Unit 2 : Design of Combinational Logic (10 hrs)

Minimal two level networks
- Minimization of POS and SOP. (2)

Design of two level gate networks. (2)
1. Two level NAND-NAND Network (1)
2. Two level NOR – NOR Network. (1)

Karnaugh maps
- Advantages and Limitations. (2)

Quine McClusky's Method. (2)

Switching function:

Expressions are Constructed a constants and variables with the Boolean Operations.

These expressions are also known as Boolean formulas. we use these expressions to describe switching functions (or) Boolean function.

Example: $(A+\bar{B})C$

$F(A,B,C) = (A+\bar{B})C$ (or) $F = (A+\bar{B})C$

Let us consider the four variable Boolean function.

$$F(A,B,C,D) = A + \underbrace{\bar{B}C}_{\text{product}} + \underbrace{AC\bar{D}}_{\text{product}}$$ → literals.

Literals: Each occurance of a variable in either a Complemented or an uncomplemented form is called a literals.

Product: product of literals

$$F(A,B,C,D) = \underbrace{(B+\bar{D})}_{\text{Sum}} \cdot \underbrace{(A+\bar{B}+C)}_{\text{Sum}} \cdot \underbrace{(\bar{A}+C)}_{\text{Sum}}.$$ Literals

There are seven literals, and three sum terms

These literals and terms are arranged in one of the two forms.

1. Sum of product form (SOP)
2. Product of sum form (POS)

Sum of Product form:

The word sum and product are derived from the symbolic representations of the OR and AND function by + and •

But these are not arithemetic operations.

Here all the product terms are sumed with the operator +.

Example:

sum

1. $F(A, B, C) = ABC + A\bar{B}C$

Product

2. $F(P, Q, R, S) = \bar{P}Q + QR + R'S$

sum

Product of sum:

* Sum terms are ANDed together.

sum

1. $F(A, B, C) = (A+B) \cdot (\bar{B}+C)$

Product

2. $F(P, Q, R, S) = (P+Q) \cdot (R+\bar{S}) \cdot (P+S)$

sum.

Standard SOP and POS form:

we can realise that in the SOP form all the individual terms do not involve all the literals.

Example $AB + AB\bar{C}$

(i) the first product term do not contain literal C.

(ii) If each term in SOP form contains all the literals then the SOP form is called standard

Example:

1. Convert the given expression in standard SOP form.

$F(A,B,C) = AC + AB + BC.$

Solution:

$$F(A,B,C) = \underbrace{AC}_{B \text{ is missing}} + \overbrace{AB}^{C \text{ is missing}} + \underbrace{BC}_{A \text{ is missing}}$$

AND product term with (missing literals + its Complement)

$$F(A,B,C) = AC\cdot\underline{(B+\bar{B})} + AB\cdot\underline{(C+\bar{C})} + BC\underline{(A+\bar{A})}$$

missing literals and their Complements

Expand the terms and reorder literals.

Expand $F(A,B,C) = ACB + AC\bar{B} + ABC + AB\bar{C} + ABC + \bar{A}BC$

Reorder $F(A,B,C) = \underline{ABC} + A\bar{B}C + \underline{ABC} + AB\bar{C} + \underline{ABC} + \bar{A}BC$

Omit repeated product terms

$$F(A,B,C) = ABC + A\bar{B}C + AB\bar{C} + \bar{A}BC$$

---

2. Convert the given expression in standard SOP form

$$F(A,B,C) = A + ABC$$

$$F(A,B,C) = A\cdot(B+\bar{B})\cdot(C+\bar{C}) + ABC$$

Expand the term and re-order

$$F(A,B,C) = AB + A\bar{B}\cdot(C+\bar{C}) + ABC$$

$$= ABC + A\bar{B}\bar{C} + AB\bar{C} + A\bar{B}\bar{C} + ABC + A\bar{B}C$$

$$= ABC + AB\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$$

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3. Express $F = B\bar{C} + AC$ in a canonical SOP form.

solution

$$F = B\bar{C} + AC$$

$$= (A+\bar{A})B\bar{C} + AC(B+\bar{B})$$

$$= AB\bar{C} + \bar{A}B\bar{C} + ABC + A\bar{B}C$$

Q1. Express F_1 in standard SOP form.

$F_1 = AB + \bar{C}D + A\bar{B}C$ Ans: $\bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + A\bar{B}\bar{C}D + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}CD + AB\bar{C}\bar{D} + ABC\bar{D} + ABCD$

Q2. Determine the canonical SOP form of

$$f(x,y,z) = (xy + \bar{z})(y + x\bar{z})$$

Ans: $xyz + xy\bar{z} + \bar{x}y\bar{z} + x\bar{y}\bar{z}$.

SOP form.

(iii) Each individual term in SOP form is called min term.

Example for standard SOP form:

$$F(A,B,C) = A\bar{B}C + ABC + \bar{A}B\bar{C}$$

All the terms consists of all literals in either complemented or uncomplemented form

Standard POS form:

(i) If each term in POS form contains all the literals then the POS form is known as Standard or Canonical POS form.

(ii) Each individual term in the standard POS form is called max term.

Example for standard POS form.

$$F(A,B,C) = (A+B+C) \cdot (A+\bar{B}+C)$$

All the terms consists of all literals in either complemented or uncomplemented form.

Steps to Convert SOP to standard SOP form:

Step1: find the missing literal in each product term if any.

Step2: AND each product term having missing literals with term form by OR ing the literals and its complement

Step3: Expand the terms by applying distributive law and reorder the literals in the product term.

Step4: Reduce the expression by omitting repeated product terms if any. Because $A+A=A$.

Convert the given expressions in standard POS form.

$F(A,B,C) = (A+B)\cdot(B+C)$.

Solution:

$= \underbrace{(A+B)}_{C \text{ is missing}} \overbrace{(B+C)}^{A \text{ is missing}}$

$= (A+B)+C\cdot\bar{C}\cdot(B+C)+A\cdot\bar{A}$

$= (A+B+C)(A+B+\bar{C})\underbrace{(A+B+C)}_{\text{repeated}}(\bar{A}+B+C)$

$= (A+B+C)(A+B+\bar{C})(\bar{A}+B+C)$.

* Convert SOP to equivalent POS.

$\bar{A}\bar{B}C + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$.

Solution.

$= \bar{A}\bar{B}C + \underbrace{\bar{A}\bar{B}C}_{\text{repeated}} + \bar{A}BC + A\bar{B}C + ABC$

$= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$.

POS form

$= (\bar{A}+\bar{B}+C)(\bar{A}+B+C)(A+\bar{B}+C)(A+B+C)$.

Convert $(A+B)(A+C)(B+\bar{C})$ into standard POS form.

Solution:

$F(A,B,C) = A+B+C\cdot\bar{C} \quad A+C+B\cdot\bar{B} \quad B+\bar{C}+A\cdot\bar{A}$

$= (A+B+C)\overbrace{(A+B+\bar{C})}^{\text{repeated}}(A+B+C)(A+\bar{B}+C)$

$(A+B+\bar{C})(\bar{A}+B+\bar{C})$. repeated

$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+\bar{C})$.

Obtain Canonical POS for $F(A,B,C) = (A+\bar{B})(B+C)(A+\bar{C})$

Solution

$= A+\bar{B}+C\cdot\bar{C} \quad B+C+A\cdot\bar{A} \quad A+\bar{C}+B\cdot\bar{B}$

$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+B+C)(\bar{A}+B+C)(A+B+\bar{C})$

$\underbrace{(A+\bar{C}+\bar{B})}_{\text{repeated}}$

$= (A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+C)(A+B+\bar{C})$

Q1: Convert the given expression in standard POS form

$F(P,Q,R) = (P+\bar{Q})(P+R)$.

Ans: $(P+\bar{Q}+R)(P+\bar{Q}+\bar{R})(P+Q+R)$

Steps to Convert POS to standard POS.

Step1: Find the missing literals in each sum term if any.

Step2: OR each sum term having missing literal/s with terms form by ANDing the literal and its Complement.

Step3: Expand the terms by applying distributive law and reorder the literals in the sum term.

Step4: Reduce the expression by omitting repeated sum terms if any, Because $A \cdot A = A$.

Examples:

1. Convert the given expression in standard POS form.

$$F(A,B,C) = (A+B)(B+C)(A+C)$$

Solution:

A is missing

$$(A+B)\ (B+C)\ (A+C)$$

C is missing B is missing.

$$F(A,B,C) = (A+B)+(C\cdot\bar{C}) \cdot B+C+A\cdot\bar{A} \cdot A+C+B\cdot\bar{B}$$

Expand and reorder.

$$= \underline{(A+B+C)}\ (A+B+\bar{C})\ \underline{(B+C+A)}\ (B+C+\bar{A})\ \underline{(A+B+C)}$$

repeated

$$(A+\bar{B}+C)$$

$$F(A,B,C) = (A+B+C)\ (A+B+\bar{C})\ (\bar{A}+B+C)\ (A+\bar{B}+C).$$

2. Convert the given expression in standard POS form.

solution:

$$y = \underline{A}\cdot(A+B+C)$$

$$y = A\cdot\ (A+B+C)$$

B, C missing.

$$= A+B\cdot\bar{B}+C\cdot\bar{C}\ (A+B+C)$$

Expand and reorder.

$$=(A+B\cdot\bar{B}+C)\ (A+B\cdot\bar{B}+\bar{C})\ (A+B+C)$$

$A+BC =$

$(A+B)(A+C)$ $= \underline{(A+C+B)}\ (A+C+\bar{B})\ (A+\bar{C}+B)\ (A+\bar{C}+\bar{B})$

$$\underline{(A+B+C)}$$

repeated

$$= (A+B+C)\ (A+C+\bar{B})\ (A+B+\bar{C})\ (A+\bar{C}+\bar{B})$$

M Notations.

Minterms and Maxterms.

* Each individual term is standard SOP form is called minterms
* Each individual term in standard POS form is called maxterms.

Three variable logical functions are expressed as $2^3 = 8$.

Variables			minterms	Maxterms
A	B	C	m_i	M_i
0	0	0	$\bar{A}\bar{B}\bar{C} = m_0$	$A+B+C = M_0$
0	0	1	$\bar{A}\bar{B}C = m_1$	$A+B+\bar{C} = M_1$
0	1	0	$\bar{A}B\bar{C} = m_2$	$A+\bar{B}+C = M_2$
0	1	1	$\bar{A}BC = m_3$	$A+\bar{B}+\bar{C} = M_3$
1	0	0	$A\bar{B}\bar{C} = m_4$	$\bar{A}+B+C = M_4$
1	0	1	$A\bar{B}C = m_5$	$\bar{A}+B+\bar{C} = M_5$
1	1	0	$AB\bar{C} = m_6$	$\bar{A}+\bar{B}+C = M_6$
1	1	1	$ABC = m_7$	$\bar{A}+\bar{B}+\bar{C} = M_7$

Example

(1) $F(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + AB\bar{C}$

$= m_0 + m_1 + m_3 + m_6$

$= \Sigma m(0,1,3,6)$

(2) $F(A,B,C) = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$

$= M_1 \cdot M_3 \cdot M_6$.

$= \Pi M(1,3,6)$

Sum of products denoted by Σ

Product of Sum denoted by Π

Complements of standard forms:

* If the given function is

$F(A,B,C) = m_0 + m_1 + m_3 + m_4 + m_6 + m_7$.

$= M_2 \cdot M_5$

$\therefore F(A,B,C) = \Sigma m(0,1,3,4,6,7) = \Pi M(2,5)$

* If the given function is

$F(A,B,C,D) = \Sigma m(0,2,4,6,8,10,12,14)$. then

$F(A,B,C,D) = \Pi M(1,3,5,7,9,11,13,15)$.

Express the switching function $f_{(BA)} = A$ in terms of minterms

$$f_{(BA)} = A$$
$$= A(B+\bar{B})$$
$$= AB + A\bar{B}$$

Express $f = A + \bar{B}C$ as sum of minterms.

$$A + \bar{B}C = A\cdot(B+\bar{B})(C+\bar{C}) + \bar{B}C(A+\bar{A})$$
$$= AB + A\bar{B}\ (C+\bar{C}) + A\bar{B}C + \bar{A}\bar{B}C.$$
$$= ABC + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C + \cancel{AB\bar{C}} + AB\bar{C}$$
$$= ABC + A\bar{B}C + AB\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C}.$$
$$f = \Sigma m(1,4,5,6,7)$$

Prove that the logical sum of all minterms of a Boolean function of 2 variables is 1.

Solution:

$$\bar{A}\bar{B},\ A\bar{B},\ \bar{A}B,\ AB.$$
$$F = \bar{A}\bar{B} + A\bar{B} + \bar{A}B + AB.$$
$$= \bar{B}(A+\bar{A}) + B(A+\bar{A})$$
$$= \bar{B} + B$$
$$= 1 \quad \text{Thus proved.}$$

Express the Boolean function $F = XY + \bar{X}Z$ in product of max terms.

$$F = xy + \bar{x}z$$
$$= xy(z+\bar{z}) + \bar{x}z(y+\bar{y})$$
$$= xyz + xy\bar{z} + \bar{x}yz + \bar{x}\bar{y}z.$$
$$= \Sigma m(7,6,3,1)$$
$$= \Pi M(0,2,4,5)$$
$$= (x+y+z)(x+\bar{y}+z)(\bar{x}+y+z)(\bar{x}+y+\bar{z})$$

Express the Boolean function in POS and SOP form

$$D = (\bar{A}+B)(\bar{B}+C) \Rightarrow \text{POS form}$$

$$D = \bar{A}\bar{B} + B\cdot\bar{B} + \bar{A}\cdot C + BC.$$
$$= \bar{A}\bar{B} + \bar{A}C + BC \Rightarrow \text{SOP form.}$$

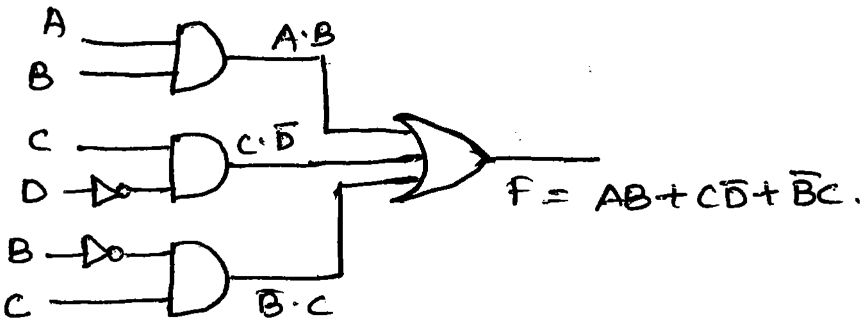
Implementation of Logic function using gates.

Boolean algebra is used to express the o/p of any Combinational network. such network is implemented using logic gates.

Example

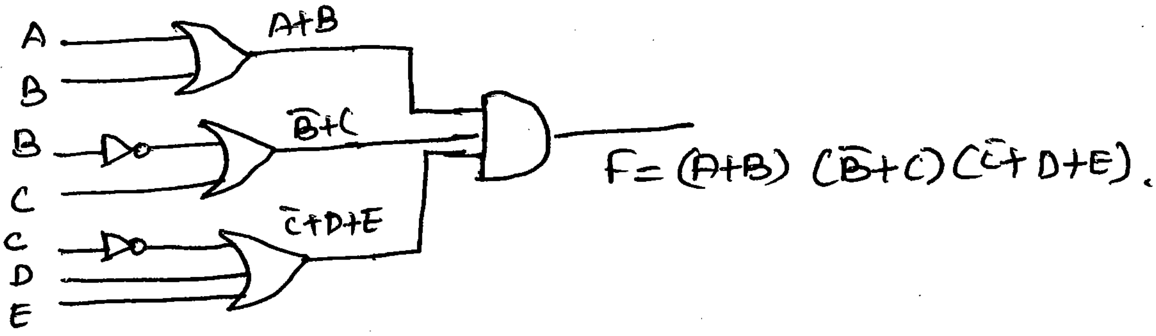
Implementation of SOP expression.

$F = AB + C\bar{D} + \bar{B}C$.



Implementation of Pos expression.

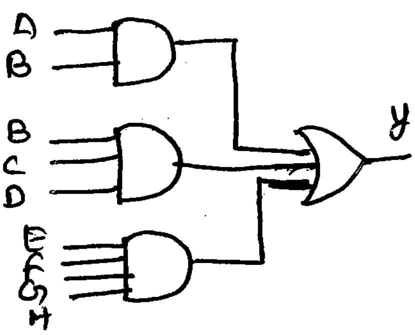
$F = (A+B)\ (\bar{B}+C)\ (\bar{C}+D+E)$

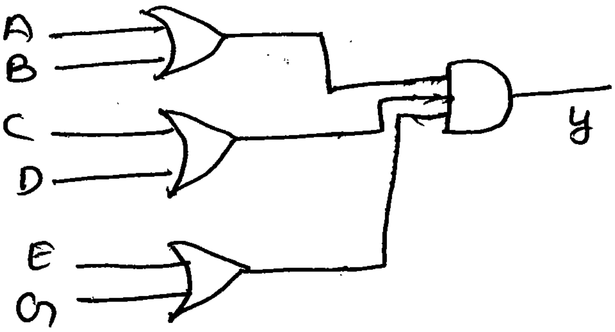


(1) Implement the expression.

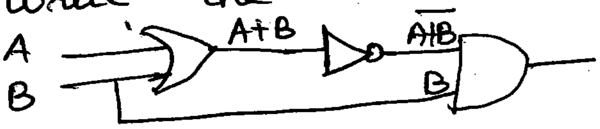
a) $AB + BCD + EFGH$.

(b) $(A+B)\ (C+D)\ (E+G)$





(2) Write the Boolean expression for the o/p of the system.

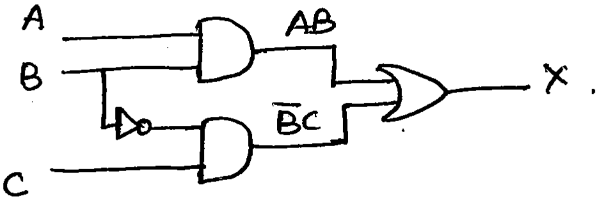


$C = \overline{(A+B)} \cdot B$

$= \bar{A} \cdot \bar{B} \cdot B$

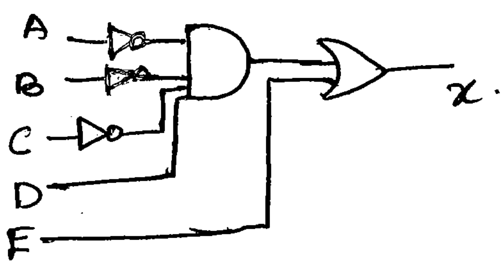
$= 0$

3. Draw the logic diagram for $x = AB + \overline{B}C$.



4. Implement the Boolean expression using gates.

$$X = (AB + C)'D + E = (\overline{BA + C}) \cdot D + E$$



$$= \overline{AB} \cdot \overline{C} \cdot D + E$$

$$= \overline{A}\overline{B}\overline{C}D + E$$

UNIVERSAL GATES:

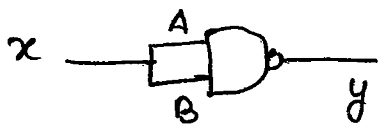
NAND and NOR gates are called universal gates because any logic functions can be implemented by these gates.

NAND gate:

It performs NOT, AND, OR function and also NOR.

(i) NOT operation:

An inverter can be made from a NAND gate by connecting all of the input together and creating a effect of single common input.

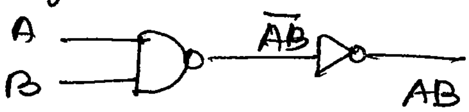


$$y = \overline{AB} = \overline{x \cdot x} = \overline{x} + \overline{x} = \overline{x}$$

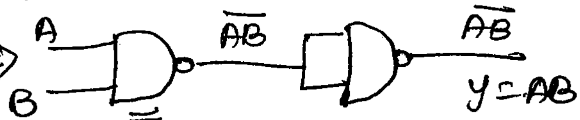
	A	B	y	
$x=0$	0	0	1	$y=1$
	0	1	1	
	1	0	1	
$x=1$	1	1	0	$y=0$

(ii) AND function:

AND operation can be performed simply by inverting the o/p of NAND gate.



$\Rightarrow$

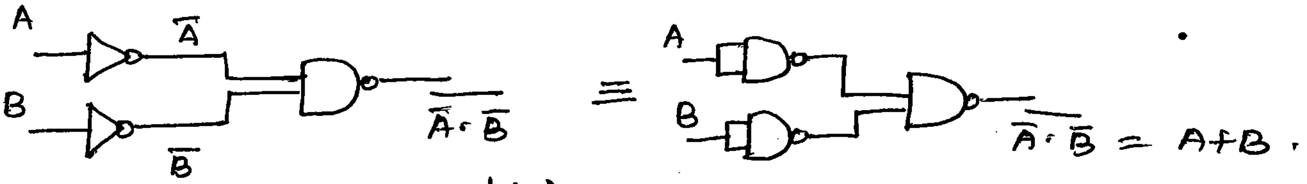


A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

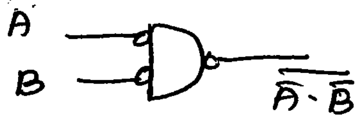
A	B	$\overline{AB}$	$\overline{\overline{AB}}$
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

OR operation.

$$y = A+B = \overline{\overline{\bar{A}+\bar{B}}} = \overline{\bar{A}\cdot\bar{B}}$$



(or)



Note: Bubble at the Input of NAND gate indicates inverted Input.

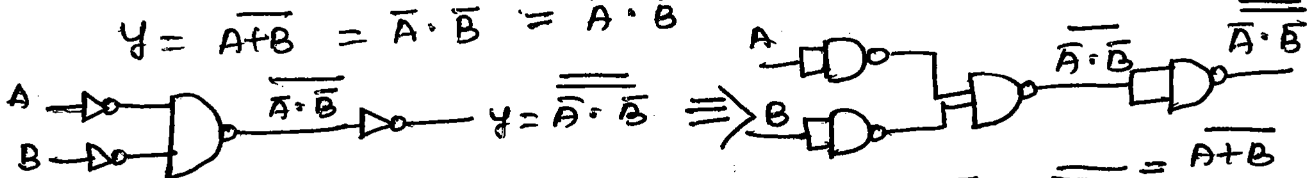
A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

⇒

A	B	$\bar{A}\cdot\bar{B}$	$\overline{\bar{A}\cdot\bar{B}}$
0	0	1	0
0	1	0	1
1	0	0	1
1	1	0	1

NOR operation.

$$y = \overline{A+B} = \bar{A}\cdot\bar{B} = \overline{\overline{\bar{A}\cdot\bar{B}}}$$



A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

=

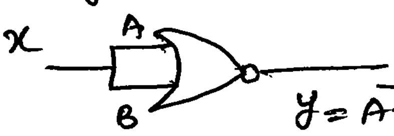
A	B	$\bar{A}\cdot\bar{B}$	$\overline{\bar{A}\cdot\bar{B}}$	$\overline{\overline{\bar{A}\cdot\bar{B}}}$
0	0	1	0	1
0	1	0	1	0
1	0	0	1	0
1	1	0	1	0

NOR Gate:

NOR gate performs NOT, AND, OR, NAND operations so called universal gate.

NOT function:

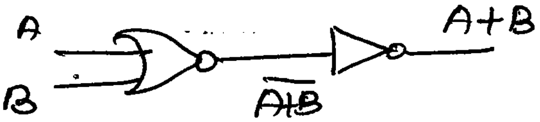
An inverter can be made from a NOR gate by connecting all the Inputs together and creating the effect of single common input.



$y = \overline{A+B} = \overline{x+x} = \bar{x}$

	A	B	$y = \overline{A+B}$
x=0	0	0	1
	0	1	0
	1	0	0
x=1	1	1	0

OR operation.



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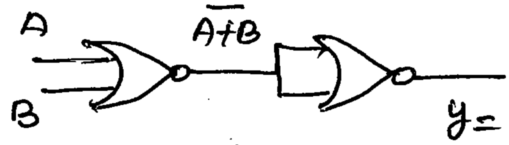


$y = \overline{\overline{A+B}}$

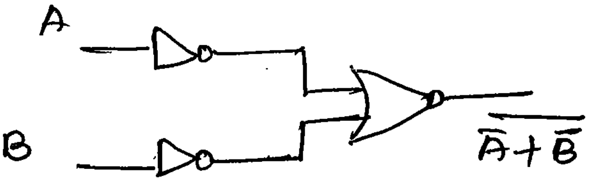
$= A+B$

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

A	B	$\overline{A+B}$	$\overline{\overline{A+B}}$
0	0	1	0
0	1	0	1
1	0	0	1
1	1	0	1

AND Operation.

$Y = A \cdot B = \overline{\overline{A}} \cdot \overline{\overline{B}} = \overline{\overline{A} + \overline{B}}$



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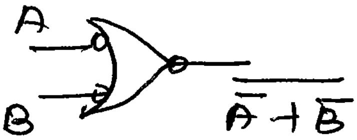
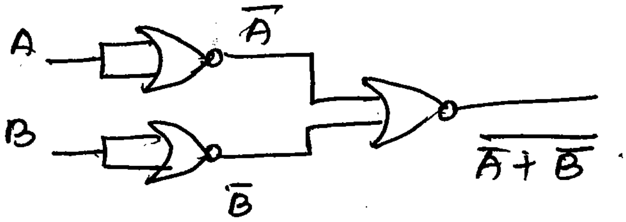


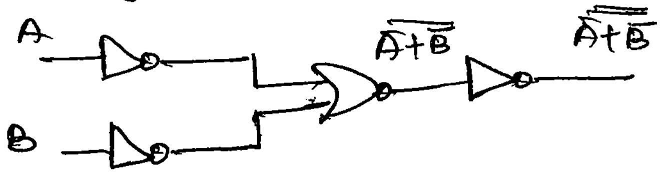


A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

A	B	$\overline{A} + \overline{B}$	$\overline{\overline{A}+\overline{B}}$
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

NAND Operation.

$y = \overline{A \cdot B} = \overline{A} + \overline{B} = \overline{\overline{\overline{A} + \overline{B}}}$



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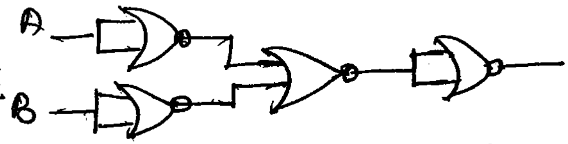


A	B	$\overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

A	B	$\overline{A} + \overline{B}$	$\overline{\overline{A}+\overline{B}}$	$\overline{\overline{\overline{A}+\overline{B}}}$
0	0	1	0	1
0	1	1	0	1
1	0	1	0	1
1	1	0	1	0

Conversion of AND/OR/NOT Logic to NAND/NOR Logic using graphical procedure.

(1) Draw AND/OR logic.

(2) If NAND hardware has been choosen, add bubbles on the output of each AND gate; and bubbles on input side to all OR gates.

