自抗扰控制设计方法的新进展

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本次课程目的：了解相关方向的一些前沿热点

自抗扰控制设计方法的新进展

- 降阶ESO的设计方法
- 基于相对阶的自抗扰控制设计
- 基于自抗扰控制的反步法设计
- 预测自抗扰控制设计方法

正常阶的ESO：(n+1)-th order

非线性
不确定系统

$$\begin{cases} \dot{x} = Ax + B\left(f(\bullet) + \bar{b}u \right) \\ y = C^T x + n \end{cases}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$x: state \quad y: output \quad u: control\ input$

$n: measurement\ noise$

$f(\bullet) \triangleq a^T x + d(t) + \left(b - \bar{b}\right) u: total\ disturbance$

线性ESO

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - \beta_1(\hat{x}_1 - y) \\ \vdots \\ \dot{\hat{x}}_n = \hat{x}_{n+1} - \beta_n(\hat{x}_1 - y) + bu \\ \dot{\hat{x}}_{n+1} = -\beta_{n+1}(\hat{x}_1 - y) \end{cases},$$

ESO带宽

$$s^{n+1} + \sum_{i=1}^{n+1} \beta_i s^{n+1-i} = (s + \omega_e)^{n+1},$$

传递函数： 从扰动到 $\hat{x}_{n+1}$ $\qquad Q_N = \dfrac{\hat{x}_{n+1}(s)}{f(s)} = \dfrac{\omega_e^{\ n+1}}{(s+\omega_e)^{n+1}},$

传递函数：从噪声到 $\hat{x}_{n+1}$ $\qquad G_N = \dfrac{\hat{x}_{n+1}(s)}{n(s)} = \dfrac{\omega_e^{\ n+1}}{(s+\omega_e)^{n+1}} s^n$

降阶ESO (RESO)：n-th order

非线性
不确定系统

$$\begin{cases} \dot{x} = Ax + B\left(f(\bullet) + \bar{b}u \right) \\ y = C^T x + n \end{cases}$$

$x: state \quad y: output \quad u: control\ input$

$n: measurement\ noise$

$f(\bullet) \triangleq a^T x + d(t) + \left(b - \bar{b}\right)u : total\ disturbance$

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

RESO

$$\begin{cases} \dot{\eta}_1 = \begin{cases} -\beta_1\eta_1 - \beta_1^2 x_1 - \beta_1 \bar{b}(t)u(t), \text{ if } n = 1. \\ -\beta_1\eta_1 + \eta_2 + (\beta_2 - \beta_1^2)y, \text{ if } n > 1. \end{cases} \\ \dot{\eta}_2 = -\beta_2\eta_2 + \eta_3 + (\beta_2 - \beta_1\beta_2)y \\ \dots \\ \dot{\eta}_{n-1} = -\beta_{n-1}\eta_1 + \eta_n + (\beta_n - \beta_1\beta_{n-1})y + \bar{b}(t)u(t) \\ \dot{\eta}_n = -\beta_n\eta_1 - \beta_1\beta_n y \end{cases}$$

$$\hat{x}_{i+1} = \eta_i + \beta_{i-1}y, \quad i = 1, 2, \dots, n,$$

RESO带宽

$$s^n + \sum_{i=1}^{n} \beta_i s^{n-i} = (s + \omega_e)^n$$

传递函数：从扰动到 $\hat{x}_{n+1}$ $\quad Q_R = \dfrac{\hat{x}_{n+1}(s)}{f(s)} = \dfrac{\omega_e^{\ n}}{(s+\omega_e)^n},$

传递函数：从噪声到 $\hat{x}_{n+1}$ $\quad G_R = \dfrac{\hat{x}_{n+1}(s)}{n(s)} = \dfrac{\omega_e^{\ n}}{(s+\omega_e)^n} s^n$

RESO V.S ESO

	ESO	RESO
传递函数：从扰动到 $\hat{x}_{n+1}$ ：	$Q_N = \dfrac{\omega_e^{n+1}}{(s+\omega_e)^{n+1}}$,	$Q_R = \dfrac{\omega_e^{n}}{(s+\omega_e)^{n}}$,
传递函数：从噪声到 $\hat{x}_{n+1}$ ：	$G_N = \dfrac{\omega_e^{n+1}}{(s+\omega_e)^{n+1}} s^n$,	$G_R = \dfrac{\omega_e^{n}}{(s+\omega_e)^{n}} s^n$

$$\angle \frac{Q_R(j\omega)}{Q_N(j\omega)} > 0$$

总扰动的跟踪

RESO > ESO

$$|G_R(j\omega)| > |G_N(j\omega)|$$

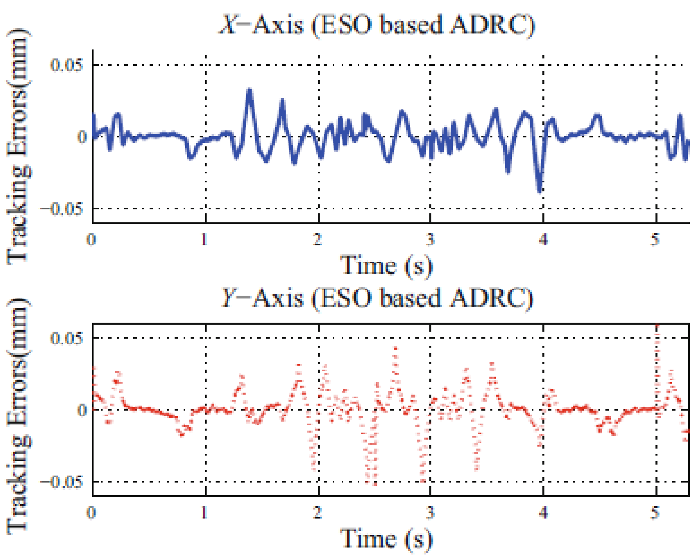
对量测噪声的滤波

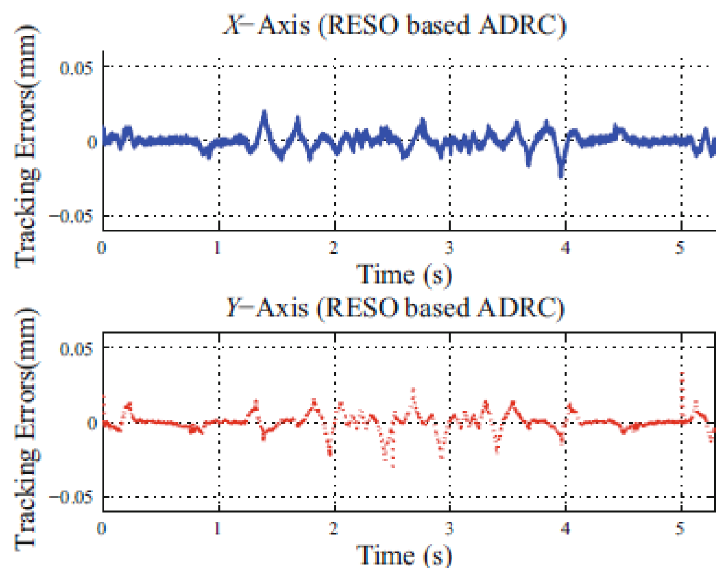
RESO < ESO

仿真结果: 伺服系统

$$
\begin{cases}
\dot{x}_1 = x_2, \\
\dot{x}_2 = x_3 + b_0 u, \\
y = x_1,
\end{cases}
\qquad x_3 = -a_1 x_2 - f_d.
$$

RESO使得跟踪误差更小；ESO使得滤波误差更小；





J. Guo, W. Xue, and T. Hu, Active Disturbance Rejection Control for PMLM Servo System in CNC Machining, Journal of Systems Science & Complexity, vol. 29, issue 1, pp. 74–98, 2016*

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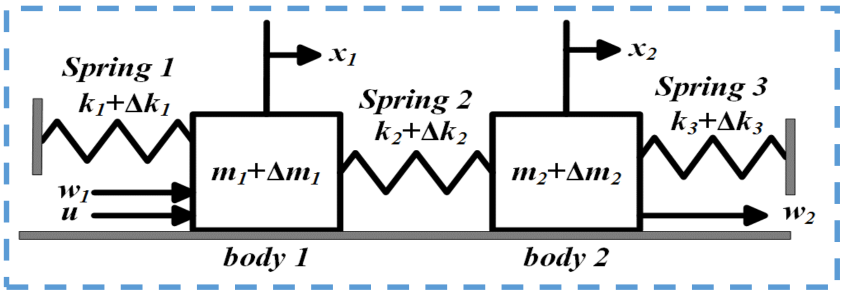
刚性+弹性耦合系统



高超声速飞行器
细长飞行器

柔性机器人

吊缆(细长绳、杆)

刚性+弹性耦合系统

$$\begin{cases} \dot{x}(t) = Ax(t) + b_u u(t) + B_f f(x,u,t) \\ y(t) = c^T x(t) = x_2(t) \end{cases}$$

扰动

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{k_1+k_2}{m_1} & \dfrac{k_2}{m_1} & 0 & 0 \\ \dfrac{k_2}{m_2} & -\dfrac{k_2+k_3}{m_2} & 0 & 0 \end{bmatrix}, \quad b_u = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{m_1} \\ 0 \end{bmatrix}, B_f = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, c = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix},$$

$$f(x,u,t) = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} \dfrac{m_1 w_1 - \Delta m_1 u + \left((k_1+k_2)\Delta m_1 - (\Delta k_1 + \Delta k_2) m_1\right) x_1 + (\Delta k_2 m_1 - k_2 \Delta m_1) x_2}{m_1 (m_1 + \Delta m_1)} \\ \dfrac{m_2 w_2 + (\Delta k_2 m_2 - k_2 \Delta m_2) x_1 + \left((k_2+k_3)\Delta m_2 - (\Delta k_2 + \Delta k_3) m_2\right) x_2}{m_2 (m_2 + \Delta m_2)} \end{bmatrix}$$

已知模型部分
(非积分器串联型)

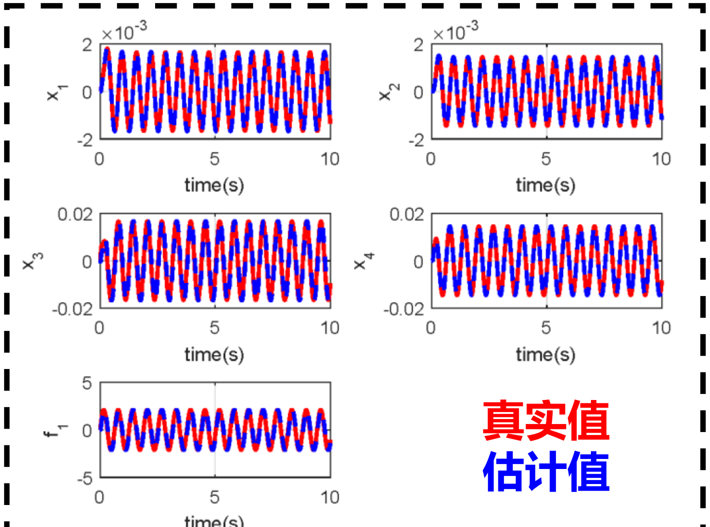
扰动f_2不匹配
扰动和输入不在一个通道

- **仅有扰动f_1**

只需要设计ESO估计f_1

$$\begin{cases}\Delta m_i = 0\,(kg), \Delta k_j = 0\,(N/m), \\ w_1 = 5\sin(10t)/m_1\,(N),\ w_2 = 0\,(N)\end{cases}$$

$$\begin{cases} f_1 = 5\sin(10t)/m_1\,(N) \\ f_2 = 0\,(N)\end{cases}$$



- **仅有扰动f_2**

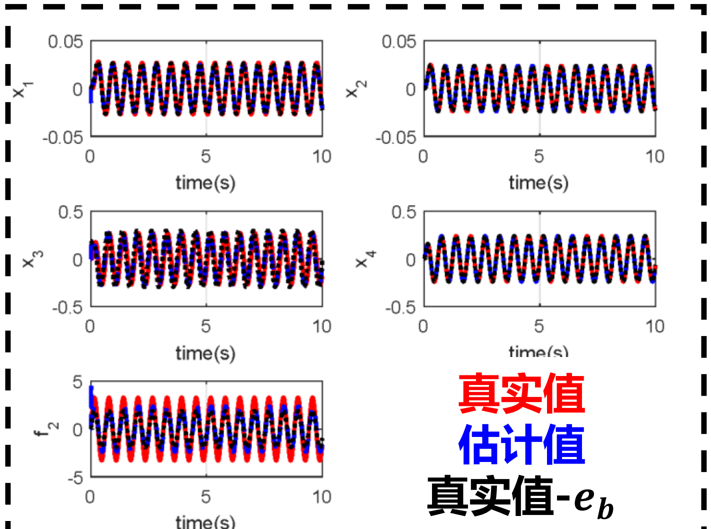
只需要设计ESO估计f_2

$$\begin{cases}\Delta m_i = 0\,(kg), \Delta k_j = 0\,(N/m), \\ w_1 = 0\,(N),\ w_2 = 5\sin(10t)/m_1\,(N)\end{cases}$$

$$\begin{cases} f_1 = 0\,(N) \\ f_2 = 5\sin(10t)/m_1\,(N)\end{cases}$$

估计偏差

$$e_b(t) = \begin{bmatrix} 0 & 1.04\times10^{-5} & 0 \\ 0 & 0 & 0 \\ -0.0038 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -0.028 & 0 \end{bmatrix} \begin{bmatrix} f_2^{(1)} \\ f_2^{(2)} \\ f_2^{(3)} \end{bmatrix}$$



- **不一定每个扰动的估计误差都收敛到0**

能观性条件不一定成立

刚性+弹性耦合系统:

$$\begin{cases} \dot{x}(t) = Ax(t) + b_u u(t) + B_f f(x,u,t) \\ y(t) = c^T x(t) = x_2(t) \end{cases}$$

扰动

$$f(x,u,t) = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} \dfrac{m_1 w_1 - \Delta m_1 u + \left((k_1+k_2)\Delta m_1 - (\Delta k_1 + \Delta k_2) m_1\right) x_1 + (\Delta k_2 m_1 - k_2 \Delta m_1) x_2}{m_1 (m_1 + \Delta m_1)} \\ \dfrac{m_2 w_2 + (\Delta k_2 m_2 - k_2 \Delta m_2) x_1 + \left((k_2+k_3)\Delta m_2 - (\Delta k_2 + \Delta k_3) m_2\right) x_2}{m_2 (m_2 + \Delta m_2)} \end{bmatrix}$$

非积分器串联型

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{k_1+k_2}{m_1} & \dfrac{k_2}{m_1} & 0 & 0 \\ \dfrac{k_2}{m_2} & -\dfrac{k_2+k_3}{m_2} & 0 & 0 \end{bmatrix}, \quad b_u = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{m_1} \\ 0 \end{bmatrix}, B_f = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, c = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

- 扰动 f_2 不匹配!
- 扰动不能观!

- 输入到输出相对阶为4
- 4阶积分器串联型内核
- 总扰动

积分器串联型内核

$$\begin{cases} \dot{\tilde{x}}_1 = \tilde{x}_2, \\ \dot{\tilde{x}}_2 = \tilde{x}_3, \\ \dot{\tilde{x}}_3 = \tilde{x}_4, \\ \dot{\tilde{x}}_4 = bu + f_{total}, \end{cases}$$

$$y = \tilde{x}_1 = x_2, b = \frac{k_2}{m_1 m_2}.$$

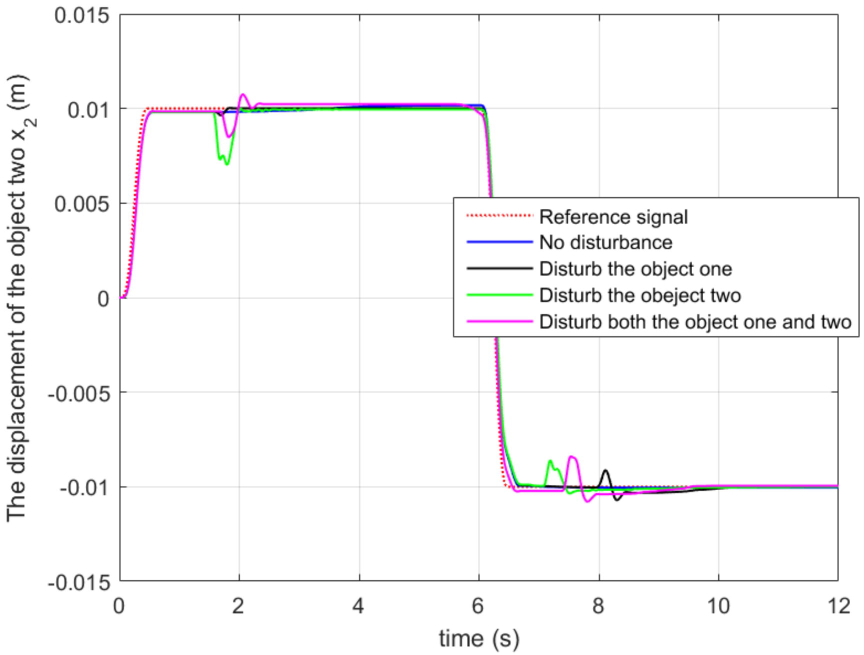
总扰动

$$\begin{aligned} f_{total} = &-\frac{(k_1+\Delta k_1)(k_2+\Delta k_2)+(k_1+\Delta k_1)(k_3+\Delta k_3)+(k_2+\Delta k_2)(k_3+\Delta k_3)}{(m_1+\Delta m_1)(m_2+\Delta m_2)}\tilde{x}_1 \\ &-\frac{(m_2+\Delta m_2)(k_1+\Delta k_1+k_2+\Delta k_2)+(m_1+\Delta m_1)(k_2+\Delta k_2+k_3+\Delta k_3)}{(m_1+\Delta m_1)(m_2+\Delta m_2)}\tilde{x}_2 \\ &+\left(\frac{k_2+\Delta k_2}{(m_1+\Delta m_1)(m_2+\Delta m_2)} - \frac{k_2}{m_1 m_2}\right)u + \frac{k_2+\Delta k_2}{m_1(m_2+\Delta m_2)}w_1 + \frac{1}{m_2+\Delta m_2}w_2^{(2)} \\ &-\frac{(k_1+\Delta k_1+k_2+\Delta k_2)(m_2+\Delta m_2)+2(k_2+\Delta k_2+k_3+\Delta k_3)(m_1+\Delta m_1)}{(m_1+\Delta m_1)(m_2+\Delta m_2)^2}w_2 \end{aligned}$$

- 5阶扩张状态观测器
- ADRC控制

实验1 （外界扰动）

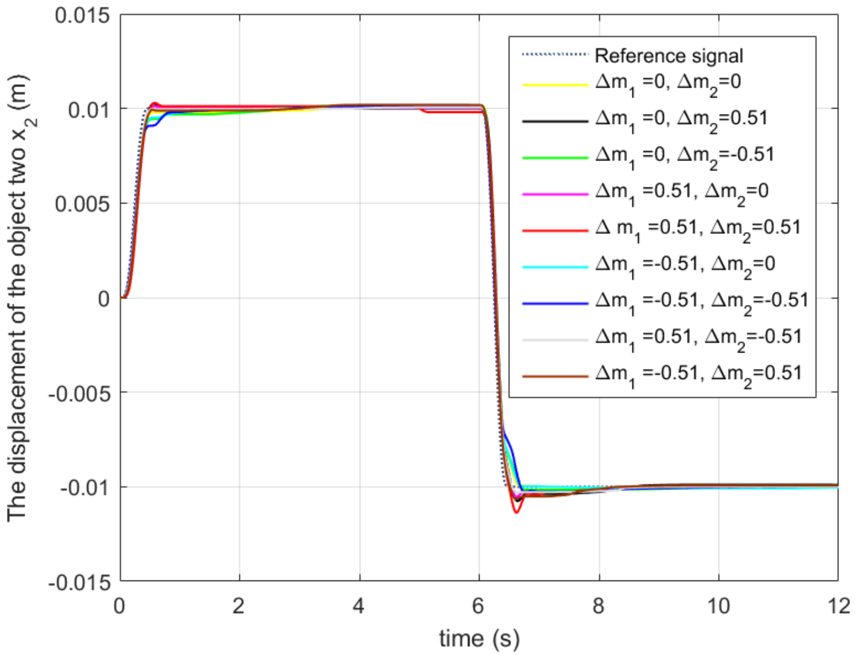
●情况1：无扰动
●情况2：扰动物体1（2秒与8秒附近）
●情况3：扰动物体2（2秒与8秒附近）
●情况4：扰动物体1与物体2（2秒与8秒附近）



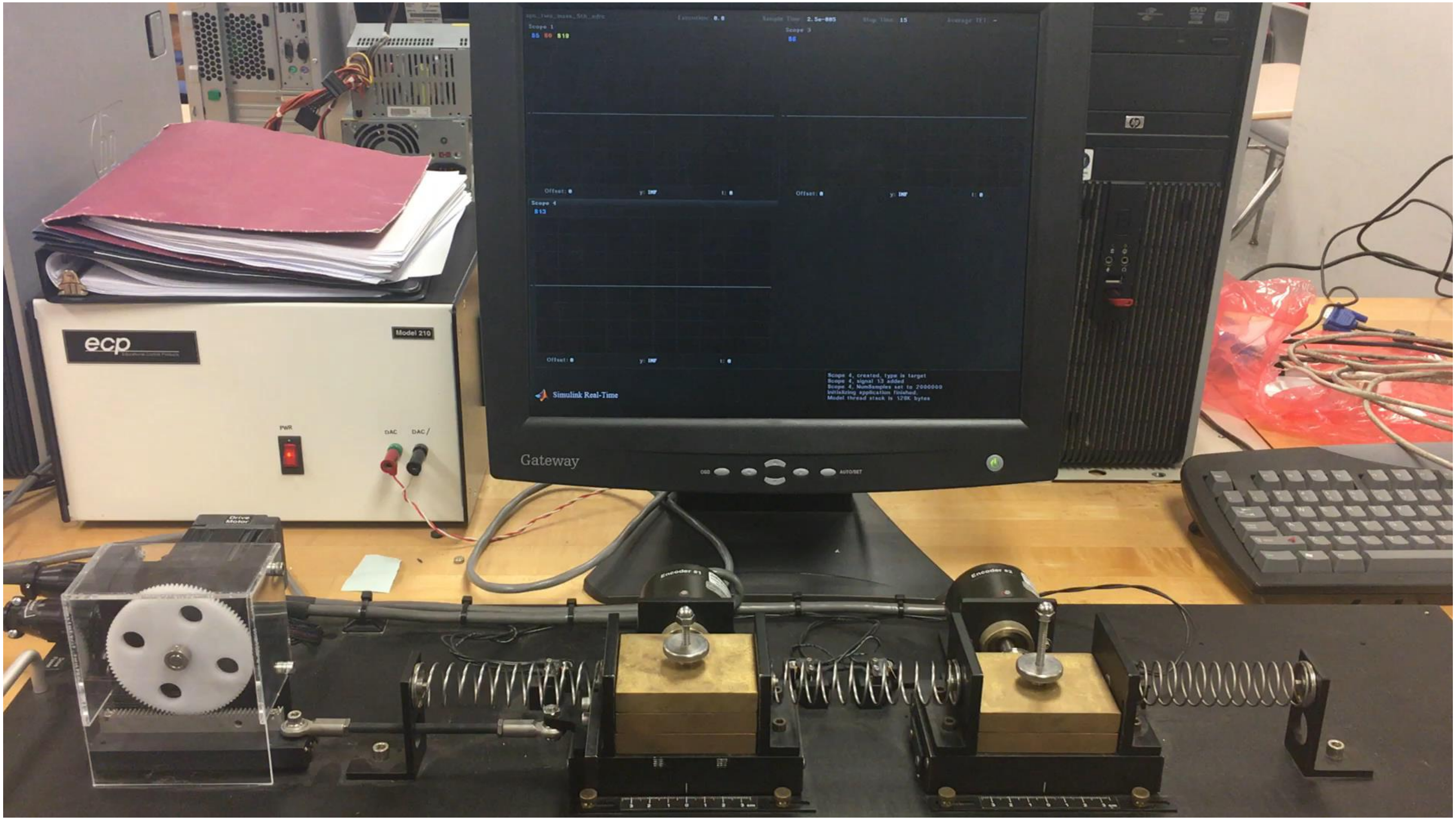
实验2 （质量偏差）

$$\{(\Delta m_1, \Delta m_2) \mid \Delta m_1 \in \Delta_m, \Delta m_2 \in \Delta_m\},$$

$$\Delta_m = \{0(kg), -0.51(kg), 0.51(kg)\}$$



实验结果


ecp
Model 210
Gateway
Simulink Real-Time
Drive Motor
Encoder #1
Encoder #2

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飞机：
攻角控制系统

X_1 : **angle of attack**
X_2 : **angular rate**

无人碾压机：
路径跟踪控制

X_1 : **cross-track error**
X_2 : **error course angle**

$$\dot{X}_1 = b_1(X_1,t)X_2 + f_1(X_1,t)$$
$$\dot{X}_2 = b_2(X_1,X_2,t)U + f_2(X_1,X_2,t)$$

