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Homework 5 – Q2

Define type one vertices ranges from one to  $n$  represents the id of columns, also define type two vertices ranges from one to  $n$  represents the id of rows.

Make a bipartite graph with all the type one vertices on the left-hand side and all the type two vertices on the right-hand side. Thus, we add a super source  $S$  and a super sink  $T$  and connect  $S$  with all the type one vertices and similarly all the type two vertices with  $T$ . Since there could only be one black rook in each column, the capacity of each directed edge from  $S$  to type one vertex is one. Similarly, there could only be one black rook in each row, the capacity of each directed edge from type two vertex to  $T$  is also one. Each square  $S_{ij}$  can now be represented by an edge from type one vertex  $j$  to type two vertex  $i$ . Notice that the edge does not exist if the corresponding coordinate is under the attack range of either of the  $k$  bishops.

Now, we run the Edmonds-Karp algorithm to find the maximal flow through such a network. The complexity of the Edmonds-Karp algorithm is  $O(|V||E|^2)$ , where  $|V|$  is the total number of vertices and  $|E|$  is the total number of edges. After the algorithm has converged, we can compute the total number of nodes result in sink  $T$ . This number is the largest number of black rooks we can place on the board so that no two rooks are in the same row or in the same column and are not under the attack of any of the  $k$  bishops.