
SATHYABAMA


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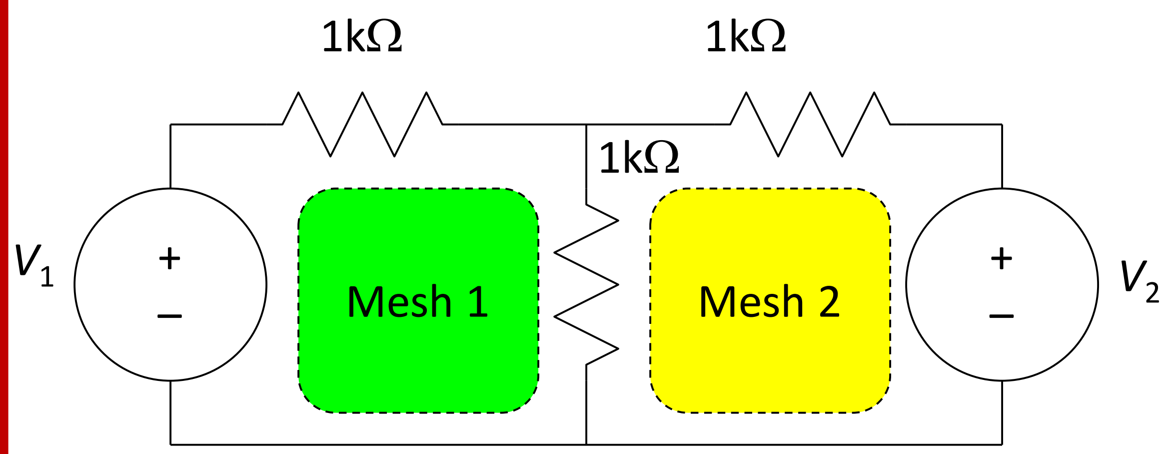
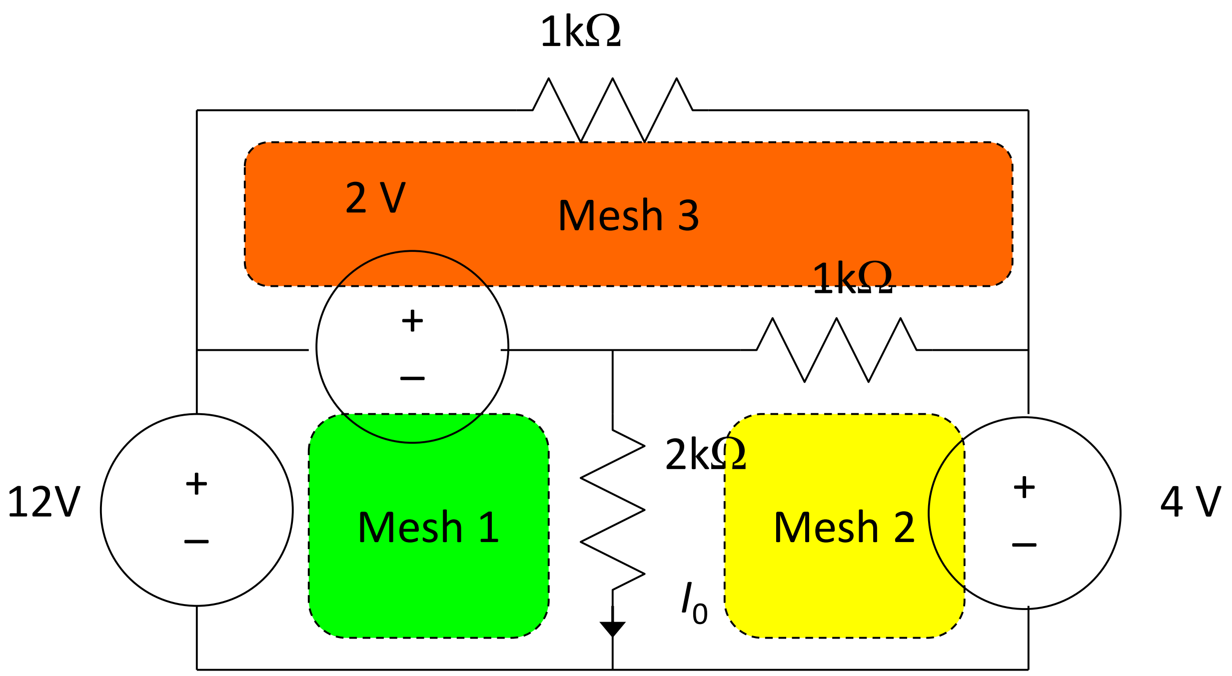
# Identifying the Meshes



By ohms law V=IR

If the circuit contains 2 loops
then matrix will be 2X2 matrix

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

If the circuit contains 3 loops
then matrix will be 3X3 matrix

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{23} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Where

$R_{11}$ ⟹ Sum of the resistances in loop 1

$R_{22}$ ⟹ Sum of the resistances in loop 2

$R_{33}$ ⟹ Sum of the resistances in loop 3

$R_{31} = R_{13}$ ⟹ Sum of the resistances in loop 1and 3

$R_{12}=R_{21}$ ⟹ Sum of the resistances in loop 1 and 2

$R_{32}=R_{23}$ ⟹ Sum of the resistances in loop 2 and 3

$V_1$ ⟹ Sum of the voltages in loop 1

$V_2$ ⟹ Sum of the voltages in loop 2

$V_3$ ⟹ Sum of the voltages in loop 3

$I_1$ ⟹ Unknown loop current in loop 1

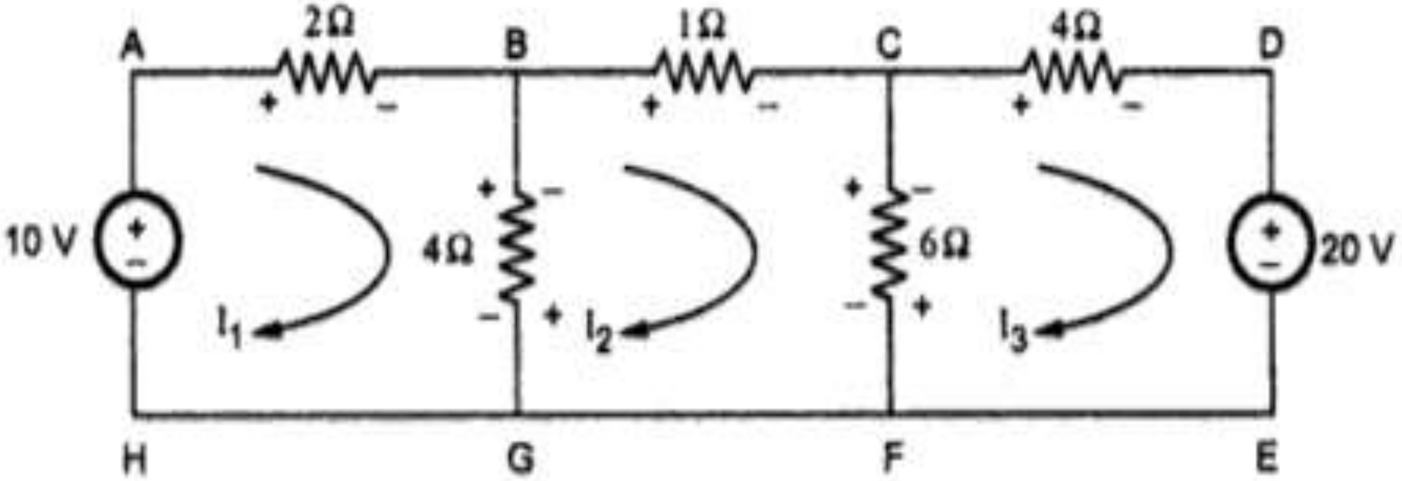
$I_2$ ⟹ Unknown loop current in loop 2

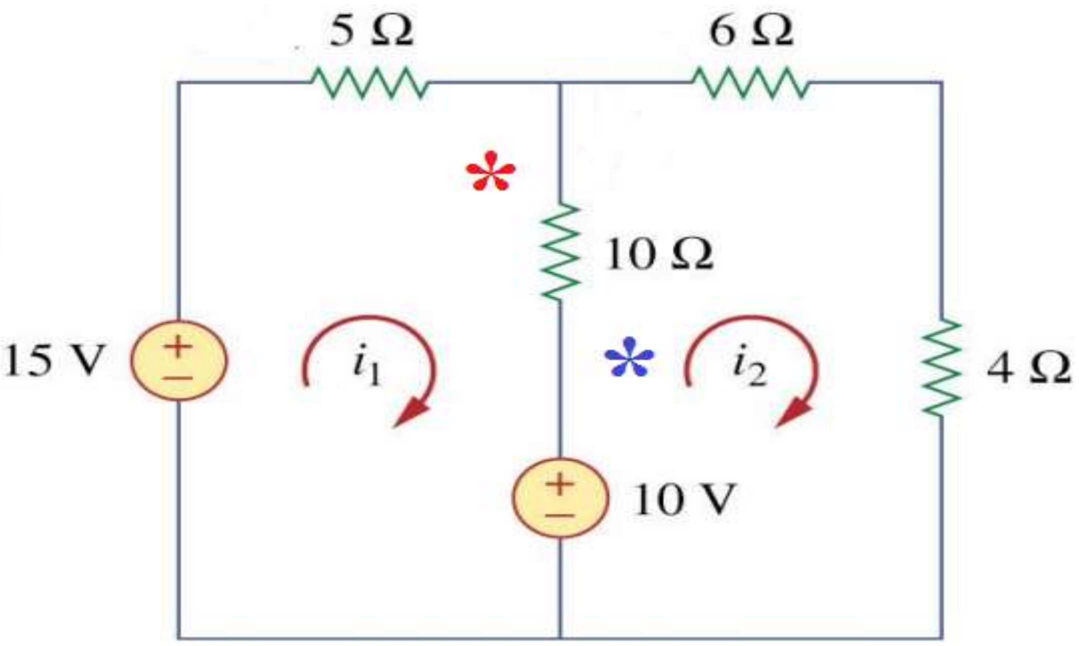
$I_3$ ⟹ Unknown loop current in loop 3

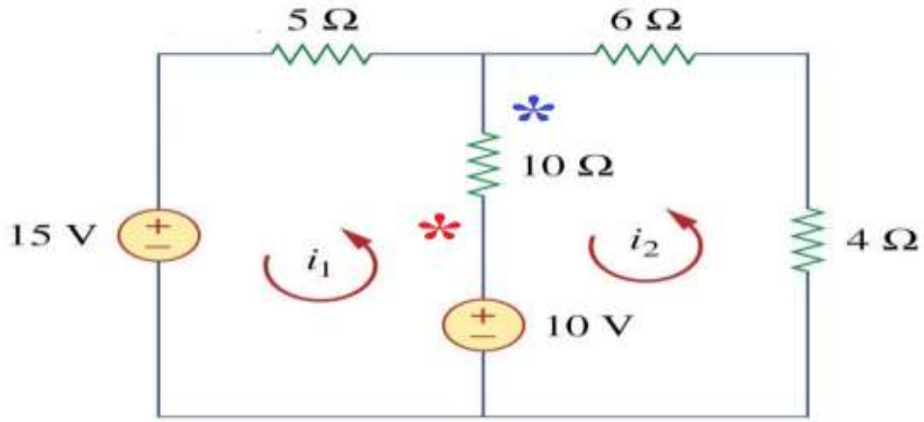
# Rules:

# All the diagonal elements are positive ($R_{11}, R_{22}, R_{33}$)

# All the Non diagonal elements($R_{12}, R_{23}, R_{13}, R_{21}, R_{32}, R_{31}$) are Negative if all the loop currents are in the same direction

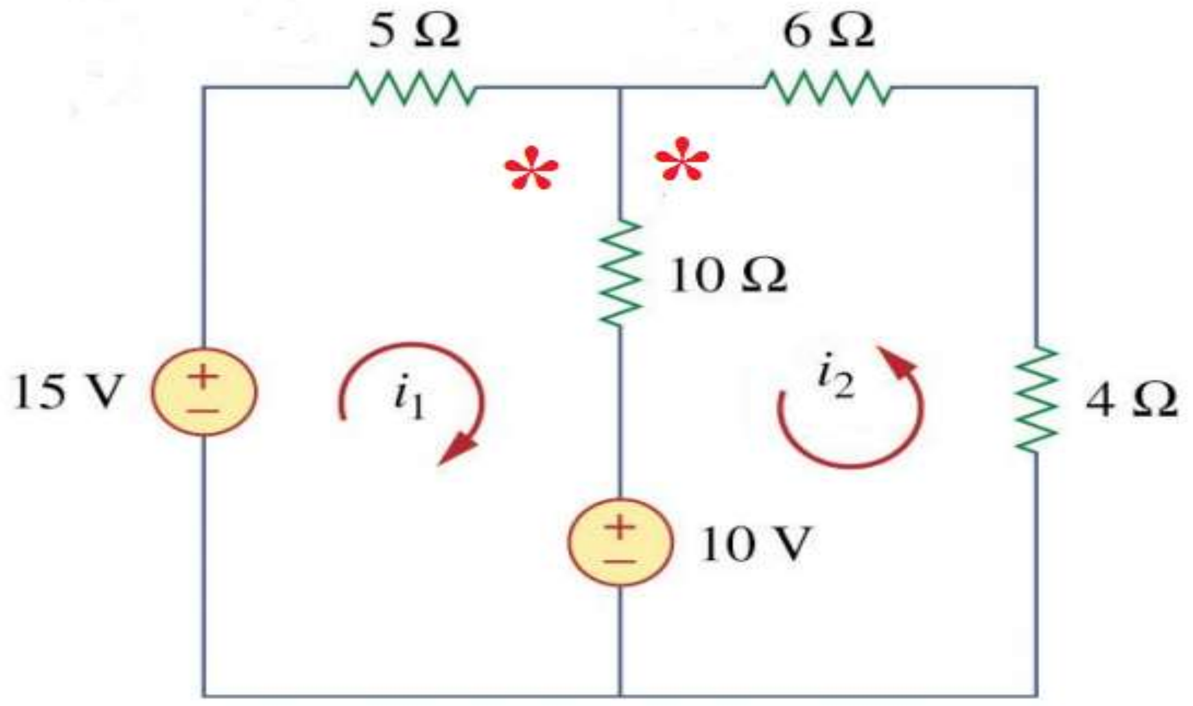


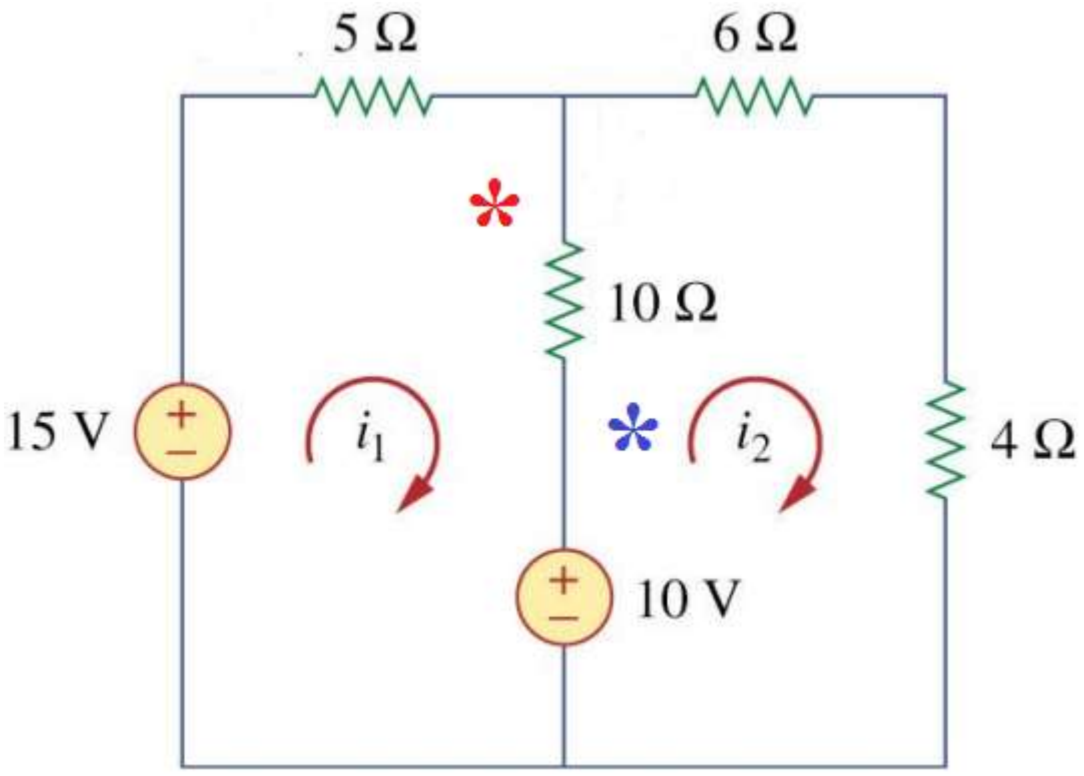
# Rules continued

# The Non diagonal elements($R_{12}$,$R_{23}$,$R_{13}$,$R_{21}$,$R_{32}$,$R_{31}$) are positive if the loop currents in the common element aid each other.



Problem No.1

Solve the given circuit and find the mesh currents and branch currents and branch voltages.




## Solution for Problem No.1

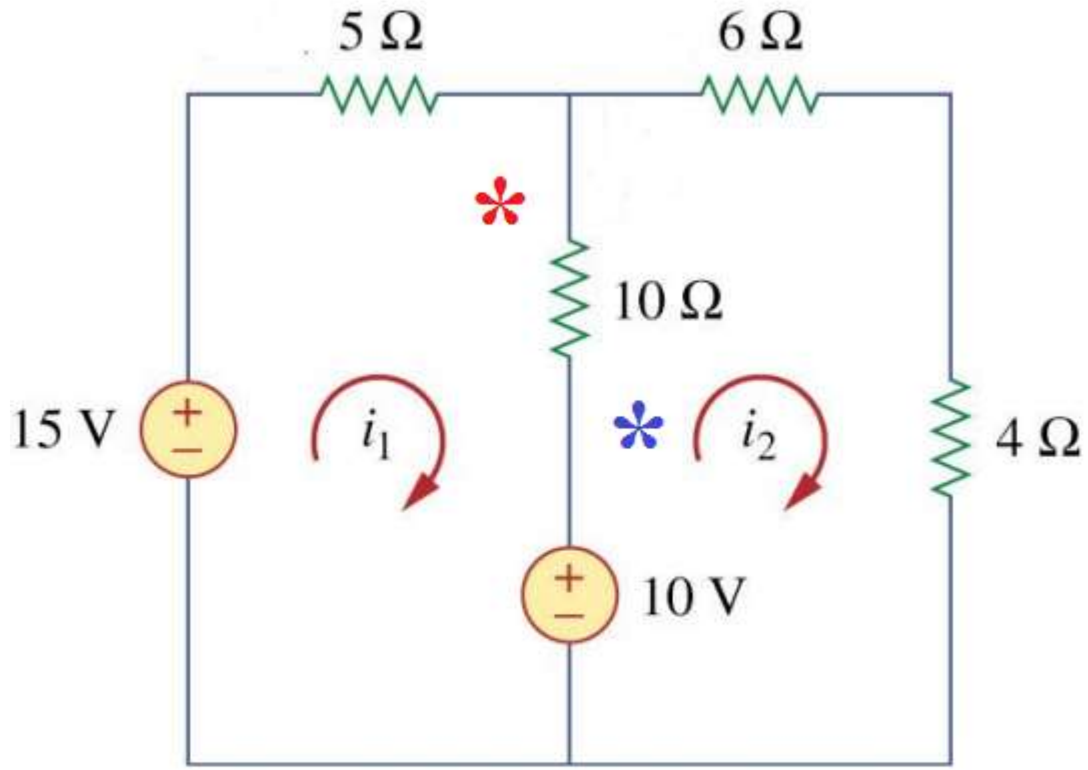
$R_{11} = 5+10 = 15$

$R_{22} = 10+6+4 = 20$

$R_{12} = R_{21} = -10$

$V_1 = 15-10 = 5$

$V_2 = 10$




$$\begin{bmatrix} 15 & -10 \\ -10 & 20 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

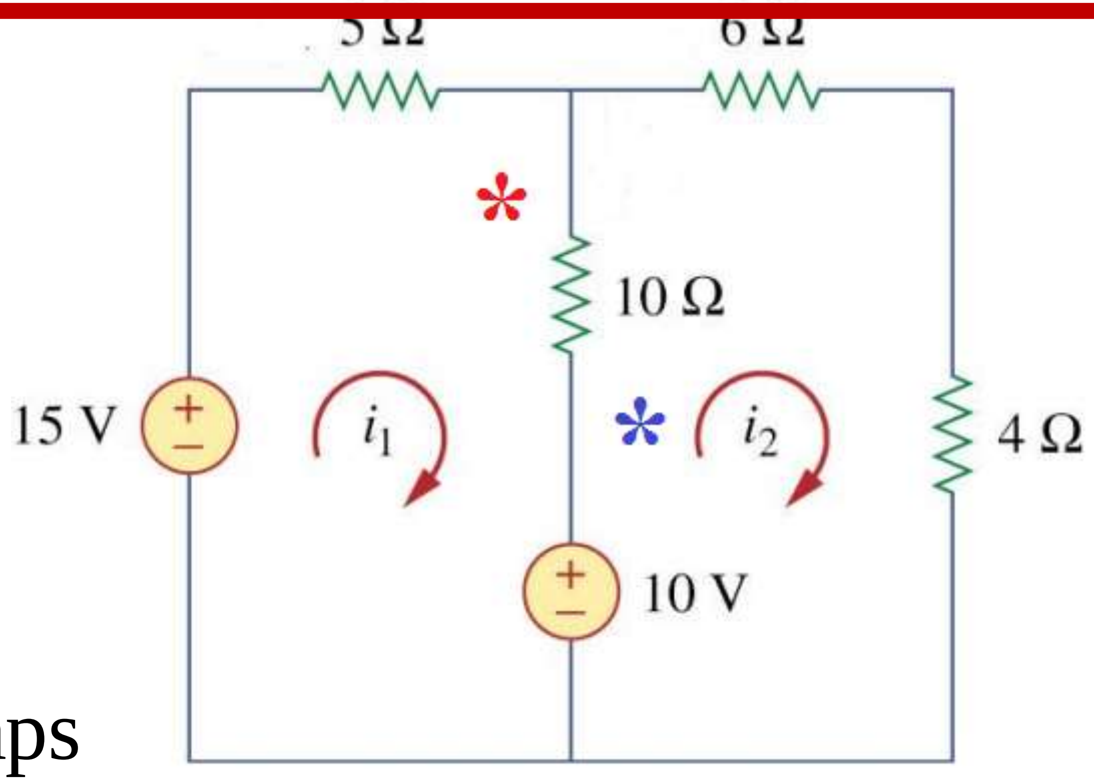
$$\Delta = \begin{bmatrix} 15 & -10 \\ -10 & 20 \end{bmatrix} = 200$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{200}{200} = 1 Amps$$

$$\Delta_1 = \begin{bmatrix} 5 & -10 \\ 10 & 20 \end{bmatrix} = 200 \qquad \Delta_2 = \begin{bmatrix} 15 & 5 \\ -10 & 10 \end{bmatrix} = 200$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{200}{200} = 1 Amps$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{200}{200} = 1 Amps$$




