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SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF MATHEMATICS

UNIT – I – DISCRETE MATHEMATICS – SMTA 1302

UNIT I: LOGIC

Statements - Truth tables - Connectives - Equivalent Propositions - Tautological Implications - Normal forms -Predicate Calculus, Inference theory for Propositional Calculus and Predicate Calculus.

Propositional Logic – Definition

A proposition is a collection of declarative statements that has either a truth value "true" or a truth value "false". A propositional consists of propositional variables and connectives. We denote the propositional variables by capital letters (A, B,…, P,Q,…). The connectives connect the propositional variables.

Some examples of Propositions are given below −

- "Man is Mortal", it returns truth value "TRUE"
- "12 + 9 = 3 − 2", it returns truth value "FALSE"

The following is not a Proposition −

- "A is less than 2". It is because unless we give a specific value of A, we cannot say whether the statement is true or false.

Connectives

In propositional logic generally we use five connectives which are − OR (∨), AND (∧), Negation/ NOT (¬), If-then/Conditional (→), If and only if/ Biconditional (↔).

OR (∨) : The OR operation of two propositions A and B (written as A ∨ B) is true if at least any of the propositional variable A or B is true.

The truth table is as follows −

A	B	A ∨ B
True	True	True
True	False	True
False	True	True
False	False	False

AND (∧) : The AND operation of two propositions A and B (written as A ∧ B) is true if both the propositional variable A and B is true.

The truth table is as follows −

A	B	A ∧ B
True	True	True
True	False	False
False	True	False
False	False	False

Negation (¬) : The negation of a proposition A (written as ¬A) is false when A is true and is true when A is false.

The truth table is as follows –

A	¬A
True	False
False	True

If-then /Conditional (→): An implication A→ B is False if A is true and B is false. The rest of the cases are true.Here A is called Hypothesis or antecedent and q is called consequent or conclusion.

The truth table is as follows −

A	B	A→ B
True	True	True
True	False	False
False	True	True
False	False	True

If and only if (↔) : A↔B is bi-conditional logical connective which is true when p and q are both false or both are true.

The truth table is as follows −

A	B	A↔B
True	True	True
True	False	False
False	True	False
False	False	True

Tautologies

A Tautology is a formula which is always true for every value of its propositional variables.

Example − Prove [(A → B) ∧ A] → B is a tautology

The truth table is as follows −

A	B	A → B	(A → B) ∧ A	[(A → B) ∧ A] → B
True	True	True	True	True
True	False	False	False	True
False	True	True	False	True
False	False	True	False	True

As we can see every value of [(A → B) ∧ A] → B is “True”, it is a tautology.

Contradictions

A Contradiction is a formula which is always false for every value of its propositional variables.

Example − Prove (A ∨ B) ∧ [(¬A) ∧ (¬B)] is a contradiction

The truth table is as follows −

A	B	A ∨ B	¬A	¬B	(¬A) ∧ (¬B)	(A ∨ B) ∧ [(¬A) ∧ (¬B)]
True	True	True	False	False	False	False
True	False	True	False	True	False	False
False	True	True	True	False	False	False
False	False	False	True	True	True	False

As we can see every value of (A ∨ B) ∧ [(¬A) ∧ (¬B)] is "False", it is a contradiction

Contingency

A Contingency is a formula which has both some true and some false values for every value of its propositional variables.

Example − Prove (A ∨ B ∨) **∧** (¬A) a contingency

The truth table is as follows −

A	B	A ∨ B	¬A	(A ∨ B) ∧ (¬A)
True	True	True	False	False
True	False	True	False	False
False	True	True	True	True
False	False	False	True	False

As we can see every value of (A ∨ B) **∧** (¬A) has both "True" and "False", it is a contingency.

Propositional Equivalences

Two statements X and Y are logically equivalent if any of the following two conditions −

- The truth tables of each statement have the same truth values.

- The bi-conditional statement X ↔Y is a tautology.

Example − Prove ¬(A ∨ B) and [(¬A) ∧ (¬B)] are equivalent

Testing by 1st method (Matching truth table)

A	B	A ∨ B	¬ (A ∨ B)	¬A	¬B	[(¬A) ∧ (¬B)]
True	True	True	False	False	False	False
True	False	True	False	False	True	False
False	True	True	False	True	False	False
False	False	False	True	True	True	True

Here, we can see the truth values of ¬ (A ∨ B) and [(¬A) ∧ (¬B)] are same, hence the statements are equivalent.