(3) If NOR hardware has been choosen, add bubbles on output of each OR gate and bubble on input of each AND gate

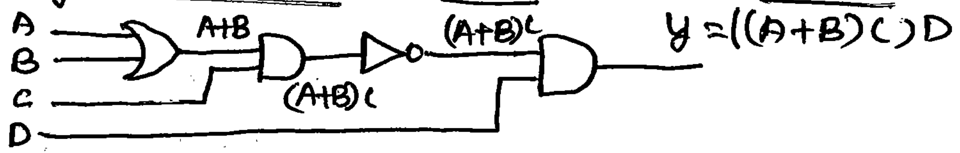
4. Add or subtract an inverter on each line that received a bubble in step 2 or 3

5. Replace bubbled OR by NAND and bubbled AND by NOR

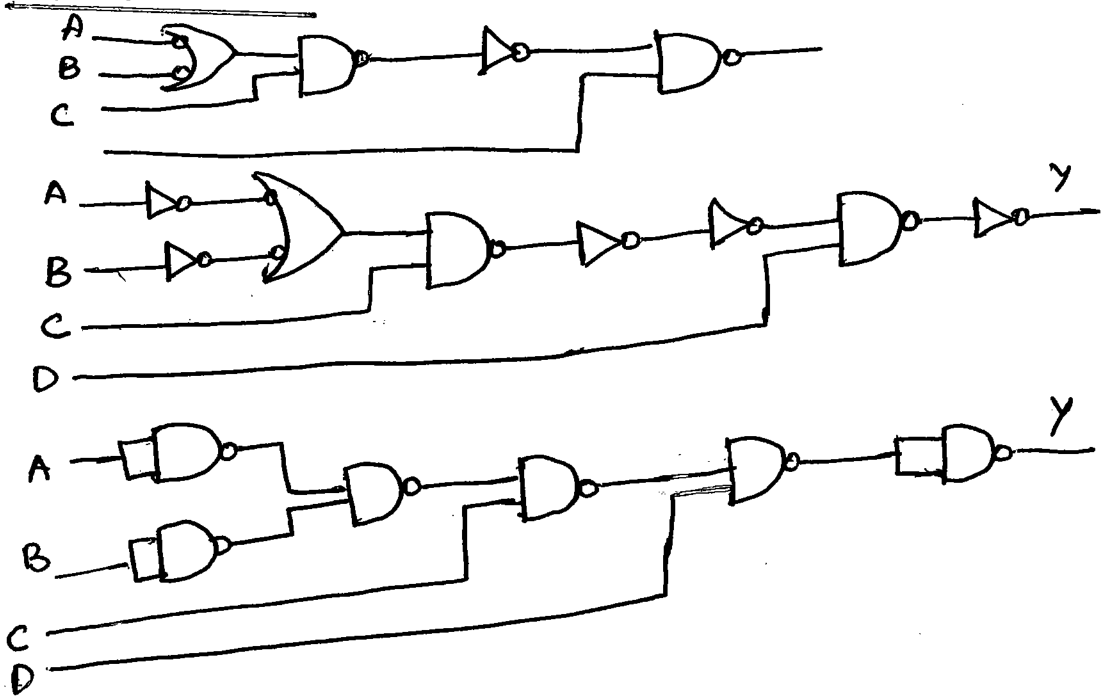
6. Eliminate double inversions.

* Boolean expression: $((\overline{(A+B)C})D)$

Original circuit:



NAND CIRCUIT:



NOR Circuit:

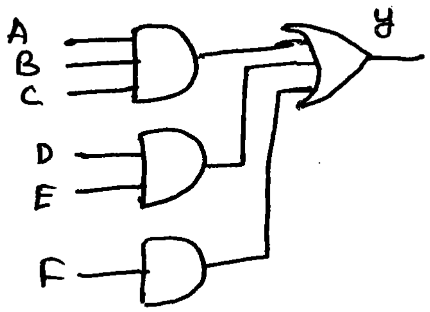
Design of two level Network:

NAND-NAND Implementation.

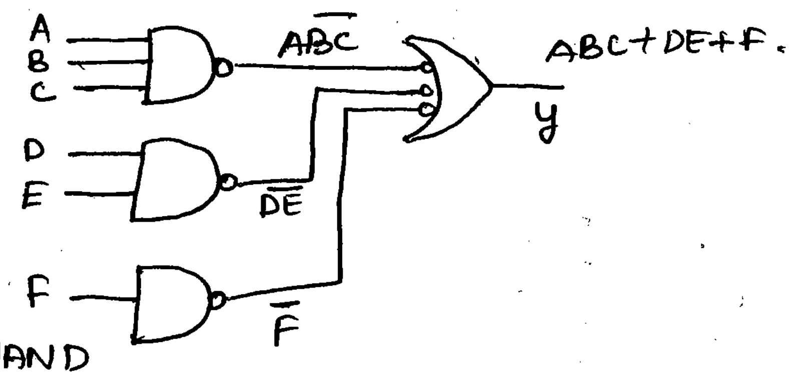
* Requires the expression simplified to SOP term
* Relation between AND-OR and NAND-NAND logic is explained

1) $y = ABC + DE + F$.

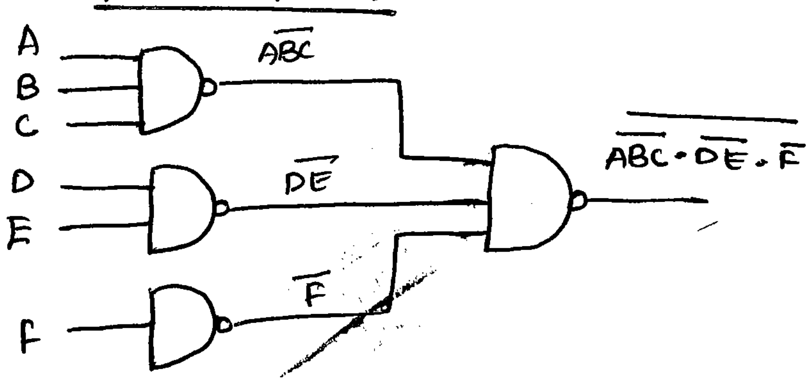
AND-OR



NAND-Bubbled OR



NAND-NAND



$$= \overline{\overline{ABC}} + \overline{\overline{DE}} + \overline{\overline{F}}$$

$$= ABC + DE + F.$$

Implement the following Boolean function with NAND – NAND logic $Y = AC + ABC + \bar{A}BC + AB + D$.

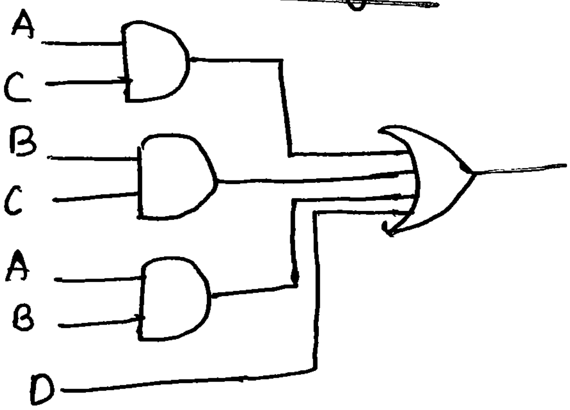
Simplify

$= AC + ABC + \bar{A}BC + \cancel{AB} + D.$

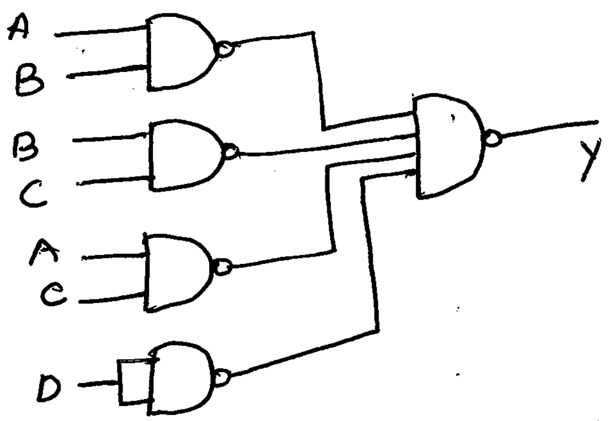
$= AC + BC(A + \bar{A}) + AB + D.$

$= AC + BC + AB + D.$

AND – OR logic



NAND – NAND



Implement the following Boolean function with NAND-NAND logic.

$F(A, B, C) = \Sigma m(0, 1, 3, 5)$

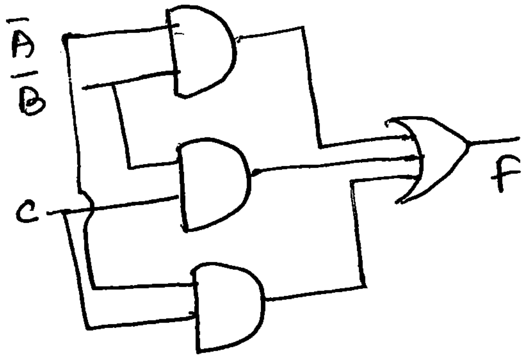
A \ BC	00	01	11	10
0	1	1	1	0
1	0	1	0	0

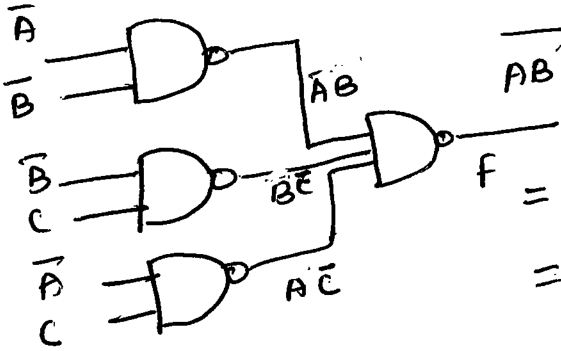
$g_1 = \bar{A}C$

$g_3 = \bar{A}\bar{B}$ $g_2 = \bar{B}C$

$F = \bar{A}\bar{B} + \bar{B}C + \bar{A}C$

AND – OR logic





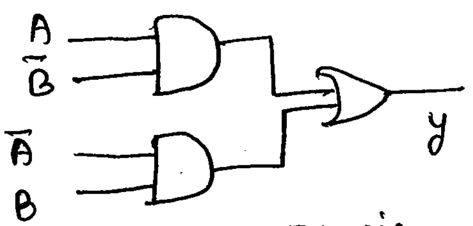
$\overline{\overline{\bar{A}\bar{B}} \cdot \overline{\bar{B}C} \cdot \overline{\bar{A}C}}$

$= \overline{\overline{\bar{A}\bar{B}}} + \overline{\overline{\bar{B}C}} + \overline{\overline{\bar{A}C}}$

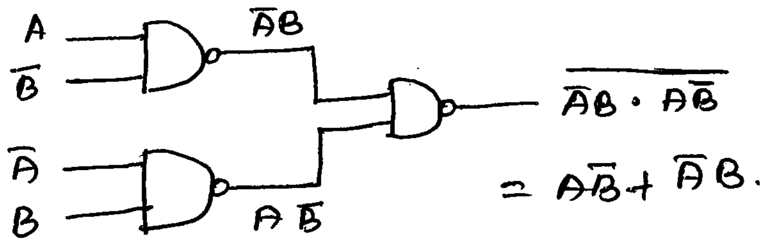
$= \bar{A}\bar{B} + \bar{B}C + \bar{A}C$

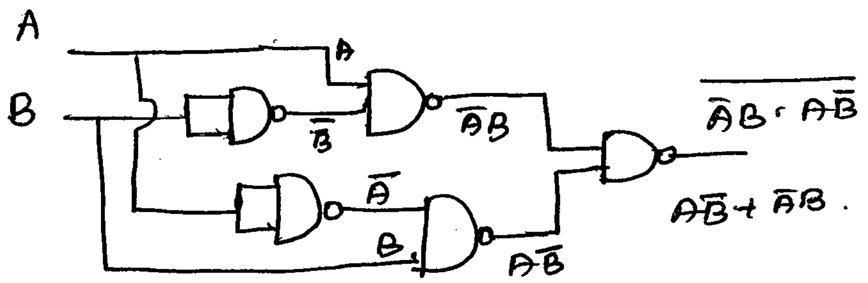
Implement EX-OR gate using only NAND gates.

$y = \bar{A}B + A\bar{B}$

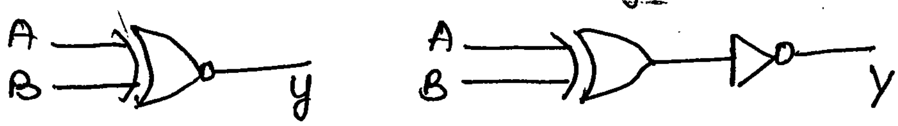


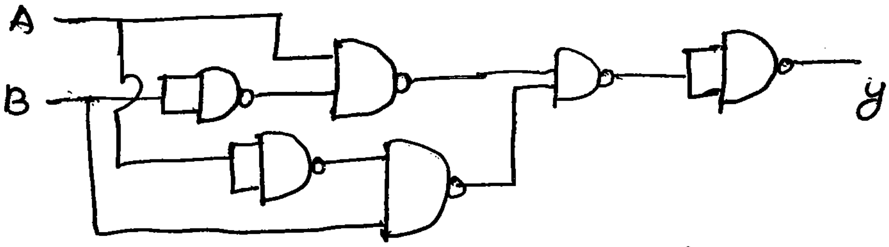
AND-OR Logic





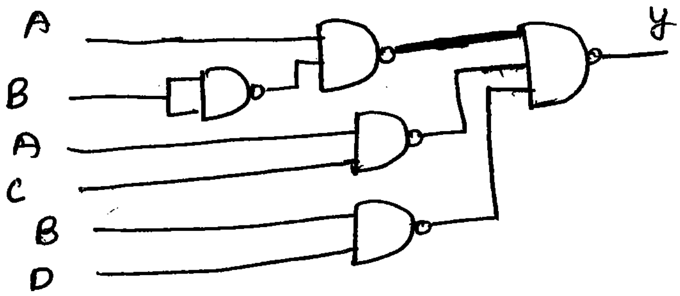
Implement EX-NOR gate using only NAND gate.





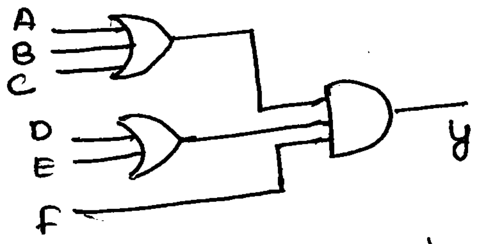
Sketch a NAND-NAND logic circuit for the Boolean expression

$y = A\bar{B} + AC + BD$



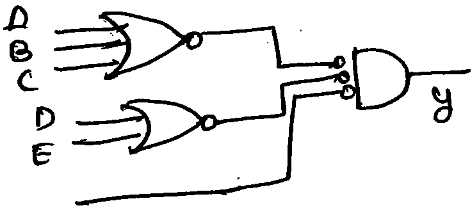
NOR - NOR Implementation:

(1) $y = (A+B+C)(D+E)F$.

