec{I}_{1,2} \text{ and } \vec{I}_B - \vec{I}_Y = \vec{I}_{1,3}$$

Hence $I_{L2} = I_P\sqrt{3}$ *and* $I_{L3} = I_P\sqrt{3}$

Thus Line current $= \sqrt{3}$ Phase current

$$I_L = I_P\sqrt{3}$$

Power relationship:
Let cosϕ be the power factor of the system.

Power consumed in one phase $= E_P\, l_P\, cos\phi$

Power consumed in three phase $= 3\,\mathrm{V}_L\left(\dfrac{I_L}{\sqrt{3}}\right)\cos\phi$

$$= \sqrt{3}V_L I_L \cos\phi\, watts$$

Reactive power in one phase $= E_p\, I_p\, sin\phi$

Total Reactive power $= 3E_p\, I_p\, cos\phi$

$$= \sqrt{3}V_L I_L \sin\phi VAR$$

Apparent power per phase = $E_p I_p$

Total Apparent Power $= 3E_p I_p = \sqrt{3}\, V_L I_L$ volt

3.7 MEASUREMENT OF POWER IN THREE PHASE CIRCUITS:

A three phase circuit supplied from a balanced three phase voltage may have balanced load or unbalanced load. The load in general can be identified as a complex impedance. Hence the circuit will be unbalanced when the load impedance in all the phase are not of same value. As a result, the current flowing in the lines will have unequal values. These line currents will have equal values when the load connected to the three phases have equal values. The two cases mentioned above can exist when the load is connected in star or delta. The three phase power can be measured by using three watt maters in each phases. The algebraic sum of the reading gives the total three phase power consumed. However three phase power can also measured using two watt meter.

Case I Star Connected load

In this section we analyse the measurement of three phase power using two wattmeter, when the load is star connected. The following assumption made:

(I) The three phase supply to which the load in connected is balanced.
(II) The phase sequence is R, Y, B.
(III) The load is balanced.
(IV) The load is R-L in nature.

Diagram 4

Figure 3.35

For Wattmeter 1

$$\text{Current measured} = \vec{I}_{L1} = \vec{I}_R$$

$$\text{Voltage measured} = \vec{V}_{RY}$$

$$\text{Phase angle between them} = 30 + \phi$$

$$\text{Power measured} = P1 = V_{RY} I_R \cos(30 + \phi)$$

For Wattmeter 2

$$\text{Current measured} = \vec{I}_{L3} = \vec{I}_B$$

$$\text{Voltage measured} = \vec{V}_{BY}$$

$$\text{Phase angle between them} = 30 - \phi$$

$$\text{Power measured} = P1 = V_{BY} I_B \cos(30 - \phi)$$

$$= \mathrm{V_L I_L} \cos(30 - \phi)$$

$$\text{Now,} \quad P1 + P2 = V_L I_L \cos(30 + \phi) + \mathrm{V_L I_L} \cos(30 - \phi)$$

$$= V_L I_L [\cos 30 \cos \phi + \sin 30 \sin \phi + \cos 30 \cos \phi - \sin 30 \sin \phi]$$

$$= \mathrm{V_L I_L} \times 2 \times \frac{\sqrt{3}}{2} \cos \phi$$

$$= \sqrt{3}\, \mathrm{V_L I_L} \cos \phi = \text{Total power in a three phase circuit}$$

$$P2-P1=V_L I_L[\cos(30-\phi)-\cos(30+\phi)]$$
$$=V_L I_L \times 2\times \sin 30 s\text{in}\,\phi$$
$$=\mathrm{V_L I_L}\sin\phi$$

$$\frac{P2-P1}{P2+P1}=\frac{V_L I_L \sin\phi}{\sqrt{3}V_L I_L \cos\phi}=\frac{\tan\phi}{\sqrt{3}}$$
$$\tan\phi=\sqrt{3}\left[\frac{P2-P1}{P2+P1}\right]$$
$$\tan\phi=\sqrt{3}\,(\mathrm{P}\,2-P1/\mathrm{P}\,2+P1)$$
$$\text{Power factor}=\cos\left\{\tan^{-1}\sqrt{3}\left[\frac{P2-P1}{P2+P1}\right]\right\}$$

Thus, two wattmeters connected appropriately in a three phase circuit can measure the total power consumed in the circuit.

Case II Delta Connected load

In this section we analyse the measurement of three phase power using two wattmeter, power when the load is star connected. The following assumption made:

(I) The three phase supply to which the load in connected is balanced.
(II) The phase sequence is R, Y, B.
(III) The load is balanced.
(IV) The load is R-L in nature.

Figure 3.36

For Wattmeter 1

$$\textit{Current measured}=\vec{I}_{1,1}=\vec{I}_R-\vec{I}_B$$
$$\textit{Voltage measured}=\vec{V}_{RY}=\vec{E}_R$$
$$\textit{Phase angle between them}=30+\phi$$
$$\textit{Power measured}=P1=V_{RY}I_{L1}\cos(30+\phi)$$
$$=V_L I_L\cos(30+\phi)$$

For Wattmeter 2

$$Current\ measured = \vec{I}_{1,3} = \vec{I}_B - \vec{I}_Y$$

$$Voltage\ measured = \vec{V}_{BY} = -\vec{E}_Y$$

$$Phase\ angle\ between\ them = 30 - \phi$$

$$Power\ measured = P1 = V_{BY} I_{1,3} \cos(30-\phi)$$

$$= \mathrm{V_L I_L} \cos(30-\phi)$$

$$Now,\quad P1 + P2 = V_L I_L \cos(30+\phi) + \mathrm{V_L I_L}\cos(30-\phi)$$

$$= V_L I_L [\cos 30 \cos\phi - \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$$

$$= \mathrm{V_L I_L} \times 2 \times \frac{\sqrt{3}}{2} \cos\phi$$

$$= \sqrt{3}\,\mathrm{V_L I_L} \cos\phi = Total\ power\ in\ a\ three\ phase\ circuit$$

$$P2 - P1 = V_L I_L [\cos(30-\phi) - \cos(30+\phi)]$$

$$= V_L I_L \times 2 \times \sin 30 s \mathrm{in}\,\phi$$

$$= \mathrm{V_L I_L} \sin\phi$$

$$\mathrm{Tan}\,\phi = \sqrt{3}\,(\mathrm{P}\,2 - P1/\mathrm{P}\,2 + P1)$$

$$\text{Power factor} = \cos\left\{\tan^{-1}\sqrt{3}\left[\frac{P2-P1}{P2+P1}\right]\right\}$$

Problems 3.30

Three similar coils of Resistance of 10Ω and inductance 0.15 Henry are connected in star across a 3Φ, 440V, 50Hz supply. Find the line and phase values of current. Also find the above values when they are connected in Delta.

Solution:

Given Data

$$V_L = 440V,\ R_{ph} = 10\Omega,\ L_{ph} = 0.15H,\ f = 50Hz$$

$$X_{L\,ph} = 2\pi f\ L_{ph} = 2 \times \pi \times 50 \times 0.15 = 47.12\ \Omega$$

$$|Z_{ph}| = \sqrt{R_{ph}{}^2 + X_{L}{}^2{}_{ph}} = \sqrt{10^2 + (47.12)^2}$$

$$= 48.17\Omega$$

In star Connection

$$I_L = I_{ph} \qquad V_L = \sqrt{3}V_{ph}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 230.95 \text{ Volt}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.95}{48.17} = 4.794A$$

$$I_L = I_{ph} = 4.794\,A$$

Active power = $3V_{ph}\,I_{ph}\cos\Phi$

$$\cos\Phi = \frac{R_{ph}}{Z_{ph}} = 0.2075$$

$$\text{Active power} = 3*230.95*4.794*0.2075$$
$$= 689.54W$$

$$\text{Reactive power} = 3V_{ph}\,I_{ph}\sin\Phi$$

$$\sin\Phi = \sqrt{1-\cos^2\Phi} = 0.9782$$

$$\text{Reactive power} = 3*230.95*4.794*0.9782$$
$$= 3249.23VAR$$

$$\text{Apparent power} = 3V_{ph}\,I_{ph} = 3*230.95*4.794$$
$$= 3321.52V$$

If it is Delta connected coils, then

$$VL = V_{ph} \quad \& \quad IL = \sqrt{3}\,I_{ph}$$

$$VL = V_{ph} = 440V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}}\frac{440}{48.17} = 9.134\,A$$

$$IL = \sqrt{3}\,I_{ph} = \sqrt{3}*9.134 = 15.82\,A$$

$$\text{Active power} = 3\,V_{ph}\,I_{ph}\cos\Phi$$
$$= 3*440*9.134*0.2075$$
$$= 2501.80\,watt$$

$$\text{Reacive power} = 3\,V_{ph}\,I_{ph}\sin\Phi$$
$$= 3*440*9.134*0.9782$$

$$\text{Apparent power} = 3V_{ph}\,I_{ph} = 3*440*9.134$$
$$= 12056.88\,VA$$

Problem 3.31

Two wattmeters connected to measure the 3Φ power indicate 1000 watts and 500 watts respectively. Calculate the power factor of the ckt.

Solution:

Given data

$$p_1 = 500 \text{ watts},\ p_2 = 1000 \text{ watts},$$

$$p_1 + p_2 = 1000 + 500 = 1500\,watts$$

$$p_2 - p_1 = 1000 - 500 = 500\,watts$$

$$p_1 = VL\,IL \cos(30 + \Phi)$$

$$p_2 = VL\,IL \cos(30 - \Phi)$$

$$\mathrm{p}_1 + \mathrm{p}_2 = \sqrt{3}\,VL\,IL \cos\Phi$$

$$p_2 - p_1 = \sqrt{3} * \frac{(p_2 - p_1)}{(p_1 + p_2)} = \frac{\sqrt{3} * 500}{1500}$$

$$= 0.5773$$

$$\Phi = 29.99°$$

Power factor cos Φ = 0.866

Problem 3.32

A balanced star connected load of (3+j4) Ω impedance is connected to 400V, three phase supply. What is the real power consumed by the load?

Solution:

Given data

$$V_L = 400\,volt$$

$$\text{Impedence / phase} = Z_{ph} = 3 + j4 = 5\angle 53°$$

$$\text{In star connection}$$

$$\mathrm{I_L} = I_{ph}\ \&\ V_{\mathrm{L}} = \sqrt{3}\,V_{ph}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\,volt$$

$$\text{Current in each } I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{5\angle 53°}$$

$$= 46.02\angle -53°\,A$$

$$\text{Line current } I_L = 46.02\,A$$

$$\text{Total power consumed in the load} = \sqrt{3}\,V_L I_L \cos\Phi$$

$$= \sqrt{3} * 400 * 46.02 * \cos(-53°)$$

$$= 19188\,watt$$

PART A – QUESTIONS

1. Define Form factor and Peak factor
2. What is meant by average value?
3. Give the relation between line voltage and phase voltage, line current and phase current for star and delta connection.
4. What are the advantages of polyphase system ?
5. Define power factor?
6. What is phase sequence?
7. Define inductance and write its unit.
8. What is meant by balanced system?
9. Write down the expression for power factor in two wattmeter method.

PART B – QUESTIONS

1. Explain with neat figures the power measurement in three phase circuits using two-wattmeter method.
2. A given load consisting of a resistor R & a capacitor C, takes a power of 4800W from 200V, 60HZ supply mains, Given that the voltage drop across the resistor is 120V, Calculate the (a) impedance (b) current (c) power factor (d) resistance (e) capacitance. Write down the equations for the current and voltage.
3. A coil of 10 ohms and inductance of 0.1H in series with a 150µF capacitor across 200V,250HZ supply. Calculate (i) inductive reactance, capacitive reactance and impedance of the circuit (ii) current (iii)power factor(iv)voltage across the coil and capacitor respectively.
4. An impedance z_1= (2.4+j3.2) ohms is in parallel with another impedance z_2= (3-j4) ohms. The combination is given a supply of 200 V. Calculate (i) total impedance (ii) individual & total currents (iii) power factor (iv) power in the circuit.
5. A balanced three phase load consists of 6 ohms resistor & 8 ohms reactor (inductive) in each phase. The supply is 230V, 3 phases, 50HZ. Find (a) phase current (b) line current (c) total power. Assume the load to be connected in star & delta.
6. A 3phase, 4 wire 208 V, ABC system supplies a star connected load in which $Z_A=10\angle 0$, $Z_B=15\angle 30$, $Z_C=10\angle -30$. Find the line currents, the neutral current and the load power.
7. A coil having R = 10Ω and L = 0.2H is connected to a 100V, 50 Hz supply. Calculate (i) the impedance of the coil (ii) the current (iii) the phase difference between the current and voltage and (iv) the power.
8. Three similar coils of resistance of 10Ω and inductance 0.15H are connected in star across a 3 phase 440V, 50 Hz supply. Find the line

and phase values of current. Also find the above values when they are connected in delta.

9. Each phase of a delta connected load comprises a resistor of Ohm and a capacitor of μF in series. Calculate the line current for a 3 - ϕ voltages of 400V at 50 Hz. Also evaluate the power factor and the total 3 - ϕ power absorbed by the load.