Testing by 2nd method (Bi-conditionality)

A	B	¬ (A ∨ B)	[(¬A) ∧ (¬B)]	[¬ (A ∨ B)] ⇔ [(¬A) ∧ (¬B)]
True	True	False	False	True
True	False	False	False	True
False	True	False	False	True
False	False	True	True	True

As [¬ (A ∨ B)] ⇔ [(¬A) ∧ (¬B)] is a tautology, the statements are equivalent.

EQUIVALENT LAWS

Equivalence	Name of Identity
$p\wedge T \equiv p$ $p\vee F \equiv p$	Identity Laws
$p\wedge F \equiv F$ $p\vee T \equiv T$	Domination Laws
$p\wedge p \equiv p$ $p\vee p \equiv p$	Idempotent Laws
$\neg(\neg p) \equiv p$	Double Negation Law
$p\wedge q \equiv q \wedge p$ $p\vee q \equiv q \vee p$	Commutative Laws
$(p\wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p\vee q) \vee r \equiv p \vee (q \vee r)$	Associative Laws
$p\wedge(q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p\vee(q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Ditributive Laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's Laws
$p\wedge(p \vee q) \equiv p$ $p\vee(p \wedge q) \equiv p$	Absorption Laws
$p\wedge\neg p \equiv F$ $p\vee\neg p \equiv T$	Negation Laws

Logical Equivalences involving Conditional Statements

$$p \to q \equiv \neg p \vee q$$
$$p \to q \equiv \neg q \to \neg p$$
$$p \vee q \equiv \neg p \to q$$
$$p \wedge q \equiv \neg(p \to \neg q)$$
$$\neg(p \to q) \equiv p \wedge \neg q$$
$$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$$
$$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$$
$$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$$
$$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$$

Logical Equivalences involving Biconditional Statements

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$
$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$
$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$
$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

A conditional statement has two parts − **Hypothesis** and **Conclusion**.

Example of Conditional Statement − “If you do your homework, you will not be punished.” Here, "you do your homework" is the hypothesis and "you will not be punished" is the conclusion.

Inverse, Converse, andContra-positive

Inverse –An inverse of the conditional statement is the negation of both the hypothesis and the conclusion. If the statement is “If p, then q”, the inverse will be “If not p, then not q”. The inverse of “If you do your homework, you will not be punished” is “If you do not do your homework, you will be punished.”

Converse −The converse of the conditional statement is computed by interchanging the hypothesis and the conclusion. If the statement is “If p, then q”, the inverse will be “If *q*, then *p*”. The converse of "If you do your homework, you will not be punished" is "If you will not be punished, you do not do your homework”.

Contra-positive –The contra-positive of the conditional is computed by interchanging the hypothesis and the conclusion of the inverse statement. If the statement is “If p, then q”, the inverse will be “If not *q*, then not *p*”. The Contra-positive of "If you do your homework, you will not be punished” is "If you will be punished, you do your homework”.

Example:

Give the converse and the Contra positve of the implication “ If it is raining then I get wet”.

Solution :

P : It is raining Q : I get wet

Converse : $Q \rightarrow P$: If I get wet, then it is raining.

Contrapositive : $\neg Q \rightarrow \neg P$: If I do not get wet, then it is not raining

DUALITY PRINCIPLE

Duality principle set states that for any true statement, the dual statement obtained by interchanging unions into intersections (and vice versa) and interchanging Universal set into Null set (and vice versa) is also true. If dual of any statement is the statement itself, it is said **self-dual** statement.

Examples : i) The dual of $(A \cap B) \cup C$ is $(A \cup B) \cap C$

ii) The dual of $P \wedge Q \wedge F$ is $P \vee Q \vee T$

Example : 1

Construct a truth table for $(p \to q) \to (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Example 2: Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent

.

Solution : The truth tables for these compound proposition is as follows.

1	2	3	4	5	6	7	8
P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$	$6 \leftrightarrow 7$
T	T	F	F	T	F	F	T
T	F	F	T	T	F	F	T
F	T	T	F	T	F	F	T
F	F	T	T	F	T	T	T

We can observe that the truth values of $\neg(p \vee q)$ and $\neg p \wedge \neg q$ agree for all possible combinations of the truth values of p and q.

Example 3: Show that $p \to q$ and $\neg p \vee q$ are logically equivalent.

Solution : The truth tables for these compound proposition as follows.

p	q	$\neg p$	$\neg p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

As the truth values of $p \to q$ and $\neg p \vee q$ are logically equivalent.

Example 4 : Determine whether each of the following form is a tautology or a contradiction or neither :

i) $(P \wedge Q) \to (P \vee Q)$

ii) $(P \vee Q) \wedge (\neg P \wedge \neg Q)$

iii) $(\neg P \wedge \neg Q) \to (P \to Q)$

iv) $(P \to Q) \wedge (P \wedge \neg Q)$

v) $[P \wedge (P \to \neg Q) \to Q]$

Solution:

i) The truth table for $(p \wedge q) \to (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Here all the entries in the last column are 'T'.

$\therefore (p \wedge q) \to (p \vee q)$ is a tautology.

ii) The truth table for $(p \vee q) \wedge (\neg p \wedge \neg q)$ is

1	2	3	4	5	6	
p	q	$p \vee q$	$\neg p$	$\neg q$	$\neg P \wedge \neg q$	$3 \wedge 6$
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	F	F
F	F	F	T	T	T	F

The entries in the last column are 'F'. Hence $(p \vee q) \wedge (\neg p \wedge \neg q)$ is a contradiction.

iii) The truth table is as follows.

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$p \rightarrow q$	$(\neg p \wedge \neg q) \rightarrow (p \rightarrow q)$
T	T	F	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Here all entries in last column are 'T'.

$\therefore$ $(\neg p \wedge \neg q) \rightarrow (p \rightarrow q)$ is a tautology.

iv) The truth table is as follows.

p	q	$\neg q$	$p \wedge \neg q$	$p \rightarrow q$	$(p \rightarrow q) \wedge (p \wedge \neg q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

All the entries in the last column are 'F'. Hence it is contradiction.

v) The truth table for $\left[p \wedge (p \to \neg q) \to q\right]$

p	q	$\neg q$	$p \to \neg q$	$p \wedge (p \to \neg q)$	$\left[p \wedge (p \to \neg q) \to q\right]$
T	T	F	F	F	T
T	F	T	T	T	F
F	T	F	T	F	T
F	F	T	T	F	T

The last entries are neither all 'T' nor all 'F'.

$\therefore$ $\left[p \wedge (p \to \neg q) \to q\right]$ is a neither tautology nor contradiction. It is a Contingency.

Example 5: Symbolize the following statement

Let p, q, r be the following statements:
p: I will study discrete mathematics
q: I will watch T.V.
r: I am in a good mood.
Write the following statements in terms of p, q, r and logical connectives.
(1) If I do not study and I watch T.V., then I am in good mood.
(2) If I am in good mood, then I will study or I will watch T.V.
(3) If I am not in good mood, then I will not watch T.V. or I will study.
(4) I will watch T.V. and I will not study if and only if I am in good mood.
Solution:

(1) $(\neg p \wedge q) \to r$

(2) $r \to (p \vee q)$

(3) $\neg r \to (\neg q \vee p)$

(4) $(q \wedge \neg p) \leftrightarrow r$

Example 6: Show that $\neg(p \vee (\neg p \wedge q))$ is logically equivalent to $\neg p \wedge \neg q$

Solution :

$$
\begin{array}{llll}
\neg(p \vee (\neg p \wedge q)) & \equiv & \neg p \wedge \neg(\neg p \wedge q) & \text{by the second De Morgan law} \\
 & \equiv & \neg p \wedge [\neg(\neg p) \vee \neg q] & \text{by the first De Morgan law} \\
 & \equiv & \neg p \wedge (p \vee \neg q) & \text{by the double negation law} \\
 & \equiv & (\neg p \wedge p) \vee (\neg p \wedge \neg q) & \text{by the second distributive law} \\
 & \equiv & F \vee (\neg p \wedge \neg q) & \text{because } \neg p \wedge p \equiv F \\
 & \equiv & (\neg p \wedge \neg q) \vee F & \text{by the commutative law} \\
 & & & \text{for disjunction} \\
 & \equiv & (\neg p \wedge \neg q) & \text{by the identity law for } \mathbf{F}
\end{array}
$$

Example 7: **Show that $\neg (p \leftrightarrow q) \equiv (p \vee q) \wedge \neg (p \wedge q)$ without constructing the truth table**

Solution :

$\neg (p \leftrightarrow q) \equiv (p \vee q) \wedge \neg (p \wedge q)$

$\neg (p \leftrightarrow q) \equiv \neg (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv \neg (\neg p \vee q) \wedge (\neg q \vee p)$

$\equiv \neg (\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)$

Elementary Product: A product of the variables and their negations in a formula is called an elementary product. If P and Q are any two atomic variables, then p, $\neg p \wedge q$, $\neg q \wedge p \wedge \neg p$ are some examples of elementary products.

Elementary Sum: A sum of the variables and their negations in a formula is called an elementary sum. If P and Q are any two atomic variables, then p, $\neg p \vee q$, $\neg q \vee p$ are some examples of elementary sums.

Normal Forms

We can convert any proposition in two normal forms −

1. Conjunctive normal form   2. Disjunctive normal form

Conjunctive Normal Form

A compound statement is in conjunctive normal form if it is obtained by operating AND among variables (negation of variables included) connected with ORs.

Examples

- $(P \cup Q) \cap (Q \cup R)$

- $(\neg P \cup Q \cup S \cup \neg T)$

Disjunctive Normal Form

A compound statement is in disjunctive normal form if it is obtained by operating OR among variables (negation of variables included) connected with ANDs.

Examples

- $(P \cap Q) \cup (Q \cap R)$
- $(\neg P \cap Q \cap S \cap \neg T)$

Predicate Logic deals with predicates, which are propositions containing variables.

Functionally Complete set

A set of logical operators is called functionally complete if every compound proposition is logically equivalent to a compound proposition involving only this set of logical operators. $\vee$, $\wedge$, and $\neg$form a functionally complete set of operators.

Minterms: For two variables p and q there are 4 possible formulas which consist of conjunctions of p,q or its negation given by $p \wedge q$, $p \wedge \neg q$, $\neg p \wedge q$ and $\neg p \wedge\neg \neg q$

Maxterms: For two variables p and q there are 4 possible formulas which consist of disjunctions of p,q or its negation given by $p \vee q$, $p \vee \neg q$, $\neg p \vee q$ and $\neg p \vee \neg q$

Principal Disjunctive Normal Form: For a given formula an equivalent formula consisting of disjunctions of minterms only is known as principal disjunctive normal form(PDNF)

Principal Conjunctive Normal Form: For a given formula an equivalent formula consisting of conjunctions of maxterms only is known as principal conjunctive normal form(PCNF)

Obtain DNF of $Q \vee (P \wedge R) \wedge \neg((P \vee R) \wedge Q)$.

Solution:

$Q \vee (P \wedge R) \wedge \neg((P \vee R) \wedge Q)$

$\Leftrightarrow (Q \vee (P \wedge R)) \wedge (\neg((P \vee R) \wedge Q)$ (Demorgan law)

$\Leftrightarrow (Q \vee (P \wedge R)) \wedge ((\neg P \wedge \neg R) \vee \neg Q)$ (Demorgan law)

$\Leftrightarrow (Q \wedge (\neg P \wedge \neg R)) \vee (Q \wedge \neg Q) \vee ((P \wedge R) \wedge \neg P \wedge \neg R) \vee ((P \wedge R) \wedge \neg Q)$

(Extended distributed law)

$\Leftrightarrow (\neg P \wedge Q \wedge \neg R) \vee F \vee (F \wedge R \wedge \neg R) \vee (P \wedge \neg Q \wedge R)$ (Negation law)

$\Leftrightarrow (\neg P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R)$ (Negation law)

Obtain Pcnf and Pdnf of the formula $(\neg P \vee \neg Q) \rightarrow (P \leftrightarrow \neg Q)$

Solution:

Let S = $(\neg P \vee \neg Q) \rightarrow (P \leftrightarrow \neg Q)$

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$P \leftrightarrow \neg Q$	S	Minterm	Maxterm
T	T	F	F	F	F	T	$P \wedge Q$	
T	F	F	T	T	T	T	$P \wedge \neg Q$	
F	T	T	F	T	T	T	$\neg P \wedge Q$	
F	F	T	T	T	F	F		$P \vee Q$

PCNF: $P \vee Q$ and PDNF: $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$

Inference Theory

The theory associated with checking the logical validity of the conclusion of the given set of premises by using Equivalence and Implication rule is called **Inference theory**

Direct Method

When a conclusion is derived from a set of premises by using the accepted rules of reasoning is called **direct method**.

Indirect method

While proving some results regarding logical conclusions from the set of premises, we use negation of the conclusion as an additional premise and try to arrive at a contradiction is called **Indirect method**

Consistency and Inconsistency of Premises

A set of formular $H_1,H_2,\ldots,H_m$ is said to be **inconsistent** if their conjunction implies Contradiction.

A set of formular $H_1,H_2,\ldots,H_m$ is said to be **consistent** if their conjunction implies Tautology.

Rules of Inference

Rule P: A premise may be introduced at any point in the derivation

Rule T: A formula S may be introduced at any point in a derivation if S is tautologically implied by any one or more of the preceeding formula.

Rule CP: If S can be derived from R and set of premises , then R S can be derived from the set of premises alone.

Rules of Inference

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore\ q$	$[p \wedge (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore\ \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore\ p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore\ q$	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
p $\therefore\ p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore\ p$	$(p \wedge q) \to p$	Simplification
p q $\therefore\ p \wedge q$	$[(p) \wedge (q)] \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore\ q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution

Rule of inference to build arguments

Example:

1. It is not sunny this afternoon and it is colder than yesterday.
2. If we go swimming it is sunny.
3. If we do not go swimming then we will take a canoe trip.
4. If we take a canoe trip then we will be home by sunset.
5. We will be home by sunset

p	It is sunny this afternoon
q	It is colder than yesterday
r	We go swimming
s	We will take a canoe trip
t	We will be home by sunset (the conclusion)

propositions

1. $\neg p \wedge q$
2. $r \rightarrow p$
3. $\neg r \rightarrow s$
4. $s \rightarrow t$
5. t

hypotheses

Example 1.Show that R is logically derived from $P \rightarrow Q$, $Q \rightarrow R$, and P

Solution.

{1}	(1)	$P \rightarrow Q$	Rule P
{2}	(2)	P	Rule P
{1, 2}	(3)	Q	Rule (1), (2) and I11
{4}	(4)	$Q \rightarrow R$	Rule P
{1, 2, 4}	(5)	R	Rule (3), (4) and I11.

Example 2.Show that $S \vee R$ tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$.

Solution .

{1}	(1)	$P \vee Q$	Rule P
{1}	(2)	$7P \rightarrow Q$	T, (1), E1 and E16
{3}	(3)	$Q \rightarrow S$	P
{1, 3}	(4)	$7P \rightarrow S$	T, (2), (3), and I13
{1, 3}	(5)	$7S \rightarrow P$	T, (4), E13 and E1
{6}	(6)	$P \rightarrow R$	P
{1, 3, 6}	(7)	$7S \rightarrow R$	T, (5), (6), and I13
{1, 3, 6)	(8)	$S \vee R$	T, (7), E16 and E1

Example 3. Show that 7Q, P→ Q => 7P

Solution .

	{1}	(1) P → Q	Rule P
	{1}	(2) 7P → 7Q	T, and E 18
	{3}	(3) 7Q	P
	{1, 3}	(4) 7P	T, (2), (3), and I11 .

Example 4 .Prove that R ∧ (P V Q) is a valid conclusion from the premises PVQ ,
Q → R, P → M and 7M.

Solution .

	{1}	(1) P → M	P
	{2}	(2) 7M	P
	{1, 2}	(3) 7P	T, (1), (2), and I12
	{4}	(4) P V Q	P
	{1, 2 , 4}	(5) Q	T, (3), (4), and I10.
	{6}	(6) Q → R	P
	{1, 2, 4, 6}	(7) R	T, (5), (6) and I11
	{1, 2, 4, 6}	(8) R ∧ (PVQ)	T, (4), (7), and I9.

Example 5 .Show that R → S can be derived from the premises
P → (Q → S), 7R V P , and Q.

Solution.

	{1}	(1) 7R V P	P
	{2}	(2) R	P, assumed premise
	{1, 2}	(3) P	T, (1), (2), and I10
	{4}	(4) P → (Q → S)	P
	{1, 2, 4}	(5) Q → S	T, (3), (4), and I11
	{6}	(6) Q	P
	{1, 2, 4, 6}	(7) S	T, (5), (6), and I11
	{1, 4, 6}	(8) R → S	CP.

Example 6. Show that $P \rightarrow S$ can be derived from the premises, $7P \vee Q$, $7Q \vee R$, and $R \rightarrow S$.

Solution.

{1}	(1)	$7P \vee Q$	P
{2}	(2)	P	P, assumed premise
{1, 2}	(3)	Q	T, (1), (2) and I11
{4}	(4)	$7Q \vee R$	P
{1, 2, 4}	(5)	R	T, (3), (4) and I11
{6}	(6)	$R \rightarrow S$	P
{1, 2, 4, 6}	(7)	S	T, (5), (6) and I11
{2, 7}	(8)	$P \rightarrow S$	CP

Predicate Logic

A predicate is an expression of one or more variables defined on some specific domain. A predicate with variables can be made a proposition by either assigning a value to the variable or by quantifying the variable.

Eg.

" x is a Man"

Here **Predicate** is " is a Man" and it is denoted by M and **subject** "x" is denoted by x.

Symbolic form is M(x).

Quantifiers

The variable of predicates is quantified by quantifiers. There are two types of quantifier in predicate logic − Universal Quantifier and Existential Quantifier.

Universal Quantifier

Universal quantifier states that the statements within its scope are true for every value of the specific variable. It is denoted by the symbol ∀.

∀x P(x) is read as for every value of x, P(x) is true.

Example − "Man is mortal" can be transformed into the propositional form ∀x P(x) where P(x) is the predicate which denotes x is mortal and the universe of discourse is all men.

Existential Quantifier

Existential quantifier states that the statements within its scope are true for some values of the specific variable. It is denoted by the symbol ∃.∃x P(x) is read as for some values of x, P(x) is true.

Example − "Some people are dishonest" can be transformed into the propositional form ∃x P(x) where P(x) is the predicate which denotes x is dishonest and the universe of discourse is some people.

Nested Quantifiers

If we use a quantifier that appears within the scope of another quantifier, it is called nested quantifier.

Eg.2.

"***Every apple is red***".

The above statement can be restated as follows

For all x, if x is an apple then x is red

Now, we will translate it into symbolic form using universal quantifier.

Define A (x) : x is an apple.

R (x) : x is red.

$\therefore$ We write (*) into symbolic form as

$$(\forall x)\ (A(x) \rightarrow R(x))$$

Eg.3. "***Some men are clever***".

The above statement can be restated as

"there is an x such that x is a man and x is clever".

We will translate it into symbolic form using Existential quantifier.

Let M (x) : x is a man

and C (x) : x is clever

$\therefore$ We write (B) into symbolic form as

$$(\exists x)\ (M(x) \wedge C(x))$$

Inference theory for Predicate calculus

Rule of Inference	Name
$\dfrac{\forall x P(x)}{\therefore P(y)}$	Rule US: Universal Specification
$\dfrac{P(c) \text{ for any c}}{\therefore \forall x P(x)}$	Rule UG: Universal Generalization
$\dfrac{\exists x P(x)}{\therefore P(c) \text{ for any c}}$	Rule ES: Existential Specification
$P(c)$ for any c $\therefore \exists x P(x)$	Rule EG: Existential Generalization

Problem : 1 Show that $(\exists x)\ M(x)$ follows logically from the premises $(x)\ (H(x) \to M(x))$ and $(\exists x)\ H(x)$

Solution :

1)	$(\exists x)\ H(x)$	rule P
2)	$H(y)$	ES
3)	$(x)\ (H(x) \to M(x))$	P
4)	$H(y) \to M(y)$	US
5)	$M(y)$	T, (2)
6)	$(\exists x)\ M(x)$	EG

