

Session Outline

- **01.** Introduction to BFS/DFS
- **02.** Problem Sets
- 03. Debrief & Q/A

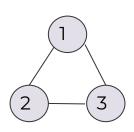


Graph

- A **graph** is a data structure with two distinct parts: a finite set of vertices, which are also called nodes, and a finite set of edges, which are references/links/pointers from one vertex to another.
- In directed graphs, the connections between nodes have a direction, and are called **arcs**; in undirected graphs, the connections have no direction and are called **edges**.
- Sometimes the nodes or arcs of a graph have weights or costs associated with them, and we are interested in finding the cheapest path.
- The characteristics of graph are strongly tied to what its vertices and edges look like. (Dense/Sparse)



Graph Representation



- As a list (or array) is called an **edge list**, and is a representation of all the edges (/E/) in the graph. [[1, 2],[2, 3],[3, 1]]
- Or, an *adjacency matrix* is a matrix representation of exactly *which* nodes in a graph contain edges between them. [[0, 1, 1],[1, 0, 1],[1, 1, 0]]
- The matrix is kind of like a lookup table: once we've determined the two nodes that we want to find an edge between, we look at the value at the intersection of those two nodes.
- The values in the adjacency matrix are like boolean flag indicators; they are either present or not present. If the value is 1, that means that there is an edge between the two nodes; if the value is 0, that means an edge does not exist between them.
- We will have a value of 0 down the diagonal, since most graphs that we're dealing with won't be referential.



Graph Representation

Adjacency Matrix					
	0	1	2		
0	0	1	1		
1	1	0	1		
2	1	1	0		

Adjacency List					
0:	1	2			
1:	0	2			
2:	0	1			

Edge List					
0:	0	1			
1:	0	2			
2:	1	2			

- Adjacency list hybrid between an edge list and an adjacency matrix.
- Each vertex is given an index in its list, and has all of its neighboring vertices stored as an linked list (which could also be an array), adjacent to it.
- We can see that, because of the *structure* of an adjacency list, it's very easy to determine all the neighbors of one particular vertex.
- The **degree** of a vertex is the number of edges that it has, which is also known as the number of neighboring nodes that it has.



Binary Tree

- One of the important types of non-linear data structures is a tree.
- Trees like linked lists are made up of nodes and links.
- Trees are undirected and connected acyclic graph. There are no cycles or loops.
- Complete binary tree A complete binary tree is a binary tree in which every level, except possibly the last, is completely filled, and all nodes in the last level are as far left as possible.
- Balanced binary tree A binary tree structure in which the left and right subtrees of every node differ in height by no more than 1.
- In-order traversal Left -> Root -> Right
- Pre-order traversal Root -> Left -> Right
- Post-order traversal Left -> Right -> Root



Binary Tree Terms

- Root: the topmost node of the tree, which never has any links or edges connecting to it
- **Neighbor** Parent or child of a node
- **Ancestor** A node reachable by traversing its parent chain
- Descendant A node in the node's subtree
- **Degree** of a tree Maximum degree of nodes in the tree
- Distance Number of edges along the shortest path between two nodes
- Level/Depth Number of edges along the unique path between a node and the root node
- Width Number of nodes in a level



Binary Tree

- If a tree has n nodes, it will always have one less number of edges (n-1).
- Trees are recursive data structures because a tree is usually composed of smaller trees often referred to as subtrees inside of it.
- A simple way to think about the depth of a node is by answering the question: how far away is the node from the root of the tree?
- The height of a node can be simplified by asking the question: how far is this node from its furthest-away leaf?
- A tree is considered to be balanced if any two sibling subtrees do not differ in height by more than one level. However, if two sibling subtrees differ significantly in height (and have more than one level of depth of difference), the tree is unbalanced.



Binary Search Tree

Properties of a BST:

- 1. Left subtree of a node N contains nodes whose values are lesser than or equal to node N's value.
- 2. Right subtree of a node N contains nodes whose values are greater than node N's value.
- 3. Both left and right subtrees are also BSTs.



Depth-first Search

Depth-first search is a graph traversal algorithm which explores as far as possible along each branch before backtracking. A stack is usually used to keep track of the nodes that are on the current search path. This can be done either by an implicit recursion stack, or an actual stack data structure. A simple template for doing depth-first searches on a matrix goes like this:

```
def dfs(matrix):
# Check for an empty matrix/graph.
if not matrix:
 return []
rows, cols = len(matrix), len(matrix[0])
visited = set()
directions = ((0, 1), (0, -1), (1, 0), (-1, 0))
def traverse(i, j):
 if (i, i) in visited:
  return
 visited.add((i, j))
 # Traverse neighbors.
 for direction in directions:
  next_i, next_j = i + direction[0], j + direction[1]
  if 0 <= next_i < rows and 0 <= next_i < cols:
   # Add in question-specific checks, where relevant.
   traverse(next_i, next_i)
for i in range(rows):
 for j in range(cols):
  traverse(i, i)
```



Breadth-first Search

Breadth-first search is a graph traversal algorithm which starts at a node and explores all nodes at the present depth, before moving on to the nodes at the next depth level. A queue is usually used to keep track of the nodes that were encountered but not yet explored.

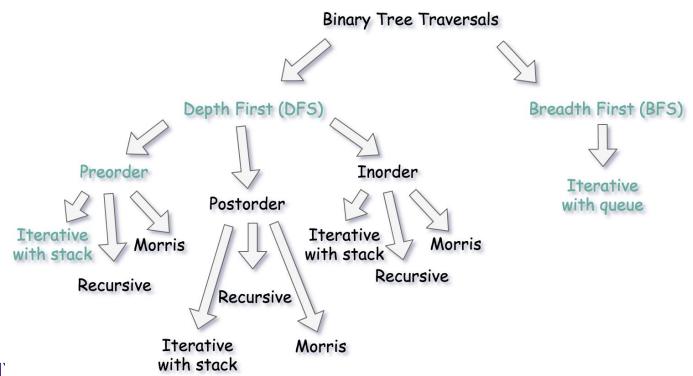
A similar template for doing breadth-first searches on the matrix goes like this. It is important to use double-ended queues and not arrays/Python lists as enqueuing for double-ended queues is O(1) but it's O(n) for arrays.

from collections import deque

```
def bfs(matrix):
# Check for an empty matrix/graph.
if not matrix:
 return []
rows, cols = len(matrix), len(matrix[0])
visited = set()
directions = ((0, 1), (0, -1), (1, 0), (-1, 0))
def traverse(i, i):
 queue = deque([(i, j)])
 while queue:
  curr_i, curr_i = queue.popleft()
  if (curr_i, curr_j) not in visited:
   visited.add((curr_i, curr_j))
   # Traverse neighbors.
   for direction in directions:
    next_i, next_j = curr_i + direction[0], curr_j + direction[1]
    if 0 <= next_i < rows and 0 <= next_i < cols:
     # Add in question-specific checks, where relevant.
     queue.append((next_i, next_j))
```



Overview

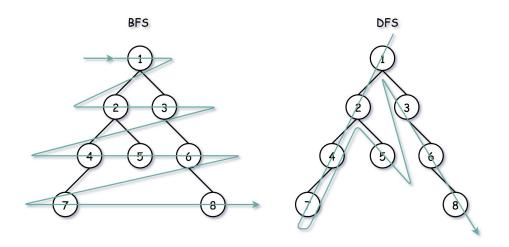




Overview

Both are starting from the root and going down, both are using additional structures, what's the difference?

Here is how it looks at the big scale: BFS traverses level by level, and DFS first goes to the leaves.





Overview

Now let's go down to the implementation. The idea is similar:

• Push root into queue (BFS) or stack (DFS).

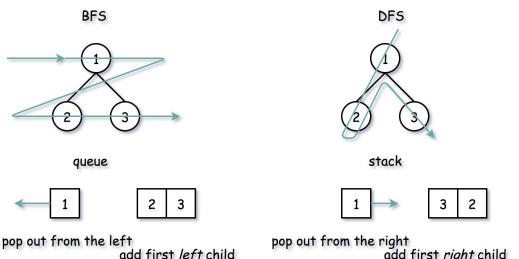
and then right child

(adding at the end)

At each step pop out one node, and push its children into stack/queue.

For **BFS**: pop out from the *left*, first push the *left* child, and then the *right* one.

For **DFS**: pop out from the *right*, first push the *right* child, and then the *left* one.





op out from the right add first *right* child and then *left* child (adding *at the end*)

Binary Trees & Graphs

Corner cases (Graphs):

- Empty graph
- Graph with one or two nodes
- Disjoint graphs
- Graph with cycles

Corner cases (Trees):

- Empty tree
- Single node
- Two nodes
- Very skewed tree (like a linked list)

Techniques:

- Use recursion
- Traversing by level
- Summation of nodes



PART 02

Problem Sets

Steps to approach the question:

Understand the problem

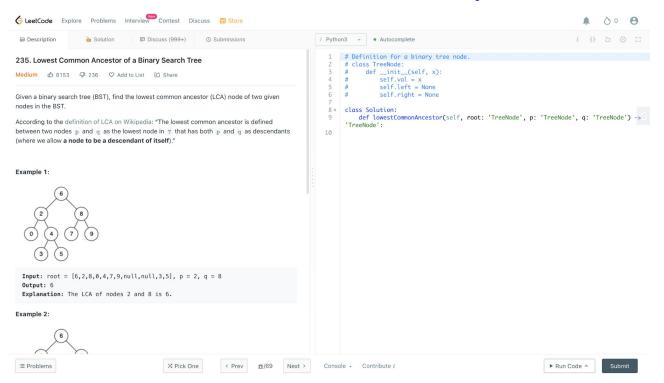
Code your solution

Manage your time

Take time to carefully read through the problem from start to finish is critical in finding the correct and complete solution to the problem in hand. Map out your solution before you write any code. Avoid too much time trying to find the perfect solution. Validate your solution early and often. Don't forget, you have multiple questions to complete within a said time. Make sure you allocate enough time to carefully consider all problems.



Problem 1: Lowest Common Ancestor of a Binary Search Tree





Approach: Recursion

```
def lowestCommonAncestor(self, root, p, q):
 # Value of current node or parent node.
 parent_val = root.val
 # Value of p
 p_val = p.val
 # Value of a
 q_val = q.val
 # If both p and q are greater than parent
 if p val > parent val and q val > parent val:
    return self.lowestCommonAncestor(root.right, p, g)
 # If both p and q are lesser than parent
 elif p_val < parent_val and q_val < parent_val:
    return self.lowestCommonAncestor(root,left, p. a)
 # We have found the split point, i.e. the LCA node.
  else:
    return root
```

Algorithm

- 1. Start traversing the tree from the root node.
- 2. If both the nodes p and q are in the right subtree, then continue the search with right subtree starting step 1.
- 3. If both the nodes p and q are in the left subtree, then continue the search with left subtree starting step 1.
- 4. If both step 2 and step 3 are not true, this means we have found the node which is common to node p's and q's subtrees. and hence we return this common node as the LCA.

Complexity Analysis

Time complexity : O(n), where N is the number of nodes in the BST. In the worst case we might be visiting all the nodes of the BST.

Space complexity: O(n), This is because the maximum amount of space utilized by the recursion stack would be N since the height of a skewed BST could be N.



Approach: Iterative

```
def lowestCommonAncestor(self, root, p, q):
    # Value of p
    p_val = p.val
    # Value of a
    q_val = q.val
    # Start from the root node of the tree
    node = root
    # Traverse the tree
    while node.
      # Value of current node or parent node.
      parent_val = node.val
      if p_val > parent_val and q_val > parent_val:
        # If both p and g are greater than parent
        node = node.riaht
      elif p_val < parent_val and q_val < parent_val:
        # If both p and g are lesser than parent
        node = node.left
      else:
        # We have found the split point, i.e. the LCA node.
        return node
```

Algorithm

The steps taken are also similar to approach 1. The only difference is instead of recursively calling the function, we traverse down the tree iteratively. This is possible without using a stack or recursion since we don't need to backtrace to find the LCA node. In essence of it the problem is iterative, it just wants us to find the split point. The point from where p and q won't be part of the same subtree or when one is the parent of the other.

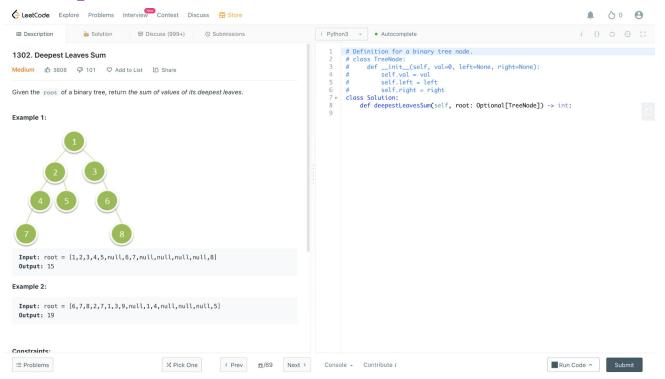
Complexity Analysis

Time complexity : O(n), where N is the number of nodes in the BST. In the worst case we might be visiting all the nodes of the BST.

Space complexity: O(7)



Problem 2: Deepest Leaves Sum





Approach: Iterative BFS Traversal

```
def deepestLeavesSum(self, root: TreeNode) -> int:
    next_level = deque([root,])

while next_level:
    # prepare for the next level
    curr_level = next_level
    next_level = deque()

for node in curr_level:
    # add child nodes of the current level
    # in the queue for the next level
    if node.left:
        next_level.append(node.left)
    if node.right:
        next_level.append(node.right)

return sum([node.val for node in curr_level])
```

Traverse level by level, to check if this level is the last one. If it's the case, return the sum of all nodes values.

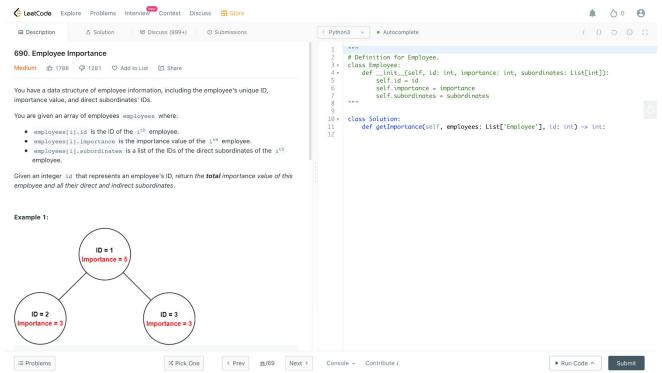
Complexity Analysis

Time complexity : O(n), since one has to visit each node.

Space complexity: up to O(N) to keep the queues. Let's use the last level to estimate the queue size. This level could contain up to N/2 tree nodes in the case of complete binary tree.



Problem 3: Employee Importance





Approach: DFS

def getImportance(self, employees, query_id):

Create a hashmap for employees. emap = {e.id: e for e in employees}

Define a recursive function
def dfs(eid):
 employee = emap[eid]
 return (employee.importance +
 sum(dfs(eid) for eid in employee.subordinates))

return dfs(query_id)

Let's use a hashmap emap = {employee.id -> employee} to query employees quickly.

Now to find the total importance of an employee, it will be the importance of that employee, plus the total importance of each of that employee's subordinates. This is a straightforward depth-first search.

Complexity Analysis

Time complexity : O(n), where N is the number of employees. We might query each employee in dfs.

Space complexity : O(N), the size of the implicit call stack when evaluating dfs.



PART 06

Q/A

Slack Hours

Join Slack Workspace!

Office Hours: Tuesday (10AM - 1PM)



Problem Assignments

- **01.** Validate Binary Search Tree (Medium)
- **02.** Pacific Atlantic Water Flow (Medium)
- **03.** Rotting Oranges (Medium)
- **04.** Minimum Knight Moves (Medium)
- **05.** Clone Graph (Medium)
- **0 6.** Number of Connected Components in an Undirected Graph (Medium)