- 下三角系统: 不匹配的扰动
- 状态: 所有都可以被量测

W. Xue, and Y. Huang, On Performance Analysis of ADRC for a Class of MIMO Lower-triangular Nonlinear Uncertain Systems, ISA Transactions, vol. 53, pp. 955-962, 2014.

S.Chen, W. Xue, Y. Lin and Y. Huang, On Active Disturbance Rejection Control for Path Following of Automated Guided Vehicle with Uncertain Velocities, ACC 2019, 2019.

对于下三角系统的多个ESO设计

$$\dot{X}_1 = b_1(X_1,t)X_2 + f_1(X_1,t)$$
$$\dot{X}_2 = b_2(X_1,X_2,t)U + f_2(X_1,X_2,t)$$

$$\dot{X}_1 = b_1(X_1,t)X_2 + f_1(X_1,t)$$

虚拟输入

第1个子系统
的总扰动

$$\begin{cases} \dot{\hat{X}}_1 = b_1(X_1,t)X_2 - \beta_{1,1}(\hat{X}_1 - X_1) + \hat{\Delta}_1 \\ \dot{\hat{\Delta}}_1 = -\beta_{1,2}(\hat{X}_1 - X_1) \end{cases}$$

第1个子系统的RESO

$$\dot{X}_2 = b_2(X_1,X_2,t)U + f_2(X_1,X_2,t)$$

第2个子系统
的总扰动

$$\begin{cases} \dot{\hat{X}}_2 = \hat{b}_2(t)U - \beta_{2,1}(\hat{X}_2 - X_2) + \hat{\Delta}_2 \\ \dot{\hat{\Delta}}_2 = -\beta_{2,2}(\hat{X}_2 - X_2) \end{cases}$$

第2个子系统的RESO

$$\bar{X}_2 = b_1(\cdot)^{-1}(-K_1X_1 - \hat{\Delta}_1(\cdot))$$

虚拟输入的参考信号

$$U = \hat{b}_2(\cdot)^{-1}\left(-K_2(X_2 - \bar{X}_2) - \hat{\Delta}_2(\cdot) + \dot{\bar{X}}_2\right)$$

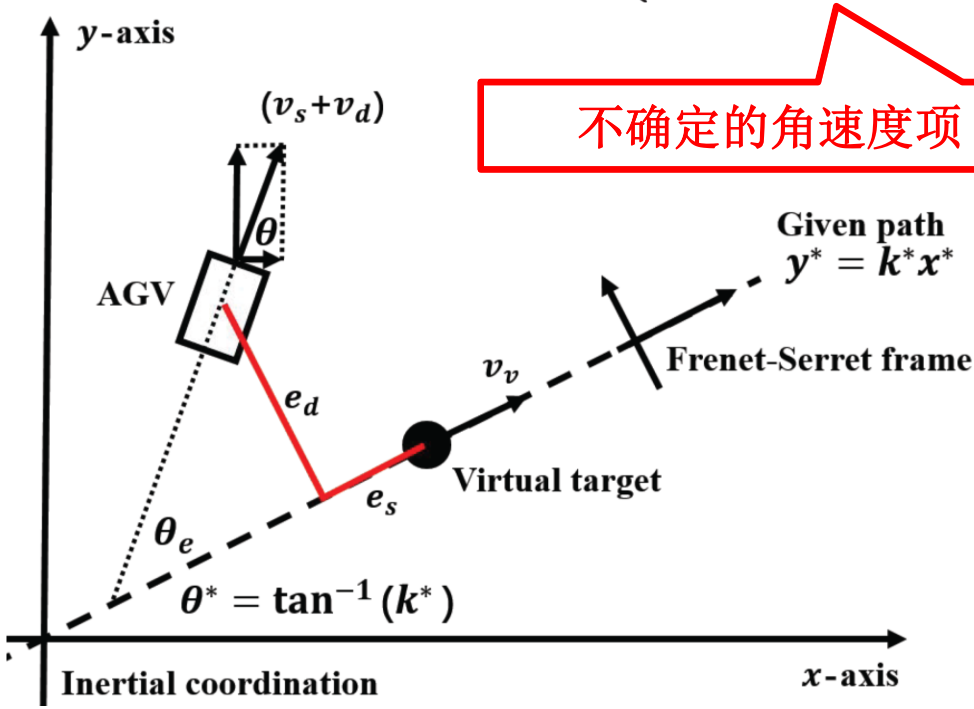
控制输入设计

无人碾压机：路径跟踪控制

$$\begin{cases} \dot{x}(t) = (v_s + v_d(t))\cos(\theta(t)), \\ \dot{y}(t) = (v_s + v_d(t))\sin(\theta(t)), \qquad t \geq t_0, \\ \dot{\theta}(t) = \omega_s(t) + \omega_d(t), \end{cases}$$

不确定的角速度项

不确定的线速度项



下三角系统

$$\boxed{\begin{cases} \dot{e}_d(t) = v_s \sin(\theta_e(t)) + f_1, \\ \dot{\theta}_e(t) = \omega_s(t) + f_2, \end{cases}}$$

S. Chen, W. Xue, Y. Lin and Y. Huang, On Active Disturbance Rejection Control for Path, 2018. Following of Automated Guided Vehicle with Uncertain Velocities, ACC 2019.

设计两个RESO

$$\begin{cases} \dot{e}_d(t) = v_s \sin(\theta_e(t)) + f_1, \\ \dot{\theta}_e(t) = \omega_s(t) + f_2, \end{cases}$$

$$\begin{cases} \dot{\xi}_1(t) = -\beta_1 \xi_1(t) - \beta_1^2 e_d(t) - \beta_1 v_s \sin(\theta_e(t)), \\ \hat{f}_1(t) = \beta_1 e_d(t) + \xi_1(t), \end{cases}$$

$$\begin{cases} \dot{\xi}_2(t) = -\beta_2 \xi_2(t) - \beta_2^2 \theta_e(t) - \beta_2 \omega_s(t), \\ \hat{f}_2(t) = \beta_2 \theta_e(t) + \xi_2(t), \end{cases}$$

虚拟输入的参考值

$$\theta_{ed}(t) = \begin{cases} \arcsin\left(\dfrac{-\hat{f}_1(t) - k_d e_d(t)}{v_s}\right), & \text{if } \left|\dfrac{-\hat{f}_1(t) - k_d e_d(t)}{v_s}\right| < 1, \\ \pi/2, & \text{if } \dfrac{-\hat{f}_1(t) - k_d e_d(t)}{v_s} > 1, \\ -\pi/2, & \text{if } \dfrac{-\hat{f}_1(t) - k_d e_d(t)}{v_s} < -1. \end{cases}$$

控制输入加入饱和

$$\omega_s(t) = -\hat{f}_2(t) - k_\theta(\theta_e(t) - \theta_{ed}(t)) + \dot{\theta}_{ed}(t),$$

控制输入设计

$$
\begin{cases}
\dot{\xi}_1(t) = -\beta_1 \xi_1(t) - \beta_1^2 e_d(t) - \beta_1 v_s \sin(\theta_e(t)), \\
\hat{f}_1(t) = \beta_1 e_d(t) + \xi_1(t),
\end{cases}
$$

$$
\begin{cases}
\dot{\xi}_2(t) = -\beta_2 \xi_2(t) - \beta_2^2 \theta_e(t) - \beta_2 \omega_s(t), \\
\hat{f}_2(t) = \beta_2 \theta_e(t) + \xi_2(t),
\end{cases}
$$

$$
\theta_{ed}(t) = \begin{cases}
\arcsin\left(\dfrac{-\hat{f}_1(t) - k_d e_d(t)}{v_s}\right), & \text{if } \left|\dfrac{-\hat{f}_1(t) - k_d e_d(t)}{v_s}\right| < 1, \\
\pi/2, & \text{if } \dfrac{-\hat{f}_1(t) - k_d e_d(t)}{v_s} > 1, \\
-\pi/2, & \text{if } \dfrac{-\hat{f}_1(t) - k_d e_d(t)}{v_s} < -1.
\end{cases}
$$

$$
\omega_s(t) = -\hat{f}_2(t) - k_\theta(\theta_e(t) - \theta_{ed}(t)) + \dot{\theta}_{ed}(t),
$$

Theorem: Under certain conditions we have that for any given $\varepsilon > 0$ and $t_\varepsilon > 0$, there exist $(k_\theta^*, \beta_1^*, \beta_2^*)$ such that

$$
\sup_{t \in [t_0, \infty)} |e_d(t) - e_d^*(t)| \leq \varepsilon, \tag{2}
$$

where

真实误差与理想轨迹之间的误差可以调节的任意小

$$
\dot{e}_d^*(t) = -k_d e_d^*(t), \quad t \geq t_0, \quad e_d^*(t_0) = e_d(t_0), \tag{3}
$$

for $\forall k_\theta \geq k_\theta^*$, $\forall \beta_1 \geq \beta_1^*$ and $\forall \beta_2 \geq \beta_2^*$.

误差的理想轨迹

无人碾压机：路径跟踪控制的仿真结果

$$\begin{cases} \dot{x}(t) = (v_s + v_d(t))\cos(\theta(t)), \\ \dot{y}(t) = (v_s + v_d(t))\sin(\theta(t)), & t \geq t_0, \\ \dot{\theta}(t) = \omega_s(t) + \omega_d(t), \end{cases} \qquad (1)$$

4种不确定性

$$\begin{cases} C1: v_d = \omega_d = 0, \\ C2: v_d = 0.1(1+2\sin(2t))v_s, \quad \omega_d = 2+3\sin(2t), \\ C3: v_d = 0.3v_s\sin(2t), \quad \omega_d = \begin{cases} -5, & 0 \leq t < 1, \\ 5, & 1 \leq t < 3, \\ -5, & 3 \leq t < 5, \\ 5, & t \geq 5, \end{cases} \\ C4: v_d = \begin{cases} -0.3v_s, & 0 \leq t < 1, \\ 0.3v_s, & 1 \leq t < 3, \\ -0.3v_s, & 3 \leq t < 5, \\ 0.3v_s, & t \geq 5, \end{cases} \quad \omega_d = 5\sin(2t), \end{cases}$$

自抗扰控制的参数固定

$$k_d = 2, \quad k_\theta = 100, \quad \beta_1 = 10, \quad \beta_2 = 20.$$

轨迹跟踪：仿真结果

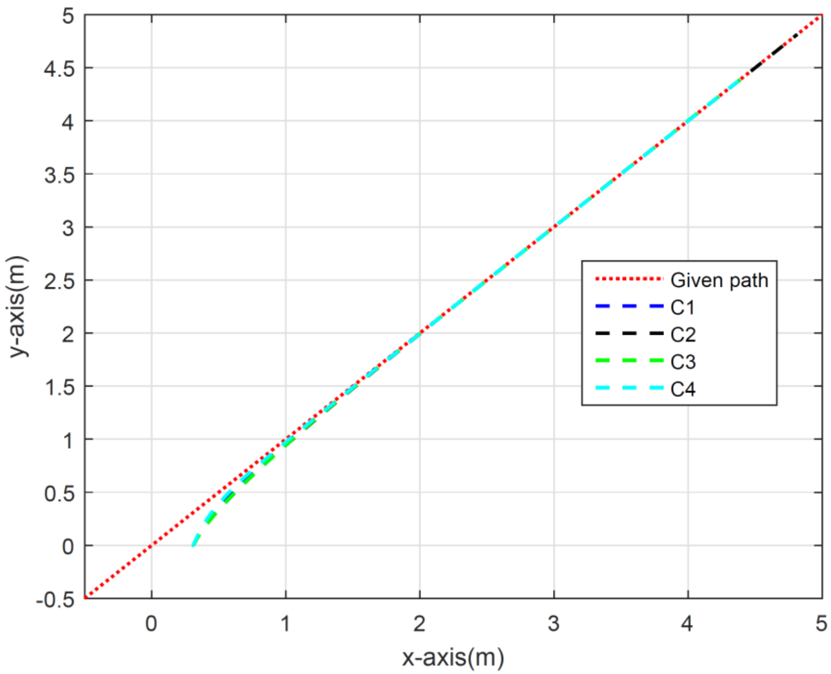


Fig. 2. Tracking results of the AGV with uncertainties (70).

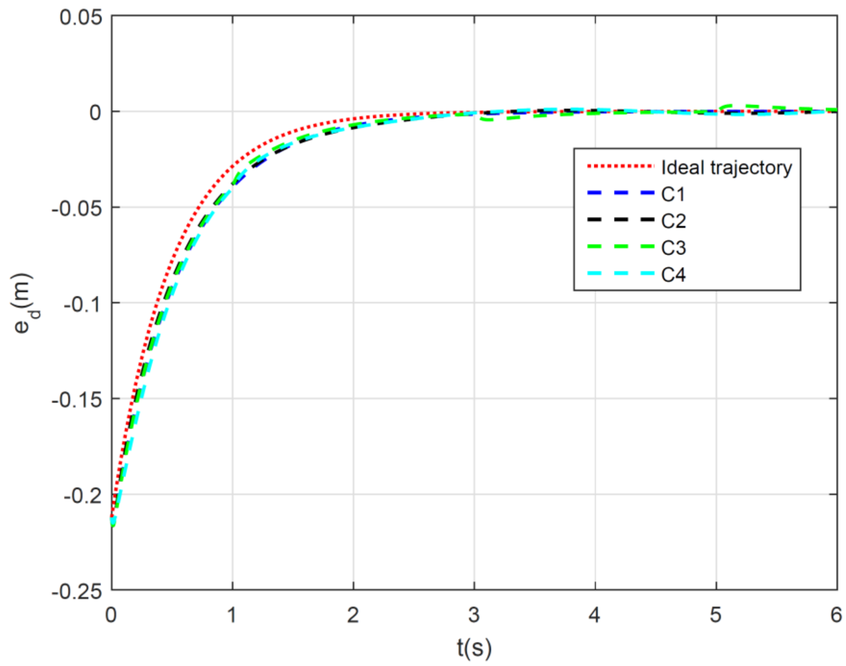


Fig. 3. Cross-track errors for four cases of uncertainties (70).

$$\dot{X}_1 = b_1(X_1,t)X_2 + f_1(X_1,t)$$

虚拟输入

第1个子系统的总扰动

$$\begin{cases} \dot{\hat{X}}_1 = b_1(X_1,t)X_2 - \beta_{1,1}(\hat{X}_1 - X_1) + \hat{\Delta}_1 \\ \dot{\hat{\Delta}}_1 = -\beta_{1,2}(\hat{X}_1 - X_1) \end{cases}$$

第1个子系统的RESO

$$\bar{X}_2 = b_1(\cdot)^{-1}(-K_1X_1 - \hat{\Delta}_1(\cdot))$$

虚拟输入的参考信号

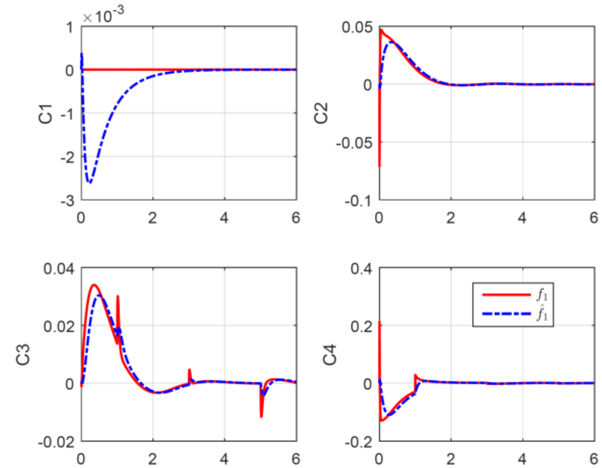


Fig. 4. The estimations of the "total disturbance" f_1 for four cases of uncertainties (70).

$$\dot{X}_2 = b_2(X_1,X_2,t)U + f_2(X_1,X_2,t)$$

第2个子系统的总扰动

$$\begin{cases} \dot{\hat{X}}_2 = \hat{b}_2(t)U - \beta_{2,1}(\hat{X}_2 - X_2) + \hat{\Delta}_2 \\ \dot{\hat{\Delta}}_2 = -\beta_{2,2}(\hat{X}_2 - X_2) \end{cases}$$

第2个子系统的RESO

$$U = \hat{b}_2(\cdot)^{-1}\left(-K_2(X_2 - \bar{X}_2) - \hat{\Delta}_2(\cdot) + \dot{\bar{X}}_2\right)$$

控制输入设计

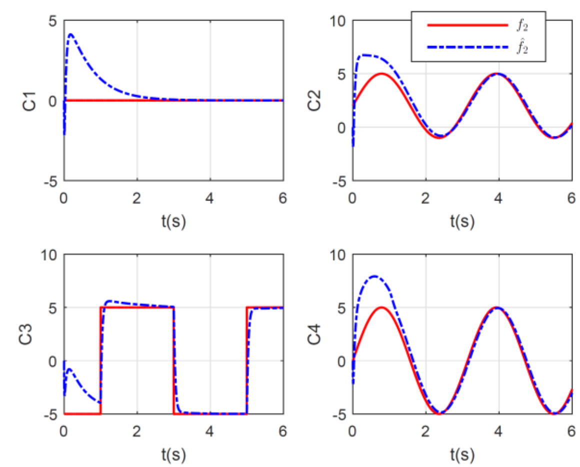


Fig. 5. The estimations of the "total disturbance" f_2 for four cases of uncertainties (70).

自抗扰控制设计方法的新进展

- 降阶ESO的设计方法
- 基于相对阶的自抗扰控制设计
- 基于自抗扰控制的反步法设计
- 预测自抗扰控制设计方法

非线性不确定时滞系统

$$\begin{cases} \dot{x}(t) = Ax(t) + B\left(u(t-\tau) + \delta\left(x(t),t\right)\right) \\ y(t) = C^T x(t) \end{cases}$$

输入延迟

总扰动

(注：输入输出延迟模型可等价变换为输入延迟模型)

总扰动：	$\delta(x,t) \in R$	控制输入：	$u(t) \in R$
时延：	$\tau \in R$	量测与被控输出：	$y(t) \in R$
系统状态：	$x(t) \in R^n$	已知模型部分：	A, B, C

理想轨线 $\boldsymbol{x}^*(\boldsymbol{t})$

$$\begin{cases} \dot{x}^*(t) = Ax^*(t) - BK^T\left(x^*(t) - r(t)\right) \\ x^*(t_0) = x(t_0) \end{cases}$$

控制参数

参考信号

$$A_K = A - BK^T$$

设计特征值

快速、无超调

控制目的：设计输入使状态跟踪理想轨线

理想控制输入

$$u^*(t) = -K^T\left(x(t+\tau) - r(t+\tau)\right) - \delta\left(x(t+\tau), t+\tau\right)$$

预估系统状态与扰动

针对时延系统改进的自抗扰控制算法

- **PO-ADRC: Predictor Observer based ADRC (基于预测观测器的自抗扰)**
- **DD-ADRC: Delayed designed ADRC (匹配时延自抗扰)**
- **PP-ADRC: Polynomial based predictive ADRC (基于多项式预测的自抗扰)**
- **SP-ADRC: Smith Predictor based ADRC (基于斯密斯预估器的自抗扰)**

Predictor Observer based ADRC (PO-ADRC)

无扰系统

$$\begin{cases}\dot{X}(t)=AX(t)+Bu(t-\tau)\\ Y(t)=C^{T}X(t)\end{cases}$$

预测观测器

$$\begin{cases}\dot{\hat{X}}(t+\tau)=A\hat{X}(t+\tau)+Bu(t)+e^{A\tau}L\left(Y(t)-\hat{Y}(t+\tau)\right)\\ \hat{Y}(t+\tau)=C^{T}\hat{X}(t)+C^{T}\int_{0}^{\tau}e^{A\theta}L\left(Y(t-\theta)-\hat{Y}(t+\tau-\theta)\right)d\theta\end{cases}$$

[13] M. Krstic. Delay Compensation for Nonlinear, Adaptive, and PDE Systems. Cambridge, MA: Birkhäuser Boston, 2009.

$$\begin{cases}\begin{bmatrix}\dot{\hat{x}}_{PO}(t+\tau)\\ \dot{\hat{\delta}}_{PO}(t+\tau)\end{bmatrix}=A_e\begin{bmatrix}\hat{x}_{PO}(t+\tau)\\ \hat{\delta}_{PO}(t+\tau)\end{bmatrix}+B_eu(t)+e^{A_e\tau}L_{e,PO}\left(y(t)-\hat{y}_{PO}(t+\tau)\right)\\ \hat{y}_{PO}(t+\tau)=C^{T}\hat{x}_{PO}(t)+C_e^{T}\int_{0}^{\tau}e^{A\theta}L_{e,PO}\left(y(t-\theta)-\hat{y}_{PO}(t+\tau-\theta)\right)d\theta\end{cases}$$

扩张状态预测观测器

$$A_e=\begin{bmatrix}A & B\\ 0 & 0\end{bmatrix},B_e=\begin{bmatrix}B\\ 0\end{bmatrix},C_e=\begin{bmatrix}C\\ 0\end{bmatrix}$$

Predictor Observer based ADRC (PO-ADRC)

扩张状态预测观测器

$$
\begin{cases}
\begin{bmatrix} \dot{\hat{x}}_{PO}(t+\tau) \\ \dot{\hat{\delta}}_{PO}(t+\tau) \end{bmatrix} = A_e \begin{bmatrix} \hat{x}_{PO}(t+\tau) \\ \hat{\delta}_{PO}(t+\tau) \end{bmatrix} + B_e u(t) + e^{A_e \tau} L_{e,PO}\left(y(t) - \hat{y}_{PO}(t+\tau)\right) \\
\hat{y}_{PO}(t+\tau) = C^T \hat{x}_{PO}(t) + C_e^T \int_0^{\tau} e^{A\theta} L_{e,PO}\left(y(t-\theta) - \hat{y}_{PO}(t+\tau-\theta)\right) d\theta
\end{cases}
$$

主动抗扰的控制设计

$$
u(t) = -K^T\left(\hat{x}_{PO}(t+\tau) - r(t+\tau)\right) - \hat{\delta}_{PO}(t+\tau)
$$

状态与扰动的预估值

[14] W. Xue, P. Liu, S. Chen, Y. Huang. On extended state predictor observer based active disturbance rejection control for uncertain systems with sensor delay. In 16th International Conference on Control, Automation and Systems, 2016.

Delayed designed ADRC (DD-ADRC)

匹配时延的扩张状态观测器

$$\begin{cases} \begin{bmatrix} \dot{\hat{x}}_{DD}(t) \\ \dot{\hat{\delta}}_{DD}(t) \end{bmatrix} = A_e \begin{bmatrix} \hat{x}_{DD}(t) \\ \hat{\delta}_{DD}(t) \end{bmatrix} + \boxed{B_e u(t-\tau)} + L_{e,DD}\left(y(t) - \hat{y}_{DD}(t)\right) \\ \hat{y}_{DD}(t) = C^T \hat{x}_{DD}(t) \end{cases}$$

时延匹配的控制输入

主动抗扰的控制设计

$$u(t) = -K^T\left(\hat{x}_{DD}(t) - r(t)\right) - \hat{\delta}_{DD}(t)$$

状态与扰动的预估值

[15] S. Zhao, Z. Gao. Modified active disturbance rejection control for time-delay systems. ISA Transactions, 2014.