Problem : 2

Symbolize the following statements:

(a) All men are mortal

(b) All the world loves a lover

(c) X is the father of mother of Y

(d)No cats has a tail

(e) Some people who trust others are rewarded

Solution:

(a) Let M(x): x is a man H(x): x is Mortal
$(\forall x)(M(x) \rightarrow H(x))$

(b) Let P(x): x is a person L(x): x is a lover R(x,y): x loves y
$(x)(P(x) \rightarrow (y)(P(y) \wedge L(y) \rightarrow R(x,y)))$

(c) Let P(x): x is a person F(x,y): x is the father of y
M(x,y): x is the mother of y $(\exists z)(P(z) \wedge F(x,z) \wedge M(z,y))$

(d) Let C(x): x is a cat T(x): x has a tail

$(\forall x)(C(x) \rightarrow \neg T(x))$

(e) Let P(x): x is a person T(x): x trust others R(x): x is rewarded

$(\exists x)(P(x) \wedge T(x) \wedge R(x))$

Problem: 3

Use the indirect method to prove that the conclusion $\exists z Q(z)$ follows from the premises $\forall x(P(x) \rightarrow Q(x))$ and $\exists y P(y)$

Solution:

1	$\neg \exists z Q(z)$	P(assumed)
2	$\forall z \neg Q(z)$	T, (1)
3	$\exists y P(y)$	P
4	$P(a)$	ES, (3)
5	$\neg Q(a)$	US, (2)
6	$P(a) \wedge \neg Q(a)$	T, (4),(5)
7	$\neg(P(a) \rightarrow Q(a))$	T, (6)
8	$\forall x(P(x) \rightarrow Q(x))$	P
9	$P(a) \rightarrow Q(a)$	US, (8)
10	$P(a) \rightarrow Q(a) \wedge \neg(P(a) \rightarrow Q(a))$	T,(7),(9) contradiction

Problem: 4

Show that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$

Solution:

1) $(\exists x)(P(x) \wedge Q(x))$	Rule P
2) $P(a) \wedge Q(a)$	ES, 1
3) $P(a)$	Rule T, 2
4) $Q(a)$	Rule T, 2
5) $(\exists x) P(x)$	EG, 3
6) $(\exists x) Q(x)$	EG, 4
7) $(\exists x) P(x) \wedge (\exists x) Q(x)$	Rule T, 5, 6

ASSIGNMENT PROBLEMS

1. Write the statement in symbolic form "Some real numbers are rational".
2. Symbolize the expression "x is the father of the mother of y"
3. Symbolize the expression "All the world loves a lover"
4. Write the negation of the statement "If there is a will, then there is a way".
5. Construct the truth table for $\neg(p \wedge q)$
6. Find the CNF and DNF of $\neg(p \vee q) \leftrightarrow (p \wedge q)$
7. Show that $P \to Q, Q \to \neg R, R, P \vee (J \wedge S)$ imply $J \wedge S$
8. Show that $P \to Q, P \to R, Q \to R, P$ are inconsistent.
9. Prove that $(\exists x)(P(x) \wedge Q(x) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$
10. Show that $\neg P(a,b)$ follows logically from $(x)(y)(P(x,y) \to W(x,y)$ and $\neg W(a,b)$
11. Show that $\neg P \vee Q, \neg Q \vee R, R \to S \Rightarrow P \to S$
12. Show that $\neg(P \wedge \neg Q) \wedge \neg Q \vee R \wedge \neg R \Rightarrow \neg P$

13. Show that P is equivalent to $\neg\neg P, P \wedge P, P \vee P, P \wedge (P \vee Q), (P \wedge Q) \vee (P \wedge \neg Q)$

14. Indicate which one are tautologies (or) contradictions

(a) $(P \wedge Q) \Leftrightarrow P$ (b) $P \rightarrow P \vee Q$

15. If R:Ram is rich, H:Ram is happy ,Write in symbolic form

(a) Ram is poor but happy (b) Ram is poor or unhappy

(c) Ram is neither rich nor happy

16. Show that the hypothesis, "It is not sunny this afternoon and it is colder than yesterday," " We will go swimming only if it is sunny," "If we do not go swimming then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset "lead to the conclusion "we will be home by sunset".