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# A Feature Extraction Technique based on Tunable Q-Factor Wavelet Transform for Brain Signal Classification

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## HIGHLIGHTS

- Electroencephalogram (EEG) signals are important for brain health monitoring applications.
- Characteristics of EEG signals are complex, being non-stationarity, aperiodic and nonlinear in nature that make them very difficult to deal with.
- A tunable Q-factor wavelet transform (TQWT) and a statistical approach are proposed to analyse various EEG recordings and classify epileptic seizures.
- This technique is tested on two different epileptic EEG databases.
- This technique has been evaluated by three popular machine learning methods and it provides better classification accuracy compared with the existing methods reported.
- The outcomes of the proposed technique can assist doctors and other health experts to identify diversified EEG categories.

## Abstract:

**Background:** Electroencephalogram (EEG) signals are important for brain health monitoring applications. Characteristics of EEG signals are complex, being non-stationarity, aperiodic and nonlinear in nature. EEG signals are a combination of sustained oscillation and non-oscillation transients that are challenging to deal with using linear approaches.

**Method:** This research proposes a new scheme based on a tunable Q-factor wavelet transform (TQWT) and a statistical approach to analyse various EEG recordings. Firstly, the proposed method decompose EEG signals into different sub-bands using the TQWT method, which is parameterized by its Q-factor and redundancy. This method depends on the resonance of a signal, instead of frequency or scaling as in the Fourier and wavelet transforms. Secondly, using a statistical feature extraction on the sub-bands to divide each sub-band into  $n$  windows, and then extract several statistical features from each window. Finally, the extracted features are forwarded to a bagging tree (BT),  $k$  nearest neighbor ( $k$ -NN), and support vector machine (SVM) as classifiers to evaluate the performance of the proposed feature extraction technique.

*Results:* The proposed method is tested on two different EEG databases: Bonn University database and Born University database. The experimental results demonstrate that the proposed feature extraction algorithm with the  $k$ -NN classifier produces the best performance compared with the other two classifiers.

*Comparison with existing methods:* In order to further evaluate the performances, the proposed scheme is compared with the other existing methods in terms of accuracy. The results prove that the proposed TQWT based feature extraction method has great potential to extract discriminative information from brain signals.

*Conclusion:* The outcomes of the proposed technique can assist doctors and other health experts to identify diversified EEG categories.

**Keywords:** Electroencephalogram (EEG) signal; Classification; Epilepsy; Tunable Q-factor wavelet transform;

## 1. Introduction

The human brain comprises billions of neurons that are connected with each other by sending tiny electrical signals. Electroencephalogram (EEG) signals are recorded by using electrodes placed on the scalp. EEG signals indicate the electrical activity of the brain, that is highly random in nature and contain useful information about the brain state to study brain function and neurological disorders (Siuly and Li, 2014; Siuly and Zhang, 2016; Siuly et al., 2013; Siuly et al., 2011; Zhu et al., 2012). Analysing brain signals is a very challenging task due to their oscillatory and non-oscillatory transients, nonlinear, aperiodic, and non-stationary dynamic behaviours (Selesnick, 2011; Siuly et al., 2017). It is challenging to extract the most representative information from raw EEG data for classification. Two types of feature extraction methods, linear and nonlinear (Zhu et al., 2014; Siuly et al., 2010), are generally used for EEG classification.

A number of methods are used to identify various types of brain disorders by EEG analysis. The most commonly used nonlinear classification methods often employ Fourier transform (FT), wavelet transform (WT), or Lyapunov exponent. FT and WT, and other techniques are applied for detecting brain disorders because EEGs contain a combination of sustained oscillation and non-oscillation transients (Selesnick, 2011; Siuly et al., 2017; Zhu et al., 2014; Gajic et al., 2014). The Fourier transform is used to transfer EEG signals from time series into frequency domain in which the most discriminative features are extracted (Kohtoh et al., 2008; Polat and Güneş, 2007; Murugappan et al., 2014). In order to extract the best features from different bands of wavelets, a wavelet transform is also applied to EEG data (Samaret al., 1999; Subasi et al., 2005; Zhang et al., 2016; Lekshmi et al., 2014; Gajic et al., 2014). Wavelet transform is an improved version of Fourier transform that can capture transient features and localize them in both time and frequency domains. A Lyapunov exponent is applied to extract most significant features from EEG data (Pritchard et al., 1995; Adeli et al., 2007; Hosseinifard et al., 2013). To explore the representative samples, an empirical mode decomposition is also used (Acharya et al., 2015; Hassan and Subasi, 2016; Hassan and Haque, 2015; Sharma et al., 2014; Pachori et al., 2015; Alickovic et al., 2018). Several researchers have used entropies from EEG signals as features (Zhang et al., 2016; Pachori and Patidar, 2014; Broberg and Lewis, 2014; Jie et al., 2014; Xiang et al., 2015). However, most of the nonlinear approaches are slow in their implementation, which makes them difficult to use in real time (Diykh et al., 2016; Zhu et al., 2014).

Most recently, the tunable Q-factor WT (TQWT) has become popular in brain signal processing (Patidar et al., 2017; Patidar and Panigrahi, 2017; Patidar et al., 2015; Hassan et al., 2016) and in the other fields (Soleymani et al., 2018; Selesnick, 2011) as a flexible and discrete wavelet transform that is applicable particularly for analysing oscillatory signals (Hassan et al., 2016). Most wavelet transforms are incapable of tuning their Q-factors. The TQWT is able to adjust its Q-factor and has thus emerged as a powerful tool for oscillatory signals analysis. By changing Q value, the wavelet transform can better reflect the signal (Selesnick, 2011). In addition, the TQWT method is developed to process the signals through employing ideal reconstruction oversampled filter banks that is developed in expression of repetition two channel filter banks, with real-valued scaling factors. However, it depends on the resonance (its oversampling rate) of a signal, instead of the frequency, while the band pass filter depends on the frequency of a signal. This study presents a nonlinear algorithm that is sufficiently accurate to analyse and classify EEG signals. This procedure is developed based on both the TQWT and a statistical feature extraction method. Three popular classifiers: bagging trees (BT),  $k$ -nearest neighbor (k-NN), and support vector machine (SVM), are employed to evaluate the performance of the proposed scheme. This approach is explained in detail in Sections 2.2. Two epileptic databases, which are Bonn University EEG Database and Bern University EEG Database (focal and non-focal EEG signals), are used to test the effectiveness of the proposed methodology in this study.

The rest of the paper is organized as follows. Section 2 provides the description of the experimental data used in this study and the proposed framework. Section 3 presents the experimental results with discussions. Section 4 provides a comparative report with state-of-the-art methods. Finally, Section 5 provides the concluding remarks about this study.

## 2. Materials and methods

### 2.1. Experimental data

In order to test the efficiency of the proposed method, this study uses two epileptic databases: a set of epileptic signals collected by Bonn University denoted as Bonn University EEG database and a set of focal and non-focal epileptic data collected by Bern University denoted as Born University EEG database as described below:

#### 2.1.1. Bonn University EEG Database:

This database is publicly available, which is widely used by researchers (Siuly and Li, 2014; Zhu et al., 2014; Al Ghayab et al., 2016; Supriya et al., 2016; Al Ghayab et al., 2017; Tzamourta et al., 2018), and was collected by Bonn University, Germany. It contains five different EEG data sets (A-E). From five healthy people, sets A and B were recorded with eyes opened and closed, respectively. Sets C-E were obtained from different five patients. Sets C and D were recorded from epileptic patients free of seizures. Set E was taken from epileptic subjects during active seizures. The EEG recordings were digitized at 173.61 Hz with 12-bit resolution. Consequently, each dataset (A, B, C, D or E) contained 100 single-channels, 4096 sample length, with each set having 23.6 seconds of time duration to avoid continuous multichannel EEG recordings after visual inspection for artifacts, (e.g., due to muscle activity or eye movements) (Andrzejak et al., 2001).

#### 2.1.2. Bern University EEG Database:

The second datasets utilized in this study are the Bern Barcelona database that were collected by Bern University and this database is publicly available (The Bern University EEG database 2012; Andrzejak et al., 2012). Multichannel EEG signals were recorded using the 10/20 system and the sampling rate of the acquisition is 512 Hz. It contains two different EEG signals, focal and non-focal and denoted  $F$  and  $N$ , respectively. Those datasets were recorded from five patients who were suffering from temporal lobe EEG epilepsy. Each recording contains 10240 observations and 3750 pairs of focal ( $F$ ) and non-focal ( $N$ ) EEG data. In this study, to evaluate the proposed method, full size of this database, 3750 pairs ( $F$  and  $N$  simultaneously) of EEG recording, respectively, with 15000 sets was utilized. This study processed the full size of data pairs and formed 7500 sets from each group of  $F$  and  $N$ , respectively. Each set having 20 seconds of time duration.

## 2.2. Methodology

A detailed description of the proposed scheme is presented in this section. The structure of the proposed TQWT based feature extraction method is shown in Fig. 1. The proposed method and three popular classifiers ( $k$ -NN, BT, and SVM) are implemented to evaluate the classification process.

### 2.2.1. Feature Extraction

Generally, a huge number of data points are included in the EEG recordings. Redundant data slow down the classification process and often cause inaccurate results. To reduce the data dimensionality and for a better performance, feature extraction techniques are often used in this stage. Fig. 2 illustrates the feature extraction process used in this study.

#### 2.2.1.1. Tunable Q-factor WT (TQWT)

The TQWT is a newly developed signal decomposition technique. It is an analogous form of the rational-dilation wavelet transform (Bayram and Selesnick, 2009; Selesnick, 2011), and it has been used to analyse EEG signals (Patidar et al., 2017; Bhattacharyya et al., 2017; Al Ghayab et al., 2017). The TQWT depends on changeable parameters: Q-factor ( $Q$ ), redundancy ( $R$ ), and decomposition level ( $J$ ). For the TQWT parameters,  $Q$  is often setting at a high value because EEG signals have more oscillations. The TQWT decomposes EEG signals into a number of decomposition levels called sub-bands (SB) using the input parameters ( $Q$ ,  $R$ , and  $J$ ).

The input signal,  $Sig[m]$ , in each level is decomposed into a low pass sub-band ( $Lsig$ ) and a high pass sub-band ( $Hsig$ ) with sampling frequencies of  $\alpha f_s$  and  $\beta f_s$ , respectively, where  $\alpha$  and  $\beta$  are low pass scaling and high pass scaling factors, respectively, and  $f_s$  refers to the sampling rate of input  $Sig[m]$ . For this process, a filter bank with two channels is repeatedly utilized and has employed this filter to  $Lsig$  by TQWT.  $Lsig$  is generated by applying a low pass filter  $h_0(\omega)$  with  $\alpha$ . Also,  $Hsig$  is generated from a high pass filter  $h_1(\omega)$  with  $\beta$  at one level of decomposing.  $Hsig$  is further decomposed into its  $Lsig$  and  $Hsig$  at  $j^{th}$  level as shown in Fig. 3. In this research, equations (1) and (2) are used to calculate  $h_0(\omega)$  and  $h_1(\omega)$  (Patidar and Panigrahi, 2017; Patidar et al., 2015; Hassan et al., 2016; Selesnick, 2011):

$$h_0(\omega) = \begin{cases} 1, & \text{if } |\omega| \leq (1 - \beta)\pi, \\ \theta \left( \frac{\omega + (\beta - 1)\pi}{\alpha + \beta - 1} \right), & \text{if } (1 - \beta)\pi < |\omega| < \alpha\pi, \\ 0, & \text{if } \alpha\pi \leq |\omega| \leq \pi, \end{cases} \quad (1)$$

where  $h_1(\omega)$  can be mathematically expressed as (Patidar et al., 2015; Bayram and Selesnick, 2009):

$$h_1(\omega) = \begin{cases} 0, & \text{if } |\omega| \leq (1 - \beta)\pi, \\ \theta \left( \frac{\alpha\pi - \omega}{\alpha + \beta - 1} \right), & \text{if } (1 - \beta)\pi < |\omega| < \alpha\pi, \\ 1, & \text{if } \alpha\pi \leq |\omega| \leq \pi, \end{cases} \quad (2)$$

where  $\theta(\omega)$  represents Daubechies filter frequency response as (Patidar et al., 2015; Hassan and Bhuiyan, 2016). R, Q, and J values in the TQWT related to the filter bank parameters,  $\alpha$  and  $\beta$ , as in the equations below (Patidar et al., 2015; Hassan et al., 2016):

$$Q = \frac{(\beta - 2)}{\beta} \quad (3)$$

$$R = \beta / (\alpha - 1) \quad (4)$$

This research used two databases as mentioned in Section 2.1, which have different oscillation transients. For instance, the Q-factor sets to high if the EEG signal has more oscillations. In this study, the Q value was set at 14 for Bonn University database and 6 for the focal and non-focal database, which are selected empirically for the both databases. The different Q values were used because the databases have different oscillatory transients. In addition, the decomposition level J was set empirically for the five sub-bands. Figs. 4(a), (b) and 5(a), (b) present a single EEG signal from the health persons and epileptic patients, and focal/non-focal epileptic EEG, and their five sub-bands which were obtained by the TQWT. From Fig. 4, the difference between F and N data are seen in the last two sub-bands of the TWQT method that means the statistical power is different in these sub-bands. However, the difference from sets A and E appeared in Fig. 5 in each sub-band of the proposed method. Clearly, the statistical power of each sub-band is different among Figs. 5 (a) and (b).

#### 2.2.1.2. Statistical Feature Extracted Method

The statistical approach is the most important part of the proposed scheme and employed to extract the representative statistical features. It influences the performance of a classifier if the features are not selected well. The statistical method includes two stages, the segmentation and the statistical feature extraction. The EEG signals are nonlinear and non-stationary in nature (Siuly et al., 2017; Al Ghayab et al., 2018), which makes analysis and classification difficult.

Two methods are employed to make EEG signals quasi-stationary. Firstly, a segmentation technique is utilized to divide each sub-band into several smaller windows that are denoted as  $W_1, W_2, \dots, W_n$ . The window size is determined by the following algorithm:

**Algorithm:**

Input the number of Sub-Bands (SB)

1. **For**  $i = 1$  to the number of SB
2.  $X = SB(i)$
3. **Initially**  $n = 0$
4. let  $p$  be the size of each window
5.  $n = n + 1$
6.  $p = X / n$
7. **Extracting** the statistical features from  $p$
8. **Put** statistical features in one set
9. **Forward** this vector to the classifier
10. **if** the classification accuracy is satisfactory then stop segmentation, go to step 12
11. **else** go to step 5
12. **end**

Output sets of statistical features

Size of each window

Secondly, ten statistical features are extracted from each window. The features are {*minimum (Min)*, *maximum (Max)*, *mean (Mean)*, *median (Med)*, *first quartile (Q1)*, *second quartile (Q2)*, *range (Ran)*, *standard deviation (SD)*, *skewness (Sk)*, and *variance (Var)*}. The statistical features can represent important information included in the EEG signals (Siuly et al., 2015a, 2015b) and can be expressed mathematically as:

$$Min = \min(x_n) \quad (5)$$

$$Max = \max(x_n) \quad (6)$$

$$Mean = \frac{1}{n} \sum_{1}^n x_n \quad (7)$$

$$Med = \left( \frac{N+1}{2} \right)^{th} \quad (8)$$

$$Q1 = \frac{1}{4(N+1)}^{th} \quad (9)$$

$$Q2 = \frac{2}{4(N+1)}^{th} \quad (10)$$

$$Ran = Max - Min \quad (11)$$

$$SD = \sqrt{\sum_{n=1}^N (x_n - SM) \frac{2}{n-1}} \quad (12)$$

$$SK = \sum_{n=1}^N (x_n - SM) \frac{3}{(N-1)SD^3} \quad (13)$$

$$Var = \sum_{n=1}^N (x_n - SM) \frac{2}{N-1} \quad (14)$$

where  $x_n = 1, 2, \dots, n$ , is a time series;  $N$  refers to the number of data points,  $SM$  is the mean of the samples.

ANOVA  $f$ -test is used to test the strength of the characteristics of the extracted features for classifying various brain signal cases, {A vs E}, {B vs E}, {C vs E}, {D vs E}, {AB vs E}, {AB vs CD vs E}, and {A vs B vs C vs D vs E} as shown in Table 1. The ANOVA  $f$ -test is tested using the one-way ANOVA that takes one random sample from each population under consideration, with a selection of a significance level ( $\alpha$ ) of 0.05.

This investigation found that five of the extracted features ( $Min$ ,  $Max$ ,  $Ran$ ,  $SD$ , and  $Var$ ) were significant, with  $f > 1.65$ . The features were labeled as “S”, reflecting their significance in representing epileptic activities as shown in Table 1. In Table 1, the  $NS$  label indicates that the features with  $f < 1.65$  are not significant for the classification. In this study, we considered the  $f$  value of 1.65 based on the hypothesis used in this paper that said that each value more than 1.65 is accepted otherwise it was rejected. Choosing value of the  $f$ -test is depended on the correlation among features. Based on the results, we found that those features gained less than 1.65, were not significant and noisy. As result, they were rejected. For details of how to choose the significant features based on the  $f$ -test, readers may refer to (Faul et al., 2007; Morgan, et al., 2004; Brown and Forsythe, 1974).

At the end, the first and second sub-bands contain 2048 observations and the rest of the sub-bands have 1024 observations. Each sub-band includes 20 data points. Furthermore, each group of the focal and non-focal EEG contains 8192 observations of the first two sub-bands and other three sub-bands have 4096 observations. Next step, we will use the cross-validation measurement technique as can see in Section 2.2.3.

## 2.2.2. Classification method

Three popular machine learning classifiers, bagging trees,  $k$ -nearest neighbors, and support vector machine are applied to evaluate the proposed features extraction technique and select the most suitable classification method. In the sections below, we present more details about those classifiers.