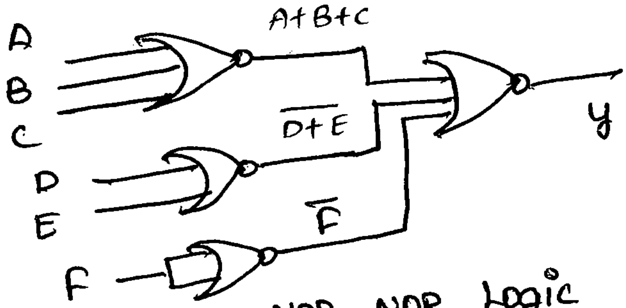


OR - AND Logic

$\equiv$



NOR - Bobbled AND.



NOR - NOR Logic

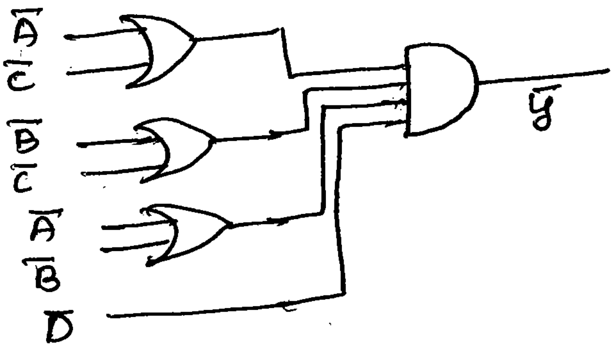
$$\overline{\overline{A+B+C} + \overline{D+E} + \overline{F}}$$
$$= \overline{\overline{(A+B+C)}} \cdot \overline{\overline{(D+E)}} \cdot \overline{\overline{F}}$$
$$= (A+B+C)(D+E)F$$

(2) Implement the following Boolean function with NOR-NOR logic $y = AC + BC + AB + D$.

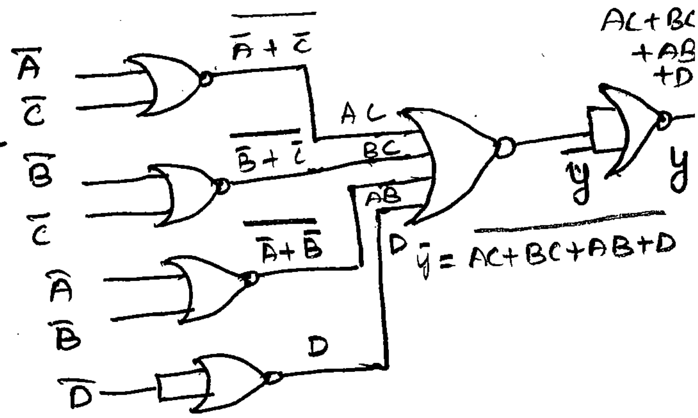
Solution: Convert to POS form

By Duality * Change $\cdot$ to $+$, $+$ to $\cdot$
* Take Complement of the function.

$$\overline{y} = (\overline{A}+\overline{C}) \cdot (\overline{B}+\overline{C}) \cdot (\overline{A}+\overline{B}) \cdot \overline{D}$$



OR - AND logic



NOR - NOR Logic

3. Implement the following Boolean function with NOR-NOR logic $F = (A,B,C) = \Pi M(0,2,4,5,6)$.

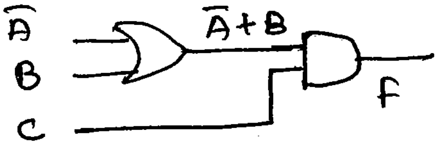
A \ BC	00	01	11	10
0	0			0
1	0	0		0

C

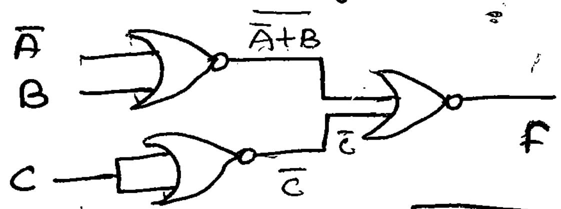
$\overline{A}+B$

$$F = (\overline{A}+B) \cdot C.$$

OR - AND Logic



NOR - NOR logic



$\overline{\overline{(\bar A+B)}+\bar C}$

$\overline{\overline{(\bar A+B)}}\cdot\overline{\overline{C}}$

$(\bar A+B)\cdot C$

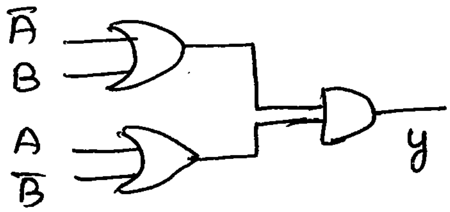
4. Implement EX-NOR gate using only NOR - gate.

$y = AB + \bar A\bar B$

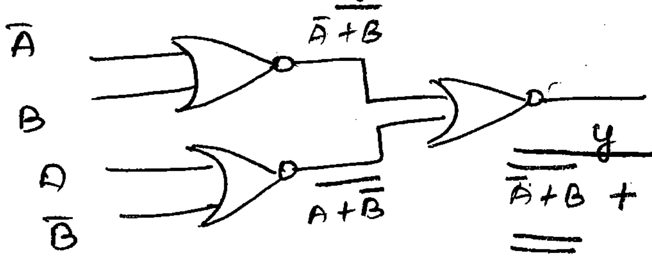
$= \overline{A\bar B + \bar A B}$

$= \overline{A\bar B}\cdot\overline{\bar A B}$

$= (\bar A+B)\cdot(A+\bar B)$



NOR - NOR Logic



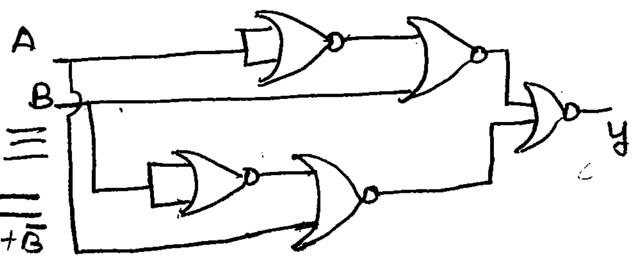
y

$\overline{\overline{\bar A+B}+\overline{A+\bar B}}$

$\overline{\overline{\bar A+B}}\cdot\overline{\overline{A+\bar B}}$

$(\bar A+B)\cdot(A+\bar B)$

$\equiv$



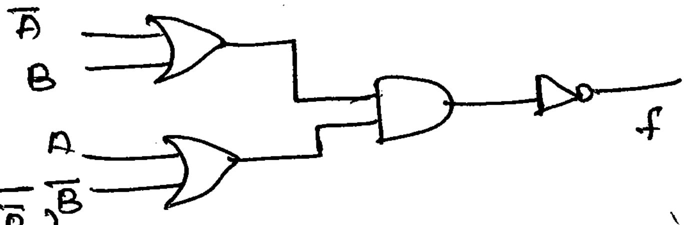
5. Implement Ex-OR using NOR only.

$F = A\bar B + \bar A B$

$\overline{\overline{F}} = \overline{\overline{A\bar B+\bar A B}}$

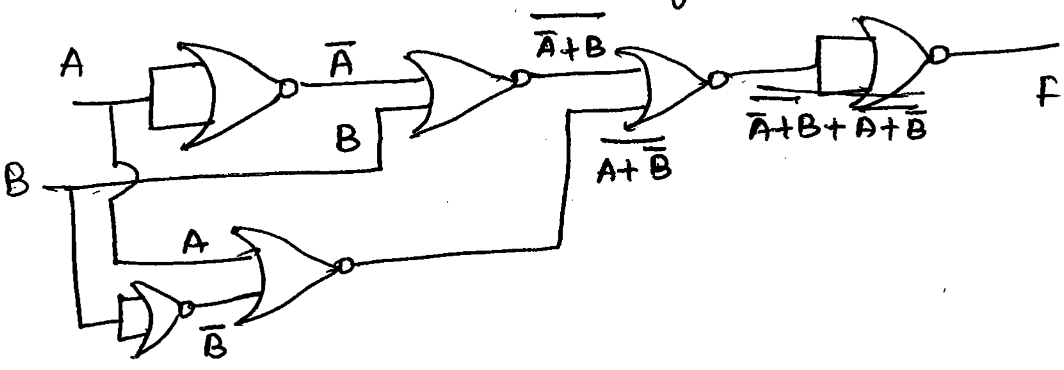
$= \overline{\overline{A\bar B}\cdot\overline{\bar A B}}$

$= \overline{(\bar A+B)\cdot(A+\bar B)}$



OR - AND - NOT Logic

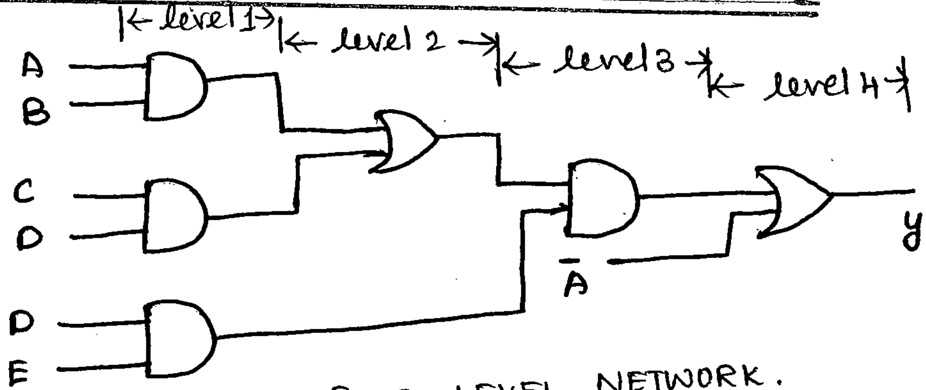
NOR - NOR logic



$\overline{\overline{\overline{\bar A+B}+\overline{A+\bar B}}}$

$\overline{(\bar A+B)(A+\bar B)}$

Multilevel Gate Implementations.

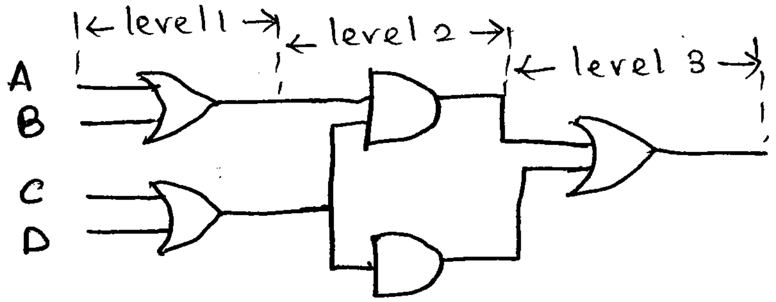


FOUR LEVEL NETWORK.

We know that logic gates can be cascaded to get the desired output. The maximum number of gates cascaded in between a network i/p and the output is referred to number of levels of gates.

* Thus the Boolean function written in SOP form and POS form are the two level gate networks.
* usually it is assumed that all variables and their complements are available as network inputs.
* Thus we will not normally count inverters connected directly to input variables as a seperate levels in the network.

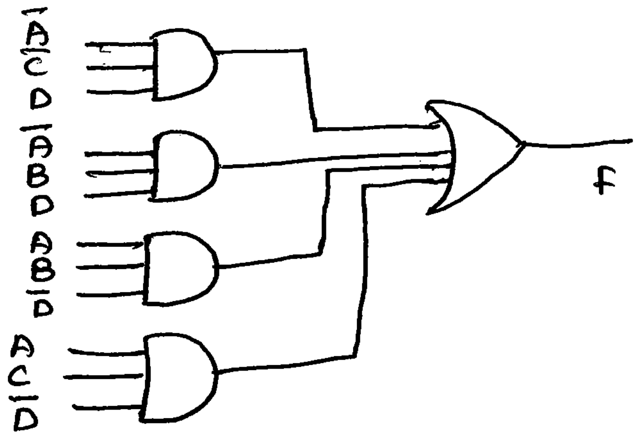
Example of Three level network.



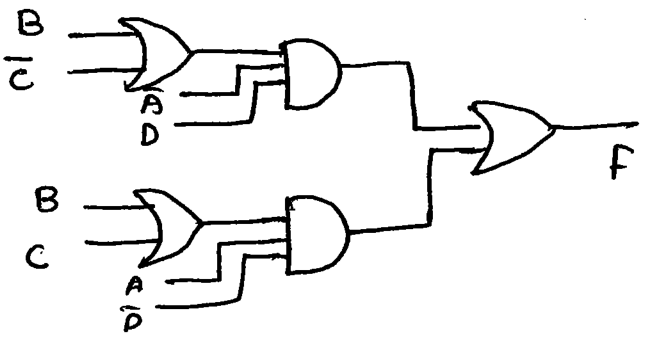
Example:

$$F = \bar{A}\bar{C}D + \bar{A}BD + AB\bar{D} + AC\bar{D}$$

* 5 gates
* 2 level
*



$$F = \bar{A}\bar{C}D + \bar{A}BD + AB\bar{D} + AC\bar{D}$$
$$= \bar{A}D(B+\bar{C}) + A\bar{D}(B+C)$$

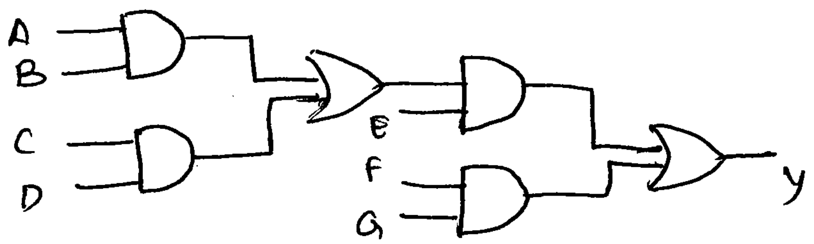


levels = 3
gates = 5
I/p's = 12

Multilevel NAND - NOR Implementations.

