D.C.CIRCUITS

Electrical /Quantities – Definitions, Symbols and / Units

- **Charge:**

A body is said to be changed positively, if it has deficit of electrons. It is said to be charged negatively if it has excess of electrons. The charge is measured in Coulombs and denoted by Q (or) q.

1 Coulomb = Charge pm 6.28×10^{18} electrons.

- Electric Potential:

When a body is charged, either electrons are supplied on it (or) removed on from it. In both cases the work is done. The ability of the charged body to do work is called electric Potential. The charged body has the capacity to do (or) by moving the other charges by either attraction Repulsion.

The greater the capacity of a charged body to do work, the greater is its electric potential And, the work done, to charge a body to 1 coulomb is the measure of electric Potential.

Electric Potential, $V = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}$

W = Work done per unit charge.

Q = Charge measured in coulombs.

Unit of electric potential is **joules / Coulomb (or) volt**. If $W_{1=}$ 1 joule; Q= 1 coulomb, then V=1/1 = 1 Volt.

A body is said to have and electric potential of 1 volt, if one joule of work is done to / charge a body to one coulomb. Hence greater the joules / coulomb on charged body, greater is electric Potential.

- **Potential Difference**:

The difference in the potentials of two charged bodies is called potential difference.

- **Electric Current:**

Flow of free electrons through a conductor is called electric current. Its unit is ampere (or) Coulomb / sec.

$$\text{Current, } (i) = \frac{\text{Charge(q)}}{\text{Time(t)}} = \frac{q}{t} \text{Coulombs / Sec}$$

In differential Form, $i = \frac{dq}{dt}$ Coulombs / Sec

- **Resistance:**

Resistance is defined as the property of the substance due to which restricts the flow of electrons through the conductor. Resistance may, also be defined as the physical Property of the substance due to which it opposes (or) Restricts the flow of electricity (ie electrons) through it. Its unit is ohms.

A wire is said to have a resistance of 1 ohm if a p.d. /of 1V across the ends causes current of 1 Amp to flow through it (or) a wire is said to have a resistance of 1 ohm if it releases 1 joule, when a current of 1A flows through it.

- **conductors**

Certain substances offer very little opposition to the flow of electric current, they are called as conductors for examples, metals, acids etc. Amongst Pure metals, Sliver, Copper and aluminium are very good conductors.

- **Insulators**

Certain substances offer very high opposition to the flow of electric current, they are called as insulators. For eg, Bakelite, mica, glass, rubber, P.V.C, dry wood etc. The substance, whose properties lies between those of Conductors and insulators are called semi-conductors, for eg, Silicon, Germanium etc.

- **Laws of Resistance:**

The electrical resistance (R) of a metallic conductor depends upon the various Factors as given below,

(i) It is directly proportional to length l, ie, $R \alpha l$

(ii) It is l inversely proportional to the Cross Sectional area of the Conductor, ie,

$$R \alpha \frac{l}{A}$$

(iii) It depends upon the nature of the matter of the Conductor.

(iv) It depends upon the temperature of the conductor.

Therefore by assuming the temperature to remain constant, we get,

$$R \alpha \frac{l}{A}$$
$$R = \rho \frac{1}{A}$$

ρ ('Rho') is a constant of proportionality called **resistivity** (or) Specific resistance of the material of the conductor. The value of ρ depend upon the nature of the material of the conductor.

Specific Resistance (or) Resistivity:

Resistance of a wire is given by $R = \rho \frac{1}{A}$

Resistivity is the property (or) nature of the material due to which it opposes the Flow of Current through it. The unit of resistivity is ohm-metre. $[\rho = \frac{RA}{l} = \frac{\Omega m^2}{m} = \Omega m(\text{ohm-metre})$

- **Conductance (or) Specific Conductance:**

Conductance is the inducement to the flow of current. Hence, Conductance is the reciprocal of resistance. It is denoted by symbol G.

$$G = \frac{1}{R} = \frac{A}{\rho l} = \sigma \frac{A}{l}$$

G is measured in mho

$$\sigma = \frac{1}{\rho}$$

Here, σ is called the conductivity (or) specific conductance of the material

- **Conductivity (or) Specific Conductance:**

Conductivity is the property (or) nature of the material due to which it allows Flow of Current through it.

$$G = \sigma \frac{A}{l} \text{ (or) } \sigma = G \frac{l}{A}$$

Substituting the units of various quantities we get

$$\sigma = \frac{\text{mho*m}}{\text{m}^2} = \text{mho/metre}$$

∴ The S.I unit of Conductivity is mho/metre.

- **Electric Power:**

The rate at which the work is done in an electric Circuit is called electric power.

$$\text{Electric Power} = \frac{\text{Work done in electric circuit}}{\text{Time}}$$

When voltage is applied to a circuit, it causes current to flow through it. The work done moving the electrons in a unit time is called electric power. The unit of electric Power is Joules/sec or Watt. $\left[P = VI = I^2R = V^2/R\right]$

- **Electrical Energy:**

The total work done in an electric circuit is called electrical energy.

ie, Electrical Energy = electric power * time.

Electrical Energy is measured in Kilowatt hour (kwh)

Problems:

1. The resistance of a conductor $1mm^2$ in Cross section and 20m long is 0.346Ω. Determine the Specific resistance of the conducting material.

Given Data

Area of Cross-Section A = $1mm^2$

Length, l = 20m

Resistance, R = 0.346Ω

Formula used: Specific resistance of the Conducting Material, $R = \frac{\rho l}{A} \Rightarrow \rho = \frac{Rl}{A}$

Solution: Area of Cross-section, $A = 1mm^2$

$= 1 * 10^{-6} m^2$

$$\rho = \frac{1*10^{-6}*0.346}{20} = 1.73*10^{-8}\,\Omega m$$

Specific Resistance of the conducting Material, $\rho = 1.738*10^{-8}\Omega m.$

2. A Coil consists of 2000turns of Copper wire having a Cross-sectional area of 1mm$^{2.}$ The mean length per turn is 80cm and resistivity of copper is 0.02μΩm at normal working temperature. Calculate the resistance of the Coil.

Given data:

No of turns = 2000

Length /turn = 80cm = 0.8m

Resistivity = ρ = 0.02μΩm = $0.02*10^{-6}$

P = $2*10^{-8}$Ωm.

Cross-Sectional area of the wire, A = 1mm^2 = $1*10^{-6}$m

Solution:

Mean length of the wire, l = 2000*0.8

l = 1600m.

We know that $R = \rho \frac{l}{A}$

Substituting the Values, $R = \frac{2*10^{-8}*1600}{1*10^{-6}} = 32\Omega$

Resistance of the Coil $= 32\Omega$

3. A silver wire of length 12m has a resistance of 0.2Ω. Find the specific Resistivity of the wire, if the cross-sectional area of the wire is 0.01cm^2.

$R = \frac{\rho l}{A}$ $\Rightarrow$ length,l=12m

Resistance, R=0.2Ω

A=0.01cm^2

$$\rho = \frac{RA}{l} = \frac{0.2*0.01*10^{-4}}{12}$$

$$\rho = 1.688*10^{-8}\Omega m$$

Ohm's law and its limitations:

The relationship between the potential difference (V), the current (I) and Resistance (R) in a d.c. Circuit was first discovered by the scientist George Simon ohm, is called ohm's law.

Statement:

The ratio of potential difference between any two points of a conductor to the current following between them is constant, provided the physical condition (eg. Temperature, etc.) do not change.

ie, $\frac{V}{I}$ = Constant

(or)

$$\frac{V}{I} = R \Rightarrow V = I*R$$

Where, R is the resistance between the two points of the conductor.

It can also be stated as, provided Resistance is kept constant, current is directly proportional to the potential difference across the ends of the conductor.

Power, $P = V*I = I^2R = \frac{V^2}{R}$

Limitations:

(i) Ohm's law does not applied to all non-metallic conductors. For eg. For Silicon carbide.

(ii) It also does not apply to non-linear devices such as zener diode, voltage Regulators.

(iii) Ohm's law is true for metal conductor at constant temperature. If the temperature changes the law is not applicable.

Problems based on ohm's law:

1.An electric heater draws 8A from 250V Supply. What is the power rating? Also find the resistance of the heater Element.

Given data:

Current, $I = 8A$

Voltage, $V = 250V$

Solution:

Power rating, $P = VI = 8*250 = 2000Watt$

Resistance $(\mathrm{R}) = \frac{V}{I} = \frac{250}{8} = 31.25\Omega$

2. What will be the current drawn by a lamp rated at 250V, 40Watt, connected to a 230V Supply.

Given Data:

Rated Power = 40W
Rated Voltage = 250V
Supply Voltage = 230V

Solution:

Resistance,

$$R = \frac{V^2}{P} = \frac{250^2}{40} = 1562.5\Omega$$

$$Current, I = \frac{V}{P} = \frac{230}{1562.5} = 0.1472A$$

3. A Battery has an emf of 12.8 volts and supplies a current of 3.24A. What is the resistance of the circuit? How many coulombs leave the battery in 5 minutes?

Solution:

$$\text{Current Resistance, R} = \frac{\mathrm{V}}{\mathrm{I}} = \frac{12.8}{3.24} = 4\Omega$$

Charge flowing in 5 minutes = Current × time in seconds

Charge flowing in 5 minutes = 3.2×5×60 = 960 coulomb

Combination of Resistors:

Resistances in series (or) series combination:

The circuit in which resistances are connected end to end so that there is one path for the current flow is called **series circuit**. The voltage source is connected across the free ends A and B.

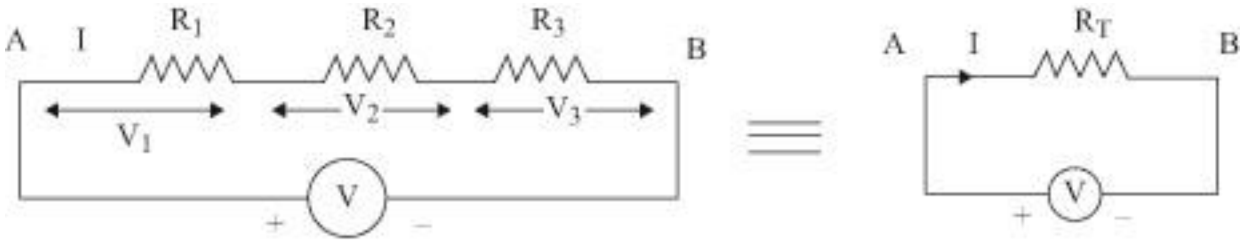


Figure 1.1

In the above circuit, there is only one closed path, so only one current flow through all the elements. In other words if the Current is same through all the resistors the combination is called series combination

To find equivalent Resistance:

Let, V = Applied voltage

I = Source current = Current through each Element

V_1, V_2, V_3 are the voltage across R_1, R_2 and R_3 respectively.

By ohms low, $V_1 = IR_1$

$$V_2 = IR_2 and\ V_3 = IR_3$$

But $V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I\left(R_1 + R_2 + R_3\right)$

$$V = I\left(R_1 + R_2 + R_3\right)$$

$$V = IR_T$$

$$\frac{V}{T} = R_T$$

The ratio of $\left(V/I\right)$ is the total resistance between points A and B and is called the total (or) equivalent resistance of the three resistances

$$R_T = R_1 + R_2 + R_3$$

Also, $\frac{1}{G_T} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$

∴ Equivalent resistance (R_T) is the sum of all individual resistances.

Concepts of series circuit:

i. The current is same through all elements.
ii. The voltage is distributed. The voltage across the resistor is directly proportional to the current and resistance.
iii. The equivalent Resistance (R_T) is greater than the greatest individual resistance of that combination.
iv. Voltage drops are additive.
v. Power are additive.
vi. The applied voltage equals the sum of different voltage drops.

Voltage Division Technique: (or) To find V_1, V_2, V_3 in terms of V and R_1, R_2, R_3:

Equivalent Resistance, $RT = R_1 + R_2 + R_3$

By ohm's low, $I = \frac{V}{RT} = \frac{V}{R_1 + R_2 + R_3}$

$$V_1 = IR_1 = \frac{V}{R_T} R_1 = \frac{VR_1}{R_1 + R_2 + R_3}$$

$$V_2 = IR_2 = \frac{V}{R_T} R_2 = \frac{VR_2}{R_1 + R_2 + R_3}$$

$$V_3 = IR_3 = \frac{V}{R_T} R_3 = \frac{VR_3}{R_1 + R_2 + R_3}$$

∴ Voltage across any resistance in the series circuit,

$$\Rightarrow V_x = \frac{R_x}{R_T} V$$

Note: If there are n resistors each value of R ohms is series the total Resistance is given by,

$$R_T = n * R$$

Applications:

* When variable voltage is given to the load, a variable Resistance (Rheostat) is connected in series with the load. Example: Fan Regulator is connected in series with the fan.
* The series combination is used where many lamp of low voltages are to be operated on the main supply. Example: Decoration lights.
* When a load of low voltage is to be Operated on a high voltage supply, a fixed value of resistance is, connected in series with the load.

Disadvantage of Series Circuit:

* If a break occurs at any point in the circuit, current coil flow and the entire circuit become useless.
* If 5 numbers of lamps, each rated 230 volts are be connected in series circuit, then the supply voltage should be 5 x 230 = 1150volts. But voltage available for lighting circuit in each and every house is only 230v. Hence, series circuit is not practicable for lighting circuits.
* Since electrical devices have different current ratings, they cannot be connected in series for efficient Operation.

Problems based on series combination:

1. Three resistors 30Ω, 25 Ω, 40 Ω are connected in series across 200v as shown in figure 1.2. Calculate (i) Total resistance (ii) Current (iii) Potential difference across each element.

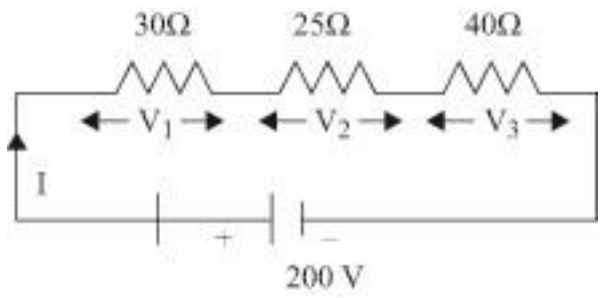


Figure 1.2

(i) Total Resistance (R_T)

(ii) Current,

(iii) Potential difference across each element,

2. An incandescent lamp is rated for 110v, 100w. Using suitable resistor how can you operate this lamp on 220v mains.

Rated current of the lamp, $I = \frac{Power}{Voltage} = \frac{100}{110}$

I = 0.909A

When the voltage across lamp is 110v, then the remaining voltage must be across R

$$
\begin{aligned}
Supply\ voltage &= V = 220 Volts \\
Voltage\ across\ R &= V - 110 Volts \\
ie,\ V_R &= 220 - 110 = 110v \\
By\ ohm's\ law,\ V_R &= IR \\
110 &= 0.909R \\
R &= 121\Omega
\end{aligned}
$$

Resistance in Parallel (or) Parallel Combination:

If one end of all the resistors is joined to a common point and the other ends are joined to another common point, the combination is said to be parallel combination. When the voltage source is applied to the common points, the voltage across each resistor will be same. Current in the each resistor is different and given by ohm's law.