Polynomial based predictive ADRC (PP-ADRC)

预测环节

泰勒展开

$$e^{\tau s}=1+\tau s+O\left(\tau^2 s^2\right)$$

量测预测

$$y_p\left(t\right)=\tau\dot{y}\left(t\right)+y\left(t\right)$$

扩张状态观测器

$$\begin{cases}\begin{bmatrix}\dot{\hat{x}}_{PP}\left(t+\tau\right)\\ \dot{\hat{\delta}}_{PP}\left(t+\tau\right)\end{bmatrix}=A_e\begin{bmatrix}\hat{x}_{PP}\left(t+\tau\right)\\ \hat{\delta}_{PP}\left(t+\tau\right)\end{bmatrix}+B_e u\left(t\right)+\underbrace{L_{e,PP}\left(y_P\left(t\right)-\hat{y}_{PP}\left(t+\tau\right)\right)}_{\text{量测预测值}}\\ \hat{y}_{PP}\left(t+\tau\right)=C^T\hat{x}_{PP}\left(t+\tau\right)\end{cases}$$

主动抗扰的控制设计

$$u\left(t\right)=-K^T\left(\hat{x}_{PP}\left(t+\tau\right)-r\left(t+\tau\right)\right)-\hat{\delta}_{PP}\left(t+\tau\right)$$

状态与扰动的预估值

[16] J. Han. Auto-Disturbances Rejection Control for Time-Delay Systems. Control engineering of China, 2008.

Smith Predictor based ADRC (SP-ADRC)

Smith预估器(预测环节)

$$y_{P,SP}(t) = y(t) - \tilde{y}_{SP}(t) + \tilde{y}_{SP}(t+\tau), \quad \begin{cases} \dot{\tilde{x}}_{SP}(t) = A\tilde{x}_{SP}(t) + Bu(t-\tau) \\ \tilde{y}_{SP}(t) = C^T \tilde{x}_{SP}(t) \end{cases}$$

量测预测

扩张状态观测器

$$\begin{cases} \begin{bmatrix} \dot{\hat{x}}_{SP}(t+\tau) \\ \dot{\hat{\delta}}_{SP}(t+\tau) \end{bmatrix} = A_e \begin{bmatrix} \hat{x}_{SP}(t+\tau) \\ \hat{\delta}_{SP}(t+\tau) \end{bmatrix} + B_e u(t) + \boxed{L_{e,SP}\left(y_{P,SP}(t) - \hat{y}_{SP}(t+\tau)\right)} \\ \hat{y}_{SP}(t+\tau) = C^T \hat{x}_{SP}(t+\tau) \end{cases}$$

量测预测值

主动抗扰的控制设计

$$u(t) = -K^T\left(\hat{x}_{SP}(t+\tau) - r(t+\tau)\right) - \hat{\delta}_{SP}(t+\tau)$$

状态与扰动的预估值

[17] Q. Zheng, Z. Gao. Predictive active disturbance rejection control for processes with time delay. ISA Transactions, 2014.

- 可处理的给定时延大小

	可处理的给定时延大小 (无扰情况)
PO-ADRC	**任意给定**
DD-ADRC	**有界**
PP-ADRC	**有界**
SP-ADRC	**任意给定**

Sen Chen, Wenchao Xue*, Sheng Zhong, and Yi Huang, On comparison of modified ADRCs for nonlinear uncertain systems with time delay, SCIENCE CHINA Information Sciences, Vol. 61 070223:1–070223:15, 2018.

- 对开环稳定条件的要求

	开环稳定条件 (有扰情况)
PO-ADRC	**非必要条件**
DD-ADRC	**非必要条件**
PP-ADRC	**非必要条件**
SP-ADRC	**必要条件**

Sen Chen, Wenchao Xue*, Sheng Zhong, and Yi Huang, On comparison of modified ADRCs for nonlinear uncertain systems with time delay, SCIENCE CHINA Information Sciences, Vol. 61 070223:1–070223:15, 2018.

● 抗扰性能(低频段)

假设参考信号与初始值为零

	抗扰性能 (低频段)
PO-ADRC	$\lim\limits_{\omega\to 0} G_{y\delta,a}(j\omega)=0$ $a=PO,DD,PP,SP$
DD-ADRC	
PP-ADRC	
SP-ADRC	

$$Y_a(s)=G_{y\delta,a}(s)\Delta_a(s),\quad a=PO,DD,PP,SP.$$

扰动到输出的传递函数

$Y_a(s)=L(y_a)$ 输出的拉普拉斯变换

$\Delta_a(s)=L(\delta_a)$ 扰动的拉普拉斯变换

Sen Chen, Wenchao Xue*, Sheng Zhong, and Yi Huang, On comparison of modified ADRCs for nonlinear uncertain systems with time delay, SCIENCE CHINA Information Sciences, Vol. 61 070223:1–070223:15, 2018.

● 抗扰性能(低频段)

	抗扰性能 (低频段, 一阶系统)
$\lim\limits_{\omega\to 0}\dfrac{G_{y\delta,\mathrm{PP}}(j\omega)}{G_{y\delta,\mathrm{DD}}(j\omega)}$	$O\left(\dfrac{1}{\omega_o}\right)$
$\lim\limits_{\omega\to 0}\dfrac{G_{y\delta,\mathrm{PP}}(j\omega)}{G_{y\delta,\mathrm{SP}}(j\omega)}$	
$\lim\limits_{\omega\to 0}\dfrac{G_{y\delta,\mathrm{PP}}(j\omega)}{G_{y\delta,\mathrm{PO}}(j\omega)}$	

PP-ADRC: 更强的低频段抗扰性能(较大观测器带宽ω_o)

$$\det\left(A_e - L_{e,a}C_e^T\right) = \left(s+\omega_o\right)^2$$

相同的观测器带宽ω_o

Sen Chen, Wenchao Xue*, Sheng Zhong, and Yi Huang, On comparison of modified ADRCs for nonlinear uncertain systems with time delay, SCIENCE CHINA Information Sciences, Vol. 61 070223:1–070223:15, 2018.

仿真分析

一阶非线性
不确定时延系统

$$\begin{cases} \dot{x}_1(t) = ax_1(t) + bu(t-\tau) + \delta(x_1, t) \\ y(t) = x_1(t) \end{cases}$$

延迟

总扰动

$$a = -6.9\times10^{-3},\ b = 4.35\times10^{-2},\ \tau = 60$$

[18] W. Tan, F. Fang, et al. Linear control of a boiler-turbine unit: analysis and design. ISA Transaction, 2008.

不确定性

$$Case\,1:\ \delta = \begin{cases} 0,\ 0\,(\mathrm{s}) \le t < 1000\,(\mathrm{s}) \\ 5,\ t \ge 1000\,(\mathrm{s}) \end{cases}$$