Current in 5Ω Resistor= $I_1$=1 Amps

Current in 6Ω Resistor= $I_2$=1 Amps

Current in 4Ω Resistor= $I_2$=1 Amps

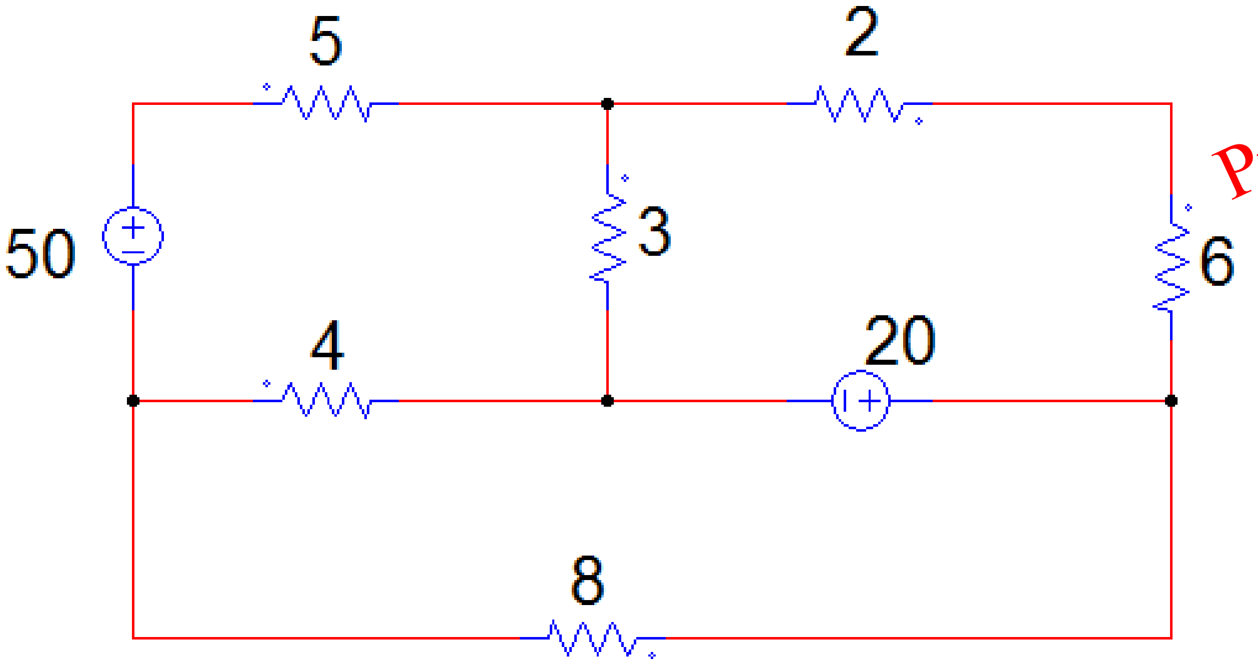
Current in 10Ω Resistor= $I_1$-$I_2$=0 Amps

Voltage in 5Ω Resistor =1*5 =5 Volts

Voltage in 6Ω Resistor= 6*1= 6 Volts

Voltage in 4Ω Resistor= 4*1= 4 Volts

Voltage in 10Ω Resistor= 0 Volts




Problem No.2

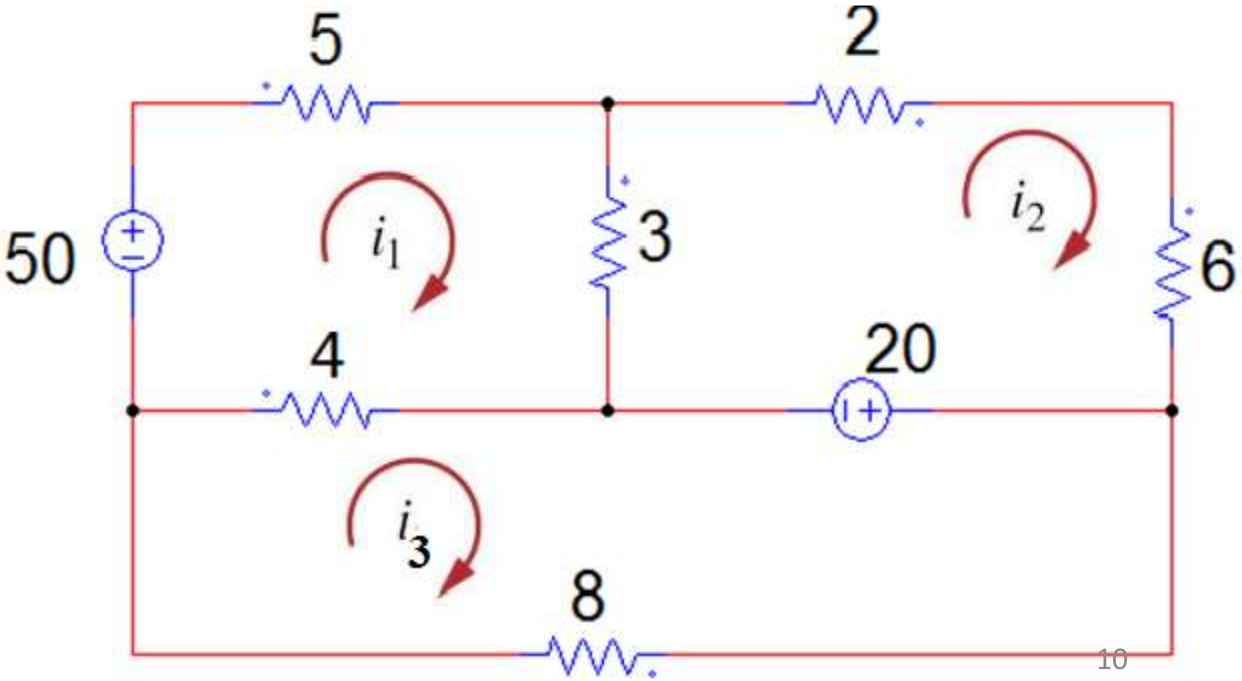
Solve the given circuit by Mesh Analysis and find the loop currents

Solution for Problem No.2

$R_{11}$=5+3+4=12

$R_{22}$=2+6+3=11

$R_{33}$=4+8=12

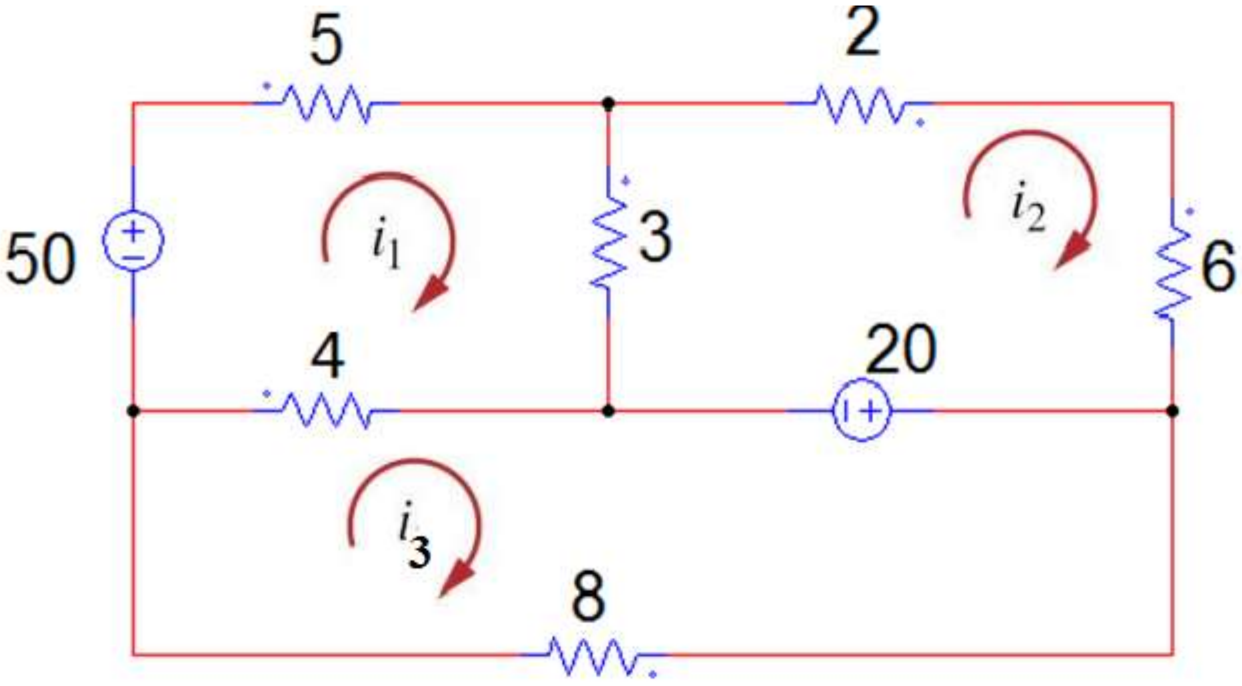


# Solution for Problem No.2

$R_{12}=R_{21}= -3$

$R_{13}=R_{31}= -4$

$R_{23}=R_{32}= 0$




$V_1= 50$
$V_2= -20$
$V_3=20$

$$\begin{bmatrix} 12 & -3 & -4 \\ -3 & 11 & 0 \\ -4 & 0 & 12 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ -20 \\ 20 \end{bmatrix}$$

# Solution for Problem No.2

$$\begin{bmatrix} 12 & -3 & -4 \\ -3 & 11 & 0 \\ -4 & 0 & 12 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ -20 \\ 20 \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{6760}{1300} = 5.2\,Amps$$

$$\Delta = \begin{bmatrix} 12 & -3 & -4 \\ -3 & 11 & 0 \\ -4 & 0 & 12 \end{bmatrix} = 1300$$

$$\Delta_2 = \begin{bmatrix} 12 & 50 & -4 \\ -3 & -20 & 0 \\ -4 & 20 & 12 \end{bmatrix} = -520$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-520}{1300} = -0.4\,Amps$$