Let R_1, R_2, R_3 be three resistors connected between the two common terminals A and B,

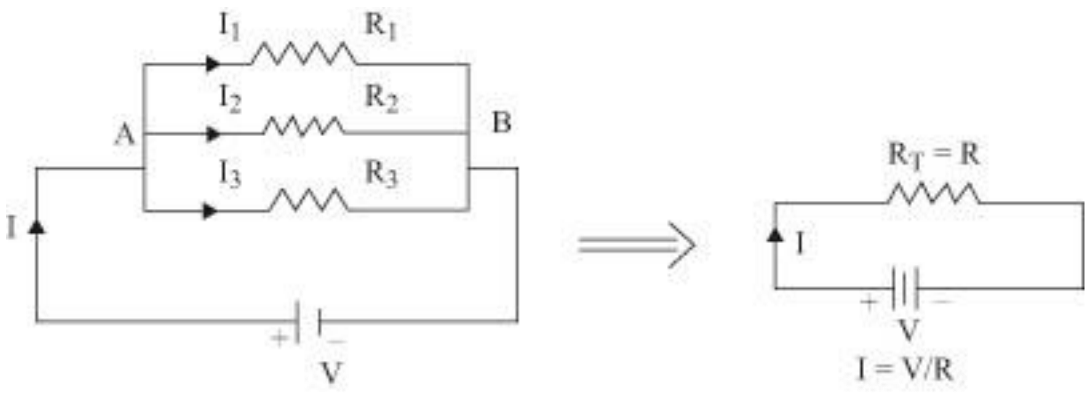


Figure 1.3

as shown in the Figure 1.3

$$I = \frac{V}{R} \text{ .... (1)}$$

Let I_1, I_2, I_3 are the currents through R_1, R_2, R_3 respectively. By ohm's law,

$$\left[I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \right] \text{................ (2)}$$

Total current is the sum of three individual currents, $I_T = I = T_1 + I_2 + I_3$ .... (3)

Substituting the above expression for the current in equation (3), $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If $R_T = R$,

Then $\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ .... (4)

Hence, in the case of parallel combination the reciprocal of the equivalent resistance is equal to the sum of reciprocals of individual resistances. Multiplying both sides of equation (4) by V^2, we get

$$\frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^3}{R_3}$$

ie,

$$\text{Power dissipated by } R_1 = \text{Power dissipated by } R_1 + \text{Power dissipated by } R_2 + \text{Power dissipated by } R_3$$

We know that reciprocal of Resistance is called as conductance.

$$\text{Conductance} = \frac{1}{\text{Resistance}} \left[G = \frac{1}{R} \right]$$

Equation (4) can be written as,

$$G = G_1 + G_2 + G_3$$

Concepts of Parallel Circuit:

- Voltage is same across all the elements.
- All elements we have individual currents, depends upon the resistance of element.
- The total resistance of a parallel circuit is always lesser than the smallest of the resistance.
- If n resistance each of R are connected in parallel, then

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \text{............} n \, terms$$

$$\frac{1}{R_T} = \frac{n}{R}$$

$$(or)\, R_T = \frac{R}{n}$$

Powers are additive.

Conductance are additive.

Branch currents are additive.

Current Division Technique:

Case (i) When two resistances are in parallel:

Two resistance $R_1 \, and \, R_2$ ohms are connected in parallel across a battery of V (volts)

Current through $R_2 \, is \, I_2$ and through $R_2 \, is \, I_2$ as shown in figure 4. The total current is I.

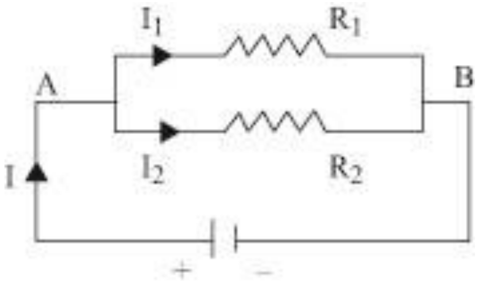


Figure 1.4

- To express I_1 and I_2 in terms of I, R_1 and R_2(or)

To find Branch Currents I_1, I_2

$$I_2R_2 = I_1R_1$$

$$I_2 = \frac{I_1R_1}{R_2} \text{ ....(5)}$$

Also, The total Current, $I = I_1 + I_2$ ....(6)

Substituting (5) in (6), $I_1 + \frac{I_1R_1}{R_2} = I$

$$\frac{I_1R_2 + I_1R_1}{R_2} = I$$

$$I_1(R \ + R_2) = IR_2$$

$$I_1 = \frac{IR_2}{(R_1 + R_2)}$$

Similarly, $I_2 = \frac{IR_1}{(R_1 + R_2)}$

To find Total equivalent Resistance, (R_T):

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_T} = \frac{R_2 + R_1}{R_1R_2}$$

$$R_T = R_1R_2$$

Hence the total value of two resistances connected parallel is equal to their product divided by their ie, Equivalent Resistance = $\frac{\text{Product of the two Resistance}}{\text{Sum of the two Resistane}}$

Case (ii) When three resistances are connected in parallel. Let R_1, R_2 and R_3 be resistors in parallel as shown in figure 1.5. Let I be the supply current (or) total curve I_1, I_2, I_3 are the current through R_1, R_2 and R_3.

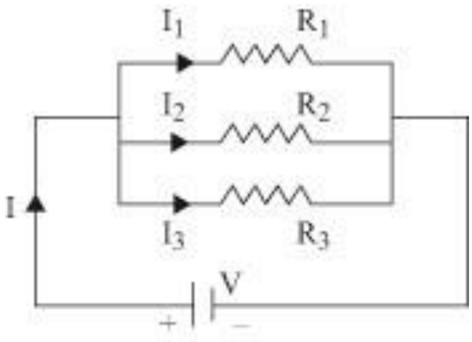


Figure 1.5

Let R_T be equivalent resistance

To find the equivalent Resistance (R_T):

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1R_2R_3}$$

$$R_T = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

To find the Branch currents I_1, I_2 and I_3:

We know that, $I_1 + I_2 + I_3 = I$ ……………(7)

Also, $I_3R_3 = I_1R_1 = I_2R_2$

From the above expression, we can get expressions for I_2 and I_3 interms of I_1 and substitute them in the equation (7)

$$I_2 = \frac{I_1R_1}{R_2}; I_3 = \frac{I_1R_1}{R_3}$$

$$I_1 + \frac{I_1R_1}{R_2} + \frac{I_1R_1}{R_3} = I$$

$$I_1\,(1+\frac{R}{R_2}+\frac{R_1}{R_3})=I$$

$$\frac{I_1\,(R_2R_3+R_3R_1+R_1R_2)}{R_2R_3}=I$$

$$I_1=\frac{I(R_2R_3)}{(R_1R_2+R_2R_3+R_3R_1)}$$

Similarly we can express I_2 and I_3 as,

$$I_2=\frac{I(R_1R_3)}{(R_1R_2+R_2R_3+R_3R_1)}$$

$$I_3=\frac{I(R_1R_3)}{(R_1R_2+R_2R_3+R_3R_1)}$$

Advantages of parallel circuits:

* The electrical appliances rated for the same voltage but different powers can be connected in parallel without affecting each other is performance.
* If a break occurs in any one of the branch circuits, it will have no effect on the other branch circuits.

Applications of parallel circuits:

* All electrical appliances are connected in parallel. Each one of them can be controlled individually will the help of separate switch.
* Electrical wiring in Cinema Halls, auditoriums, House wiring etc.

Comparison of series and parallel circuits:

Series Circuit	Parallel Circuit
The current is same through all the elements.	The current is divided,
The voltage is distributed. It is proportional to resistance.	The voltage is the same across each element in the parallel combination.
The total (or) equivalent resistance is equal	Reciprocal of resistance is equal to sum of

to sum of Individual Resistance, ie. $R_T = R_1 + R_2 + R_3$ Hence, the total resistance is greater than the greatest resistance in the circuit.	reciprocals of individual resistances, ie, $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ Total resistance is lesser than the smallest of the resistance.
There is only one path for the flow of current.	There are more than one path for the flow of current.

Problems based on parallel Combinations:

1. what is the value of the unknown resistor R in Figure 6, If the voltage drop across the 500Ω resistor is 2.5V. All the resistor are in ohms.

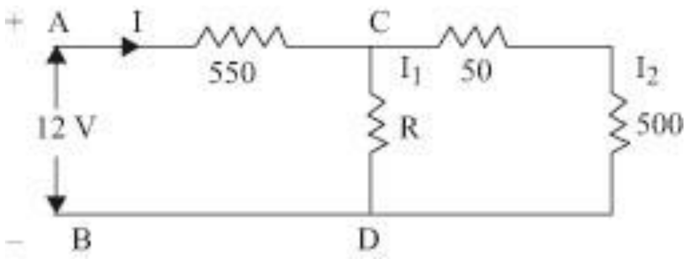


Figure1.6

Given Data:

$V_{500} = 2.5V$

$$I_2 = \frac{V_{550}}{R} = \frac{2.5}{500} = 0.005A$$

$V_{50} =$ Voltage across 50Ω

$V_{50} = IR = 0.005*50 = 0.25V$

$VCD = V_{50} + V500 = 0.25+2.5 = 2.75V$

$V_{550} =$ Drop across 550Ω = 12-2.75 = 9.25V

$$I = \frac{V_{550}}{R} = \frac{9.25}{550} = 0.0168A$$

$$I = I_1 + I_2 \rightarrow I_1 = I - I_2 = 0.0168 - 0.005$$

$$I_1 = 0.0118A$$

$$R = \frac{V_{CD}}{I} = \frac{2.75}{0.018} = 232.69\Omega$$

$$R = 232.69\Omega$$

2. Three resistors 2Ω, 3Ω, 4Ω are in parallel. How will a total current of 8A is divided in the circuit shown in figure1.7.

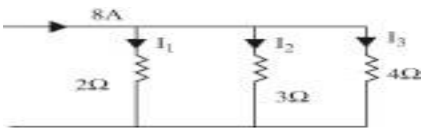


Figure1.7

3Ω and 4Ω are connected in parallel. Its equivalent resistances are, $\frac{3*4}{3+4} = \frac{12}{7} = 1.714\Omega$

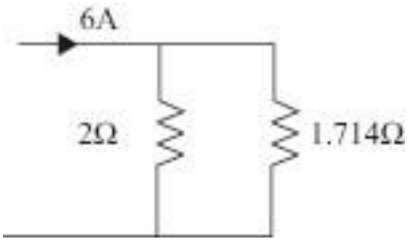


Figure1.8

1.714Ω and 2Ω are connected in parallel, its equivalent resistance is 0.923Ω

$$\frac{1.714*2}{2+1.714} = 0.923\Omega$$

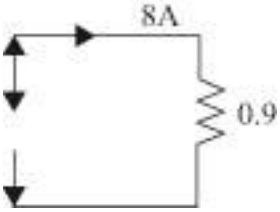


Figure 1.9

$$V = IR = 8*0.923$$
$$V = 7.385V$$

Branch Currents, $I_1 = \frac{V}{R_1} = \frac{7.385}{2} = 3.69$

$$I_2 = \frac{V}{R_1} = \frac{7.385}{3} = 2.46A$$

$$I_3 = \frac{V}{R_3} = \frac{7.385}{4} = 1.84A$$

Total current, I = 8A is divided as 3.69A, 2.46A, 1.84A.

3. what resistance must be connected in parallel with 10Ω to give an equivalent resistance of 6Ω R is connected in parallel with 10Ω Resistor to given an equivalent Resistance of 6Ω.

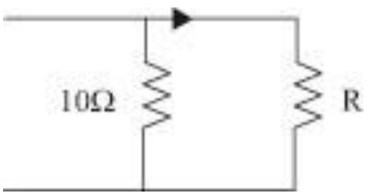


Figure 1.10

$$\frac{10 * R}{10 + R} = 6$$

$$10R = (10 + R)6$$
$$10R = 60 + 6R$$

$$10R - 6R \rightarrow 4R = 60$$

$$R = \frac{60}{4} = 15\Omega$$

$$R = 15\Omega$$

4. Calculate the current supplied by the battery the given circuit as shown in the figure 1.11.

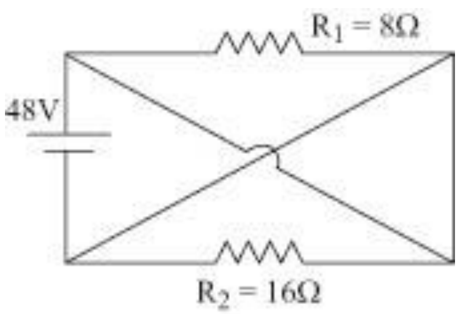


Figure 1.11

Solution: The above given Circuit can be redrawn as as shown in figure 1.12.

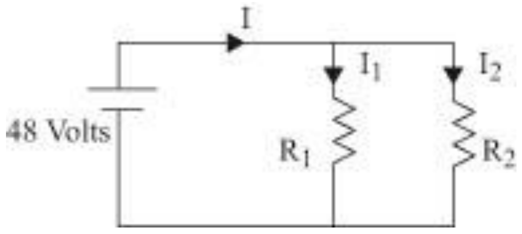


Figure 1.12

R_1 and R_2 are in Parallel across the voltage of 48volts.

Equivalent Resistance, $R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{8*16}{8+16} = \frac{16}{3}\Omega$

$$R_T = 5.33A$$

$$I = \frac{V}{R} = \frac{48}{5.33} = 9A$$

5. Calculate the total resistance and battery current in the given circuit shown in figure 1.13.

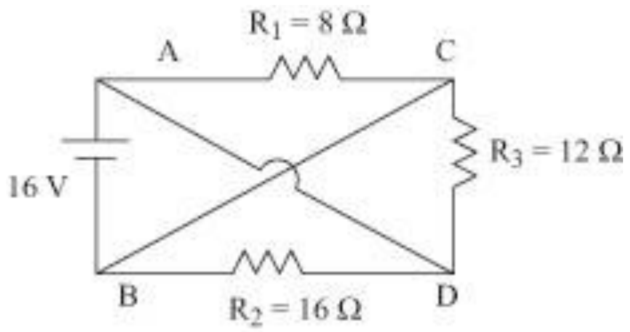


Figure1.13

The given above circuit can be re-drawn as shown in figure1.14.