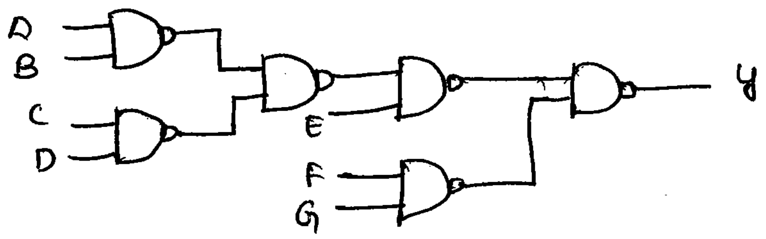
AND- OR Network can be Converted to NAND-NAND and also to NOR- NOR Network. from OR- AND

(i)

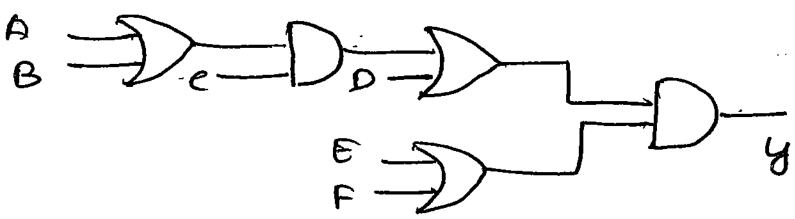


AND-OR- Network.

NAND- NAND Implementations.

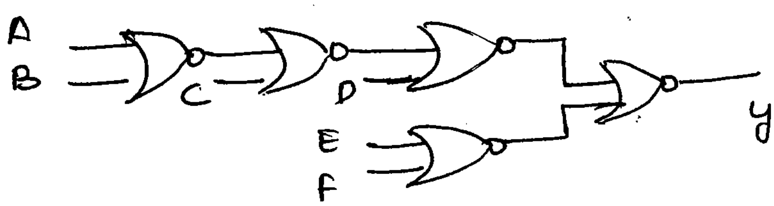


(ii)



OR-AND NETWORK.

NOR- NOR Implementation.



Multiple Output Implementation.

1. Simplify the following functions and draw the logic diagram for the same.

$$F_1 = F(A,B,C) = \Sigma(1,2,3,5)$$
$$F_2 = F(A,B,C) = \Sigma(1,3,5,7)$$
$$F_3 = F(A,B,C) = \Sigma(2,3,4,5)$$

For F1

A \ BC	00	01	11	10
0		1	1	1
1		1		

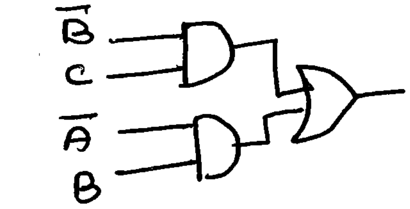
for F2

A \ BC	00	01	11	10
0		1	1	
1		1	1	

for F3

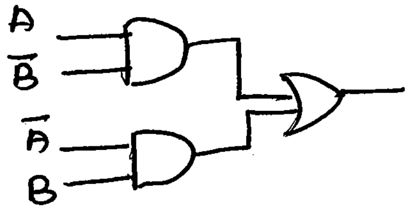
A \ BC	00	01	11	10
0			1	1
1	1	1		

$F_1 = \bar{B}C + \bar{A}B$

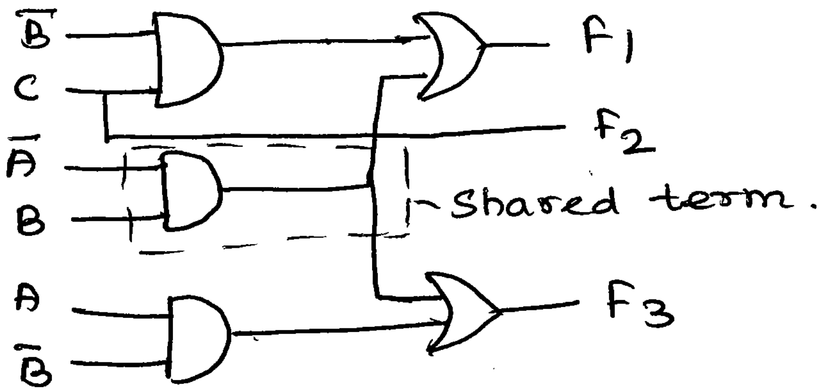


$F_2 = C$

$F_3 = A\bar{B} + \bar{A}B$



Combined circuit



2. Simplify the following functions and draw the logic diagram for the same.

$$F_1 = F(A,B,C) = \Sigma(2,3,7)$$
$$F_2 = F(A,B,C) = \Sigma(0,1,3).$$

for F1

A \ BC	00	01	11	10
0			1	1
1			1	

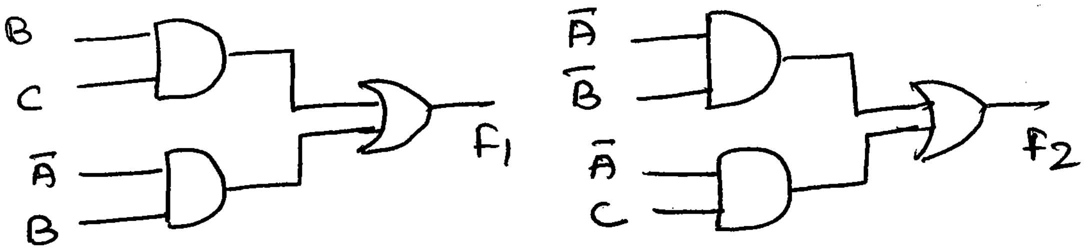
for F2

A \ BC	00	01	11	10
0	1	1	1	
1				

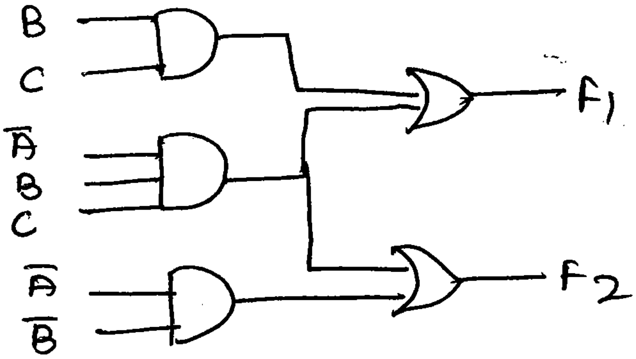
$F_1 = BC + \bar{A}B$ (or)

$F_2 = \bar{A}\bar{B} + \bar{A}C$

F_1



multiple o/p considered.

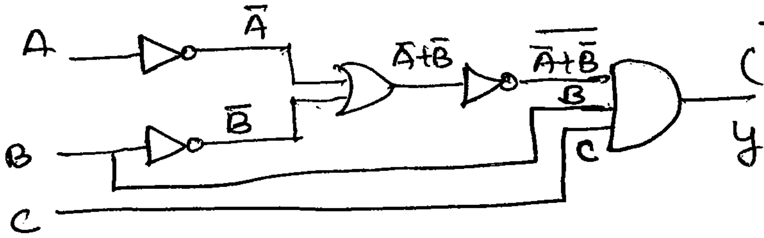


common term has to be found for multiple output Implementation.

Note: If multiple outputs are considered together does not gurantee a minimum no. of IC's.

Extra:

1. write a Boolean expression for the output y and determine the truth table.



$(\overline{\overline{A}+\overline{B}}) \cdot B \cdot C$

$y = \overline{\overline{A}} \cdot \overline{\overline{B}} \cdot B \cdot C$

$= \overline{\overline{A}} \cdot B \cdot B \cdot C$

$= A \cdot B \cdot C$

Truth table.

A	B	C	y = ABC
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Karnaugh Map

Boolean expressions can be simplified by Boolean algebra needs better understanding of Boolean laws, rules and theorems. during simplification we have to predict each sucessive steps.

We can also simplify the expressions by mapping also by systamatic approach.

1. one variable 2. Two variable 3. three variable. 4. four variable.

2 variable map contains - $2^2 = 4$ cells.
3 variable map Contains - $2^3 = 8$ cells
4 variable map Contains - $2^4 = 16$ cells.

1 variable map

A	0	1

2 variable map

A \ B	0	1
0		
1		

3 variable map

A \ BC	00	01	11	10
0				
1				

4 variable map.

AB \ CD	00	01	11	10
00				
01				
11				
10				

* Product terms are assigned to the cells.

1 variable map

A	
$\bar{A}$	$\bar{A}$
A	A

A \ B	$\bar{B}$	B
$\bar{A}$	$\bar{A}\bar{B}_0$	$\bar{A}B_1$
A	$A\bar{B}_2$	AB_3

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	$\bar{A}\bar{B}\bar{C}_0$	$\bar{A}\bar{B}C_1$	$\bar{A}BC_3$	$\bar{A}B\bar{C}_2$
A	$A\bar{B}\bar{C}_4$	$A\bar{B}C_5$	ABC_7	$AB\bar{C}_6$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}\bar{D}$ 0	$\bar{A}\bar{B}\bar{C}D$ 1	$\bar{A}\bar{B}CD$ 3	$\bar{A}\bar{B}C\bar{D}$ 2
$\bar{A}B$	$\bar{A}B\bar{C}\bar{D}$ 4	$\bar{A}B\bar{C}D$ 5	$\bar{A}BCD$ 7	$\bar{A}BC\bar{D}$ 6
AB	$AB\bar{C}\bar{D}$ 12	$AB\bar{C}D$ 13	$ABCD$ 14	$ABC\bar{D}$ 15
$A\bar{B}$	$A\bar{B}\bar{C}\bar{D}$ 8	$A\bar{B}\bar{C}D$ 9	$A\bar{B}CD$ 11	$A\bar{B}C\bar{D}$ 10

ways to represent for SOP expression.

A	0
0	m_0
1	m_1

A \ B	0	1
0	m_0	m_1
1	m_2	m_3

A \ BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

ways to map the sum terms.

A	A
$\bar{A}$	$\bar{A}$

A \ B	B	$\bar{B}$
A	$A+B$	$A+\bar{B}$
$\bar{A}$	$\bar{A}+B$	$\bar{A}+\bar{B}$

A \ BC	$B+C$	$B+\bar{C}$	$\bar{B}+\bar{C}$	$\bar{B}+C$
A	$A+B+C$	$A+B+\bar{C}$	$A+\bar{B}+\bar{C}$	$A+\bar{B}+C$
$\bar{A}$	$\bar{A}+B+C$	$\bar{A}+B+\bar{C}$	$\bar{A}+\bar{B}+\bar{C}$	$\bar{A}+\bar{B}+C$

AB \ CD	$C+D$	$C+\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$
$A+B$	$A+B+C+D$	$A+B+C+\bar{D}$	$A+B+\bar{C}+\bar{D}$	$A+B+\bar{C}+D$
$A+\bar{B}$	$A+\bar{B}+C+D$	$A+\bar{B}+C+\bar{D}$	$A+\bar{B}+\bar{C}+\bar{D}$	$A+\bar{B}+\bar{C}+D$
$\bar{A}+\bar{B}$	$\bar{A}+\bar{B}+C+D$	$\bar{A}+\bar{B}+C+\bar{D}$	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$	$\bar{A}+\bar{B}+\bar{C}+D$
$\bar{A}+B$	$\bar{A}+B+C+D$	$\bar{A}+B+C+\bar{D}$	$\bar{A}+B+\bar{C}+\bar{D}$	$\bar{A}+B+\bar{C}+D$

ways to represent POS expression.

	0
0	M_0
1	M_1

A \ B	0	1
0	M_0	M_1
1	M_2	M_3

A \ BC	00	01	11	10
0	M_0	M_1	M_3	M_2
1	M_4	M_5	M_7	M_6

AB \ CD	00	01	11	10
00	M_0	M_1	M_3	M_2
01	M_4	M_5	M_7	M_6
11	M_{12}	M_{13}	M_{15}	M_{14}
10	M_8	M_9	M_{11}	M_{10}

Plotting of K-Map.

(1) for Truth table

(2) SOP expression.

(3) POS expression.

Truth Table:

(1) 2-Variable.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

A \ B	0	1
0	0	1
1	1	0

A \ B	$\bar{B}$	B
$\bar{A}$	0	1
(or) A	1	0

(2) 3 Variable.

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

A \ BC	00	01	11	10
0	0	0	0	1
1	1	1	1	0

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	0	0	1
A	1	1	1	0

4 variable.

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

AB \ CD	00	01	11	10
00	1	0	1	0
01	1	1	1	0
11	1	0	1	1
10	0	0	0	1

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	1	0
$\bar{A}B$	1	1	1	0
AB	1	0	1	1
$A\bar{B}$	0	0	0	1

Representation of SOP on K-map.

* Boolean expressions in the SOP form can be plotted on K-map by placing '1' in the map for the terms in the expression and remaining zero.

Example: Plot Boolean expression $Y = AB\bar{C} + ABC + \bar{A}\bar{B}C$ on the K-map.

Solution: $AB\bar{C} = 110$, $ABC = 111$; $\bar{A}\bar{B}C = 001$.

A \ BC	00	01	11	10
0		1 $\bar{A}\bar{B}C$		
1			1 ABC	1 $AB\bar{C}$

(or)

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$		1 $\bar{A}\bar{B}C$		
A			1 ABC	1 $AB\bar{C}$

Plot Boolean expression

$y = \bar{A}B\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}BC\bar{D} + A\bar{B}CD + AB\bar{C}D$ on K-map.

AB \ CD	00	01	11	10
00	0	0	0	0
01	1 ($\bar{A}B\bar{C}\bar{D}$)	0	0	1 ($\bar{A}BC\bar{D}$)
11	0	1 ($AB\bar{C}D$)	0	0
10	0	0	1	1 ($A\bar{B}C\bar{D}$)

$\bar{A}B\bar{C}\bar{D} = 0100$

$A\bar{B}C\bar{D} = 1010$

$\bar{A}BC\bar{D} = 0110$

$A\bar{B}CD = 1011$

$AB\bar{C}D = 1101$

Representation of POS on K-map.

* Sum terms in the expression are represented by '0' in K map.
* remaining by '1'

Example: plot Boolean expression $y = (A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)(A+B+\bar{C})$ on the K-map.

Solution:

$A+\bar{B}+C \Rightarrow 010 \Rightarrow M_2$

$A+\bar{B}+\bar{C} \Rightarrow 011 \Rightarrow M_3$

$\bar{A}+\bar{B}+C \Rightarrow 110 \Rightarrow M_6$

$A+B+\bar{C} \Rightarrow 001 \Rightarrow M_1$

A \ BC	00	01	11	10
0	1	0	0	0
1	1	1	1	0

Plot Boolean expression $y = (A+B+C+\bar{D})(A+\bar{B}+\bar{C}+D)(A+B+\bar{C}+\bar{D})(\bar{A}+\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+\bar{C}+D)$. on K-map.

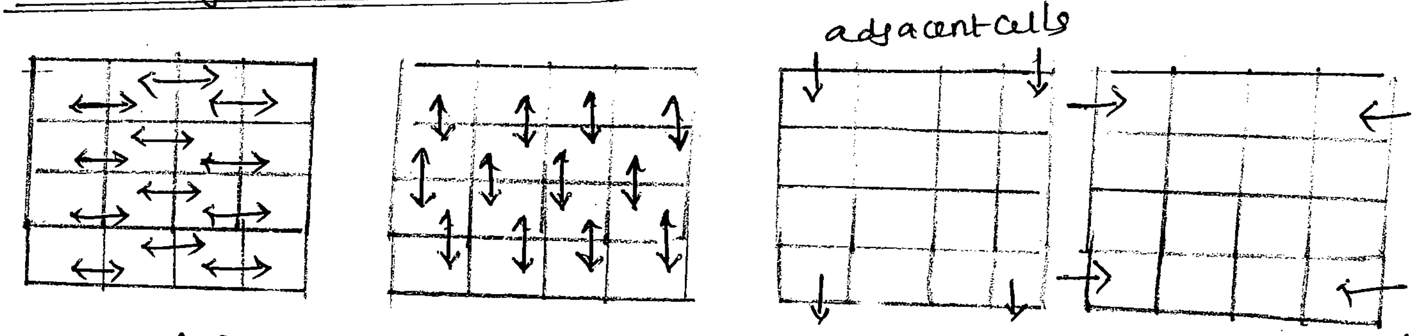
$A+B+C+\bar{D} = 0001 \Rightarrow M_1$ $\quad \bar{A}+\bar{B}+C+\bar{D} \Rightarrow 1101 \Rightarrow M_{13}$

$A+\bar{B}+\bar{C}+D = 0110 \Rightarrow M_6$ $\quad \bar{A}+\bar{B}+\bar{C}+D \Rightarrow 1110 \Rightarrow M_{14}$

$A+B+\bar{C}+\bar{D} = 0011 \Rightarrow M_3$

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	1	1	0
11	1	0	1	0
10	1	1	1	1

Grouping of Cells for simplification. (Adjacent cells)



Example:

(a) $Y = \bar{A}\bar{B}C + \bar{A}BC$

$= \bar{A}C(B+\bar{B})$

$= \bar{A}C$

adjacent cells

A \ BC	00	01	11	10
0	0	1	1	0
1	0	0	0	0

$\bar{A}(\bar{B}C+BC)$
$= \bar{A}C$

(b) $Y = \bar{A}BC + ABC$

$= BC(A+\bar{A})$

$= BC$

$= BC$

A \ BC	00	01	11	10
0	0	0	1	0
1	0	0	1	0

(c) $Y = A\bar{B}\bar{C} + AB\bar{C}$

$= A\bar{C}(\bar{B}+B)$

$= A\bar{C}$

A \ BC	00	01	11	10
0				
1	1			1

d) $Y = \bar{A}\bar{B}\bar{C}D + A\bar{B}\bar{C}D$.

$= \bar{B}\bar{C}D(A+\bar{A})$

$= \bar{B}\bar{C}D$.

	AB \ CD	00	01 ($\bar{C}D$)	11	10
$\bar{A}\bar{B}$	00	0	1	0	0
$(\bar{B})$	01	0	0	0	0
	11	0	0	0	0
$A\bar{B}$	10	0	1	0	0

e) $Y = \bar{A}\bar{B}C + \bar{A}BC + ABC$

$= 001 + 011 + 111 = \bar{A}\bar{B}C + \bar{A}BC + ABC + \bar{A}BC$

$\bar{A}+\bar{A} = \bar{A}$ $\quad = \bar{A}C(\bar{B}+B) + ABC$

$= \bar{A}C + ABC + \bar{A}BC$

$= \bar{A}C + BC(A+\bar{A})$

$= \bar{A}C + BC$.

$g_2 = \bar{A}C$

A \ BC	00	01	11	10
$\bar{A}$ 0		1	1	
A 1			1	

$g_1 = BC$

f. $Y = \bar{A}\bar{B}C + \bar{A}BC + ABC + AB\bar{C}$

$= AB(C+\bar{C}) + \bar{A}C(\bar{B}+B)$

$= AB + \bar{A}C$

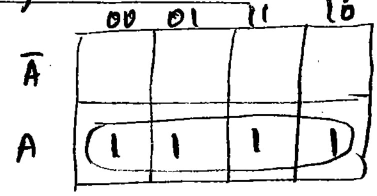
$g_1 = \bar{A}C$

A \ BC	00	01	11	10
0		1	1	
1			1	1

g_2 AB

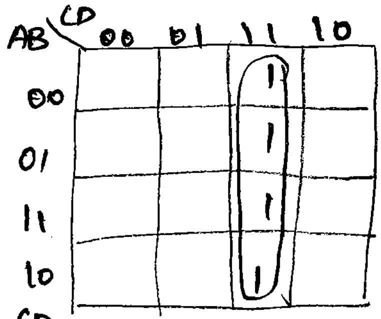
Grouping four Adjacent Ones (quad)

Example 1



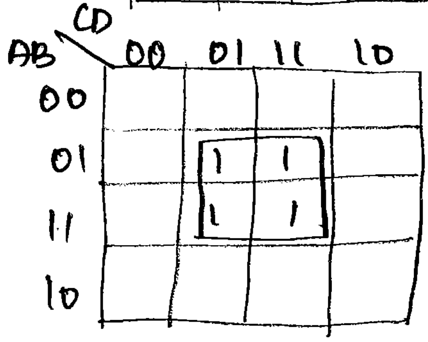
$Y = A$

Example 2.

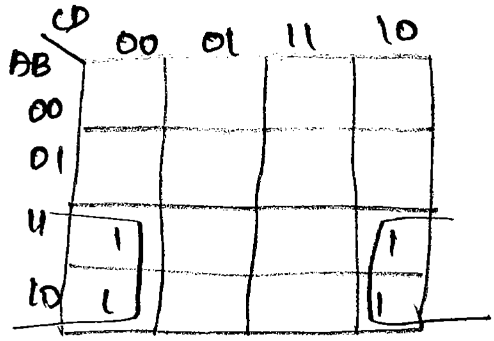


$y = CD$

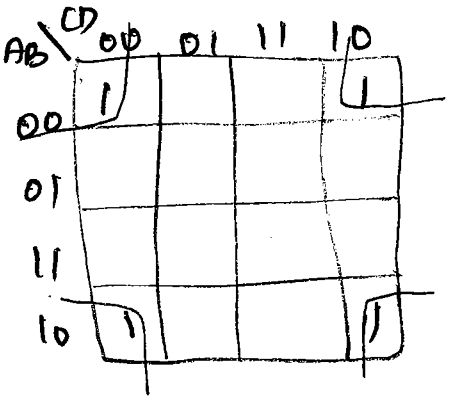
Example 3



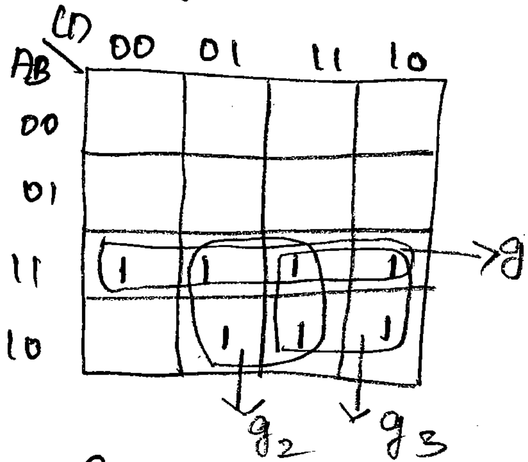
$Y = BD$



$y = A\bar{D}$



$y = \bar{B}\bar{D}$



$g_1 = AB \quad g_2 = AD \quad g_3 = AC$

$y = AB + AD + AC$

Grouping of 8 adjacent Ones:

$$y = \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D + ABCD + ABC\bar{D}$$

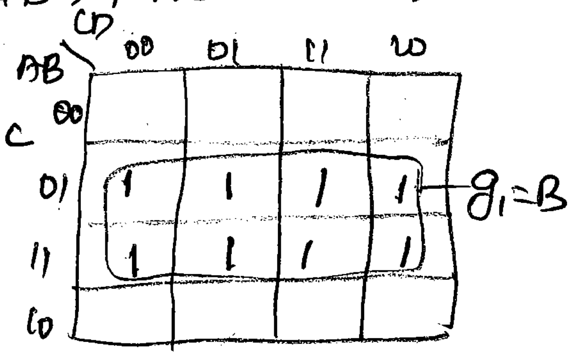
$$= \bar{A}B\bar{C}(D+\bar{D}) + \bar{A}BC(D+\bar{D}) + AB\bar{C}(\bar{D}+D) + ABC(D+\bar{D})$$

$$= \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC$$

$$= \bar{A}B(\bar{C}+C) + AB(\bar{C}+C)$$

$$= \bar{A}B + AB.$$

$$= B(\bar{A}+A) = B$$



Example 2

AB \ CD	00	01	11	10
00		1	1	
01		1	1	
11		1	1	
10		1	1	

$y = D$

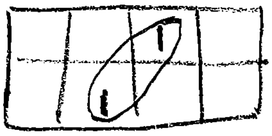
AB \ CD	00	01	11	10
00	1	1	1	1
01				
11				
10	1	1	1	1

$y = \overline{B}$

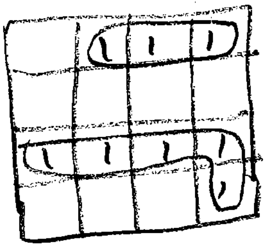
AB \ CD	00	01	11	10
00	1			1
01	1			1
11	1			1
10	1			1

$y = \overline{D}$

illegal grouping:



diagonal grouping



Grouping of odd number of cells is illegal.

Simplification of SOP expression:

Example 1: minimize the expression $y = A\overline{B}C + \overline{A}\overline{B}C + \overline{A}BC + A\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C}$

solution: map plotted for the given variable.

A \ BC	$\overline{B}\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$ 0	1 (0)	1 (1)	1 (3)	0 (2)
A 1	1 (4)	1 (5)	0 (7)	0 (6)

$A\overline{B}C = 101$ $\quad A\overline{B}\overline{C} = 100$

$\overline{A}\overline{B}C = 001$ $\quad \overline{A}\overline{B}\overline{C} = 000$

$\overline{A}BC = 011$

$g_1 = \overline{B}$ $\quad g_2 = C\overline{A}$. $\quad$ Ans: $y = C\overline{A} + \overline{B}$

Example 2: minimize the expression.

$$y = \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D + AB\overline{C}\overline{D} + AB\overline{C}D + A\overline{B}\overline{C}D + \overline{A}\overline{B}C\overline{D}$$

AB \ CD	00	01	11	10
00	0	0	0	1
01	1	1	0	0
11	1	1	0	0
10	0	1	0	0

$\overline{A}B\overline{C}\overline{D} = 0100$ $\quad \overline{A}\overline{B}C\overline{D} = 0010$

$\overline{A}B\overline{C}D = 0101$

$AB\overline{C}\overline{D} = 1100$

$AB\overline{C}D = 1101$

$A\overline{B}\overline{C}D = 1001$

Ans: $y = \overline{A}\overline{B}C\overline{D} + A\overline{C}D + B\overline{C}$

$g_1 = B\overline{C}$ $\quad g_2 = A\overline{C}D$ $\quad g_3 = \overline{A}\overline{B}C\overline{D}$

Example 3 :

Reduce the following four variables function to its minimum sum of products form.

$$y = \bar{A}\bar{B}C\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D} + A\bar{B}CD + A\bar{B}\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}\bar{D}.$$

Solution:

AB \ CD	00	01	11	10
00	1 (0)	0 (1)	1 (3)	1 (2)
01	0 (4)	0 (5)	0 (7)	0 (6)
11	1 (12)	0 (13)	0 (15)	1 (14)
10	1 (8)	0 (9)	1 (11)	1 (10)

$g_3 = \bar{B}\bar{D}$

$g_1 = A\bar{D}$ $\quad g_2 = \bar{B}C$.

$\bar{A}\bar{B}C\bar{D} = 0010$

$ABC\bar{D} = 1110$

$A\bar{B}C\bar{D} = 1010$

$A\bar{B}CD = 1011$

$A\bar{B}\bar{C}\bar{D} = 1000$

$AB\bar{C}\bar{D} = 1100$

$\bar{A}\bar{B}CD = 0011$

$\bar{A}\bar{B}\bar{C}\bar{D} = 0000$

Solution: $y = \bar{B}\bar{D} + A\bar{D} + \bar{B}C$.

Example 4 :

Reduce the following function to its minimum sum of products form.

$$y = \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D + ABCD + A\bar{B}CD$$

AB \ CD	00	01	11	10
00	0 (0)	1 (1)	0 (3)	0 (2)
01	0 (4)	1 (5)	1 (7)	1 (6)
11	1 (12)	1 (13)	1 (15)	0 (14)
10	0 (8)	0 (9)	1 (11)	0 (10)

$g_1 = \bar{A}\bar{C}D$

g_2 $AB\bar{C}$

$g_3 = ACD$

$g = \bar{A}BC$

$\bar{A}\bar{B}\bar{C}D = 0001 = 1$ $\quad AB\bar{C}D = 1101 = 13$

$\bar{A}B\bar{C}D = 0101 = 5$ $\quad ABCD = 1111 = 15$

$\bar{A}BCD = 0111 = 7$ $\quad A\bar{B}CD = 1011 = 11$

$\bar{A}BC\bar{D} = 0110 = 6$

$AB\bar{C}\bar{D} = 1100 = 12$

Answer: $\bar{A}BC + ACD + AB\bar{C} + \bar{A}\bar{C}D$

Example 5 :

Reduce the following function using K-map technique and implement using gates:

$$F(ABCD) = \bar{A}\bar{B}D + AB\bar{C}\bar{D} + \bar{A}BD + ABC\bar{D}.$$

$$= \bar{A}\bar{B}D(C+\bar{C}) + AB\bar{C}\bar{D} + \bar{A}BD(C+\bar{C}) + ABC\bar{D}.$$

$$= \bar{A}\bar{B}DC + \bar{A}\bar{B}D\bar{C} + AB\bar{C}\bar{D} + \bar{A}BDC + \bar{A}B\bar{C}D + ABC\bar{D}.$$

$$= \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}D + AB\bar{C}\bar{D} + \bar{A}BCD + \bar{A}B\bar{C}D + ABC\bar{D}.$$

$$= 0011 + 0001 + 1100 + 0111 + 0101 + 1110$$

$$= 3 + 1 + 12 + 7 + 5 + 14$$

AB \ CD	00	01	11	10
00		1	1	
01		1	1	
11	1			1
10				

$g_1 = \bar{A}D$

$g_2 = AB\bar{D}$

Solution: $y = AB\bar{D} + \bar{A}D$.

6. Reduce the following function using K-map techniques.

$f(A,B,C,D) = \Sigma m(0,1,4,8,9,10)$.

AB \ CD	00	01	11	10
00	1	1	0	0
01	1	0	0	0
11	0	0	0	0
10	1	1	0	1

$g_1 = \bar{A}\bar{C}\bar{D}$

$g_2 = A\bar{B}\bar{D}$

$g_3 = \bar{B}\bar{C}$

Ans: $F(A,B,C,D) = \bar{A}\bar{C}\bar{D} + \bar{B}\bar{C} + A\bar{B}\bar{D}$

7. Plot the following Boolean function on a Karnaugh map and simplify it.

$F(w,x,y,z) = \Sigma(0,1,2,4,5,6,8,9,12,13,14)$.

AB \ CD	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	1	1	0	1
10	1	1	0	0

$g_2 = \bar{A}\bar{D}$

$g_3 = B\bar{D}$

$g_1 = \bar{C}$

Answer $= B\bar{D} + \bar{A}\bar{D} + \bar{C}$

$= x\bar{z} + \bar{w}\bar{z} + \bar{y}$

8. Show the Karnaugh map with the encircled groups for the Boolean function $f = \bar{C} + \bar{A}\bar{D} + A\bar{B}\bar{D}$.

AB \ CD	00	01	10	11
00	1	1		1
01	1	1		1
11	1	1		
10	1	1		1

$\bar{A}\bar{D}$

$A\bar{B}\bar{D}$

$\bar{C}$

Incompletely specified functions (Don't care terms)

* In some logic circuits, certain input conditions never occur, therefore the corresponding output never appears.
* The output level is not defined, either high or low.
* indicated by 'x' or 'd' called don't care conditions or incompletely specified function.

Example:

$F(A,B,C,D) = \sum m(1,2,4,7,8) + d(10,11,12,13,14,15).$

Minimization of incompletely specified functions.

* don't care condition, it is either '0' or '1'.

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	x
1	1	1	x

A \ BC	00	01	11	10
0	0	1	1	0
1	0	1	X	X

⇓

A \ BC	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$y = C$

Example: 1.

Find the reduced SOP form of the following function.

$F(A,B,C,D) = \sum m(1,3,7,11,15) + \sum d(0,2,4).$

AB \ CD	00	01	11	10
00	X (0)	1 (1)	1 (3)	X (2)
01	X (4)	(5)	1 (7)	(6)
11	(12)	(13)	1 (15)	(14)
10	(8)	(9)	1 (11)	(10)

AB \ CD	00	01	11	10
00	0	1	1	0
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

$g_1 = \bar{A}\bar{B} + CD.$

2. Reduce the following function using karnaugh map

$F(A,B,C) = \sum m(0,1,3,7) + \sum d(2,5).$

A \ BC	00	01	11	10
0	1	1	1	X
1		X	1	

⇒

A \ BC	00	01	11	10
0	1	1	1	1
1		1	1	

$g_1 = \bar{A}$

$y = \bar{A} + C$

$g_2 = C$

3 Reduce the following function using Karnaugh map technique.

$$F(A,B,C,D) = \Sigma m(5,6,7,12,13) + \Sigma d(4,9,14,15).$$

AB \ CD	00	01	11	10
00	0	1	3	2
01	X 4	1 5	1 7	1 6
11	1 12	1 13	X 15	X 14
10	8	X 9	11	10

Ans. $f = B$.

AB \ CD	00	01	11	10
00				
01	1	1	1	1
11	1	1	1	1
10		0		

$g_1 = B$

4 Reduce the following function using Karnaugh map

$$f(w,x,y,z) = \Sigma m(0,7,8,9,10,12) + \Sigma d(2,5,13).$$

AB \ CD	00	01	11	10
00	1			X
01		X	1	
11	1	X		
10	1	1		1

$g_3 = \bar{A}BD$

$g_1 = \bar{B}\bar{D}$ $g_2 = A\bar{C}$

$$f(w,x,y,z) = \bar{x}\bar{z} + w\bar{y} + \bar{w}xz$$

5 Solve $g(ABCD) = \Sigma m(1,3,4,6,11) = \Sigma d(0,8,10,12,13).$

AB \ CD	00	01	11	10
00	X	1	1	
01	1			1
11	X	X		
10	X		1	X

$g_1 = \bar{A}\bar{B}D$

$g_2 = \bar{A}B\bar{D}$

$\bar{B}CD$

$$g(A,B,C,D) = \bar{A}\bar{B}D + \bar{A}B\bar{D} + \bar{B}CD$$

6. Express the following function as the minimal sum of Product using a k-map. $f(a,b,c,d) = \Sigma(0,2,4,5,6,8,10,15) + \Sigma\phi(7,13,14).$

ab \ cd	00	01	11	10
00	1			1
01	1	1	X	1
11		X	1	X
10	1			1

$g_1 = \bar{a}b01$

$g_2 = bd11$

$g_3 = \bar{b}\bar{d}$

$f(a,b,c,d) = \bar{a}b + bd + \bar{b}\bar{d}$

ab \ cd	00	01	11	10
00	1			1
01	1	1	X	1
11		X	1	X
10	1			1

$= \bar{b}\bar{d} + \bar{a}b + bc$

ab \ cd	00	01	11	10
00	1			1
01	1	1	X	1
11		X	1	X
10	1			1

$= bd + \bar{b}\bar{d} + \bar{a}\bar{d}$

Note:

* After grouping the cells, the sum terms which appear in the K-map are called prime implicants group.

* A circuit designer is free to make the o/p of any don't care conditions either a '0' or a '1' in order to produce the simplest o/p expression.

Simplification of POS Expressions.

* the sum terms in the expression is marked as zero in K-map.

* grouping of '0' as to be done.

<u>Example:</u>

1. minimize the expression

$$Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+C)(A+B+C).$$

A \ B+C	00	01	11	10
0	0	0	0	
1	0		0	

$A+B+\bar{C} = 001 \Rightarrow M_1$

$A+\bar{B}+\bar{C} = 011 \Rightarrow M_3$

$\bar{A}+\bar{B}+\bar{C} = 111 \Rightarrow M_7$

$\bar{A}+B+C = 100 \Rightarrow M_4$

$A+B+C = 000 \Rightarrow M_0$

$g_1 = B+C$ $\quad g_2 = \bar{B}+\bar{C}$ $\quad g_3 = A+\bar{C}$

Answer: $Y = (B+C)(\bar{B}+\bar{C})(A+\bar{C})$

2. Minimize the following expressions in POS form.

$$Y = (\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{C}+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})(\bar{A}+B+C+D)(A+\bar{B}+\bar{C}+D)(A+\bar{B}+\bar{C}+\bar{D})(A+B+C+D)(\bar{A}+\bar{B}+C+\bar{D})$$

A+B \ C+D	00	01	11	10
00	0			
01			0	0
11	0	0	0	0
10	0			

$\bar{A}+\bar{B}+C+D = 1100 = 12$

$\bar{A}+\bar{B}+\bar{C}+D = 1110 = 14$

$\bar{A}+\bar{B}+\bar{C}+\bar{D} = 1111 = 15$

$\bar{A}+B+C+D = 1000 = 8$

$A+\bar{B}+\bar{C}+D = 0110 = 6$

$A+\bar{B}+\bar{C}+\bar{D} = 0111 = 7$

$A+B+C+D = 0000 = 0$

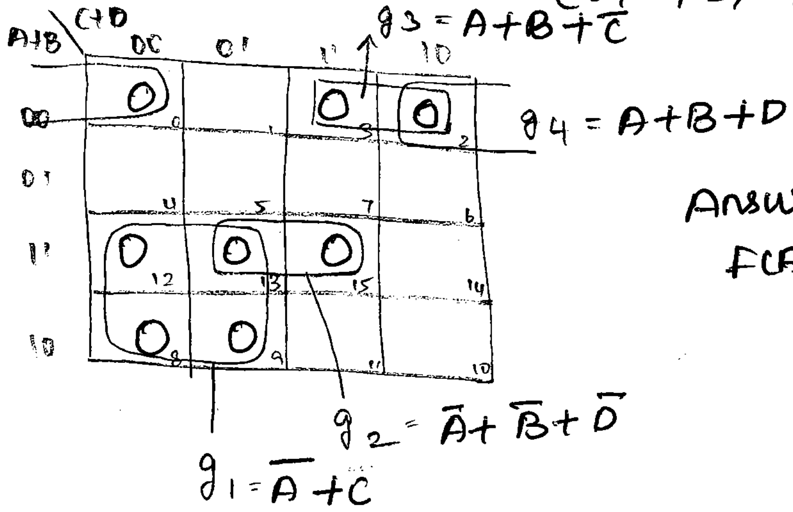
$\bar{A}+\bar{B}+C+\bar{D} = 1101 = 13$

$g_1 = \bar{B}+\bar{C}$ $\quad g_2 = \bar{A}+\bar{B}$ $\quad g_3 = (B+C+D)$

Ans: $Y = (B+C+D)(\bar{B}+\bar{C})(\bar{A}+\bar{B})$

3. Reduce the following function using k-map technique.

$F(A,B,C,D) = \pi M(0,2,3,8,9,12,13,15)$.

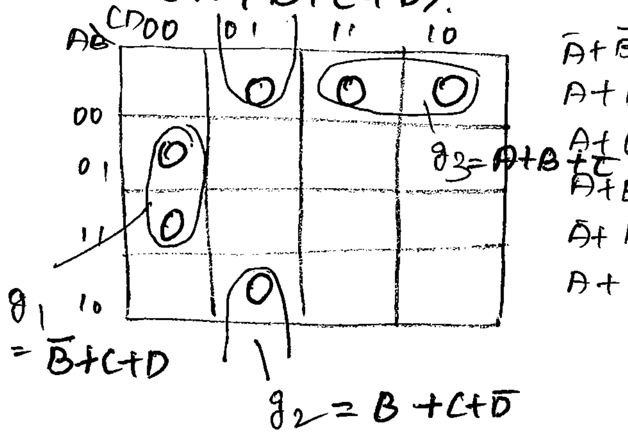


Answer:

$F(A,B,C,D) = (A+B+\overline{C})(\overline{A}+C)(\overline{A}+\overline{B}+\overline{D})(A+B+D)$

4. Simplify using k-map to obtain a minimum POS exp.

$(\overline{A}+\overline{B}+C+D)(A+\overline{B}+C+D)(A+B+C+\overline{D})(A+B+\overline{C}+\overline{D})(\overline{A}+B+C+\overline{D})(A+B+\overline{C}+D)$.



$\overline{A}+\overline{B}+C+D = 1100 = M_{12}$

$A+\overline{B}+C+D = 0100 = M_4$

$A+B+C+\overline{D} = 0001 = M_1$

$A+B+\overline{C}+\overline{D} = 0011 = M_3$

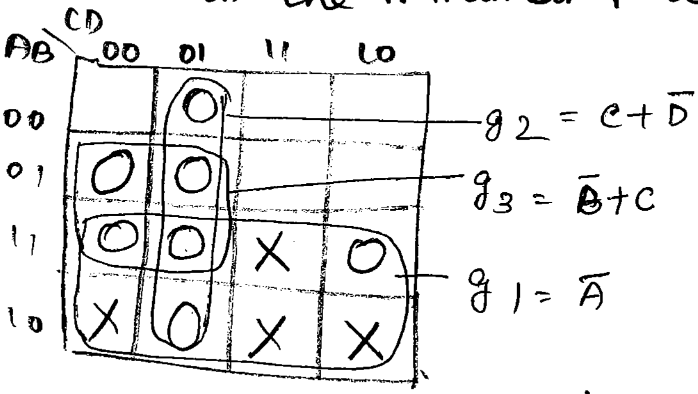
$\overline{A}+B+C+\overline{D} = 1001 = M_9$

$A+B+\overline{C}+D = 0010 = M_2$

Answer.

$y = (A+B+\overline{C})(\overline{B}+C+D)(B+C+\overline{D})$.

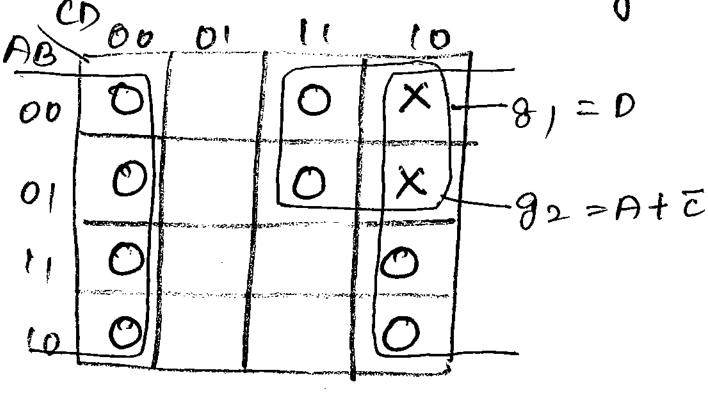
5. Obtain the minimal product of sum for $F = \Sigma(0,2,3,6,7) + d(8,10,11,15)$.



Answer:

$f = \overline{A} \cdot (\overline{B}+C) \cdot (C+\overline{D})$.

6. Reduce the following using k-map $f(A,B,C,D) = \pi(0,3,4,7,8,10,12,14) + d(2,6)$.



Answer

$F(A,B,C,D) = (A+\overline{C}).D$.

Rules for simplifying logic function using K-map.

1. group should not include any cell containing a zero.
2. The number of cells in a group must be a power of 2, such as 1, 2, 4, 8 or 16.
3. Group may be horizontal, vertical but not diagonal.
4. Cell containing 1 must be included in atleast one group
5. Groups may overlap.
6. Each group should be as large as possible to get maximum simplification.
7. Groups may be wrapped around the map.
8. Cell may be grouped more than once.

or

9. we need not group all don't care cells.
10. the above rules are for SOP and for POS except '0' the rest are same.

Limitations of K-Map:

* Convenient as long as the no. of variables is 5 or 6
* for 7, 8, 9 variables it is a impossible tasks.
* Another important point is that the K-map technique simplification is manual technique and simplification process is heavily depends on human abilities.
* To meet this need we go for Tabulation method to simplify called "Quine Mc cluskey or tabular method".

Quine - Mccluskey Method of Minimization.

1. Simplify the following Boolean function by using a Quine Mccluskey method.

$$F(A,B,C,D) = \sum m(0,2,3,6,7,8,10,12,13).$$

Solution.

minterms	Binary representation.					
m_0	0000	m_0	0000	0, 2	00-0	✓
m_2	0010	m_2	0010	0, 8	-000	✓
m_3	0011	m_8	1000	2, 3	001-	✓
m_6	0110	m_3	0011	2, 6	0-10	✓
m_7	0111	m_6	0110	2, 10	-010	✓
m_8	1000	m_{10}	1010	8, 10	10-0	✓
m_{10}	1010	m_{12}	1100	8, 12	1-00	
m_{12}	1100	m_7	0111	3, 7	0-11	✓
m_{13}	1101	m_{13}	1101	6, 7	011-	✓
				12, 13	110-	

0, 2, 8, 10	-0-0
2, 3, 6, 7	0-1-
0, 8, 2, 10	-0-0
2, 3, 6, 7	0-1-
8, 12	1-00
12, 13	110-

0, 2, 8, 10	-0-0
2, 3, 6, 7	0-1-
8, 12	1-00
12, 13	110-

		0	2	3	6	7	8	10	12	13
8, 12	$A\bar{C}\bar{D}$						X		X	
12, 13 ✓	$AB\bar{C}$								X	X
0, 2, 8, 10 ✓	$\bar{B}\bar{D}$	X	X				X	X		
2, 3, 6, 7 ✓	$\bar{A}C$		X	X	X	X				

The final expression is

$$F(A,B,C,D) = AB\bar{C} + \bar{B}\bar{D} + \bar{A}C$$

2. Minimize the expression using McCluskey method.

$$y = \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D}.$$

$$= 0100 + 0101 + 1100 + 1101 + 1001 + 0010.$$

$$= m_4 + m_5 + m_{12} + m_{13} + m_9 + m_2.$$

m_4	0100	m_4	0100	(4,5)	010–	4,5,12,13	–10–
m_5	0101	m_2	0010	(12,13)	110–	(4,12,5,13)	–10–
m_{12}	1100	m_{12}	1100	(4,12)	–100	9,13	1–01
m_{13}	1101	m_5	0101	(5,13)	–101		
m_9	1001	m_9	1001	(9,13)	1–01		
m_2	0010	m_{13}	1101				

4,5,12,13	–10–	$B\bar{C}$
9,13	1–01	$A\bar{C}D$
2	0010	$\bar{A}\bar{B}C\bar{D}$

		2	4	5	9	12	13
$B\bar{C}$	4,5,12,13 ✓		X	X		X	X
$A\bar{C}D$	9,13 ✓				X		X
$\bar{A}\bar{B}C\bar{D}$	2 ✓	X					

Final expression is

$$f = \bar{A}\bar{B}C\bar{D} + A\bar{C}D + B\bar{C}.$$

3. Reduce the following equation using the tabular method.

$$F = m_0 + m_2 + m_3 + m_5 + m_8 + m_{10} + m_{11} + m_{13}.$$

m_0	0000	m_0	0000	(0,2)	00–0
m_2	0010	m_2	0010	(0,8)	–000
m_3	0011	m_8	1000	(2,3)	001–
m_5	0101	m_3	0011	(2,10)	–010
m_8	1000	m_5	0101	(8,10)	10–0
m_{10}	1010	m_{10}	1010	(10,11)	101–
m_{11}	1011	m_{11}	1011	(3,11)	–011
		m_{13}	1101	(5,13)	–101

0,2,8,10	-0-0
0,8,2,10	-0-0
2,3,10,11	-01-
(3,11)	-011
(5,13)	-101

0,2,8,10	-0-0
2,3,10,11	-01-
3,11	-011
5,13	-101

	0	2	3	5	8	10	11	13
$\bar{B}\bar{D}$ (0,2,8,10) ✓	⊗	X			⊗	X		
$\bar{B}C$ (2,3,10,11)		X	X			X	X	
$\bar{B}CD$ (3,11)			X				X	
$B\bar{C}D$ (5,13) ✓				⊗				⊗

minterms not-covered (2,3,10,11) -01- 2 variables → selected (minimum variable)

(3,11) -011 3 variables

Final expression is

$$F = \bar{B}C + \bar{B}\bar{D} + B\bar{C}D.$$

4. Simplify $F(w,x,y,z) = \Sigma m(1,2,3,5,9,12,14,15) + \Sigma d(4,8,11)$.

m_1	0001
m_2	0010
m_3	0011
m_5	0101
m_9	1001
m_{12}	1100
m_{14}	1110
m_{15}	1111
dm_4	0100
dm_8	1000
dm_{11}	1011

m_1	0001
m_2	0010
dm_4	0100
dm_8	1000
m_3	0011
m_5	0101
m_9	1001
m_{12}	1100
m_{14}	1110
dm_{11}	1011
m_{15}	1111

(1,3)	00-1 ✓
(1,5)	0-01
(1,9)	-001 ✓
(2,3)	001-
(4,5)	010-
(4,12)	-100
(8,9)	100-
(8,12)	100-
(3,11)	-011 ✓
(11,15)	1-11
(14,15)	111-
(12,14)	11-0
(9,11)	10-1 ✓

(1,3,9,11)	-0-1
(1,9,3,11)	-0-1

(1, 3, 9, 11) $-0-1$

(1, 5) $0-01$

(2, 3) $001-$

(4, 5) $010-$

(4, 12) -100

(8, 9) $100-$

(8, 12) $1-00$

(14, 15) $111-$

(11, 15) $1-11$

(12, 14) $11-0$

		✓	✓						d	d	d
	1	2	3	5	9	12	14	15	4	8	11
✓ 1, 3, 9, 11	X		X		X						X
✓ 1, 5	X			X							
✓ 2, 3		X	X								
4, 5				X					X		
4, 12						X			X		
8, 9					X					X	
8, 12						X				X	
✓ 14, 15							X	X			
11, 15								X			X
✓ 12, 14						X	X				

2 → checked once so select (2, 3) for final expression.

1 → " twice so select (1, 3, 9, 11), (1, 5) for final exp.

3 → already selected

5 → already selected

9 → already selected

12 → Checked thiree so select (14, 12), not (8, 12) or (4, 12)
4, 8, → don't car

15 → Select (14, 15) only not (11, 15)
don't care.

The final expression is

$$f = \bar{B}D + \bar{A}\bar{C}D + \bar{A}\bar{B}C + ABC + AB\bar{D}$$

Exercise:

1. Simplify following logical expression using karnaugh maps.
$$y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + AB\bar{C}$$ Ans: $\bar{A}\bar{B} + \bar{C}$

2. Simplify the following function.
$$f_1(A,B,C,D) = \Sigma m(0,3,5,6,9,10,12,15)$$ Ans: $(A\oplus B)\odot(C\oplus D)$

3. Simplify the following function.
$$f_3(A,B,C,D) = \Sigma m(0,1,2,3,11,12,14,15)$$ Ans: $\bar{A}\bar{B} + AB\bar{D} + ACD$

4. Simplify the following using k-map.
$$X = \bar{A}B + \bar{A}\bar{B}C + AB\bar{C} + A\bar{B}\bar{C}.$$ Ans: $\bar{A}C + B = X$

5. Simplify the following function using k-map technique.
$$F(A,B,C,D) = \Sigma(0,2,3,6,7,8,10,12,13)$$ Ans $AB\bar{C} + \bar{B}\bar{D} + \bar{A}C$

6. Simplify the following Boolean function using 4-variable map
$$F(w,x,y,z) = \Sigma(2,3,10,11,12,13,14,15)$$ Ans: $wx + \bar{x}y$.

7. Simplify the following switching function using k-map
$$f(A,B,C,D) = \Sigma(0,5,7,8,9,10,11,14,15) + \phi(1,4,13)$$ Ans: $\bar{B}\bar{C} + BD + AC$

8. Determine the minimal sum of product form of
$$f(w,x,y,z) = \Sigma m(4,5,7,12,14,15) + d(3,8,10).$$ Ans: $\bar{w}x\bar{y} + xyz + w\bar{z}$

9. Simplify the following Boolean function for minimum POS form.
$$f(w,x,y,z) = \Pi M(4,5,6,7,8,12) + d(1,2,3,9,11,14)$$ Ans $= (w+\bar{x})(\bar{w}+y+z)$

10. Reduce using k-map.
$$f(A,B,C) = \Pi M(0,1,2,3,4,7)$$ Ans $= A\cdot(\bar{B}+\bar{C})\cdot(B+C)$

11. Simplify the following using Boolean algebra.
$$f(x,y,z) = \Pi M(3,5,7).$$ Ans: $(\bar{y}+\bar{z})(\bar{x}+\bar{z})$

12. Reduce using Tabulation method

(1) $F(A,B,C,D) = \Sigma(0,1,2,3,4,6,8,10,12,14).$ Ans: $\bar{A}\bar{B} + \bar{D}$

(2) $F(A,B,C) = \Sigma(0,1,4,6,7)$ Ans $\bar{A}\bar{B} + B\bar{C} + A\bar{C} + AB$

(3) $F(A,B,C,D) = \Sigma(2,4,6,7,9,11,13)$ Ans $+ \bar{A}BC + A\bar{B}D + A\bar{C}D +$

4. $F(A,B,C,D) = \Sigma(0,1,2,3,5,6,7,8,11,13)$.

Ans: $\bar{A}\bar{B} + \bar{A}D + \bar{A}C + \bar{B}CD + B\bar{C}D + \bar{B}\bar{C}\bar{D}$

5. $F(A,B,C,D) = \Sigma m(2,3,5,7,9,11,12,13,14,15)$

Ans $\bar{A}\bar{B}C + BD + AD + AB$.

6. $F(A,B,C,D) = \Sigma m(0,1,2,3,6,7,13,15)$

Ans: $ABD + \bar{A}\bar{B} + \bar{A}C$

7. $F(A,B,C,D) = \Sigma m(0,1,2,3,4,7,9,10)$

Ans: $\bar{A}\bar{C}\bar{D} + \bar{B}\bar{C}D + \bar{B}C\bar{D} + \bar{A}CD + \bar{A}\bar{B}$

8. $f(A,B,C,D) = \Sigma m(0,1,3,6,7,8,9,13,15)$

Ans: $\bar{B}\bar{C} + ABD + \bar{A}BC + \bar{A}CD$

9. $F(A,B,C,D) = \Sigma m(3,4,5,7,9,13,14,15)$

Ans: $BD + ABC + A\bar{C}D + \bar{A}CD + \bar{A}B\bar{C}$

10. $F = m_0 + m_2 + m_4 + m_8 + m_9 + m_{10} + m_{11} + m_{12} + m_{13}$

Ans: $\bar{B}\bar{D} + \bar{C}\bar{D} + A\bar{C} + A\bar{B}$

Part - A.

1. Give the steps for simplification of SOP expression.
2. what do you mean by essential prime implicants
3. what is don't care Conditions?
4. Give the steps for simplification of POS exp.
5. State the rules for K-map simplification.
6. Explain the limitations of Karnaugh map.
7. State the advantage and disadvantage of Qunine Mc Clusky method of simplification.

Possible Questions:

1. Define switching function
2. Define literal, product term and sum term.
3. Explain sum of product form.
4. What do you mean by standard SOP and POS form.
5. Explain how to convert SOP or POS expressions in their standard forms.
6. What do you mean by minterms and maxterms.

---

1. Express F_1 in standard SOP form

$$F_1 = AB + \bar{C}D + A\bar{B}C$$

2. Determine the canonical SOP form of

$$F(x, y, z) = (xy + \bar{z})(y + x\bar{z})$$

3. Convert the given expression in standard POS form

$$F(P, Q, R) = (P + \bar{Q})(P + R)$$

Questions:

1. Implement the following function with NAND gates
$$f(x,y,z) = \Sigma(0,6)$$
2. Design a logic circuit to simulate the function $F(A,B,C) = A(B+C)$ by using only NAND gates.
3. Realize (i) AND gate (ii) NOR gate using only NAND gates.
4. Realize (i) OR gate (ii) EX-OR gate using NAND gates.
5. Realize (i) OR gate (ii) AND gate using only NOR gates.
6. Write a short note on multilevel NAND and NOR Implementations.
7. Write a short note on multioutput gate Implementation.
8. Show that $y = AB\bar{C}$ can be implemented with one two input NOR and one two input NAND gate

2 - D