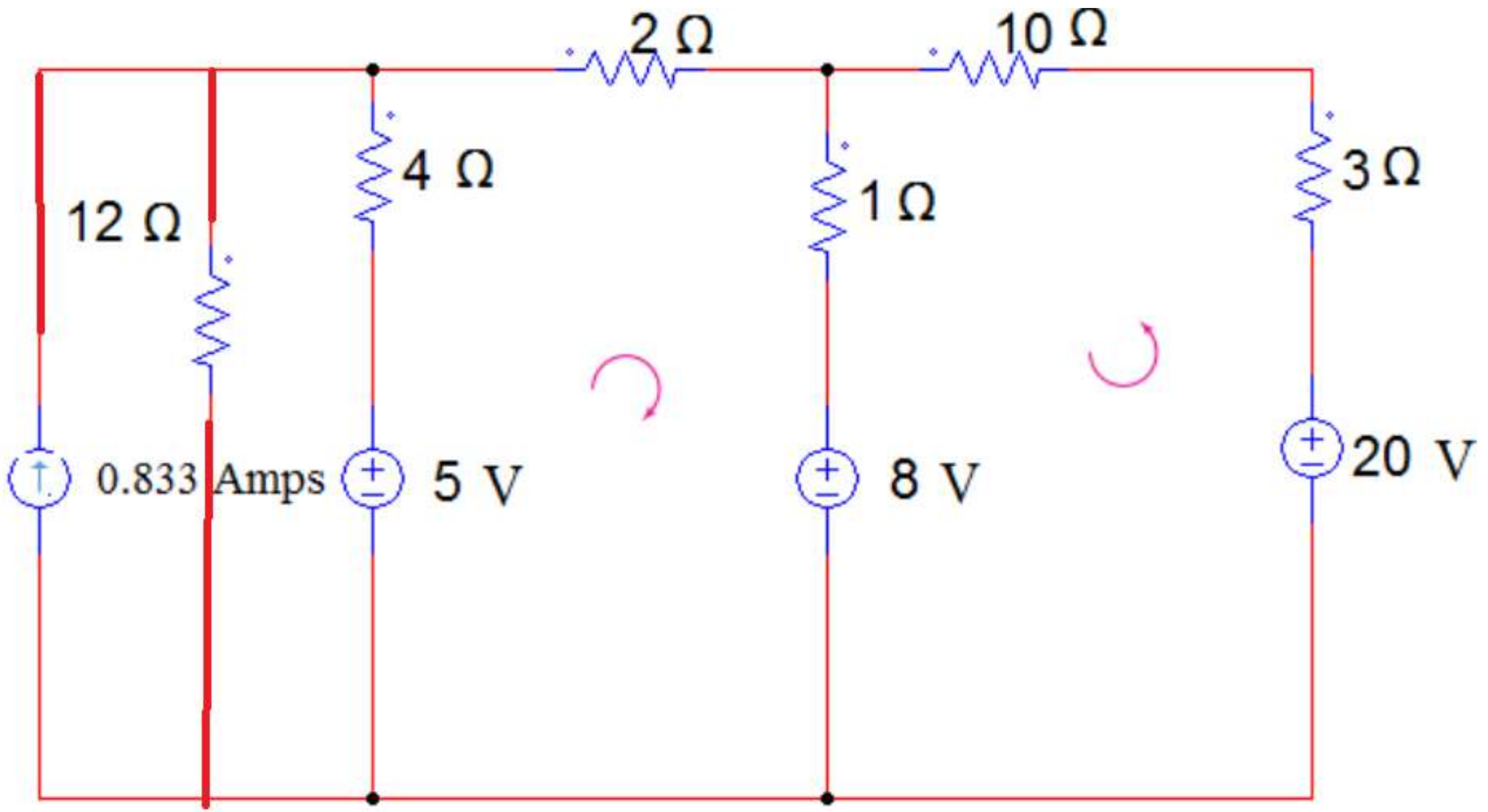
$$\Delta_1 = \begin{bmatrix} 50 & -3 & -4 \\ -20 & 11 & 0 \\ 20 & 0 & 12 \end{bmatrix} = 6760$$

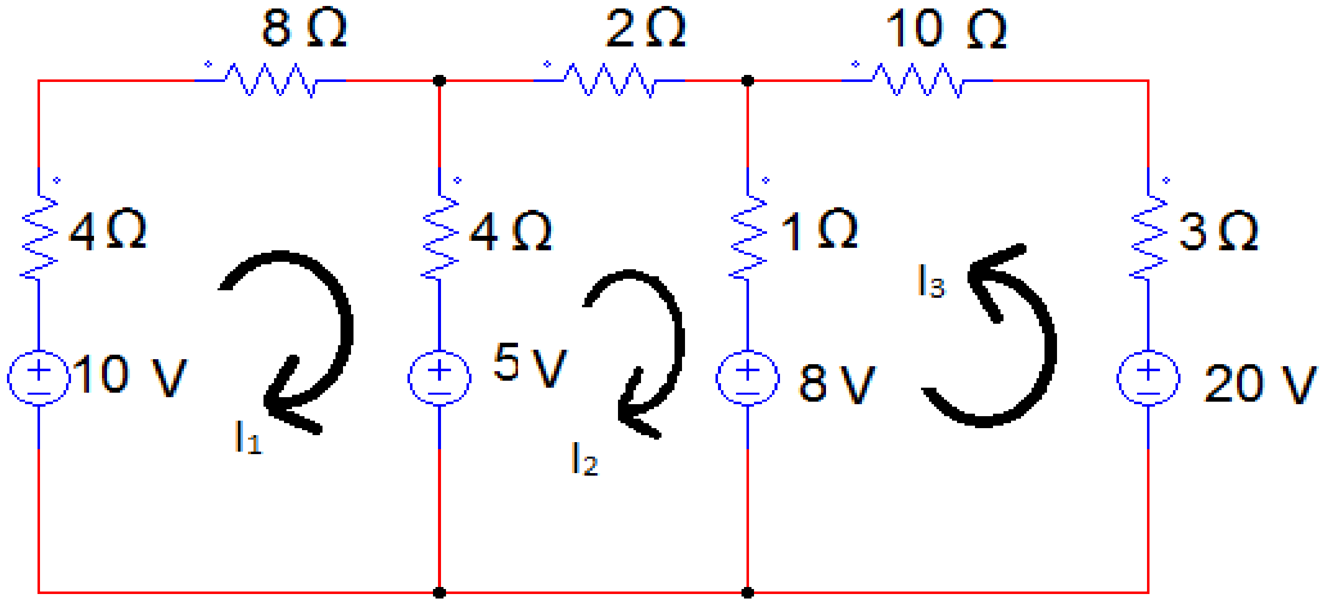
$$\Delta_3 = \begin{bmatrix} 12 & -3 & 50 \\ -3 & 11 & -20 \\ -4 & 0 & 20 \end{bmatrix} = 4420$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{4420}{1300} = 3.4\,Amps$$

Problem No.3

# Find the mesh currents by using mesh analysis






Find the mesh currents by using mesh analysis

$R_{11}$=4+8+4=16

$R_{22}$=4+2+1=7

$R_{33}$=1+10+3=14

$R_{12}$=$R_{21}$= -4

$R_{23}$=$R_{32}$= 1

$R_{13}$=$R_{31}$=0

$V_1$= 10-5 =5

$V_2$ = 5-8 = -3

$V_3$ = -8 + 20 =12

$$\begin{bmatrix} 16 & -4 & 0 \\ -4 & 7 & 1 \\ 0 & 1 & 14 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 12 \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} 5 & -4 & 0 \\ -3 & 7 & 1 \\ 12 & 1 & 14 \end{bmatrix} = 269$$

$$\Delta = \begin{bmatrix} 16 & -4 & 0 \\ -4 & 7 & 1 \\ 0 & 1 & 14 \end{bmatrix} = 1328$$

$$\Delta_2 = \begin{bmatrix} 16 & 5 & 0 \\ -4 & -3 & 1 \\ 0 & 12 & 14 \end{bmatrix} = -584$$

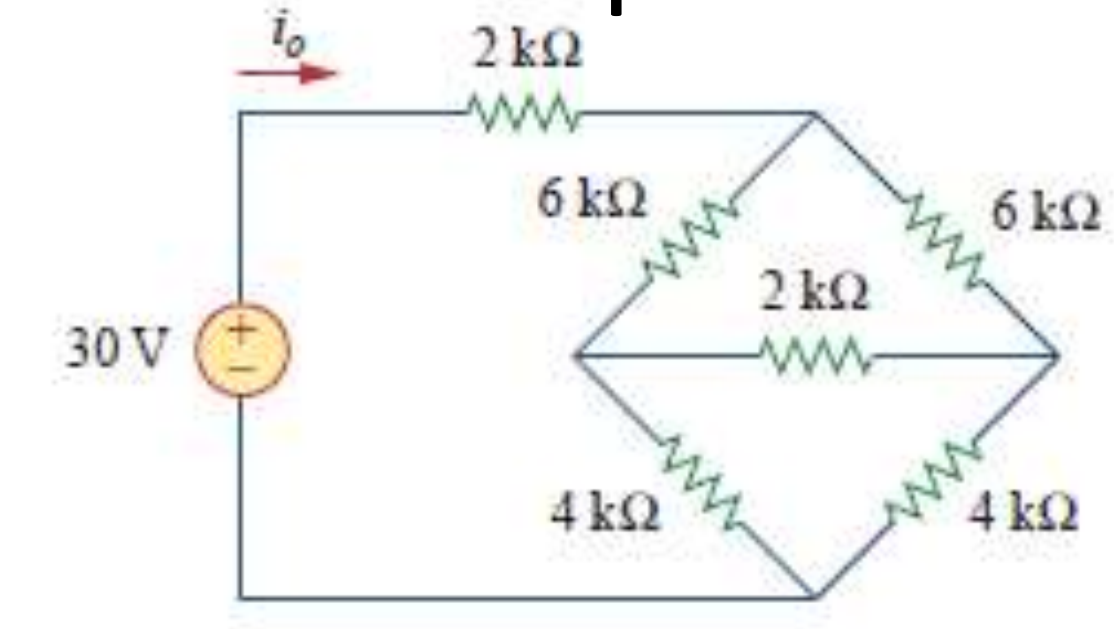
$$\Delta_3 = \begin{bmatrix} 16 & -4 & 5 \\ -4 & 7 & -3 \\ 0 & 1 & 12 \end{bmatrix} = 1180$$

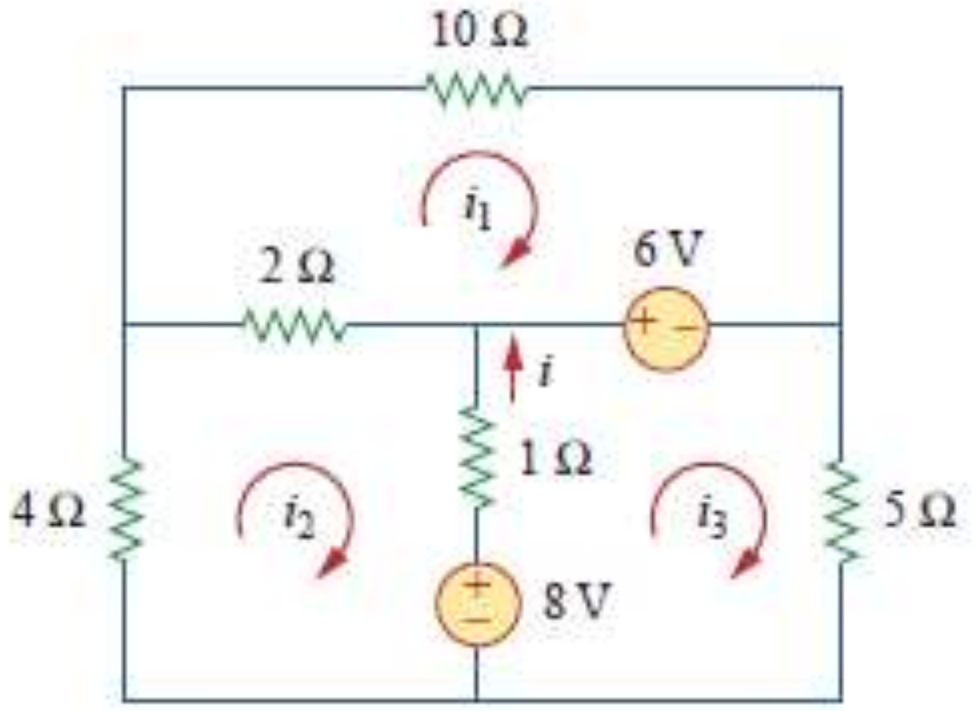
$$I_1 = \frac{\Delta_1}{\Delta} = \frac{269}{1328} = 0.2026\,Amps$$

