



Course Design Instruction of Mechanical Theory

Robot design for Jansen linkage

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2020-12-25

Abstract

The purpose of the course design is to design a new structure of Jansen linkage by using the theory of machines and mechanisms. The recreation is based on the study of various structures which now exist. To begin with, we analyze existing mechanism, search corresponding papers, draw their kinematic diagrams and analyze their motion. After that, we decide the overall dimensions by using graphical and analytic methods. Finally, use Adams and Inventor to simulate and analyze the new designed model, use matlab and mathematical model to check out the simulation results.

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Chapter 1 Topic Introduction

1.1 Background

With the acceleration of the development of the realm of robotics, our daily life is more convenient with their help. As one of the members of robot, quadruped robots are commonly used in our daily life and have a better prospect, because they have high mobility, adaptability, and strong load capacity.

1.2 Development

Quadruped robot has a significant influence among all types of robots, through the continuous exploration of researchers from all over the world, the quadruped robot has developed from a single configuration and simple movement to a diversified complex comprehensive movement. In the last century, world's first two four-legs working robots named 'Phony pony' and 'Mosher' were made in American, the control system of the "Mosher" quadruped robot is controlled by the operator, which is a monument to the development of quadruped robot theory and technology. However, due to the limitation of the level of science and technology, the Phony pony's control system only uses simple logic circuits, so the four-legged robot can only perform some simple movements such as working and turning, which has no practical application significance. In 1990s, robots named 'Patrush' and 'TITAN-VIII' were made in Japan. The "Patrush" quadruped robot has the ability to jump, and the "TITAN-VIII" quadruped robot has the characteristics of being able to walk in a variety of gaits, the development of these two robots provided a significant tribute to the theory of the quadruped robot. In 2013, 'Dynamics' made a quadruped robot which can realize some difficult actions, such as emergency braking, steering and sprinting, it was originally designed for the purpose of transporting materials, therefore, this robot has the ability to walk stably on complex roads, and also has the characteristics of being able to withstand larger loads, and the robot can quickly and automatically recover stability after being hit by a side status. What's more, the same company made a new type of robot named 'Little Dog' which is controlled by AI can speed up to 46 kilometer per hour in 2010. Meanwhile, MIT has developed the MIT Cheetah series of robots in the period of 2012-2019, the leg of the series has adapted four-bar linkage mechanism and each leg has two degrees of

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freedom. China has done some relative research since the early 1990s, the first quadruped robot named 'QW-1' was made in Tsinghua University, each leg joint of the robot is driven by a servo motor, which laying the foundation for the follow-up research on quadruped robots in China. Subsequently, Shanghai Jiaotong University developed a series of "JTWM" quadruped robots. The control system of the "JTWM-III" quadruped robot uses artificial neural networks and fuzzy algorithms, however, the robot has a low walking speed. At the beginning of this century, Northwestern Polytechnical University and University of Science and Technology of China launched bionic dog robots and the "TIM-1" quadruped robot. These two quadruped robots are highly adaptable to the environment, and the control system is also advanced compared to other robots in the same period. In the following years, Harbin Institute of Technology, Shanghai Jiaotong University, Shandong University and other research institutions have carried out higher-level research and manufacturing of quadruped robots, and have achieved rich results. Among them, the "SCalf-1" quadruped robot developed by Shandong University uses hydraulic drive to provide power, and the quadruped robot can walk at 1.8m/s. In the period of 2016-2019, ZJU has developed a high stability, light weight and durable quadruped robot, more and more complex and stable robots are made in the last years.

To sum up, with the continuous development of control, materials, energy and other technologies, quadruped robot technology around the world is also improved day by day, and the application of quadruped robots is becoming more and more extensive in modern times.

1.3 Classification

Mobile robot can be divided into wheeled robots, crawler robots and walking robots, the biggest difference between them is their moving part, which are consist of wheels, crawlers, and links. Generally, stationary robot can easily fulfill some actions given by the commands, for example, industrial robot is commonly used in factories. On the contrary, the quadruped robot is a kind of bionic robot, which has great advantages when crossing terrain obstacles. They have very low requirements for walking on the road, consequently, it can cross obstacles and drive on rough and uneven roads. The most commonly used mobile robot in daily life is serving robots, which work in the region of catering and welcome industry.

1.4 Characteristic

For quadruped robots, the placement of leg shape and joint posture is extremely necessary for their dynamic characteristic. Their leg shape can be divided into parallel shape and series shape, generally the parallel shape sacrifices the mobile range to get a more stable stress distribution. The power of the quadruped robots is generated by brushless motor structure, and the transmission of the robot uses gear trains to control the inner shaft and the outer shaft. Compared with other types of robot, characteristics such as high stability, strong capacity make quadruped robot widely used in the area of material transportation and emergency rescue.

1.5 Topic description

We choose quadruped robot as our subject, after observing and surveying some kinds of quadruped robot mechanisms, we want to summary its general type and working principles, then we decide to design a new type of quadruped robot mechanism to achieve a specified functions (step length:108mm, height of each step:10mm, walking velocity: 72mm/s).

1.6 Design requirements

- 1) Understand the basic composition of the quadruped robot, which consists of the base, main body and legs.
- 2) Try to save space by using as less linkages as we can, and try to make the overall appearance elegant.
- 3) Under the condition of moving smoothly and stably, making the operation as simple as we can.
- 4) Designing requirements should meet the standard of that of practical and engineering.
- 5) The length of each step is 108mm, height of each step is 10mm, and the mechanism can walk 72mm/s.

Chapter 2 Analysis of existing mechanisms

2.1 Analysis of the existing walking robot mechanism

As a product of effective combination of bionics and robotics, the legged robot is a robotic system designed by mimicking the structure and movement of the legs and feet of mammals, insects, and amphibians.

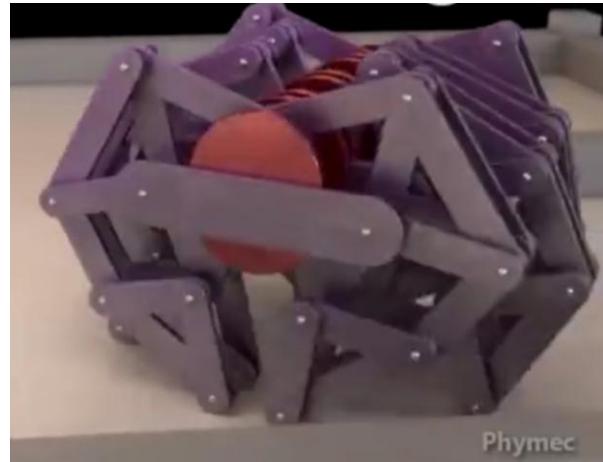
Pic 2.1.1 is a typical walking robot. It has following features:

1, Compared with other walking robots, it is more stable because it is composed of many groups of legs.

2, It can realize some actions, such as rotation, brake. Its movement is supplied by a power engine.

3, Overall appearance is elegant and clever. Such mechanism is made by a

foreign named Theo Jansen, who has created many such kind artworks.



Pic.2.1.1

4, Though the mechanism seems tough to assemble, its part is easy to be manufactured.

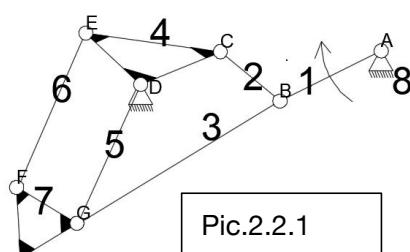
5, It has used Stephenson six-bar linkage, and can be conveniently used to increase force, expand stroke and realize long-distance transmission.

6, More linkages in mechanism means more connecting rod curves, which will make the movement of the mechanism more beautiful.

2.2 Draw kinematic diagram of the existing mechanism

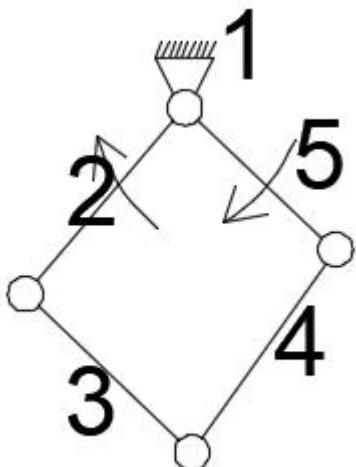
The walking robot's kinematic diagram is shown in Pic 2.2.1, we can calculate its DOF.

To begin with, let's count down the number of its links, which is 7. Then we count down its lower pairs, which is 10. Finally, $DOF = 3 * 7 - 2 * 10 = 1$, which means there is only one driving link in this mechanism. What's more, from the diagram we can clearly see that it is a typical Stephenson link.

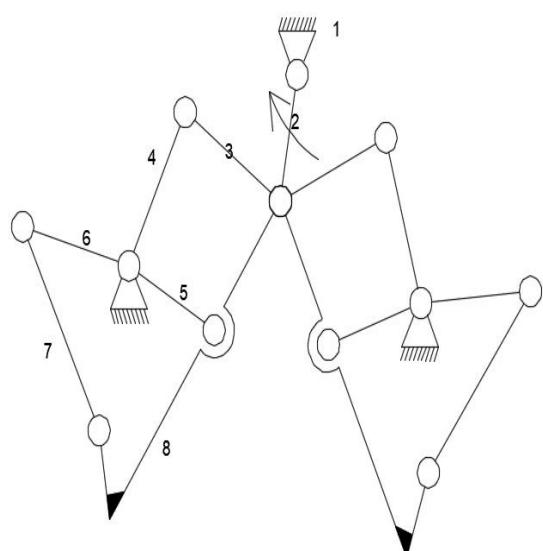
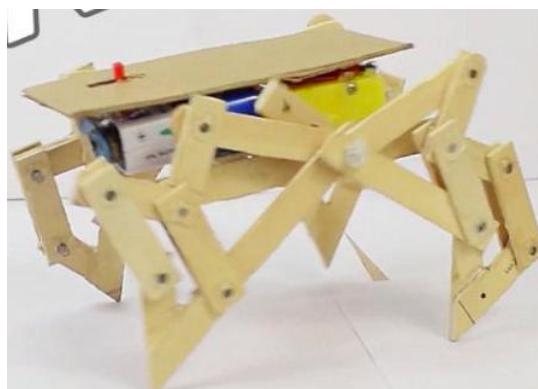


Pic.2.2.1

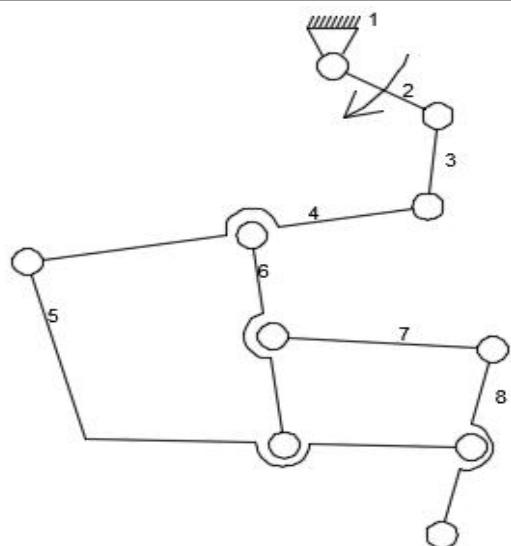
2.3 Preliminary investigation



$$DOF = 3 \times 4 - 2 \times 5 = 2$$

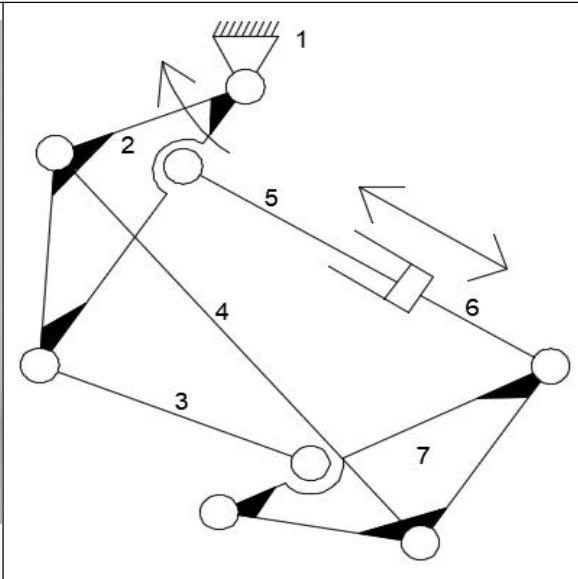
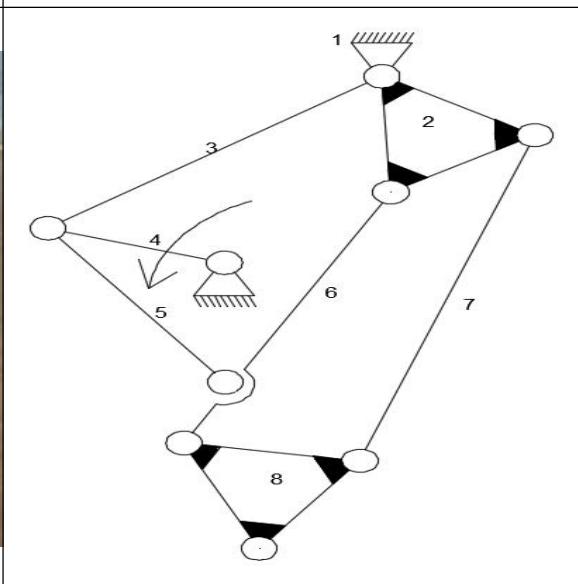
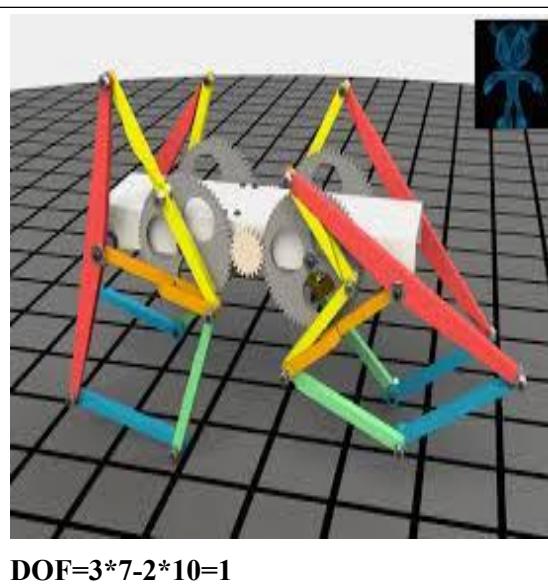
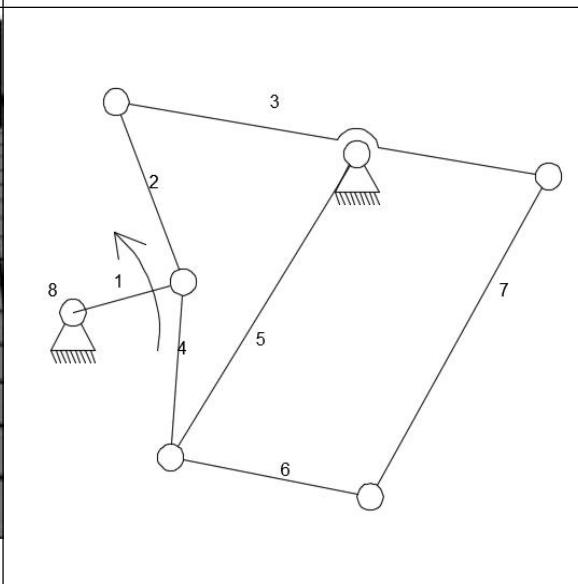


$$DOF = 3 \times 7 - 2 \times 10 = 1$$



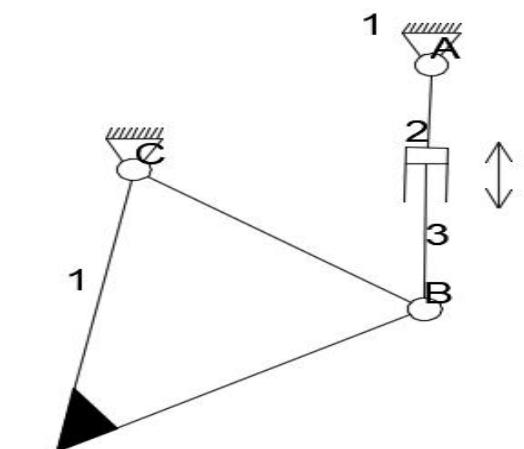
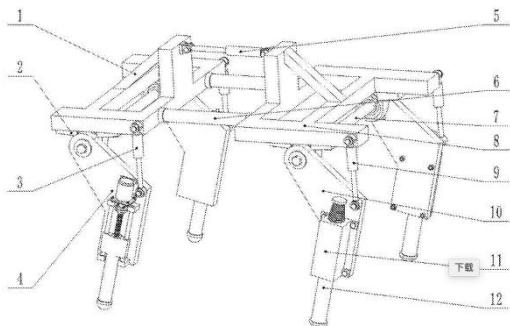
$$DOF = 3 \times 7 - 2 \times 10 = 1$$

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	 $DOF = 3 \cdot 6 - 2 \cdot 8 = 2$
	 $DOF = 3 \cdot 7 - 2 \cdot 10 = 1$
	 $DOF = 3 \cdot 7 - 2 \cdot 10 = 1$

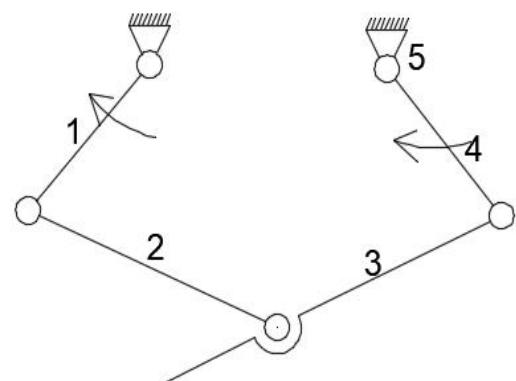
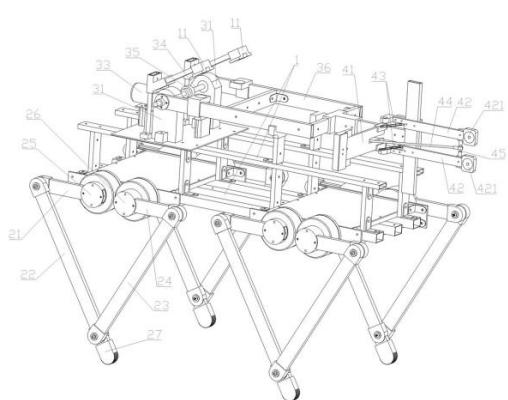
2.4 Patent search

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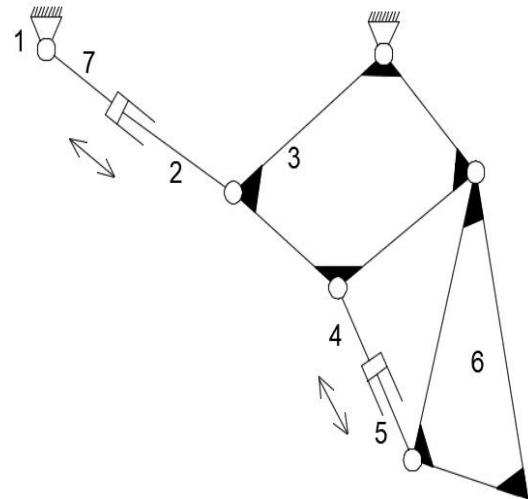
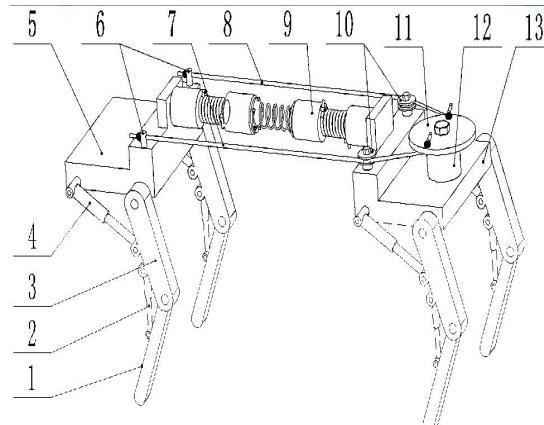
$$DOF = 3 \times 3 - 2 \times 4 = 1$$

专利申请号: CN201920727251.8



$$DOF = 3 \times 4 - 2 \times 5 = 2$$

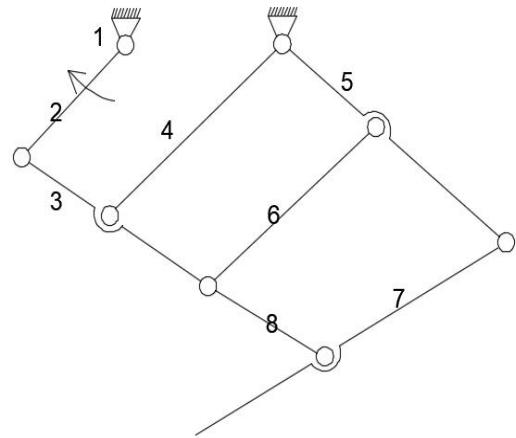
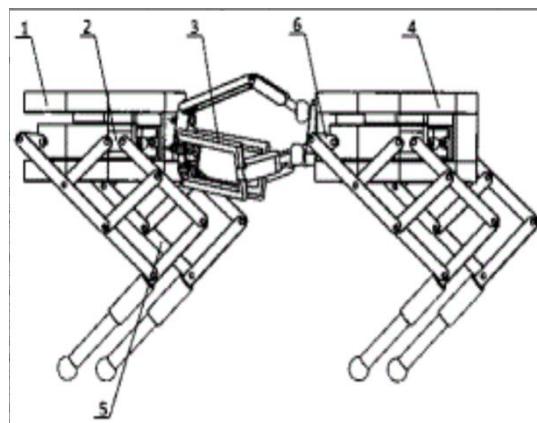
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$$DOF = 3 \times 6 - 2 \times 8 = 2$$

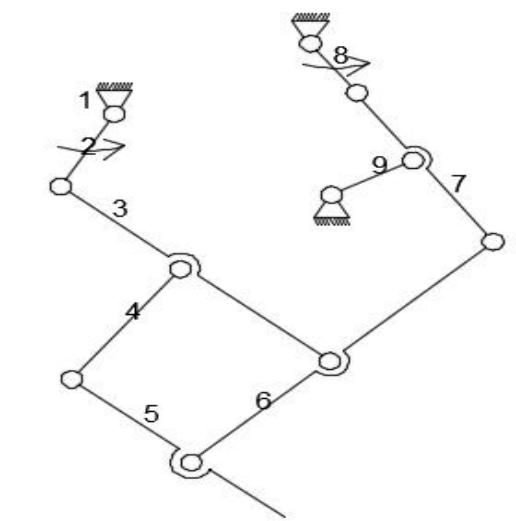
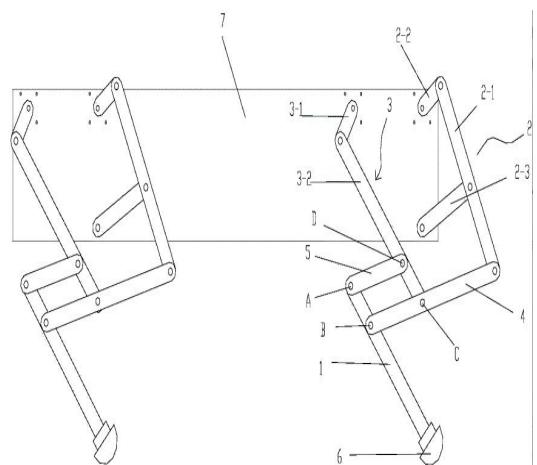
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$$DOF = 3 \times 7 - 2 \times 10 = 1$$

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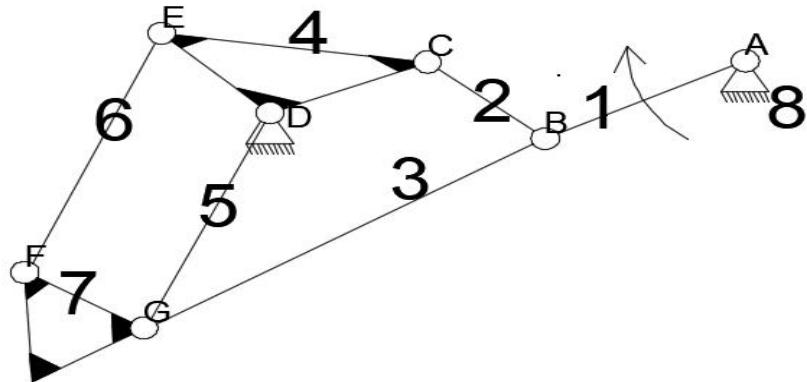


$$DOF = 3 \times 8 - 2 \times 11 = 2$$

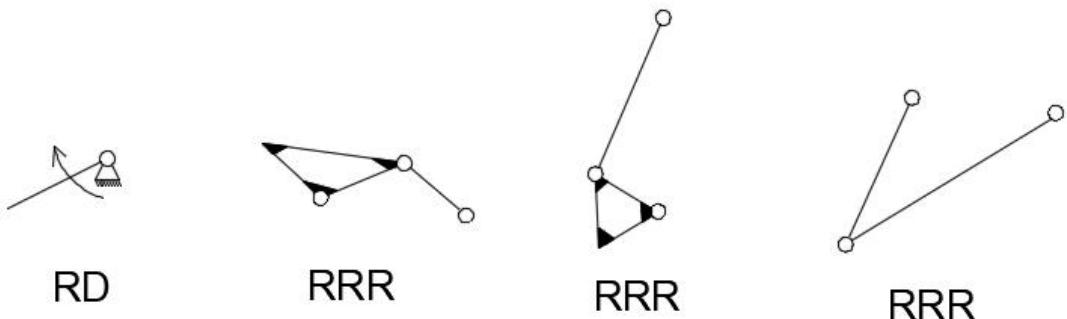
Chapter 3 Composition analysis of the existing mechanism and new types' generation

3.1 Analysis of the existing mechanism

The structure of the existing mechanism is shown in the below picture:



From the picture we can see that the DOF of the structure is 1, and there is no virtual constraints or local degrees of freedom. What's more, the original moving part is determined to be component 1. After splitting the rod group, the result can be obtained in the below picture. According to the result of the splitting of the rod group, the highest level of the rod group is level II, so the mechanism is level II.



Advantages: The structure is a jansen eight bar linkages, it can accomplish a complex movement with only one input force.

3.2 Assur group replace/add for the case structure

There are 5 second grade assur-groups, showing in the following picture.

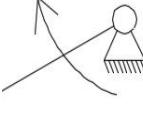
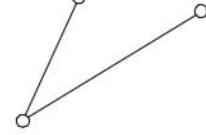
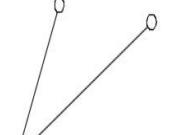
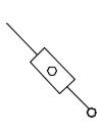
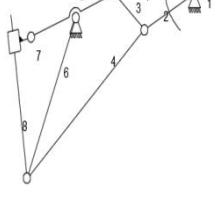
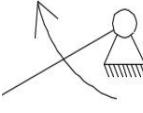
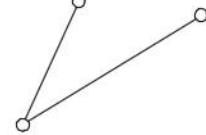
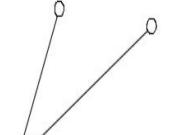
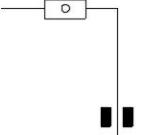
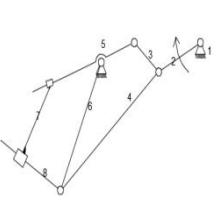
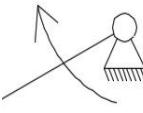
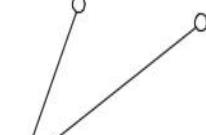
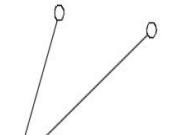
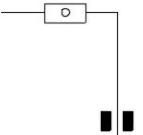
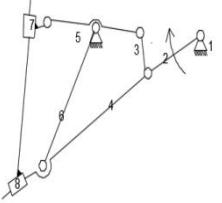
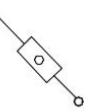
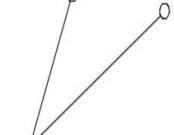
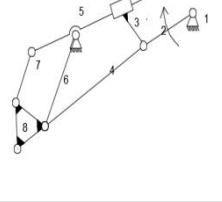
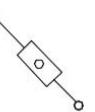
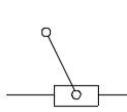
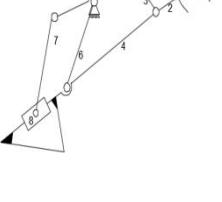
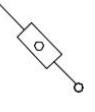
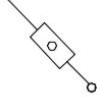
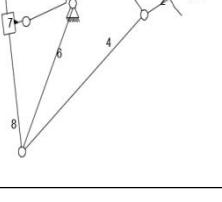
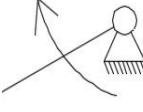
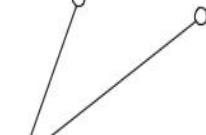
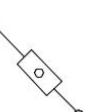
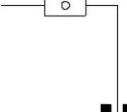
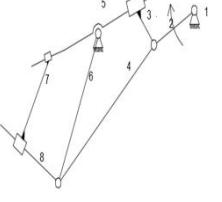
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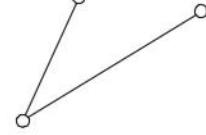
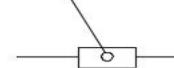
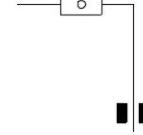
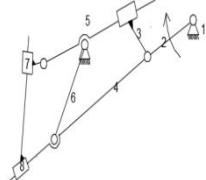
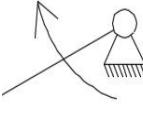
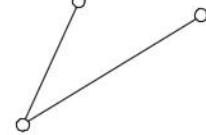
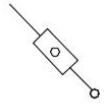
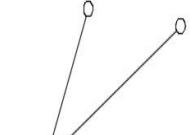
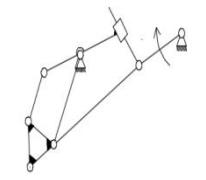
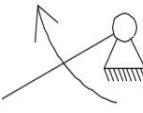
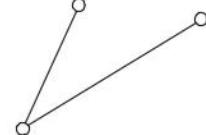
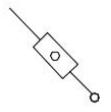
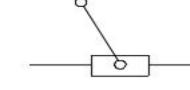
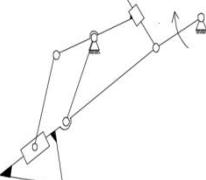
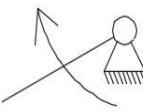
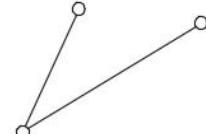
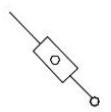
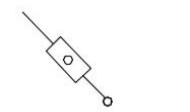
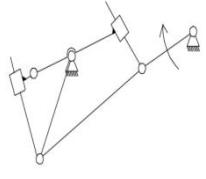
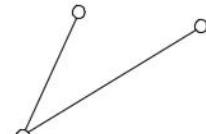
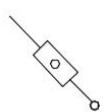
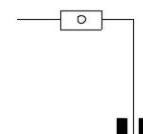
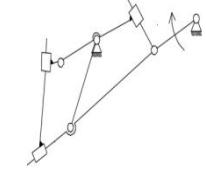
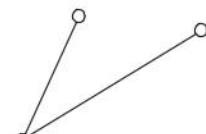
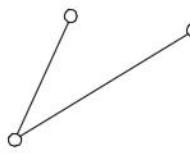
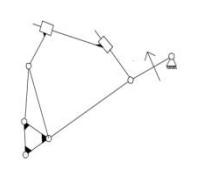
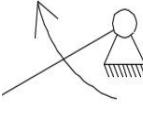
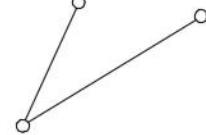
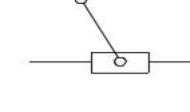
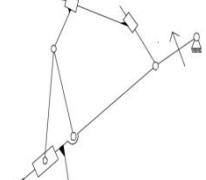
After replacing the original assur-groups of the case structure, we can get following conditions:

Series	Driving Link	Basic assur-group 1	Basic assur-group 2	Basic assur-group 3	Scheme sketch
1					
2					
3					
4					
5					
6					

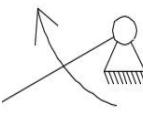
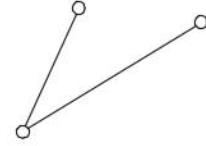
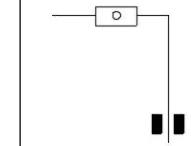
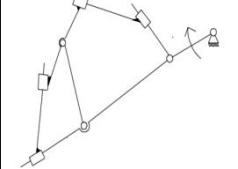
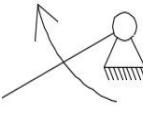
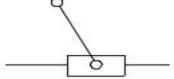
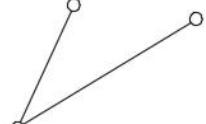
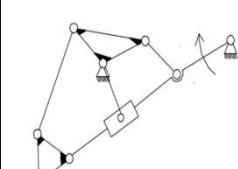
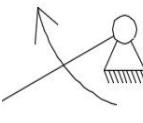
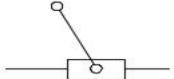
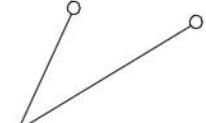
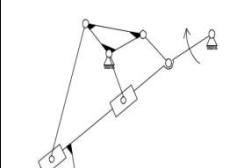
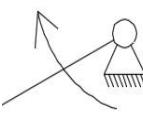
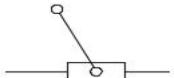
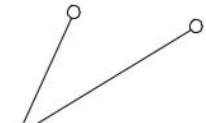
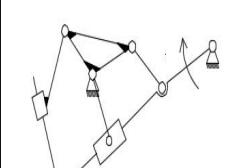
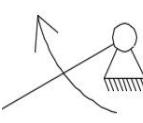
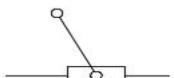
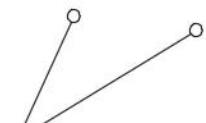
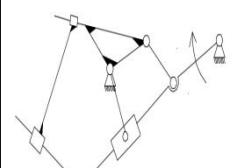
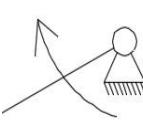
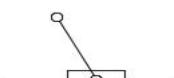
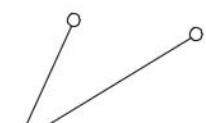
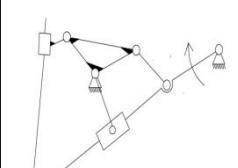
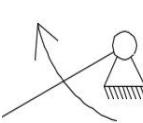
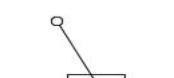
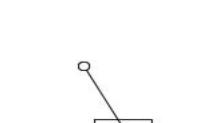
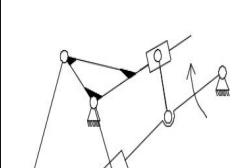
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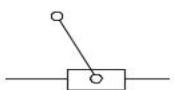
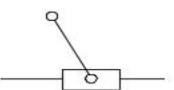
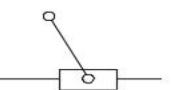
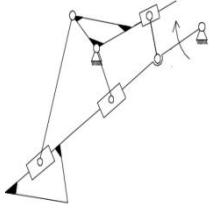
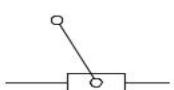
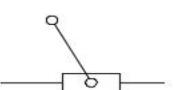
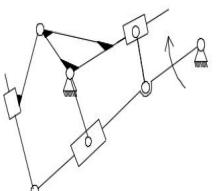
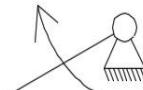
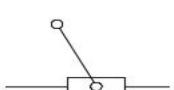
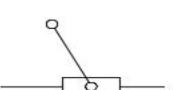
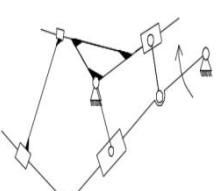
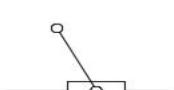
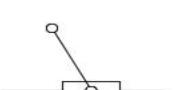
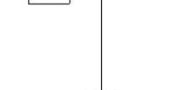
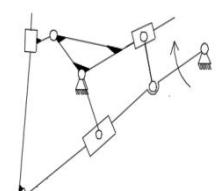
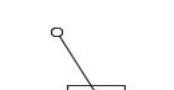
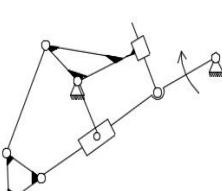
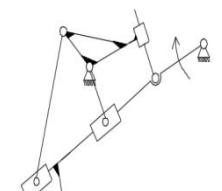
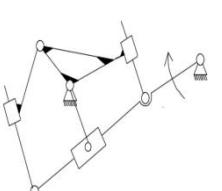
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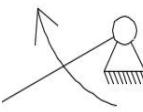
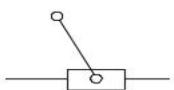
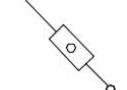
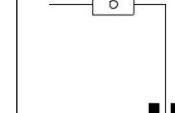
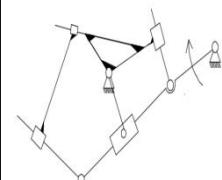
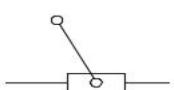
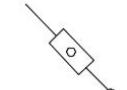
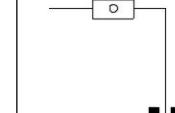
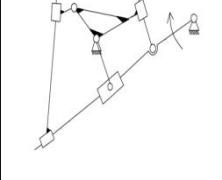
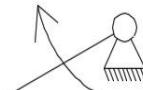
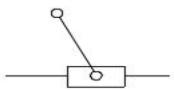
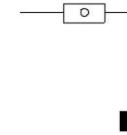
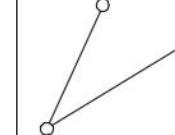
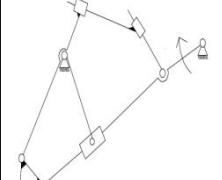
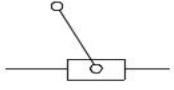
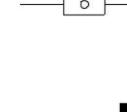
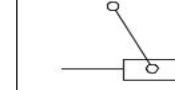
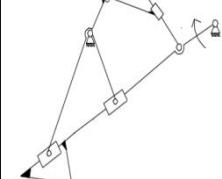
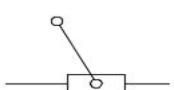
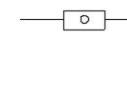
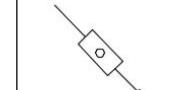
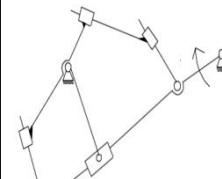
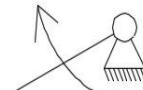
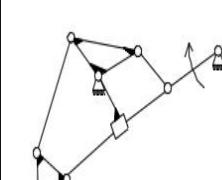
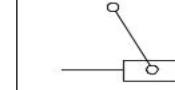
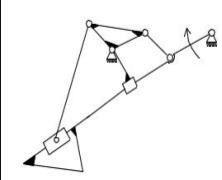
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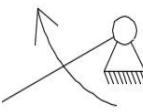
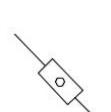
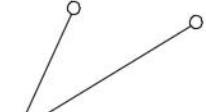
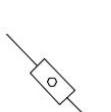
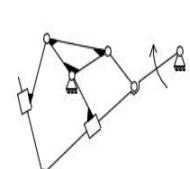
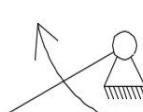
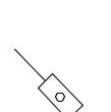
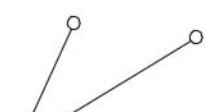
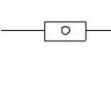
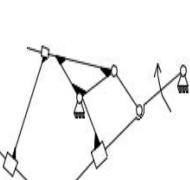
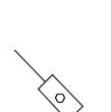
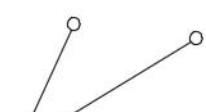
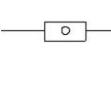
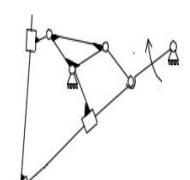
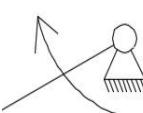
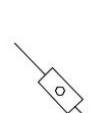
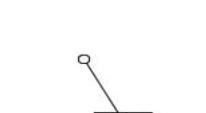
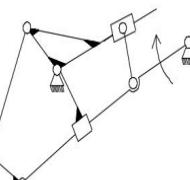
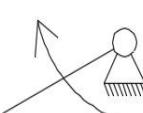
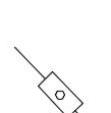
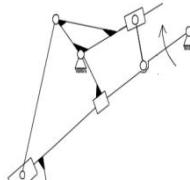
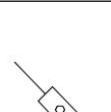
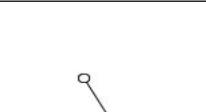
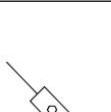
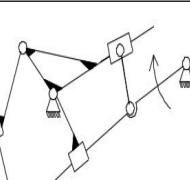
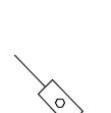
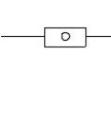
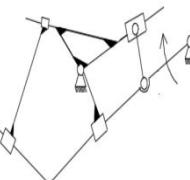
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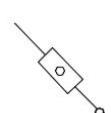
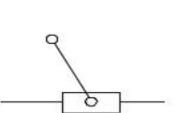
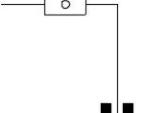
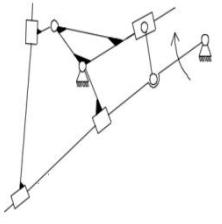
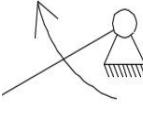
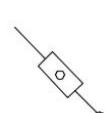
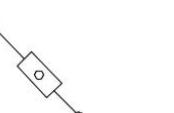
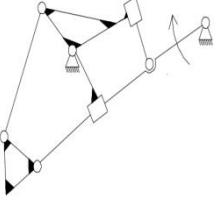
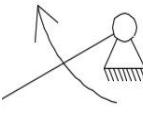
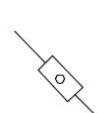
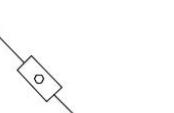
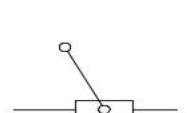
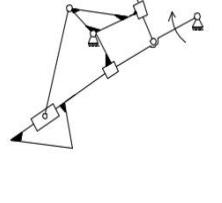
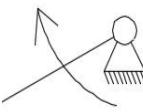
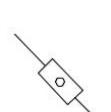
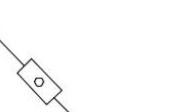
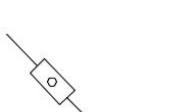
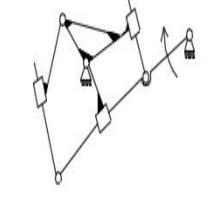
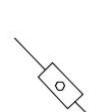
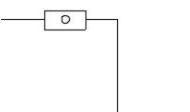
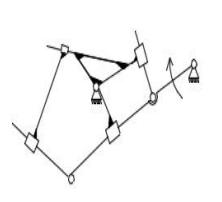
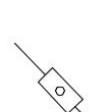
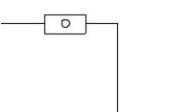
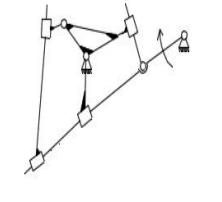
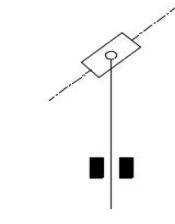
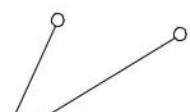
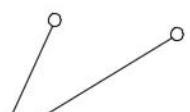
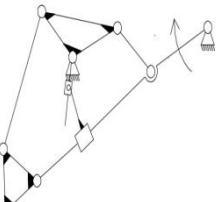
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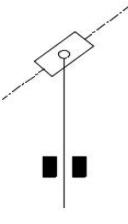
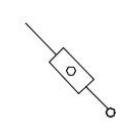
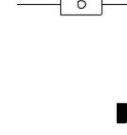
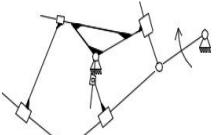
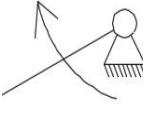
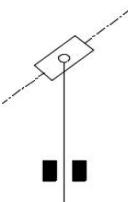
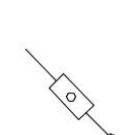
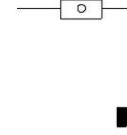
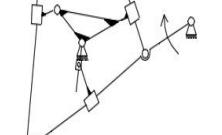
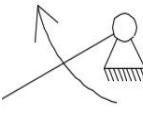
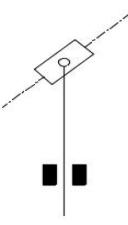
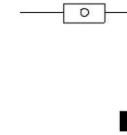
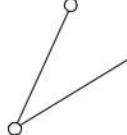
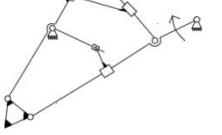
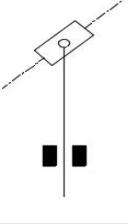
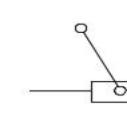
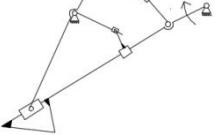
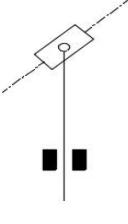
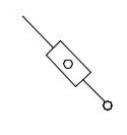
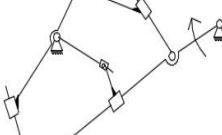
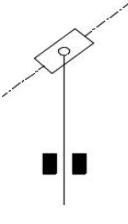
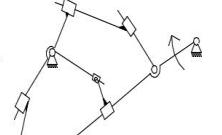
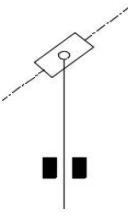
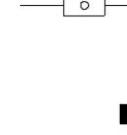
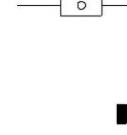
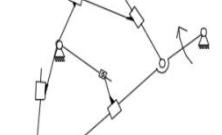
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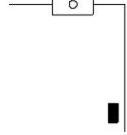
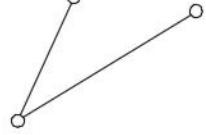
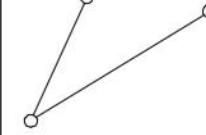
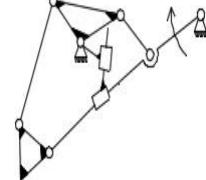
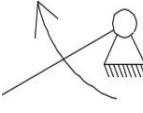
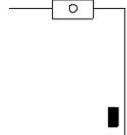
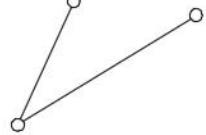
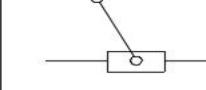
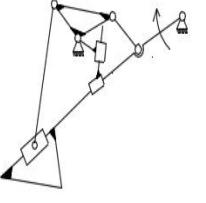
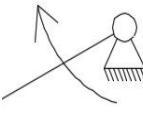
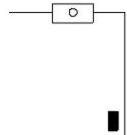
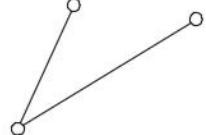
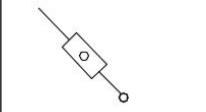
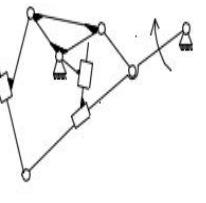
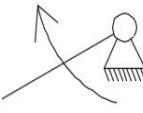
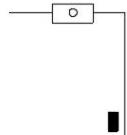
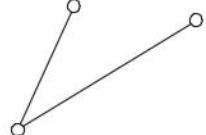
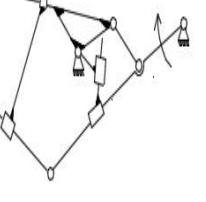
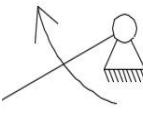
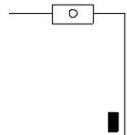
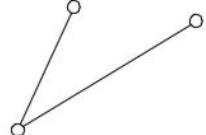
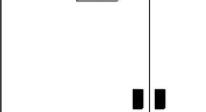
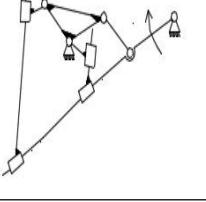
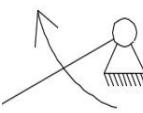
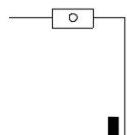
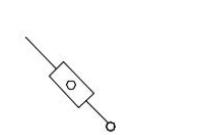
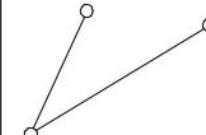
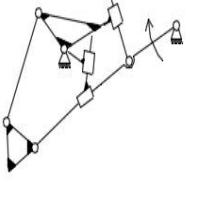
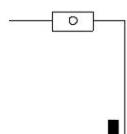
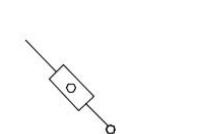
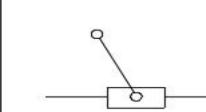
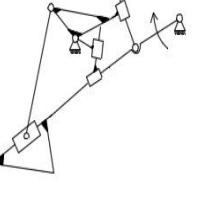
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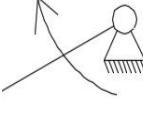
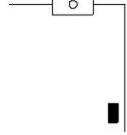
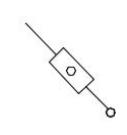
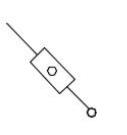
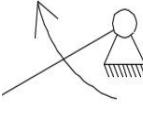
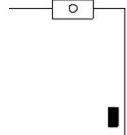
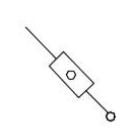
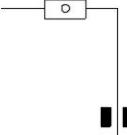
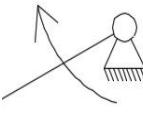
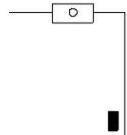
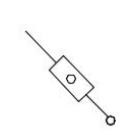
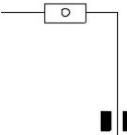
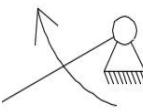
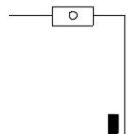
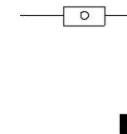
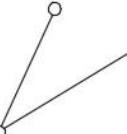
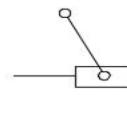
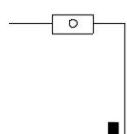
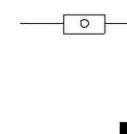
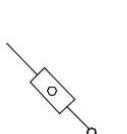
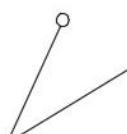
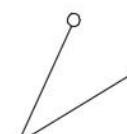
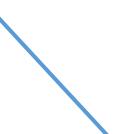
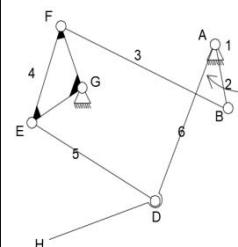
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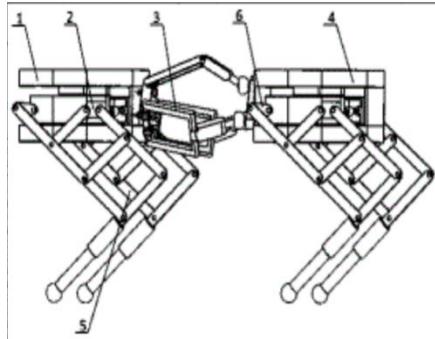
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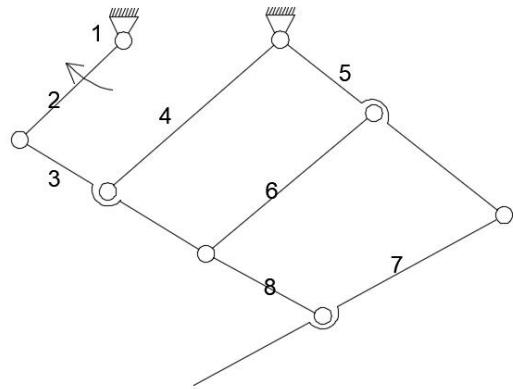
3.3 Analysis of the patent mechanism

