FE5112

Stochastic Calculus and Quantitative Methods

Project Report

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1 Introduction

The project aims to obtain the price of an Asian American put option by using the Least-Squared Monte Carlo simulation method. We also have explored various techniques to reduce the variance of the results produced and to increase the stablity of the results.

2 Problem Statement

The market is complete and arbitrage free. The risk free interest rate is 8%. The price of a non-dividend-paying stock S_t at time t is governed by the following SDE under the risk neutral probability measure $\mathbb{Q}$:

$$\begin{cases} dS_t = 0.08\, S_t\, dt + \sigma(S_t) \cdot S_t\, dW_t, t > 0 \\ S_0 = 100 \end{cases}$$

where W_t is a Brownian motion under $\mathbb{Q}$ and $\sigma(\cdot)$ is a function given as the following:

$$\sigma(x) = \begin{cases} 0.25 + 0.02 \times \left(1.0 - \dfrac{x}{S_0}\right), \text{if } x \leq 100 \\ \max\left(0.001, 0.25 - 0.01 \times \left(\dfrac{x}{S_0} - 1\right)\right), \text{if } x > 100 \end{cases}$$

We are to price a one-year-non-exercise American Asian put option with the strike price $K = 108$ and the maturity $T = 2$ in years. The option is not exercisable in the first year. The option is exercisable once and only once in the second year. If the option is exercised on a day n, where $366 \leq n \leq 730$, the option buyer will receive a payout of $(K - A_n)^+$, where A_n is given by:

$$A_n = \frac{1}{60} \sum_{i=n-60}^{n-1} S_{\frac{i}{365}}$$

We assume that there are 365 days in a year and all days are business days.

3 LSMC with no variance reduction

3.1 Settings

The variables are set up as the folloging:

```
T = 2                       # Duration in years
L = 730                     # Number of time intervals
M = 500                     # Number of asset paths
rf = 0.08                   # Annualized risk-free rate
S0 = 100.0                  # Beginning stock price
dt = T/L                    # Time discretization (dt = 1/365)
dayEx = 730                 # Maximum days before the option exercises
discf = math.exp(-rf * dt)  # Discount factor to backwardize by one day
    .
```

3.2 The Algorithm

Our algorithm follows the following step:

1. Generate M sample paths.
2. Generate a table of stock price data S_L for 730 time intervals with 500 asset price paths. $S(i, j)$ would refer to the stock price on the i^{th} path on the j^{th} day.
3. Calculate the option payoff on the final day, the L^{th} day or day 730.
4. Starting with the $(L-1)^{\text{th}}$ day, we do a backward propagation for each simulated path of the asset. For day i from $L-1$ back to 366:
   (a) Skip all the paths where it is not in the money on the i^{th} day.
   (b) If some path is in the money,
5. The prices on all the sample pathsa re discounted to day 0. We simply take the average of the paths to obtain the fair value of the option.

3.3 Analysis of results

3.4 Conclusion

4 Variance reduction methods applied to LSMC

4.1 Antithetic Method

4.2 Control Variate Method

4.3 Importance Sampling Method

5 Conclusion

6 Final Words

The contributions of our group members are as of the following:

- Kai Jit: Variance reduction with Antithetic method.
- Penghao: Base code for LSMC and analysis.
- Zequn: Variance reduction with importance sampling; Visualization of price paths; Refactor code base.
- Yihao: Variance reduction with control variate.
- Joanna: