Network Diversity and Economic Resilience to Financial Crises

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Abstract

This paper argues that an economy becomes more resilient to nationwide shocks such as financial crises when the input-output network has high entropy. In networks, entropy increases as each node has widely and evenly distributed relationships with other nodes. I construct a measure of economic diversity based on entropy and empirically investigate how this measure relates to the aftermath of financial crises. It turns out that countries with high domestic upstream or downstream diversity were less affected by financial crises. The role of the network changes in a global economy: Upstream diversification in the global market does not serve as insurance, but downstream diversification still plays a crucial role in reducing the negative impact. This implies that geographic export diversification is essential to reduce the adverse effects of financial crises. I construct a multisector model with financial constraints and show that the volatility of GDP in response to productivity shocks or financial shocks shrinks if each industry has widely and evenly distributed relationships with its input suppliers. When demand shocks hit the economy, the volatility of GDP declines if each industry has widely and equally distributed relationships with its customers.

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1 Introduction

There is a considerable variation in the way economies respond to financial crises, as shown by Romer and Romer (2017). Financial crises have catastrophic effects on the economy, but a resilient economy recovers quickly from adverse shocks. For example, some countries, such as South Korea and Australia, were not seriously affected by the global financial crisis of 2008. In addition, among those countries that experienced sharp drops in GDP, the USA and the UK started to recover within a year after the crisis while many countries in Europe suffered for a longer period. Then, what makes an economy under financial crises more resilient than others?

To answer this question, I focus on the structure of the input-output network. There is an extensive amount of literature on the role of the input-output network in the propagation of shocks over the economy (e.g., Gabaix (2011), Acemoglu et al. (2012), Acemoglu et al. (2017), Carvalho and Grassi (2019)). These studies illustrate how idiosyncratic shocks can be amplified to cause aggregate fluctuations. In these studies, interdependencies provide channels through which cascades occur. However, a diversified relationship can also serve as insurance against shocks, as Elliott et al. (2014) indicated. In reality, most of the sectors are exposed to severe shocks at the same time when a financial crisis hits a country. Therefore, I try to exhibit the insurance role of the input-output network when nationwide shocks hit all sectors simultaneously. This approach will effectively expose the relationship between the network structure and the aftermath of financial crises.

To do this, I rely upon network theory. Among the numerous measures that exhibit various features of networks, I focus on the entropy-based measure of centrality that is designed to capture the distribution of flows from a node in a network.[1] Entropy is a measure of disorder or randomness in a system. Based on this centrality measure, a node is central if a flow from this node stops with nearly equal probabilities at all other nodes. When applied to the input-output network, this measure takes a high value for industries that have widely and evenly dispersed input-output relationships with other industries. For example, high downstream entropy means that an industry

[1] See Tutzauer (2007) for more details of the entropy-based centrality measure.

supplies intermediate goods and services to many different industries, and the flows are equally distributed among the customers. On the other hand, low entropy means that the flow is concentrated largely toward a few industries so that the degree of randomness is low. Gabaix (2011) showed that the Herfindahl-Hirschman Index (HHI) of sales shares is the sufficient statistic for aggregate volatility. I show that the entropy-based centrality provides virtually the same information as the inverse of the HHI regarding the structure of network.

I construct a measure of economic diversity by taking the weighted average of sectoral entropy, using the output shares as weights. Therefore, this measure reflects both the distribution of sectoral output and entropy of each sector.[2] The value of this diversity measure becomes close to one if a country has major industries being connected to many other industries with equal distribution. In other words, industries in this country are densely connected around the major industries. Contrarily, the diversity measure goes to zero if the flows from (or to) major industries are connected to only a few other industries. In this case, the country has a sparsely connected input-output network around the major industries.

Then, I examine the relationship between the diversity measure and the aftermath of financial crises for 24 advanced countries, using the local projection method (Jord, 2005). I take the baseline specifications and data used by Romer and Romer (2017, 2018). They construct a new measure of financial distress, scaled from 0 (no financial distress) to 15 (extreme crisis) for 24 advanced OECD countries in the postwar period.[3] I utilize this measure to identify financial disruptions and investigate the responses of real GDP to the realization of financial crises for high- and low-diversity countries.[4]

The empirical results illustrate the distinct roles of domestic and global input-output linkages. In terms of the domestic network, both upstream and downstream diversities play a critical role in shaping responses of real GDP after financial crises. On average, real GDP declines up to 6 percent

[2] Gabaix (2011) and Acemoglu et al. (2012) indicate the importance of the output distribution to understand the granular origins of aggregate fluctuations.

[3] A country is considered to be in a crisis if its value is equal to or greater than 7.

[4] The high-diversity countries take values of diversity that are one standard deviation above the sample mean, while the low-diversity countries take those that are one standard deviation below the mean.

after financial crises when we ignore the level of diversity. However, the real GDP of countries with high upstream or downstream diversity was greater than that of the low-diversity countries by up to around 5.4 and 5.6 percentage points, respectively. This implies that upstream and downstream diversity played an important role in reducing the negative impact of financial crises. When we use the global input-output network, the insurance role of upstream diversity disappears, and there is no significant difference between high- and low-diversity countries. However, countries with a high downstream global diversity experienced a much smaller decline in real GDP than others. The difference in real GDP between the two groups reaches 8 percentage points at the peak.

Theoretically, I build a macroeconomic model with input-output linkages and financial constraints, following Long and Plosser (1983), Acemoglu et al. (2012), Acemoglu et al. (2015), and Bigio and LaO (2020). The model predicts that the upper bound of aggregate output volatility is minimized when each industry has evenly distributed flows of intermediate inputs passing through it. Specifically, the volatility of GDP in response to the productivity shock or the financial shock shrinks if each industry has widely and equally distributed relationships with its input suppliers, i.e., upstream industries. On the other hand, the volatility of GDP in response to demand shocks declines if each industry has widely and evenly distributed relationships with its customers, i.e., downstream industries. This implies that widely and evenly distributed suppliers and customers in the network serve as insurance by reducing an industry's dependence on a specific counterpart.

Furthermore, I explore the propagation of shocks in a multi-country setting with trades in intermediate goods following Johnson (2014). The role of the input-output network changes when we have multiple countries. I show that the same theoretical predictions can be applied to the global network if worldwide shocks simultaneously hit all countries and sectors. However, financial crises usually hit only a small number of countries in the global economy, or at least there is extensive heterogeneity in the degree of exposure. This weakens the relationship between the network structure and the volatility of GDP, undermining especially the insurance role of upstream diversification in the global network. Still, the downstream diversification in the global network can serve as insurance by ensuring stable cash flows and signaling good financial health (Greenaway et al., 2007; Myles Shaver, 2011).

Related literature. The recent literature about the aftermath of financial crises includes Bordo et al. (2001), Hoggarth et al. (2002), Cerra and Saxena (2008), Reinhart and Rogoff (2009a,b, 2014), Claessens et al. (2010), Schularick and Taylor (2012), Gourinchas and Obstfeld (2012), Jord et al. (2013), Romer and Romer (2017, 2018). Especially, the empirical part of this paper is closely related to Romer and Romer (2018). They find that the degree of monetary and fiscal policy space prior to financial distress is an important factor in explaining the aftermath of financial crises. This paper contributes to this literature by identifying the importance of the input-output structure in reducing the impact of the financial crisis.

Since the pioneering work of Leontief (1941), there have been extensive studies regarding the propagation of shocks through the input-output linkages: Long and Plosser (1983), Horvath (1998, 2000), Shea (2002), Gabaix (2011), Acemoglu et al. (2017), Atalay (2017), Baqaee and Farhi (2019), Bigio and LaO (2020), and Carvalho et al. (2021).[5] These studies generally use degree centrality and the law of large numbers. For example, Acemoglu et al. (2012) show that the distribution of network centrality matters for the macro effect of micro shocks. Unlike these studies, this paper highlights the distribution of flows around each sector and empirically investigates the role of the input-out network in mitigating the impact of financial crises rather than depicting how micro shocks lead to macro volatility.[6] Exceptionally, Miranda-Pinto (2021) investigates the relationship between network diversification and aggregate volatility. However, he focuses on volatility over business cycles rather than financial crises. In addition, this paper extends the input-output model by including multiple countries.

This paper focuses on the input-output network between sectors. There also exists literature on financial contagion using financial networks or both, including but not limited to Allen and Gale (2000), Freixas et al. (2000), Dasgupta (2004), Shin (2008), Gai et al. (2011), Elliott et al. (2014),

[5]See Carvalho and Tahbaz-Salehi (2019) for a review on production networks in macroeconomics.

[6]There are several other lines of research using the input-output network. For example, Foerster et al. (2011), Carvalho and Gabaix (2013), and di Giovanni et al. (2014) investigate the role of common shocks and micro shocks. La'O and Tahbaz-Salehi (2020), Carvalho et al. (2021), and Pasten et al. (2021) investigate aggregate responses to sectoral shocks and optimal monetary policy using sectoral price rigidity.

Acemoglu et al. (2015), Glasserman and Young (2015), Heipertz et al. (2019), Luo (2020), and Altinoglu (2021). Some of these studies investigate the relationship between financial networks and economic stability. For example, Elliott et al. (2014) indicate that integration and diversification face trade-offs in terms of financial contagion.[7] Diversification connects the network initially, allowing cascades to occur, but as it increases further, organizations are better insured against one anothers failures. This paper's findings are consistent with this prediction. However, this paper uses the network of intermediate inputs and shows different roles of the domestic and global network in terms of upstream and downstream relationship.

Entropy is originally a scientific concept introduced in physics, but since Shannon (1948), this concept has been extensively used in the field of social science. Many economists also apply this concept usually to measure the amount of information or uncertainty. Some recent studies using entropy include Weitzman (2000), Sims (2003), Hong and White (2005), Cabrales et al. (2013), and Matjka (2015). This paper applies entropy to the input-output analysis and empirically investigates the relationship between entropy and volatility of GDP in various contexts.

This paper is organized as follows. Section 2 introduces the measure of economic diversity based on entropy in the input-output network. Section 3 uses the local projection method to show the relationship between economic diversity and the aftermath of financial crises. Section 4 provides a model with input-output linkages and financial constraints and discusses its implications regarding the relationship between the structure of the input-output network and the volatility of GDP. I conclude in Section 5.

2 Entropy and Economic Diversity

In this section, I construct a measure of economic diversity based on Gabaix (2011) and Acemoglu et al. (2012). Gabaix (2011) shows that the HHI of sales shares is the sufficient statistic for aggregate volatility, and Acemoglu et al. (2012) show the importance of the structure of input-

[7] Integration means greater dependence on counterparties, and diversification means more counterparties per organization.

output networks in determining aggregate volatility. I show that the entropy-based centrality for network analysis provides practically the same information as the inverse of the HHI regarding the structure of networks.[8] Then, I build a diversity measure based on the entropy-based centrality and discuss its properties.

2.1 Entropy-Based Measure of Centrality

In statistical mechanics, entropy is determined by the number of random microstates. In other words, entropy is a measure of disorder or randomness in a system. In information theory, Shannon (1948) introduced a measure of information, choice, and uncertainty based on entropy. In a communication system with n symbols, an ith symbol is transmitted with probability p_i. Then, the information or entropy of the system is defined as

$$H = -\sum_{i=1}^{n} p_i \log p_i.$$

If only one p_i is unity and all others are zero, $H = 0$. This means that there is no uncertainty. This measure increases as the probabilities become close to each other and reaches its maximum, $\log n$, when all the probabilities are identical (i.e., $p_i = 1/n$). This is the most uncertain situation. If all the probabilities are identical, H increases monotonically with the total number of symbols n.

Tutzauer (2007) proposed an entropy-based measure of centrality that can be applied to a network of flows and nodes. In a network with total n nodes, the entropy of node i is given by

$$C_H(i) = -\sum_{j=1}^{n} p_{ij} \log p_{ij} \tag{1}$$

where p_{ij} is the probability that a flow originated from node i stops at node j. This measure is analogous to Shannon's entropy. Intuitively, a node is central if a flow from this node stops with nearly equal probabilities at all other nodes.

[8] Appendix A.3 provides additional reasons why the entropy-based centrality is appropriate for input-output network analysis.

This measure can be normalized to have a value between 0 and 1 by dividing by the maximum entropy $\log n$.

$$C'_H(i) = \frac{C_H(i)}{\log n}. \tag{2}$$

Here, what we need to know to calculate the entropy, $C'_H(i)$, is the number of nodes in the network, n, and the probabilities, p_{ij}, for all possible combinations of $i = \{1,2,3,\ldots,n\}$ and $j = \{1,2,3,\ldots,n\}$.

2.2 Entropy with Input-Output Networks

Flows originated from a node with high entropy stop at other nodes with almost identical probabilities. When we apply this measure to the input-output network, nodes represent industries, and flows imply transfers of intermediate inputs. The flip side of the intermediate input flow is the flows of money running in the opposite direction. It means that an industry with high entropy has diverse routes of revenues running to or from the industry.

The input-output matrix **A** represents the direct first-order network of industries in a country, and the Leontief inverse matrix **L** represents the cumulative network effect. The probabilities in equation (1) can be calculated either from the input-output matrix or the Leontief inverse matrix. If the Leontief inverse matrix is used, the probabilities represent the full network effect, including higher-order connections. In other words, the entropy-based measure of centrality can take into account the full higher-order connections.

In addition, the entropy-based measure of centrality can be applied to both upstream and downstream flows. Upstream industries are input suppliers, and downstream industries are customers of intermediate inputs. Equations (1) and (2) can represent upstream entropy if target industries, j, represent supplier industries, and downstream entropy if j denotes customer industries. When the full network effect is considered, the downstream entropy for sector i uses columns of the Leontief inverse matrix.

Definition 1 *The downstream entropy of sector i is given by*

$$C_H(i) = -\sum_{j=1}^{n} p_{ji} \log p_{ji} \text{ where } p_{ji} = \frac{l_{ji}}{\sum_{j=1}^{n} l_{ji}}. \tag{3}$$

where l_{ji} represents elements of the Leontief inverse matrix.

The column sum of the Leontief inverse matrix, $\sum_j l_{ji}$, is out-degree and represents the importance of industry i as an input supplier. The upstream entropy for industry i can be defined similarly, using rows of the Leontief matrix.[9]

Definition 2 *The upstream entropy of sector i is given by*

$$C_H(i) = -\sum_{j=1}^{n} p_{ij} \log p_{ij} \text{ where } p_{ij} = \frac{l_{ij}}{\sum_{j=1}^{n} l_{ij}}. \tag{4}$$

where l_{ij} represents elements of the Leontief inverse matrix.

The row sum of the Leontief matrix, $\sum_j l_{ij}$, is in-degree and represents the importance of industry i as a customer of intermediate goods. Equations (3) and (4) use the Leontief inverse matrix and reflect the full network effects.

This paper uses the OECD harmonized national input-output tables for empirical analysis. The national input-output tables present matrices of inter-industrial flows of goods and services in current prices (USD million) from 1995 to 2018. The OECD input-output tables take the industry $\times$ industry approach with 45 industry categories covering all sectors of the economy for 66 countries.[10] The rows of the tables represent supplier industries, and the columns represent customer industries. So, I transpose the input-output matrix and its Leontief inverse to apply the results of this section to the OECD input-output tables.[11]

[9] I calculate the Leontief matrix following the model in Section 4: $\mathbf{L} = [\mathbf{I} - \boldsymbol{\mu}\boldsymbol{\iota}' \circ \mathbf{A}]^{-1}$. The share of intermediate input for each sector is calculated from the input-output matrix by dividing expenditures on total intermediate inputs with values of output.

[10] One of the industries, *Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use*, is omitted from the following analysis since this industry does not have any input-output relationships with other industries.

[11] The Leontief inverse matrix is calculated from the input-output matrix including all 66 countries.

Table A1 shows the list of industries with their average entropy and ranks across 24 advanced OECD countries in 2018.[12] On average, *Wholesale and retail trade; repair of motor vehicles* have the highest downstream entropy (0.91), and *Fishing and aquaculture* have the lowest downstream entropy (0.09) domestically. Regarding the upstream flows, entropy is the highest (0.61) for *Air transport* and the lowest (0.37) for *Financial and insurance activities*. It is noteworthy that the industries with high downstream entropy tend to have low upstream entropy and vice versa. In addition, the variation of domestic entropy across sectors is much smaller for upstream flows than downstream flows.

The topology of entropy does not change significantly when considering the global input-output network. *Wholesale and retail trade; repair of motor vehicles* have the highest downstream and upstream entropy (0.66), and *Fishing and aquaculture* have the lowest downstream entropy (0.07) globally. In terms of global upstream flows, *Electronic equipment* has the highest entropy (0.46), and *Real estate activities* have the lowest entropy (0.22).[13] The global network includes much more nodes in the domestic network, and many of them have zero values in the global network. Therefore, the value of entropy is generally lower in the global network.

2.3 A Measure of Economic Diversity

I now construct a measure of economic diversity based on entropy. Because entropy can be calculated for each industry, I give weights to industries with their output shares. In the case of a low-entropy industry that makes up a large portion of GDP, the entropy measure fails to capture the importance in the economy because it does not take into account the size of the total output of industries. By aggregating the entropy of each industry with their output shares, I take into account

[12]The countries include Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Finland (FIN), France (FRA), Germany (DEU), Greece (GRC), Iceland (ISL), Ireland (IRL), Italy (ITA), Japan (JPN), Luxembourg (LUX), Netherlands (NLD), New Zealand (NZL), Norway (NOR), Portugal (PRT), Spain (ESP), Sweden (SWE), Switzerland (CHE), Turkey (TUR), United Kingdom (GBR), United States (USA).

[13]The values of entropy for every sector and country are available upon request.

both the size of each industry and its entropy.[14]

Definition 3 *A measure of economic diversity of a country can be defined as*

$$D_H = \sum_{i=1}^{n} C'_H(i) w(i) \tag{5}$$

where $w(i) = y(i)/\sum_{i=1}^{n} y(i)$ is the output share of industry i, and $C'_H(i)$ is given by equations (2), (3), and (4).

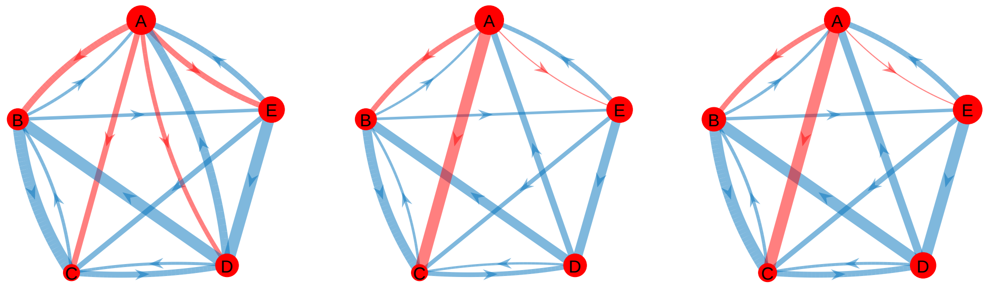
High diversity D_H means that the nation's major industries, in terms of output shares, have equally strong relationships with all other industries. Therefore, the industries in this country are densely connected around the major industries. In other words, the major industries are well diversified. On the other hand, low D_H implies that the nation's major industries are isolated to have strong relationships with only a few industries. So, the economy has a sparsely connected network around the major industries.

Figure 1 provides three different networks to illustrate the properties of the diversity measure D_H. Let's focus on node A and its downstream entropy based on outflows from the node. In the first two networks, node A has an equal output share. However, outflows from node A are unevenly distributed in the second network, while they are equally distributed in the first network. Thus, node A's downstream entropy $C'_H(A) = 0.96$ for the first network and $C'_H(A) = 0.73$ for the second network.[15] The second and third networks have equal downstream entropy for node A because they have the same distribution of probabilities p_{ij}. However, the weighted entropy (=$\sum$ output share $\times$ entropy) is smaller for the third network due to the lower output share of A. This shows that the diversity measure reflects both the size and entropy of each industry.

Table A2 shows entropy of industries weighted by their output shares. Since we take into account the size of industries, the ranks change significantly from Table A1. Now, *Wholesale and*

[14] The entropy of each industry is aggregated by the preference weight in Proposition 2 in Section 4. An industry is more likely to have a larger output share if its preference weight is bigger. On the other hand, entropy is aggregated with the share of labor in Proposition 4. In the empirical section, I test robustness with equal weight. The results do not change qualitatively when equal weight is used.

[15] This difference in the distribution of flows is not captured by degree centrality, which is used by Acemoglu et al. (2012) and Pasten et al. (2021).



Output share = 0.25 Output share = 0.25 Output share = 0.21
Entropy = 0.96 Entropy = 0.73 Entropy = 0.73
Weighted entropy = 0.24 Weighted entropy = 0.18 Weighted entropy = 0.15

Figure 1: Star networks with different outputs and entropy

Note. This figure shows three different networks. Node A has the equal output share for the first two stars, but the distribution of downstream flows is less evenly distributed for the second star. The second and third stars have equal entropy for node A, but the output share is smaller for the third network. The weighted entropy is calculated by multiplying output shares by entropy.

retail trade; repair of motor vehicles have the highest values of weighted entropy for both upstream and downstream flows in the domestic and global network. Also, *Real estate activities*, *Construction*, *Professional, scientific and technical activities*, and *Financial and insurance activities* are major industries contributing the most to enhancing economic diversity. On the other hand, *Mining support service activities*, *Fishing and aquaculture*, and *Postal and courier activities* are minor industries contributing the least to enhancing economic diversity of a nation. Note that, in terms of downstream flows, unweighted entropy plays a more important role than the output size in determining the rank for weighted entropy because the variation in unweighted entropy is large. However, the output size plays a more important role for the upstream rank since the variation in unweighted entropy is relatively small.

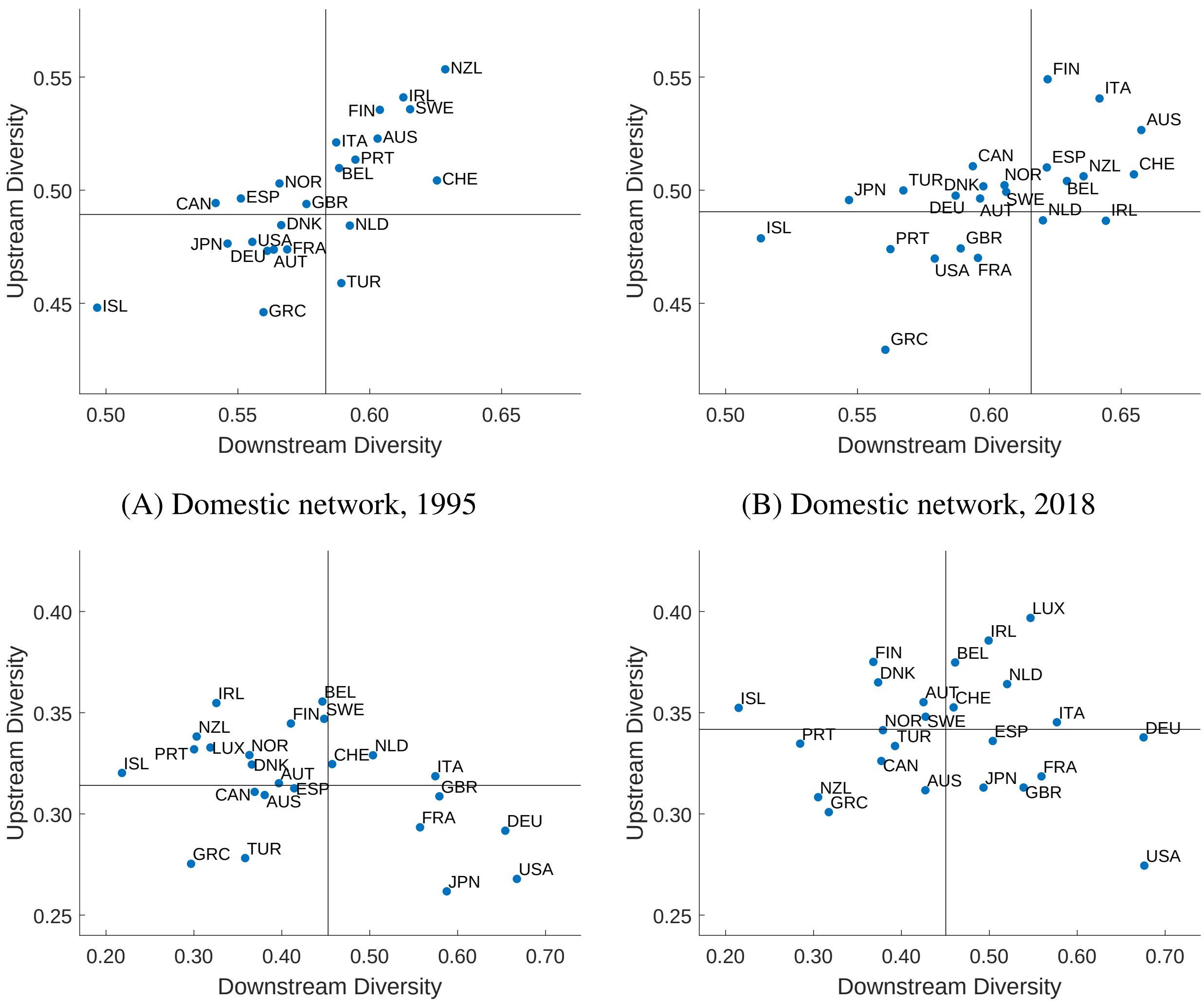
2.4 Properties of the Diversity Measure

Figure 2 shows the relationships between the upstream and downstream diversities for 24 advanced OECD countries in 1995 and 2018: panels (A) and (B) for the domestic network and panels (C) and (D) for the global network. The domestic network exhibits a strong positive correlation between the upstream and downstream diversities across countries, while the global network displays no strong correlation between the upstream and downstream diversities.

For the domestic network, the average level of downstream diversity has increased over time, while there is almost no change in the average level of upstream diversity. In contrast, the average upstream diversity has increased over time while the average downstream diversity stays stable. This implies that countries now import inputs from more diversified sources than before.

In addition, the diversity measures show a large variation across countries. In terms of the domestic network, Italy, Australia, and Finland have high upstream and downstream diversities in 2018, whereas Greece, Iceland, Portugal have low diversities. In terms of the global network in 2018, Luxembourg, Ireland, Netherlands have high upstream and downstream diversities, whereas Greece and New Zealand have low upstream and downstream diversities. The economic diversity also has evolved differently for those countries. The positions of Greece and the United States have not changed much over time, but Luxembourg and New Zealand have changed their positions significantly over time.

Table 1 shows the summary statistics of the diversity measures for 24 countries from 1995 to 2018. The second column shows the (unweighted) mean of the diversity measures for domestic and global networks. The downstream diversity is higher on average than the upstream diversity, and the domestic diversity is higher than the global diversity. The third and fourth columns show the cross-country and time variations of the diversity measure, respectively. The cross-country variation is calculated using the (unweighted) time average of cross-country standard deviation over 24 years. The time variation is the (unweighted) average of the over-time standard deviation of each country's diversity across 24 countries. The average standard deviations show that the cross-country variations of the diversity measures are much larger than time variations. In other words, the degree of diversity varies widely across countries but stays relatively stable over time for each country.



(A) Domestic network, 1995

(B) Domestic network, 2018

(C) Global network, 1995

(D) Global network, 2018

Figure 2: Cross-country correlation of the diversity measures

Note. This figure shows the upstream and downstream diversity measures for 24 countries. The straight lines indicate the average values.

Table 1: Summary statistics of the diversity measure

		Mean	Standard Deviation	
			Cross-country variation	Time variation
Domestic	Downstream	0.604	0.045	0.016
	Upstream	0.499	0.044	0.013
Global	Downstream	0.448	0.116	0.017
	Upstream	0.334	0.029	0.012

Note. This table shows the summary statistics of the diversity measure for 24 countries from 1955 to 2018. The unweighted mean of all countries and periods are used. The cross-country variation is calculated using the (unweighted) time average of cross-country standard deviation over 17 years. The time variation is the (unweighted) average of the over-time standard deviation of each country's diversity.

3 The Empirical Analysis

Romer and Romer (2017) show that there is substantial variation in the aftermath of financial crises, using the newly constructed measure of financial distress for 24 advanced countries in the postwar period. Romer and Romer (2018) also show that the monetary and fiscal policy space prior to financial distress is an important factor in explaining the heterogeneous effects of financial crises. This paper adopts the approach of Romer and Romer (2018) to show the role of the input-output network in the aftermath of financial crises.

3.1 Data

This paper uses the new scaled measure of financial distress constructed by Romer and Romer (2017) for 24 advanced countries from 1967 to 2012. This measure captures the cost of credit intermediation which reduces the supply of credit. This measure is based on qualitative evidence in OECD Economic Outlook and scaled to have a value between 0 (no financial distress) and 15 (extreme crisis). A country is considered to be in a crisis if its value is equal to or greater than 7.

Among other macroeconomic variables, this paper will focus on the behavior of real GDP as the measure of real economic activity. As the financial distress measure is semiannual, the quarterly real GDP from the OECD is modified to semiannual by taking the values for the second and fourth quarters of each year as used by Romer and Romer (2017, 2018).

3.2 Econometric Model

Following Romer and Romer (2017), this paper uses the local projection method (Jord, 2005) to identify the behavior of real economic activity after financial crises. In particular, the responses of macroeconomic variables to financial disruptions are estimated by

$$y_{j,t+i} = \alpha_j^i + \gamma_t^i + \beta^i F_{j,t} + \sum_{k=1}^{4} \phi_k^i F_{j,t-k} + \sum_{k=1}^{4} \theta_k^i y_{j,t-k} + e_{j,t}^i, \tag{6}$$

where $y_{j,t+i}$ is the measure of economic activity (log real GDP) for country j at time $t+i$. The i superscript indicates the horizon from 0 to 10 half-years. The main explanatory variable $F_{j,t}$ represents the financial distress for country j at time t. The four lags of the financial distress variable and the variable of real economic activity are also included as controls to capture the serial correlation. The country fixed effects, α_j^i, are included to control for country heterogeneity. The time fixed effects, γ_t^i, are also included to control for shocks common to all countries in a given period. The coefficient of the financial distress variable, β^i, represents the response of the dependent variable to variation in the financial distress variable at time t.

Furthermore, Romer and Romer (2018) investigate the role of monetary and fiscal policy space before the crises in explaining the substantial variation in the aftermath of financial crises. This paper follows their approach as a baseline specification to investigate the role of the input-output network in explaining the variation in the aftermath of financial crises. The regression model is given by

$$\begin{aligned} y_{j,t+i} =& \alpha_j^i + \gamma_t^i + \beta^i F_{j,t} + \delta^i \left(F_{j,t} \cdot D_{j,t}\right) + \zeta^i D_{j,t} + \sum_{k=1}^{4} \phi_k^i F_{j,t-k} \\ &+ \sum_{k=1}^{4} \rho_k^i D_{j,t-k} + \sum_{k=1}^{4} \omega_k^i \left(F_{j,t-k} \cdot D_{j,t-k}\right) + \sum_{k=1}^{4} \theta_k^i y_{j,t-k} + e_{j,t}^i, \end{aligned} \tag{7}$$

where $D_{j,t}$ is the diversity measure for country j at time t. Equation (7) also include the previous values of $F_{j,t}$, $D_{j,t}$, and their interaction term $F_{j,t} \cdot D_{j,t}$. Because the degree of diversity can evolve over time, reflecting the economic condition, $D_{j,t}$ is lagged a year (i.e., two half-years) to avoid the endogeneity problem. Therefore, $D_{j,t}$ represents the previous year's diversity.[16] This specification includes the contemporaneous relationship between economic activity and financial distress, following previous literature. The assumption is that distress is not affected by economic activity contemporaneously, but economic activity may be affected by distress within the same period. The contemporaneous marginal effect of financial distress on economic activity, which is equal to $\beta^i + \delta^i D_{j,t}$, depends on the degree of diversity.

The financial distress measures and the macroeconomic variable are taken from Romer and Romer (2017). I estimate equation (6) for the full time period from 1967. Equation (7) is estimated using available data from 1995 to 2015. Though the input-output tables are available through 2018, I make use of the GDP and diversity data through 2015 since the financial distress measure is available through 2012 and I include 4 lags of the financial distress measure. Though most of the events during this period are related to the 2008 financial crises, this sample period also includes the financial distress in Japan and Turkey in the 1990s and 2000s as well as a credit disruption in the United States in 1998.[17]

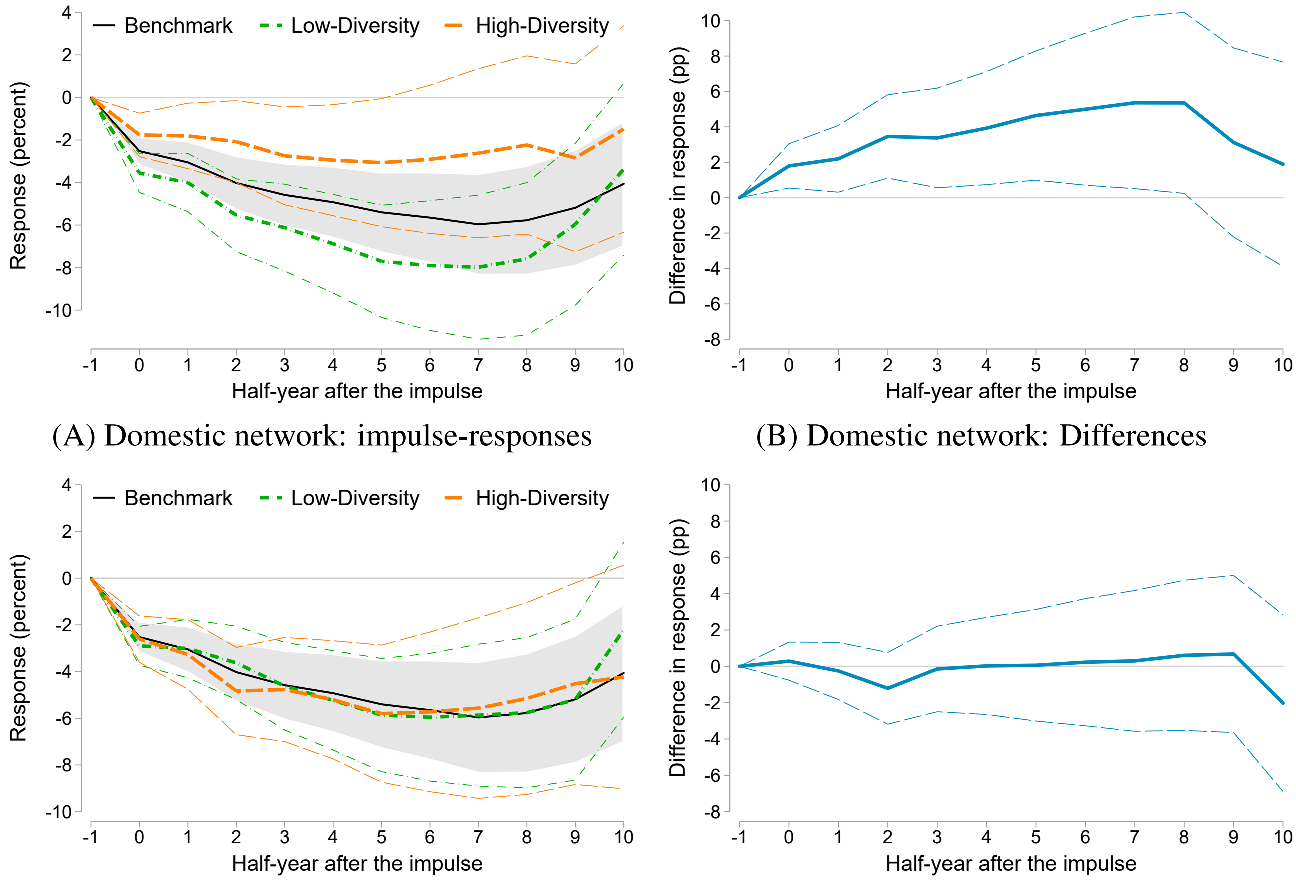
3.3 Results

This section presents the impulse responses for two different cases based on the direction of flows: upstream and downstream.

Upstream. Panel (A) and (B) of Figure A2 show the impulse-response of log real GDP to the financial distress shocks for the domestic and global network, respectively. Following Romer and Romer (2017, 2018), I multiply the responses by 7, which is the value of the financial distress measure corresponding to the start of the moderate crisis. The solid line represents the coefficient,

[16] The diversity measure is annual, and all other variables are semiannual.

[17] As Romer and Romer (2017) indicated, there were essentially no episodes of financial distress in the 1970s and 1980s.



(A) Domestic network: impulse-responses

(B) Domestic network: Differences

(C) Global network: impulse-responses

(D) Global network: Differences

Figure 3: The role of upstream diversity

Note. Panels (A) and (C) show the impulse-response function of log real GDP to financial distress shocks. Panels (B) and (D) show the differences between high- and low-diversity countries. The 95% confidence intervals are added to each line.

β_i, in Equation (6). This is the case when we do not consider the input-output network. The shaded area is its 95% confidence interval for five years. The long-dash line and the dash-dot line represent the responses of real GDP in countries with high and low downstream diversity, respectively. These responses are produced from Equation (7). In particular, the long-dash line shows the marginal effect of the financial distress, $F_{j,t}$, on the real GDP when the country's degree of diversity is one standard deviation above the mean. The dash-dot line shows the responses of real GDP in countries with low downstream diversity, or with diversity values of one standard deviation below the mean.

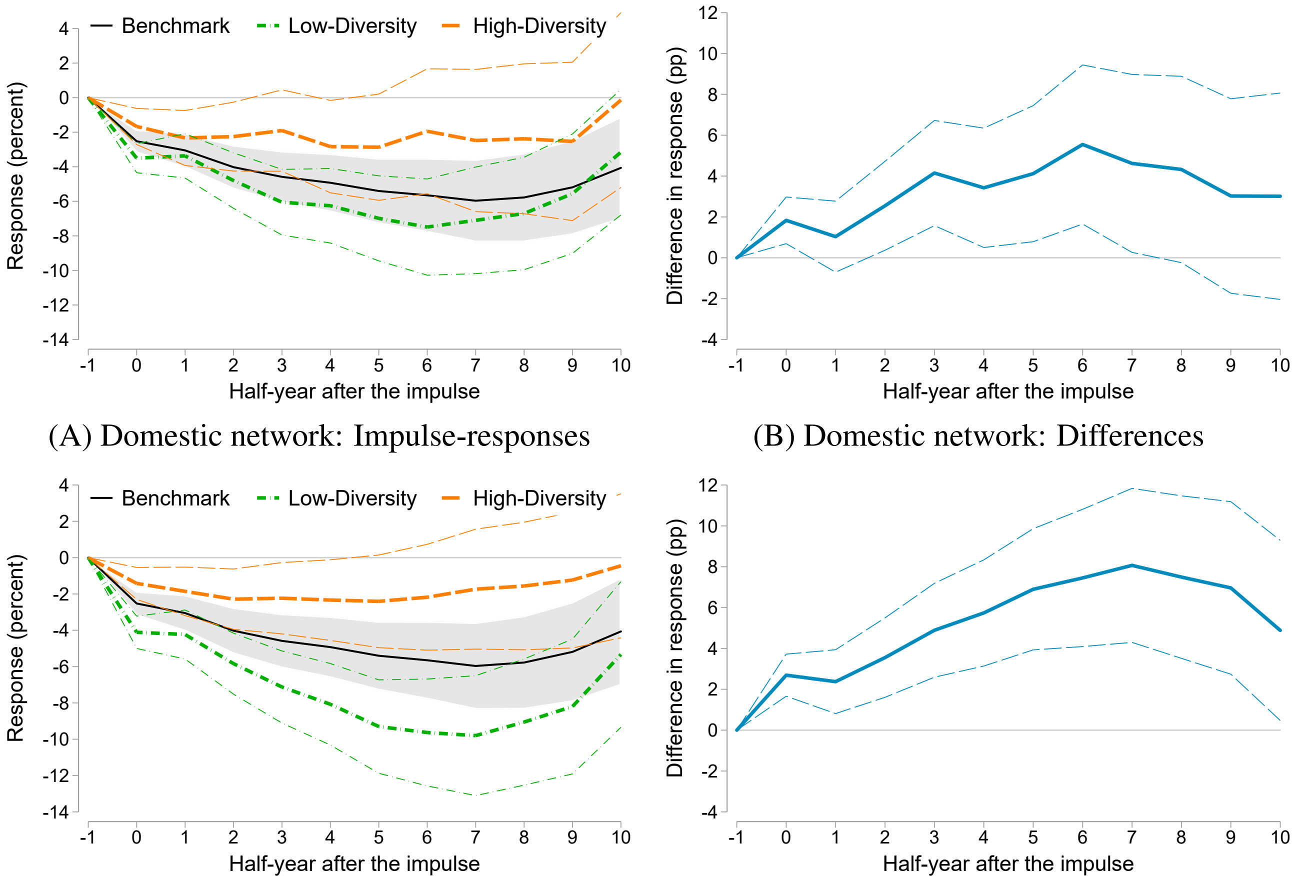
When we consider only the domestic network, countries with high upstream diversity are less affected by financial crises. The GDP reaches -3.1 percent in the 5th half-year for high-diversity countries and -8.0 percent in the 7th half-year for the low-diversity countries. This result implies that financial crises had much deeper and longer impacts on low-diversity countries. Countries with high upstream diversity not only experienced a smaller initial impact of financial crises but also recovered quickly compared to other countries. Panel (B) of Figure A2 shows that the differences in impulse responses between the two groups of countries are significantly different from zero. The maximum difference is around 5.4 percentage points in the 7th half-year.

However, panels (C) and (D) demonstrate that impulse-responses are almost identical for high- and low-diversity countries when we consider the global input-output network. This implies that having diversified input suppliers in the global market has not helped countries reduce the negative impact of the financial crisis.

Downstream. Panels (A) and (C) of Figure A3 show the impulse-response of log real GDP to the financial distress shocks for the downstream relationship. When we consider the domestic network, countries with high downstream diversity are less affected by financial distress shocks. The bottom of GDP reaches at -2.5 percent in the 9th half-year for high-diversity countries and at -7.5 percent in the 6th half-year for the low-diversity countries. Panel (B) shows that the difference between the two groups of countries reaches its maximum, 5.6 percentage points in the 6th half-year.

It turns out that the downstream diversity plays an important role in reducing the negative impact of financial crises when we consider the global input-output network. The maximum difference between high- and low-diversity countries reach 8.1 percentage points in the 7th half-year. Having diversified output customers in the global market has helped countries recover quickly from financial crises.

I also provide results with the Driscoll and Kraay (1998) standard errors in Figure A1. The Driscoll-Kraay standard errors are assumed to be heteroskedastic, autocorrelated, and possibly correlated between the countries. The Driscoll-Kraay standard errors are comparable to the OLS standard errors and do not change the results qualitatively from the OLS results. Appendix A.2 provides other checks for robustness.



(A) Domestic network: Impulse-responses

(B) Domestic network: Differences

(C) Global network: Impulse-responses

(D) Global network: Differences

Figure 4: The role of downstream diversity

Note. Panels (A) and (C) show the impulse-response function of log real GDP to financial distress shocks. Panels (B) and (D) show the differences between high- and low-diversity countries. The 95% confidence intervals are added to each line.

Summary. The empirical results with the domestic input-output network show that financial crises have been less painful for countries with a dense input-output network, such that major industries have more widely and evenly dispersed relationships with other industries. On the other hand, countries with low diversity suffered from deeper recessions due to financial distress.

However, widely and equally distributed input suppliers in the global market does not act as insurance against large nationwide shocks. When a financial crisis hits a country, its industries are already exposed to adverse shocks through their domestic input suppliers. Having widely distributed foreign suppliers cannot prevent the shock from spreading in this situation.[18]

[18] An alternative diversity measure based on the Herfindahl-Hirschman Index also produces similar results as shown in A.2.1.

Notably, widely and equally distributed customers in the global market play an essential role as insurance against large nationwide shocks such as financial crises. The following section aims to provide a theoretical explanation of the empirical findings presented in this section.

4 Theoretical Model

In this section, I build a multisector model to clarify the role of input-output linkages in the propagation of shocks. The model is closely related to Long and Plosser (1983), Acemoglu et al. (2012), and Acemoglu et al. (2017). To simplify the model I remove the time delay in production so that the time subscript disappears from all equations. Also, I impose financial constraints on the sector's purchase of intermediate inputs, following Bigio and LaO (2016).

4.1 A Closed Economy Model

Firms. Consider an economy composed of N industries. The representative firm in industry $i \in \{1..N\}$ produces output according to the following Cobb-Douglas production function:

$$y_i = e^{z_i} \zeta_i l_i^{1-\mu_i} \left(\prod_{j=1}^{N} x_{ij}^{a_{ij}} \right)^{\mu_i},$$

where x_{ij} is the quantity of goods produced by industry j used as inputs by industry i, l_i is labor working in sector i, $\zeta_i > 0$ is a normalization constant[19], $\mu_i \in [0,1]$ is the share of intermediate goods in production, and z_i is a sector-specific productivity shock representing all factors affecting productivity. The *input-output matrix* $\mathbf{A} = [a_{ij}]$ represents the intersectoral input-output linkages. Constant returns to scale imply $\sum_{j=1}^{N} a_{ij} = 1$ for all i.

Each firm's financial constraint takes the following stylized form, where $\chi_i \in [0,1]$ describes

[19] $\zeta_i = (1-\mu_i)^{-(1-\mu_i)} \prod_{j=1}^{N} (\mu_i a_{ij})^{-(\mu_i a_{ij})}$. This constant is introduced to simplify the expression. The results are not affected qualitatively by this constant.

the tightness of firm i's financial constraint.

$$l_i + \sum_{j=1}^{N} p_j x_{ij} \leq \chi_i p_i y_i,$$

where p_j is the price of industry j's good and the wage level is normalized to 1. Firms choose labor and intermediate goods to maximize profits subject to their financial constraint. The financial constraint is binding when $\chi_i < 1$.

The financial constraint inserts a wedge $\phi_i = min\{\chi_i, 1\}$ between firm i's total expenditure $u_i = l_i + \sum_{j=1}^{N} p_j x_{ij}$ and total revenue $g_i = p_i y_i$ as

$$u_i = \phi_i g_i.$$

Given prices and the real wage, the firm's optimal expenditures on labor and intermediate goods are proportional to its total expenditure as

$$p_j x_{ij} = \mu_i a_{ij} u_i \quad \text{and} \quad l_i = (1 - \mu_i) u_i.$$

Households. The representative household maximizes utility

$$u(c,l) = \frac{c^{1-\gamma}}{1-\gamma} - \frac{l^{1+\varepsilon}}{1+\varepsilon}$$

subject to

$$c = \prod_{i=1}^{N} c_i^{\theta_i}$$

and the budget constraint

$$\sum_{i=1}^{N} p_i c_i = l + \sum_{i=1}^{N} \pi_i,$$

where $\theta_i \in [0,1]$ denotes the consumption share of good i, normalized such that $\sum_{i=1}^{N} \theta_i = 1$, and π_i is profit from the firm in industry i.

Market Clearing. The market-clearing conditions for labor and good i are given by

$$\sum_{i=1}^{N} l = 1 \quad \text{and} \quad y_i = c_i + \sum_{j=1}^{N} x_{ji} + v_i,$$

where v_i is the demand shock. The demand shock may include exogenous changes in consumer preference, government purchases, or firms' demand for the intermediate good produced in industry i.[20]

Equilibrium. The competitive equilibrium is defined as a sequence of prices and quantities such that (i) firms maximize profits while taking the prices and the wage as given; (ii) the representative household maximizes her utility; and (iii) all markets clear.

4.2 The Network Structure and Propagation of Shocks

Now, I characterize the equilibrium GDP as a function of financial constraints and microeconomic shocks. Then, I investigate the relationship between the structure of the input-output network and the response of GDP to different types of shocks.[21]

Proposition 1 *Suppose an economy without demand shocks. Let ι denote a vector of ones. The log of equilibrium real GDP is defined as the following function of productivity shocks and financial frictions:*

$$\log GDP = \beta \boldsymbol{\theta}' \boldsymbol{L}(z + \log \phi) + \left(\beta - \frac{1}{\varepsilon + \gamma}\right) \log(1 + \iota'(\iota - \phi) \circ \boldsymbol{a}(\phi) \boldsymbol{\theta}) + \beta \boldsymbol{\theta}' \log \boldsymbol{\theta},$$

where $\beta = \frac{\varepsilon+1}{\varepsilon+\gamma}$, $\boldsymbol{L} = [\boldsymbol{I} - \boldsymbol{\mu}\boldsymbol{\iota}' \circ \boldsymbol{A}]^{-1}$, *and* $\boldsymbol{a}(\phi) = \left[\boldsymbol{I} - \boldsymbol{\theta}(\boldsymbol{\iota} - \phi)' - ((\boldsymbol{\mu} \circ \phi)\boldsymbol{\iota}' \circ \boldsymbol{A})'\right]^{-1}$.

[20] After identifying the sources of demand shocks, we can modify the household and firm maximization problems accordingly. If the demand shock represents domestic government purchases, we must include lump-sum taxes to satisfy the government budget constraint. These adjustments do not change how shocks propagate through the input-output network.

[21] The proofs of the following propositions and corollaries are given in Appendix A.4.

This proposition shows that the equilibrium GDP can be characterized by productivity shocks and financial frictions.[22] Note that the Leontief inverse matrix, L, represents all direct and indirect effects of shocks on GDP through the input-output network as $L = I + (\boldsymbol{\mu}\iota' \circ \mathbf{A}) + (\boldsymbol{\mu}\iota' \circ \mathbf{A})^2 + \cdots$. The following corollary shows how GDP responds to productivity shocks.

Corollary 1 *The response of log real GDP to productivity shocks is given by*

$$d \log GDP = \beta \boldsymbol{\theta}' \boldsymbol{L} \, dz$$

where the Domar weight is given by $\beta \boldsymbol{\theta}' \boldsymbol{L}$.

This corollary is a well-known property of a multisector model as indicated by Hulten (1978), Gabaix (2011), and Acemoglu et al. (2017) and shows that aggregate output is a linear combination of sectoral shocks weighted by shares of sectoral sales in GDP. This corollary shows that the Hulten's theorem holds in the absence of financial frictions. The Domar weight involves the column sum of the Leontief inverse matrix. This implies that productivity shocks have downstream effects on the customer industries but have no upstream impact on the supplier industries.

Corollary 1 implies that the variability of GDP is related to the structure of the input-output network. If an industry i is hit by a shock, this not only affects industry i but also affects other industries through the input-output network. The following proposition now explains how the variability of GDP depends on the structure of the input-output network.

Proposition 2 *(1) If the productivity shock, z_i, is i.i.d across sectors and from a common distribution with mean zero and finite variance σ_z, the standard deviation of log real GDP to a one-time productivity shock and its upper bound are given by*

$$\sigma_z \beta \sqrt{\sum_{i=1}^{n} \left(\sum_{j=1}^{n} \theta_j l_{ji} \right)^2} \le \sigma_z \beta \sum_{j=1}^{n} \theta_j \sqrt{\sum_{i=1}^{n} l_{ji}^2}.$$

[22] This result is a special case of Bigio and LaO (2016) when the production function has the property of constant returns to scale.

(2) Given the preference shares, θ_j, the upper bound is minimized when the elements in each row of the Leontief inverse are evenly distributed across industries.

The elements in each row of the Leontief inverse matrix represent flows of intermediate goods from upstream industries to each industry represented by the row. It is worth noting that the upper bound is analogous to the weighted average of the HHI. Given the inverse relationship between the HHI and the entropy-based centrality, this proposition implies that the GDP volatility decreases as the economic diversity increases. Therefore, Proposition 2 implies that the volatility of GDP in response to productivity shocks decreases as industries have widely and evenly distributed relationships with their upstream industries.

This result is related to Acemoglu et al. (2012), which show that aggregate volatility scales with the Euclidean norm of the Domar weights as the economy becomes more disaggregated. They show that the impact of shocks on aggregate output increases if each sector's degree centrality of an economy exhibits high variability.[23] Proposition 2 extends this result and suggests that not only the distribution of the size of sectors (measured by the degree centrality) but also the distribution of flows between upstream and downstream sectors plays an important role in determining the variability of aggregate output.

Proposition 3 *Suppose an economy without productivity shocks. $D_{\boldsymbol{x}}$ denotes a diagonal matrix with the vector $\boldsymbol{x}$ as the diagonal elements. The equilibrium nominal GDP is defined as the following function of demand shocks and financial frictions:*

$$GDP = \tilde{\eta}\tilde{\boldsymbol{\kappa}}'\tilde{\boldsymbol{L}}'\tilde{\boldsymbol{v}},$$

where $\tilde{\eta}$ is a constant defined as $\tilde{\eta} = \frac{1}{1-\iota'(D_{(\iota-\boldsymbol{\mu})}+D_{\boldsymbol{\phi}}^{-1}-\boldsymbol{I})\tilde{\boldsymbol{L}}'\boldsymbol{\theta}}$, $\tilde{\boldsymbol{\kappa}}'$ is a row vector defined as $\tilde{\boldsymbol{\kappa}}' = \iota'(D_{(\iota-\boldsymbol{\mu})}+D_{\boldsymbol{\phi}}^{-1}-\boldsymbol{I})$, $\tilde{\boldsymbol{L}}' = \left[D_{\boldsymbol{\phi}}^{-1} - (\boldsymbol{\mu}\iota' \circ \boldsymbol{A})'\right]^{-1}$, and $\tilde{\boldsymbol{v}}$ is given in Appendix A.4.

This proposition shows that demand shocks propagate through the input-output network, but the mechanism is different from productivity shocks as the transpose of the input-output matrix is used

[23] The variability is measured by the coefficient of variation of the degree centrality.

in the (pseudo) Leontief inverse matrix $\tilde{\mathbf{L}}'$. This implies that demand shocks only have upstream effects on the supplier industries with no downstream effects on the customer industries, as indicated by Acemoglu et al. (2015). If there are no financial frictions, the equation can be simplified further as shown in the following corollary.

Corollary 2 *When there are no financial frictions ($D_\phi = \boldsymbol{I}$), the response of nominal GDP to nominal demand shocks is given by*

$$dGDP = \eta \boldsymbol{\kappa}' \boldsymbol{L}' \, d\tilde{\boldsymbol{v}},$$

where $\eta = \frac{1}{1-(\iota-\boldsymbol{\mu})'\boldsymbol{L}'\boldsymbol{\theta}}$, $\boldsymbol{\kappa}' = (\iota - \boldsymbol{\mu})'$, *and* $\boldsymbol{L}' = [\boldsymbol{I} - (\boldsymbol{\mu}\iota' \circ \boldsymbol{A})']^{-1}$.

The following proposition explains how the variability of aggregate output due to demand shocks depends on the structure of the input-output network.

Proposition 4 *(1) If the demand shock, v_i, is i.i.d across sectors and from a common distribution with mean zero and finite variance σ_v, the standard deviation of the response of nominal GDP to a one-time demand shock and its upper bound are given by*

$$\sigma_v \eta \sqrt{\sum_{i=1}^{n} \left(\sum_{j=1}^{n} \kappa_j l_{ij} \right)^2} \leq \sigma_v \eta \sum_{j=1}^{n} \kappa_j \sqrt{\sum_{i=1}^{n} l_{ij}^2}.$$

(2) Given the labor shares, $1 - \mu_j$ ($= \kappa_j$), the upper bound is minimized when the elements in each column of the Leontief inverse are evenly distributed across industries.

The elements in each column of the Leontief inverse matrix represent flows of intermediate goods to downstream industries from each industry represented by the column. Therefore, this proposition implies that the volatility of GDP due to demand shocks decreases as industries have widely and evenly distributed relationships with their downstream industries.

Now, I investigate the relationship between the response of GDP to shocks of financial tightening and the structure of the input-output network. The following corollary is an extension of Proposition 1 and illustrates how GDP responds to the tightening of the financial constraint.

Corollary 3 *Suppose an economy without supply or demand shocks. The response of log real GDP to the tightening of the financial constraint is given by*

$$d\log GDP = \beta\boldsymbol{\theta}'\boldsymbol{L}\, d\log\phi + \left(\beta - \frac{1}{\varepsilon+\gamma}\right) d\log(1+\iota'(\iota-\phi)\circ\boldsymbol{a}(\phi)\boldsymbol{\theta}). \tag{8}$$

The first term on the right side of the equation shows that the shocks of financial tightening propagate the economy through the input-output network similarly to productivity shocks. The second term represents the nonlinear effect of financial tightening shocks on GDP.[24] Because of this nonlinear effect, it is impossible to show analytically the relationship between the structure of the input-output network and the volatility of GDP.

Thus, I conduct a Monte Carlo simulation to investigate the response of GDP to the financial tightening shock using equation (8). I set $\gamma = 0$ to shut down the wealth effect and set $\varepsilon = 0.5$, which means a Frisch elasticity of labor supply of 2 following Hall (2009).[25] The share of labor in the production function is set to 0.4 ($\mu = 0.6$) for all sectors. I simulate 1,000 input-output matrices with different network structures by drawing the elements of **A** from the lognormal distribution with mean 0 and standard deviation 1.2. The number of sectors is set to 50. The initial value of ϕ is equal to 1, which means no financial frictions for all sectors. I simulate 5,000 series of financial shocks for every input-output network from a uniform distribution ranging from 0.5 to 0.9. Then, I calculate the standard deviation of log real GDP using equation (8). Figure 5 shows the relationship between the structure of the input-output network and the standard deviation of log real GDP. The x-axis represents the degree of diversity of input-output flows as defined in Section 2. This measure increases as sectors have widely and evenly distributed flows with other sectors. Panels (A) and (B) show the standard deviation of the first (linear) and the second (nonlinear) part of equation (8), respectively. As expected, the linear part works similarly to productivity shocks such that the standard deviation of GDP decreases as the upstream diversity measure increases, i.e., industries have widely and evenly distributed relationships with upstream industries. On the other

[24] Note that this non-linear effect is different from the higher-order terms in Baqaee and Farhi (2019).

[25] The simulation result does not change qualitatively with greater values of gamma or lower labor supply elasticities.

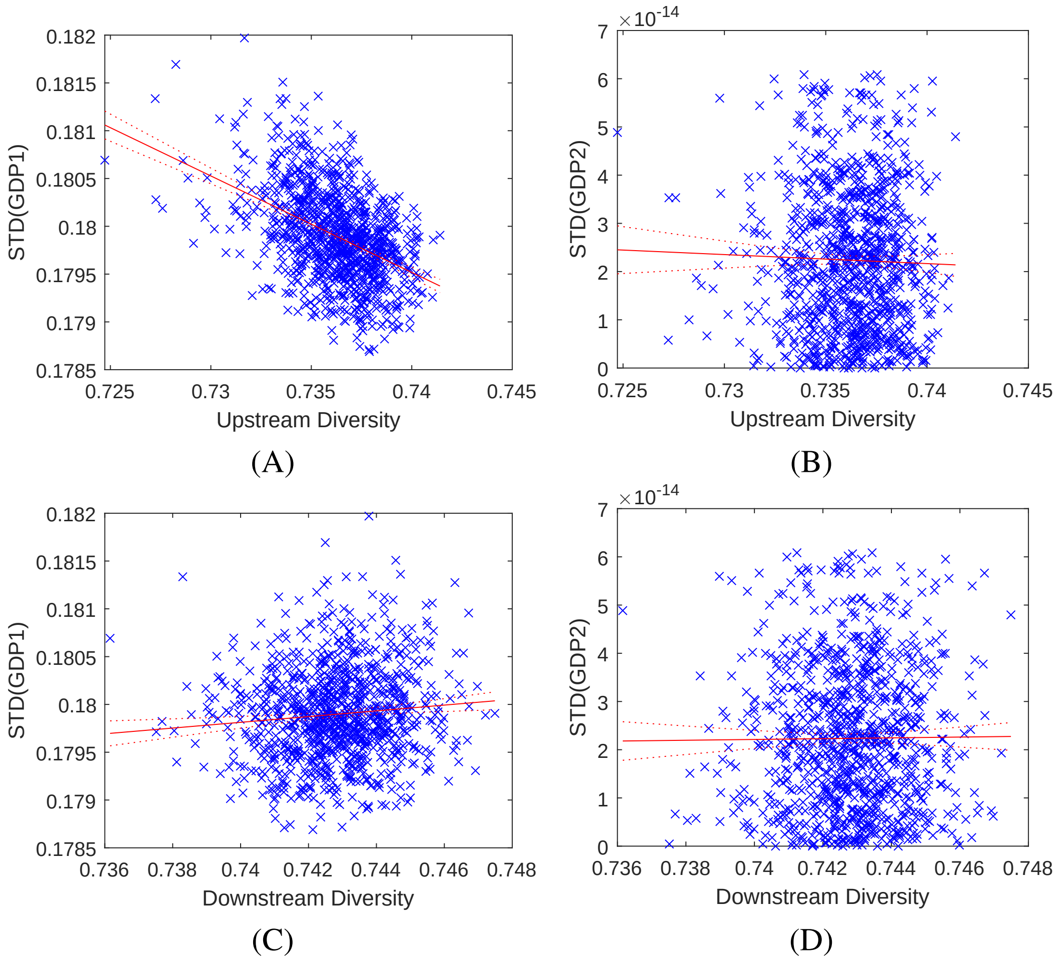


Figure 5: Response of GDP to financial shocks

Note. This figure shows the Monte Carlo simulation result. The y-axis represents the standard deviation of log GDP when financial shocks hit the economy: panels (A) and (B) for the relationship with the upstream diversity and panels (C) and (D) for the relationship with the downstream diversity. GDP1 and GDP2 represent the first and the second part of equation (8), respectively. The linear fit line and 95% confidence intervals are included.

hand, the nonlinear part does not show any systematic relationship with the upstream diversity. Regarding the downstream diversity (panels (C) and (D)), both parts do not show any systematic relationship. The following proposition summarizes the relationship between the network structure and the standard deviation of the response of GDP when financial shocks hit the economy.

Proposition 5 *(1) The standard deviation of log real GDP to the financial tightening shock decreases as industries have widely and evenly distributed relationships with their upstream industries. (2) The distribution of downstream flows does not affect the volatility of log real GDP due to* *the financial tightening shock.*

4.3 Propagation of Shocks in a multi-country setting

In the previous section, I investigated the role of the domestic input-output structure in propagation of shocks within a country. I now explore how the relationship changes when we have multiple countries.

Let's assume that the total number of countries is K and the number of sectors in each country is N. Then, the total number of sectors in the global economy becomes KN. The first N industries from 1 to N for the first country, the next N industries from $N+1$ to $2N$ for the second country, and so on. If we assume a representative firm in each industry $i \in \mathfrak{K} = \{1 \,..\, KN\}$, the input-output matrix $\mathbf{A}$ becomes a matrix of size $KN \times KN$. The Leontief matrix is also a matrix of size $KN \times KN$. In this economy firms can export and import intermediate inputs to produce the final output. Therefore, the production function is given by

$$y_i = e^{z_i} \zeta_i l_i^{1-\mu_i} \left(\prod_{j=1}^{KN} x_{ij}^{a_{ij}} \right)^{\mu_i}.$$

Similarly, all other equations regarding firms can be used after changing the number of sectors from N to KN.

I assume the representative household for each country. The representative household allocates consumption into N sectors within its country based on θ_i (normalize to $\sum_{i \in k} \theta_i = 1 \; \forall k \in \mathfrak{K}$).[26] To simplify the problem, I assume that the representative household supplies one unit of labor inelastically. Then, the aggregate consumption and the budget constraint for the k-th country are given by

$$c_k = \prod_{i \in k} c_i^{\theta_i}$$

[26] Following Johnson (2014) I assume that there is no international trade for final goods.

and

$$\sum_{i \in k} p_i c_i = l_k + \sum_{i \in k} \pi_i.$$

where $l_k = 1$. With these changes, I can derive the response of GDP to various types of shocks as the following propositions.[27]

Proposition 6 *With international trades, the responses of log real GDP to productivity shocks for the k-th country is given by*

$$d \log \boldsymbol{GDP}_k = \hat{\boldsymbol{\theta}}_k \boldsymbol{L} \, d\boldsymbol{z},$$

where $\hat{\boldsymbol{\theta}}_k$ is the k-th row of the $K \times KN$ matrix $\hat{\boldsymbol{\theta}}$ which is given in Appendix A.4.

This proposition corresponds to Corollary 1 and illustrates that the results with the domestic economy apply to the economy with international trades. First, productivity shocks have downstream effects on the customer industries but have no upstream effects on the supplier industries. Second, the volatility of GDP in response to productivity shocks decreases as industries have widely and evenly distributed relationships with their upstream industries in the global economy.

Proposition 7 *With international trades, if there are no financial frictions, the responses of nominal GDP to nominal demand shocks for the k-th country are given by*

$$d\boldsymbol{GDP}_k = \hat{\boldsymbol{\eta}}_k \hat{\boldsymbol{\kappa}} \tilde{\boldsymbol{L}}' \, d\tilde{\boldsymbol{v}},$$

where $\hat{\boldsymbol{\eta}}_k$ is the k-th row of $\hat{\boldsymbol{\eta}} \equiv \left[I_K - \hat{\iota} D_{(\iota-\mu)} \tilde{\boldsymbol{L}}' \tilde{\boldsymbol{\theta}} \right]^{-1}$, $\hat{\boldsymbol{\kappa}} \equiv \hat{\iota} D_{(\iota-\mu)}$, $\hat{\iota}$ is a matrix of size $K \times KN$ such that $\hat{\iota} \equiv I_K \otimes \iota_N'$ with $\otimes$ denoting the Kronecker delta, and $\tilde{\boldsymbol{\theta}}$ is a matrix of size $KN \times K$, which is given in Appendix A.4.

This proposition corresponds to Corollary 2 and demonstrates that the previous results regarding demand shocks apply to the economy with international trades. First, demand shocks only have

[27] In this simplified model for international trades, the wage levels and exchange rates are normalized to one for all countries. The critical point in this setup is that foreign sectors are not directly affected by nationwide domestic shocks. Foreign countries without financial crises are indirectly affected through the global input-output network.

upstream effects on the supplier industries with no downstream effects on the customer industries. Second, the volatility of GDP due to demand shocks decreases as industries have widely and evenly distributed relationships with their downstream industries in the global economy.

Proposition 8 *When there are no supply or demand shocks, the response of log real GDP to the tightening of the financial constraint is given by*

$$d\log \boldsymbol{GDP}_k = \hat{\boldsymbol{\theta}}_k \boldsymbol{L}\, d\log \boldsymbol{\phi} + \, d\log(1 + \widehat{(\boldsymbol{\iota} - \boldsymbol{\phi})}_k \tilde{\boldsymbol{a}}(\boldsymbol{\phi})\boldsymbol{\theta}),$$

where $\tilde{\boldsymbol{a}} \equiv \left[\boldsymbol{I}_{KN} - \tilde{\boldsymbol{\theta}}\widehat{(\boldsymbol{\iota} - \boldsymbol{\phi})} - ((\boldsymbol{\mu} \circ \boldsymbol{\phi})\boldsymbol{\iota}' \circ \boldsymbol{A})'\right]^{-1}$*, and* $\widehat{(\boldsymbol{\iota} - \boldsymbol{\phi})}_k$ *is the k-th row of the* $K \times KN$ *matrix* $\widehat{(\boldsymbol{\iota} - \boldsymbol{\phi})}$ *which is given in Appendix A.4.*

This proposition corresponds to Corollary 3. Again, the results for the closed economy apply to the open economy with international trade. The first term on the right side shows that the financial tightening shock propagates to downstream industries similar to productivity shocks. The second term represents the nonlinear effect of the financial tightening shock on GDP. A simulation can show that the volatility of GDP decreases as industries have widely and evenly distributed relationships with their upstream industries in the global economy.

4.4 Discussion

Until now, we have found similarities between the closed economy and the open economy in the way how shocks propagate through the input-output network. However, there is an important difference in the nature of shocks between the closed and open economies. When we assume a closed economy, most sectors would face large negative shocks if a country experiences a financial crisis. In contrast, when we consider the global input-output network, only a small portion of countries and sectors experience large negative shocks in the global economy unless all countries are simultaneously confronted by financial crises. For example, if we assume 100 countries included in the input-output network and two countries experience financial crises, only 2% of sectors fall under the direct influence of the crisis. This implies that a network benefits from diversification by reducing an industry's dependence on a specific counterpart when we consider international trades.[28] This also implies that diversification in the global market weakens the relationship between the structure of the global input-output network and volatility of GDP when a country experiences nationwide shocks.

Another important benefit of international trade in time of financial crises is that participation in the global market improves firms' financial condition. Greenaway et al. (2007) investigate UK manufacturing firms and find strong evidence that participation in export markets improves firms' ex-post financial health.[29] In addition, Myles Shaver (2011) examines Spanish manufacturers and finds that geographic sales diversification (i.e., exporting) mitigates investment liquidity constraints. He focuses on geographic sales diversification rather than product diversification and argues that firms can relax financial constraints by geographically diversifying sales because exporting signals more stable expected cash flows and firm quality. These studies indicate that firms with more diversified downstream customers in the global market can mitigate the negative impact of the financial crisis. In other words, the financial constraint χ is a function of downstream diversification, $\chi = \chi(\Gamma)$, where Γ represents downstream diversification such that $\chi' < 0$.

Therefore, I can summarize the theoretical explanations for the empirical relationship between the volatility of GDP and the network structure as follows:

(Domestic network, Upstream) Countries having a domestic network with more diversified flows from upstream suppliers will experience less volatility of GDP when nationwide productivity shocks or financial shocks hit the economy.

(Domestic network, Downstream) Countries having a domestic network with more diversified flows to downstream customers will experience less volatility of GDP when nationwide demand shocks hit the economy.

[28] Elliott et al. (2014) indicate the trade-off between integration and diversification in terms of financial contagion. Integration means greater dependence on counterparties, and diversification means more counterparties per organization.

[29] Greenaway et al. (2007) find no evidence that firms having better ex-ante financial health are more likely to export.

(Global network, Upstream) Having more diversified flows from upstream suppliers of inputs in the global network will not significantly affect the volatility of GDP when nationwide productivity shocks or financial shocks hit the economy.

(Global network, Downstream) Having more diversified flows to downstream customers in the global network will reduce the volatility of GDP when nationwide financial shocks hit the economy.

I obtain the first and second explanations directly from propositions 1, 2, and 3. The third explanation is based on the fact that global diversification weakens the relationship between the network structure and the volatility of GDP when nationwide shocks hit an economy. The fourth explanation comes from the fact that exporting to globally diversified customers improves firms' financial health.

5 Conclusion

This paper constructs a measure of economic diversity based on literature and the theoretical model, using the entropy-based measure of centrality. The degree of diversity increases when a country's major industries, in terms of the output share, have widely and evenly distributed input-output relationships with other industries.

The empirical evidence shows that the structure of the input-output network, captured by the diversity measure, plays a key role in explaining the substantial variation in the aftermath of financial crises. Specifically, the aftermath of financial crises is much worse when a country has low diversity than when it has high diversity. In other words, an economy with major industries connected to many other industries is more resilient to financial distress. Both downstream and upstream relationships are important factors in reducing the pains of financial crises, athough global upstream diversity does not serve as insurance against nationwide shocks.

This paper develops a dynamic multisector model with input-output networks and shows that the volatility of GDP depends on the distribution of flows through industries. When productivity shocks or financial shocks hit the economy, aggregate volatility declines if an economy has widely and evenly distributed upstream flows. When demand shocks hit the economy, economies with widely and equally distributed downstream flows experience smaller aggregate volatility. We also need to consider the effect of global diversification to understand the empirical facts regarding international input-output linkages.

The information about the structure of the input-output network can be useful for policymakers. The obvious policy implication of this research is that industrial policies should aim at encouraging a more diversified economy, in order to increase the economic resilience to financial crises. By targeting industries based on their importance and positions in the network structure, the policy impact on the economy's ability to endure financial crises would be even stronger and more effective. In other words, given the limited amount of budget, policymakers can provide some industries with delicate support to strengthen the economic resilience to financial crises.

A limitation of this research is that a fixed number of industries are used to analyze the structure of input-output networks. The results might be different if we used a more segregated industry definition. This is because flows become distributed more evenly across industries as the number of nodes decreases. When we increase the number of industries sufficiently large, a different structure can emerge. This remains an interesting topic for future research.

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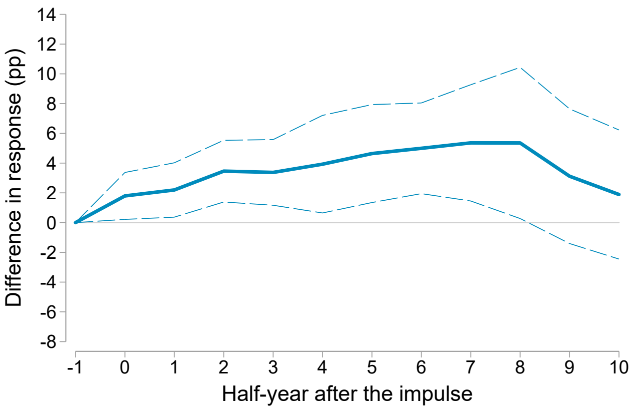
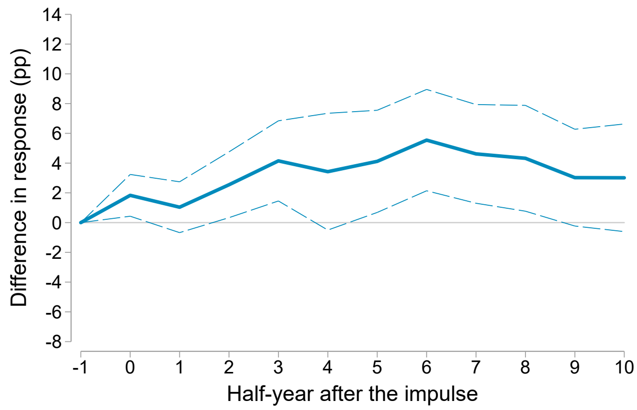
A Appendix

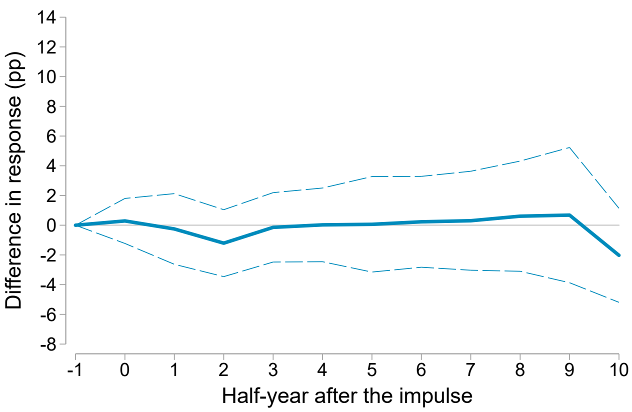
A.1 Figures and tables



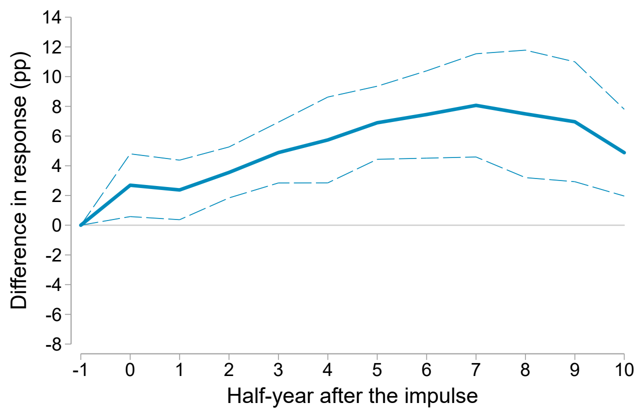
(A) Domestic network, upstream diversity



(B) Domestic network, downstream diversity



(C) Global network, upstream diversity



(D) Global network, downstream diversity

Figure A1: Driscoll-Kraay standard errors

Note. This figure shows the difference in impulse-responses of log real GDP to financial distress shocks between high- and low-diversity countries. The 95% confidence intervals are added to each line.

Table A1: The average entropy of industries

Industry	Domestic network				Global network			
	Downstream		Upstream		Downstream		Upstream	
	Rank	Mean	Rank	Mean	Rank	Mean	Rank	Mean
Wholesale and retail trade; repair of motor vehicles	1	0.91	33	0.50	1	0.66	32	0.32
Professional, scientific and technical activities	2	0.87	40	0.45	2	0.60	40	0.30
Administrative and support services	3	0.82	38	0.48	3	0.58	36	0.31
Financial and insurance activities	4	0.79	44	0.37	4	0.56	42	0.25
Land transport and transport via pipelines	5	0.72	23	0.54	6	0.52	24	0.36
Real estate activities	6	0.71	43	0.37	12	0.46	44	0.22
Electricity, gas, steam and air conditioning supply	7	0.68	37	0.48	13	0.45	37	0.31
Warehousing and support activities for transportation	8	0.65	26	0.53	8	0.48	28	0.35
Chemical and chemical products	9	0.62	18	0.56	5	0.52	9	0.43
IT and other information services	10	0.62	41	0.44	9	0.47	38	0.31
Coke and refined petroleum products	11	0.62	21	0.55	10	0.46	20	0.37
Construction	12	0.60	14	0.57	19	0.38	19	0.38
Fabricated metal products	13	0.59	13	0.57	14	0.45	10	0.42
Basic metals	14	0.58	12	0.58	7	0.51	5	0.44
Mining and quarrying, energy producing products	15	0.56	36	0.49	32	0.27	29	0.33
Machinery and equipment, nec	16	0.54	10	0.58	11	0.46	6	0.44
Manufacturing nec; repair and installation of machinery and equipment	17	0.54	2	0.61	15	0.40	8	0.43
Rubber and plastics products	18	0.50	4	0.60	16	0.39	4	0.44
Food products, beverages and tobacco	19	0.49	7	0.59	18	0.39	17	0.40
Accommodation and food service activities	20	0.46	16	0.56	28	0.30	26	0.36
Agriculture, hunting, forestry	21	0.46	24	0.54	23	0.33	22	0.37
Publishing, audiovisual and broadcasting activities	22	0.45	25	0.53	20	0.36	23	0.36
Paper products and printing	23	0.45	15	0.57	17	0.39	16	0.41
Water supply; sewerage, waste management and remediation activities	24	0.44	31	0.51	29	0.30	35	0.32
Telecommunications	25	0.44	32	0.50	24	0.33	30	0.33
Public administration and defence; compulsory social security	26	0.44	34	0.49	30	0.28	39	0.30
Computer, electronic and optical equipment	27	0.42	17	0.56	22	0.35	7	0.43
Electrical equipment	28	0.41	3	0.61	21	0.35	1	0.46
Other non-metallic mineral products	29	0.36	6	0.60	31	0.28	14	0.41
Air transport	30	0.35	1	0.61	27	0.31	11	0.42
Mining and quarrying, non-energy producing products	31	0.34	27	0.52	37	0.22	27	0.35
Motor vehicles, trailers and semi-trailers	32	0.34	8	0.59	25	0.31	2	0.46
Water transport	33	0.33	5	0.60	26	0.31	12	0.42
Other service activities	34	0.31	30	0.51	38	0.20	34	0.32
Postal and courier activities	35	0.31	35	0.49	34	0.24	31	0.33
Wood and products of wood and cork	36	0.29	9	0.58	35	0.24	15	0.41
Other transport equipment	37	0.27	11	0.58	33	0.26	3	0.44
Education	38	0.27	42	0.38	40	0.18	43	0.23
Pharmaceuticals, medicinal chemical and botanical products	39	0.24	20	0.55	36	0.23	18	0.39
Textiles, textile products, leather and footwear	40	0.22	19	0.55	39	0.18	13	0.41
Arts, entertainment and recreation	41	0.22	29	0.52	41	0.16	33	0.32
Human health and social work activities	42	0.17	39	0.46	43	0.11	41	0.29
Mining support service activities	43	0.16	28	0.52	42	0.11	25	0.36
Fishing and aquaculture	44	0.09	22	0.54	44	0.07	21	0.37

Note. This table shows the list of industries with their average entropy and rank across 24 advanced OECD countries in 2018. The industries are sorted by the domestic-downstream rank.

Table A2: The weighted entropy of industries

Industry	Domestic network				Global network			
	Downstream		Upstream		Downstream		Upstream	
	Rank	Mean	Rank	Mean	Rank	Mean	Rank	Mean
Wholesale and retail trade; repair of motor vehicles	1	8.97	1	4.86	1	6.55	1	3.17
Real estate activities	2	5.47	3	2.81	2	3.51	4	1.67
Construction	3	4.81	2	4.45	4	2.96	2	2.97
Professional, scientific and technical activities	4	4.73	6	2.40	3	3.35	6	1.59
Financial and insurance activities	5	4.02	8	1.86	5	2.93	8	1.26
Administrative and support services	6	2.97	9	1.69	6	2.13	9	1.11
Public administration and defence; compulsory social security	7	2.18	7	2.34	8	1.47	7	1.43
Food products, beverages and tobacco	8	2.11	5	2.46	7	1.59	3	1.68
Electricity, gas, steam and air conditioning supply	9	1.82	12	1.21	11	1.19	16	0.78
Land transport and transport via pipelines	10	1.71	11	1.24	10	1.25	14	0.81
IT and other information services	11	1.67	16	1.13	9	1.36	12	0.83
Accommodation and food service activities	12	1.37	10	1.63	16	0.90	10	1.04
Machinery and equipment, nec	13	1.19	14	1.17	12	1.15	11	0.88
Agriculture, hunting, forestry	14	1.15	13	1.18	17	0.79	15	0.80
Basic metals	15	1.06	18	0.96	14	0.98	17	0.75
Warehousing and support activities for transportation	16	1.04	20	0.83	18	0.78	23	0.54
Coke and refined petroleum products	17	0.98	19	0.84	19	0.77	22	0.57
Chemical and chemical products	18	0.96	21	0.83	15	0.91	19	0.65
Human health and social work activities	19	0.94	4	2.55	21	0.64	5	1.61
Fabricated metal products	20	0.84	23	0.80	20	0.69	20	0.59
Education	21	0.83	15	1.16	26	0.57	18	0.69
Motor vehicles, trailers and semi-trailers	22	0.80	17	1.05	13	1.01	13	0.83
Manufacturing nec; repair and installation of machinery and equipment	23	0.80	22	0.82	24	0.61	21	0.59
Mining and quarrying, energy producing products	24	0.71	37	0.40	22	0.62	38	0.25
Publishing, audiovisual and broadcasting activities	25	0.69	26	0.66	27	0.56	26	0.46
Pharmaceuticals, medicinal chemical and botanical products	26	0.65	24	0.71	23	0.62	24	0.54
Telecommunications	27	0.58	27	0.64	29	0.45	28	0.42
Computer, electronic and optical equipment	28	0.57	25	0.69	25	0.60	25	0.53
Water transport	29	0.49	31	0.54	28	0.45	27	0.43
Paper products and printing	30	0.47	30	0.55	30	0.43	31	0.39
Water supply; sewerage, waste management and remediation activities	31	0.43	35	0.46	33	0.29	35	0.29
Rubber and plastics products	32	0.40	34	0.47	32	0.36	34	0.34
Other service activities	33	0.39	28	0.63	34	0.25	30	0.39
Electrical equipment	34	0.39	32	0.52	31	0.40	29	0.40
Air transport	35	0.38	33	0.50	35	0.24	33	0.36
Other non-metallic mineral products	36	0.26	36	0.41	38	0.21	36	0.28
Arts, entertainment and recreation	37	0.25	29	0.59	39	0.19	32	0.37
Mining and quarrying, non-energy producing products	38	0.20	41	0.21	40	0.17	42	0.13
Textiles, textile products, leather and footwear	39	0.19	40	0.32	37	0.21	39	0.23
Wood and products of wood and cork	40	0.19	39	0.33	41	0.16	40	0.23
Other transport equipment	41	0.16	38	0.36	36	0.22	37	0.28
Postal and courier activities	42	0.12	43	0.18	42	0.10	43	0.12
Fishing and aquaculture	43	0.09	42	0.20	43	0.06	41	0.14
Mining support service activities	44	0.06	44	0.08	44	0.05	44	0.05

Note. This table shows the list of industries with their average entropy weighted by their output shares and rank across 24 advanced OECD countries in 2018. The industries are sorted by the domestic-downstream rank.

A.2 Robustness Check

A.2.1 The Alternative Diversity Measure using the HHI

Gabaix (2011) showed that the HHI of sales shares is the sufficient statistic for aggregate volatility. I construct a measure of a country's economic diversity by taking the weighted average of sectoral Herfindahl-Hirschman Index (HHI), using output shares as weights.

Definition 4 *The downstream and upstream diversity of sector i is given by*

$$C_{down}(i) = \left(\sum_{i=1}^{n} p_{ji}^2\right)^{1/2} \quad and \quad C_{up}(i) = \left(\sum_{i=1}^{n} p_{ij}^2\right)^{1/2},$$

where $p_{ji} = \frac{l_{ji}}{\sum_{j=1}^{n} l_{ji}}$, $p_{ij} = \frac{l_{ij}}{\sum_{j=1}^{n} l_{ij}}$, *and* l_{ji} *represents elements of the Leontief inverse matrix. Then, a measure of a country' economic diversity is defined as*

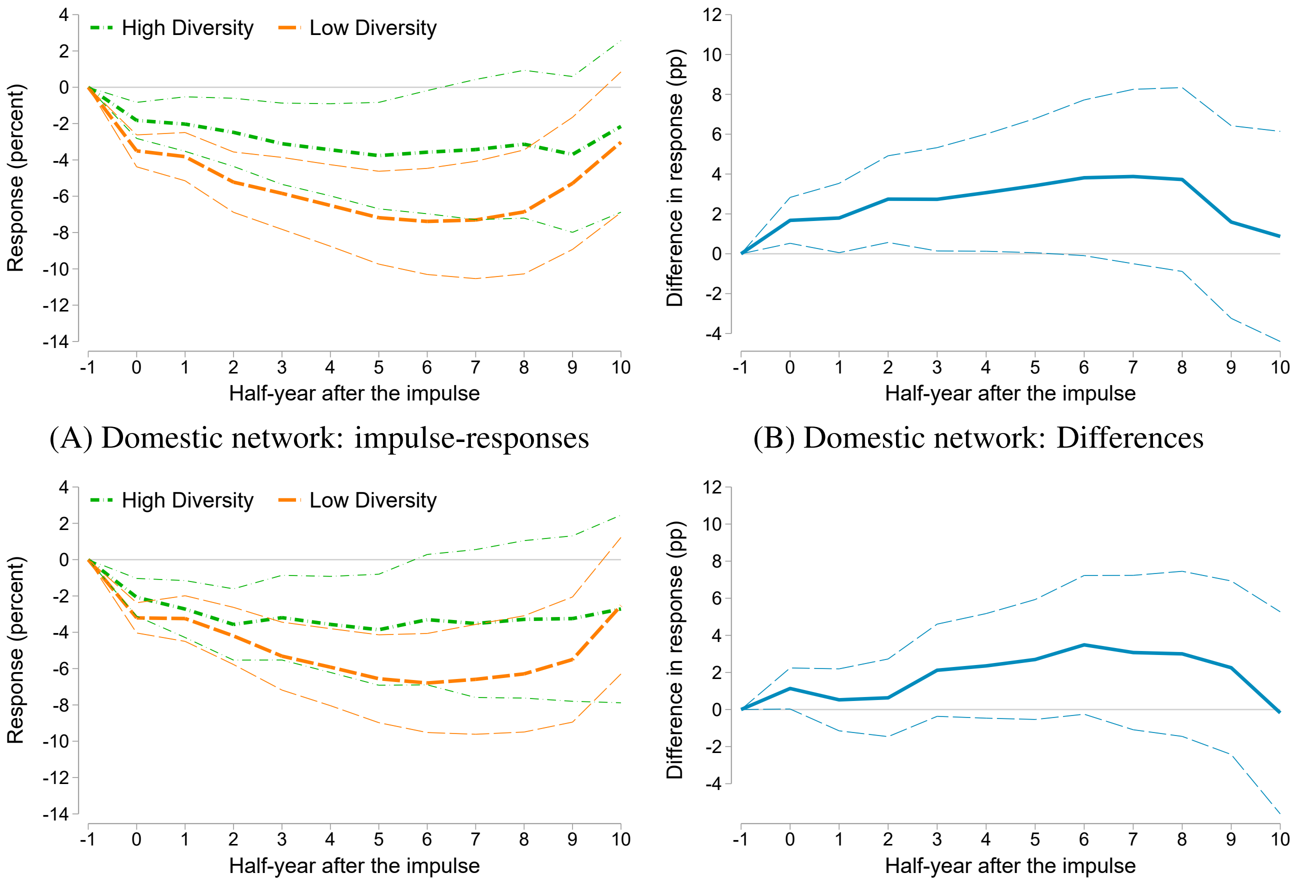
$$D_H = \sum_{i=1}^{n} C_H'(i) w(i) \tag{9}$$

where $w(i) = y(i)/\sum_{i=1}^{n} y(i)$ *is the output share of industry i, and* $H \in \{down, up\}$.

High diversity D_H means that the nation's major industries, in terms of output shares, have equally strong relationships with all other industries. Therefore, this country's industries are well diversified.

Panels (A) and (C) of Figure A2 illustrate the impulse response of log real GDP to financial distress shocks for the domestic and global networks, respectively. The figure shows that financial crises have a greater impact on low-diversity countries. The difference is marginally statistically significant and disappears within five years for the domestic network, as shown in panel (B). However, the difference is not statistically significant for the global network, as shown in panel (D).