The following figure shows a picture of one of the patent mechanisms:

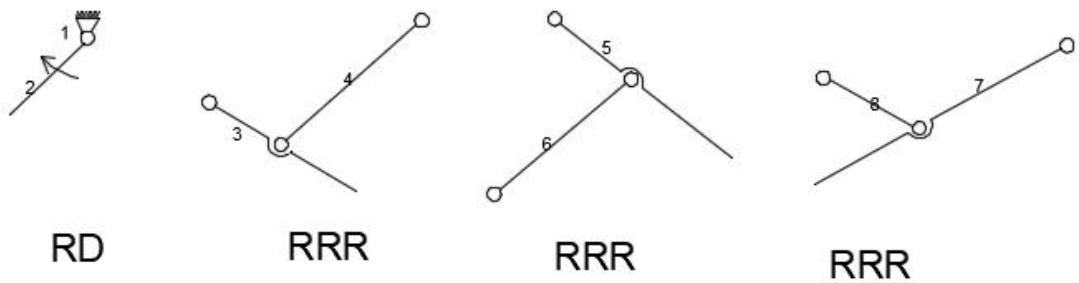


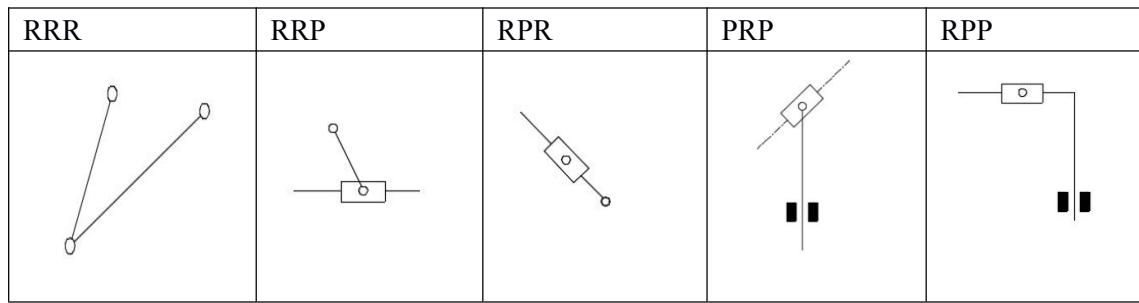
专利申请号: CN201810627660.0

The structure of the corresponding patent is given:

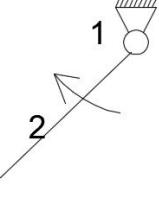
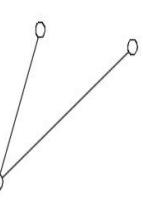
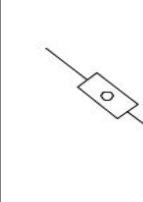
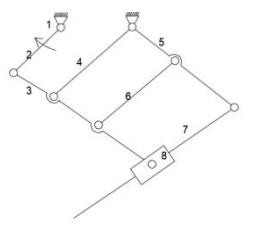
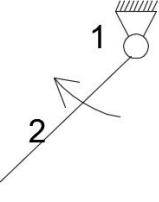
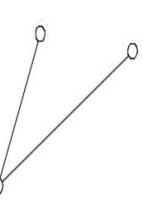
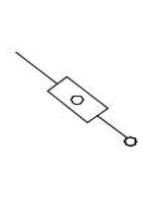
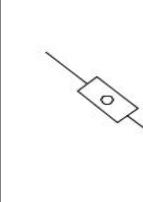
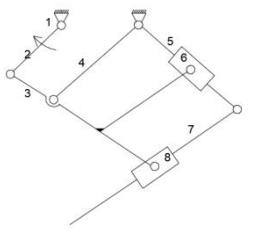
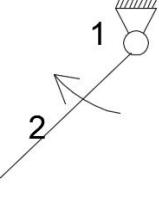
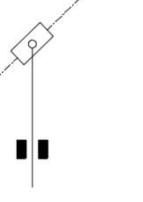
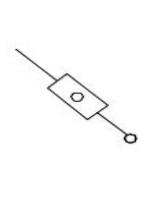
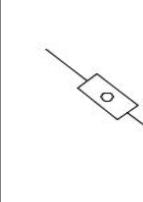
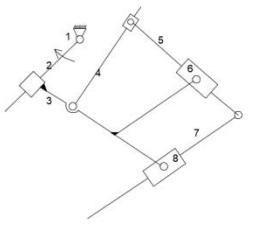
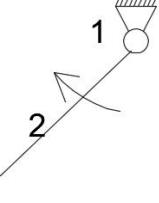
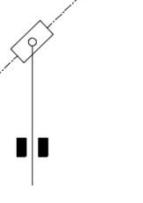
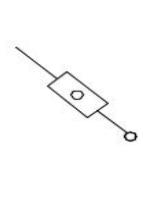
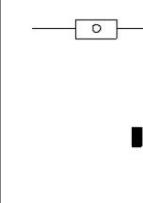
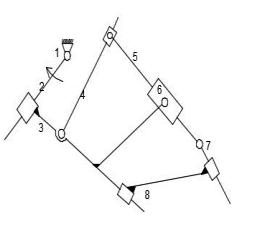
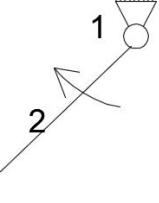
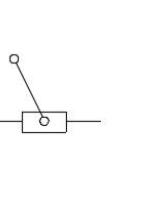
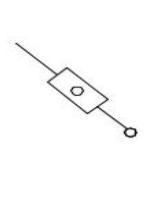
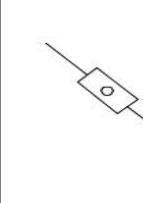
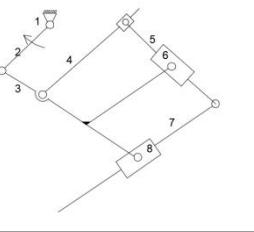


From the picture we can see that the DOF of the structure is 1, and there is no virtual constraints or local degrees of freedom. Besides, the driving links is component 1. After splitting the rod group, the result can be obtained in the below picture. According to the result of the splitting of the rod group, the highest level of the rod group is level II, so the mechanism is level II.

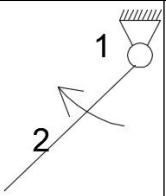
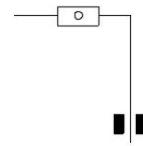
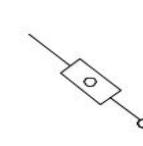
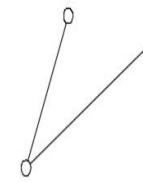
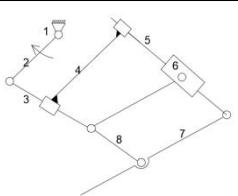
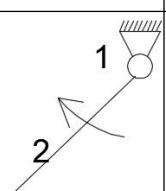
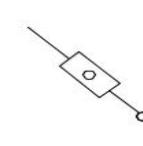
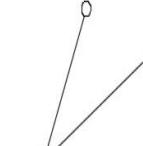
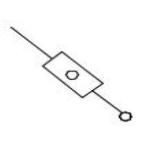
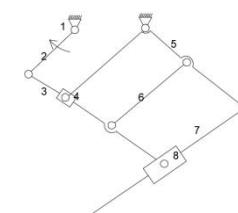
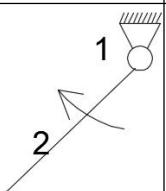
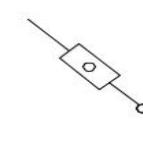
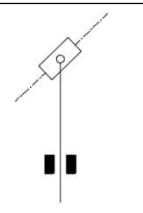
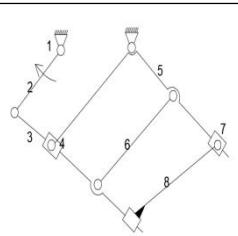




3.4 Assur group replace/add for the patent structure

Series	Driving Link	Basic assur-group 1	Basic assur-group 2	Basic assur-group 3	Scheme sketch
1					
2					
3					
4					
5					

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6	 			
7	 			
8	 			

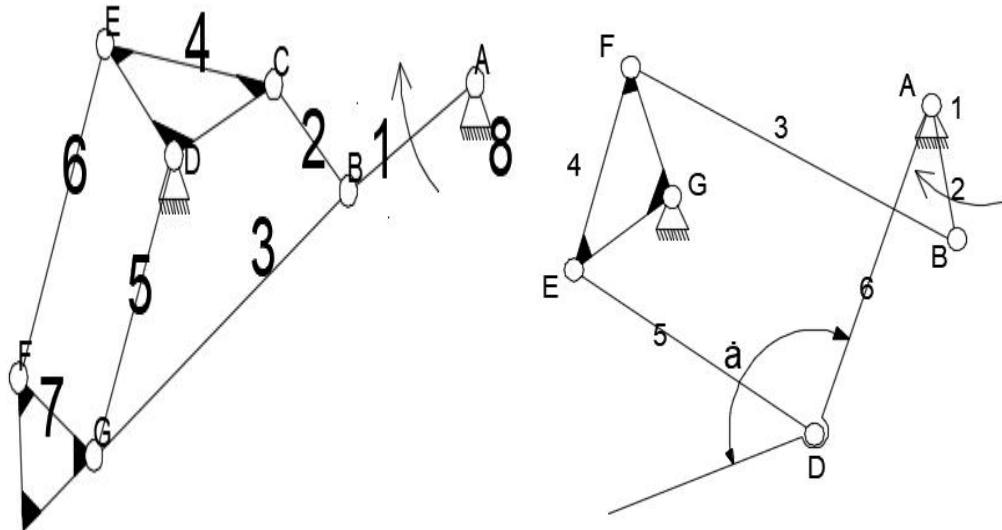
Chapter 4 Dimensional design of a selected mechanism

According to the previously determined plan, the mechanism is an eight-bar mechanism, which means it is composed of three assur-group. In order to design the size of the mechanism to meet the requirements of the system, we should make the system more simple by reducing one assur-group.

The following picture:

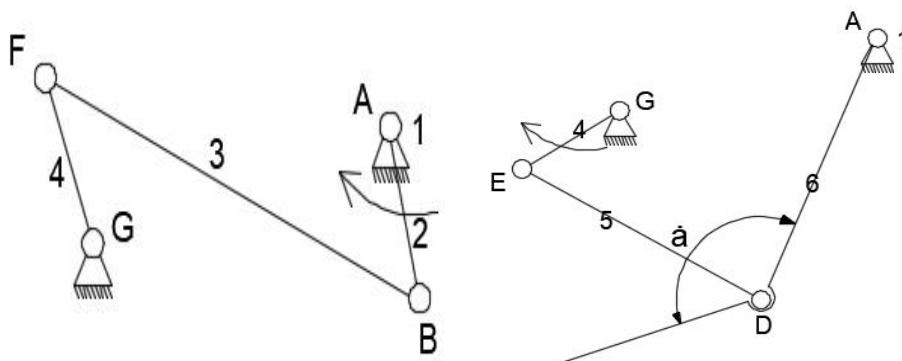
Left: eight-bar mechanism without simplification.

Right: six-bar mechanism by reducing one RRR assur-group.



4.1 Graphical method

To begin with, we split the new six-bar mechanism into two parts, which is shown in the below pictures, the left hand one has a quick return characteristic (quick return coefficient $K=1.4$), and the right hand one can walk 108mm in a step by adjusting the length of AD(DE) and angle a .



Then we read some papers about mechanical design, because we want to make our mechanism achieve a great gait analysis when the rotation bar 1 rotates one cycle.

4.1.1 Preconditions:

1, The required Jensen mechanism has a quick return characteristic, and its quick return coefficient is 1.4.

2, The trajectory of the designed Jensen mechanism is an arc.

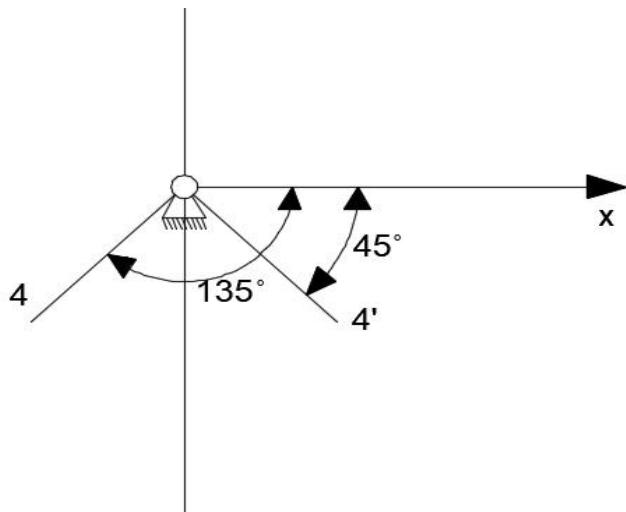
4.1.2 Known conditions:

- 1, the length of bar 1 is 50mm, the straight length of AG is 85mm, FG=EG=50mm.
- 2, we want the output x distance of point F to be 108mm.
- 3, rotation angle of bar 4 is from -135° to -45° .

4.1.3 Determine the quick return mechanism:

- 1, the length of driving link AB and the position of A:

Step1: Draw the limited region of bar 4, which is shown in the following picture:



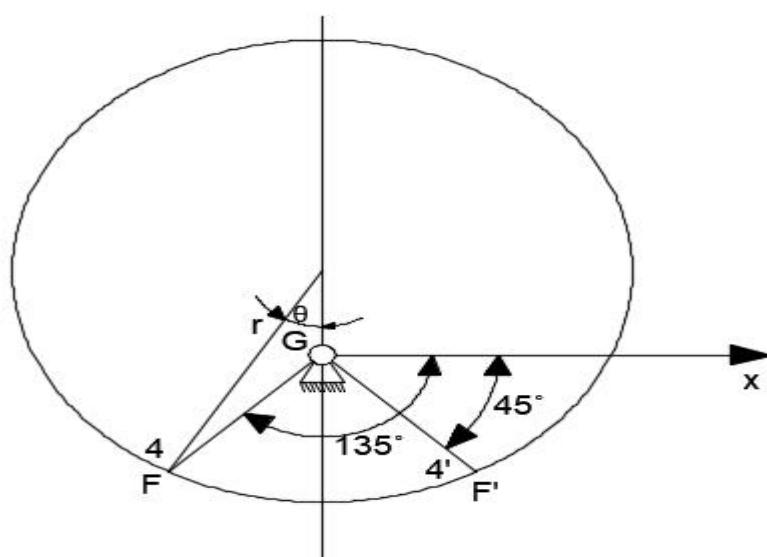
Step 2: By using the quick return coefficient $K=1.4$, we can get the radius of the circle which passes through A.

$$\Theta = \pi * \frac{K - 1}{K + 1}$$

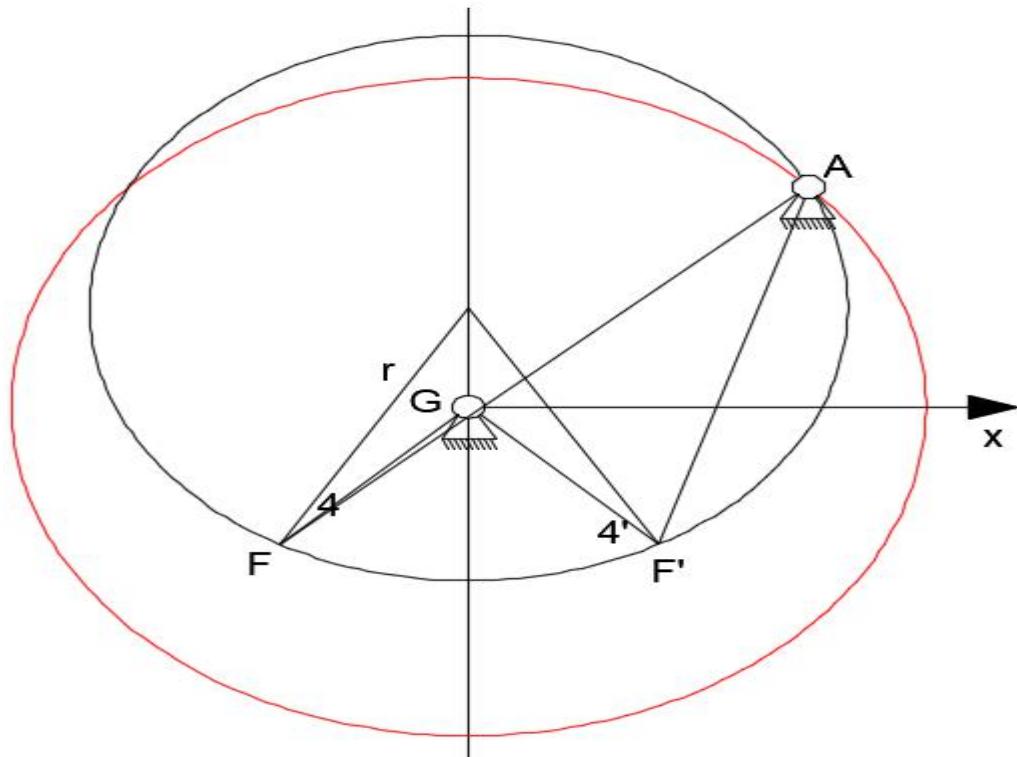
We can get $\theta = 30^\circ$.

$$\frac{FG}{\sin(\theta)} = \frac{r}{\sin(135)}$$

FG = 50mm, we can get $r = 71mm$.



Step 3: The distance between G and A is 85 mm, we then draw a circle passing through point G with radius of 85mm. The coincident point of the two circle is A. Since GF and GF' are limited position, we can get the length of AB and BF.



The length of $AB = (AF - AF')/2 = 29\text{mm}$, the length of $BF = AF - AB = 101\text{mm}$.

Finally, we can get the length of the quick return part, $AB=29\text{mm}$, $BF=101\text{mm}$, $FG=50\text{mm}$, $AG=85\text{mm}$. Since $AB + BF < FG + AG$, which satisfies the Grashof kinematic chain conditions. The coordinate of A(70,48).

4.1.4 Determine the output mechanism:

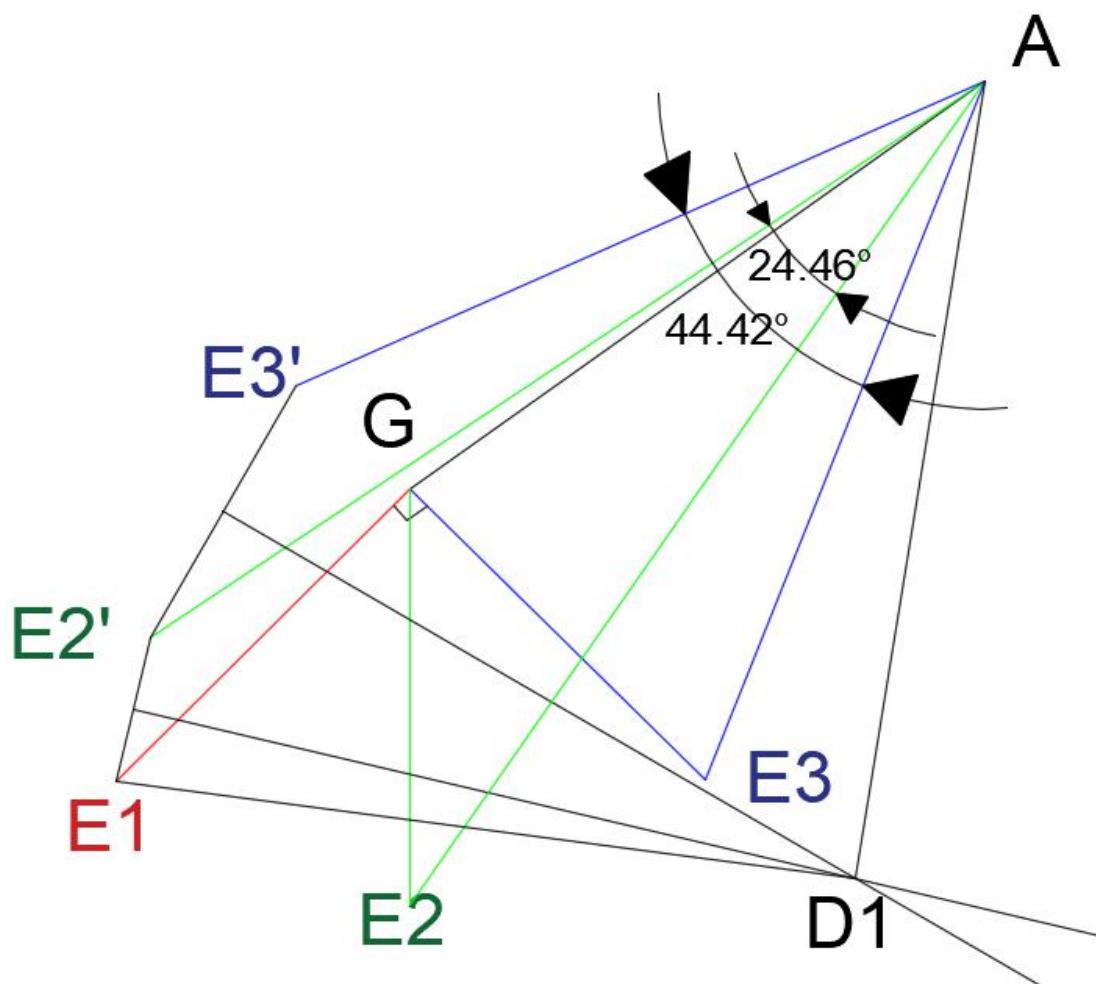
We will use function generation method to determine the size of the second four bar linkage, from the first four bar linkage we can know that the limited position of bar 4. Then, we can use frame reverse method to determine the length of bar 5 and bar 6, which is shown below: ($a_1=45^\circ, \Phi_1=24.46^\circ$; $a_2=90^\circ, \Phi_2=44.42^\circ$)

Step1: Set a relative coordinate system.

Step2: Fix the connecting rod AD, and rotate AD_2 clockwise with angle Φ_1 , get E_2' .
(same to rotate AE_2 clockwise with angle Φ_1)

Step3: Fix the connecting rod AD, and rotate AD_3 clockwise with angle Φ_2 , get E_3' .
(same to rotate AE_3 clockwise with angle Φ_2)

Finally, we draw the plumb line of E_1E_2' and $E_2'E_3'$, the coincident point D1 is the living hinge that we want, and the length of bar 5 and bar 6 can be determined from the picture.

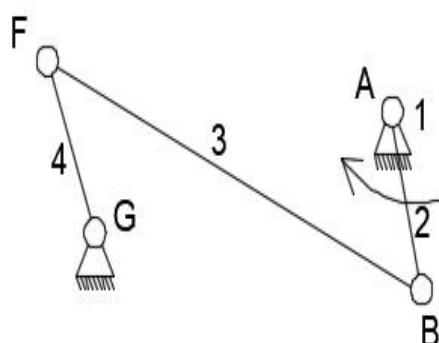


$ED = 98\text{mm}$ and $AD = 100\text{mm}$, to make the whole structure's size more clearly and symmetrically, we set $ED = AD = 100 \text{ mm}$.

Verify: We just use analytic method to prove the correctness of the size of the second four bar linkage.

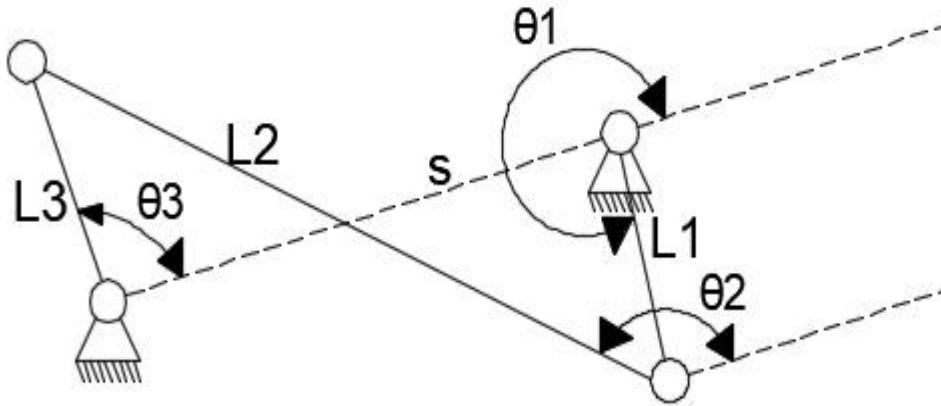
4.2 Analytic method to prove

4.2.1 Analytic method to prove the input four-bar linkage



Given that $FG = 50 \text{ mm}$, and $AG = 85 \text{ mm}$.

By given dimensions and limited angles, we can find the relationship between 4 bars. The process of deduction is as follows:



$$S + L1 * \exp(i * \theta_1) + L2 * \exp(i * \theta_2) = L3 * \exp(i * \theta_3) \quad (4.1)$$

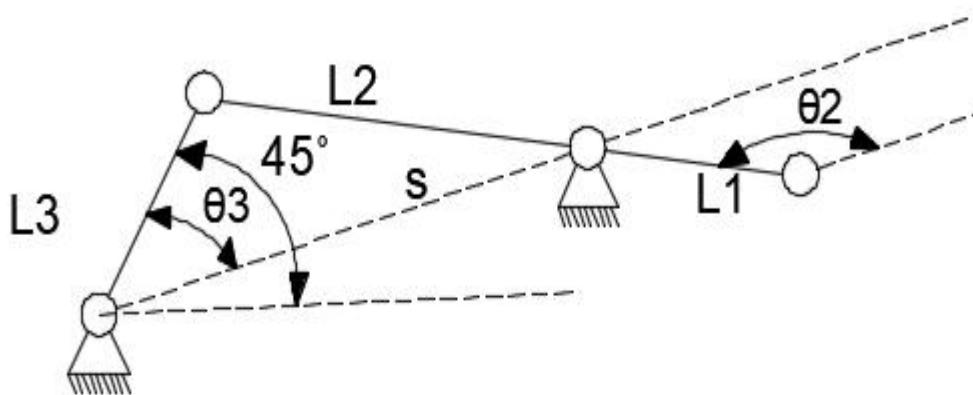
Writing the real part and imaginary part of equation (4.1) respectively, we get:

$$S + L1 * \cos(\theta_1) + L2 * \cos(\theta_2) = L3 * \cos(\theta_3) \quad (4.2)$$

$$L1 * \sin(\theta_1) + L2 * \sin(\theta_2) = L3 * \sin(\theta_3) \quad (4.3)$$

Finding the length relationship of bar L1 and L2 when L3 is in limited position, which is shown in the below pictures:

(1): position 1, where the angle between L3 and horizontal is 45° .

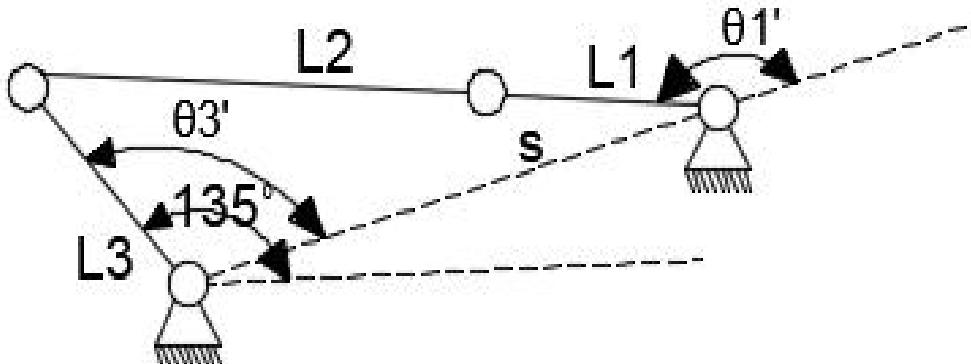


Condition 1: $\theta_1 = \theta_2 + \pi$

Condition 2:

$$\frac{50}{\sin(\theta_2)} = \frac{L2 - L1}{\sin(\theta_3)} \quad (4.4)$$

(2): position 2, where the angle between L3 and horizontal is 135° .



Condition 1: $\theta_1' = \theta_2'$

Condition 2:

$$\frac{50}{\sin(\theta_1')} = \frac{L_1 + L_2}{\sin(\theta_3 + \pi/2)} \quad (4.5)$$

(3): Let $L_1 = 29$ mm and $L_2 = 101$ mm, then we can check whether the K satisfy.

$$\frac{\theta_1' + 2\pi - \theta_1}{\theta_1 - \theta_1'} = \frac{7}{5}$$

Then we can get

$$\theta_1 - \theta_1' = 5\pi / 6 \quad (4.6)$$

(4): Finally, we get 4 equations with four unknown variables. We can use matrix to solve the unknowns or substituting the $L_1=29$ mm and $L_2=100$ mm to check whether the quick return coefficient is 1.4.

We choose the second method, and get following equations:

$$\begin{aligned} 85 + 29\cos(\theta_1) + 101\cos(\theta_2) &= 50\cos(\theta_3) \\ 29\sin(\theta_1) + 101\sin(\theta_2) &= 50\sin(\theta_3) \\ \frac{82}{\sin(\theta_3)} &= \frac{50}{\sin(\theta_2)} \\ \frac{130}{\cos(\theta_3)} &= \frac{50}{\sin(\theta_1')} \end{aligned}$$

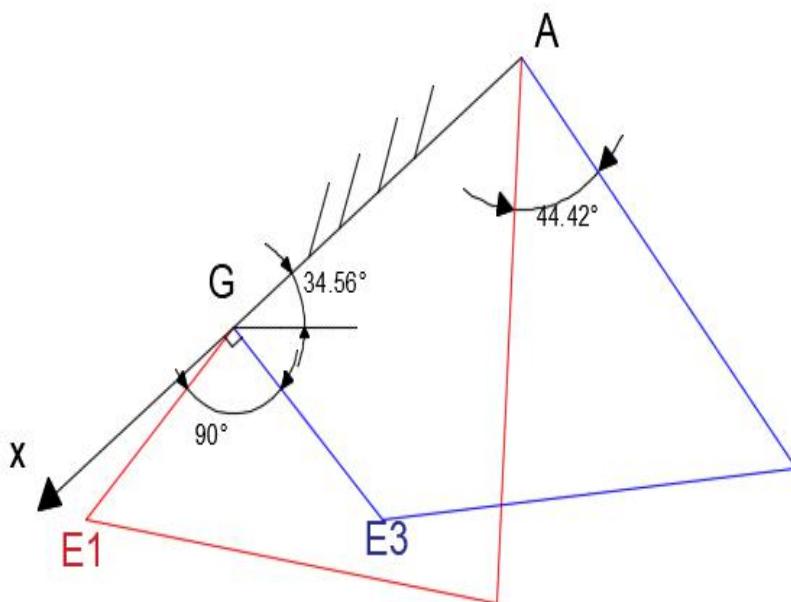
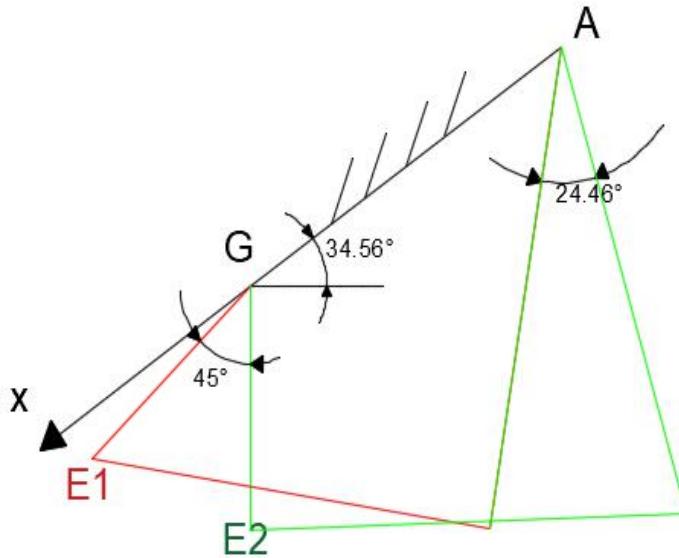
By solving the equations, we can get $\theta_1 = 326.4^\circ$ and $\theta_1' = 116.4^\circ$, which satisfy the equation (4.6).

4.2.2 Analytic method to prove the output four-bar linkage

Known conditions: the angle between AG and GE₁ is 10.44° , the angle between AG and GD₁ is 51.61° .

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We can list four equations corresponding to two positions of the second four bar linkage:



Let $l = ED/EG$, $m = AD/EG$, $n = AG/AE = 1.7$, we can list the following equations:

$$l \cos(\theta_{2i}) = n + m \cos(51.61 + \theta_{3i}) - \cos(10.44 + \theta_{1i})$$

$$l \sin(\theta_{2i}) = m \sin(51.61 + \theta_{3i}) - \sin(10.44 + \theta_{1i})$$

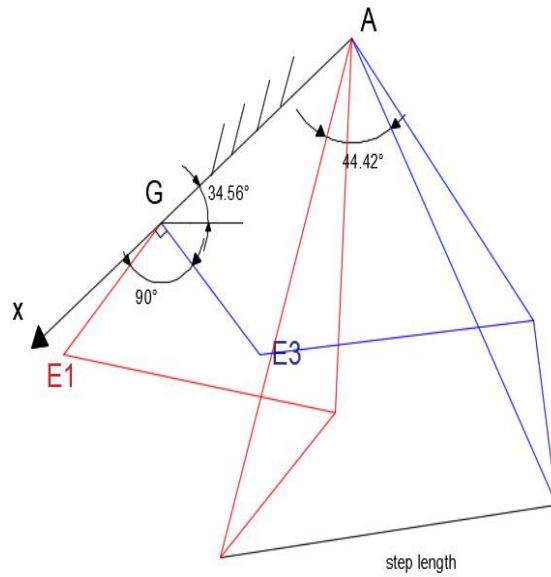
In the first limited position, $\theta_{3i} = 24.26^\circ$, $\theta_{1i} = 45^\circ$.

In the second limited position, $\theta_{3i} = 44.42^\circ$, $\theta_{1i} = 90^\circ$.

Finally, we can solve that $AD = 98.03\text{mm}$ and $ED = 98\text{mm}$, which is similar to that of the graphical method.

As for the angle a , because we want each step to be 108mm , so we can list the equation with angle a . We can use the isosceles triangle AF_1F_2 to determine the angle a :

$\cos(F_1AF_2) = (AF_1 + AF_2 - F_1F_2) / (2 * AF_1 * AF_2)$ we can solve that $AF = 143.5\text{mm}$, by using the law of cosines, we can get $a = 144^\circ$.



4.3 Overall dimensions:

Finally, the new mechanism's overall dimension is certain, $AG=85\text{mm}$, $AB=29\text{mm}$, $AD=DE=100\text{mm}$, $EG=FG=50\text{mm}$, $FB=101\text{mm}$, $\alpha=144^\circ$.

After doing a simulation about its motion, we find that it can achieve the requirements.

4.4 Design of driving mechanism:

4.4.1 Design and calculation of gear reducer

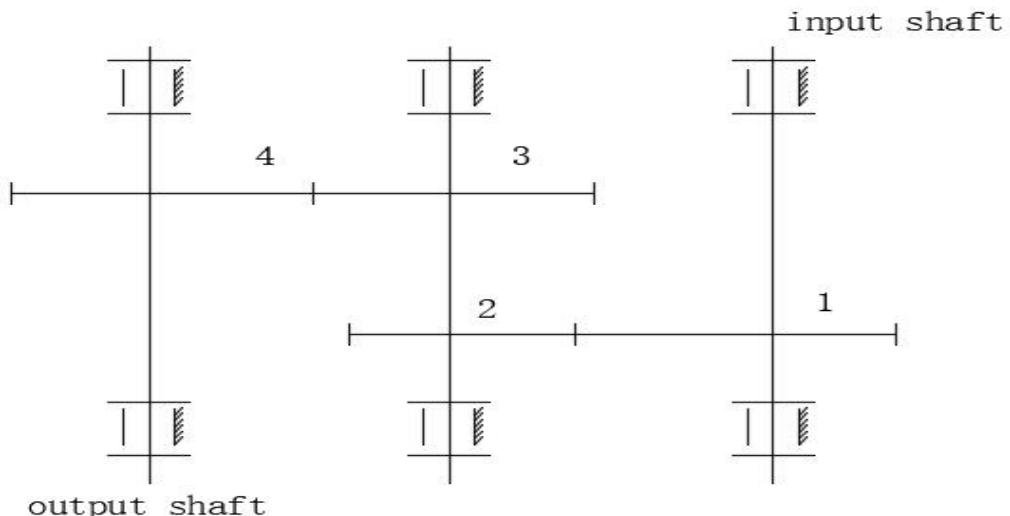
Because the output speed of the motor is very large, we need to use gear reducer to adjust the output speed of the motor to a reasonable value.

Known conditions:

- 1, the output speed of the motor is 490r/min .
- 2, the crank needs to rotate 40 turns per minute, which means the mechanism can make 40 steps per minute. (40r/min)
- 3, the gear reducer should be easy to fabricate, and have a high efficiency.

4.4.2 Gear train design

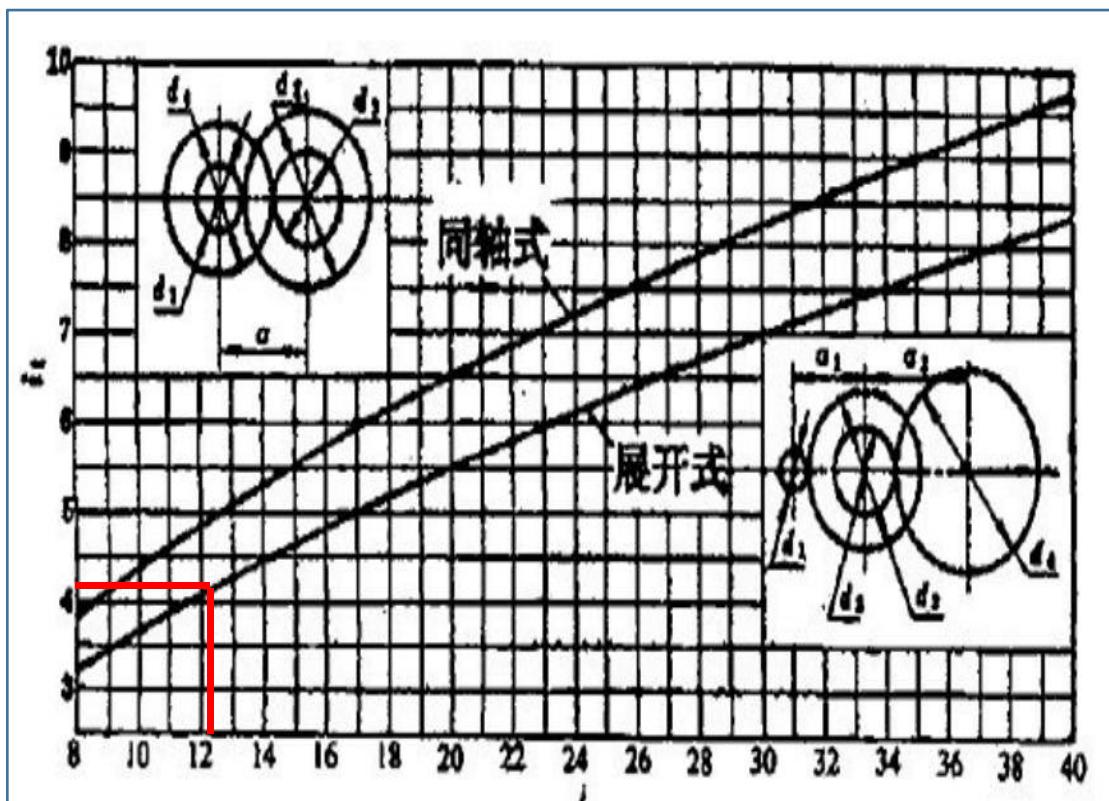
Two-stage expansion cylindrical gear transmission has a high transmission efficiency, wide range of applicable power and speed, and long service life. We have known the motor's speed is 490r/min and the corresponding output speed that we need is 40 r/min , so the gear reducer's transmission ratio is $i = 12.25$ ($8 \sim 60$), two-stage reducer is satisfied. In the gear reducer, we use the expansion gear train, which is shown in the following picture:



From the fixed-axis gear train, we can get:

$$i_{14} = w_1/w_4 = i_{12} * i_{34} = (z_2 * z_4) / (z_1 * z_3)$$

Then we need to assign the transmission ratio, in order to make the service life all transmission gears close to each other, we need to obey the principle of equal strength to make the gear train, so the high-speed transmission ratio is slightly larger than that of the low-speed. The transmission ratio of each level should be determined in accordance with the principle of gradual reduction. From the distribution diagram of transmission ratio of two-stage cylindrical reducer, we can approximately choose $i_{12} = 4.375$, $i_{34} = 2.8$.