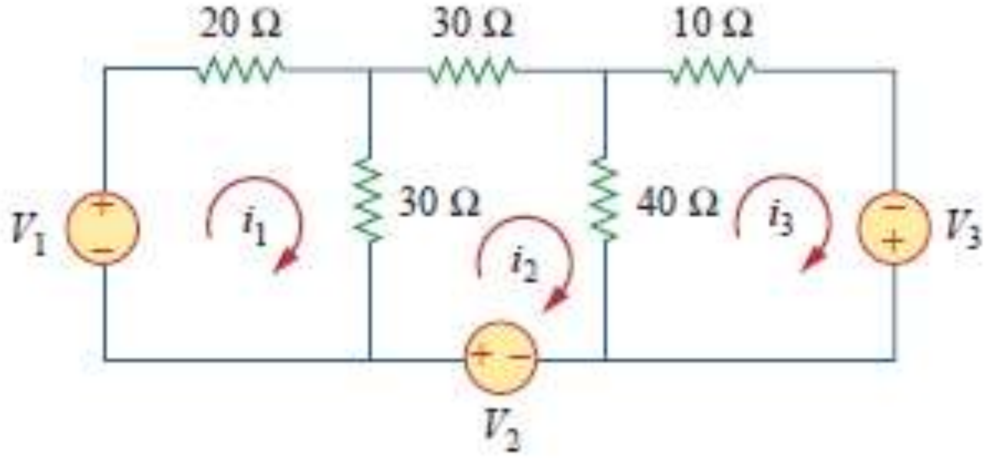
$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-584}{1328} = -0.4398\,Amps$$

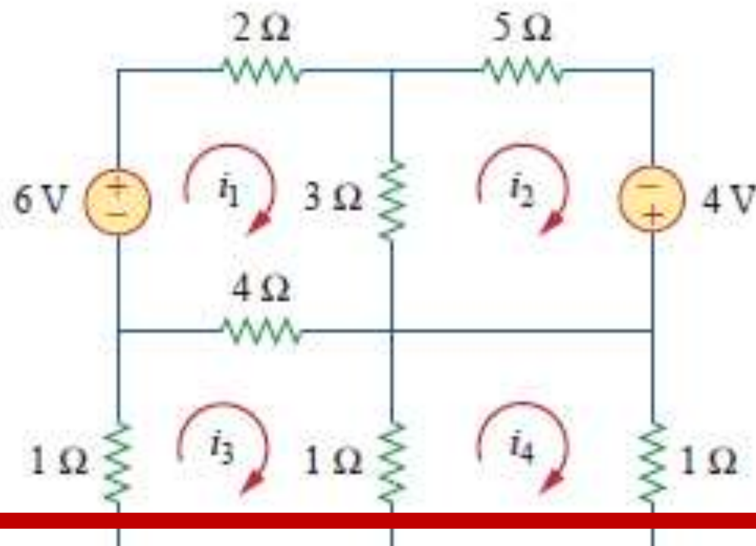
$$I_3 = \frac{\Delta_3}{\Delta} = \frac{1180}{1328} = 0.886\,Amps$$

# Additional problems

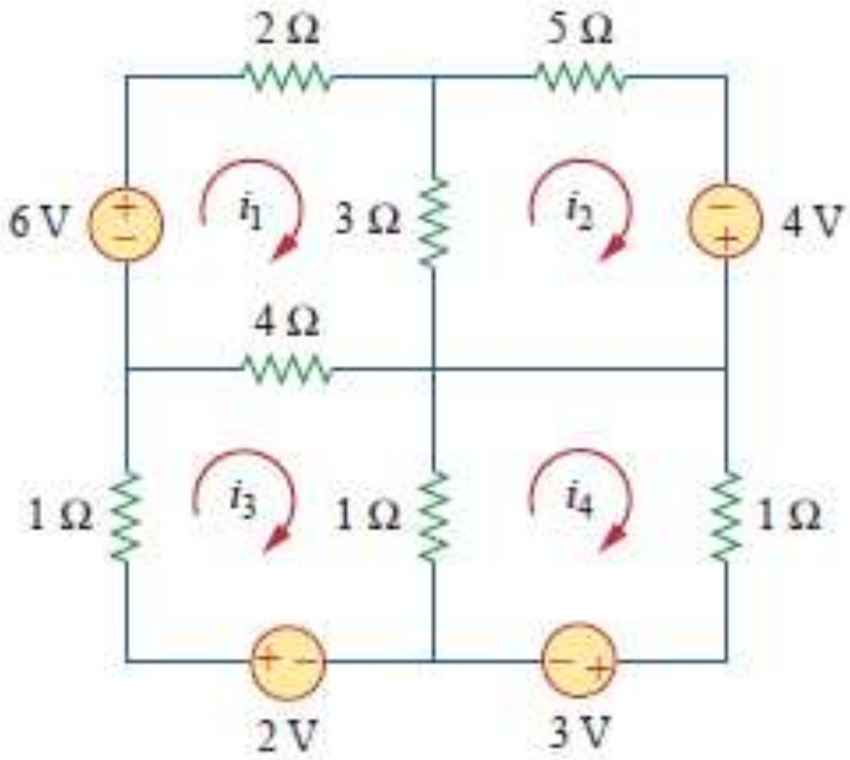


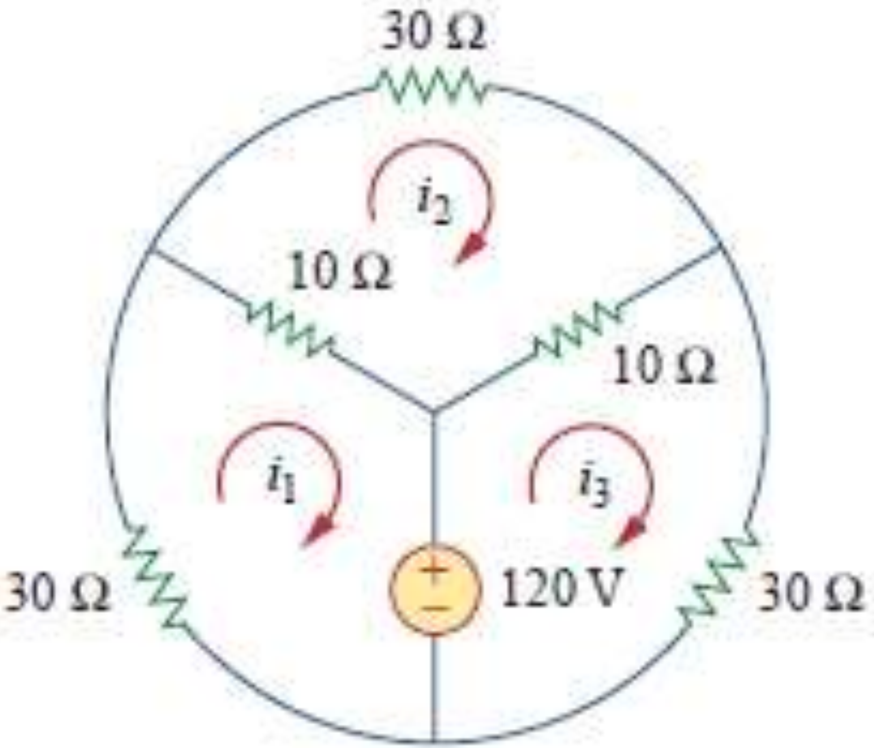



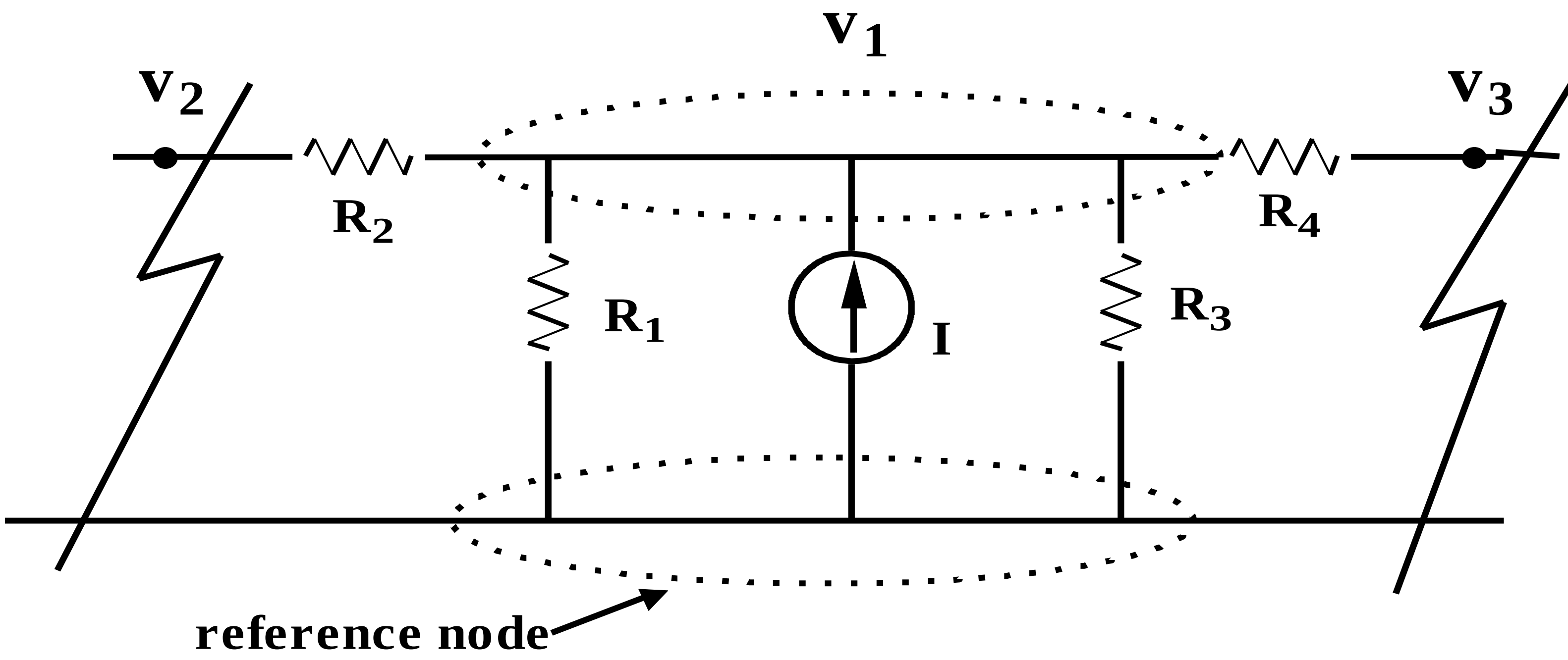
# Additional problems



# Nodal Analysis Concept

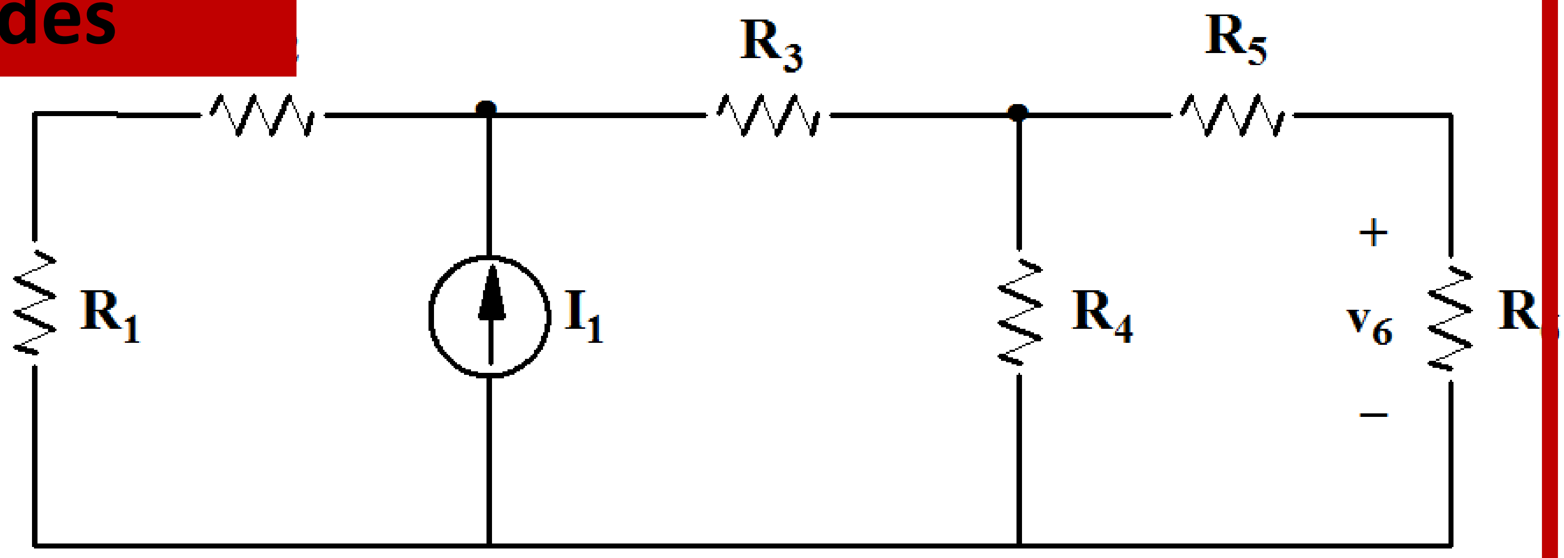
**Three are more elements joined together is called as Node.**