Figure A3 shows the impulse response of log real GDP to financial distress shocks and the associated differences with respect to the downstream relationship. The effect of diversified output customers in the domestic market is comparable to that observed in the upstream cases. However, downstream diversity in the global market plays a much more important role in mitigating the adverse effects of financial crises. The maximum difference between high and low diversity countries



(A) Domestic network: impulse-responses

(B) Domestic network: Differences

(C) Global network: impulse-responses

(D) Global network: Differences

Figure A2: The role of upstream diversity

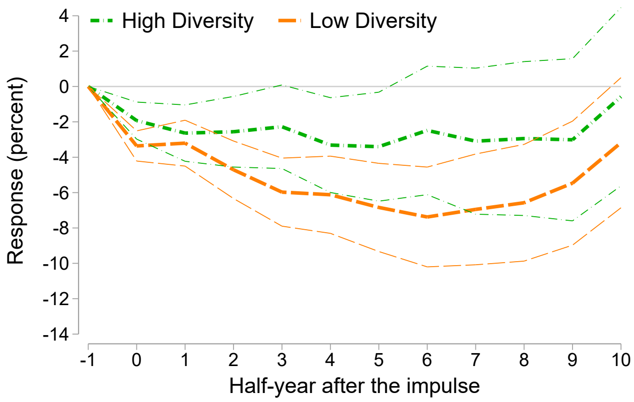
Note. Panels (A) and (C) show the impulse response function of log real GDP to financial distress shocks. Panels (B) and (D) show the differences between high and low diversity countries. The 95% confidence intervals are added.

reaches about 8 percentage points in the seventh half year, and the difference remains significant up to five years.

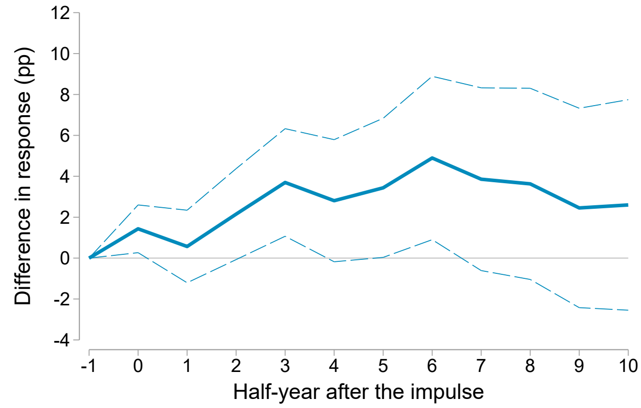
In summary, the alternative diversity measure based on the HHI produces results similar to those of the main analysis.

A.2.2 The Role of Industrial Output Distribution

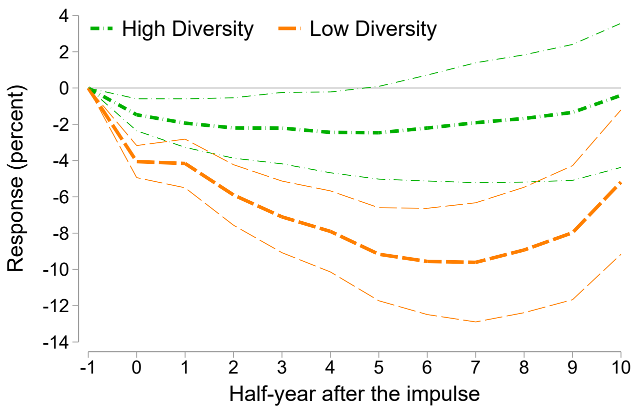
The measure of economic diversity given by equation 9 is calculated from the multiplication of each industry's output share and entropy. The output share and entropy of each industry are not uniformly distributed within a country. Then, what component is driving the result? The results could be mainly driven by the skewed distribution of outputs across industries rather than



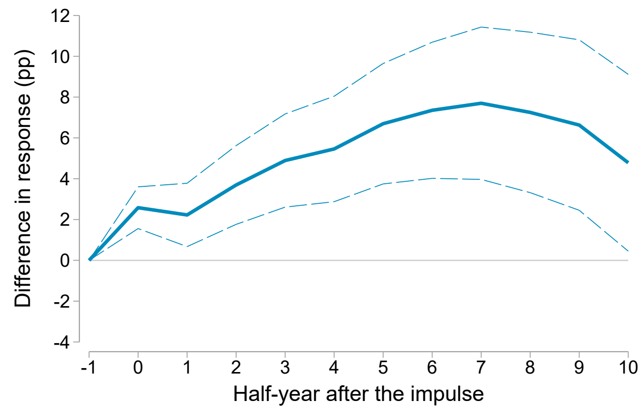
(A) Domestic network: Impulse-responses



(B) Domestic network: Differences



(C) Global network: Impulse-responses



(D) Global network: Differences

Figure A3: The role of downstream diversity

Note. Panels (A) and (C) show the impulse response function of log real GDP to financial distress shocks. Panels (B) and (D) show the differences between high and low diversity countries. The 95% confidence intervals are added.

by the distribution of entropy. To answer this question, I construct the diversity measure with equal weight across industries for each country. Thus, the alternative diversity measure is an unweighted average of each industry's entropy.

Table A3 shows the point estimates of the differences in the impulse-response function of log real GDP between high- and low-diversity countries at the 6th or 7th half-year period when the differences reach their maximum. The third column shows the baseline results from the original diversity measures. The fourth column shows the results with the alternative diversity measures. Weighting does not change the main findings qualitatively, and diversity is still an important factor in explaining the variation in the aftermath of financial crises. However, the weighting process

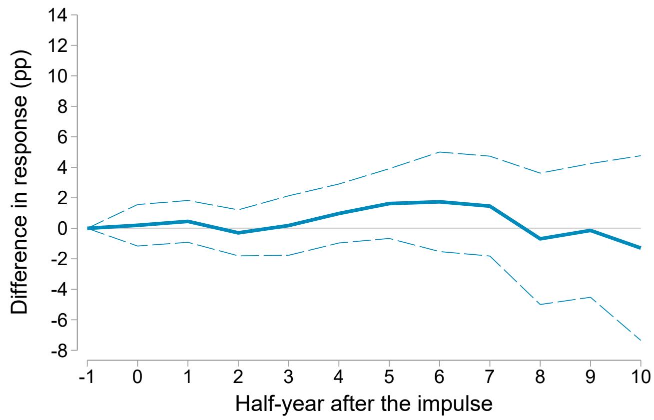


Figure A4: The role of the coefficient of variation of out-degree

Note. This figure shows the difference in impulse responses of log real GDP to financial distress shocks between two groups of countries when the coefficient of variation of the degree centrality is used. The 95% confidence intervals, using the Driscoll-Kraay standard errors, are added to each line.

moderately reduces domestic diversity's importance while strengthening the importance of global downstream diversity. This is because major industries with large output shares, such as *Wholesale*, have dispersed domestic counterparts similarly in any country. However, there is a large difference across countries about whether major countries have widely dispersed counterparts in the global market or not.

Additionally, I check the aftermath of financial crises after grouping countries based on the coefficient of variation of out-degree, following Acemoglu et al. (2012). Theorem 2 in Acemoglu et al. (2012) says that the variability of aggregate output becomes high when the coefficient of variation of the degree centrality is high. This is because the volatility of aggregate output decays

Table A3: Estimates of differences with equal weights

		Weighted	Unweighted
Domestic	Upstream	5.36 (2.48)	6.88 (1.88)
	Downstream	5.55 (1.99)	7.38 (1.88)
Global	Upstream	0.23 (1.79)	0.64 (1.85)
	Downstream	8.07 (1.92)	7.00 (1.76)

Note. This table shows the point estimates of the differences in impulse responses between high- and low-diversity countries at the peak. The units are percentage points. The OLS standard errors are in parentheses.

at a slower rate when a small fraction of industries have large outputs, even if the law of large numbers holds. Figure A4 shows that the industrial output distribution does not make a critical difference in impulse responses.[30] This is consistent with the limited role of output weights as shown above but seems inconsistent with the prediction of Acemoglu et al. (2012). However, this paper focuses on the aftermath of nationwide shocks, such as financial crises, while Acemoglu et al. (2012) describe the condition under which sector-specific shocks lead to aggregate volatility. This result implies that the diversity measure developed in this paper is better in capturing the structural properties of networks that help reduce the adverse impact of a financial crisis.

A.2.3 The Role of Variation in the Diversity Measure

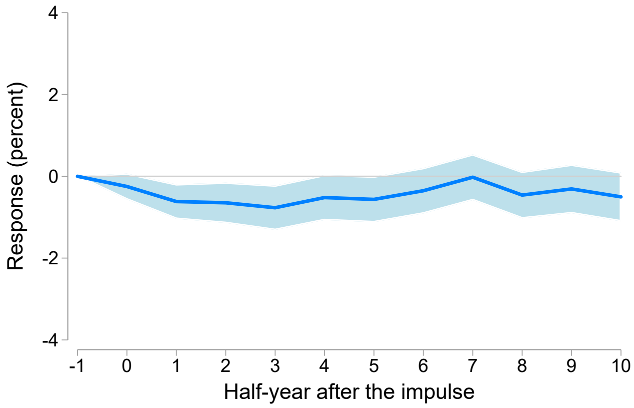
In equation (7), the diversity measure is lagged a year to avoid the endogeneity problem. However, the structures of input-output networks could be correlated with other factors, even *ex-ante*, affecting output in the wake of financial crises. If this is true, the results could be contaminated by these factors.

A simple way to check whether the diversity measures are really affected by other factors is to look at the impulse-response functions of the diversity measures to financial shocks. If there are some factors that affect both GDP and the network structures, the diversity measures will exhibit a systematic behavior in the wake of financial crises. To test this, I use the diversity measure as a response variable, instead of GDP, in equation (6). I also multiply the responses by 7, which is the value of the financial distress measure corresponding to the start of the moderate crisis.

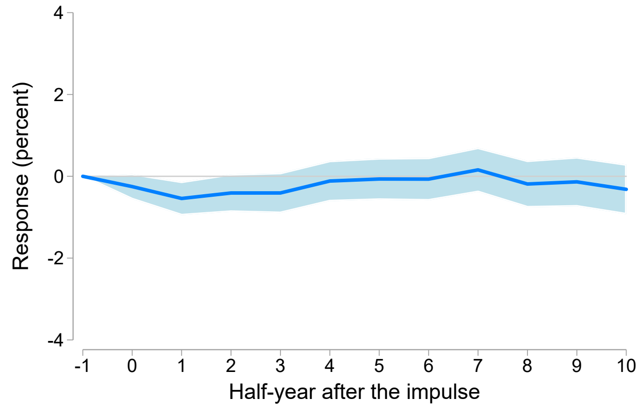
Figure A5 shows that the diversity measures do not respond strongly to financial crises.[31] It seems that domestic upstream and downstream diversity decreases slightly when a financial crisis starts. However, the effect is minimal and disappears quickly. Also, the global diversity measure is not affected significantly by financial crises. This result is consistent with the small standard deviation of the diversity measures over time, as shown in Table 1.

[30] As countries with a high coefficient of variation of out-degree experienced deeper drops in real GDP, the estimated differences between two country groups with high and low coefficients of variation go negative. Therefore, I reverse the sign in Figure A4 to facilitate comparison with the previous result.

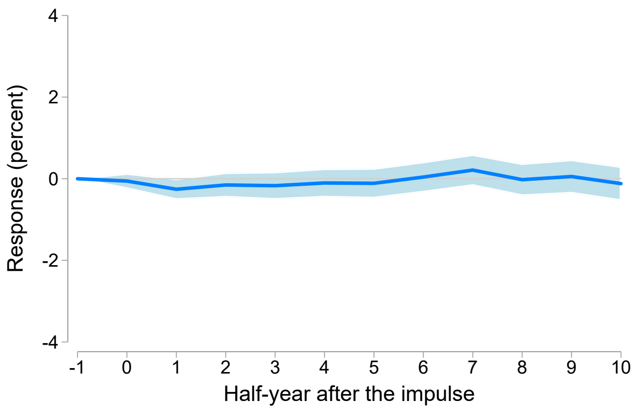
[31] I set the scale of y-axes from -4 to 4 percent to make this figure comparable to Figures A2 and A3.



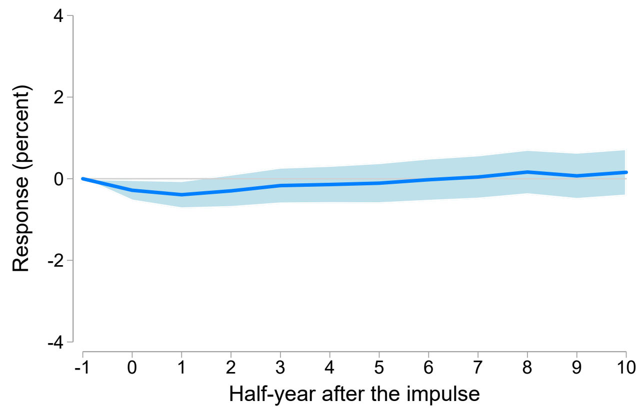
(A) Domestic, upstream diversity



(B) Domestic, downstream diversity



(C) Global, upstream diversity



(D) Global, downstream diversity

Figure A5: impulse-responses of diversity

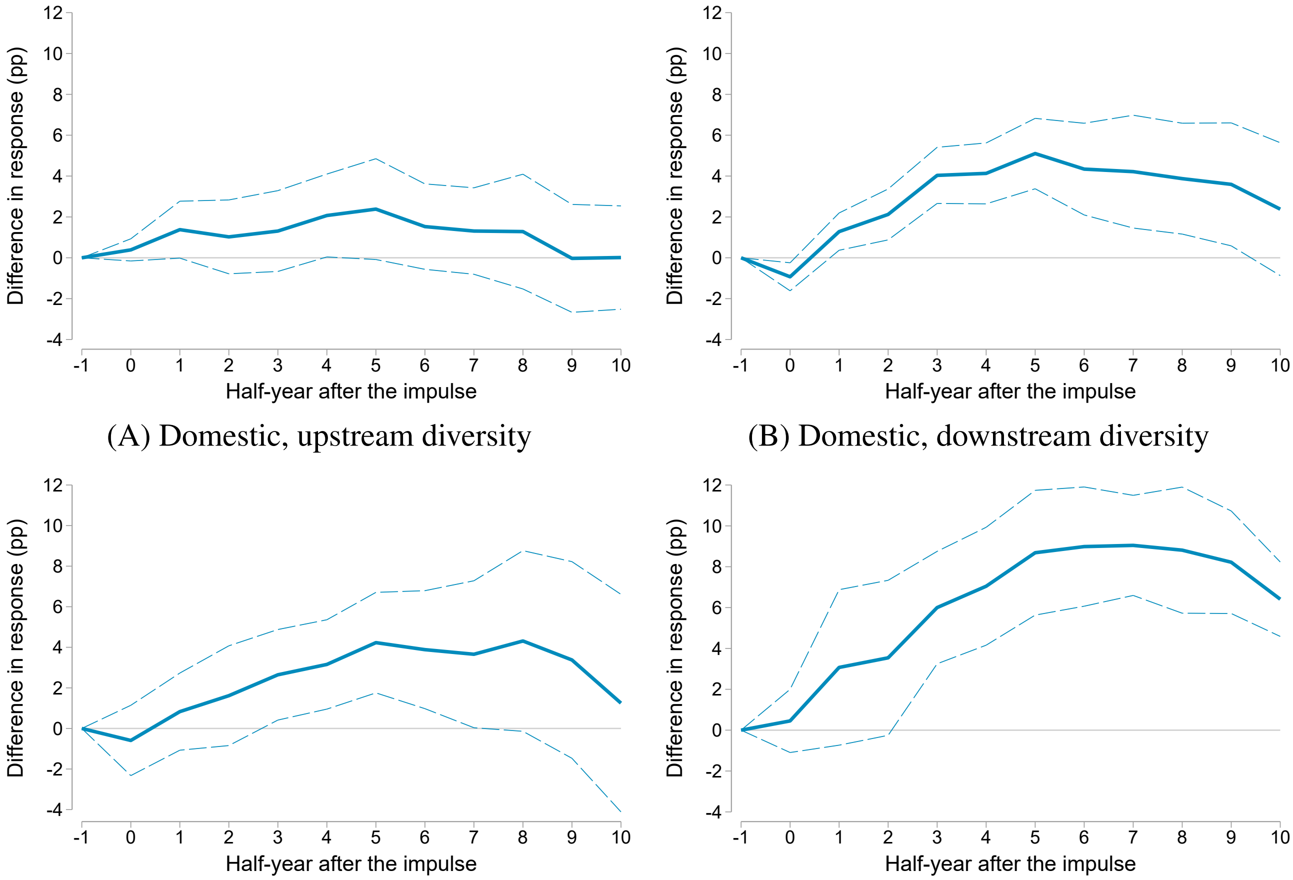
Note. This figure shows the impulse response function of the diversity measure to financial distress shocks. The shaded area represents the 95% confidence intervals.

A.2.4 Alternative Crisis Chronologies

I check the results with alternative crisis chronologies defined by the IMF (Laeven and Valencia, 2013). Following Romer and Romer (2017), I use a dummy variable equal to one when a given country is in a crisis so that the results are comparable to the baseline impulse response functions to the realization of the Romer and Romer financial distress measure of 7 (a moderate crisis).[32]

Figure A6 shows that countries with high diversity were less affected by financial crises than those with low diversity. The result confirms that the downstream diversity plays a crucial role in reducing the adverse impact of financial crises. However, there are differences in the role of

[32] The alternative chronologies transformed to the half-year dummy variable are taken from Romer and Romer (2017). The IMF chronology is available for a slightly shorter period from 1970 to 2011.



(A) Domestic, upstream diversity

(B) Domestic, downstream diversity

(C) Global, upstream diversity

(D) Global, downstream diversity

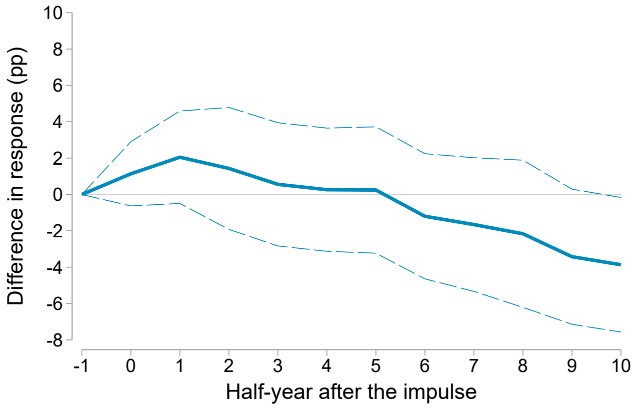
Figure A6: IMF chronology

Note. This figure shows the difference in responses of log real GDP to the realization of systemic financial crises defined by the IMF. The 95% confidence intervals, using the Driscoll-Kraay standard errors, are added to each line.

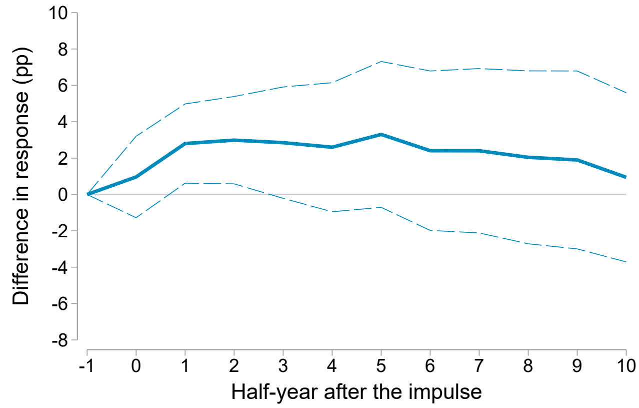
the upstream diversity. First, the domestic upstream diversity does not play an important role. This may imply that the IMF crisis chronology more effectively captures domestic demand shocks rather than technology or financial shocks. Second, the global upstream diversity plays a more important role with the IMF crisis chronologies than in the benchmark specification. This might mean that the IMF chronology better captures regional or global financial crises.

I also check the results using 42 countries with the IMF chronology.[33] The differences between high- and low-diversity countries are presented in Figure A7. Including less developed countries widens the confidence intervals. The role of upstream diversity is not significant and disappears

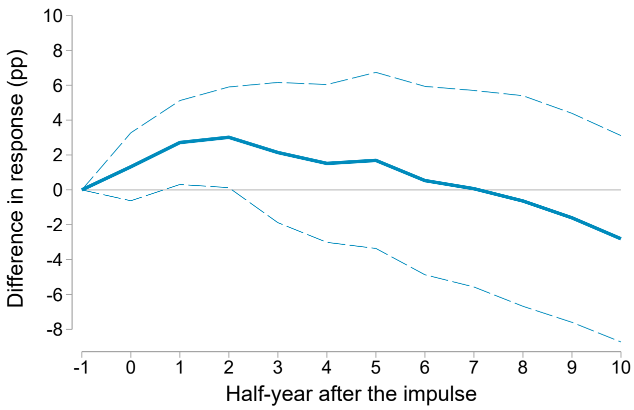
[33]The additional countries include Argentina, Brazil, Chile, Colombia, Costa Rica, Czech Republic, Estonia, Hungary, India, Indonesia, Israel, Korea, Latvia, Lithuania, Mexico, Poland, Slovakia, Slovenia.



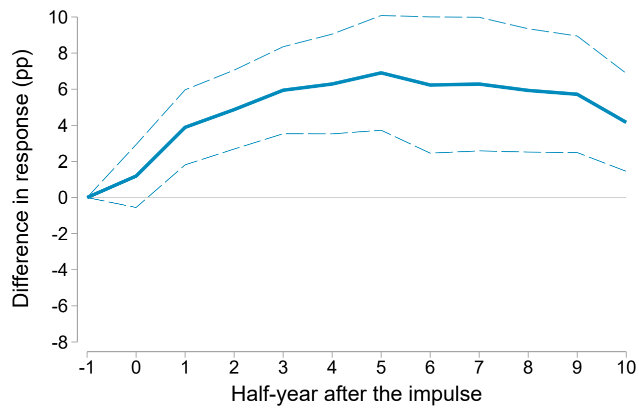
(A) Domestic, upstream diversity



(B) Domestic, downstream diversity



(C) Global, upstream diversity



(D) Global, downstream diversity

Figure A7: Additional countries with IMF chronology

Note. This figure shows the difference in responses of log real GDP to the realization of systemic financial crises defined by the IMF with 42 countries. The 95% confidence intervals, using the Driscoll-Kraay standard errors, are added to each line.

quickly. However, downstream diversity, especially in terms of the global network, still plays a critical role as insurance against financial crises. This implies that geographic diversification in exports is the most important factor for developing countries to reduce the adverse impact of financial crises.

A.3 Network Analysis

A.3.1 Definitions and Notation

Consider a network with n nodes. A network is a graph represented by its nodes or vertices $V = \{1, 2, \ldots, n\}$ and adjacency matrix $\Theta \in \mathbb{R}^{n \times n}$. The element of the adjacency matrix $\theta_{ij} \in \{0, 1\}$ indicates the existence of an *edge* connecting nodes i and j in an unweighted network. For a weighted network, θ_{ij} represents the importance of the edge between nodes i and j. An edge is a *link* if $i \neq j$ and a *loop* if $i = j$.

Two nodes are *adjacent* if they are incident with the same edge. Adjacent nodes are called *neighbors*. The *degree* of a node i is the number of edges involving node i in an undirected network. For a directed network, *out-degree* and *in-degree* can be separately defined based on the direction of flows involving node i.

A *walk* of length L from i to j is an ordered sequence of nodes $W = \{v_0, v_1, \ldots, v_L\}$ with $v_0 = i, v_L = j$ and $a_{ll+1} \neq 0$ for all $l = 0, 1, 2, \ldots, L-1$. If a walk includes only distinct nodes, then W is a *path*. A *trail* allows non-distinct nodes but requires that every ordered pair of consecutive nodes appears only once in the sequence. A *geodesic* is the shortest path from i to j such that no other path between them involves a smaller number of edges.

The *distance* between node i and j is the number of edges involved in a geodesic between i and j. If nodes i and j are disconnected, the distance is infinity.

A.3.2 Typology of Centrality Measures

Borgatti (2005) provides a typology of network flows based on the types of trajectories that traffic follows and the method of spread. As defined above, there are four kinds of trajectories: geodesics, paths, trails, and walks. The method of spread includes parallel duplication, serial duplication, and transfer. Traffic may spread by parallel duplication if it flows to neighbors simultaneously like an email broadcast. Traffic may spread by serial duplication if it flows to other nodes one at a time, like a viral infection. Finally, traffic may spread by transfer if it moves from a node to another, like a package delivery.

Based on the two dimensions, Borgatti (2005) assigned centrality measures to 12 ($= 4 \times 3$) categories. Degree centrality is assigned to {paths, parallel}, {trails, parallel}, and {walks, parallel} categories. Closeness centrality is assigned to {paths, parallel}, [trails, parallel}, {walks, parallel}, {geodesics, serial}, and {geodesics, transfer}. Betweenness centrality is assigned to {geodesics, transfer}. Finally, eigenvector centrality is assigned to {walks, parallel}.

Borgatti (2005) also shows that centrality measures can be misleading when they are not applied to the specific flow processes they are designed for. This implies that the four best-known measures of centrality are not appropriate for the input-output network analysis. The flows of goods and services in the input-output network are spread by transfer. Also, intermediate inputs move to other industries following the production process and do not seek the shortest path (geodesics) to the final target industry. Therefore, we need a centrality measure designed for {paths,transfer}, {trails, transfer}, or {walks, transfer}. In addition, we need a measure of centrality that can be used for directed networks, as the input-output network has upstream (to input-supplying industries) and downstream (to customer industries) flows.

A.4 Proofs

Proof of Proposition 1. I first solve the firm's optimization problem. Let x_i is a composite of sectoral inputs:

$$x_i = l_i^{1-\mu_i} \left(\prod_{j=1}^{N} x_{ij}^{a_{ij}} \right)^{\mu_i}$$

Then, the production function and budget constraint of sector i are given by

$$y_i = e^{z_i} \zeta_i x_i$$

$$q_i x_i \leq \chi_i p_i y_i$$

where q_i is a composite of input costs and $q_i x_i$ is the total expenditure on inputs, including labor and intermediate goods.

When the financial constraint is not binding, the firm's optimality implies $q_i x_i = p_i y_i$. However, if the constraint is binding, we have $q_i x_i = \chi_i p_i y_i$. Thus, we can write the firm's optimality as

$$q_i x_i = \phi_i p_i y_i, \quad \text{where} \quad \phi_i = min\{\chi_i, 1\}. \tag{A.1}$$

The financial constraint generates the wedge $\phi_i \in [0,1]$ between the firm's marginal cost and marginal revenue.

Given the amount of output, the firm minimizes expenditures on inputs. The first order conditions are given by

$$x_{ij}: \ p_j x_{ij} = \lambda_i \mu_i a_{ij} x_i$$

$$l_i: \qquad l_i = \lambda_i (1-\mu_i) x_i.$$

By summing over the expenditure on all inputs, we get $\lambda_i = q_i$. This leads to

$$p_j x_{ij} = \mu_i a_{ij} u_i \tag{A.2}$$

$$l_i = (1-\mu_i) u_i. \tag{A.3}$$

Now, I solve the household's optimization problem. Let $\bar{p}$ is the composite of final good prices and $u_0 = \bar{p}c$ is the household's total expenditure. The first order conditions yield

$$\frac{c^{-\gamma}}{l^{\varepsilon}} = \bar{p}. \tag{A.4}$$

Given the amount of final consumption, the household's expenditure minimization problem yields the following condition

$$\theta_i c = \lambda_0 p_i c_i \quad \text{for all } i.$$

By summing over the expenditure over all goods, I get $\lambda_0 = 1/\bar{p}$. This leads to $p_i c_i = \theta_i \bar{p} c = \theta_i u_0$. Using this result, we can calculate the price index as follows:

$$c = \prod_{i=1}^{N} \left(\frac{\theta_i}{p_i} \bar{p} c \right)^{\theta_i} = \prod_{i=1}^{N} \left(\frac{\theta_i}{p_i} \right)^{\theta_i} \bar{p} c.$$

Thus, we get

$$\bar{p} = \prod_{i=1}^{N} \left(\frac{p_i}{\theta_i} \right)^{\theta_i}. \tag{A.5}$$

For now, suppose that there are no demand shocks. From the market clearing condition, we obtain

$$p_i y_i = p_i c_i + \sum_{j=1}^{N} p_i x_{ji}.$$

Using the firm's and household's optimality conditions, we can rewrite the above equation as

$$g_i = \theta_i u_0 + \sum_{j=1}^{N} \mu_j a_{ji} u_j.$$

I rewrite this equation in a vector from as follows:

$$\mathbf{g} = \boldsymbol{\theta} u_0 + (\boldsymbol{\mu}\boldsymbol{\iota}' \circ \mathbf{A})' \mathbf{u}. \tag{A.6}$$

where $\circ$ denotes the Hadamard product and $\boldsymbol{\iota}$ is a vector of ones with total N elements.

Using the fact that $\pi_i = g_i - u_i$, the household's expenditure, u_0, must satisfy the budget constraint as follows:

$$u_0 = l + \iota'(\mathbf{g} - \mathbf{u}).$$

Substituting this expression into the market clearing equation (A.6) leads to

$$\mathbf{g} = \boldsymbol{\theta}(l + \boldsymbol{\iota}'(\mathbf{g} - \mathbf{u})) + (\boldsymbol{\mu}\boldsymbol{\iota}' \circ \mathbf{A})'\mathbf{u}.$$

From the firm's optimality condition (A.1) we know that $\mathbf{u} = \boldsymbol{\phi} \circ \mathbf{g}$. Thus, with a bit of algebraic manipulation we can rewrite the above equation as

$$\mathbf{g} = \left[\mathbf{I} - \boldsymbol{\theta}(\boldsymbol{\iota} - \boldsymbol{\phi})' - \left((\boldsymbol{\mu} \circ \boldsymbol{\phi})\boldsymbol{\iota}' \circ \mathbf{A}\right)'\right]^{-1} \boldsymbol{\theta} l. \tag{A.7}$$

where $\mathbf{I}$ is the identity matrix of size $N \times N$. We may simplify this equation to $\mathbf{g} = \mathbf{a}(\boldsymbol{\phi})\boldsymbol{\theta} l$, where $\mathbf{a}(\boldsymbol{\phi}) = \left[\mathbf{I} - \boldsymbol{\theta}(\boldsymbol{\iota} - \boldsymbol{\phi})' - ((\boldsymbol{\mu} \circ \boldsymbol{\phi})\boldsymbol{\iota}' \circ \mathbf{A})'\right]^{-1}$.

Substituting the firm's first-order conditions (A.2) and (A.3) into the production function and multiplying prices, we obtain

$$g_i = p_i \left[e^{z_i} u_i \zeta_i (1 - \mu_i)^{\mu_i} \mu_i^{\mu_i} \prod_{j=1}^{N} \left(\frac{a_{ij}}{p_j} \right)^{\mu_i a_{ij}} \right].$$

Taking logs of the above equation, we get

$$\begin{aligned} \log g_i = & \log p_i + z_i + \log u_i + \log \zeta_i + (1 - \mu_i)\log(1 - \mu_i) \\ & + \mu_i \log \mu_i + \mu_i \sum_{j=1}^{N} a_{ij} \log a_{ij} - \mu_i \sum_{j=1}^{N} a_{ij} \log p_j. \end{aligned}$$

By defining the constant $\zeta_i = (1 - \mu_i)^{-(1-\mu_i)} \mu_i^{-\mu_i} \prod_{j=1}^{N} a_{ij}^{-a_{ij}}$, the above equation results in

$$\log g_i = \log p_i + z_i + \log u_i - \mu_i \sum_{j=1}^{N} a_{ij} \log p_j.$$

I rewrite the above equation in the vector form as

$$\log \mathbf{g} = \log \mathbf{p} + \mathbf{z} + \log \mathbf{u} - (\boldsymbol{\mu}\boldsymbol{\iota}' \circ \mathbf{A}) \log \mathbf{p}.$$

Using the fact that $\log \mathbf{u} = \log \boldsymbol{\phi} + \log \mathbf{g}$, we may simplify the above equation to

$$\log \mathbf{p} = -\left[\mathbf{I} - \boldsymbol{\mu}\boldsymbol{\iota}' \circ \mathbf{A}\right]^{-1} (\mathbf{z} + \log \boldsymbol{\phi}). \tag{A.8}$$

The inverse matrix on the right side of the above equation represents the Leontief inverse matrix: $\mathbf{L} = [\mathbf{I} - \boldsymbol{\mu}\iota' \circ \mathbf{A}]^{-1}$. Also, we can derive an expression for the price index $\bar{p}$ in the vector form after taking the log of equation (A.5):

$$\log \bar{p} = \boldsymbol{\theta}' \log \mathbf{p} - \boldsymbol{\theta}' \log \boldsymbol{\theta}. \tag{A.9}$$

Now I derive an expression for real GDP from household consumption and the budget constraint, $\bar{p}c = l + \sum_{i=1}^{N} \pi_i$, as follows:

$$c = \bar{p}^{-1}(l + \iota'(\mathbf{g} - \mathbf{u})).$$

Using equation (A.7) and $\mathbf{u} = \boldsymbol{\phi} \circ \mathbf{g}$, we may rewrite the above equation as

$$c = \bar{p}^{-1}\left[1 + \iota'(\iota - \boldsymbol{\phi}) \circ \mathbf{a}(\boldsymbol{\phi})\boldsymbol{\theta}\right] l \tag{A.10}$$

By taking logs of the above equation and substituting in equations (A.8) and (A.9), I obtain the following expression for real GDP:

$$\log GDP = \boldsymbol{\theta}'\mathbf{L}(\mathbf{z} + \log \boldsymbol{\phi}) + \boldsymbol{\theta}' \log \boldsymbol{\theta} + \log(1 + \iota'(\iota - \boldsymbol{\phi}) \circ \mathbf{a}(\boldsymbol{\phi})\boldsymbol{\theta}) + \log l. \tag{A.11}$$

Now I use the household's optimality condition to get an expression for equilibrium labor. Substituting equation (A.10) into equation (A.4), we get

$$\frac{(\bar{p}^{-1}\left[1 + \iota'(\iota - \boldsymbol{\phi}) \circ \mathbf{a}(\boldsymbol{\phi})\boldsymbol{\theta}\right] l)^{-\gamma}}{l^{\varepsilon}} = \bar{p}$$

With a bit of manipulation, the above equation results in

$$l = \bar{p}^{\frac{\gamma-1}{\varepsilon+\gamma}}(1 + \iota'(\iota - \boldsymbol{\phi}) \circ \mathbf{a}(\boldsymbol{\phi})\boldsymbol{\theta})^{-\frac{\gamma}{\varepsilon+\gamma}}.$$

We may get the following equation after taking logs of the above equation:

$$\log l = -\frac{1-\gamma}{\varepsilon+\gamma} \log \bar{p} - \frac{\gamma}{\varepsilon+\gamma} \log(1 + \iota'(\iota - \boldsymbol{\phi}) \circ \mathbf{a}(\boldsymbol{\phi})\boldsymbol{\theta}).$$

Substituting equations (A.8) and (A.9) into the above equation, we may obtain

$$\log l = \frac{1-\gamma}{\varepsilon+\gamma}\boldsymbol{\theta}'\mathbf{L}(\mathbf{z}+\log\boldsymbol{\phi}) + \frac{1-\gamma}{\varepsilon+\gamma}\boldsymbol{\theta}'\log\boldsymbol{\theta} - \frac{\gamma}{\varepsilon+\gamma}\log(1+\iota'(\iota-\phi)\circ\mathbf{a}(\phi)\boldsymbol{\theta}).$$

Let $\beta = \frac{\varepsilon+1}{\varepsilon+\gamma}$ and substitute the above equation to equation (A.11). Then, we get the expression for log real GDP in Proposition 1:

$$\log GDP = \beta\boldsymbol{\theta}'\mathbf{L}(\mathbf{z}+\log\boldsymbol{\phi}) + \left(\beta - \frac{1}{\varepsilon+\gamma}\right)\log(1+\iota'(\iota-\phi)\circ\mathbf{a}(\phi)\theta) + \beta\boldsymbol{\theta}'\log\boldsymbol{\theta}. \tag{A.12}$$

QED.

Proof of Corollary 1. If there are no financial frictions, $\phi_i = 1$ and $\log\phi_i = 0$ for all i. Therefore, the response of log real GDP to productivity shocks is given by

$$d\log GDP = \beta\boldsymbol{\theta}'\mathbf{L}\,d\mathbf{z}.$$

We can define the Domar weight $\boldsymbol{\nu}$, i.e., the vector of sales shares in GDP, as $\boldsymbol{\nu} = \mathbf{g}/u_0$. The market-clearing condition is given by

$$\mathbf{g} = \boldsymbol{\theta}u_0 + (\boldsymbol{\mu}\boldsymbol{\iota}'\circ\mathbf{A})'\mathbf{u}.$$

In addition, $\mathbf{g} = \mathbf{u}$ if financial constraints are not binding. This implies that the Domar weight is given by

$$\boldsymbol{\nu} = \left[\mathbf{I} - (\boldsymbol{\mu}\boldsymbol{\iota}'\circ\mathbf{A})'\right]^{-1}\boldsymbol{\theta} = \mathbf{L}'\boldsymbol{\theta}.$$

QED.