（突变外扰）

$$Case\,2:\ \delta = (ax_1)^2 + \sin\left(x_1 / 2\pi\right) + e^{a(x_1 - 300)}$$

（非线性未建模动态）

$$Case\,3:\ \delta = 0.2ax_1 / b$$

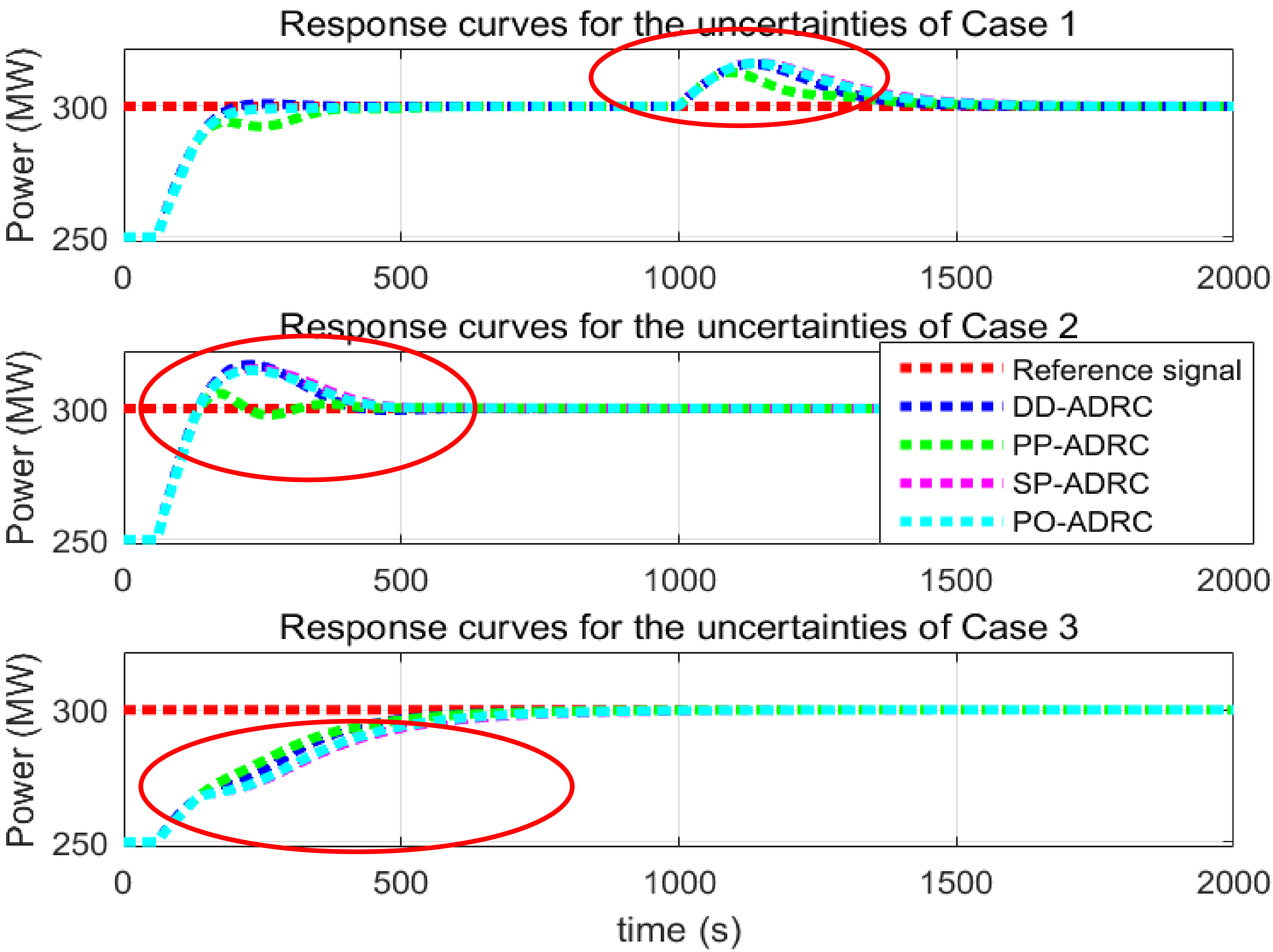
（参数不确定）

参考信号： $r = 300$

控制器参数： $K = 0.014,\ \omega_o = 0.015$

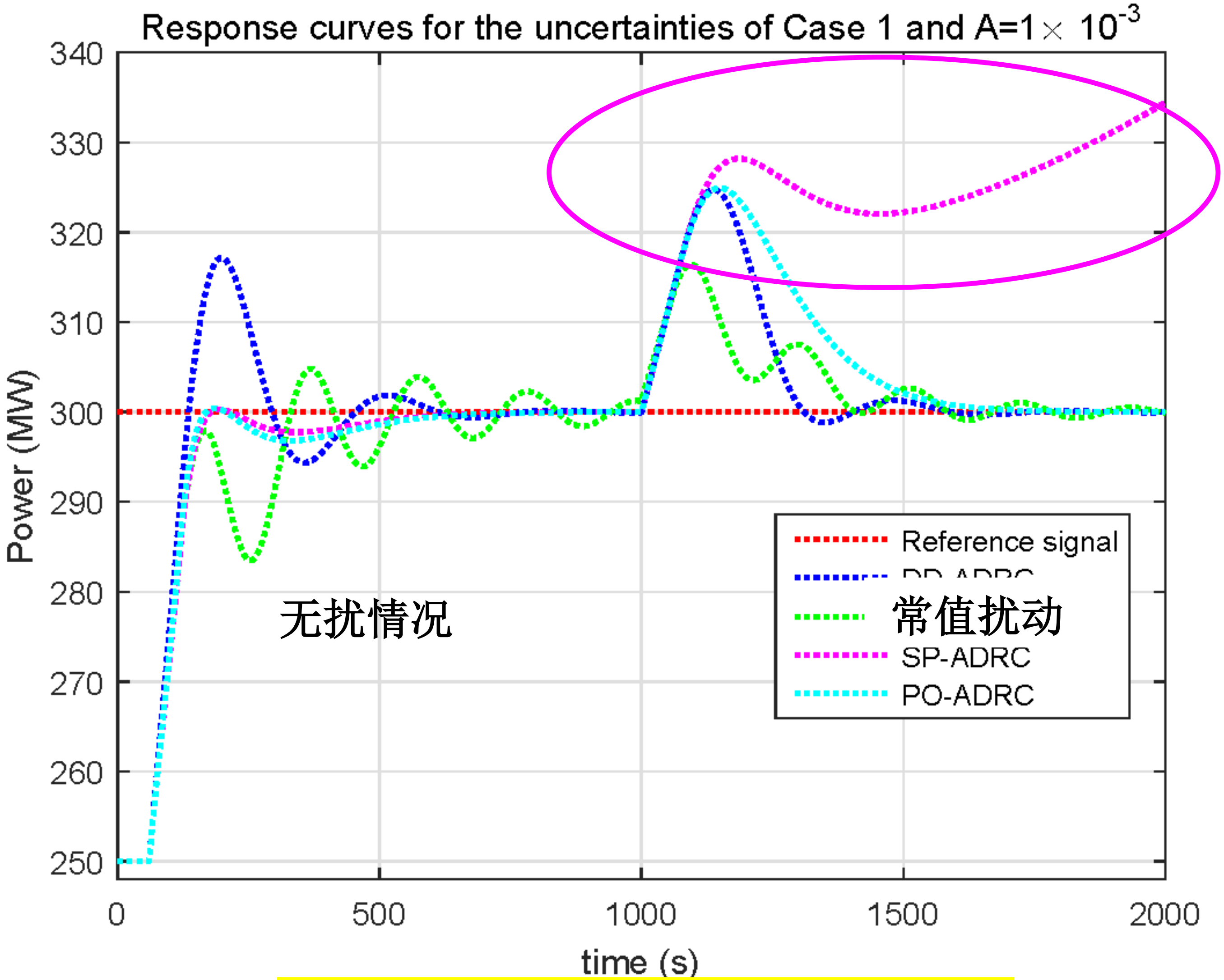
陈森，非线性不确定系统的自抗扰控制研究[D]，中国科学院大学, 2019.

四种针对时延系统改进ADRC的仿真结果(三种不确定性)



陈森，非线性不确定系统的自抗扰控制研究[D]，中国科学院大学, 2019.

四种针对时延系统改进ADRC的仿真结果(对象开环不稳定)



$a = 1 \times 10^{-3}$ (对象开环不稳定)

陈森，非线性不确定系统的自抗扰控制研究[D]，中国科学院大学, 2019.

作业

请找一个来自于实际控制系统的模型，利用自抗扰控制方法进行控制器设计，并通过仿真验证控制方法的有效性。

1. **可以通过书籍、论文等找到相关控制系统的数学模型；**
2. **可以用线性自抗扰控制方法，也可以用非线性自抗扰方法；**
3. **建议在仿真中考虑外部扰动及模型的不确定性；**
4. **可以用自抗扰控制器中的一部分，比如ESO等设计控制器。**

例子：

刚体机械臂转动控制

$$\begin{cases} \ddot{\theta} = -20.169\dot{\theta} - 8.255u + \delta(\dot{\theta}, \theta, t) \\ y = \theta \end{cases}$$

θ : 机械臂角度

$\delta(\dot{\theta}, \theta, t)$: 总扰动

u : 控制输入

y : 量测值

$\theta^* = 20\text{deg}$: 机械臂角度的设定值

Wenchao Xue, etc., Add-on Module of Active Disturbance Rejection for Set-Point Tracking of Motion Control Systems, IEEE Transactions on Industry Applications, vol. 53, no. 4, pp. 4028-4040, 2017.

谢谢！