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Actual transmission ratio is 12.25, and the transmission ratio error is:

$$\delta = |i_{14}-i_{14}^*|/i_{14} * 100\% = (12.25-4.375*2.8)/12.25 * 100\% = 0 < 0.5\%$$

which is satisfied the corresponding requirements. And we can calculate the corresponding number of teeth, which is $Z_1=16$, $Z_2=70$, $Z_3=20$, $Z_4=56$.

4.4.3 Gear parameters design

The gear parameters include modification coefficient, indexing circle diameter, addendum circle diameter, dedendum circle diameter, tooth thickness and so on.

4.4.4 Determine parameters of high-speed gear

We have known the parameters of high speed gears, which are: number of teeth: $Z_1=16$, $Z_2=100$, pressure angle is 20 degree, addendum height coefficient $ha^*=1$, coefficient of bottom clearance $c^*=0.25$, modulus $m=1$ mm, since the number of teeth of gear 1 is less than that of the minimum number of undercutting teeth, high-speed gears need to be modified and adopt positive drive.

(1): Determine the minimum modification coefficient $X_{1\min}$:

$$X_{1\min} = \frac{Z_{\min} - Z_1}{Z_{\min}} = \frac{1}{17} = 0.0588$$

X_1 is taken as 0.1.

X_2 is taken as 0.03.

(2): Pitch diameter:

$$d_1 = m * z_1 = 1 * 16 = 16 \text{ mm}$$

$$d_2 = m * z_2 = 1 * 70 = 70 \text{ mm}$$

(3): Diameter of base circle:

$$d_{b1} = d_1 * \cos(a) = 16 * \cos(20^\circ) = 15.04 \text{ mm}$$

$$d_{b2} = d_2 * \cos(a) = 70 * \cos(20^\circ) = 65.78 \text{ mm}$$

(4): Addendum:

$$ha_1 = (ha^* + x_1) * m = (1 + 0.1) * 1 = 1.1 \text{ mm}$$

$$ha_2 = (ha^* + x_2) * m = (1 + 0.03) * 1 = 1.03 \text{ mm}$$

(5): Dedendum:

$$hf_1 = (hf^* + c^* - x_1) * m = (1.25 - 0.1) * 1 = 1.15 \text{ mm}$$

$$hf_2 = (hf^* + c^* - x_2) * m = (1.25 - 0.03) * 1 = 1.22 \text{ mm}$$

(6): Tooth depth:

$$h_1 = ha_1 + hf_1 = 1.1 + 1.15 = 2.25 \text{ mm}$$

$$h_2 = ha_2 + hf_2 = 1.03 + 1.22 = 2.25 \text{ mm}$$

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(7): Diameter of addendum circle:

$$d_{a1} = d_1 + 2(ha^* + x_1) * m = 16 + 2(1 + 0.1) * 1 = 18.2 \text{ mm}$$

$$d_{a2} = d_2 + 2(ha^* + x_2) * m = 70 + 2(1 + 0.03) * 1 = 72.06 \text{ mm}$$

(8): Diameter of dedendum circle:

$$d_{f1} = d_1 - 2(ha^* + c^* - x_1) * m = 16 - 2(1.25 - 0.1) * 1 = 13.7 \text{ mm}$$

$$d_{f2} = d_2 - 2(ha^* + c^* - x_2) * m = 70 - 2(1.25 - 0.03) * 1 = 67.56 \text{ mm}$$

(9): Circular pitch:

$$P = \pi * m = 3.14 \text{ mm}$$

(10): Indexing tooth thickness:

$$s_1 = (\pi * m) / 2 + 2 * x_1 * m * \tan(a) = 1.644 \text{ mm}$$

$$s_2 = (\pi * m) / 2 + 2 * x_2 * m * \tan(a) = 1.593 \text{ mm}$$

Both gears' indexing tooth thickness is larger than $0.4 * m = 0.4 \text{ mm}$, which satisfy the corresponding requirements.

(11): Indexing tooth width:

$$e_1 = (\pi * m) / 2 - 2 * x_1 * m * \tan(a) = 1.498 \text{ mm}$$

$$e_2 = (\pi * m) / 2 - 2 * x_2 * m * \tan(a) = 1.549 \text{ mm}$$

4.4.5 Determine parameters of low-speed gear

We have known the parameters of low speed gears, which are: number of teeth: $Z_3=20$, $Z_2=56$, pressure angle is 20 degree, addendum height coefficient $ha^*=1$, coefficient of bottom clearance $c^*=0.25$, modulus $m=1 \text{ mm}$, since the number of both teeth is larger than that of the minimum number of undercutting teeth, low-speed gears don't need to be modified.

(1): Pitch diameter:

$$d_3 = m * z_3 = 1 * 20 = 20 \text{ mm}$$

$$d_4 = m * z_4 = 1 * 56 = 56 \text{ mm}$$

(2): Diameter of base circle:

$$d_{b3} = d_3 * \cos(a) = 20 * \cos(20^\circ) = 18.79 \text{ mm}$$

$$d_{b4} = d_4 * \cos(a) = 56 * \cos(20^\circ) = 52.62 \text{ mm}$$

(3): Addendum:

$$ha_3 = ha_4 = ha^* * m = 1 * 1 = 1 \text{ mm}$$

(4): Dedendum:

$$hf_3 = hf_4 = (hf^* + c^*) * m = 1.25 * 1 = 1.25 \text{ mm}$$

(5): Tooth depth:

$$h_3 = h_4 = ha_1 + hf_1 = 2.25 \text{ mm}$$

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(6): Diameter of addendum circle:

$$d_{a3} = d_3 + 2 * ha^* * m = 20 + 2 * 1 * 1 = 22 \text{ mm}$$

$$d_{a4} = d_4 + 2 * ha^* * m = 56 + 2 * 1 * 1 = 58 \text{ mm}$$

(7): Diameter of dedendum circle:

$$d_{f3} = d_3 - 2(ha^* + c^*) * m = 20 - 2 * 1.25 * 1 = 17.5 \text{ mm}$$

$$d_{f4} = d_4 - 2(ha^* + c^*) * m = 56 - 2 * 1.25 * 1 = 53.5 \text{ mm}$$

(8): Circular pitch:

$$P = \pi * m = 3.14 \text{ mm}$$

(9): Indexing tooth thickness:

$$s_3 = s_4 = (\pi * m) / 2 = 1.571 \text{ mm}$$

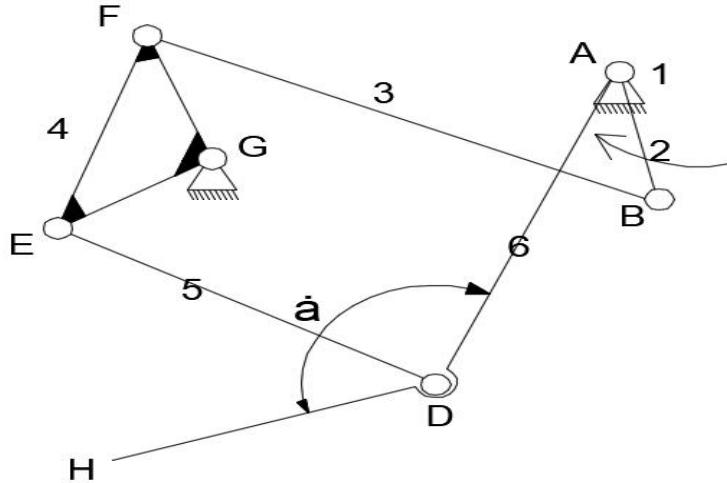
Both gears' indexing tooth thickness is larger than $0.4 * m = 0.4 \text{ mm}$, which satisfy the corresponding requirements.

(11): Indexing tooth width:

$$e_3 = e_4 = (\pi * m) / 2 = 1.571 \text{ mm}$$

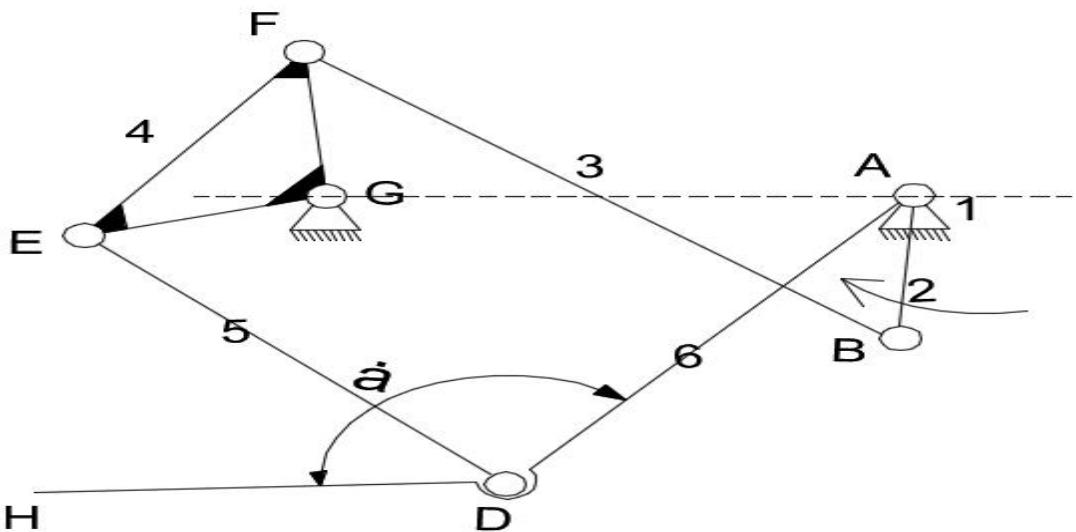
Chapter 5 Analysis and Simulation of the Mechanism

After determining the corresponding size of our optimized model, we will use a variety of professional simulation software(Adams,Inventor) to model, simulate and analyze whether it can meet the relevant requirements we have previously determined.

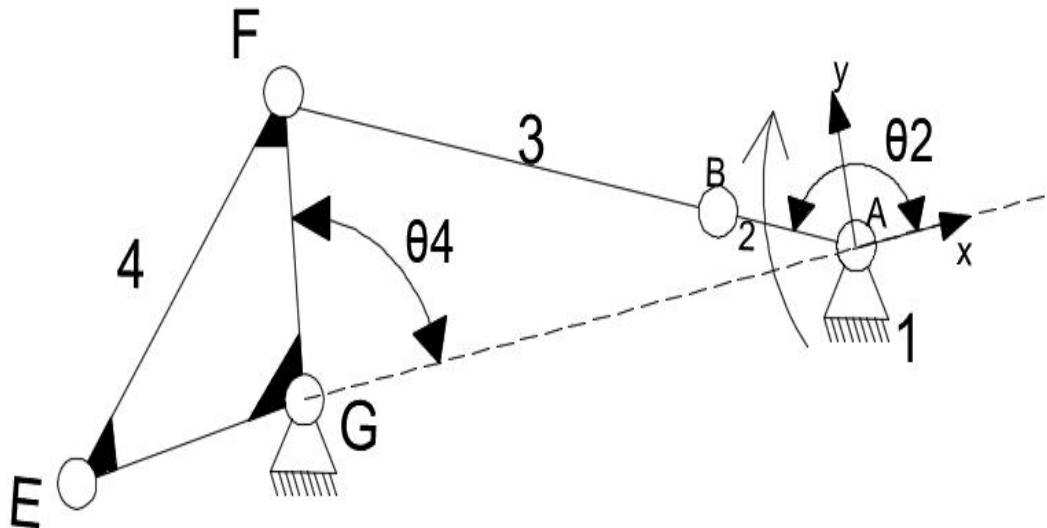


The length of link AB is 29mm, BF is 101mm, FG=EG=50mm, angle between FG and EF is 112° , ED=AD=100mm, AG=85mm, HD = 50mm, $\dot{\alpha}=144^\circ$. We want to calculate the displacement of point H by separating the mechanism into two mechanisms, one is four bar linkage(ABFG) and the other is four bar linkage(ADEG). To simplify calculation, we just rotate the whole system, making the two ground in the same horizontal line.

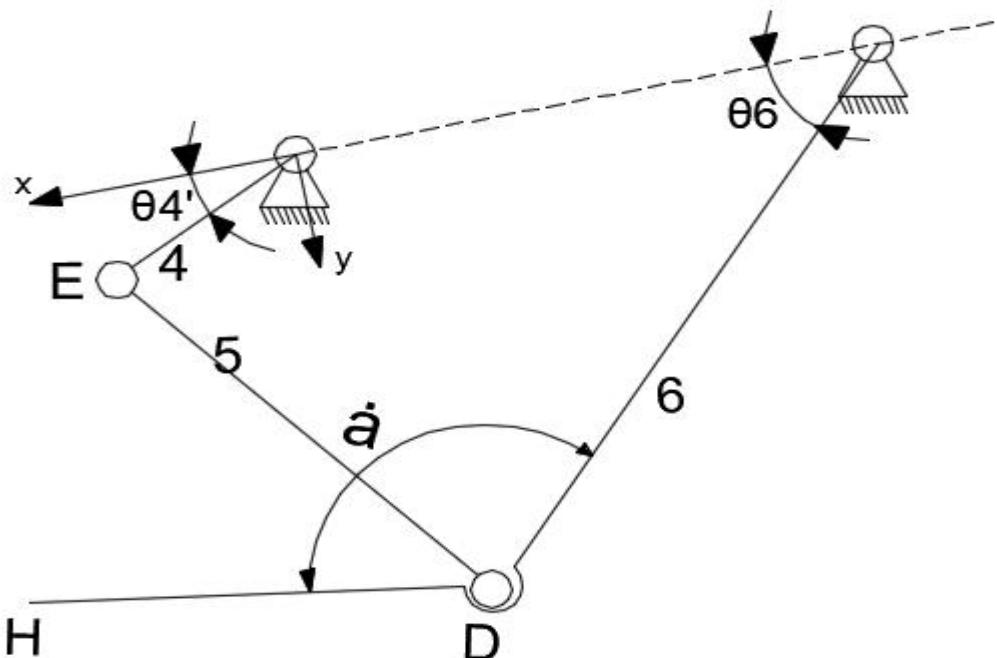
5.1 Building Mathematical Model



To begin with, we just separate the mechanism into two parts, which are shown in the following pictures:



(5.1.1)



(5.1.2)

5.1.1 Analyze the output trace by mathematical method

For picture (5.1.1), the input is the rotation of bar 2 with an angular velocity w_1 , and we want to find the output angular velocity of bar 4, which is the input angular velocity of picture (5.1.2). Finally we can get the displacement of point F, which can represent the displacement of the output trace.

$$\left\{ \begin{array}{l} r_2 * \cos\theta_2 + r_3 * \cos\theta_3 = s + r_4 * \cos\theta_4 \\ r_2 * \sin\theta_2 + r_3 * \sin\theta_3 = r_4 * \sin\theta_4 \end{array} \right.$$

$$\left\{ (r_2 * \cos\theta_2 + r_3 * \cos\theta_3 - s)^2 + (r_2 * \sin\theta_2 + r_3 * \sin\theta_3)^2 - r_4^2 = 0 \right\}$$

$$(r_2 * \cos\theta_2 - s)^2 + r_3^2 - r_4^2 + 2r_3(r_2 * \cos\theta_2 - s)\cos\theta_3 + (r_2 * \sin\theta_2)^2 + 2r_3r_2\sin\theta_2\sin\theta_3 = 0$$

$$s^2 + r_2^2 + r_3^2 - r_4^2 - 2sr_2\cos\theta_2 + 2r_3(r_2 * \cos\theta_2 - s)\cos\theta_3 + 2r_3r_2\sin\theta_2\sin\theta_3 = 0$$

$$2r_3r_2\sin\theta_2\sin\theta_3 + 2r_3(r_2 * \cos\theta_2 - s)\cos\theta_3 + s^2 + r_2^2 + r_3^2 - r_4^2 - 2sr_2\cos\theta_2 = 0$$

$$A = 2r_3r_2\sin\theta_2$$

$$B = 2r_3(r_2 * \cos\theta_2 - s)$$

$$C = s^2 + r_2^2 + r_3^2 - r_4^2 - 2sr_2\cos\theta_2$$

$$\theta_3 = 2\tan^{-1} \frac{-A \pm \sqrt{A^2 + B^2 - C^2}}{C - B}$$

In the same way, we can get the other angle

$$\left\{ \begin{array}{l} r_2 * \cos\theta_2 + r_4 * \cos\theta_4 = s + r_3 * \cos\theta_3 \\ r_2 * \sin\theta_2 + r_4 * \sin\theta_4 = r_3 * \sin\theta_3 \end{array} \right.$$

$$\left\{ (-r_2 * \cos\theta_2 + r_4 * \cos\theta_4 + s)^2 + (-r_2 * \sin\theta_2 + r_4 * \sin\theta_4)^2 - r_3^2 = 0 \right\}$$

$$(-r_2 * \cos\theta_2 + s)^2 + r_4^2 - r_3^2 + 2r_4(-r_2 * \cos\theta_2 + s)\cos\theta_4 + (r_2 * \sin\theta_2)^2 - 2r_4r_2\sin\theta_2\sin\theta_4 = 0$$

$$s^2 + r_2^2 + r_4^2 - r_3^2 - 2sr_2\cos\theta_2 + 2r_4(-r_2 * \cos\theta_2 + s)\cos\theta_4 + 2r_4r_2\sin\theta_2\sin\theta_4 = 0$$

$$2r_4r_2\sin\theta_2\sin\theta_4 + 2r_4(-r_2 * \cos\theta_2 + s)\cos\theta_4 + s^2 + r_2^2 + r_4^2 - r_3^2 - 2sr_2\cos\theta_2 = 0$$

$$A1 = 2r_4r_2\sin\theta_2$$

$$B1 = 2r_4(-r_2 * \cos\theta_2 + s)$$

$$C1 = s^2 + r_2^2 + r_4^2 - r_3^2 - 2sr_2\cos\theta_2$$

$$\theta_4 = 2\tan^{-1} \frac{-A1 \pm \sqrt{A1^2 + B1^2 - C1^2}}{C1 - B1}$$

By differentiating the close loop function, we can get the output angular velocity:

$$\begin{aligned} & \left\{ \begin{array}{l} -r_2 * w_1 * \sin \theta_2 = -r_3 * w_3 * \sin \theta_3 - r_4 * w_4 * \sin \theta_4 \\ r_2 * w_1 * \cos \theta_2 = r_3 * w_3 * \cos \theta_3 + r_4 * w_4 * \cos \theta_4 \end{array} \right\} \\ & \left\{ \begin{array}{l} -r_2 * w_1 * \sin \theta_2 \cos \theta_3 = -r_3 * w_3 * \sin \theta_3 \cos \theta_3 - r_4 * w_4 * \sin \theta_4 \cos \theta_3 \\ r_2 * w_1 * \cos \theta_2 \sin \theta_3 = r_3 * w_3 * \cos \theta_3 \sin \theta_3 + r_4 * w_4 * \cos \theta_4 \sin \theta_3 \end{array} \right\} \\ & \{ r_2 * w_1 * (\cos \theta_2 \sin \theta_3 - \sin \theta_2 \cos \theta_3) = r_4 * w_4 * (\cos \theta_4 \sin \theta_3 - \sin \theta_4 \cos \theta_3) \} \\ & w_4 = \frac{r_2}{r_4} * \frac{\sin(\theta_3 - \theta_2)}{\sin(\theta_3 - \theta_4)} * w_1 \end{aligned}$$

Similarly, we can get w_3 :

$$w_3 = -\frac{r_2}{r_3} * \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_4 - \theta_3)} * w_1$$

Following steps are functions to solve the output angular acceleration a_4 , a_1 is 0:

$$\begin{aligned} & \left\{ \begin{array}{l} -r_2 * a_1 * \sin \theta_2 - r_2 * w_1^2 * \cos \theta_2 - r_3 * a_3 * \sin \theta_3 - r_3 * w_3^2 * \cos \theta_3 = -r_4 * a_4 * \sin \theta_4 - r_4 * w_4^2 * \cos \theta_4 \\ r_2 * a_1 * \cos \theta_2 - r_2 * w_1^2 * \sin \theta_2 + r_3 * a_3 * \cos \theta_3 - r_3 * w_3^2 * \sin \theta_3 = r_4 * a_4 * \cos \theta_4 - r_4 * w_4^2 * \sin \theta_4 \end{array} \right\} \\ & \left\{ \begin{array}{l} -r_2 * a_1 * \sin \theta_2 - r_2 * w_1^2 * \cos \theta_2 - r_3 * w_3^2 * \cos \theta_3 + r_4 * w_4^2 * \cos \theta_4 = -r_4 * a_4 * \sin \theta_4 + r_3 * a_3 * \sin \theta_3 \\ r_2 * a_1 * \cos \theta_2 - r_2 * w_1^2 * \sin \theta_2 - r_3 * w_3^2 * \sin \theta_3 + r_4 * w_4^2 * \sin \theta_4 = r_4 * a_4 * \cos \theta_4 - r_3 * a_3 * \cos \theta_3 \end{array} \right\} \\ & \begin{pmatrix} r_3 * \sin \theta_3 & -r_4 * \sin \theta_4 \\ -r_3 * \cos \theta_3 & r_4 * \cos \theta_4 \end{pmatrix} \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} -r_2 * a_1 * \sin \theta_2 - r_2 * w_1^2 * \cos \theta_2 - r_3 * w_3^2 * \cos \theta_3 + r_4 * w_4^2 * \cos \theta_4 \\ r_2 * a_1 * \cos \theta_2 - r_2 * w_1^2 * \sin \theta_2 - r_3 * w_3^2 * \sin \theta_3 + r_4 * w_4^2 * \sin \theta_4 \end{pmatrix} \\ & \left\{ \begin{array}{l} a_3 = [-w_2^2 * r_2 * \cos(\theta_2 - \theta_4) + w_4^2 * r_4 - w_3^2 * r_3 * \cos(\theta_3 - \theta_4)] / r_4 * \sin(\theta_3 - \theta_4) \\ a_4 = [w_2^2 * r_2 * \cos(\theta_2 - \theta_3) + w_3^2 * r_3 - w_4^2 * r_4 * \cos(\theta_4 - \theta_3)] / r_4 * \sin(\theta_4 - \theta_3) \end{array} \right. \end{aligned}$$