Proof of Proposition 2. (1) If the productivity shock, z_i, is i.i.d across sectors and from a common distribution with mean zero and finite variance σ_z, the standard deviation of log real GDP to a one-time productivity shock and its upper bound are given by

$$\sigma_z\beta\sqrt{\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\theta_j l_{ji}\right)^2}.$$

Given that θ_j and l_{ji} are nonnegative numbers for all i and j, the Cauchy-Schwarz inequality implies that

$$\sigma_z \beta \sqrt{\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\theta_j l_{ji}\right)^2} \leq \sigma_z \beta \sum_{j=1}^{n}\theta_j \sqrt{\sum_{i=1}^{n} l_{ji}^2}. \tag{A.13}$$

(2) Using the Cauchy-Schwarz inequality again, we can show that

$$\sum_{i=1}^{n} l_{ji}^2 \geq \frac{1}{n}\left(\sum_{i=1}^{n} l_{ji}\right)^2,$$

where the equality holds if and only if l_{ji} are equal over i. Therefore, given the values of θ_j, the right side of inequality (A.13) is minimized when l_{ji} is equally distributed across i. QED.

Proof of Proposition 3. With demand shocks, the market-clearing condition can be written as

$$\mathbf{g} = \boldsymbol{\theta} u_0 + (\boldsymbol{\mu}\boldsymbol{\iota}' \circ \mathbf{A})'\mathbf{u} + \tilde{\mathbf{v}},$$

where $\tilde{\mathbf{v}}$ denotes the vector of nominal demand shocks ($\tilde{\mathbf{v}} = \mathbf{p} \circ \mathbf{v}$).

Let's assume that $D_{\mathbf{x}}$ is a diagonal matrix with the vector $\mathbf{x}$ as the diagonal elements. Then, substituting $\mathbf{g} = D_{\phi}^{-1}\mathbf{u}$ into the above equation, we get

$$D_{\phi}^{-1}\mathbf{u} = \boldsymbol{\theta} u_0 + (\boldsymbol{\mu}\boldsymbol{\iota}' \circ \mathbf{A})'\mathbf{u} + \tilde{\mathbf{v}}.$$

With a bit of algebraic manipulation, we obtain

$$\frac{\mathbf{u}}{u_0} = \tilde{\mathbf{L}}'\boldsymbol{\theta} + \tilde{\mathbf{L}}'\frac{\tilde{\mathbf{v}}}{u_0}, \tag{A.14}$$

where $\tilde{\mathbf{L}}' = \left[D_{\phi}^{-1} - (\boldsymbol{\mu}\boldsymbol{\iota}' \circ \mathbf{A})'\right]^{-1}$. If there are no financial frictions, $\tilde{\mathbf{L}}'$ becomes the transpose of the Leontief inverse matrix: $\mathbf{L}' = [\mathbf{I} - (\boldsymbol{\mu}\boldsymbol{\iota}' \circ \mathbf{A})']^{-1}$.

By summing both sides of equation (A.3), we get $l = \iota' D_{(\iota-\mu)}\mathbf{u}$. I substitute this expression for labor and $\mathbf{g} = D_{\phi}^{-1}\mathbf{u}$ into the following budget constraint of the household:

$$\begin{aligned} u_0 = \bar{p}c &= l + \iota'(\mathbf{g} - \mathbf{u}) \\ &= \iota' D_{(\iota-\mu)}\mathbf{u} + \iota'(\mathbf{g} - \mathbf{u}) \\ &= \iota'(D_{(\iota-\mu)} + D_{\phi}^{-1} - \mathbf{I})\mathbf{u}. \end{aligned}$$

If we multiply $\iota'(D_{(\iota-\mu)} + D_{\phi}^{-1} - \mathbf{I})$ to equation (A.14), we get the following equation.

$$1 = \iota'(D_{(\iota-\mu)} + D_{\phi}^{-1} - \mathbf{I})\tilde{\mathbf{L}}'\boldsymbol{\theta} + \iota'(D_{(\iota-\mu)} + D_{\phi}^{-1} - \mathbf{I})\tilde{\mathbf{L}}'\frac{\tilde{\mathbf{v}}}{u_0}.$$

The first term on the right side is a constant. With some algebraic manipulation, we obtain the following expression for nominal GDP:

$$GDP = u_0 = \tilde{\eta}\tilde{\boldsymbol{\kappa}}'\tilde{\mathbf{L}}'\tilde{\mathbf{v}}, \tag{A.15}$$

where $\tilde{\eta}$ is a constant defined as

$$\tilde{\eta} = \frac{1}{1 - \iota'(D_{(\iota-\mu)} + D_{\phi}^{-1} - \mathbf{I})\tilde{\mathbf{L}}'\boldsymbol{\theta}},$$

and $\tilde{\boldsymbol{\kappa}}'$ is a row vector defined as

$$\tilde{\boldsymbol{\kappa}}' = \iota'(D_{(\iota-\mu)} + D_{\phi}^{-1} - \mathbf{I}).$$

QED.

Proof of Corollary 2. If there are no financial constraints, $D_{\phi} = \mathbf{I}$. Then, from equation (A.15) nominal GDP is given by

$$GDP = \eta\boldsymbol{\kappa}'\mathbf{L}'\tilde{\mathbf{v}},$$

where $\eta = \frac{1}{1-(\iota-\mu)'\mathbf{L}'\boldsymbol{\theta}}$, $\boldsymbol{\kappa}' = (\iota - \boldsymbol{\mu})'$, and $\mathbf{L}' = [\mathbf{I} - (\boldsymbol{\mu}\iota' \circ \mathbf{A})']^{-1}$.

Therefore, we can derive the response of nominal GDP to nominal demand shocks as follows:

$$dGDP = \eta\boldsymbol{\kappa}'\mathbf{L}'\, d\tilde{\mathbf{v}}.$$

QED.

Proof of Proposition 4. (1) If the nominal demand shock, $\tilde{v}_i$, is i.i.d across sectors from a common distribution with zero mean and finite variance σ_v, the standard deviation of the response of nominal GDP is

$$\sigma_v \eta \sqrt{\sum_{i=1}^{n} \left(\sum_{j=1}^{n} \kappa_j l_{ij} \right)^2}.$$

Given that l_{ji} are nonnegative numbers for all i and j, the Cauchy-Schwarz inequality implies that

$$\sigma_v \eta \sqrt{\sum_{i=1}^{n} \left(\sum_{j=1}^{n} \kappa_j l_{ij} \right)^2} \leq \sigma_v \eta \sum_{j=1}^{n} \kappa_j \sqrt{\sum_{i=1}^{n} l_{ij}^2}. \tag{A.16}$$

(2) Using the Cauchy-Schwarz inequality, I can show that

$$\sum_{i=1}^{n} l_{ij}^2 \geq \frac{1}{n} \left(\sum_{i=1}^{n} l_{ij} \right)^2,$$

where the equality holds if and only if l_{ij} are equal over i. Therefore, given the values of κ_j, the right side of inequality (A.16) is minimized when l_{ij} is equally distributed across i. QED.

Proof of Corollary 3. Suppose an economy without supply or demand shocks. From equation (A.12), I can derive the response of log real GDP to the tightening of the financial constraint as follows:

$$d \log GDP = \beta \theta' \mathbf{L} \, d \log \phi + \left(\beta - \frac{1}{\varepsilon + \gamma} \right) d \log(1 + \iota'(\iota - \phi) \circ \mathbf{a}(\phi) \theta).$$

QED.

Proof of Proposition 6. Suppose an economy without demand shocks. With international trades in intermediate goods, the market-clearing condition can be written in the vector form as follows.

$$\mathbf{g} = \tilde{\theta} \mathbf{u}_0 + (\mu \iota' \circ \mathbf{A})' \mathbf{u}, \tag{A.17}$$

where $\mathbf{u}_0$ is the vector of u_0 with K elements corresponding to K countries, and $\tilde{\theta}$ is a matrix of size $KN \times K$ such that

$$\begin{bmatrix} \theta_1 & 0 & \dots & & 0 \\ \theta_2 & 0 & & & 0 \\ \vdots & \vdots & & & \vdots \\ \theta_N & 0 & & & 0 \\ 0 & \theta_{N+1} & 0 & & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & \theta_{2N} & 0 & & \\ & & \ddots & & 0 \\ & & & 0 & \theta_{(K-1)N+1} \\ \vdots & & & & \vdots \\ 0 & & & 0 & \theta_{KN} \end{bmatrix}.$$

The household's expenditure must satisfy the following budget constraint for every country.

$$\mathbf{u}_0 = \mathbf{l} + \hat{\imath}(\mathbf{g} - \mathbf{u}) = \mathbf{l} + \widehat{(\boldsymbol{\iota} - \boldsymbol{\phi})}\mathbf{g}, \tag{A.18}$$

where $\mathbf{l}$ is the vector of labor with K elements corresponding to K countries, $\hat{\imath}$ is a matrix of size $K \times KN$ such that $\hat{\imath} = I_K \otimes \iota'_N$ with $\otimes$ denoting the Kronecker delta, and $\widehat{(\boldsymbol{\iota} - \boldsymbol{\phi})}$ is a matrix of size $K \times KN$ such that

$$\begin{bmatrix} 1-\phi_1 & \dots & 1-\phi_N & 0 & \dots & & & & & \dots & 0 \\ 0 & \dots & 0 & 1-\phi_{N+1} & \dots & 1-\phi_{2N} & 0 & & \dots & & \\ & & & & & & \ddots & & & & \\ 0 & \dots & & & & \dots & 0 & 1-\phi_{(K-1)N+1} & & \dots & 1-\phi_{KN} \end{bmatrix}.$$

By substituting equation (A.18) to equation (A.17) and doing some algebra, I get the following expression for firms' sales:

$$\mathbf{g} = \left[\mathbf{I}_{KN} - \tilde{\boldsymbol{\theta}}\widehat{(\boldsymbol{\iota} - \boldsymbol{\phi})} - \left((\boldsymbol{\mu} \circ \boldsymbol{\phi})\boldsymbol{\iota}' \circ \mathbf{A}\right)'\right]^{-1} \tilde{\boldsymbol{\theta}}\mathbf{l}. \tag{A.19}$$

where $\mathbf{I}_{KN}$ is the identity matrix of size $KN \times KN$. I define $\tilde{\mathbf{a}}(\boldsymbol{\phi})$ as the inverse matrix on the right side of equation (A.19).

Now I derive an expression for real GDP from household consumption and the budget constraint for country k, $\bar{p}_k c_k = l_k + \sum_{i \in k} \pi_i$, as follows:

$$c_k = \bar{p}_k^{-1}(l_k + \hat{\imath}_k(\mathbf{g} - \mathbf{u})),$$

where $\hat{\imath}_k$ is the k-th row of $\hat{\imath}$. Using equation (A.19) and $\mathbf{u} = \boldsymbol{\phi} \circ \mathbf{g}$, we may rewrite the above equation as

$$c_k = \bar{p}_k^{-1} \left[\hat{\imath}_k + \widehat{(\boldsymbol{\iota} - \boldsymbol{\phi})}_k \tilde{\mathbf{a}}(\boldsymbol{\phi}) \tilde{\boldsymbol{\theta}} \right] \mathbf{l}, \tag{A.20}$$

where $\widehat{(\boldsymbol{\iota} - \boldsymbol{\phi})}_k$ is the k-th row of $\widehat{(\boldsymbol{\iota} - \boldsymbol{\phi})}$.

The price index for country k is given by

$$\bar{p}_k = \prod_{i \in k} \left(\frac{p_i}{\theta_i} \right)^{\theta_i}.$$

By taking the log of the above equation, we get

$$\log \bar{p}_k = \hat{\boldsymbol{\theta}}_k \log \mathbf{p} - \hat{\boldsymbol{\theta}}_k \log \boldsymbol{\theta}, \tag{A.21}$$

where $\hat{\boldsymbol{\theta}}_k$ denotes the k-th row of $\hat{\boldsymbol{\theta}}$, and $\hat{\boldsymbol{\theta}}$ is given by

$$\begin{bmatrix} \theta_1 & \dots & \theta_N & 0 & \dots & & & & \dots & 0 \\ 0 & \dots & 0 & \theta_{N+1} & \dots & \theta_{2N} & 0 & \dots & & \\ & & & & & & \ddots & & & \\ 0 & \dots & & & & \dots & 0 & \theta_{(K-1)N+1} & \dots & \theta_{KN} \end{bmatrix}.$$

By taking logs of equation (A.20) and substituting in equations (A.8) and (A.21), we obtain the following expression for real GDP:

$$\log \mathbf{GDP} = \hat{\boldsymbol{\theta}} \mathbf{L} (\mathbf{z} + \log \boldsymbol{\phi}) + \hat{\boldsymbol{\theta}} \log \boldsymbol{\theta} + \log(\hat{\imath} + \widehat{(\boldsymbol{\iota} - \boldsymbol{\phi})} \tilde{\mathbf{a}}(\boldsymbol{\phi}) \tilde{\boldsymbol{\theta}}) \mathbf{l}, \tag{A.22}$$

where **GDP** is a vector with K elements corresponding to K countries. If the representative agents in each country supply one unit of labor inelastically ($l_k = 1 \, \forall k \in \{1 \, .. \, K\}$), the k-th country's GDP is given by

$$\log \mathbf{GDP}_k = \hat{\boldsymbol{\theta}}_k \mathbf{L} (\mathbf{z} + \log \boldsymbol{\phi}) + \hat{\boldsymbol{\theta}}_k \log \boldsymbol{\theta} + \log(1 + \widehat{(\boldsymbol{\iota} - \boldsymbol{\phi})}_k \tilde{\mathbf{a}}(\boldsymbol{\phi}) \boldsymbol{\theta}). \tag{A.23}$$

Therefore, the response of GDP to the productivity shock is given by

$$d \log \mathbf{GDP}_k = \hat{\boldsymbol{\theta}}_k \mathbf{L} \, d\mathbf{z}.$$

QED.

Proof of Proposition 7. The market clearing condition for an economy with international trades and demand shocks are given by

$$\mathbf{g} = \tilde{\boldsymbol{\theta}}\mathbf{u}_0 + (\boldsymbol{\mu}\boldsymbol{\iota}' \circ \mathbf{A})'\mathbf{u} + \tilde{\mathbf{v}}.$$

With a bit of algebra, similar to equation (A.14), we obtain

$$\mathbf{u} = \tilde{\mathbf{L}}'\tilde{\boldsymbol{\theta}}\mathbf{u}_0 + \tilde{\mathbf{L}}'\tilde{\mathbf{v}}, \tag{A.24}$$

where $\tilde{\mathbf{L}}' = \left[D_{\phi}^{-1} - (\boldsymbol{\mu}\boldsymbol{\iota}' \circ \mathbf{A})'\right]^{-1}$.

By summing both sides of equation (A.3) and stacking this condition for K countries, we get $\mathbf{l} = \hat{\imath} D_{(\iota-\mu)}\mathbf{u}$. Substituting this expression into $\mathbf{u}_0 = \mathbf{l} + \hat{\imath}(\mathbf{g} - \mathbf{u})$, we have $\mathbf{u}_0 = \hat{\imath}(D_{(\iota-\mu)} + D_{\phi}^{-1} - I_{KN})\mathbf{u}$. Now, by multiplying $\hat{\imath}(D_{(\iota-\mu)} + D_{\phi}^{-1} - I_{KN})$ to equation (A.24), we may obtain

$$\left[I_K - \hat{\imath}(D_{(\iota-\mu)} + D_{\phi}^{-1} - I_{KN})\tilde{\mathbf{L}}'\tilde{\boldsymbol{\theta}}\right]\mathbf{u}_0 = \hat{\imath}(D_{(\iota-\mu)} + D_{\phi}^{-1} - I_{KN})\tilde{\mathbf{L}}'\tilde{\mathbf{v}}.$$

Therefore, we have the expression for nominal GDP as follows:

$$\mathbf{GDP} = \left[I_K - \hat{\imath}(D_{(\iota-\mu)} + D_{\phi}^{-1} - I_{KN})\tilde{\mathbf{L}}'\tilde{\boldsymbol{\theta}}\right]^{-1}\hat{\imath}(D_{(\iota-\mu)} + D_{\phi}^{-1} - I_{KN})\tilde{\mathbf{L}}'\tilde{\mathbf{v}},$$

where $\mathbf{GDP}$ is a vector with K elements corresponding to K countries. With no financial frictions ($D_{\phi} = I_{KN}$), the k-th country's nominal GDP is given by

$$\mathbf{GDP}_k = \hat{\boldsymbol{\eta}}_k\hat{\boldsymbol{\kappa}}\tilde{\mathbf{L}}'\tilde{\mathbf{v}},$$

where $\hat{\boldsymbol{\eta}}_k$ is the k-th row of $\left[I_K - \hat{\imath} D_{(\iota-\mu)}\tilde{\mathbf{L}}'\tilde{\boldsymbol{\theta}}\right]^{-1}$, and $\hat{\boldsymbol{\kappa}} = \hat{\imath} D_{(\iota-\mu)}$.

Therefore, the response of nominal GDP to the nominal demand shock is given by

$$d\mathbf{GDP}_k = \hat{\boldsymbol{\eta}}_k\hat{\boldsymbol{\kappa}}\tilde{\mathbf{L}}'\, d\tilde{\mathbf{v}}.$$

QED.

Proof of Proposition 8. Suppose an economy without supply or demand shocks. From equation

(A.23) the response of log real GDP to the tightening of the financial constraint is given by

$$d \log \mathbf{GDP}_k = \hat{\boldsymbol{\theta}}_k \mathbf{L} \, d \log \boldsymbol{\phi} + \, d \log(1 + \widehat{(\boldsymbol{\iota} - \boldsymbol{\phi})}_k \tilde{\mathbf{a}}(\boldsymbol{\phi}) \boldsymbol{\theta}).$$

QED.