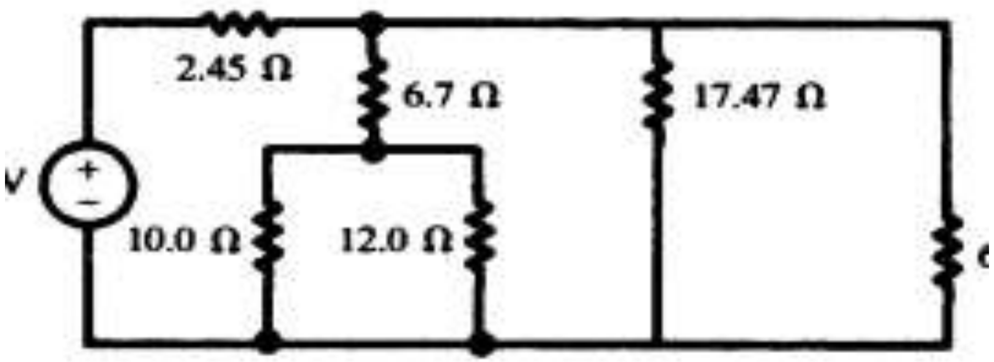


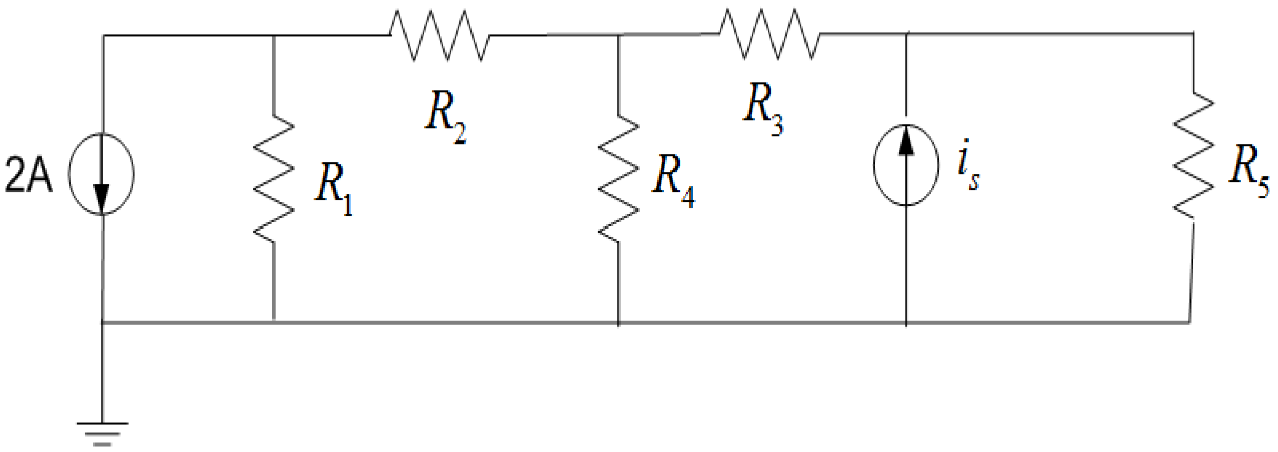

$$\frac{V_1 - V_2}{R_2} + \frac{V_1}{R_1} + \frac{V_1}{R_3} + \frac{V_1 - V_3}{R_4} = I$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)V_1 - \left(\frac{1}{R_2}\right)V_2 - \left(\frac{1}{R_4}\right)V_3 = I$$

# Identify the number of nodes



# Nodal Analysis by Inspection method

**By ohms law** V=IR

I=1/R*V

I=GV

If the circuit contains 2 major nodes then matrix will be 2X2 matrix

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

If the circuit contains 3 major nodes then matrix will be 3X3 matrix

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{23} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Where

$G_{11} \rightarrow$ Sum of the conductance's in node No. 1

$G_{22} \rightarrow$ Sum of the conductance's in node No. 2

$G_{33} \rightarrow$ Sum of the conductance's in node No. 3

$G_{31} = G_{13} \rightarrow$ Sum of the conductance's connected between node No. 1and 3

$G_{12}=G_{21} \rightarrow$ Sum of the conductance's connected between node No. 1 and 2

$G_{32}=G_{23} \rightarrow$ Sum of the conductance's connected between node No. 2 and 3

$I_1 \rightarrow$ Sum of the currents flowing towards node no. 1

$I_2 \rightarrow$ Sum of the currents flowing towards node no. 2

$I_3 \rightarrow$ Sum of the currents flowing towards node no.3

$V_1 \rightarrow$ Unknown node voltage of node no. 1

$V_2 \rightarrow$ Unknown node voltage of node no. 2

$V_3 \rightarrow$ Unknown node voltage of node no. 3

Rules:

# All the diagonal($G_{11}$,$G_{22}$,$G_{33}$ )elements are positive.

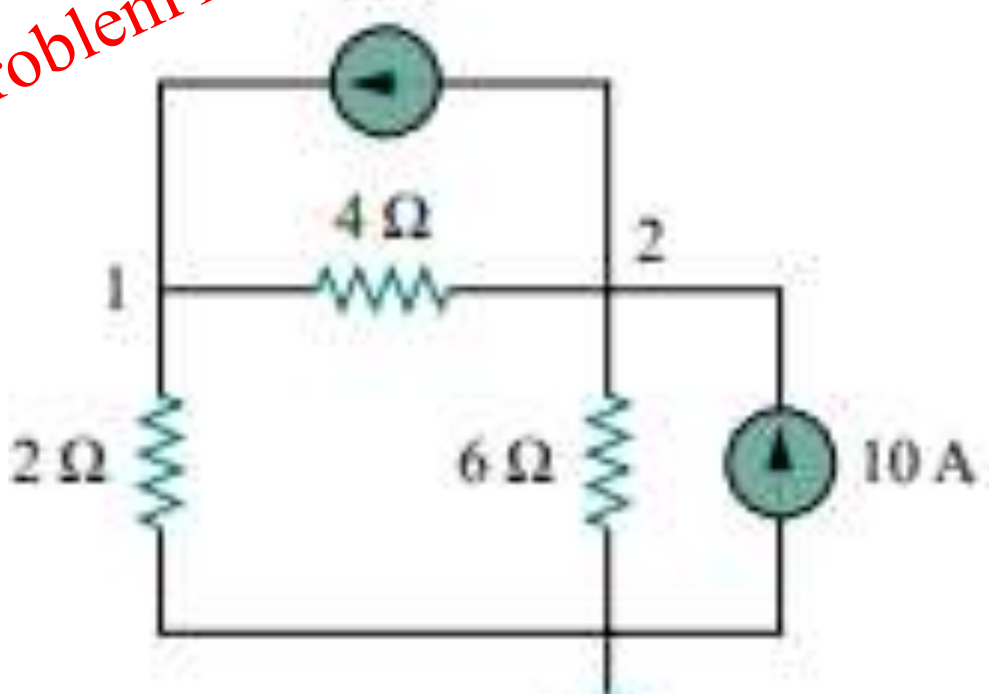
# All the Non diagonal($G_{12}$,$G_{13}$,$G_{23}$,$G_{32}$,$G_{21}$,$G_{31}$) elements are negative.

Note:

For inspection method the circuit should contain only current source. If voltage source is present convert in to current source and then apply inspection method.

Problem No.1

# Find the node voltages and branch currents.




Since there are 2 nodes we have 2x2 matrix

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

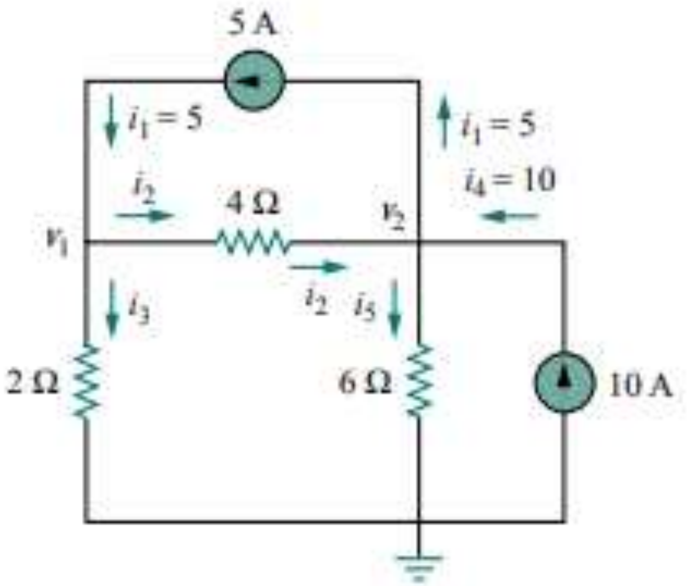
$$G_{11} = \frac{1}{2} + \frac{1}{4} = 0.25 + 0.5 = 0.75$$

$$G_{22} = \frac{1}{4} + \frac{1}{6} = 0.25 + 0.1666 = 0.4166$$

$$G_{12} = G_{21} = \frac{1}{4} = 0.25 = -0.25$$

$I_1 = 5$

$I_2 = -5 + 10 = 5$




$$\begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.4166 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.4166 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.4166 \end{bmatrix} = 0.24995$$