The next step is to use the output velocity and angular acceleration as the input of the another four-bar mechanism (5.1.2):

We just use the conclusion from the above equations, and we can get:

$$r_4 * e^{i\theta_4} + r_5 * e^{i\theta_5} = s + r_6 * e^{i\theta_6}$$

$$\left\{ \begin{array}{l} r_4 * \cos\theta_4 + r_5 * \cos\theta_5 = s + r_6 * \cos\theta_6 \\ r_4 * \sin\theta_4 + r_5 * \sin\theta_5 = r_6 * \sin\theta_6 \end{array} \right.$$

$$s^2 + r_4^2 + r_5^2 - r_6^2 - 2sr_4\cos\theta_4 + 2r_5(r_4 * \cos\theta_4 - s)\cos\theta_5 + 2r_5r_4\sin\theta_5\sin\theta_4 = 0$$

$$2r_5r_4\sin\theta_4\sin\theta_5 + 2r_5(r_4 * \cos\theta_4 - s)\cos\theta_5 + s^2 + r_4^2 + r_5^2 - r_6^2 - 2sr_4\cos\theta_4 = 0$$

$$A = 2r_5r_4\sin\theta_4$$

$$B = 2r_5(r_4 * \cos\theta_4 - s)$$

$$C = s^2 + r_4^2 + r_5^2 - r_6^2 - 2sr_4\cos\theta_4$$

$$\theta_5 = 2\tan^{-1} \frac{-A \pm \sqrt{A^2 + B^2 - C^2}}{C - B}$$

$$A1 = 2r_6r_4\sin\theta_4$$

$$B1 = 2r_6(-r_4 * \cos\theta_4 + s)$$

$$C1 = s^2 + r_4^2 + r_6^2 - r_5^2 - 2sr_4\cos\theta_4$$

$$\theta_6 = 2\tan^{-1} \frac{-A1 \pm \sqrt{A1^2 + B1^2 - C1^2}}{C1 - B1}$$

Corresponding angular velocity of bar 5 and bar 6:

$$\omega_6 = \frac{r_4}{r_6} * \frac{\sin(\theta_5 - \theta_4)}{\sin(\theta_5 - \theta_6)} * \omega_4$$

$$\omega_5 = -\frac{r_4}{r_5} * \frac{\sin(\theta_6 - \theta_4)}{\sin(\theta_6 - \theta_5)} * \omega_4$$

The output angular acceleration is given in the following steps:

$$\begin{pmatrix} r_5 * \sin\theta_5 & -r_6 * \sin\theta_6 \\ -r_5 * \cos\theta_5 & r_6 * \cos\theta_6 \end{pmatrix} \begin{pmatrix} a_5 \\ a_6 \end{pmatrix} = \begin{pmatrix} -r_4 * a_4 * \sin\theta_4 - r_4^2 * w_4^2 * \cos\theta_4 - r_5^2 * w_5^2 * \cos\theta_5 + r_6^2 * w_6^2 * \cos\theta_6 \\ r_4 * a_4 * \cos\theta_4 - r_4^2 * w_4^2 * \sin\theta_4 - r_5^2 * w_5^2 * \sin\theta_5 + r_6^2 * w_6^2 * \sin\theta_6 \end{pmatrix}$$

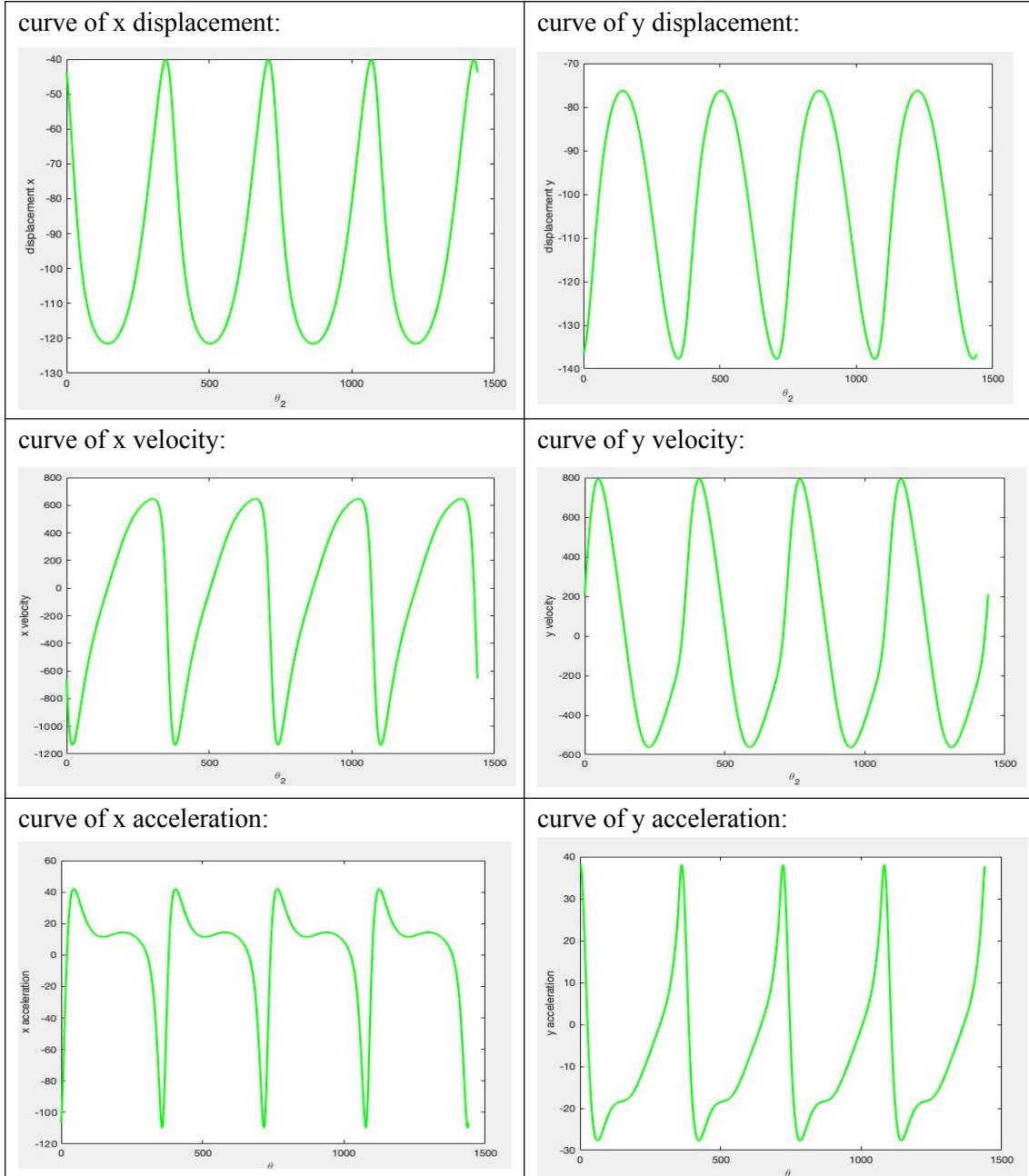
$$\begin{cases} a_5 = [r_4 * a_4 * \sin(\theta_6 - \theta_4) - w_4^2 * r_4 * \cos(\theta_4 - \theta_6) + w_6^2 * r_6 - w_5^2 * r_5 * \cos(\theta_5 - \theta_6)] / r_6 * \sin(\theta_5 - \theta_6) \\ a_6 = [r_4 * a_4 * \sin(\theta_4 - \theta_5) + w_4^2 * r_4 * \cos(\theta_4 - \theta_5) + w_5^2 * r_5 - w_6^2 * r_6 * \cos(\theta_6 - \theta_5)] / r_6 * \sin(\theta_6 - \theta_5) \end{cases}$$

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Finally, we can use mathematical method to get the corresponding angular velocity and angular acceleration of the output.

Simulation by Matlab to check the corresponding curve of displacement, velocity and acceleration:

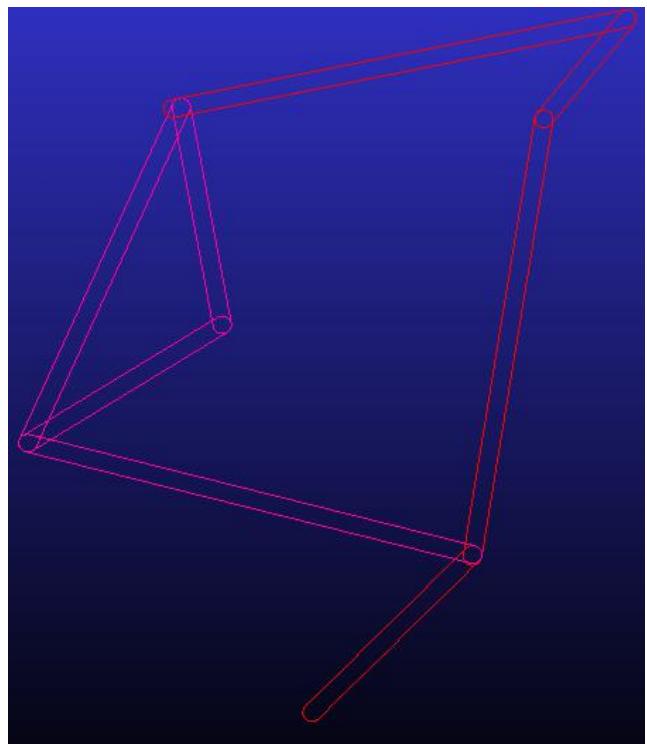
The known conditions are: the input angular velocity w_1 is 1rad/s, r_2 is 29mm, r_3 is 101mm, r_4 is 50mm, $r_5 = r_6 = 100$ mm, output angle is 144° .



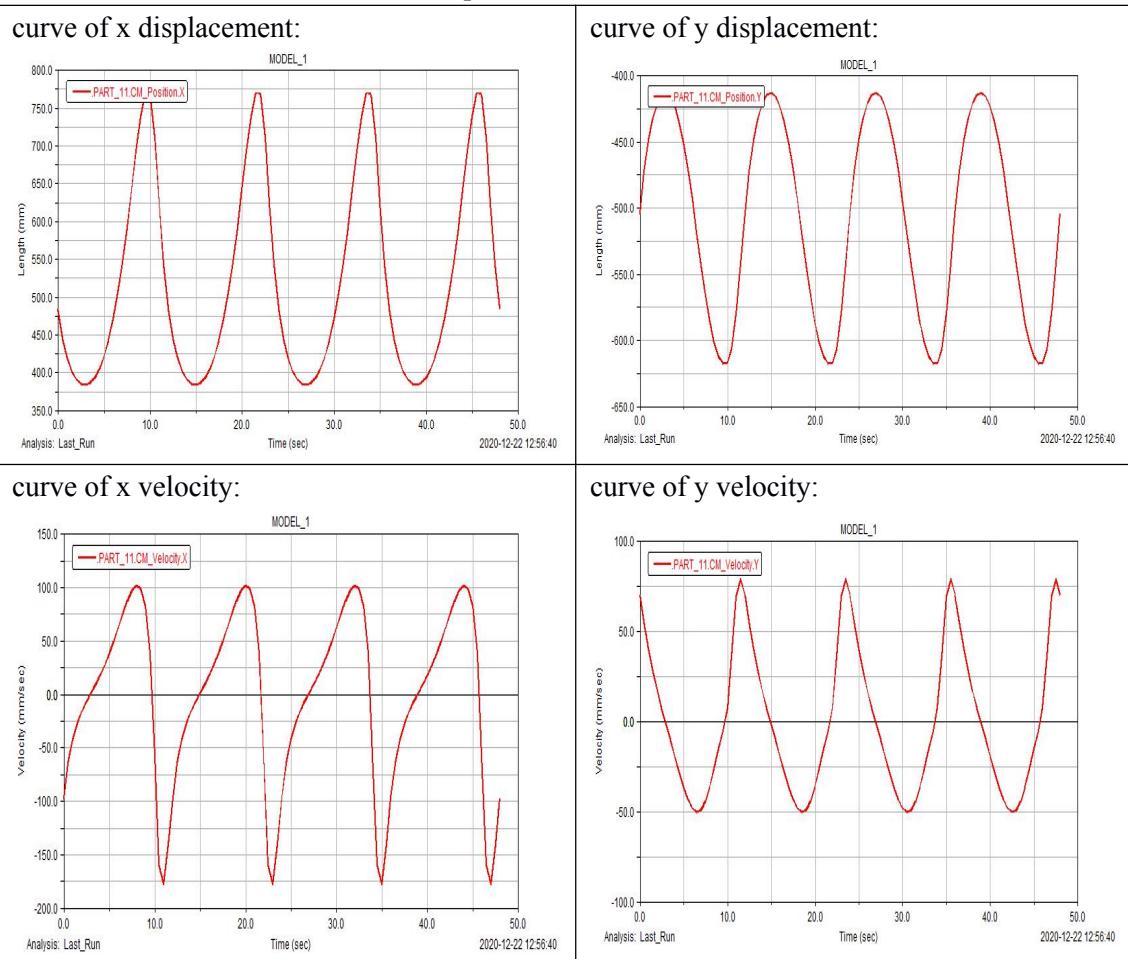
5.1.2 Model building and simulation by Adams

Use the calculation dimensions and built model in ADAMS, showing in the following pictures:

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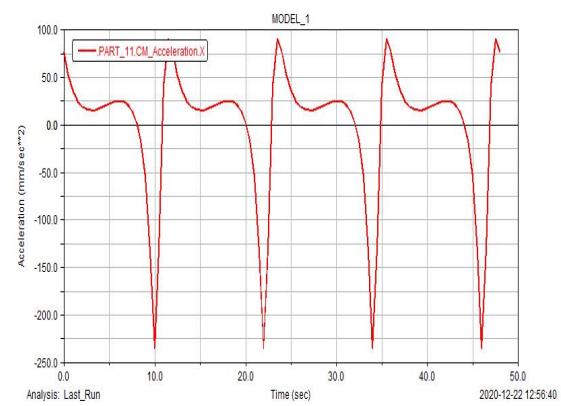


Drive the model and measure the parameters:

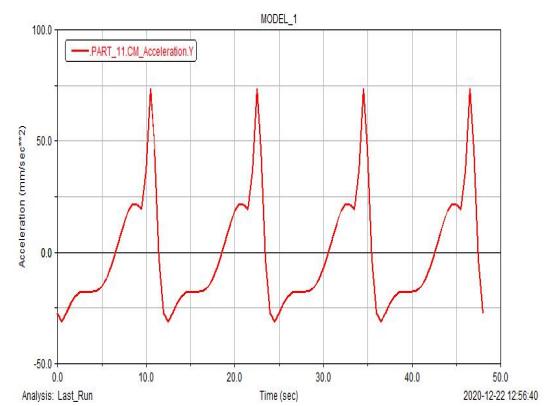


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curve of x acceleration:



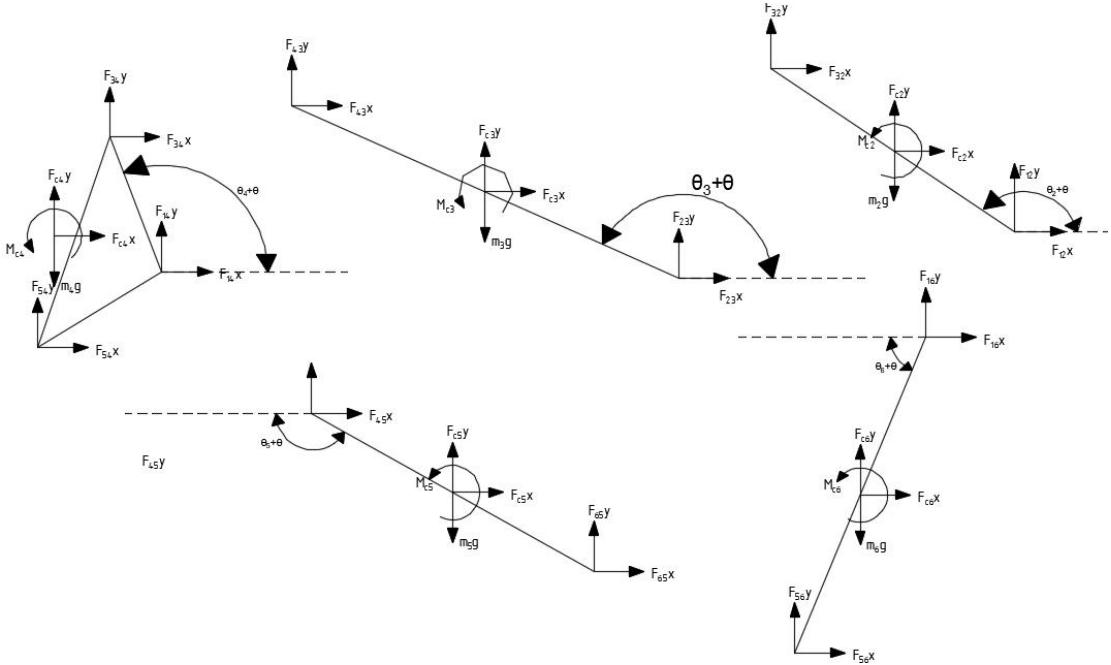
curve of y acceleration:



Chapter 6 Dynamic analysis and dynamic balance

6.1 Force analyze of mechanism

The stress distribution of the corresponding mechanism is shown in the following picture, from the picture, we can see that the end of the crank shaft is subjected to a balancing moment, each component will subject to internal force and gravity, and there is interaction and friction between each component.



Assumptions for connection and self-properties:

- 1, The friction force at the joint of each component can be ignored.
- 2, The force acting on the connecting rod is concentrated force and the inertial force acting on the center of mass as a concentrated force.
- 3, The connecting rod force acts as a concentrated force at the middle point of the crank pin and the internal force acts as a concentrated force at the mass center of the crank.
- 4, The mass distribution of the connecting rod is uniform, 0.5kg per 10mm.

6.2 Known parameters and undetermined parameters

(1): Mass and moment of inertia of each components: \$m_2=1.5\$ kg, \$m_3=5\$ kg, \$m_4=9.15\$ kg, \$m_5=m_6=5\$ kg; \$J_3=0.00417\$ kg*m², \$J_4=0.00345\$ kg*m², \$J_5=J_6=0.00417\$ kg*m².

(2): Length parameters of each components:

\$L_2=29\$ mm, \$L_{c2}=14.5\$ mm.

\$L_3=101\$ mm, \$L_{c3}=50.5\$ mm.

\$L_4=50\$ mm, \$L_{c4}=44.7\$ mm. (mass center is located at the intersection of three central lines)

\$L_5=100\$ mm, \$L_{c5}=50\$ mm.

\$L_6=100\$ mm. \$L_{c6}=50\$ mm.

The angle between the connecting line of two frames and the positive half axis of x-axis is 34.56 degree.

(3): motion parameters of each components:

Through kinematic analysis, we have found:

angular displacement of link 2: \$\theta_2\$ and we can get the acceleration of the mass center of link 2:

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$$a_{c2x} = -(w_2)^2 * (L_2/2) * \cos(\theta_2 + \theta), \quad a_{c2y} = -(w_2)^2 * (L_2/2) * \sin(\theta_2 + \theta)$$

angular displacement of link 3: θ_3 and we can get the acceleration of the mass center of link 3:

$$a_{c3x} = w_2^2 * L_2 * \cos(\theta + \theta_2) - w_3^2 * (L_3/2) * \cos(\theta + \theta_3) + a_3 * (L_3/2) * \sin(\theta + \theta_3)$$

$$a_{c3y} = w_2^2 * L_2 * \sin(\theta + \theta_3) + w_3^2 * (L_3/2) * \sin(\theta + \theta_3) - a_3 * (L_3/2) * \cos(\theta + \theta_3)$$

angular displacement of link 4: θ_4 and we can get the acceleration of the mass center of link 4:

$$a_{c4x} = w_2^2 * L_2 * \cos(\theta + \theta_2) - w_3^2 * (L_3) * \cos(\theta + \theta_3) + a_3 * (L_3) * \sin(\theta + \theta_3) + w_4^2 * r * \cos(\theta + \theta_4 - 56^\circ) - a_4 * r * \sin(\theta + \theta_4 - 56^\circ)$$

$$a_{c4y} = w_2^2 * L_2 * \sin(\theta + \theta_3) + w_3^2 * (L_3) * \sin(\theta + \theta_3) - a_3 * (L_3) * \cos(\theta + \theta_3) + w_4^2 * r * \sin(\theta + \theta_4 - 56^\circ) - a_4 * r * \cos(\theta + \theta_4 - 56^\circ)$$

angular displacement of link 5: θ_5 and we can get the acceleration of the mass center of link 5:

$$a_{c5x} = w_6^2 * L_6 * \cos(\theta + \theta_6) - a_6 * L_6 * \sin(\theta + \theta_6) - w_5^2 * (L_5/2) * \cos(\theta + \theta_5) + a_5 * (L_5/2) * \sin(\theta + \theta_5)$$

$$a_{c5y} = w_6^2 * L_6 * \sin(\theta + \theta_6) + a_6 * L_6 * \cos(\theta + \theta_6) + w_5^2 * (L_5/2) * \sin(\theta + \theta_5) + a_5 * (L_5/2) * \cos(\theta + \theta_5)$$

angular displacement of link 6: θ_6 and we can get the acceleration of the mass center of link 6:

$$a_{c6x} = w_6^2 * (L_6/2) * \cos(\theta + \theta_6) - a_6 * (L_6/2) * \sin(\theta + \theta_6)$$

$$a_{c6y} = w_6^2 * (L_6/2) * \sin(\theta + \theta_6) + a_6 * (L_6/2) * \cos(\theta + \theta_6)$$

What's more, the corresponding moment of inertia can also be determined:

$$M_{c3} = -J_3 * a_3; M_{c4} = -J_4 * a_4; M_{c5} = -J_5 * a_5;$$

(4): parameters to be solved:

Unknown forces of each pairs:

the force between the frame and link 2: F12x and F12y;

the force between the frame and link 6: F16x and F16y;

the force between the frame and link 4: F14x and F14y;

reaction force between link 2 and link 3: F23x and F23y;

reaction force between link 3 and link 4: F34x and F34y;

reaction force between link 4 and link 5: F45x and F45y;

reaction force between link 5 and link 6: F56x and F56y;

Unknown balance torque Mc2;

6.3 Force analyze of each component

6.3.1 Force analysis of link 2:

Link 2 is subjected to seven forces and a balance torque.

1, force between the frame and connecting rod 2: F12x and F12y;

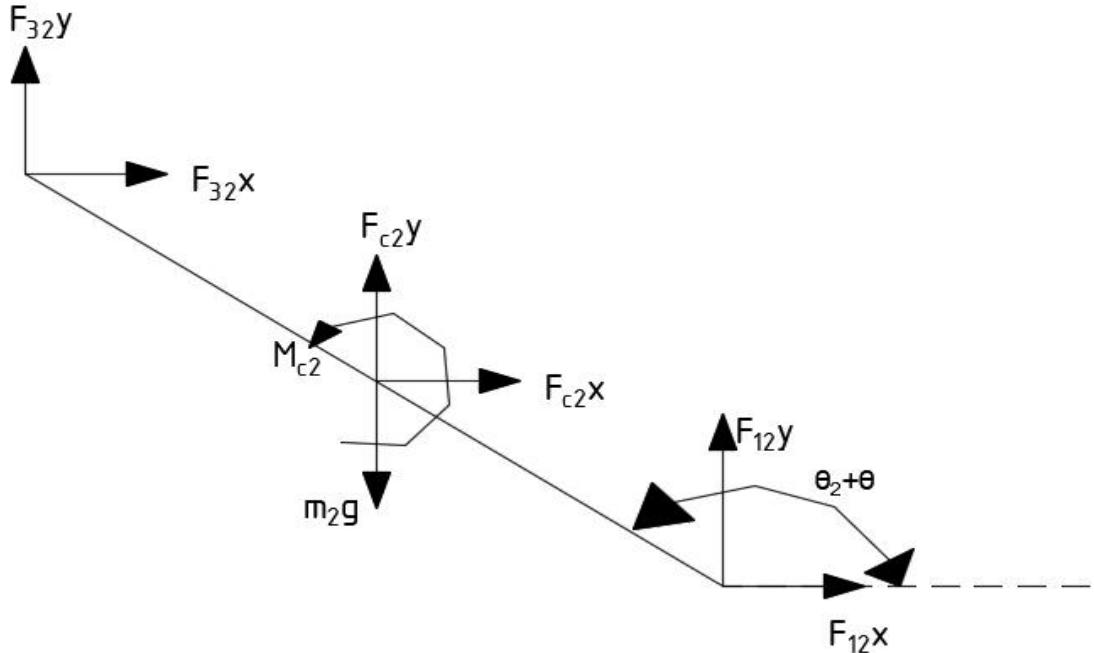
2, connecting rod's inertia force Fc2x and Fc2y;

3, reaction force between link 2 and 3: F32x and F32y;

4, balance torque Mc2;

5, gravity m2g;

Corresponding schematic diagram of link 2 is shown below:



List the corresponding balance equation:

$$F_{12}x + F_{c2}x + F_{32}x = 0 \quad (1)$$

$$F_{12}y + F_{c2}y + F_{32}y - m_2g = 0 \quad (2)$$

$$Mc_2 + (L_2/2) * F_{23}x * \sin(\theta_2 + \theta) - (L_2/2) * F_{23}y * \cos(\theta_2 + \theta) + (L_2/2) * F_{12}x * \sin(\theta_2 + \theta) - (L_2/2) * F_{12}y * \cos(\theta_2 + \theta) = 0 \quad (3)$$

Notice that:

$$F_{c2}x = m_2 * ac_{2x}; \quad F_{c2}y = m_2 * ac_{2y}; \quad F_{32}x = -F_{23}x; \quad F_{32}y = -F_{23}y$$

6.3.2 Force analysis of link 3:

Link 3 is subjected to six forces and a centroid moment of inertia.

1, reaction force between link 4 and link 3: $F_{43}x$ and $F_{43}y$;

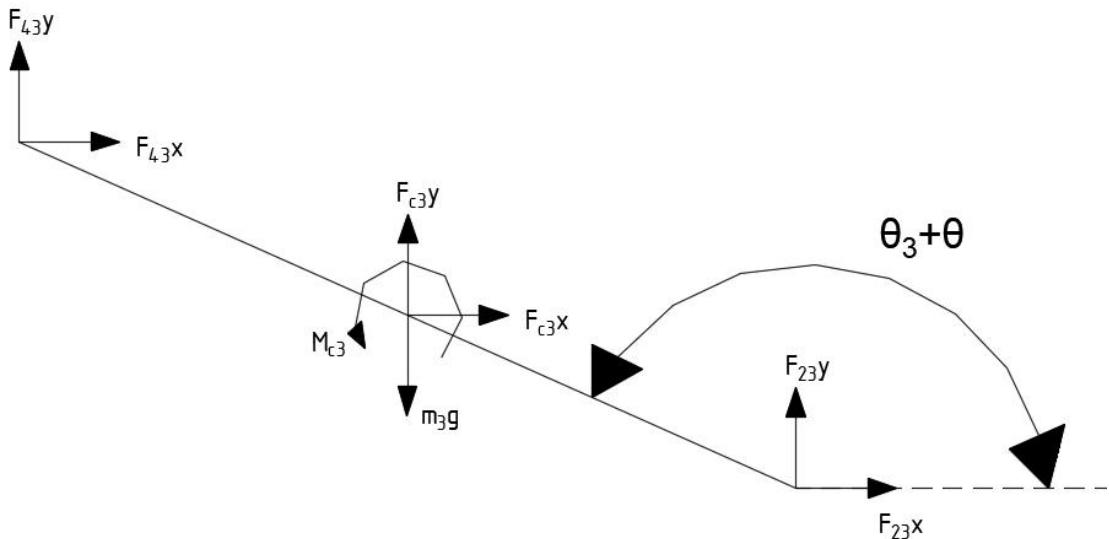
2, connecting rod's inertia force $F_{c3}x$ and $F_{c3}y$;

3, reaction force between link 2 and 3: $F_{23}x$ and $F_{23}y$;

4, centroid moment of inertia Mc_3 ;

5, gravity m_3g ;

Corresponding schematic diagram of link 2 is shown below:



List the corresponding balance equation:

$$F_{43}x + F_{c3}x + F_{23}x = 0 \quad (4)$$

$$F_{43}y + F_{c3}y + F_{23}y - m_3g = 0 \quad (5)$$

$$\begin{aligned} & M_{c3} + (L_3/2) * F_{23}x * \sin(\theta_3 + \theta) - (L_3/2) * F_{23}y * \cos(\theta_3 + \theta) - (L_3/2) * F_{43}x * \sin(\theta_3 + \theta) \\ & + (L_3/2) * F_{43}y * \cos(\theta_3 + \theta) = 0 \end{aligned} \quad (6)$$

Notice that:

$$F_{c3}x = m_3 * a_{c3}x; \quad F_{c3}y = m_3 * a_{c3}y;$$

6.3.1 Force analysis of link 4:

Link 4 is subjected to nine forces and a centroid moment of inertia.

1, reaction force between link 4 and link 3: $F_{34}x$ and $F_{34}y$;

2, connecting rod's inertia force $F_{c4}x$ and $F_{c4}y$;

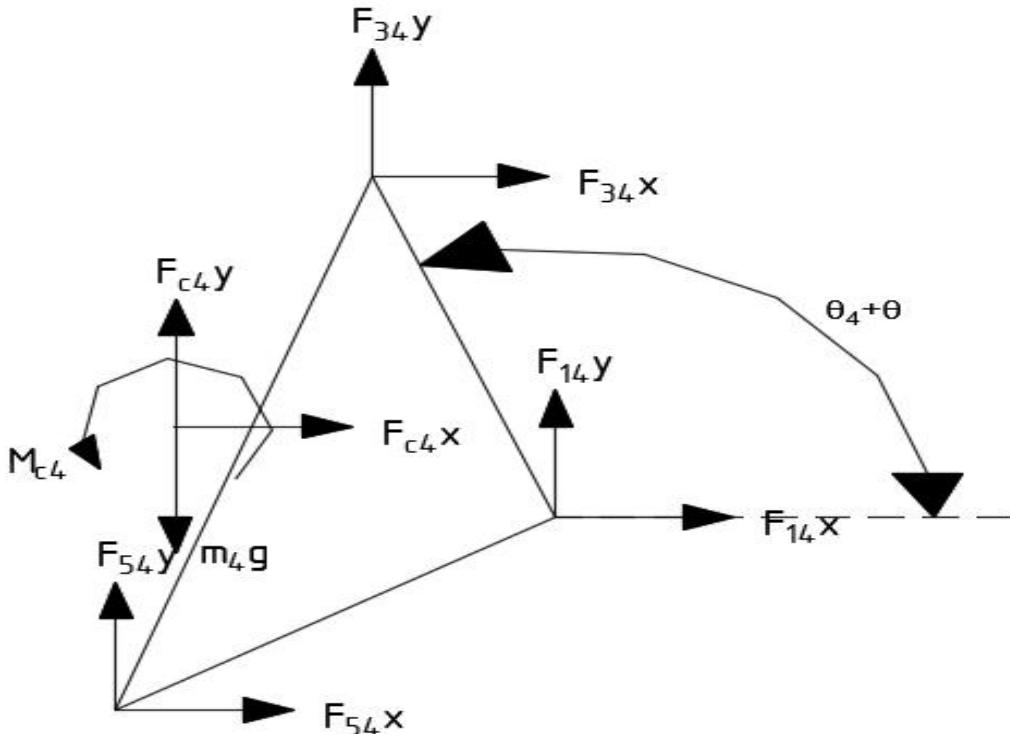
3, reaction force between link 4 and 5: $F_{54}x$ and $F_{54}y$;

4, centroid moment of inertia M_{c4} ;

5, force between the frame and connecting rod 4: $F_{14}x$ and $F_{14}y$;

6, gravity m_4g ;

Corresponding schematic diagram of link 4 is shown below:



List the corresponding balance equation:

$$F_{14x} + F_{c4x} + F_{34x} + F_{54x} = 0 \quad (7)$$

$$F_{14y} + F_{c4y} + F_{34y} + F_{54y} - m_4 g = 0 \quad (8)$$

$$M_{c4} + r * F_{14x} * \sin(90.56 + \theta_4) - r * F_{14y} * \cos(90.56 + \theta_4) + r * F_{54x} * \sin(22.56 + \theta_4) - r * F_{54y} * \cos(22.56 + \theta_4) - r * F_{34x} * \sin(201.44 - \theta_4) - r * F_{34y} * \cos(201.44 - \theta_4) = 0 \quad (9)$$

Notice that:

$$F_{c4x} = m_4 * a_{c4x}; \quad F_{c4y} = m_4 * a_{c4y}; \quad F_{54x} = -F_{45x}; \quad F_{54y} = -F_{45y}$$

6.3.2 Force analysis of link 5:

Link 5 is subjected to six forces and a centroid moment of inertia.

1, reaction force between link 4 and link 5: F_{45x} and F_{45y} ;

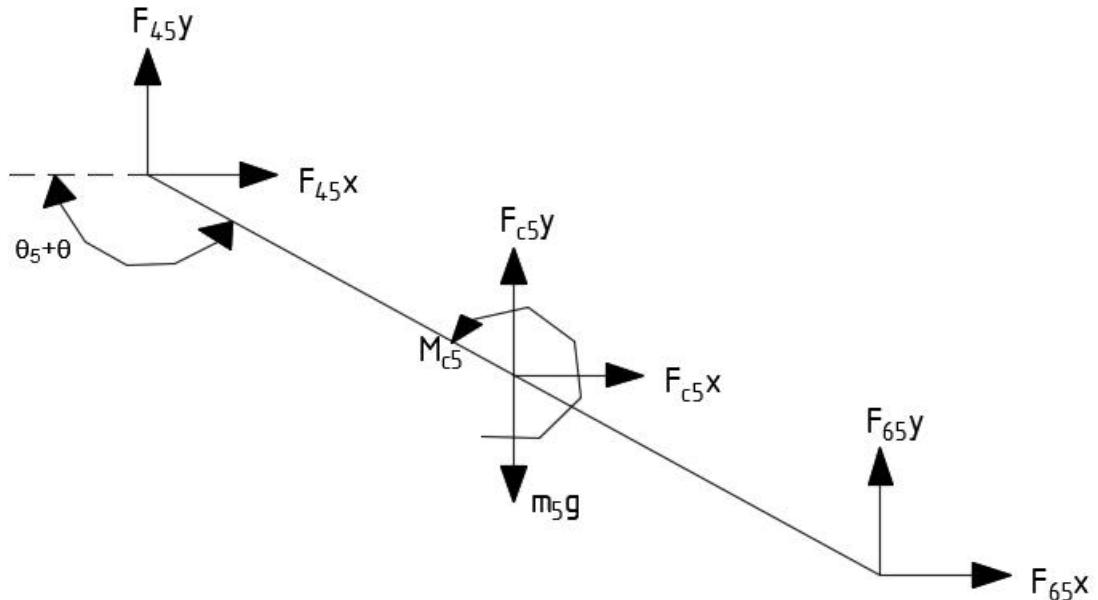
2, connecting rod's inertia force F_{c5x} and F_{c5y} ;

3, reaction force between link 5 and 6: F_{65x} and F_{65y} ;

4, centroid moment of inertia M_{c6} ;

5, gravity $m_5 g$;

Corresponding schematic diagram of link 5 is shown below:



List the corresponding balance equation:

$$F_{45x} + F_{c5x} + F_{65x} = 0 \quad (10)$$

$$F_{45y} + F_{c5y} + F_{65y} - m_5g = 0 \quad (11)$$

$$M_{c5} - (L/2) * F_{45x} * \sin(\theta+0.05) + (L/2) * F_{45y} * \cos(\theta+0.05) + (L/2) * F_{65x} * \sin(\theta+0.05) - (L/2) * F_{65y} * \cos(\theta+0.05) = 0 \quad (12)$$

Notice that:

$$F_{c5x} = m_5 * a_{c5x}; \quad F_{c5y} = m_5 * a_{c5y}; \quad F_{65x} = -F_{56x}; \quad F_{65y} = -F_{56y}$$

6.3.2 Force analysis of link 6:

Link 6 is subjected to seven forces and a centroid moment of inertia.

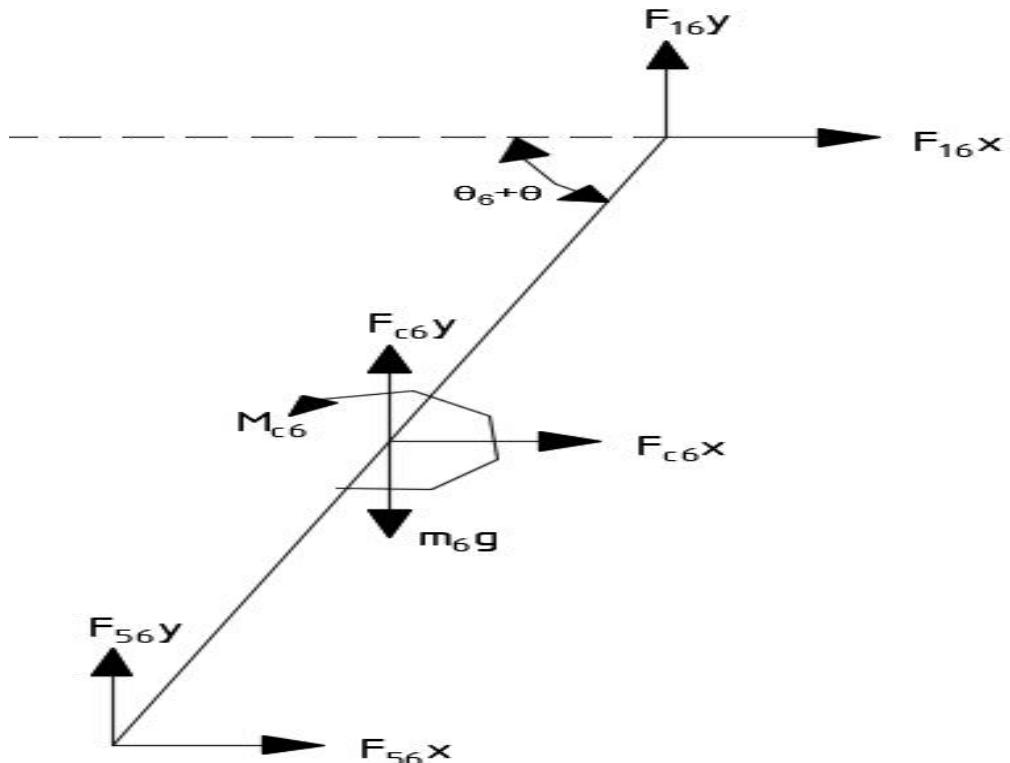
1, force between the frame and connecting rod 6: F_{16x} and F_{16y} ;

2, connecting rod's inertia force F_{c6x} and F_{c6y} ;

3, reaction force between link 5 and 6: F_{56x} and F_{56y} ;

4, gravity m_6g ;

Corresponding schematic diagram of link 6 is shown below:



List the corresponding balance equation:

$$F_{56x} + F_{c6x} + F_{16x} = 0 \quad (13)$$

$$F_{56y} + F_{c6y} + F_{16y} - m_6 g = 0 \quad (14)$$

$$M_{c6} - (L/2) * F_{16x} * \sin(\theta+6) + (L/2) * F_{16y} * \cos(\theta+6) + (L/2) * F_{56x} * \sin(\theta+6) - (L/2) * F_{56y} * \cos(\theta+6) = 0 \quad (15)$$

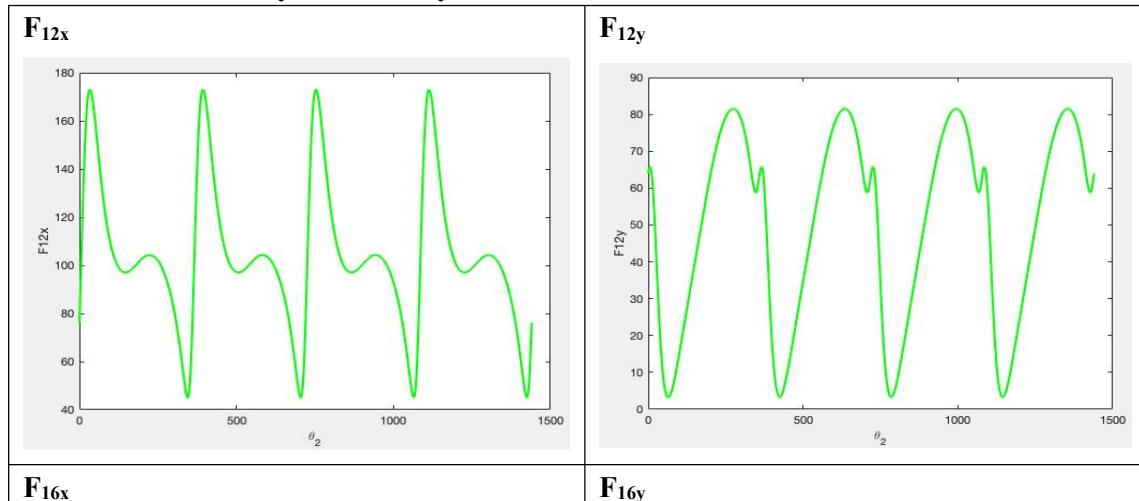
Notice that:

$$F_{c6x} = m_6 * a_{c6x}; \quad F_{c6y} = m_6 * a_{c6y};$$

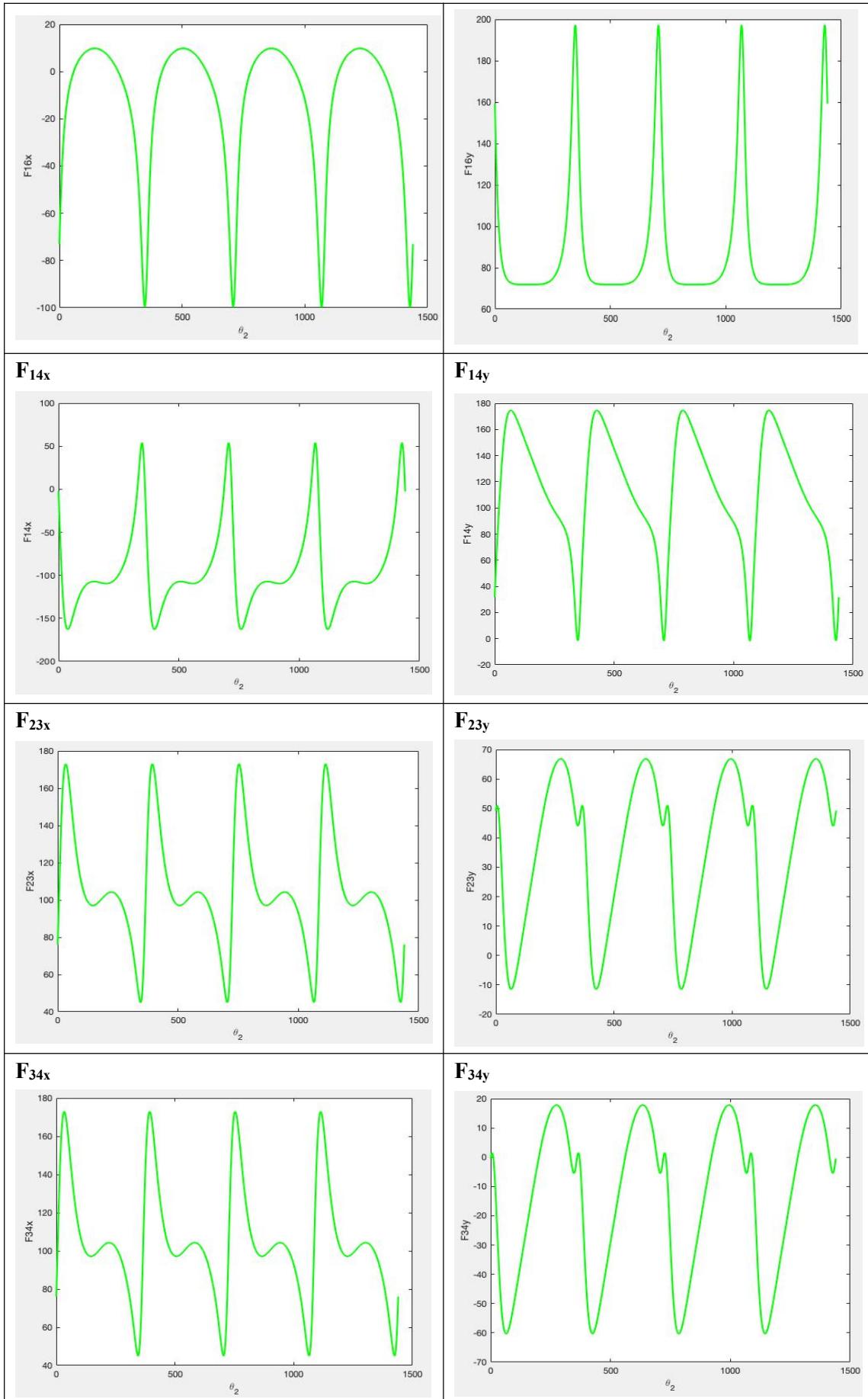
6.4 Force analysis results

There are fifteen unknowns and fifteen equations, so we can use Matlab and Adams to get the corresponding solutions:

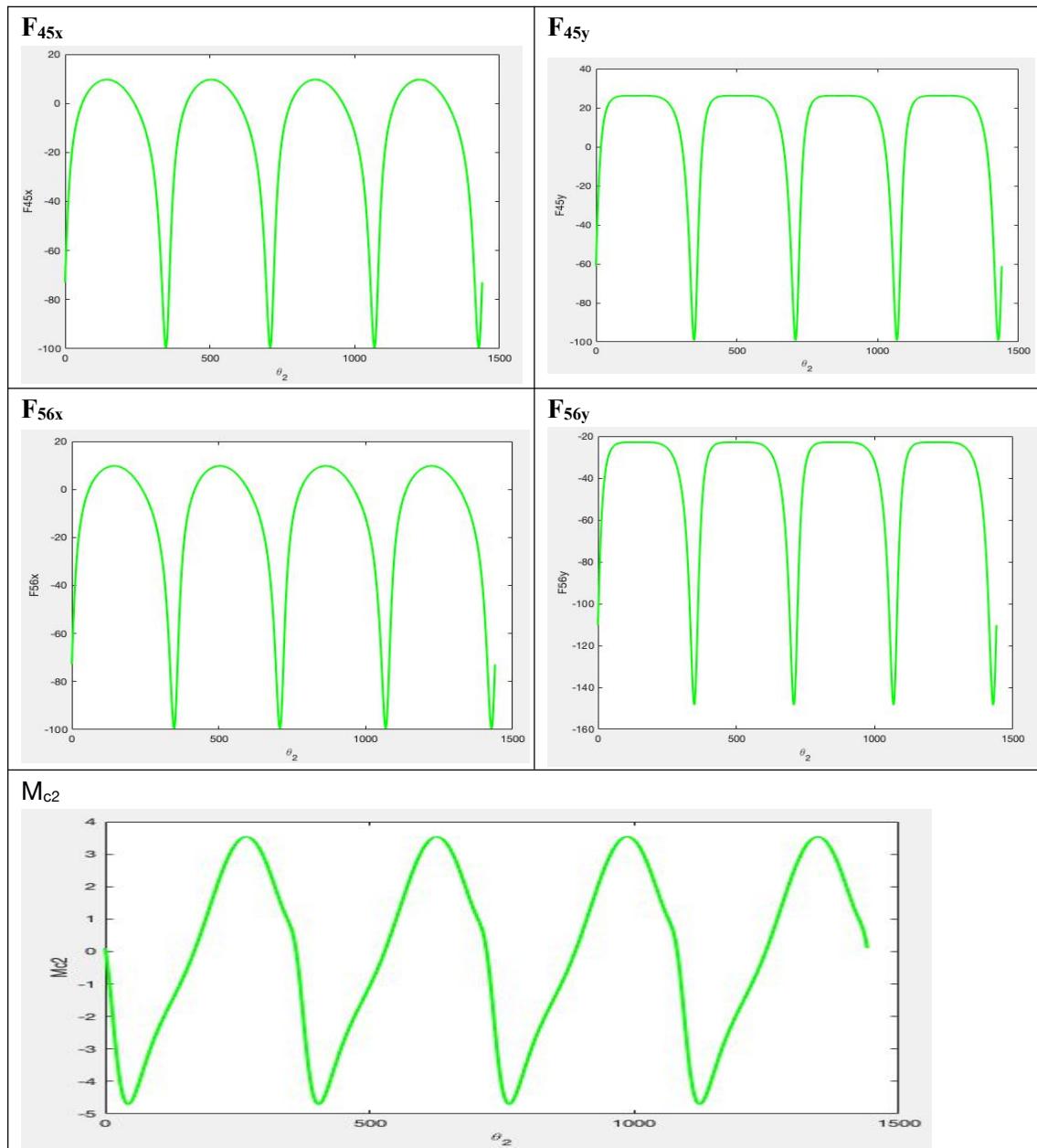
6.4.1 Force analysis results by Matlab



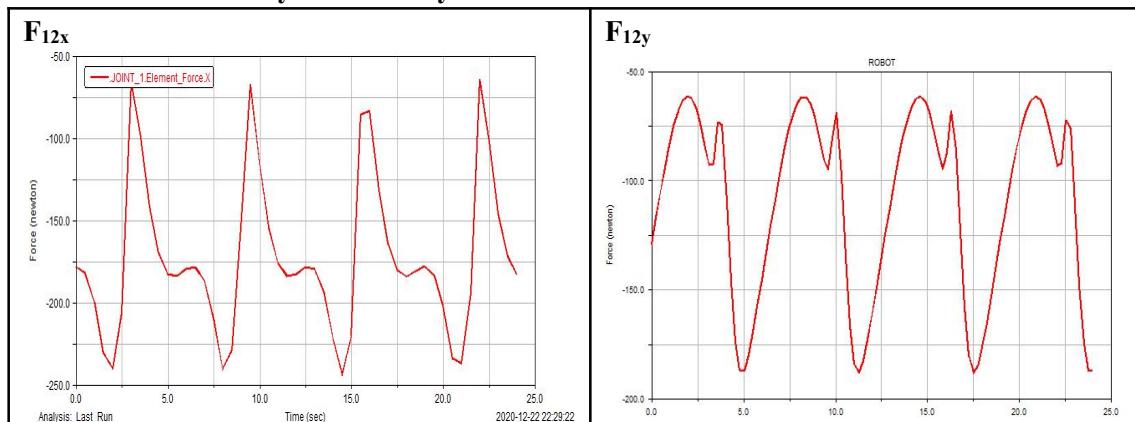
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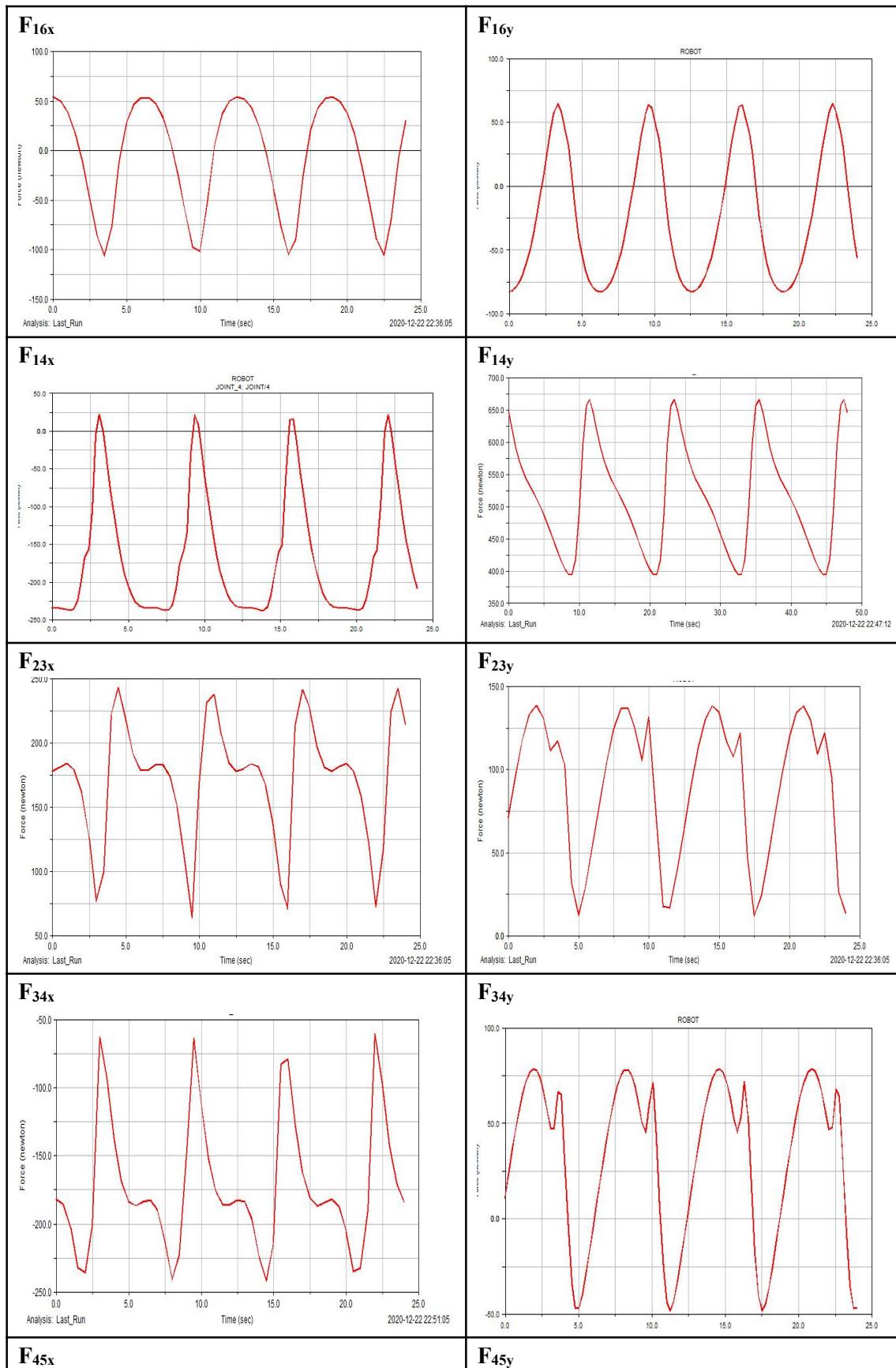
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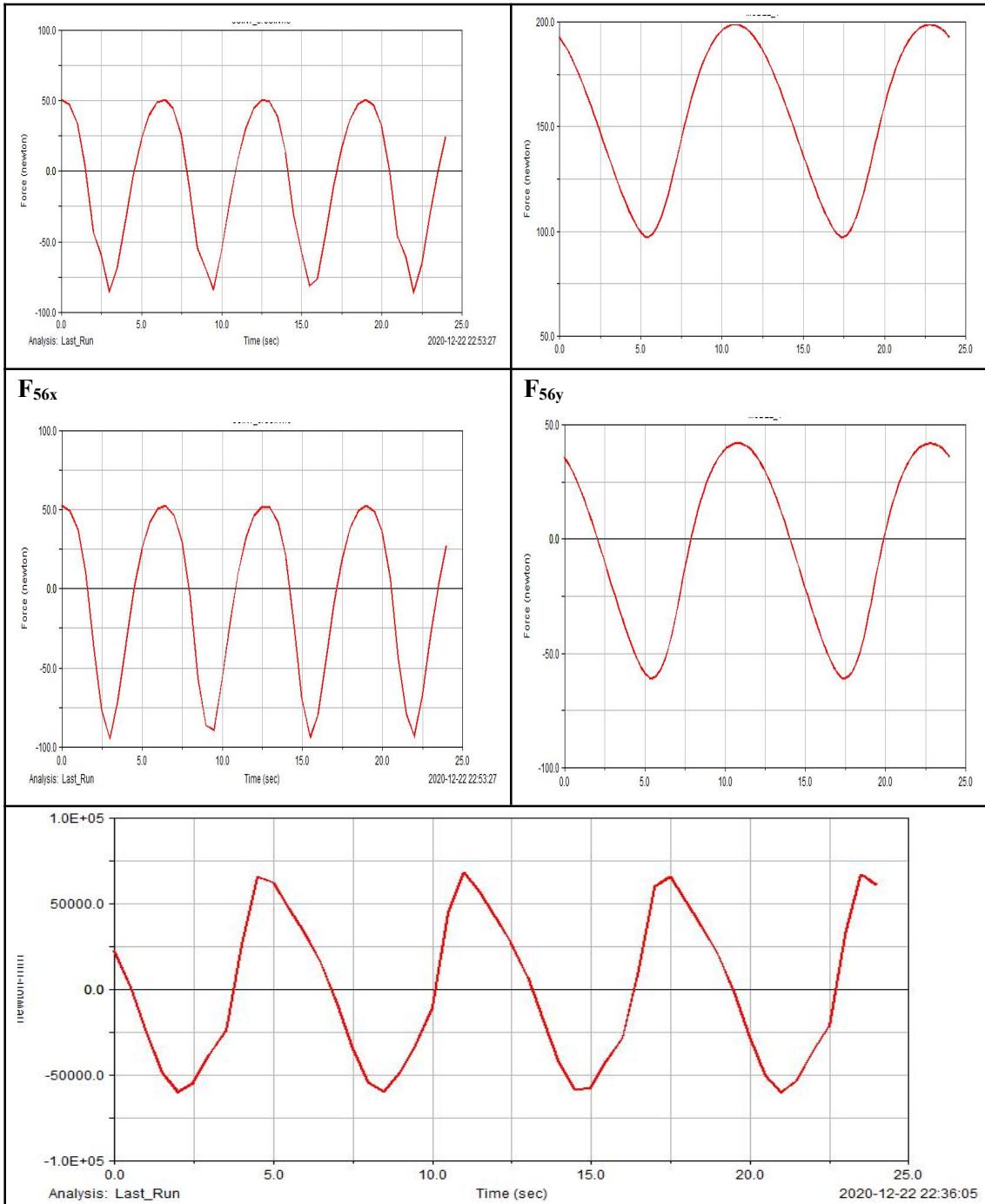
6.4.2 Force analysis results by Adams



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6.5 Dynamic Balance of the mechanism

When the mechanism is moving, it will generate inertia force and cause the component wear out, and affect motion accuracy, reliability and mechanical efficiency, finally, the mechanism can't work normally.

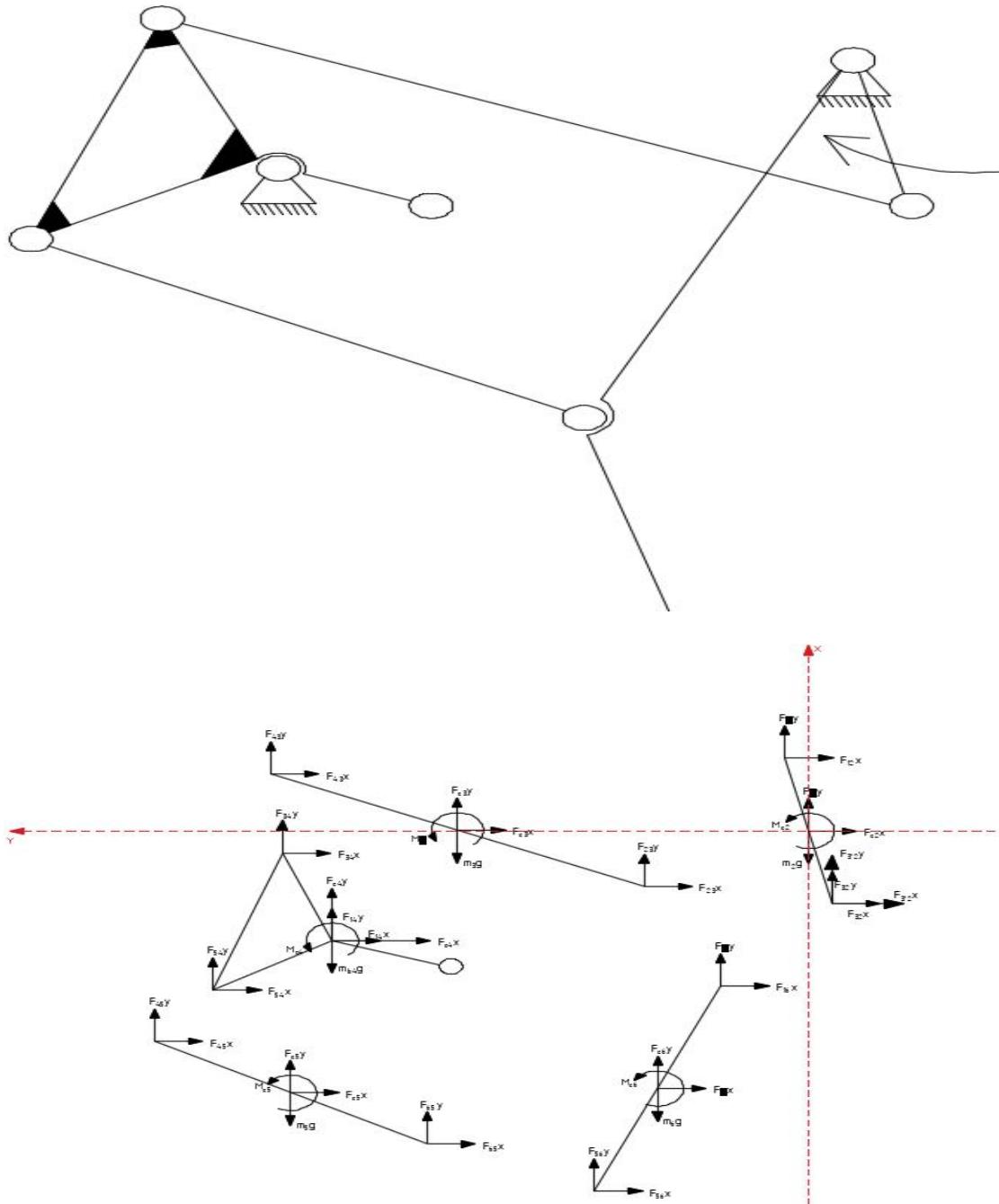
The aim of balancing is to eliminate or reduce the inertial force by using methods such as symmetrical arrangement of mechanisms to improve the moving efficiency.

6.5.1 Counterweight method

Compared with symmetric method, the counterweight method is less complex, after checking out which link's inertia force influences the overall inertial force most, we can add counterweight

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to its opposite direction to balance the inertia force. However, the new adding counterweight will increase the shaking force and shaking moment, which is harmful to the frame:



Total shaking force on the fixed support:

$$F_x = F_{12x} + F_{16x} + F_{14x}$$

$$F_y = F_{12y} + F_{16y} + F_{14y}$$

Total balancing moment on the fixed support:

$$M = Mc^2$$

Total inertial force on the fixed support:

$$F_{IX} = F_{c2X} + F_{c3X} + F_{c5X} + F_{c6X} + F_{c4X}$$

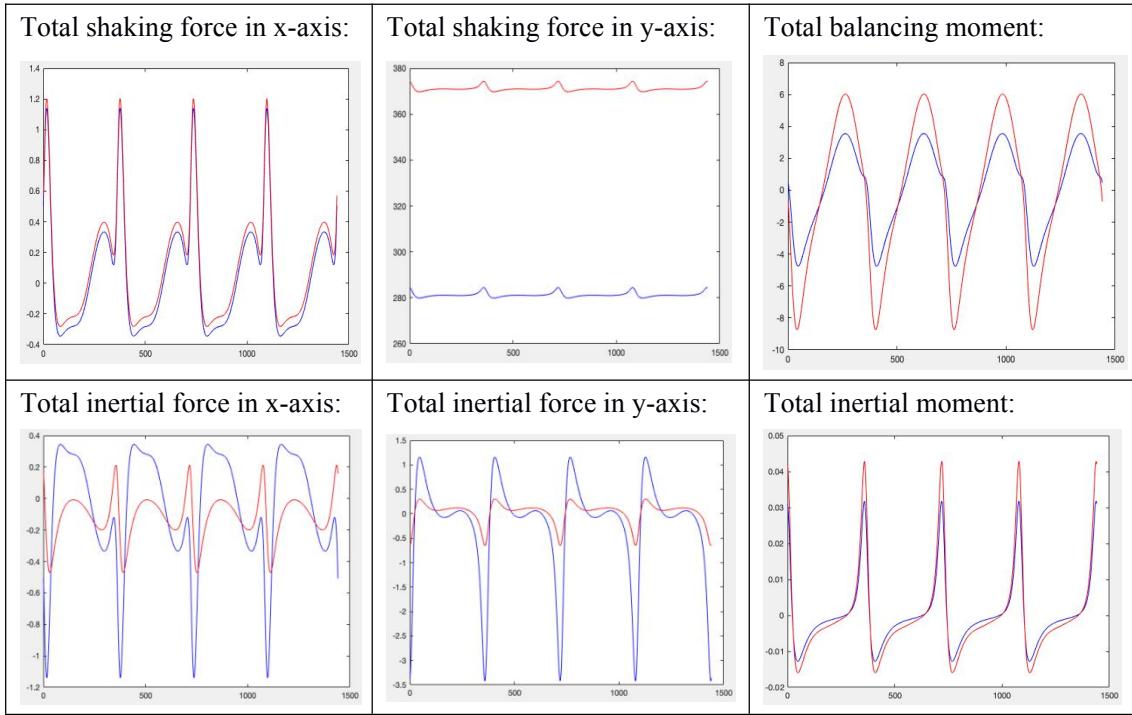
$$F_{iy} = F_{c2y} + F_{c3y} + F_{c5y} + F_{c6y} + F_{c4y}$$

Total inertial moment on the fixed support:

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$$M = M_{c3} + M_{c4} + M_{c5} + M_{c6}$$

Corresponding results are shown in the following pictures:



From the picture we can see that it eliminates major shaking force of the frame, however, by adding counter weight into the mechanism, the total balancing moment and total shaking force is increase.

Conclusion

It is a tough task to finish the curriculum design, because I need to not only use the knowledge about theory of machines and mechanisms, but also skilled grasp the simulation software like Adams, Matlab and Inventor. Hard though it is, I still overcome it by doing corresponding requirements step by step.

Recall all the steps of the course design carefully, I firstly analyze the initial structure of the Jansen linkage, departing it into three parts, based on that, drawing the kinematic diagrams, and recreating structures by replacing the assur-groups, finally getting over eighty reasonable results. What's more, I decide corresponding requirements seriously to make sure the execution mechanism achieve the design goal, and also design the driven mechanism carefully by searching some books to ensure the mechanism can be driven successfully and reasonably. After finishing the basic part, designing the dimensions of the selected execution structure by searching corresponding papers, books and using graphical method to solve some key dimensions. This is the main part and also the most important part of the curriculum design. Finally, writing down the corresponding equations set about displacement, velocity, acceleration and force and list the related equation set on Matlab, comparing the results on Matlab and Adams, ensuring the final designed structure meet the requirements. This is how I have done the jobs.

Although I have designed a new mechanism which has achieved a large step and simplified the initial Jansen linkage, there still exist some problems to be solved. For example, the output trace is an arc, which may make the structure unstable in some case where the opposite two legs of the four legs robot contact the ground simultaneously. I believe I will overcome the difficult problems after learning some knowledge about robots' walking analysis simulation in the future.

Personal feeling about the course design: To begin with, the task is really a tough thing and I have failed many times until I finally got the most reasonable result, There are so many endeavors needed to enable me to do a real designing project. What's more, the course design also makes me stronger when meeting some difficulties, what I have learned is more than corresponding engineering knowledge, it has improved my perseverance and courage to solve difficulties. Besides, I'd like to thanks for the course design for helping me learn and practice corresponding knowledge about mechanism, and it is also a significant opportunity for me to combine the theory with engineering design, I have used some software which is commonly used in engineering and search papers on some engineering reference books or journals. The course design also improved my ability in different aspects and helped me learn to consider in a comprehensive way. At last, I'd like to thanks the teacher and TA for guiding me when I meet some problems and make some mistakes, you give me an opportunity to improve my strength and make me stronger than before.

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