Team Control Number

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Problem Chosen

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2012 Mathematical Contest in Modeling (MCM) Summary Sheet

Random Process: A Simulation Model of River Rafting

In this paper, in order to select the best schedule of Big Long River rafting during the tourism season, we need to make a linear river rafting model to take different factors into consideration. One is the maximum carrying capacity of the river. Another is the average encounters per day. These are all the judgment criteria of different schedules. And the best schedule should include the type of transportation and travel plans. In this problem, we treat the whole process as a stochastic procedure.

Firstly, in order to obtain the number of boats dispatching for a common day, we employ Multi-Channel Single-Phase queuing theory when modeling the system, in which we treat rafting time as waiting time, camping time as service time. As a result, we obtain the average launching times per day.

Next, with the parameters we get, we consider more factors as random variables, which satisfy specific distribution, such as the velocity of a boat, the duration of every-day rafting, and the starting time of a boat in a day. Then we record the time and spatial situation of a boat in a matrix, and make a cycling program to simulate the day-by-day rafting process.

To optimize the schedule, we consider 5 schemes with different ratio of oars to motors for simulation in hopes of maximizing the average encounters and minimizing the rate of utilization of campsites. Then we compare the results of these schemes and decide the best schedule with half motors and half oars and the average number of encounter in one day of 0.8722 per ship.

We define the carrying capacity of the river as the maximum number of trips of one day in 180 days, and the rate of utilization as the ratio of the carrying capacity to the campsites number Y. After the simulation of day-by-day rafting process and the comparison of those schemes, we conclude that the value of carrying capacity is 63 trips a day, and 867 more boat trips to be added to the Big Long River's rafting season.

On the one hand, our model has a lot of strengths: We treat each trip as stochastic procedure, which enable a lot of flexibility to the model; we have definite judgment criteria helping us to screen the best one; we use mature models like queuing theory for simulation. On the other hand, limitations are inevitable. Our queuing system is not closely tally with real case. Our schemes are too limited to find the most desirable one. Also, this model needs more validation from the factual testing.

Random Process: A Simulation Model of River Rafting

February 14, 2012

Abstract

The most important characteristic in rafting is the random nature. In our linear river rafting model, we treat the whole process as a stochastic procedure. In questing for average common day dispatching, we employ Multi-Channel Single-Phase queuing theory. Then with the programming simulation, we obtained mean number of trips launching per day is 6. To make an optimal management, we present 5 schedules with different ratio of travel modes and utilize MATLAB to make a program to simulate the real procedure of rafting. According to the results, we compare the numbers of encounters between trips and the rate of utilization of campsites. Finally, we find the optimal arrangement of half motors and half oars, with the average number of encounter in one day of 0.8722 per shipmaximizing the average encounters and minimizing the rate of utilization of campsites. We define the carrying capacity of the river as the maximum number of trips of one day in 180 days,and determine the value as 63 trips a day, and 867 more boat trips to be added to the Big Long River's rafting season.In the end,the memo is included in the APPENDIX B.

KEYWORDS:Queuing theory, stochastic process, simulation, MATLAB programming.

Contents

1 BACKGROUND

Today, understanding and managing the interactions between human and environment can be greatly facilitated by mathematical models that systematically and accurately specify the interrelationships of human and environmental variables. However, there are many obstacles to the development of these models, such as the large number of the potential relevant environmental variables, the randomness of these variables. Thus, modeling human-environment interactions is a matter of discovering the relationships between the two factors with many interrelated variables[1].

Computers simulation in tourism dated back to 1970s. Stankey(1972) raised opinion that the degree of visitors' satisfactory was in inverse proportion to the number of encounters during trips. Fisher and Krutilla(1972) further stressed that the optimum level of use in a wilderness area to be the point at which the incremental benefit of an additional party is just offset by the decrease in the benefits of the parties encountered[2]. Under these theoretical principles, people did researches on tourism simulation models to investigate recreation behavior. The earliest tourism simulation was the Wilderness Use Simulation Model (WUSM). Then it bifurcated into two representative models—- Extend and RBsim. 1990s witnessed the birth of Agent-Based Modeling (ABM)[3]. Currently, the platform of tourism simulation became more mature and powerful.

The WUSM has great applications varying from linear river systems to tri-dimensional wilderness place. Wang and Manning(1999) used an object-oriented dynamic simulation package to model carriage-road use in Acadia National Park. Gimblett et al.(2000) adapted geo-referenced temporal data in tourism simulation of a frequently visited natural spot in Sedona, Arizona. Daniel and Gimblett(2000) simulated river rafting on the Colorado River in the Grand Canyon with autonomous agent-based model. All the improvements of new models made us aware the development of wilderness simulation has renewed itself tremendously in less than three decades[2].

2 INTRODUCTION

As the popularity of the Big Long River rafting, management become more and more complex. To help the park managers make optimal decisions, the paper take more factors into consideration and use different kinds of logistic methods, by turning to the computer simulation models for analysis and testing the alternative management plans. In this paper, we model one linear river rafting. Our treatment of the problem will employ queuing theory in questing for a common day dispatching. And we will also adapt simulation methods to investigate several feasible schemes. As a result of these simulations, we will yield the average encounters and carrying capacity of river, which are also the judgment criteria of the best scheme.

3 QUEUEING THEORY

3.1 Assumptions

Before adapting queuing theory, we would like to links with our current problem and make some assumptions about them.

In queuing theory, the X/Y/Z/A/B/C notation designates a queuing system:

X — Arrival time distribution
Y — Service time distribution
Z — Number of servers
A — System capacity
B — Size of calling population
C — Queue discipline

For example, "G/D/1" would indicate a General (may be anything) arrival process, a Deterministic (constant time) service process and a single server.

In our problem, we model a common day of Big Long River rafting. The X/Y/Z/A/B/C notation designates a rafting system:

X — Trips launching time distribution
Y — Camping time distribution
Z — Number of camps
A — System capacity (We assume infinite)
B — The maximum number of trips (We assume infinite)
C — Discipline (First come first serve)

Other assumptions are:

1. The river rafting time we assume is 6:00 to 18:00 local time, after 18:00 all groups must camp on the corridor. In other words, we only model 12 hours per day.

2. In the simple model, we use all oar- powered rubber rafts for the maximum 18-day trip, with the velocity of 4 miles per hour.

3. Launching times takes on average rate of two per hour, but can vary from group to group (assume the arrival rate is described by a Poisson distribution). Camping times (in the daytime) takes on 2.5 per hour, but can vary (assume negative exponential distribution). The total number of Camp sites is 75 (we will discuss this later). That is we can define the parameters of our simple model system(λ=2, μ=2.5, s=75).

4. We adapt the Multi-Channel, Single Phase queuing model in this simple model, because we assume that if a group wants to camp in one site which has been occupied by other group before, it can certainly find other camp sites if there is any empty sites. In this way, this model act like Multi-Channel, Single Phase queuing model (Fig 1).

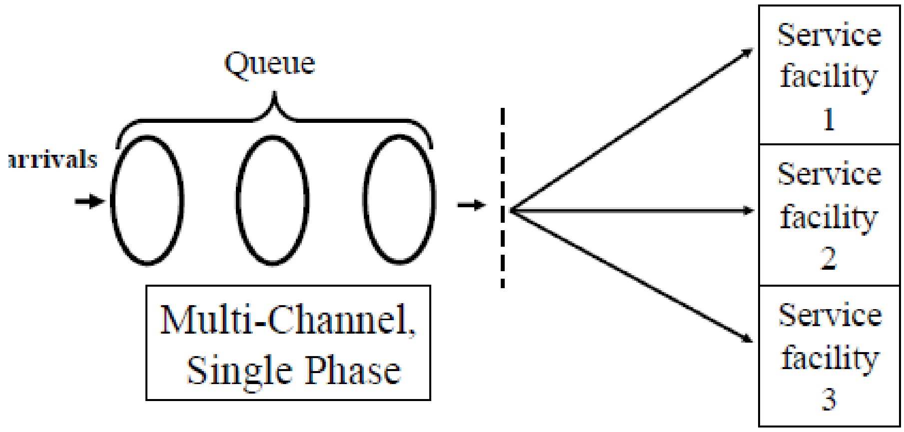

Fig 1: Multi-Channel,Single Phase Queuing System Configuration

3.2 Results & Analysis

We use the flowchart to simulate Multi-Channel, Single-Phase queuing model realization.(see Fig 2-4)

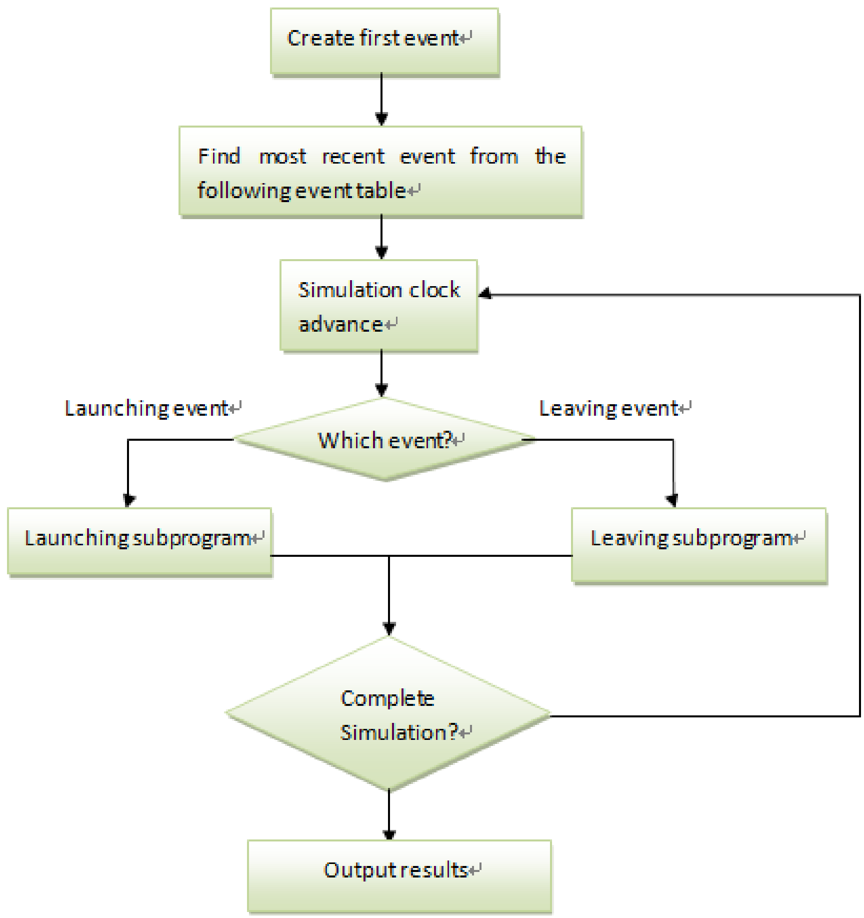

Fig 2:Simulation of main program

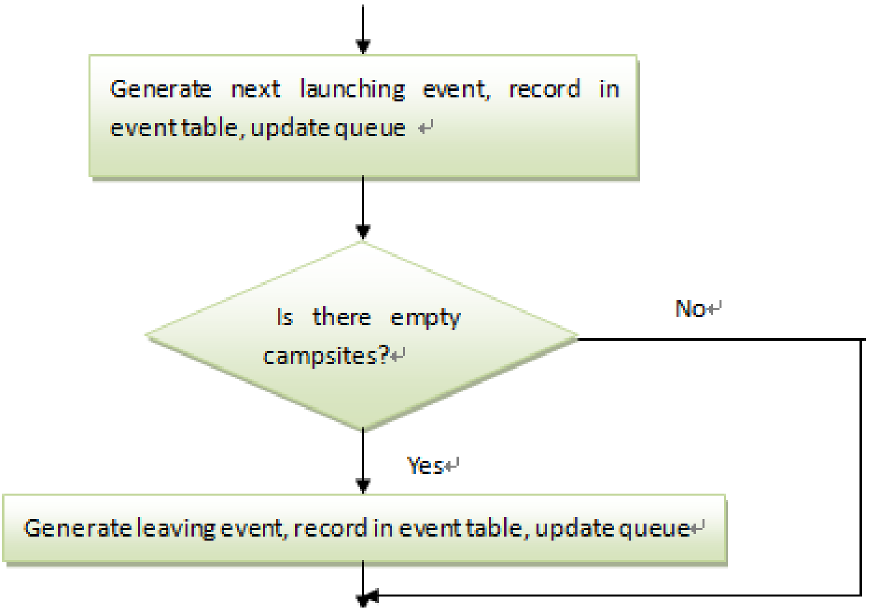

Fig 3:Launching subprogram

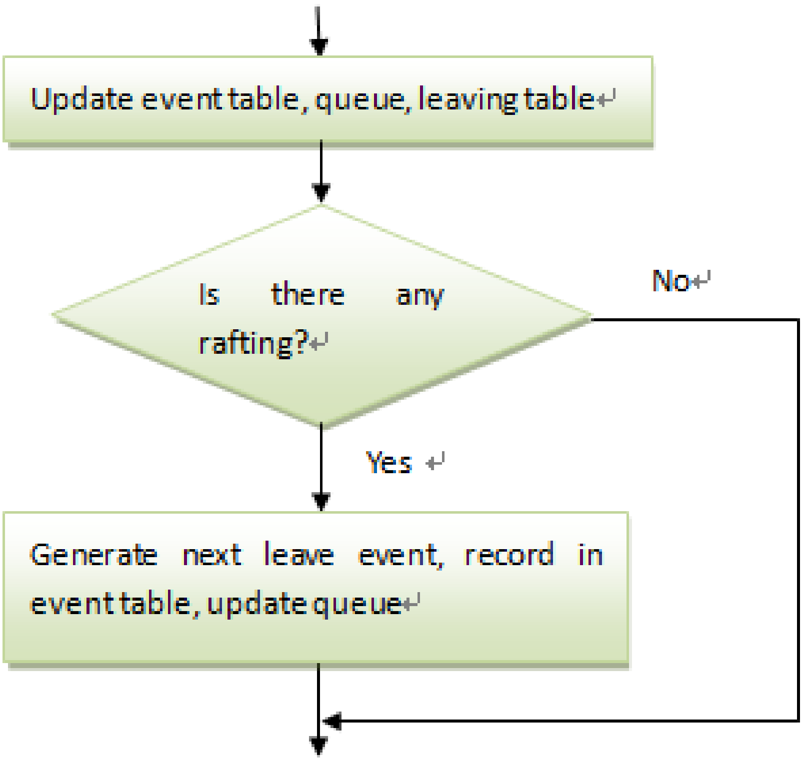

Fig 4:Leaving subprogram

We utilize MATLAB to simulate queuing process. (The MATLAB program is in the Appendix)

We can get the useful results as follows:

1. Mean number of trips launching per day is 6.
2. At the terminate time, all groups on the river can find the campsite.
3. Mean time spent on the camp sites per group is 2.4 hours.

4 SCHEMES & SIMULATION

4.1 Assumptions

As a result of the queuing theory modeling, we assume that the optimal arrangement of the number of every-day launches is 6 on average. And because of the limit of length of rafting (6-18 nights), we arrange no launches in the last 6 days of this 6-month tourist season. Besides, we consider that the average rafting duration is 12 days.

Then we would like to specify some symbols of variables:

1. We determine that the velocity of a rubber raft is a random variable, which satisfies the normal distribution. The motorized boats have an average velocity of 8 mph and a standard deviation of 0.8. Meanwhile, the oar-powered have an average velocity of 4 mph and a standard deviation of 0.4. The symbol v represents the velocity.
2. The selection of the traveling mode (oar or motor) is a random decision.
3. The duration of every-day rafting on river is a random variable, which satisfy the normal distribution with a mean number of 4 and a standard deviation of 0.4. It is presented by t.

