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Homework 5 - Q1

First, for each city, determine the set of cities the inhabitant can reach within $X$ days. As we have the road map of Krypton, we can apply breadth first search to do this. Determine the cities can be obtained in one day first, and then in two days… Finally, up to $X$ days.

Since the inhabitant always chooses the pods of their own city first, we now consider the situations after their initial choices. For each city, if the number of population $p_i$ is greater than the number of pods $q_i$, we denote this city as type one vertex, the number of the population left is $p_i - q_i$. If the number of population $p_i$ is less than the number of pods $q_i$, we denote this city as type two vertex, the number of pods left is $q_i - p_i$. Otherwise, if $q_i$ is equal to $p_i$, neither people nor pods exists, thus, we can ignore this case.

Construct a flow network as a directed graph where the cities are the vertices of the graph. The type one cities will play the role of multiple sources and the type two cities will play the role of multiple sinks. If there is a path from a source to a sink and the path can be achieved in X days, then connect these two points following the direction. Thus, we add a super source S and a super sink T and connect S with all the type one cities and similarly all the type two cities with T. Each directed edge from type one city to type two city has infinite capacity, and we need to figure out all possible paths. The capacity of each directed edge from S to type one city is the population left, however, the capacity of each directed edge from type two city to T is the pods left.

Now, we run the Edmonds-Karp algorithm to find the maximal flow through such a network. The complexity of the Edmonds-Karp algorithm is $O(|V||E|^2)$, where |V| is the total number of vertices and |E| is the total number of edges. After the algorithms have converged, we can obtain that for each type two city $i$, the maximum number of the out-of-town population it can transport to earth after its local population has been transported, which is denoted as $F_i$. For each type two city $i$, compute its local population that has been transported which is $\min\{p_i, q_i\}$, and denote it as $L_i$.

Assume there are $N$ sinks, the largest number of invaders the earth will have to deal with is $\Sigma_{i=1}^{N}(F_i + L_i)$.