Homework

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Question1

Assumption

Assume $x=[\begin{matrix} d \\ \theta \end{matrix}]$, accordingly $x_t=[\begin{matrix} d_t \\ \theta_t \end{matrix}]$

State Space

Initial Value

$$Bel(\theta_0=0, d=0)=\frac{1}{180\times 200}$$

$$Bel(\theta_0=\frac{2\pi}{180}\times 1, d=\frac{0.5}{200})=\frac{1}{180\times 200}$$

$$\dots$$

$$Bel(\theta_0=\frac{2\pi}{180}\times 180, d=\frac{0.5}{200}\times 200)=\frac{1}{180\times 200}$$

Prediction step

$for-\theta$

$$\overline{bel(\theta_{t+0.2})}=bel(\theta_t)$$

,where $bel(\theta_{t+0.2})$ and $bel(\theta_t)$ is 360*1 matrix

$\overline{bel(\theta_{t+0.2})}$ remain the same after curve

$for-d$

$$\overline{bel(d_{t+0.2})} = bel(d_t)$$

Update Step

$$\overline{bel(X_{t+0.2})} = p(z_t \mid X_t) bel(X_t) = p(z_t \mid d_t, \theta_t) bel(X_t)$$

,where $p(z_t \mid d_t, \theta_t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(z_t^k - d_{t+0.2})}{\sigma^2}}$

$$\sigma = 0.2$$

,among which $\Delta\theta = \frac{1m/s \times 0.2s}{0.5m} = 0.4radian$

$$d_{t+0.2} = r\cos(\theta_t + \Delta\theta) - r\cos(\theta_t)$$

Beam function reference: *ProbabilisticRobotics* Page 125-126, (a)Gaussian distribution related

In practice, the values measured by the range sensor are limited to the interval $[0; z_{\max}]$, where $z_{\max}$ denotes the maximum sensor range. Thus, the measurement probability is given by

$$p_{\text{hit}}(z_t^k \mid x_t, m) = \begin{cases} \eta\, \mathcal{N}(z_t^k; z_t^{k*}, \sigma_{\text{hit}}^2) & \text{if } 0 \le z_t^k \le z_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{6.4}$$

where z_t^{k*} is calculated from x_t and m via ray tracing, and $\mathcal{N}(z_t^k; z_t^{k*}, \sigma_{\text{hit}}^2)$ denotes the univariate normal distribution with mean z_t^{k*} and variance σ_{hit}^2:

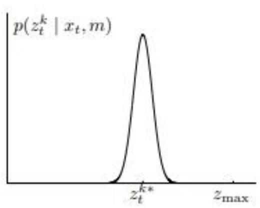
$$\mathcal{N}(z_t^k; z_t^{k*}, \sigma_{\text{hit}}^2) = \frac{1}{\sqrt{2\pi\sigma_{\text{hit}}^2}} e^{-\frac{1}{2}\frac{(z_t^k - z_t^{k*})^2}{\sigma_{\text{hit}}^2}} \tag{6.5}$$

The normalizer η evaluates to

$$\eta = \left(\int_0^{z_{\max}} \mathcal{N}(z_t^k; z_t^{k*}, \sigma_{\text{hit}}^2)\, dz_t^k \right)^{-1} \tag{6.6}$$

The variance σ_{hit} is an intrinsic noise parameter of the measurement model. Below we will discuss strategies for setting this parameter.

(a) Gaussian distribution p_{hit}



Result

• Plot the estimate at those 6 time-steps

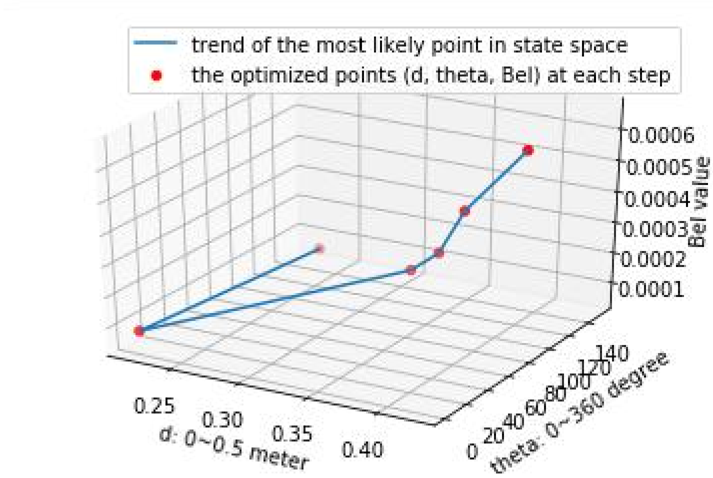


Fig1 3d plot of each optimized point at each step

Axis-x represents for the distance from the point to the wall. (unit: meter)

Axis-y represents for the angle.(unit: degree)

Axis-z represents for the distance from the point to the wall. (unit: meter)

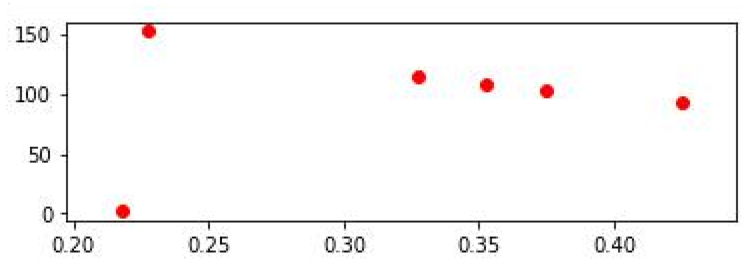


Fig2 2d plot of each optimized point at each step

Step1	distance = 0.2275	degree = 152.	Bel_value = 5.5408650043099036e-05
Step2	distance = 0.2175	degree = 2.	Bel_value = 8.339012147323719e-05
Step3	distance = 0.3275	degree = 114.	Bel_value = 0.0001458040047563614
Step4	distance = 0.3525	degree = 108.	Bel_value = 0.00023685848635039
Step5	distance = 0.375	degree = 102.	Bel_value = 0.00040340493973957907
Step6	distance = 0.425	degree = 92.	Bel_value = 0.0006512483741328149

• What is the most likely states at time =1

Time=1 means the 5th step above.(underlined)

The most likely states is:

d = 0.375m

Theta = 102 degree

Question2