### 2.2.2.1. Bagged trees (BT)

The BT classifier, developed by Breiman (1996), is one of the statistical learning methods. It is employed for the EEG classification in this paper. The equation of the BT classifier is presented below (Breiman, 1996; James et al., 2014):

$$A_{avg}(x) = \frac{1}{P} \sum_{i=1}^P A^i(x) \quad (15)$$



where  $A_{avg}(x)$  is the average of the accuracy of  $x$ ;  $P$  is a separate training sets;  $A^i(x)$  is the tree's prediction at input  $x$ . In this investigation, 80 training sets were selected from the extracted features to reduce the variance and hence increase the prediction accuracy. The main benefit of this classifier is to gather fitted values from a huge size of bootstrap samples and calculate the average of each fitted values with a low bias and high variance. This way aggregates the fitted values based on other statistical learning developments (Breiman, 1996).

#### 2.2.2.2. $K$ nearest neighbors ( $k$ -NN)

The  $k$ -NN is one of the most common nonparametric methods. It is considered to be the simplest method among all the machine learning algorithms (Duda et al., 2012). The classifier is applied to classify the extracted features based on the nearest training features. It can classify the unlabeled input features to its  $k$  nearest neighbors (Ergen, 2016). To make the  $k$ -NN classifier work properly,  $k$  should be selected carefully (Duda et al., 2012; Ergen, 2016; Cover and Hart, 1967). In this paper,  $k=1$  is used. The advantage to use the  $k$ -NN classifier is that this method applies effective techniques to reduce the noise appeared in input data, which are improving the accuracy of the  $k$ -NN classifier (Cunningham and Delany, 2007).

#### 2.2.2.3. Support vector machines (SVM)

The SVM is one of the well-known classification methods that was developed by Cortes and Vapnik (1995). A variation of the SVM is a quadratic kernel free non-linear support vector machine (denoted as QSVM) that was proposed by Dagher (2008), is used in this study. This function separates the feature sets nonlinearly as can be found in (Dagher, 2008). The main reason to apply the SVM classifier in this study is that the SVM grants a better generalization solution if its parameters are well chosen. In choosing appropriate parameters, the SVM can be robust (Auria, and Moro, 2008).

#### 2.2.3. Performance Measurements

Several measurements were used for evaluating the performance of the proposed scheme. The  $f$ -fold cross validation method was used. The datasets were divided into  $f$  subsets/folds (Siuly and Li, 2015b). In each implementation, one-fold is used as a testing set and  $f-1$  folds are utilized as a training set. In each process, the number of features in the training and testing sets are shown in Table 2.

An average accuracy is obtained for the whole process in the cross-validation method. All the sub-sets are used for testing separately, and the average of classification accuracy is calculated by using 5-fold cross validation as shown in equation (16).

$$ACA = \frac{\sum_1^n perfo}{f}, \quad n = 1, 2, \dots, 5 \quad (16)$$

where  $ACA$  refers to the average of classification accuracy,  $f$  is the number of folds, and  $perfo$  refers to the performance of each fold.

The classification accuracy (*Acc*) was applied to evaluate the proposed method. It is defined as the percentage of the correctly classification result by the classifier for the testing dataset as depicted in equation (17) (Zhu et al., 2014; Breiman, 1996; Al Ghayab et al., 2018):

$$Acc = \frac{\sum TP + \sum TN}{\sum AS} \times 100 \quad (17)$$

where *TP* is the true positives, *TN* refers to the true negatives, *FP* is the false positives, *FN* is the false negatives, and *AS* refers to all the samples.

Another statistical measurement used in this study was sensitivity (*SE*). This measurement takes into consideration the true positive rate divided by the positive samples in the test data as shown in equation (20) (Al Ghayab et al., 2016):

$$SE = \frac{\sum TP}{\sum P} \times 100 \quad (18)$$

Specificity (*SP*) was applied in this study to evaluate the performance of the algorithm. *SP* considers true negatives divided by the negative samples of the test to obtain a result (Al Ghayab et al., 2016). *SP* is defined as:

$$SP = \frac{\sum TN}{\sum N} \times 100 \quad (19)$$

### 3. Experimental results and discussions

In this research, a set of experiments were implemented by using two different databases, which are described in Section 2.1. The proposed new technique was applied to analyse and extract the key features from both databases (Bonn University EEG and Bern University EEG databases). The TQWT based feature extraction technique was implemented. Two methods were used to decompose the epileptic EEG signals into five sub-bands and extract the most representative features. Firstly, the TQWT nonlinear method was applied to each dataset of EEG signals to decompose the EEG signals based on the resonance of five sub-bands. Secondly, each sub-band was divided into smaller windows, and ten statistical features were extracted from each window. The key features were obtained and forwarded to three classifiers (BT, k-NN and SVM), separately, to select the most suitable one. In this study, several cases from the two databases were tested. Also, the experiments were conducted using MATLAB R2017b on a computer with Intel (R) core i7-7700, 3.60 GHz CPU, RAM capacity of 16 GB.

#### 3.1. Experiment results for the Bonn University EEG database

This experiment used seven epileptic cases, which are {A vs E}, {B vs E}, {C vs E}, {D vs E}, {AB vs E}, {AB vs CD vs E}, and {A vs B vs C vs D vs E} as shown in Table 2. The proposed scheme was implemented and evaluated using a variety of evaluation methods. Table 3 shows that the average accuracies of the measurement of each epileptic case were achieved from each sub-band by using the k-NN, BT, and SVM classifiers. In Table 3, the TQWT based algorithm decomposed the EEG signals into five sub-bands, which have chosen empirically as mentioned in Section 2.2.1.1, to validate the proposed method in different sub-bands. The overall classification accuracy of the TQWT method with k-NN for Bonn University database is 100% in variety cases and difference

sub-bands, which was the highest accuracy compared with the BT and SVM classifiers in each sub-band from the seven experimental cases for the epileptic EEG data. In contrast, the lowest accuracy was obtained by using the TQWT with SVM as shown in Table 3.

On the other hand, Tables 4 and 5 illustrate the confusion matrix and accuracy for the cases of {A vs B vs C vs D vs E} and {AB vs CD vs E}, respectively, from Bonn University database. Based on the yielded results there are significant differences in the classification accuracies among the three classifiers ( $k$ -NN, BT and SVM) compared with the proposed method. The accuracy of the proposed method with  $k$ -NN classifier in Tables 4 and 5 is 100% overall classification rate, which is the highest accuracy compared with the BT and SVM classifiers in Table 3. The results reveal that the TQWT based algorithm combined with  $k$ -NN classifier is effective to classify the epileptic EEG signals. Tables 4 and 5 show the confusion matrices between the results by the proposed and the expert's scoring, and their accuracies for the relevant pairs.

The suggested approach based on the  $k$ -NN classifier gained the highest score with 100% average of sensitivity in all the sub-bands for the seven epileptic cases. At the same time, the proposed method with the BT achieved a 100% sensitivity rate with most of the sub-bands for all epileptic cases as can be seen in Fig. 6. Figs. 6 (a)-(g) provide a comparison of using the proposed feature extraction method with the three classifiers on seven epileptic states in terms of the sensitivity measurement. The lowest sensitivity yielded was 46.23% and 32.32% for the epileptic cases of {AB vs CD vs E} and {A vs B vs C vs D vs E}, respectively. It was from the SVM classifier and the results are presented in Fig. 6. The two cases of {AB vs CD vs E} and {A vs B vs C vs D vs E} that are multi-classification, were similar to each other, which led to the lowest sensitivity results compared with the results for other cases.

Furthermore, Table 6 shows the average of the specificity rates from each sub-band by the proposed technique for different cases of the epileptic EEG signals. As can be seen in Table 6, the highest results for specificity were achieved by using the suggested approach combined with the  $k$ -NN classifier that was 100% specificity rate in all sub-bands for all cases. In contrast, the TQWT with the other two classifiers (BT and SVM) got low results in some sub-bands. The experiment results from this study demonstrated that the TQWT coupled with the  $k$ -NN classifier was the best method to analyse and classify the epileptic EEG signals.

### 3.2. Experiment results for the Born University EEG database

In this research, we used focal vs non-focal case of Born University database as seen in Table 2. The proposed scheme was implemented and evaluated using the above evaluation measures.

Table 7 presents the accuracy rates that were gained by the same proposed TQWT based scheme and classifiers ( $k$ -NN, BT, and SVM) with the full size of Born University database (refer to Section 2.1.2). In these experiments, the TQWT feature extraction method with the  $k$ -NN gained the highest overall accuracy of 100% for each sub-band, while the BT classifier with the proposed approach achieved > 99% classification accuracies from all the sub-bands. In contrast, the proposed technique with the SVM classifier achieved about 92% overall classification accuracies from all the sub-bands for the  $F$  vs  $N$  case with large datasets. It seems that they are the lowest rate among all the three classifiers for the proposed scheme as can be seen from Table 7.

The sensitivity and specificity were used to evaluate the implementation of the TQWT based algorithm with three popular classifiers for the  $F$  and  $N$  two datasets. This scheme based on the  $k$ -NN classifier obtained a 100% average of sensitivity and specificity for the five sub-bands for  $F$  vs  $N$  case with large datasets, which was the highest outcome. However, the proposed scheme combined with the BT yielded more than 99% sensitivity and specificity rates for the same sub-bands in the  $F$  vs  $N$  case, which are the second highest results as shown in Table 7. In addition, the low results were from the TQWT and the SVM with results of 92.35% to 97.59% for sensitivity, and 92.59% to 97.23% for specificity for the sub-bands from the  $F$  vs  $N$  case in Bonn University database.

#### 4. Comparative study

In this section, we formulated two types of performance comparisons to evaluate the implementation of the proposed technique. Firstly, the classification accuracies of the TQWT based feature extraction method and  $k$ -NN classifier on the epileptic EEG pairs in Table 3 were compared with five recent techniques. Secondly, the accuracy results of the focal and non-focal epileptic data in Table 7 was also compared with the existing methods. In this investigation, the comparisons were made between the proposed scheme and the existing approaches, which used the same databases and the same EEG channels.

1. The comparison among the proposed method and other epileptic EEG classification methods: the results from the proposed method were compared with the results from other studies in (Zhu et al., 2014; Patidar and Panigrahi, 2017; Zhu et al., 2013; Samiee et al., 2015; Alcin et al., 2016). Table 8 shows a comparison of this technique with five other existing methods. From Table 8, Zhu et al. (2013) provides a new technique based on a sample entropy and multi-scale K-means method to classify the epileptic seizures. They achieved an overall classification accuracy of 100%, 99% and 99.1% for the cases of {A vs E}, {B vs E}, and {AB vs CDE}, respectively. However, the proposed scheme in this study gained a 100% accuracy rate for all cases, which means that it achieved a higher accuracy than those by (Zhu et al., 2013). Samiee et al. (2015) applied a rational discrete short time Fourier transform combined with a multilayer perceptron classifier. That paper reported a 99.8%, 99.3%, 98.5%, and 94.9% classification accuracy for the cases of {A vs E}, {B vs E}, {C vs E}, and {D vs E}, respectively. Based on those results, our proposed method outperformed Samiee et al. (2015).

Another study was made by Zhu et al. (2014) in which fast weighting horizontal visibility graph and  $k$ -NN classifier were utilized. The authors reported a 100%, 93%, 95.3% overall accuracy for {A vs E}, {D vs E}, and {ABCD vs E}, respectively. The results in our approach were higher than those by (Zhu et al., 2014). Alcin et al. (2016) obtained a 96.4% average classification accuracy for all the data sets using time frequency image combined with a filter vector and extreme learning machine, compared with our proposed scheme and achieved 100% classification rates. Patidar et al. (2017) applied a TQWT based on Kraskov entropy and least square support vector machine to classify epileptic seizures. An average accuracy of 97.75% for seizure free classes vs epileptic seizure was reported.

Some of the existing methods were conducted with two sets of Bonn University database and achieved an average of accuracy between 99% and 100% compared with the proposed technique that tested using the whole Bonn University database. In Table 8, the best outcome among all the methods was highlighted in bold font. Based on those results, the TQWT based algorithm with  $k$ -NN produced the highest accuracy among all the existing methods.

2. The comparison between the proposed TQWT method and other focal and non-focal epileptic EEG classification methods: the outcomes of this method based on  $k$ -NN classifier were compared with those of other five existing methods, which are used the same focal and non-focal EEG database. Sharme et al. (2015) implemented an empirical mode decomposition and least square support vector machine to classify focal and non-focal epileptic EEG data. Totally, they achieved an accuracy of 87%. However, the proposed TQWT based algorithm gained a higher rate than those by Sharme et al. (2015). Das and Bhuiyan (2016) also applied the empirical mode decomposition combined with the discrete wavelet transform and  $k$ -NN classifier. Their approach obtained an 89.4% accuracy for  $\{F$  vs  $N\}$ . Based on those results, the proposed TQWT method outperformed Das and Bhuiyan (2016). Different entropies, such as approximate entropy, sample entropy, and Reyni's entropy, have been used as features by Arunkumar et al. (2017). To evaluate their features, they employed non-nested generalized exemplars classifier and achieved an average classification accuracy of 98%. Another study was made by Arunkumar et al. (2018) in which a number of entropies based on normal inverse Gaussian and non-nested generalized exemplars were utilized. Overall, their method obtained 99% classification rate compared with our method, which used the same database and achieved 100% accuracy. Sharma et al. (2017) applied orthogonal wavelet filter banks and entropies. In order to select the most discriminative features, they used  $t$ -test method. The selected features were forwarded to the least square support vector machine. They reported a 94.25% accuracy for  $F$  vs  $N$  sets. The obtained results in our scheme were higher than those by Sharma et al. (2017). Some of the existing methods were conducted with smaller datasets excerpted from Bonn University database and achieved an average of accuracy between 87% and 99%. We highlighted in bold font the best classification accuracy among all the compared techniques in Table 9.