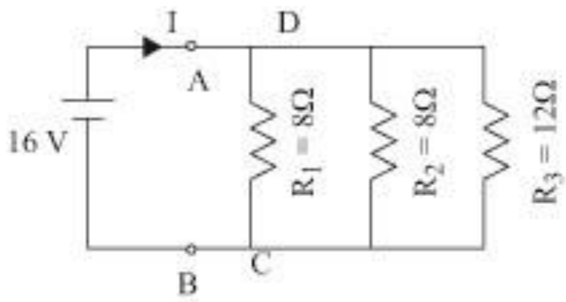


Figure1.14

8Ω, 16Ω, 12Ω are connected in parallel. Its equivalent Resistance, $R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$

$$R_T = \frac{8*6*12}{128+192+96} = 3.692\Omega$$

$$R_T = 3.692\Omega$$

$$I = \frac{V}{R} = \frac{16}{3.692} = 4.33A$$

Series — Parallel Combination

As the name suggests, this Circuit is a Combination of series and parallel Circuits. A simple example of such a Circuit is illustrated in Figure1.15. R_3 and R_2 are resistors connected in parallel with each other and both together are connected in series with R_1.

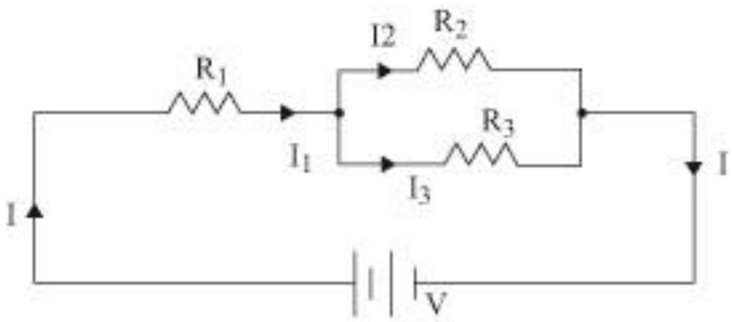


Figure 1.15

Equivalent Resistance: R_T for parallel combination.

$$R_p = \frac{R_2 R_3}{R_2 + R_3}$$

Total equivalent resistance of the Circuit is given by,

$$R_T = R_1 + R_P$$

$$R_T = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

Voltage across parallel Combination = $I * \frac{R_2 R_3}{R_2 + R_3}$

Problems based on Series – Parallel Combination:

1. In the Circuit shown in figure 1.16, find the Current in all the resist Also calculate the supply voltage.

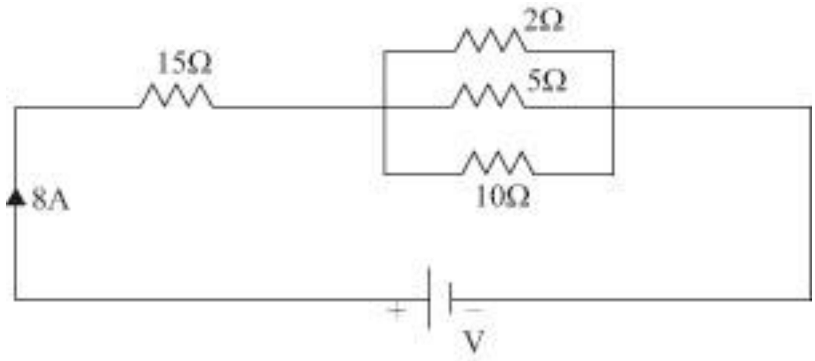


Figure 1.16

Solution: V=130 Volts.

Current through 2Ω resistor, is 5A.

Current through 5Ω Resistor, 2A

Current through 10Ω Resistor, 1A.

1. Find the equivalent resistance between the terminals A and B in the figure 1.17.

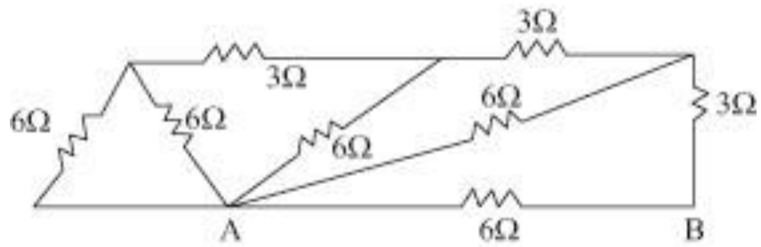


Figure 1.17

Solution: R_{AB} =3Ω.

3. Determine the value of R if the power dissipated in 10Ω Resistor is 90W in figure 1.18.

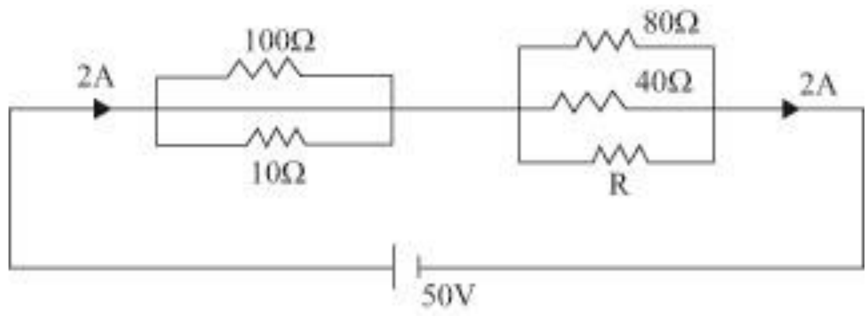


Figure1.18

Solution: R = 39.4Ω.

Kirchhoff's Law

Circuit Basics

$\boxed{V=IR}$ – Ohm's Law; applies to both the entire circuit and any resistive element within it.

The voltage across any two elements in parallel is the same;

The current through any two elements in series is the same.

The voltage across elements in series adds up to the total voltage;

The current through any elements in parallel add up to the total current.

$\boxed{R=\frac{\rho L}{A}}$; Resistance of a conductor of length L, area, A and resistivity, □.

$\boxed{\begin{aligned} R_s &= R_1 + R_2 + \ldots \\ \frac{1}{R_p} &= \frac{1}{R_1}+\frac{1}{R_2}+\ldots \end{aligned}}$ Resistors in series and parallel;

$\boxed{Q=CV}$ - definition of capacitance; applies to the entire circuit and any capacitor within the circuit;

The charge on any capacitors in series is the same and the voltage across these elements adds up to the total voltage; the charge on any capacitors in a parallel adds up to the total charge and the voltage across these elements is the same.

$\boxed{C=\frac{\kappa\varepsilon_o A}{d}}$ - Capacitance of a parallel plate capacitor of area A, separation d and dielectric.

$\boxed{\begin{aligned} C_p &= C_1 + C_2 + \ldots \\ \frac{1}{C_s} &= \frac{1}{C_1}+\frac{1}{C_2}+\ldots \end{aligned}}$ Capacitors in series and parallel;

$\boxed{P=IV}$ - the power dissipated or generated by any element is equal to the current through it multiplied by the voltage across it.

Kirchhoff's Rules

- Used when a circuit is not a simple series and parallel combination of resistors;
- Often used when there is more than one voltage source in a circuit.

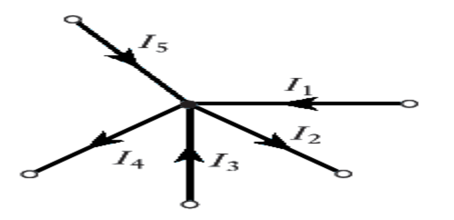
Kirchhoff's First Law:

The algebraic sum of the currents flowing through a junction is zero.

Sum of the Currents approaching the junction are equal to the sum of the currents going away from the junction are Equal.

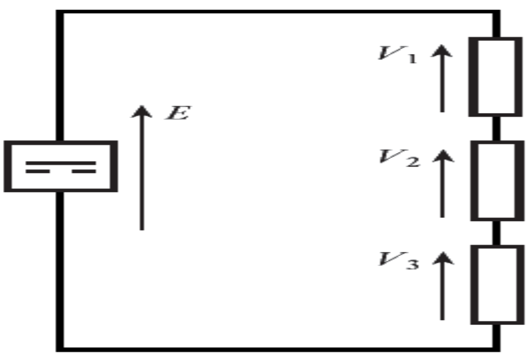
Kirchhoff's Second Law:

The algebraic sum of the potential differences in a circuit loop must be zero. Potential rises are equal to potential drops in a closed circuit.



$I_1 - I_2 + I_3 - I_4 + I_5 = 0$

Kirchhoff's First Law



$E - V_1 - V_2 - V_3 = 0$

Kirchhoff's Second Law

Figure 1.19 Figure 1.20

Point Rule: the sum of all the currents into any point in the circuit is equal to the sum of all of the currents out of the same point. $\boxed{\sum I_{in} = \sum I_{out}}$

Loop Rule: the sum of the voltage drops around any closed loop is zero. $\boxed{\sum \Delta V = 0}$

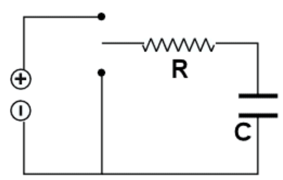
Problem Solving Strategy:

1) Label all junctions and corners in order to make solution easier.
2) Guess current directions in each branch of the circuit.
3) Apply point rule to n-1 junctions where n is the number of junctions.
4) Apply loop rule to as many loops as necessary such the total number of equations is equal to the number of unknown currents.
   a. When going with the current, $\Delta V = -IR$ through a resistor; when going against the current, $\Delta V = +IR$
   b. When going from the positive terminal to the negative terminal of a battery, □V=-V and when going from negative to positive terminal, □V=+V.
5) Solve the system of equations for the unknown currents.

Rarely are the students asked to solve an entire system on an AP exam, but they are often asked to apply these principles to parts of circuits on multiple choice problems.

RC Circuits

The simplest RC circuit contains a single resistor R in series with a capacitor C and a battery of voltage V.



Apply Kirchhoff's loop rule to the loop.

Figure 1.21

$V = V_R + V_C$

$V = iR + \frac{q}{C}$ - substituting Ohm's Law and the definition of capacitance;

$V = R\frac{dq}{dt} + \frac{q}{C}$ - substituting the relationship between current and charge;

Solve the differential equation by method of separation of variables:

$$\frac{dq}{VC - q} = \frac{dt}{RC}$$

$$\int \frac{dq}{VC - q} = \int \frac{dt}{RC}$$

$$-\ln(VC - q) + const = t / RC$$

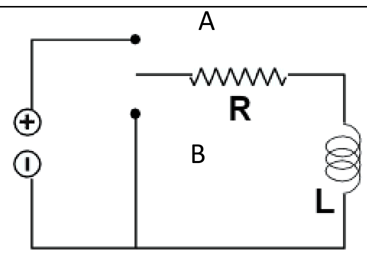
$q(t) = VC(1 - e^{-t/RC})$ The constant comes from the initial conditions: q=0 at t=0.

For a discharging circuit, the method of solution is similar: the only difference is that V=0. Plug that in, make the same substitutions as above, solve by separation of variables and the result is

$q(t) = VCe^{-t/RC}$.

In circuits with more than one branch, or a resistor in parallel with the capacitor, it helps to teach the students about the initial and steady state behavior of the capacitor. When the capacitor is empty, it offers no "resistance" to the flow of current; when it is full, no current will flow in the branch with the capacitor in it.

RL Circuits



The picture at right represents a resistor in series with an inductor and a battery.

The switch is initially moved to position A and the inductor is "charged."

Figure 1.22

The voltage across the inductor varies (according to Faraday's Law) with the rate of change of current according to the equation $V_L = L\frac{di}{dt}$ (magnitude only, the sign due to Lenz's law has been omitted).

Again we start with Kirchhoff's Loop Rule.

$V = V_R + V_L$

$$V = iR + L\frac{di}{dt}$$

Solve by separation of variables:

$$V - iR = L\frac{di}{dt}$$
$$\frac{di}{V/R - i} = \frac{Rdt}{L}$$
$$\int \frac{di}{V/R - i} = \int \frac{Rdt}{L}$$

$-\ln(\frac{V}{R} - i) = \frac{Rt}{L} + const$ - the constant comes from the initial conditions: i=0 at t=0.

$$i(t) = \frac{V}{R}(1 - e^{-Rt/L})$$

When the switch is flipped to position B, the inductor begins to "discharge." Again, for the discharging equation, simply start with V=0 in the loop equation as above. Then solve by separation of variables as above to get the equation $i(t) = \frac{V}{R}e^{-Rt/L}$ where the initial current in the inductor is the same as it was just before the switch was opened causing the inductor to discharge.

Again, for more complex circuits involving more than one resistor in a circuit with an inductor, either in series or parallel with the inductor, it is important for the students to understand the initial behavior of the circuit after a switch is moved, and the long-term behavior after the switch is moved (either opened or closed). Initially, it is useful for the students to think of the inductor as an open switch: it prevents any sudden changes in current so if there was no current just before the switch is moved, there will be no current immediately after. However, the ideal inductor behaves like a wire when the current has reached a steady state. These two situations will help us determine the current in any part of the circuit just after a switch is moved and in steady state conditions.

Problems on Kirchhoff's Law:

- Solve this circuit, find the value of 5 branch currents using Kirchhoff's laws and elimination method (or maybe called elimination by substitution BUT not using any matrix method)

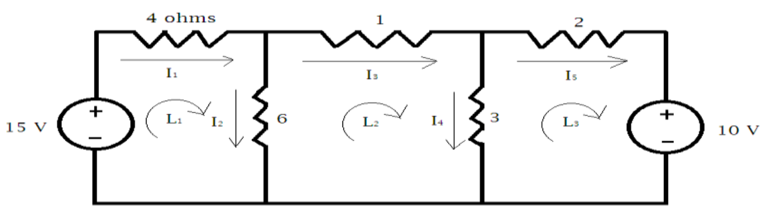


Figure 1.23

Solution:

Loop 1:15−4I1−6I2=0
Loop 2:6I2−I3−3I4=0
Loop 3:3I4−2I5−10=0

I1=1.89
I2=1.24
I3=0.65
I4=2.26
I5=−1.61

- Determine the value current in 40 Ohms resistance

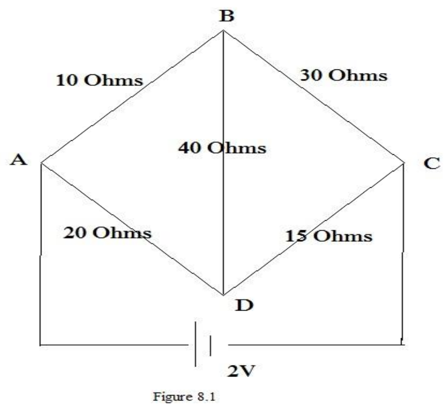


Figure 1.24

Solution:

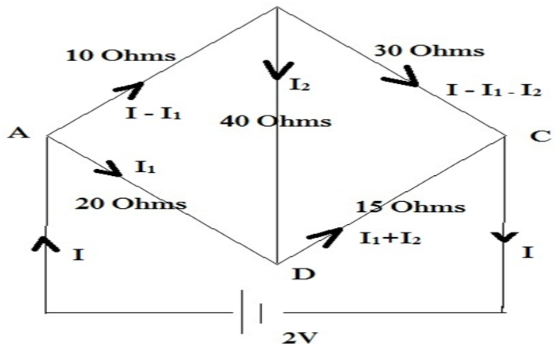


Figure 1.25

Note that there are three unknown variables (I , I1 , I2) and therefore we need to solve three equations simultaneously to find this unknown variables.

Apply KCL to Circuit ABDA

(I - I1)*10 + I2*40 - 20I = 0

10*I - 30*I1 +40*I2 = 0 -------(1)

Apply KCL to Circuit BCDB

(I - I1 - I2)*30 - (I1+I2)*15 - I2*40 = 0

30*I - 45*I1 - 85*I2 = 0 --------------(2)

Apply KCL to Circuit ADCEA

I1*20 + (I1+I2)*15 = 2

35*I1 + 15*I2 = 2 -------(3)

Now solve these three equations

I = 87/785 A

I1 = 41/785 A

I2 = 9/785 A

So the current in 40 Ohms resistance is 9/785 A From B to D

- Determine the current flowing in each of the batteries and the voltage difference between 10 Ohms resistance. Refer the following figure.

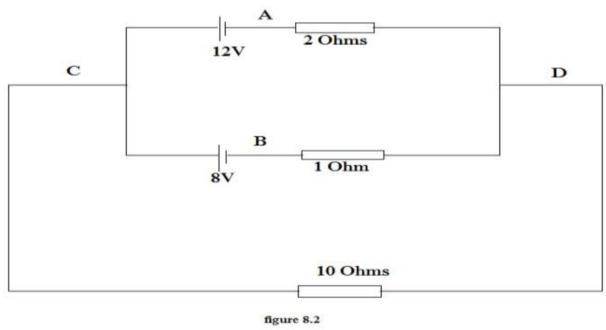


Figure 1.26

First we have to apply KCL 1 to the network.

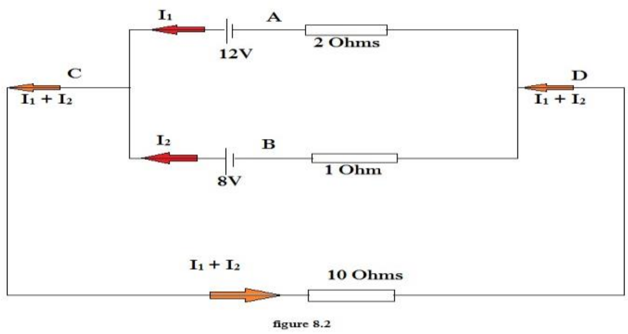


Figure 1.27

Applying KCL to loop CEDAC

(I1+I2)*10 + I1*2 = 12

12*I1 + 10*I2 = 12 ----------(1)

Applying KCL to loop CEDBC

(I1+I2)*10 + I2*1 = 8

10*I1 + 11*I2 = 8-------------(2)

By solving (1) & (2)

I1 = 1.625A

I2 = -0.750A

Current through 10 Ohms resistance is = I1 + I2 = 0.875A

Voltage across 10 Ohms resistance is = 0.875A x 10 Ohms = 8.750V

- Determine the current in 4 Ohms resistance.

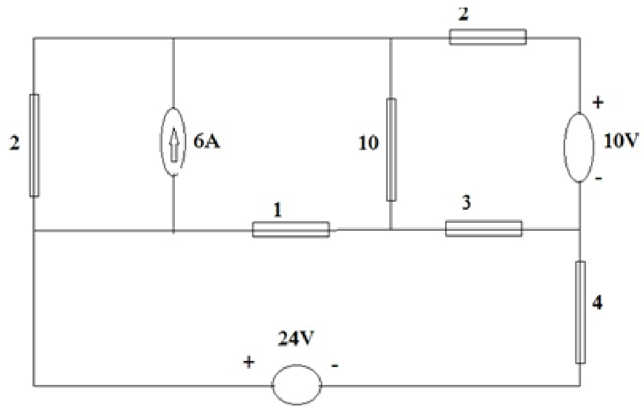


Figure 1.28

Note: This Network has a current source. So this question will teach you something new. To study about Current sources, see my previous post about "voltage and current sources".

Solution:

First we have to apply KCL 1 to the network

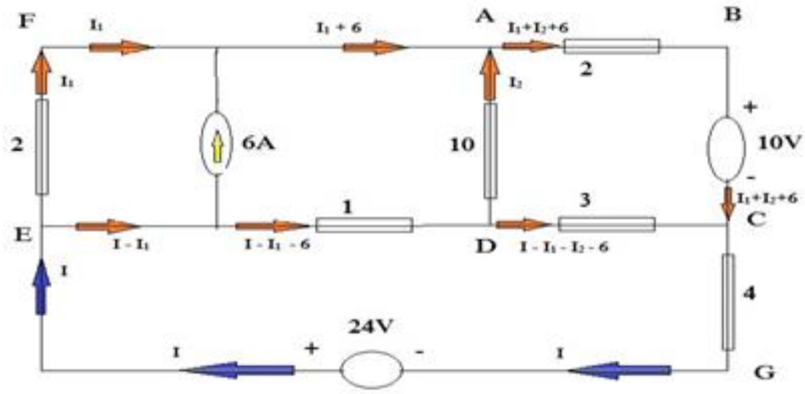


Figure 1.29

Applying KCL to loop EFADE

I1*2 - I2*10 - (I - I1 - 6)*1 = 0

I - 3*I1 + 10*I2 = 6 ------------(1)

Applying KCL to loop ABCDA

(I1+I2+6)*2 - 3*(I - I1 - I2 - 6) + I2*10 = -10

3*I - 5*I1 +15*I2 = 6 ---------(2)

Applying KCL to loop EDCGE

(I-I1-6)*1 + (I-I1-I2-6)*3 + I*4 = 24

8*I - 4*I1 3*I2 = 48 ----------(3)

By solving these three equations

I = 4.1A

So the current in 4Ohms resistance = 4.1A . Once we find the mesh currents we can use them to calculate any other currents or voltages of interest.

MESH-CURRENT METHOD

- Mesh-Current method is developed by applying KVL around meshes in the circuit.
- A mesh is a loop which doesn't contain any other loops within it.

- Loop (mesh) analysis results in a system of linear equations which must be solved for unknown currents.
- Reduces the number of required equations to the number of meshes.
- Can be done systematically with little thinking.
- As usual, be careful writing mesh equations – follow sign convention.
- Powerful analysis method which applies KVL to find unknown currents.
- It is applicable to a circuit with no branches crossing each other.

The various steps involved in Mesh method

1. Clearly label all circuit parameters and distinguish the unknown parameters from the known.
2. Identify all meshes of the circuit.
3. Assign mesh currents and label polarities.
4. Apply KVL at each mesh and express the voltages in terms of the mesh currents.
5. Solve the resulting simultaneous equations for the mesh currents.
6. Now that the mesh currents are known, the voltages may be obtained from Ohm's law.

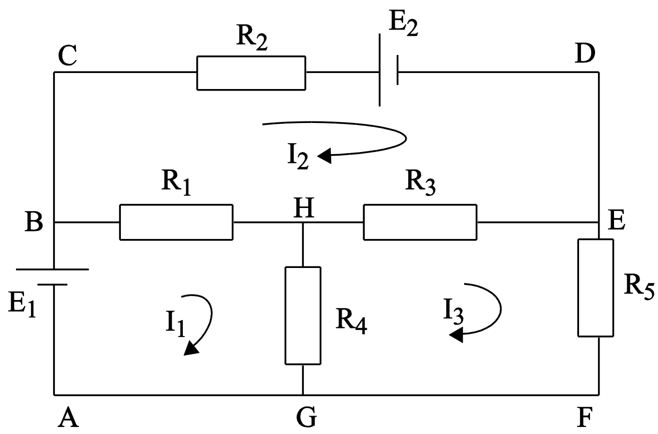


Figure 1.30

In the above figure we shall name the three loop currents I_1 , I_2 and I_3. The directions of the loop current are arbitrarily chosen. Note that the actual current flowing through R_4 is (I_1- I_3) in a downward direction and R_1 is (I_1- I_2) from left to Right.

Apply KVL for the first loop ABHGA,

$E_1 - R_1 (I_1 - I_2) - R_4 (I_1 - I_3) = 0$

$R_1 (I_1 - I_2) + R_4 (I_1 - I_3) = E_1$

$\therefore (R_1 + R_4) I_1 - R_1 I_2 - R_4 I_3 = E_1$…………….. (1)

Apply KVL for the loop BEDC,

$-R_2 I_2 - E_2 - R_3 (I_2 - I_3) - R_1 (I_2 - I_1) = 0$

$R_2 I_2 + R_3 (I_2 - I_3) + R_1 (I_2 - I_1) = -E_2$

$\therefore -R_1 I_1 + (R_1 + R_2 + R_3) I_2 - R_3 I_3 = -E_2$ ……………. (2)

Apply KVL for the loop HEFGH,

$R_3 (I_3 - I_2) + R_5 I_3 + R_4 (I_3 - I_1) = 0$

$-R_4 I_1 - R_3 I_2 + (R_3 + R_4 + R_5) I_3 = 0$ ……………. (3)

Equation (1) to (3) can be arranged in a matrix form as,

$$\begin{bmatrix} R_1 + R_4 & -R_1 & -R_4 \\ -R_1 & R_1 + R_2 + R_3 & -R_3 \\ -R_4 & -R_3 & R_3 + R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ -E_2 \\ 0 \end{bmatrix}$$

In general equation (4) will be written as

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{……………. (5)}$$

It can be seen that the diagonal elements of the matrix is the sum of the resistances of the mesh, where as the off diagonal elements are the negative of the sum of the resistance common to the loop.

Thus,

R_{ii} = the sum of the resistances of loop **i**.

$$R_{ij} = \begin{cases} -\sum(\text{Resistance common to the loop i and loop j,} \\ \quad \text{if } I_i \text{ and } I_j \text{ are in opposite direction in common resistances)} \\ +\sum(\text{Resistance common to the loop i and loop j,} \\ \quad \text{if } I_i \text{ and } I_j \text{ are in same direction in common resistances)} \end{cases}$$

The above equation is only true when all the mesh currents are taken in clockwise direction. The sign of voltage vector is decided by the considered current direction. If the mesh current is entering into the positive terminal of the voltage source, the direction of voltage vector elements will be negative otherwise it will be positive.

Equation (5) can be solved by Cramer's rule as

$$\Delta_1 = \begin{bmatrix} E_1 & R_{12} & R_{13} \\ E_2 & R_{22} & R_{23} \\ E_3 & R_{32} & R_{33} \end{bmatrix}; \quad \Delta_2 = \begin{bmatrix} R_{11} & E_1 & R_{13} \\ R_{21} & E_2 & R_{23} \\ R_{31} & E_3 & R_{33} \end{bmatrix};$$

$$\Delta_3 = \begin{bmatrix} R_{11} & R_{12} & E_1 \\ R_{21} & R_{22} & E_2 \\ R_{31} & R_{32} & E_3 \end{bmatrix}; \quad \Delta = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta}; \quad I_2 = \frac{\Delta_2}{\Delta}; \quad I_3 = \frac{\Delta_3}{\Delta}$$

Problems:

1) Find the branch currents of fig using Mesh current method

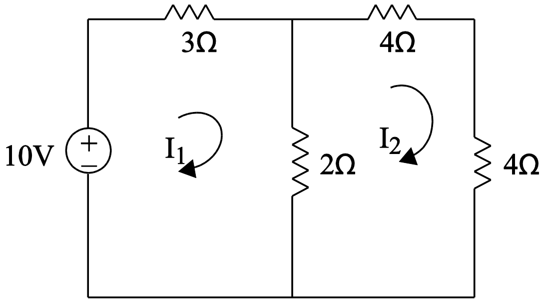


Figure 1.31

Solution

R_{11} = Sum of resistances of loop 1 = 3+2 =5Ω

R_{12} = - (common resistance between loop 1 and loop 2) = -2Ω

= R_{21}

R_{22} = Sum of resistance is loop 2 = 4+ 4+ 2 =10

E_1 = 10V

E2 = 0

$$\begin{bmatrix} 5 & -2 \\ -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5 & -2 \\ -2 & 10 \end{vmatrix} = 50 - 4 = 46$$

$$\Delta_1 = \begin{vmatrix} 10 & -2 \\ 0 & 10 \end{vmatrix} = 100$$

$$\Delta_2 = \begin{vmatrix} 5 & 10 \\ -2 & 0 \end{vmatrix} = 20$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100}{46} = 2.174A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{20}{46} = 0.435A$$

2) Find the current in 3Ω resistor using Mesh Analysis.

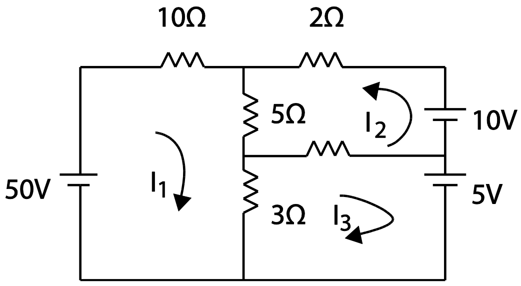


Figure 1.32

Solution

For loop 1,

$10I_1 + 5(I_1 + I_2) + 3(I_1 - I_3) = 50$

$18I_1 + 5I_2 - 3I_3 = 50$ ……………….. (1)

For loop 2,

$2I_2 + 5(I_2 + I_1) + 1(I_2 + I_3) = 10$

$5I_1 + 8I_2 + I_3 = 10$ ………………. (2)

For loop 3,

$3(I_3 - I_1) + 1(I_3 + I_2) = -5$

$-3I_1 + I_2 + 4I_3 = -5$ ………………… (3)

$$\begin{bmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 10 \\ -5 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{bmatrix}$$

$$= 18\,(32-1)\ -5\,(20+3)\ -3(5+24)$$

$$= 356$$

$$\Delta_3 = \begin{vmatrix} 18 & 5 & 50 \\ 5 & 8 & 10 \\ -3 & 1 & -5 \end{vmatrix}$$

$$= 18\,(-40-10)\ -5\,(-25+30)\ +\ 50(5+24)$$

$$= -900 - 25 + 1450$$

$$= 525$$

$$I_3 = \frac{\Delta I_3}{\Delta} = \frac{525}{356} = 1.47A$$

3) Determine the currents in various elements of the bridge circuit as shown below.