   The average rafting time on one day is 4 hours, which is calculated as follows: 225m/12d = 18.75m/d. 18.75 m/5mph $\approx$ 4 h/d.
4. We record the time and spatial values of one boat in [x, y, z].

   Symbol x represents the date. For example, if x=2, it means the second day of total 180 days.

   Symbol y represents the starting time of one boat on the day x after the first launch. For the convenience of simulation, this time we treat the starting time as a random variable satisfying the normal distribution, which has a mean number of 12 and a standard deviation of 0.4.

   Symbol z represents the coordinate of the starting point of one boat on the day x after the first launch, which is measured by the distance from the origin of the river to the starting location of this boat.

5. Symbol A represents a matrix whose columns mean the ordinal date of 180 days and rows mean the 12 periods of one day from 6 a.m. to 18 p.m. for rafting opening hours. Each value in matrix A is selected from integer numbers 0, 1, 2. 0 means there is no launch in that period, and 1 stands for a launch of oar-powered boat, and 2 stands for a launch of motorized boat.

6. Symbol B represents a matrix whose rows and columns have the same meaning with A's. Each value B(i ,j) means the coordinate of the ending point of one boat in that period i of day j , which is the same as the starting point next day.

7. We identify an encounter if B(i ,j)<B(k ,j), and i<k ,and B(i ,j) is non-zero. Since the latter boat has overtaken the former one.

8. CCR , short for the carrying capacity of the river is the maximum number of trips of one day in 180 days. We identify the rate of utilization as the ratio of the carrying capacity to the campsites number Y.

9. We assume that if two boats have the same ending coordinate, one of them will wander for a while searching for another campsite.Then no two sets of campers can occupy the same site at the same time

Then we utilize Matlab to make a program to simulate the procedure of the Big Long River's rafting on the basis of these assumptions noted above, and to find a optimal scheme.(The MATLAB program is in the Appendix)

4.2 Schemes

We research some articles for reference. According to the article [4],we combine the realistic situation and give the 5 different schedules as follows.

1) Traveling mode: oar-powered boats only.

Schedule: 35 launches per 6 days, average 5 to 6 launches a day, repeating 29 times, and no launches in the last 6 days of the total 180 days.

This simulation is chosen because there may be a case that phase out all the motors and make all trips oars for some reasons.

2) Traveling mode: 30% motors and 70% oars.

Schedule: 34 launches per 6 days, average 5 to 6 launches a day, repeating 29 times, and no launches in the last 6 days of the total 180 days.

This arrangement is selected to show the impact of a relatively large number of oar-powered boats.

3) Traveling mode: half oars and half motors.

Schedule: 33 launches per 6 days, average 5 to 6 launches a day, repeating 29 times, and no launches in the last 6 days of the total 180 days.

This arrangement is selected to show the impact of equal motors and oars.

4) Traveling mode: 70% motors and 30% oars.

Schedule is the same as that noted above in 3).

This arrangement is selected to show the impact of a relatively large number of motorized boats.

5) Traveling mode: motorized boats only.

Schedule: 34 launches per 6 days, average 5 to 6 launches a day, repeating 29 times, and no launches in the last 6 days of the total 180 days.

This simulation is chosen because there may be a case that phase out all the oars and make all trips motors for some reasons, such as the weather.

	1	2	3	4	5	6
Rubber Rafts	6:00 8:00 10:00 13:00 15:00	6:00 7:00 10:00 12:00 13:00 15:00	7:00 8:00 10:00 13:00 14:00 15:00	6:00 8:00 9:00 11:00 13:00 14:00	7:00 9:00 10:00 12:00 14:00 15:00	6:00 9:00 11:00 13:00 15:00 16:00

Table 1:Plan A

	1	2	3	4	5	6
Rubber Rafts	9:00 11:00 15:00	6:00 10:00 13:00 14:00	6:00 15:00 16:00	7:00 11:00 14:00 15:00	6:00 8:00 13:00 16:00	7:00 9:00 11:00
Motorized Boats	6:00 13:00	8:00 12:00	8:00 13:00	9:00 12:00	7:00 11:00	6:00 14:00

Table 2:Plan B

	1	2	3	4	5	6
Rubber Rafts	8:00 11:00 13:00	8:00 10:00 15:00	9:00 14:00	7:00 10:00 15:00	9:00 12:00	9:00 11:00 13:00
Motorized Boats	7:00 10:00 16:00	6:00 14:00	8:00 11:00 15:00	6:00 9:00 12:00	7:00 11:00 16:00	6:00 8:00 15:00

Table 3:Plan C

	1	2	3	4	5	6
Rubber Rafts	8:00 13:00	9:00 11:00 14:00	9:00 12:00	6:00	9:00 15:00	9:00
Motorized Boats	6:00 10:00 12:00	6:00 8:00 12:00	7:00 11:00 14:00 16:00	8:00 11:00 13:00 15:00	6:00 8:00 11:00 14:00	7:00 12:00 14:00 16:00

Table 4:Plan D

	1	2	3	4	5	6
Motorized Boats	6:00 8:00 10:00 12:00 13:00 16:00	6:00 7:00 8:00 11:00 13:00 15:00	6:00 8:00 11:00 14:00 16:00	6:00 7:00 10:00 12:00 15:00	7:00 9:00 11:00 12:00 14:00 16:00	6:00 8:00 9:00 11:00 13:00 15:00

Table 5:Plan E

4.3 Simulations & Analysis

By means of Matlab, the five simulations shown in Table 1-5 are run to demonstrate how the model will predict the impact of management actions on the river. These impacts are measured in terms of encounters and utilization level. With the different input, it will show the number of every-day contacts between trips and the maximum number of trips in one day. Then, we can compare the results to make a optimal choice. The results are shown in the Table 6.

		Plan A	Plan B	Plan C	Plan D	Plan E
Average Encounter(/d)	Total	6.6889	5.7000	5.2333	5.8611	6.3722
	Per Ship	1.1148	0.9500	0.8722	0.9769	1.0620
CCR	Number	65	63	61	61	63
	Utilization	86.67%	84.00%	81.33%	81.33%	84.00%

Table 6:Comparition

In Table 6, Plan A, with oars only, has a larger average encounters per ship and a higher rate of utilization, since the slower speeds and more days on trip. As the reduction of the ratio of oars to motors, Plan B has a smaller number in comparison with Plan A. The results of Plan C, with half oars and half motors, are the lowest of all plans. If the number of motors is relatively large, the results are higher again.

In order to decide the optimal schedule, we need to make a balance between the two conditions, maximizing the average encounters and minimizing the rate of utilization. Hence we would like to compare the ratio of the average encounters of per ship to the rate of utilization. The ratio of each plan: A: 1.286, B: 1.131, C: 1.072, D: 1.201, F: 1.264.

Then we have drawn a conclusion that the Plan C, with half oars and half motors, is the optimal schedule.

Overall speaking, the average encounter of per ship is around once a day. The average maximum number of trips and the rate of utilization in one day are around 63 and 83.47%.

Given that the Y campsites are distributed fairly uniformly throughout the river corridor, Y can be divided by 225. Since there are around 63 trips in the river on one day, Y must be larger than 63. Hence we determine Y as 75. Besides, we have found a picture for reference in a website called Canyon Exploration[5]. It is a true item of a travel agency. And we find that the long river in the Fig 5 has the same length as the one in Problem B, and there are about 40 attraction sites along the river. If we arrange about 1-2 campsites round each attraction site, there may be about 60-80 campsites. It can also demonstrate to us that setting 75 campsites is a rational arrangement. Furthermore, by the way of the queuing theory model, the two results coincide with each other, which also verify the correctness of the former model.

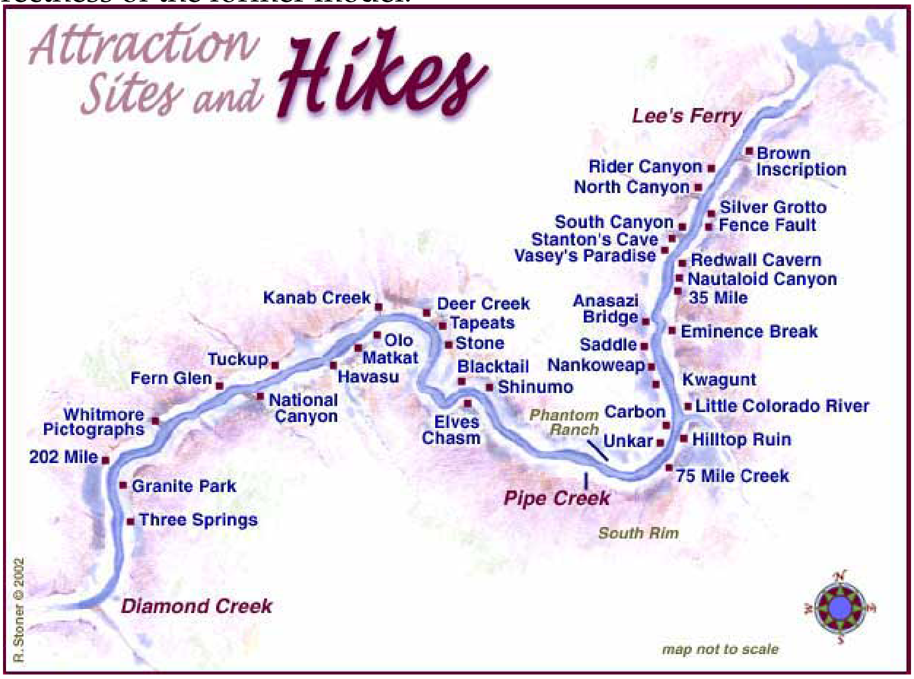

Fig 5:Hikes & Sites of the Canyon

Given that X, the number of trips at present during the tourist season, is uncertain, we determine X on the basis of the real data in Fig 6 from the website noted above. Finally, we determine that X is around 90.

Length/Price*							
6 Day-$1790 UPPER			6/11-6/16 **P** 6/28-7/3	7/16-7/21 **P** 7/22-7/27			
7 Day-$2110 UPPER	4/23-4/29	5/8-5/14 5/11-5/17 **P** 5/20-5/26 5/31-6/6	6/7-6/13 6/16-6/22	7/6-7/12 7/12-7/18	8/4-8/10 8/9-8/15	9/2-9/8 9/7-9/13 **P** 9/19-9/25	10/2-10/8
8 Day-$2415 UPPER	4/16-4/23 **HS**					9/22-9/29 **HS**	
8 Day-$2415 LOWER			6/16-6/23 **P** 6/22-6/29	7/3-7/10 7/18-7/25 7/21-7/28 **P** 7/27-8/3	8/10-8/17 8/15-8/22		
9 Day-$2710 LOWER	4/23-5/1 **HS** 4/29-5/7	5/14-5/22 5/17-5/25 **P** 5/26-6/3	6/6-6/14 6/13-6/21	7/12-7/20		9/8-9/16 9/13-9/21 **P** 9/25-10/3 9/29-10/7	10/8-10/16
10 Day-$2990 LOWER						9/29-10/8 **HS**	
13 Day-$3335 FULL			6/11-6/23 **P** 6/28-7/10	7/16-7/28 **P** 7/22-8/3			
14 Day-$3535 FULL			6/16-6/29	7/12-7/25	8/4-8/17 8/9-8/22		
15 Day-$3735 FULL	4/23-5/7	5/8-5/22 5/11-5/25 **P** 5/20-6/3 5/31-6/14	6/7-6/21 6/20-7/4 **SQ**	7/6-7/20	8/31-9/14	9/2-9/16 9/7-9/21 **P** 9/19-10/3	10/2-10/16
16 Day-$3935 FULL	4/16-5/1 **HS**	5/6-5/21 **HS**				9/14-9/29 **HS** 9/28-10/13 **HS**	10/6-10/21 **HS**
17 Day-$4135 FULL						9/22-10/8 **HS**	

Fig 6:2011 Dates/Fares

Then, we can calculate the additional number of trips that can be added to the Big Long River's rafting season. According to the arrangement of schedule, there are 33 trips per 6 days and no trip on the last 6 days. Hence the total number of trips is 967 (=33×29). The additional number is 867.

Overall, we arrange the schedule as Plan C and determine the carrying capacity of the river as 63 trips a day. And there are 867 more boat trips could be added to the Big Long River's rafting season. The number of encounter of per ship is 0.8722 on average, less than once.

5 FURTHER IMPROVEMENTS ABOUT MODEL

5.1 Single Channel,Multi-phase Queuing system

In the previous simple model of queuing system, we devise multi-phase queuing methods, and assume if there is any empty camp sites, the ship can immediately land on the available place. Also, we oversimplify camping event, we just assume after each of them stop camping on the one site, these groups quit the whole system without moving to the next destination. These all limit the application of our queuing model.

However, in the real case, most trips need to raft or drive for several time (waiting time) to get another available site when they encounter an occupied site. And after they stop camping and start rafting again, they enter the waiting system again, and wait for the next camp site (service facility). In this, we find a more accurate and sophisticated queuing model — Single Channel, Multi-phase Queuing System (See Fig 7).

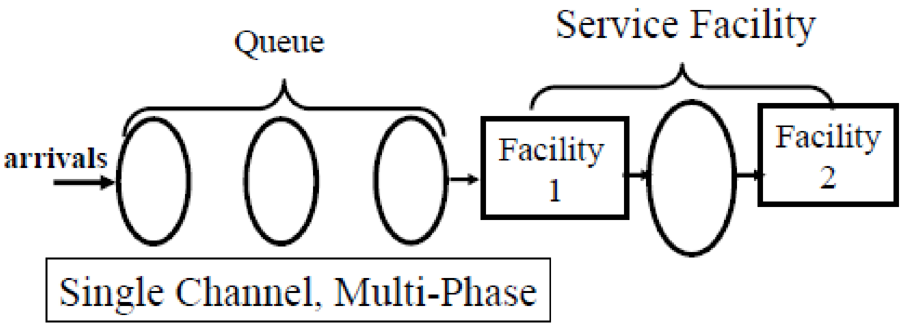

Fig 7:Single Channel,Multi-Phase Queuing System Configuration

When it comes to Single Channel, Multi-phase Queuing system, there are a lot of complicated factors we should take into consideration, many of which still await investigation. For example, the output distribution of the first facility and the input distribution of the following facilities are most likely unknown.

5.2 Grand Canyon River Trip Simulator (GCRTSim)

For our simulation model, limited by small database, we only discuss several schemes and compare these methods. We cannot devise a comprehensive model to screen the best schedule.

But a computer program called the Grand Canyon River Trip Simulator (GCRTSim) could solve the launch schedule problems. GCRTSim consists of a database, simulator, and extensive analysis tools. The simulator provides methods to set prospective launch schedules for rafting trips and simulate rafting seasons using these launch calendars. Both the trip diary database and the results of the simulations can be analyzed using graphing tools. The analysis can provide insight into use levels that could impact both the recreational experiences and resources along the river(See Fig 8 and Fig 9)[6][7].

Fig 8:Grand Canyon River Trip Simulator (GCRTS)

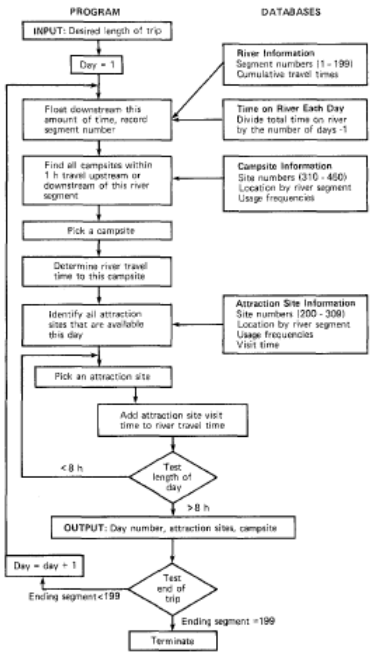

Fig 9:Microcomputer Flowchart

6 EXECUTIVE SUMMARY

6.1 Overview: Strengths & Weakness of Model

In this problem of scheduling ship launching, we present models to select the best time table. Our algorithm firstly adapts queuing theory to create one day trip. Then using the parameter obtained, we present several scheme and utilize programming simulation to determine the carrying capacity of the river (CCR) and the average encounters during trips. The model has several key strengths:

- **We treat the whole process as a total stochastic procedure.** In our schemes, we just set the launching time and ratio of two type transportation intentionally. As to the duration of trips, the detailed schedule of each trips, and even the velocity are all random. It fits the reality more. Every group can plan on their own even after the trip begins. For example, they can raft slowly when encountering beautiful sceneries, and drive quickly when they are in hurry to reach campsites. In a word, all schedules are flexible.

- **We adapt mature theory for modeling.** In the simple model of queuing theory, we indentify our model fitting for the multi-channel single-phase queuing system. By simplifying several factors, we are able to figure out all the parameters for queuing theory. Then we can investigate deep inside for a more sophisticated model with the considerations of many factors we disregard before.

- **We have definite and clear judgment criteria.** When we make schemes and compare those schemes to screen the best one, we set feasible and right criteria. That is the carry capacity of the river (maximum occupying rate of whole camp sites) and average encounters each day per ship. The optimum level of a scheme is carrying the most CCR (for the benefit of tour operators) and least encountering times (on the behalf of travelers). In the simulation results of our model, we all reach a great level for both of two rules.

  At the same time, our model contains several inevitable limitations:

- **The queuing theory model doesn't fits closely with real case.** As stated in further improvement of our model, the queuing theory model we adapt is multi-channel single-phase model, which is too simple to fit for the real case. Since our research shows that a more fit model-single channel, multi-phase queuing model is more complicated, unpredictable, and influenced by many other factors.

- **Our schemes are too limited to find a most desirable schedule.** It is supposed to be that we find a scheme which can excel all the other ones. A more abundant data resource can guarantee a better result in our models.

Limited by small database and random nature of our model, we cannot obtain such a superior model.

- **The model has not been justified by large amounts of factual testing.** In real life, we can hardly find random variables exactly subject to standard distribution. In other words, we need more factual validation.

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APPENDIX A

1.Here are queuing theory programmes to calculate the Carrying Capability of the River we used in our model as following.

Input matlab source:

```
function out=MMSkteam(s,k,mu1,mu2,T)

%s-number of campsites
%k-maximum number of trips per day
%T-the whole time
```

```
%mu1−arrival rate is described by a Poisson distribution
%mu2−camping time is described by negative exponential distribution
%event  table
%    arrive_time−launching events
%    leave_time−leaving events
%mintime−the most recent event in the event table
%current_time
%L−queue length
%tt−time series
%LL−queue length series
%c−launching time series
%b−camping beginning time series
%e−camping end time series
%a_count−total launching number
%b_count−total camping number
%e_count−losing trips

% initialization

arrive_time=exprnd(mu1);
leave_time=[];
current_time=0;
L=0;
LL=[L];
tt=[current_time];
c=[];
b=[];
e=[];
a_count=0;
b_count=0;
e_count=0;
%loop
while min([arrive_time,leave_time])<T
    current_time=min([arrive_time,leave_time]);
    tt=[tt,current_time];      %record time series
    if current_time==arrive_time              %lauching time subprocedure
        arrive_time=arrive_time+exprnd(mu1); % update launching events
        a_count=a_count+1; %adding total launching number
        if  L<s              %available camp sites
            L=L+1;          %update queue length
            b_count=b_count+1;%add total camping number
            c=[c,current_time];%record launching time series
            b=[b,current_time];%record camping beginning time series
            leave_time=[leave_time,current_time+exprnd(mu2)];%creat new group leaving
                events
            leave_time=sort(leave_time);%sort leaving events
        elseif  L<s+k                  %if there  are  empty camp sites
            L=L+1;          %update queue length
            b_count=b_count+1;%add number of camping group
            c=[c,current_time];%record launching time series
        else                      %lose groups
            e_count=e_count+1;%add losing groups
```

```
        end
    else                          %leaving subprocedure
            leave_time(1)=[];%delete leaving event from event table
            e=[e,current_time];%record leaving series
            if L>s    %if there still group rafting
                L=L-1;        %update queue length
                b=[b,current_time];%record camping beginning time series
                leave_time=[leave_time,current_time+exprnd(mu2)];
                leave_time=sort(leave_time);%sort leaving events
            else      %no waiting
                L=L-1;        %update queue length
            end
    end
    LL=[LL,L];   %record queue length
end
Ws=sum(e-c(1:length(e)))/length(e);
Wq=sum(b-c(1:length(b)))/length(b);
Wb=sum(e-b(1:length(e)))/length(e);
Ls=sum(diff([tt,T]).*LL)/T;
Lq=sum(diff([tt,T]).*max(LL-s,0))/T;
fprintf('launching trips:%d\n',a_count)
fprintf('camping trips:%d\n',b_count)
fprintf('losing trips:%d\n',e_count)
fprintf('mean staying time:%f\n',Ws)
fprintf('mean waiting time:%f\n',Wq)
fprintf('mean camping time:%f\n',Wb)
fprintf('average queue length:%f\n',Ls)
fprintf('average rafting trips number:%f\n',Lq)
if k~=inf
   for i=0:s+k
     p(i+1)=sum((LL==i).*diff([tt,T]))/T;
     fprintf('probability of queue length equals %d is:%f\n',i,p(i+1));
   end
else
   for i=0:3*s
     p(i+1)=sum((LL==i).*diff([tt,T]))/T;
     fprintf('probability of queue length equals %d is:%f\n',i,p(i+1));
   end
end
fprintf('probability of groups cannot reach a camp is:%f\n',1-sum(p(1:s)))
out=[Ws,Wq,Wb,Ls,Lq,p];
```

2.Here are simulation programmes to calculate the number of contact we used in our model as following.(Take plan A for example)

Input matlab source:

```
A= [ 1    1    0    1    0    1
     0    1    1    0    1    0
     1    0    1    1    0    0
     0    0    0    1    1    1
     1    1    1    0    1    0
     0    0    0    1    0    1
```

```
     0    1    0    0    1    0
     1    1    1    1    0    1
     0    0    1    1    1    0
     1    1    1    0    1    1
     0    0    0    0    0    1
     0    0    0    0    0    0];
D=zeros(12,6);
E=repmat(A,1,29);
A=[E,D];
t=normrnd(4,0.4,12,180);
v=normrnd(4,0.4,12,180);
B=A.*t.*v;
B(B>=225)=inf;
F=[];
for k=1:11
  for j=1:180
      for i=1:12
        x=j+k;y=normrnd(12,0.4);
        z=B(i,j)*(normrnd(4,0.4)*normrnd(4,0.4))^k;
        if z~=inf && z~=0
         F=[F;x,y,z];
        end
     end
  end
end

[m,n]=size(F);
Y=zeros(180,1);
Y(1)=6;
for k=2:180
    for i=1:m
        if F(i,1)==k
            Y(k)=Y(k)+1;
        end
     end
end
Y
max(Y)
max(Y)/75
```

3.Here are simulation programmes to calculate the carrying capacity of the river we used in our model as following.(Take plan A for example,too)

Input matlab source:

```
A= [ 1    1    0    1    0    1
     0    1    1    0    1    0
     1    0    1    1    0    0
     0    0    0    1    1    1
     1    1    1    0    1    0
     0    0    0    1    0    1
     0    1    0    0    1    0
     1    1    1    1    0    1
```

```
        0       0       1       1       1       0
        1       1       1       0       1       1
        0       0       0       0       0       1
        0       0       0       0       0       0];
D=zeros(12,6);
E=repmat(A,1,29);
A=[E,D];
t=normrnd(4,0.4,12,180);
v=normrnd(4,0.4,12,180);
B=A.*t.*v;
B(B>=225)=inf;
for i=1:12
    for j=1:180
        x=j+1;y=normrnd(12,0.4);
        z=B(i,j);
        if z==inf && z~=0
        continue;
        else [x,y,z];

        end
    end
end


for j=1:180
    for i=1:12
        for k=1:12
          if k~=i && B(i,j)==B(k,j)&&B(i,j)~=0
             B(k,j)=B(k,j)+normrnd(12,1.2)*normrnd(6,0.6);
          end
        end
    end
end
B

 C=zeros(1,180);
for j=1:180
    for i=1:12
         for k=i+1:12
             if B(i,j)<B(k,j) && B(i,j)~=0
                 C(j)=C(j)+1;
             end
         end
    end
end
C
sum(C)/180
```

APPENDIX B

Memo

DATE: February 14th, 2012

TO: managers of Big Long River

FROM: anonymous authors

SUBJECT: Best Schedule of Big Long River Rafting

This memo is in response to your quest for the best schedule of Big Long River rafting during tourism season. The memo will firstly explain the methods of modeling and specify the key findings during our model simulation. Then, it will present the best schedule for both you and your customers.

1. Model

   To be more fitting for the real case, we treat the whole process after ship launching to be stochastic. But we assume all the random variables are subject to organized distributions. For example, the launching time and camping time are all subject to negative exponential distribution. So that we can employ Queuing theory in order to get a common day dispatching schedule, in which, we particularly use Multi-Channel Single-Phase queuing system to mimic the process. As a result of MATLAB programming, we obtain mean number of trips launching per day is 6.

2. Schedule

   With the result from queuing theory, we continue to make simulations. For the convenience of simulation, this time we treat all the variables as random variables satisfying the normal distribution, such as the velocity of a boat, the duration of every-day rafting, and the starting time of a boat in a day. Then we record the time and spatial situation of a boat in a matrix, and make a cycling program by Matlab to simulate the day-by-day rafting process. In this session, limited by space, we present only 5 schemes for comparison. Schemes differ from each other by transportation mode and the number of launches. Those are: 1) all oar-powered boats, 35 launches per 6 days, on average 5 to 6 launches a day (resulted from queuing theory model). No launches in the last 6 days. 2) 30% motors and 70% oars, 34 launches per 6 days, no launches in the last 6 days. 3) 50% motors and 50% oars, 33 launches per 6 days, no launches in the last 6 days. 4) 70% motors and 30% oars, 34 launches per 6 days, no launches in the last 6 days, 5) all motorized boats, 34 launches per 6 days, no launches in the last 6 days. Finally we compare the results of these schemes and decide the best schedule with half motors and half oars and the average number of encounter in one day of 0.8722 per ship.

   Hence we suggest that the schedule 3) is the optimal scheme, and about 867 more boat trips can be added to the Big Long River's rafting season.

   Let me know if you have any further questions about our model or the best scheme.