FE5112

Stochastic Calculus and Quantitative Methods

Project Report

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1 Introduction

The project aims to obtain the price of an Asian American put option by using the Least-Squared Monte Carlo simulation method. We also have explored various techniques to reduce the variance of the results produced and to increase the stablity of the results.

2 Problem Statement

The market is complete and arbitrage free. The risk free interest rate is 8%. The price of a non-dividend-paying stock S_t at time t is governed by the following SDE under the risk neutral probability measure $\mathbb{Q}$:

$$\begin{cases} dS_t = 0.08\, S_t\, dt + \sigma(S_t) \cdot S_t\, dW_t, t > 0 \\ S_0 = 100 \end{cases}$$

where W_t is a Brownian motion under $\mathbb{Q}$ and $\sigma(\cdot)$ is a function given as the following:

$$\sigma(x) = \begin{cases} 0.25 + 0.02 \times \left(1.0 - \dfrac{x}{S_0}\right), \text{if } x \leq 100 \\ \max\left(0.001, 0.25 - 0.01 \times \left(\dfrac{x}{S_0} - 1\right)\right), \text{if } x > 100 \end{cases}$$

We are to price a one-year-non-exercise American Asian put option with the strike price $K = 108$ and the maturity $T = 2$ in years. The option is not exercisable in the first year. The option is exercisable once and only once in the second year. If the option is exercised on a day n, where $366 \leq n \leq 730$, the option buyer will receive a payout of $(K - A_n)^+$, where A_n is given by:

$$A_n = \frac{1}{60} \sum_{i=n-60}^{n-1} S_{\frac{i}{365}}$$

We assume that there are 365 days in a year and all days are business days.

3 LSMC with no variance reduction

3.1 Settings

The variables are set up as the folloging:

```
T = 2                        # Duration in years
L = 730                      # Number of time intervals
M = 500                      # Number of asset paths
rf = 0.08                    # Annualized risk-free rate
S0 = 100.0                   # Beginning stock price
dt = T/L                     # Time discretization (dt = 1/365)
dayEx = 730                  # Maximum days before the option exercises
discf = math.exp(-rf * dt)   # Discount factor to backwardize by one day.
```

3.2 The Algorithm

Our algorithm follows the following step:

1. Generate M sample paths.
2. Generate a table of stock price data S_L for 730 time intervals with 500 asset price paths. $S(i, j)$ would refer to the stock price on the i^{th} path on the j^{th} day.
3. Calculate the option payoff on the final day, the L^{th} day or day 730.
4. Starting with the final day, we do a backward propagation for each simulated path of the asset:
   (a) Skip all the paths where

3.3 Analysis of results

3.4 Conclusion

4 Variance reduction methods applied to LSMC

4.1 Antithetic Method

4.2 Control Variate Method

4.3 Importance Sampling Method

5 Conclusion

6 Final Words

The contributions of our group members are as of the following:

- Kai Jit: Variance reduction with Antithetic method.
- Penghao: Base code for LSMC and analysis.
- Zequn: Variance reduction with importance sampling; Visualization of price paths; Refactor code base.
- Yihao: Variance reduction with control variate.
- Joanna: