2. 课本 322 页习题 7.16

解：(a)初始尺度 $j_0 = 1$，k 的取值为 0，1，基函数的值为 1，即 $\varphi(n) = \{1,1,1,1\}$，因此可得：

$$\varphi_{1,0}(n) = \sqrt{2}\varphi(2n-0) = \sqrt{2}\{1,1,0,0\}$$
$$\varphi_{1,1}(n) = \sqrt{2}\varphi(2n-1) = \sqrt{2}\{0,0,1,1\}$$
$$\psi_{1,0}(n) = \sqrt{2}\psi(2n-0) = \sqrt{2}\{1,-1,0,0\}$$
$$\psi_{1,1}(n) = \sqrt{2}\psi(2n-1) = \sqrt{2}\{0,0,1,-1\}$$

因此 DWT 变换对可进行如下计算：

$$W_\varphi\{1,0\} = \frac{\sqrt{2}}{2}[1*1+4*1-3*0+0*0] = \frac{5\sqrt{2}}{2}$$

$$W_\varphi\{1,1\} = \frac{\sqrt{2}}{2}[1*0+4*0-3*1+0*1] = -\frac{3\sqrt{2}}{2}$$

$$W_\psi\{1,0\} = \frac{\sqrt{2}}{2}[1*1-4*1-3*0+0*0] = -\frac{3\sqrt{2}}{2}$$

$$W_\psi\{1,1\} = \frac{\sqrt{2}}{2}[1*0+4*0-3*1-0*1] = -\frac{3\sqrt{2}}{2}$$

(b)f(x)可由以下公式计算得出：

$$f(x) = \frac{1}{\sqrt{M}}\sum_k W_\varphi(j_0,k)\varphi_{j_0,k}(x) + \frac{1}{\sqrt{M}}\sum_{j=j_0}^{\infty}\sum_k W_\psi(j,k)\psi_{j,k}(x)$$

对于本题，要计算 f(1)，即 x=1 可进行如下计算：

$$f(1) = \frac{1}{2}[W_\varphi(1,0)\varphi_{1,0}(x) + W_\varphi(1,1)\varphi_{1,1}(x) + W_\psi(1,0)\psi_{1,0}(x) + W_\psi(1,1)\psi_{1,1}(x)]$$
$$= \frac{\sqrt{2}}{2}[\frac{5\sqrt{2}}{2}*1 - \frac{3\sqrt{2}}{2}*0 - \frac{3\sqrt{2}}{2}(-1) - \frac{3\sqrt{2}}{2}*0]$$
$$= 4$$