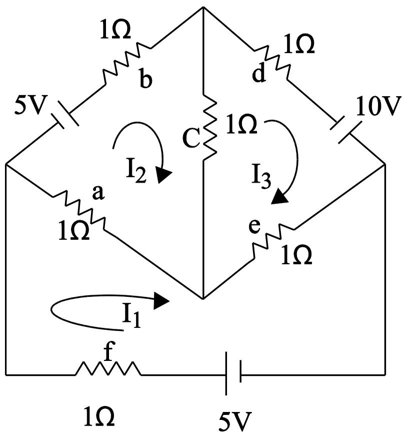


Figure 1.33

Solution

For loop 1,

$1I_1 + 1(I_1 - I_2) + 1(I_1 - I_3) = 5$

$3I_1 - I_2 - I_3 = 5$ ……………….. (1)

For loop 2,

$1I_2 + 1(I_2 - I_3) + 1(I_2 - I_1) = 5$

$-I_1 + 3I_2 - I_3 = 5$………………. (2)

For loop 3,

$1I_3 + 1(I_3 - I_1) + 1(I_3 - I_2) = 10$

$-I_1 - I_2 + 3I_3 = 10$ ………………… (3)

$$\rightarrow \begin{bmatrix} -3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}$$

$= 3(9-1) + 1(-3-1) - 1(1+3)$

$= 16$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & -1 \\ 5 & 3 & -1 \\ 10 & -1 & 3 \end{vmatrix}$$

$= 40 + 25 + 35$

$= 100$

$$\Delta_2 = \begin{vmatrix} 3 & 5 & -1 \\ -1 & 5 & -1 \\ -1 & 10 & 3 \end{vmatrix}$$

= 3 (15 + 10) -5 (-3 - 1) -1 (-10 + 5)

= 100

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 5 \\ -1 & 3 & 5 \\ -1 & -1 & 10 \end{vmatrix}$$

= 3 (30 + 5) + 1 (-10 + 5) + 5 (1 + 3)

= 120.

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100}{16} = 6.25A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{100}{16} = 6.25A$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{120}{16} = 7.5A$$

$I_a = I_1 - I_2$ = 6.25 - 6.25 = 0 A

$I_b = I_2$ = 6.25 A.

$I_c = I_2 - I_3$ = 6.25-7.5 = -1.25 A

$I_d = I_3$ = 7.5 A

$I_e = I_1 - I_3$ = 6.25-7.5 = -1.25 A.

$I_f = I_1$ = 6.25 A.

NODE ANALYSIS

This method is mainly based on Kirchhoff's Current Law (KCL). This method uses the analysis of the different nodes of the network. Every junction point in a network, where two or more branches meet is called a node. One of the nodes is assumed as reference node whose potential is assumed to be zero. It is also called zero potential node or datum node. At other nodes the different voltages are to be measured with respect to this reference node. The reference node should be given a number zero and then the equations are to be written for all other nodes by applying KCL. The advantage of this method lies in the fact that we get (n – 1) equations to solve if there are 'n' nodes. This reduces calculation work.

STEPS FOR THE NODE ANALYSIS

1. Choose the nodes and node voltages to be obtained.
2. Choose the currents preferably leaving the node at each branch connected to each node.
3. Apply KCL at each node with proper sign convention.
4. If there are super nodes, obtain the equations directly in terms of node voltages which are directly connected through voltage source.
5. Obtain the equation for the each branch current in terms of node voltages and substitute in the equations obtained in step 3.
6. Solve all the equations obtained in step 4 and step 5 simultaneously to obtain the required node voltages.

CASE I.

Consider the below fig. Let the voltages at nodes a and b be V_a and V_b. Applying Kirchoff's current law (KCL) at node a we get

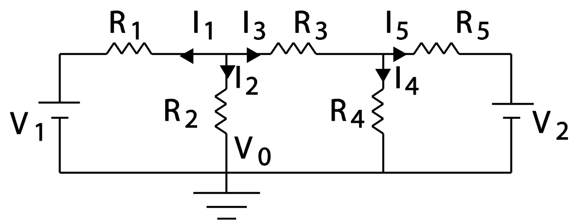


Figure 1.34

$I_1 + I_2 + I_3 = 0$ …………….. (1)

Where

$$I_1 = \frac{V_a - V_1}{R_1}; I_2 = \frac{V_a - V_0}{R_2}; I_3 = \frac{V_a - V_b}{R_3};$$

Substituting in equ .(1)

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_0}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

On simplifying [V_o = 0]

$$\frac{V_a}{R_1} - \frac{V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a}{R_3} - \frac{V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - V_b\left[\frac{1}{R_3}\right] = \frac{V_1}{R} \quad \ldots\ldots\ldots\ldots\ldots.. (2)$$

Similarly for node b we have

$$I_4 + I_5 = I_3 \quad \ldots\ldots\ldots\ldots\ldots.. (3)$$

$$I_4 = \frac{V_b - V_o}{R_4} ; I_5 = \frac{V_b - V_2}{R_5}$$

On substituting in equ (3)

$$\frac{V_b - V_o}{R_4} + \frac{V_b - V_2}{R_5} = \frac{V_a + V_b}{R_3}$$

WKT

V_o = 0 [reference node]

$$V_b\left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right] - V_a\left[\frac{1}{R_3}\right] = \frac{V_2}{R_5} \quad \ldots\ldots\ldots\ldots\ldots.. (4)$$

Solving equations (2) and (4) we get the values as V_a and V_b.

Method for solving V_a and V_b by Cramers rule.

$$\begin{pmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{pmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} \frac{V_1}{R_1} \\ \frac{V_2}{R_2} \end{bmatrix}$$

$$\Delta = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) - \left(\frac{-1}{R_3}\right)\left(\frac{-1}{R_3}\right)$$

To find Δ_1

$$\begin{pmatrix} \frac{V_1}{R_1} & \frac{-1}{R_3} \\ \frac{V_2}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}$$

$$\Delta_1 = \left(\frac{V_1}{R_1}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right) - \left(\frac{-1}{R_3}\right)\left(\frac{V_2}{R_5}\right)$$

To find Δ_2,

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{V_1}{R_1} \\ \frac{-1}{R_3} & \frac{V_2}{R_5} \end{pmatrix}$$

$$\Delta_2 = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{V_2}{R_5}\right) - \left(\frac{-1}{R_3}\right)\left(\frac{V_1}{R_1}\right)$$

To find v_a: To find v_b:

$$V_a = \frac{\Delta_1}{\Delta}; \qquad V_b = \frac{\Delta_2}{\Delta}$$

Hence V_a and V_b are found.

CASE II:

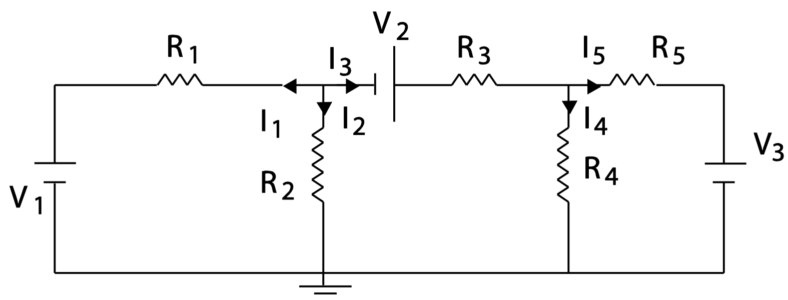


Figure 1.35

Consider the above fig

Let the voltages at nodes a and b be V_a and V_b.

The node equation at node a are

$$I_1 + I_2 + I_3 = 0$$

Where $I_1 = \frac{V_a - V_1}{R_1}$; $I_2 = \frac{V_a}{R_2}$; $I_3 = \frac{V_a + V_2 - V_b}{R_3}$

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a + V_2 - V_b}{R_3} = 0$$

Simplifying

$$\frac{V_a}{R_1}-\frac{V_1}{R_1}+\frac{V_a}{R_2}+\frac{V_a}{R_3}+\frac{V_2}{R_3}-\frac{V_b}{R_3}=0$$

Combining the common terms.

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-V_b\left[\frac{1}{R_3}\right]=\frac{V_1}{R_1}-\frac{V_2}{R_3} \quad \text{……………. (5)}$$

The nodal equations at node b are

$$I_3 = I_4 + I_5$$

$$\frac{V_a+V_2-V_b}{R_3}=\frac{V_b}{R_4}+\frac{V_b-V_3}{R_5}$$

On simplifying

$$\frac{V_a}{R_3}+\frac{V_2}{R_3}-\frac{V_b}{R_3}=\frac{V_b}{R_4}+\frac{V_b}{R_5}-\frac{V_3}{R_5}$$

$$\frac{V_a}{R_3}-V_b\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]=-\frac{V_3}{R_5}-\frac{V_2}{R_3}$$

$$-\frac{V_a}{R_3}+V_b\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]=\frac{V_3}{R_5}+\frac{V_2}{R_3} \quad \text{……………. (6)}$$

Solving equ (5) and (6) we get V_a and V_b

Method to solve V_a and V_b.

Solve by cramers rule.

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{-1}{R_3} \\ \frac{-1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}\begin{bmatrix} V_a \\ V_b \end{bmatrix}=\begin{bmatrix} \frac{V_1}{R_1}-\frac{V_2}{R_3} \\ \frac{V_2}{R_3}+\frac{V_3}{R_5} \end{bmatrix}$$

$$\Delta=\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(-\frac{1}{R_3}\right)$$

$$\Delta_1=\begin{pmatrix} \frac{V_1}{R_1}-\frac{V_2}{R_3} & -\frac{1}{R_3} \\ \frac{V_2}{R_3}+\frac{V_3}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}$$

$$\left(\frac{V_1}{R_1}-\frac{V_2}{R_3}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(\frac{V_2}{R_3}+\frac{V3}{R_5}\right)$$

$$\Delta_2 = \begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{V_1}{R_1}-\frac{V_2}{R_3} \\ \frac{-1}{R_3} & \frac{V_2}{R_3}+\frac{V_3}{R_5} \end{pmatrix}$$

$$\Delta_2 = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{V_2}{R_3}+\frac{V3}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(\frac{V_1}{R_1}-\frac{V_2}{R_3}\right)$$

$$\Delta_a = \frac{\Delta_1}{\Delta}; \qquad \Delta_b = \frac{\Delta_2}{\Delta}$$

Hence V_a and V_b are found.

Case iii

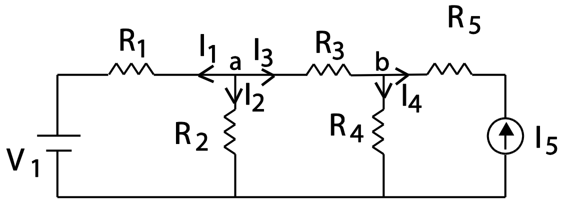


Figure 1.36

Let the voltages at nodes a and b be V_a and V_b as shown in fig

Node equations at node a are

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_2}\right] - V_b\left[\frac{1}{R_3}\right] = \frac{V_1}{R_1} \text{ .... (7)}$$

Similarly Node equations at node b

$$I_3 + I_5 = I_4$$

$$\frac{V_a - V_b}{R_3} + I_5 = \frac{V_b}{R_4}$$

$$I_5 = V_b\left[\frac{1}{R_3}+\frac{1}{R_4}\right] - V_a\left[\frac{1}{R_3}\right] \text{ ....(8)}$$

Solving eqn (7) and (8)

V_a and V_b has been found successfully.

Problems

1) Write the node voltage equation and calculate the currents in each branch for the network.

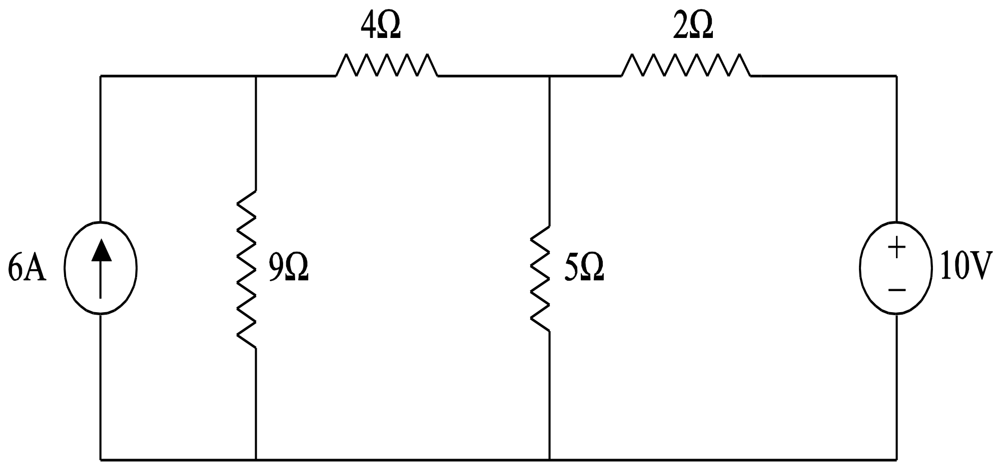


Figure 1.37

Step 1: To assign voltages at each node

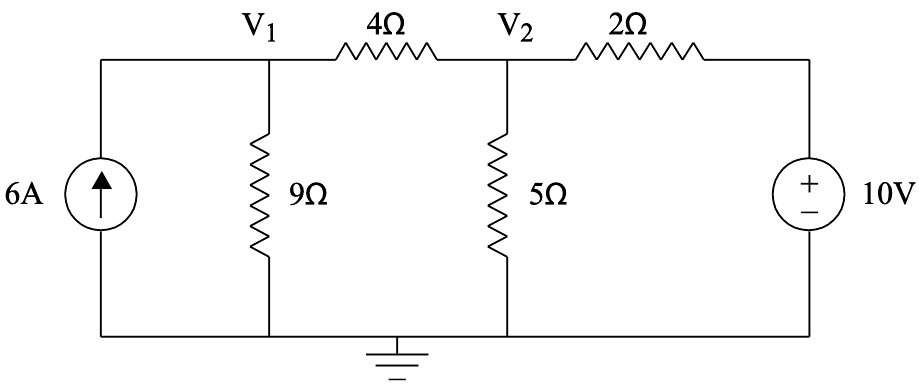


Figure 1.38

V_1 & V_2 are active nodes

V_3 is a reference node on datum node.

Hence $V_3 = 0$.

Step 2: Mark the current directions in all the branches.

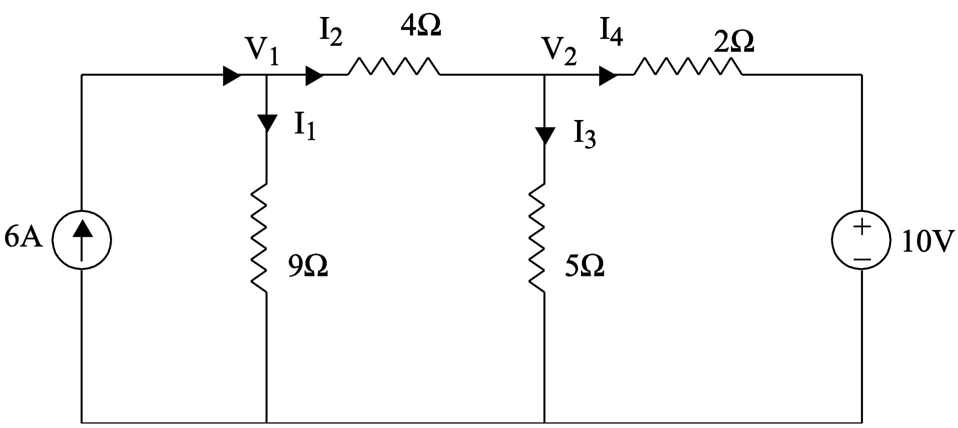


Figure 1.39

Step 3: Write the node equations for node (1) and (2)

Node 1

$I_1 + I_2 = 6$

$$\frac{v_1}{9} + \frac{v_1 - v2}{4} = 6$$

$$V_1\left[\frac{1}{9} + \frac{1}{4}\right] - V_2\left[\frac{1}{4}\right] = 6 \quad \ldots\ldots\ldots\ldots\ldots \text{(1)}$$

Node 2:

$I_2 = I_3 + I_4$

$$\frac{V_1 - V_2}{4} = \frac{V_2}{5} + \frac{V_2 - 10}{2}$$

$$V_1\left[\frac{1}{4}\right] = V_2\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2}\right] - \frac{10}{2}$$

$$-V_1\left[\frac{1}{4}\right] + V_2\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2}\right] = \frac{10}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Step 4: Solving equ (1) and (2) and finding V_1 and V_2 by Cramers rule,

form the matrix

$$\begin{bmatrix} \frac{1}{9}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{5}+\frac{1}{4}+\frac{1}{2} \end{bmatrix}\left[\frac{V_1}{V_2}\right] = \left[\frac{6}{5}\right]$$

$$\begin{pmatrix} .36 & -.25 \\ -.25 & .95 \end{pmatrix}\left[\frac{V_1}{V_2}\right] = \left[\frac{6}{5}\right]$$

$\Delta = 0.2795$

To find Δ_1

$$\begin{pmatrix} 6 & -.25 \\ 5 & .95 \end{pmatrix} = 6.95$$

$$V_1 = \frac{6.95}{.279} = 24.86\upsilon$$

To find Δ_2

$$\begin{bmatrix} .36 & 6 \\ -.25 & 5 \end{bmatrix} = 3.3$$

$$V_{22}\,\frac{3.3}{.2795} = 11.8V$$

$I_{9\Omega} = \frac{V_1}{9} = \frac{24.86}{9} = 2.76A$

$I_{4\Omega} = \frac{V_1 - V_2}{4} = \frac{24.86 - 11.8}{4} = 3.26A$

$I_{5\Omega} = \frac{V_2}{5} = \frac{11.86}{5} = 2.37A$

$I_{2\Omega} = \frac{V_2 - 10}{2} = \frac{11.86 - 10}{2} = 0.93A$

Hence currents in all the branches are found.

2) Use the Nodal Method to find V_{ba} and current through 30Ω resistor in the circuit shown.

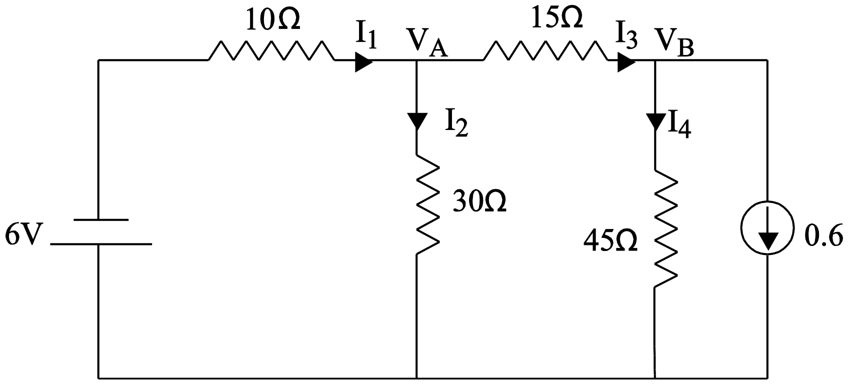


Figure 1.40

At node A

$$\frac{V_A + 6}{10} + \frac{V_A}{30} + \frac{V_A - V_B}{15} = 0$$

$$V_A\left[\frac{1}{10} + \frac{1}{30} + \frac{1}{15}\right] - \frac{V_B}{15} = -0.6$$

At node B

$$\frac{V_B - V_A}{15} + \frac{V_B}{45} \quad + 0.6 = 0$$

$$V_B\left[\frac{1}{15} + \frac{1}{45}\right] - \frac{V_A}{15} = -0.6$$

$$\begin{pmatrix} \frac{1}{10} + \frac{1}{30} + \frac{1}{15} & -\frac{1}{15} \\ -\frac{1}{15} & \frac{1}{15} + \frac{1}{45} \end{pmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -0.6 \\ -0.6 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0.2 & -0.066 \\ -0.066 & 0.088 \end{bmatrix} = \left[0.0176 - 4.35 \times 10^{-3}\right]$$

$\Delta = [0.01324]$

$\Delta = 0.01324$

$$\Delta_1 = \begin{pmatrix} -0.6 & -\frac{1}{15} \\ -0.6 & \frac{1}{15} + \frac{1}{45} \end{pmatrix} = -0.093$$

$\Delta_1 = [-0.053 - 0.04] = -0.093$

$$V_A = \frac{\Delta_1}{\Delta} = -\frac{0.093}{0.01324} = -7.02V$$

$$\Delta_2 = \begin{bmatrix} 0.2 & -0.6 \\ -0.066 & -0.6 \end{bmatrix}$$

$\Delta_2 = [-0.12 - 0.0396]$

$\Delta_2 = -0.1596$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-0.1596}{0.01324} = -12.05V$$

$V_{ba} = V_A - V_B = -7 + 12 = 5V$

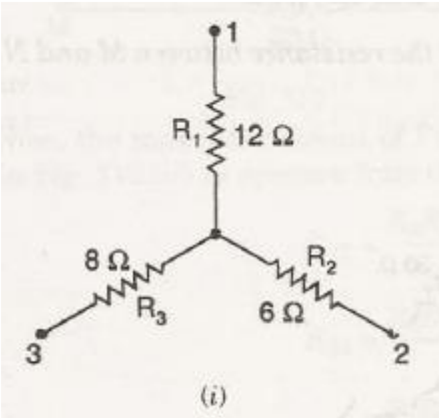
$$I_2 = \frac{V_A}{30} = \frac{-7}{30} = -0.233A$$

$I_2 = -0.233A$

Star- Delta Transformation

1. In the figure shown below a number of resistances connected in delta and star. Using star/delta conversion method complete the network resistance measured between (i) Land M (ii) M and N and (iii) N and L.

Solution. Three resistances 12 Ω, 6 Ω and 8 Ω are star connected. Transform them into delta with ends at the same points as before.



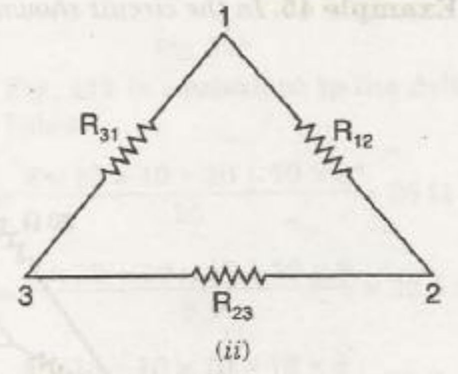
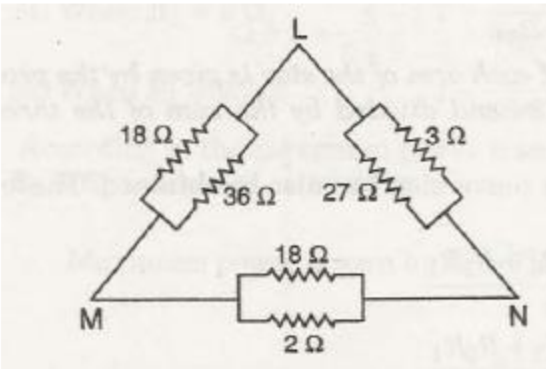


Figure 1.41 Figure 1.42

$R_{12} = R_1 R_2 + R_2 R_3 + R_3 R_1 / R_3 = 12 \times 6 + 6 \times 8 + 8 \times 12 / 8 = 27\ \Omega$

$R_{23} = R_1 R_2 + R_2 R_3 + R_3 R_1 / R_1 = 12 \times 6 + 6 \times 8 \times 12 / 12 = 18\ \Omega$



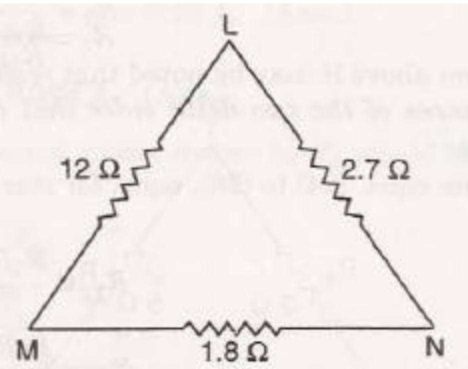


Figure 1.43 Figure 1.44

the transformed circuit connected to original delta connected resistances in the circuit 18 Ω, 3 Ω and 2 Ω.

here 18 Ω and 36 Ω are in parallel;

3 Ω and 27 Ω are in parallel, and

2 Ω and 18 Ω are in parallel.

These resistances are equivalent to:

$18 \times 36 / 18 + 36 = 12\ \Omega$; $3 \times 27 / 3 + 27 = 2.7\ \Omega$ and $2 \times 18 / 2 + 18 = 1.8\ \Omega$

(i) Resistance between L and M,

$R_{LM} = 12 \times (2.7 + 1.8) / 12 + (2.7 + 1.8) = 3.27\ \Omega$

(ii) Resistance between M and N,

$R_{MN} = 1.8 \times (12 + 1.8) / 1.8 + 12 + 2.7 = 1.6\ \Omega$

(iii) Resistance between N and L,

$R_{NL} = 2.7 \times (12 + 1.8) / 2.7 + (12 + 1.8) = 2.25\ \Omega$

2. In the circuit shown in Figure, find the resistance between M and N.

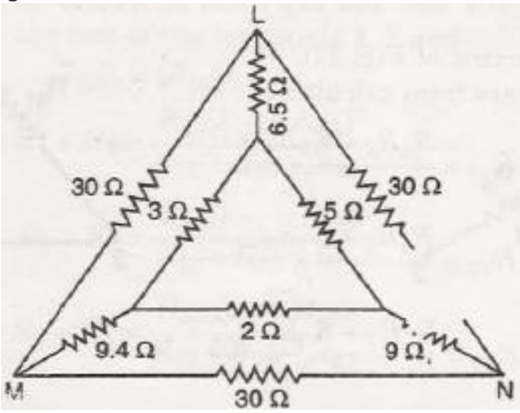


Figure 1.45

Solution.

$R_1 = R_{12}\, R_{31} / R_{12} + R_{23} + R_{31} = 5 \times 3 / 5 + 2 + 3 = 1.5$

$R_2 = R_{23}\, R_{12} / R_{12} + R_{23} + R_{31} = 2 \times 5 / 5 + 2 + 3 = 1$

$R_3 = R_{31}\, R_{23} / R_{12} + R_{23} + R_{31} = 3 \times 2 / 5 + 2 + 3 = 0.6$

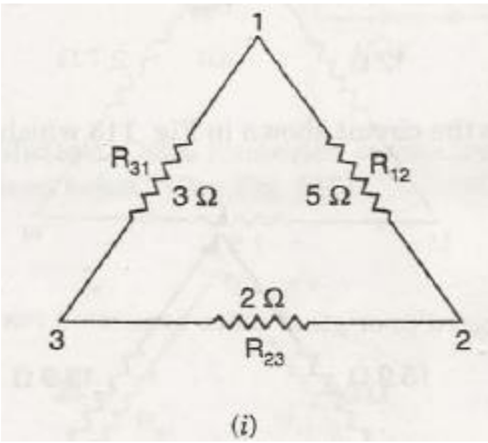


Figure 1.46

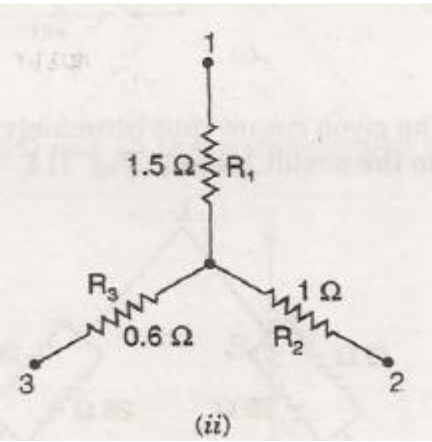


Figure 1.47

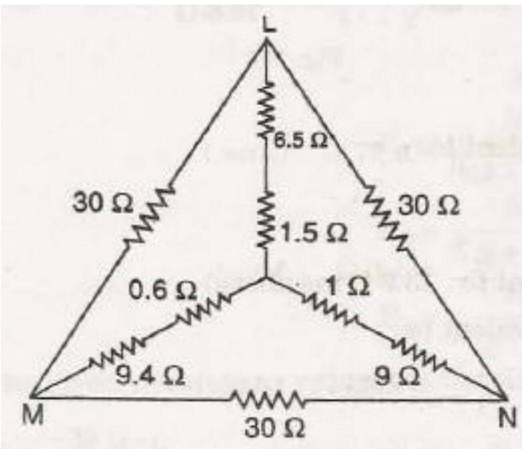


Figure 1.48

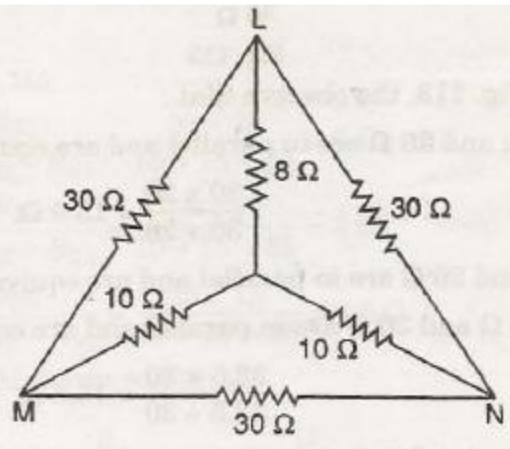


Figure 1.49

$R_{12} = R_1 R_2 + R_2 R_3 + R_3 R_1 / R_3 = 8 \times 10 + 10 \times 10 + 10 \times 8 / 10 = 26\ \Omega$

$R_{23} = R_1 R_2 + R_2 R_3 + R_3 R_1 / R_1 = 8 \times 10 + 10 \times 10 \times 10 / 8 = 32.5\ \Omega$

$R_{31} = 8 \times 10 + 10 \times 10 \times 10 \times 8 / 10 = 26\ \Omega$

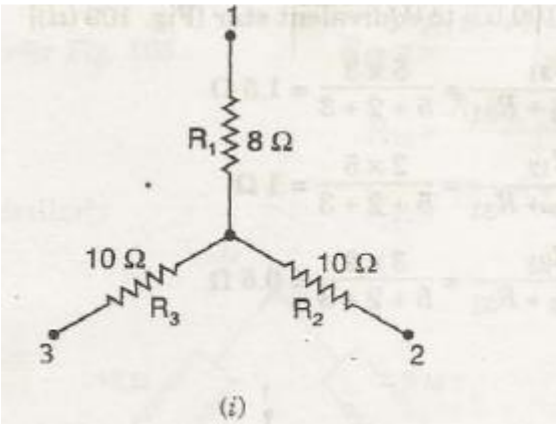


Figure 1.50

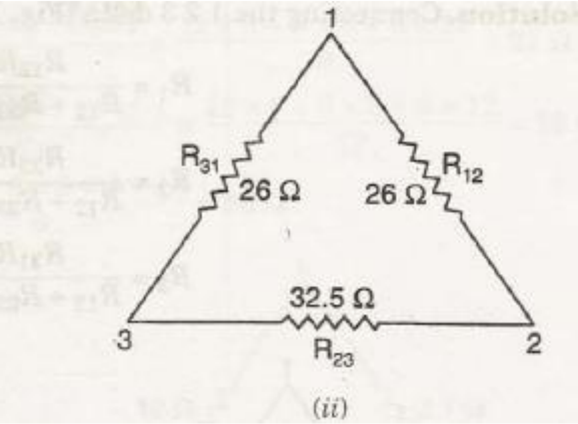


Figure 1.51

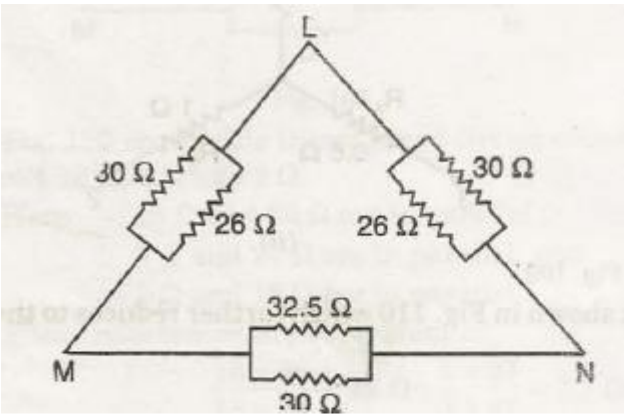


Figure 1.52

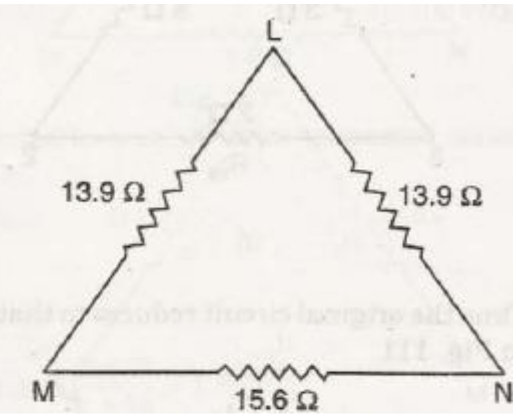


Figure 1.53

30 Ω and 26 Ω are in parallel and are equivalent to :

30 x 26 / 30 + 26 = 13.9

30 and 26 Ω are in parallel and are equivalent to : 13.9 Ω (as above)

32.5 Ω and 30 Ω are in parallel and are equivalent to :

32.5 x 30 / 32.5 + 30 = 15.6

Hence total resistance between M and N,

$R_{MN} = 15.6 \times (13.9 + 13.9) / 15.6 + (13.9 + 13.9)$

= 433.69 / 43.4 = 9.99

Example 3. Find the current I supplied by the battery, using delta/star transformation.

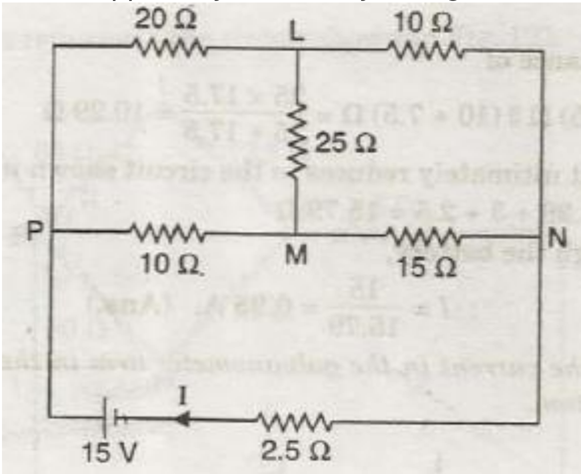
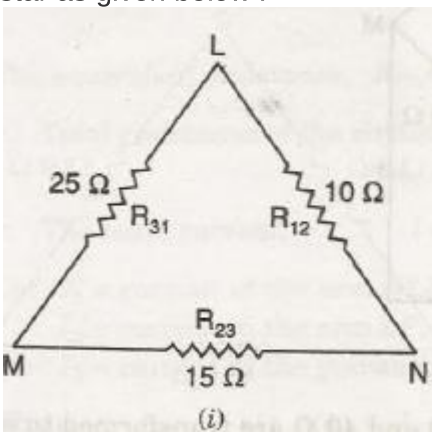


Figure 1.54

Solution. Delta connected resistances 25 Ω, 10 Ω and 15 Ω are transformed to equivalent star as given below :



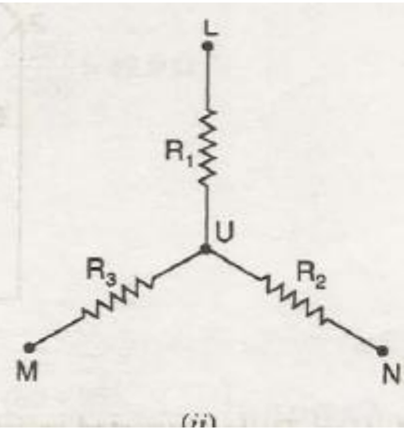
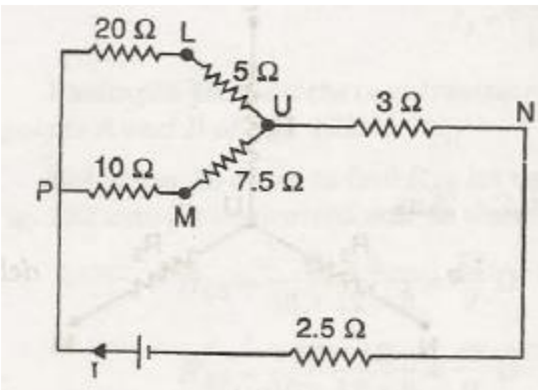


Figure 1.55 Figure 1.56

$R_1 = R_{12}\ R_{31} / R_{12} + R_{23} + R_{31} = 10 \times 25 / 10 + 15 + 25 = 5$

$R_2 = R_{23}\ R_{12} / R_{12} + R_{23} + R_{31} = 15 \times 10 / 10 + 15 + 25 = 3$

$R_3 = R_{31}\ R_{23} / R_{12} + R_{23} + R_{31} = 25 \times 15 / 10 + 15 + 25 = 7.$



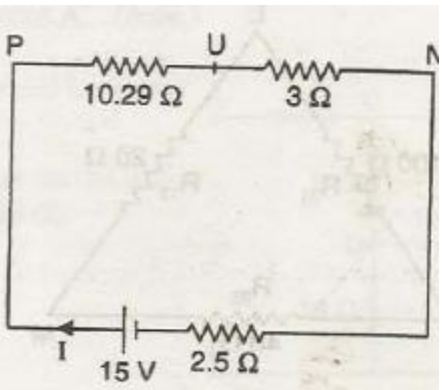


Figure 1.57 Figure 1.58

The equivalent resistance of

(20 + 5) Ω || (10 + 7.5) Ω = 25 x 17.5 / 25 + 17.5 = 10.29 Ω

Total resistance = 10.29 + 3 + 2.5 = 15.79 Ω

Hence current through the battery,

I = 15 / 15.79 = 0.95 A

Example 4. Using Delta-star conversion find resistance between terminals AB

Solution.

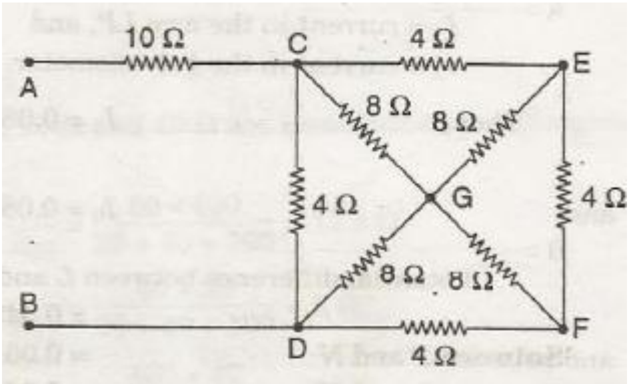
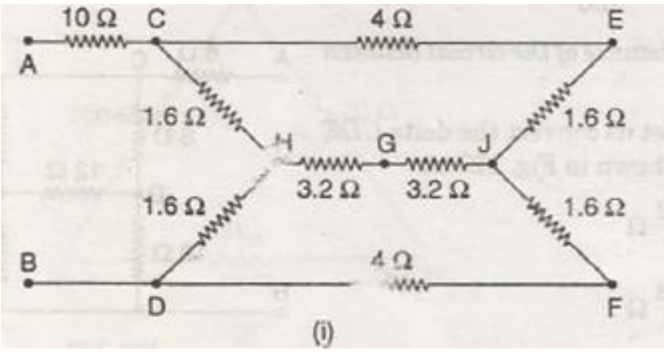


Figure 1.59

Using series-parallel combinations, we have

R_{AB} = (10 + 0.66) + [(0.752 + 1.6) || (2.64 + 5.6)]



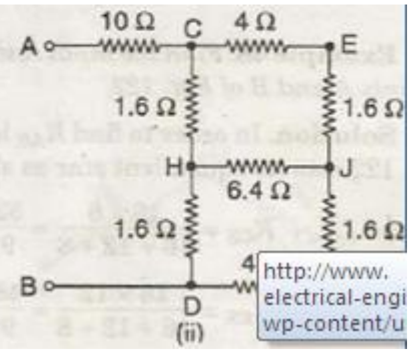


Figure 1.60 Figure 1.61

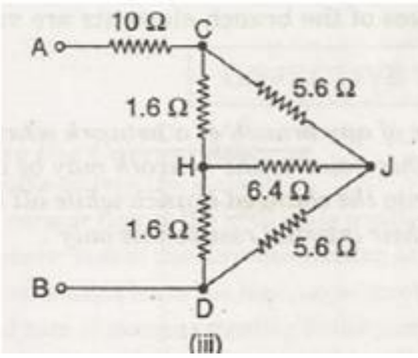


Figure 1.62

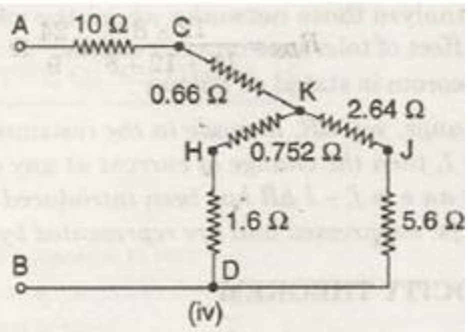


Figure 1.63

= 10.66 + 2.352 x 8.24 / (2.352 + 8.24 = 12.49 Ω

Example 5. Find the resistance at the A-B terminals in the electric circuit, using Δ-Y transformations.

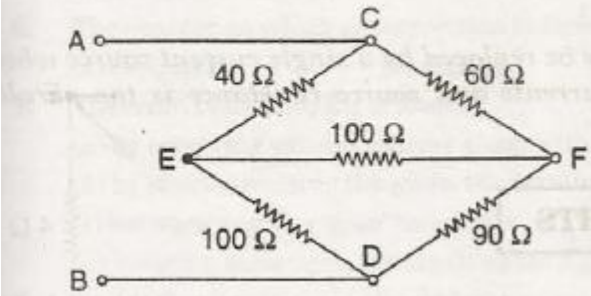


Figure 1.64

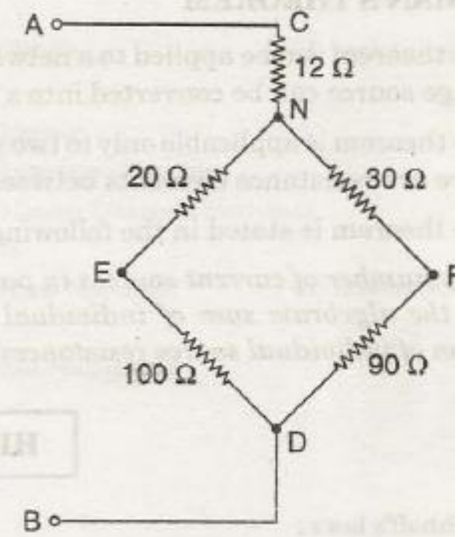


Figure 1.65

Solution. Converting CEF into star connections,

CN = 60 x 40 / 60 + 40 + 100 = 24 Ω

NE = 40 x 100 / 60 + 40 + 100 = 20 Ω

NF = 60 x 100 / 60 + 40 + 100 = 30 Ω

R_{ND} = (100 + 20) x (90 x 30) / 120 + 120 = 60 Ω

R_{AB} = 12 + 60 = 72 Ω

Questions

Part – A

1. What is voltage ?
2. Define resistance?
3. State ohm's law?
4. Explain the kirchoff's law?
5. Write the conditions of series circuit?
6. Write the condition of parallel circuit?
7. State and explain Kirchhoff's First law.
8. Give the formulas for current dividing between two parallel connected resistors.
9. Give the formulas for voltage dividing among two series connected resistors.
10. State and explain Kirchhoff's Second law.
11. State few application of Kirchhoff's law
12. What is a node?
13. What is a supernode?
14. Define mesh.
15. Define supermesh.
16. Write the objective of star delta transformation.
17. Given a delta circuit having resistors, write the required expressions to transform the circuit to a star circuit.
18. Given a star circuit having resistors, write the required expressions to transform the circuit to a delta circuit.
19. Three equal value resistors of value 5Ω are connected in star configuration. What is the resistance in one of the arms of equivalent delta network?
20. Three equal values resistors of value 9 Ω are connected in delta configuration. What is the resistance in one of the arms of equivalent star network?

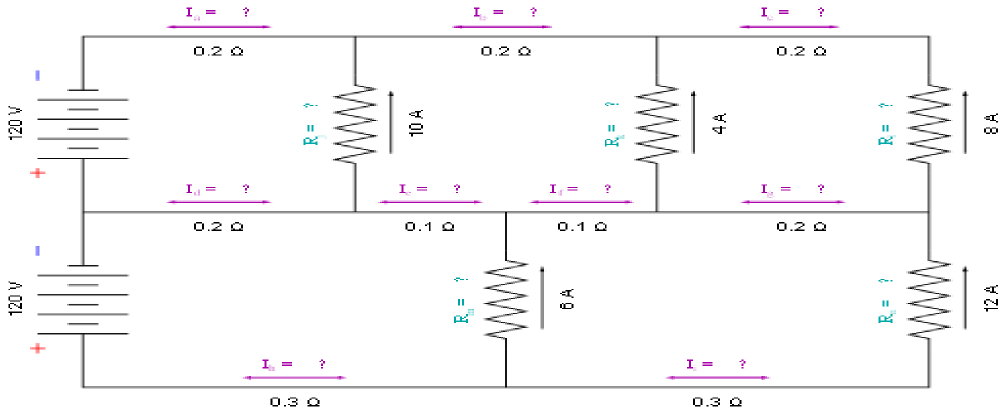
Part-B

1. A fairly complicated three-wire circuit is shown below. The source voltage is 120 V between the center (neutral) and the outside (hot) wires. Load currents on the upper half of the circuit are given as 10 A, 4 A, and 8 A for the load resistors j, k, and l, respectively. Load currents on the lower half of the circuit are given as 6 A and 12 A for the load resistors m and n, respectively. The resistances of the connecting wires a, b, c, d, e, f, g, h, and i are also given. Determine…

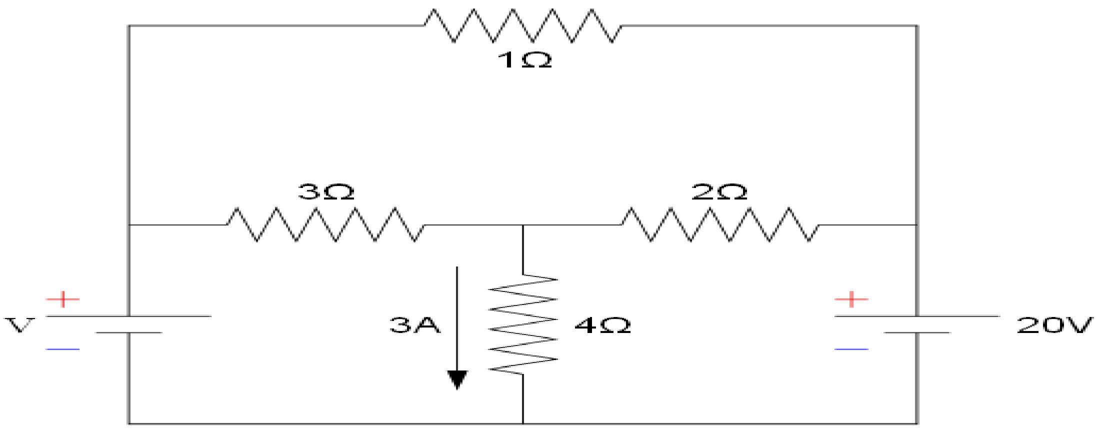
the current through each of the connecting wires (a, b, c, d, e, f, g, h, i) with the direction (left, right);

the voltage drop across each load element (j, k, l, m, n); and

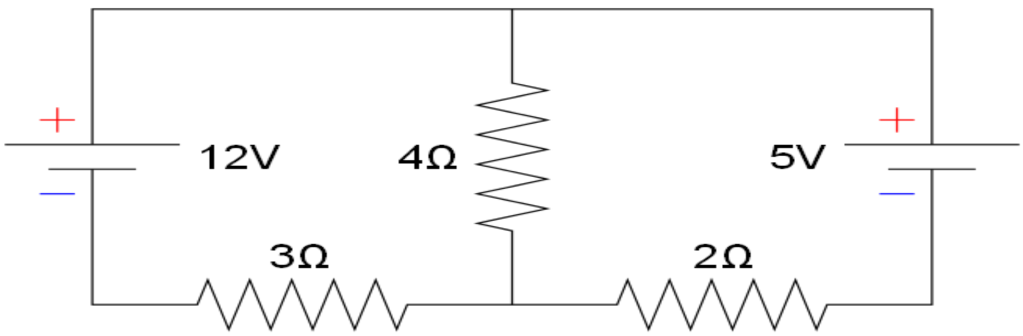
the resistance of each load element (j, k, l, m, n).



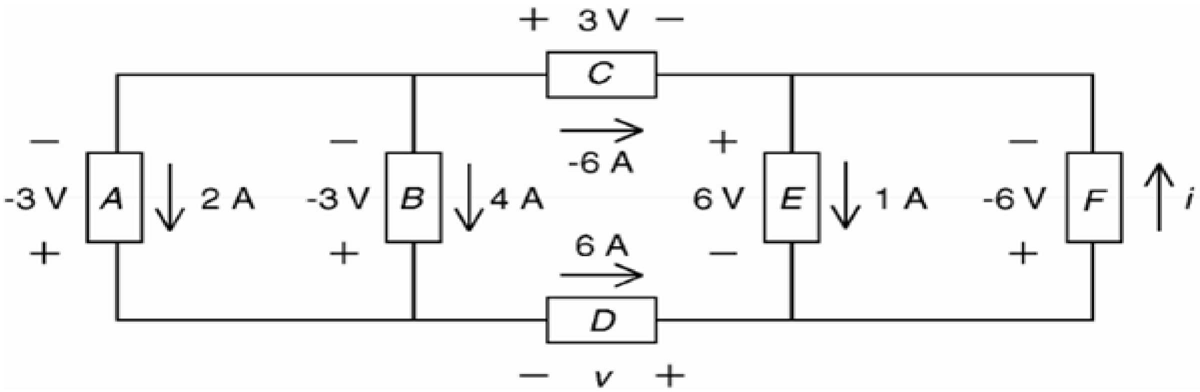
2. Given the circuit below with 3 A of current running through the 4 Ω resistor as indicated in the diagram to the right. Determine. The current through each of the other resistors, the voltage of the battery on the left, and the power delivered to the circuit by the battery on the right.



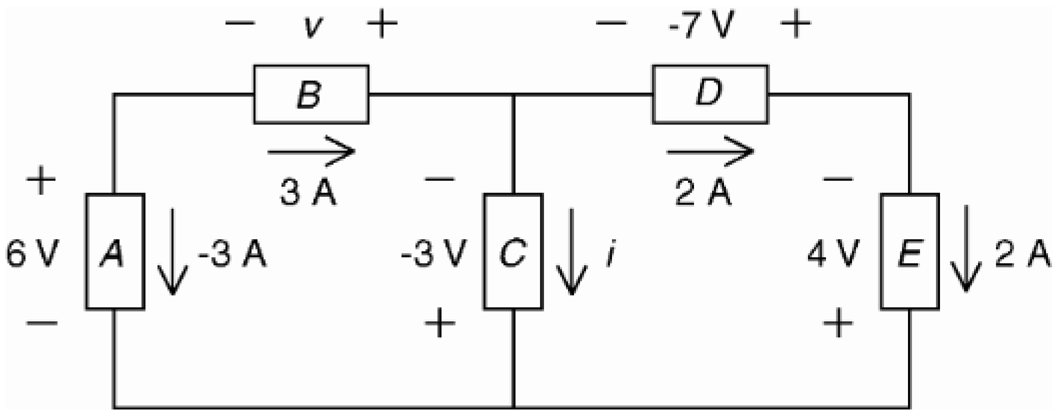
3. Determine the current through each resistor in the circuit shown below.



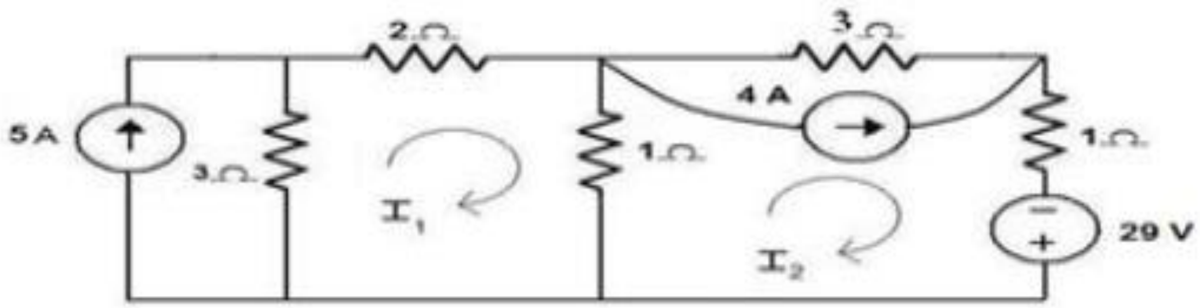
4. Consider the circuit shown below. Determine the power supplied by element D and the power received by element F.



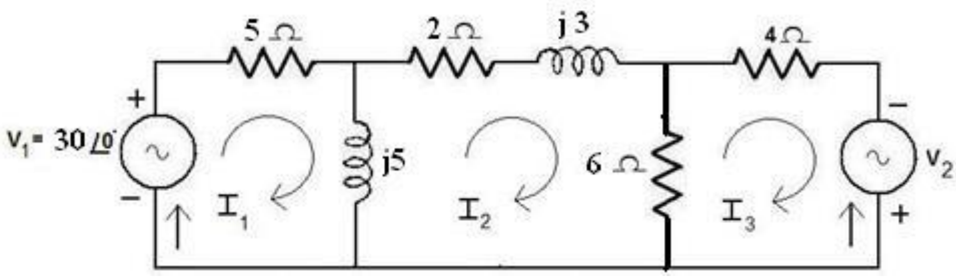
5. Consider the circuit shown below. Determine the power supplied by element B and the power supplied by element C.



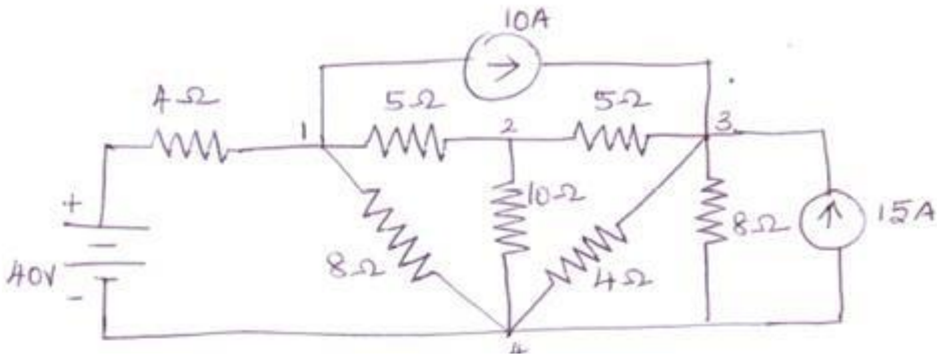
6.Determine the mesh currents I1 and I2 for the given circuit shown below



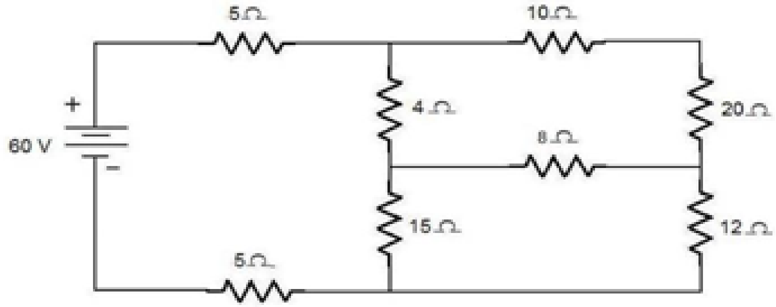
7.Determine the value of V2 such that the current through the impedance (2+j3) ohm is zero.



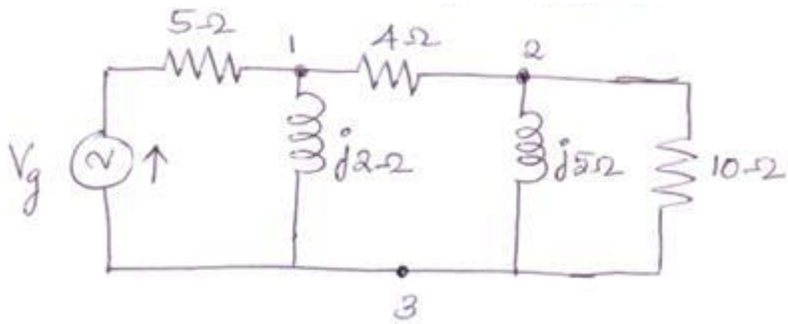
8.Use Nodal Voltage method and find thepower dissipated in the 10 Ω resistance on the circuit shown below



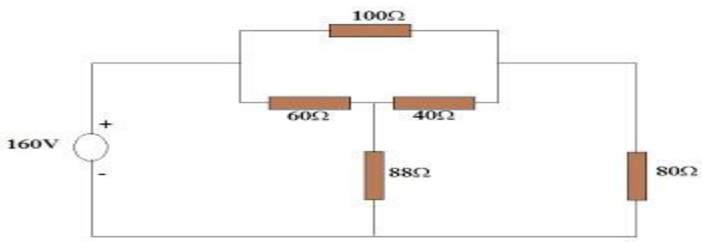
9.In the network shown below, find the current delivered by the battery.



10.Given the nodes 1 and 2 in network of figure, Find the ratio of voltage V1 / V2

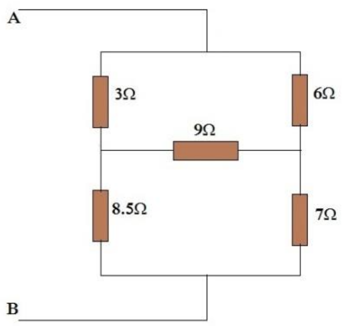


11. Determine the resistance between the terminals A&B and hence find the current through the voltage source. Refer figure1



12. Derive the expression for conversion of delta into star with neat Diagram.

13. Find the total resistance between A&B terminals for the network shown in figure 2



14. Derive the expression for conversion of star into delta with the neat diagram

15. Use star to delta transformation to find resistance between terminals A and B