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Homework 2 – Q2

Use the substitution $y = x^{100}$ to reduce $P(x)$ to $P(y) = A_0 + A_1 y + A_2 y^2$. The product $Q(y) = P(y) \cdot P(y)$ of these two polynomials is of degree 4 so we need 5 values to uniquely determine $Q(y)$. We choose the smallest possible 5 integer values: -2, -1, 0, 1, 2. Thus, we compute $P(y)$ at these 5 points.

$$P(-2) = A_0 - 2A_1 + 4A_2$$
$$P(-1) = A_0 - A_1 + A_2$$
$$P(0) = A_0$$
$$P(1) = A_0 + A_1 + A_2$$
$$P(2) = A_0 + 2A_1 + 4A_2$$

Therefore, we can obtain $Q(y)$ at these 5 points which requires 5 large integer multiplications.

$$Q(-2) = P(-2) \cdot P(-2) = (A_0 - 2A_1 + 4A_2)^2$$
$$Q(-1) = P(-1) \cdot P(-1) = (A_0 - A_1 + A_2)^2$$
$$Q(0) = P(0) \cdot P(0) = A_0 \cdot A_0$$
$$Q(1) = P(1) \cdot P(1) = (A_0 + A_1 + A_2)^2$$
$$Q(2) = P(2) \cdot P(2) = (A_0 + 2A_1 + 4A_2)^2$$

If we represent the product $Q(y) = P(y) \cdot P(y)$ in coefficient form as $Q(y) = Q_4 y^4 + Q_3 y^3 + Q_2 y^2 + Q_1 y + Q_0$ and then

$$Q_4(-2)^4 + Q_3(-2)^3 + Q_2(-2)^2 + Q_1(-2) + Q_0 = Q(-2)$$
$$Q_4(-1)^4 + Q_3(-1)^3 + Q_2(-1)^2 + Q_1(-1) + Q_0 = Q(-1)$$
$$Q_4 0^4 + Q_3 0^3 + Q_2 0^2 + Q_1 0 + Q_0 = Q(0)$$
$$Q_4 1^4 + Q_3 1^3 + Q_2 1^2 + Q_1 1 + Q_0 = Q(1)$$
$$Q_4 2^4 + Q_3 2^3 + Q_2 2^2 + Q_1 2 + Q_0 = Q(2)$$

Simplify the left-hand side we get

$$16Q_4 - 8Q_3 + 4Q_2 - 2Q_1 + Q_0 = Q(-2)$$

$$Q_4 - Q_3 + Q_2 - Q_1 + Q_0 = Q(-1)$$

$$Q_0 = Q(0)$$

$$Q_4 + Q_3 + Q_2 + Q_1 + Q_0 = Q(1)$$

$$16Q_4 + 8Q_3 + 4Q_2 + 2Q_1 + Q_0 = Q(2)$$

Solve this system of linear equations for Q_0, Q_1, Q_2, Q_3, Q_4 we obtain

$$Q_0 = Q(0)$$

$$Q_1 = \frac{Q(-2)}{12} - \frac{2Q(-1)}{3} + \frac{2Q(1)}{3} - \frac{Q(2)}{12}$$

$$Q_2 = -\frac{Q(-2)}{24} + \frac{2Q(-1)}{3} - \frac{5Q(0)}{4} + \frac{2Q(1)}{3} - \frac{Q(2)}{24}$$

$$Q_3 = -\frac{Q(-2)}{12} + \frac{Q(-1)}{6} - \frac{Q(1)}{6} + \frac{Q(2)}{12}$$

$$Q_4 = \frac{Q(-2)}{24} - \frac{Q(-1)}{6} + \frac{Q(0)}{4} - \frac{Q(1)}{6} + \frac{Q(2)}{24}$$

With these 5 coefficients provided, we can now form the polynomial $Q(y) = Q_4y^4 + Q_3y^3 + Q_2y^2 + Q_1y + Q_0$. Then substitute back y with x^{100} to obtain the square of $P(x)$.