$$\Delta_1 = \begin{bmatrix} 5 & -0.25 \\ 5 & 0.4166 \end{bmatrix} = 3.333$$

$$\Delta_2 = \begin{bmatrix} 0.75 & 5 \\ -0.25 & 5 \end{bmatrix} = 5$$

$$V_1 = \frac{\Delta_1}{\Delta} = 13.33 Volts$$

$$V_2 = \frac{\Delta_2}{\Delta} = 20 Volts$$

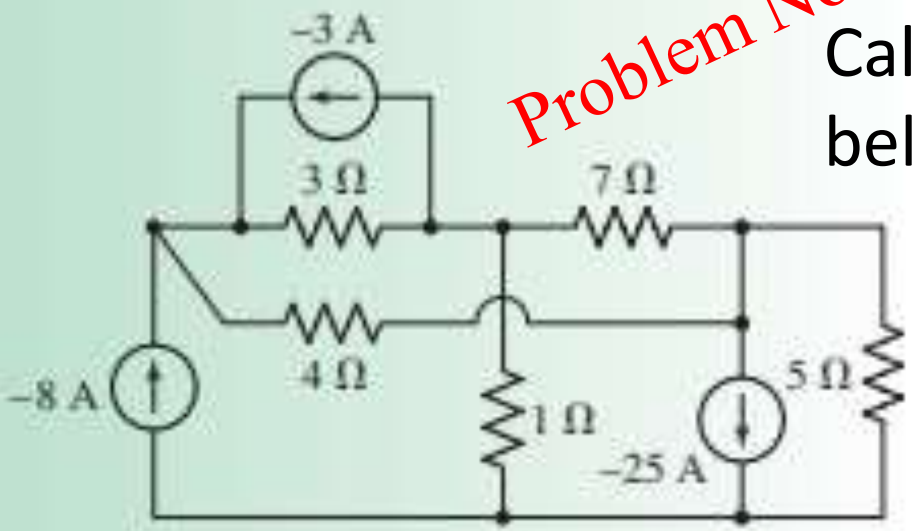
$$I_{2\Omega} = \frac{V_1}{2} = 6.665 Amps$$

$$I_{4\Omega} = \frac{V_2 - V_1}{4} = 1.6675 Amps$$

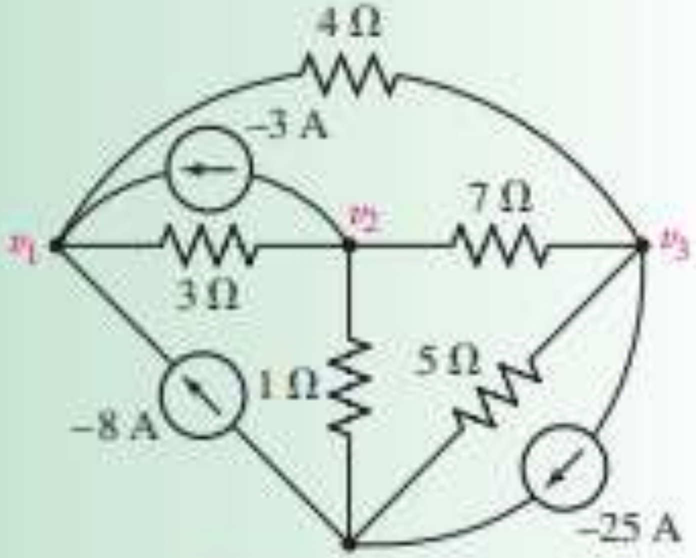
$$I_{6\Omega} = \frac{V_2}{6} = 3.33 Amps$$

Problem No.2

# Calculate the node voltages for the circuit given below



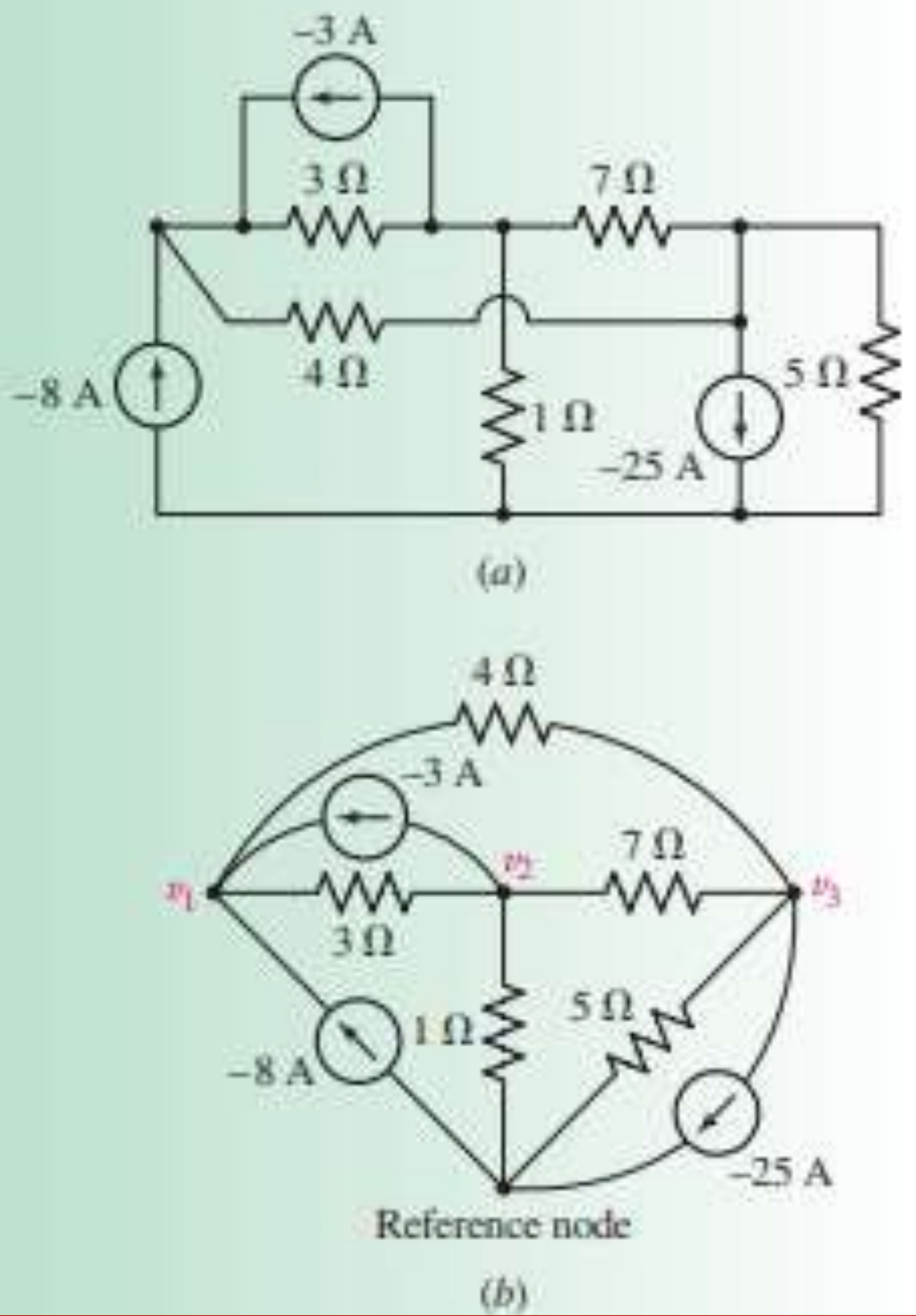

(a)




(b)

SOLUTION: Since there are three nodes we have 3x3 matrix

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{23} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$




$$G_{11}=\frac{1}{3}+\frac{1}{4}=0.5833$$

$$G_{22}=\frac{1}{3}+\frac{1}{1}+\frac{1}{7}=1.4762$$

$$G_{33}=\frac{1}{5}+\frac{1}{7}+\frac{1}{4}=0.5929$$

$$G_{13}=G_{31}=\frac{1}{4}=0.25$$

$$I_1=-3-8=-11$$

$$I_2=-(-3)=3$$

$$I_3=-(-25)=25$$

$$G_{12}=G_{21}=\frac{1}{3}=0.33$$

$$G_{32}=G_{23}=\frac{1}{7}=0.1429$$

$$\begin{bmatrix}0.5833 & -0.33 & -0.25\\ -0.33 & 1.4762 & -0.1429\\ -0.25 & -0.1429 & 0.5929\end{bmatrix}\begin{bmatrix}V_1\\ V_2\\ V_3\end{bmatrix}=\begin{bmatrix}-11\\ 3\\ 25\end{bmatrix}$$

$$v_1 = \frac{\begin{vmatrix} -11 & -0.3333 & -0.2500 \\ 3 & 1.4762 & -0.1429 \\ 25 & -0.1429 & 0.5929 \end{vmatrix}}{\begin{vmatrix} 0.5833 & -0.3333 & -0.2500 \\ -0.3333 & 1.4762 & -0.1429 \\ -0.2500 & -0.1429 & 0.5929 \end{vmatrix}} = \frac{1.714}{0.3167} = 5.412\ \text{V}$$

Similarly,

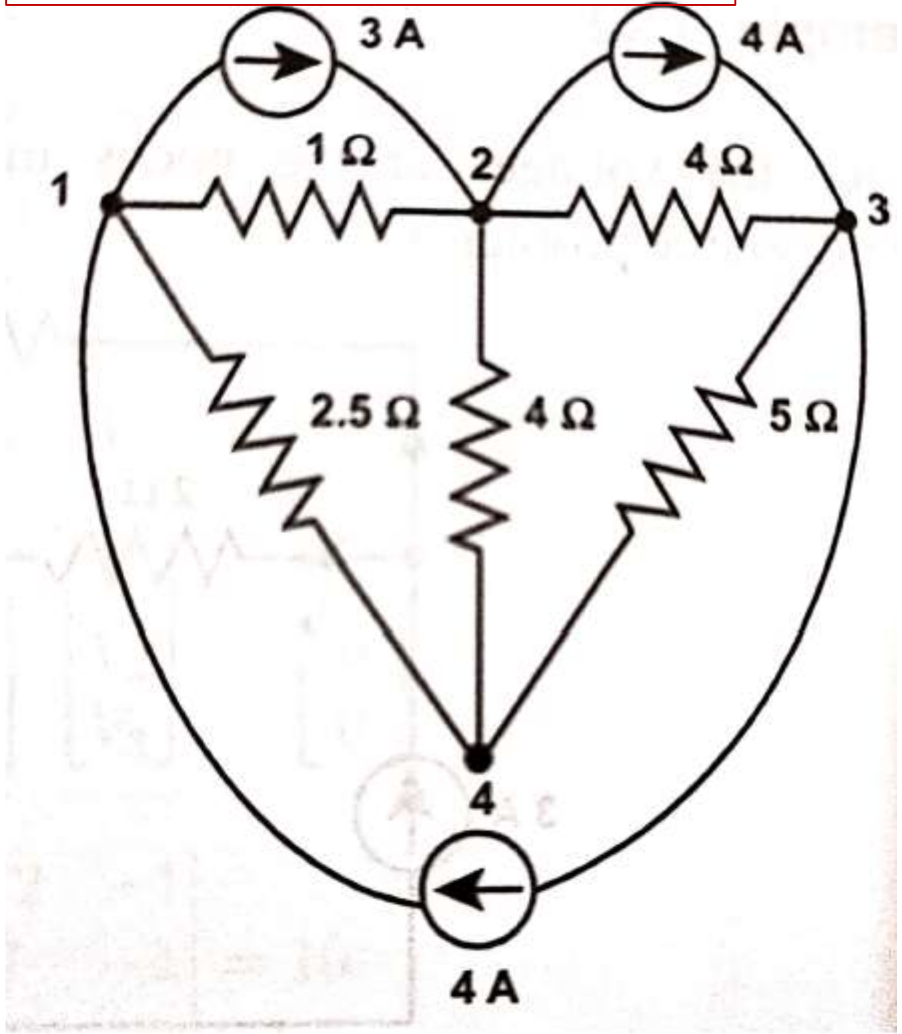
$$v_2 = \frac{\begin{vmatrix} 0.5833 & -11 & -0.2500 \\ -0.3333 & 3 & -0.1429 \\ -0.2500 & 25 & 0.5929 \end{vmatrix}}{0.3167} = \frac{2.450}{0.3167} = 7.736\ \text{V}$$

and

$$v_3 = \frac{\begin{vmatrix} 0.5833 & -0.3333 & -11 \\ -0.3333 & 1.4762 & 3 \\ -0.2500 & -0.1429 & 25 \end{vmatrix}}{0.3167} = \frac{14.67}{0.3167} = 46.32\ \text{V}$$

$$\begin{bmatrix} 0.5833 & -0.33 & -0.25 \\ -0.33 & 1.4762 & -0.1429 \\ -0.25 & -0.1429 & 0.5929 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 3 \\ 25 \end{bmatrix}$$

## Problem No.3




**Find the voltage across the node 2 and 4 by nodal analysis**

$$G_{11} = \frac{1}{1} + \frac{1}{2.5} = 1.4$$

$$G_{22} = \frac{1}{4} + \frac{1}{1} + \frac{1}{4} = 1.5$$

$$G_{33} = \frac{1}{4} + \frac{1}{5} = 0.45$$

$$G_{13} = G_{31} = 0$$

$$G_{12} = G_{21} = \frac{1}{1} = 1 = -1$$

$$G_{32} = G_{23} = \frac{1}{4} = 0.25 = -0.25$$

$$I_1 = 4 - 3 = 1$$

$$I_2 = 3 - 4 = -1$$

$$I_3 = 4 - 4 = 0$$

$$\begin{bmatrix} 1.4 & -1 & 0 \\ -1 & 1.5 & -0.25 \\ 0 & -0.25 & 0.45 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} 1.4 & 1 & 0 \\ -1 & -1 & -0.25 \\ 0 & 0 & 0.45 \end{bmatrix} = -0.28$$

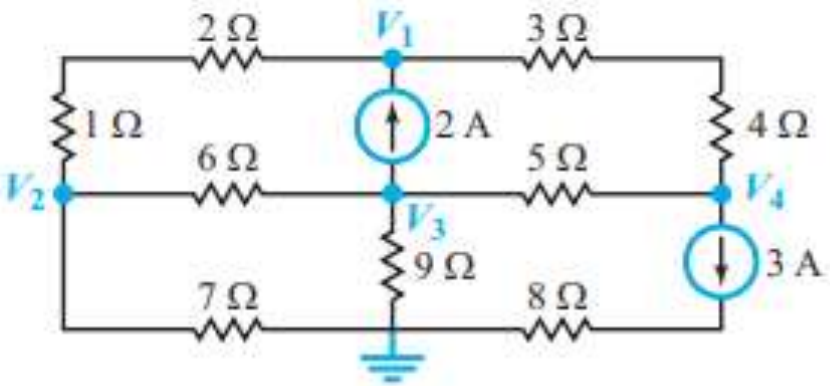
$$\Delta = \begin{bmatrix} 1.4 & -1 & 0 \\ -1 & 1.5 & -0.25 \\ 0 & -0.25 & 0.45 \end{bmatrix} = 0.665$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-0.28}{0.665} - 0.421 Volts$$

$$V_{4\Omega} = V_2 - V_4 = -0.421 - 0 = -0.421 Volts$$

Problem No.3

Use the by-inspection method to establish a node-voltage matri
equation for the circuit
to find $V_1$ to $V_4$.




$$G_{11} = \frac{1}{2+1} + \frac{1}{3+4} = 0.476$$

$$G_{12} = G_{21} = -\frac{1}{2+1} = -0.333$$

$$G_{13} = G_{31} = 0$$

$$G_{14} = G_{41} = -\frac{1}{3+4} = -0.143$$

$$G_{22} = \frac{1}{1+2} + \frac{1}{7} + \frac{1}{6} = 0.643$$

$$G_{23} = G_{32} = -\frac{1}{6} = -0.167$$

$$G_{24} = G_{42} = 0$$

$$G_{33} = \frac{1}{5} + \frac{1}{6} + \frac{1}{9} = 0.478$$

$$G_{34} = G_{43} = -\frac{1}{5} = -0.2$$

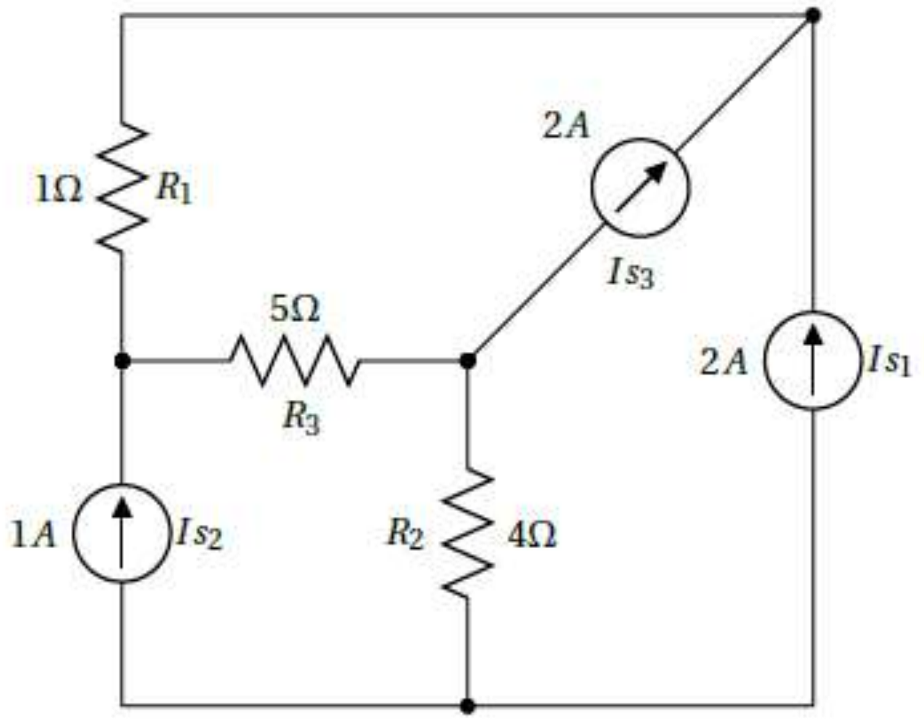
$$G_{44} = \frac{1}{3+4} + \frac{1}{5} = 0.343$$

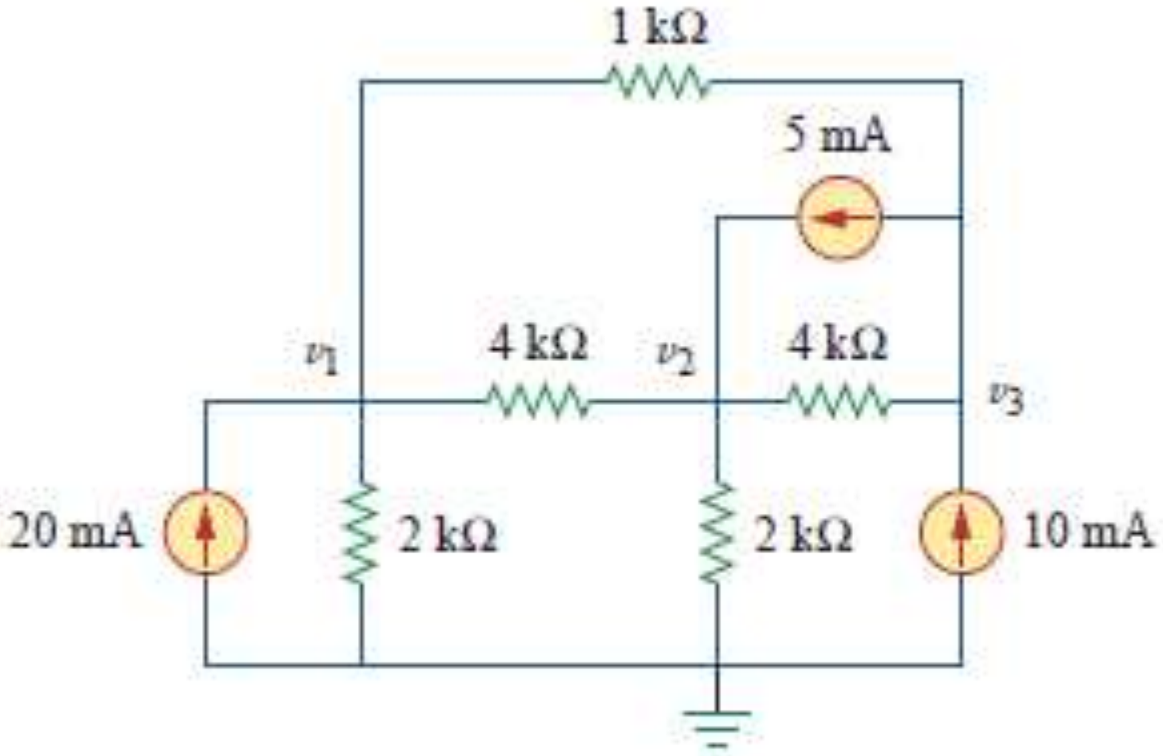
$$\begin{bmatrix} 0.476 & -0.333 & 0 & -0.143 \\ -0.333 & 0.643 & -0.167 & 0 \\ 0 & -0.167 & 0.478 & -0.2 \\ -0.143 & 0 & -0.2 & 0.343 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -2 \\ -3 \end{bmatrix}$$

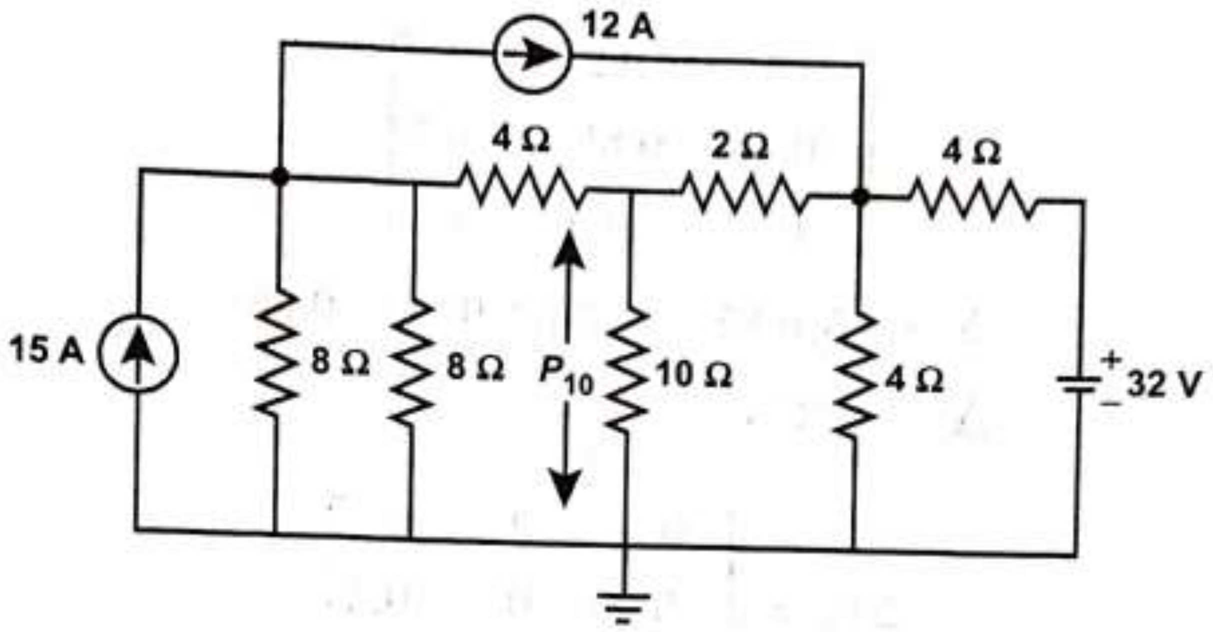
Matrix inversion gives:

$V_1 = -8.1689$ V, $V_2 = -8.4235$ V, $V_3 = -16.155$ V, $V_4 = -21.5748$ V.

# Additional problems




12 A
4 Ω
2 Ω
4 Ω
15 A
8 Ω
8 Ω
$P_{10}$
10 Ω
4 Ω
+
32 V
_

# THANK YOU