Obviously, based on Tables 8 and 9, the proposed scheme achieved the highest accuracy for both two EEG databases, the Bonn University and Born University, compared with the existing methods which used the same databases.

One of the limitations of the proposed method is that the proposed method may not work well for real time applications, due to it needs further processing to remove the artifacts. However, the proposed method was implemented with two off-line databases collected by Bonn University and Born University. In addition, the main advantage of the TQWT based feature extraction method is: the technique can reduce a huge amount of EEG data into small sets selecting the best representative data points from every segmentation of a dataset considering the variability of the observations. Due to the reduction of data, this approach can handle a large amount of data with less computation cost compared to the existing methods.

## 5. Conclusion

This article developed an innovative technique to extract the most significant features from two different EEG databases, Bonn University EEG and Born University EEG databases. The TQWT was applied to decompose the EEG signals into five sub-bands based on its parameters,  $Q$ ,  $R$ , and  $J$  level. After signal decomposition, ten statistical features were used to segment the sub-bands and the most representative features were extracted. These features were put in one set for each sub-band and then forwarded to three popular machine-learning classifiers,  $k$ -NN, BT, and SVM. The experimental results reveal that the proposed TQWT based feature extraction method

combined with the k-NN classifier is capable to differentiate the epileptic EEG signals with an excellent performance, compared to the existing methods. In the future, we will endeavour to enhance this method and implement it online.

### **Ethical Standard Agreement**

**Human participants and/or animals rights:** The used databases of EEG signals are available online.

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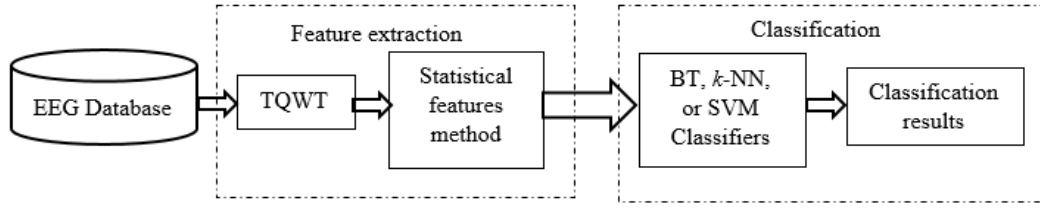
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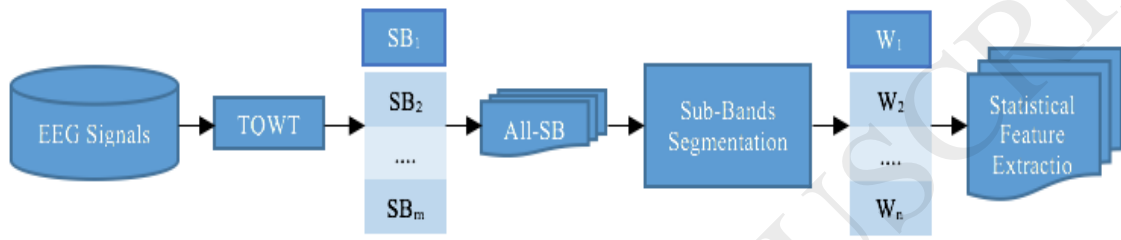
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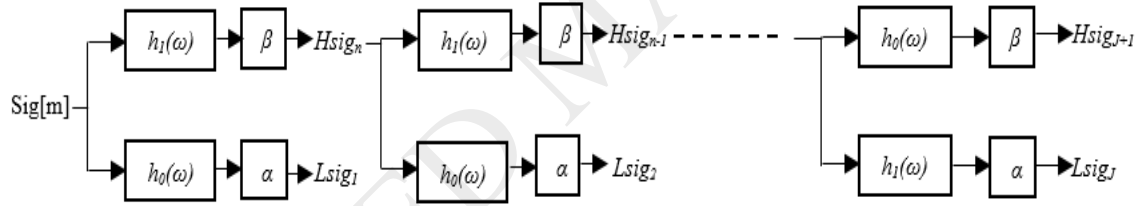
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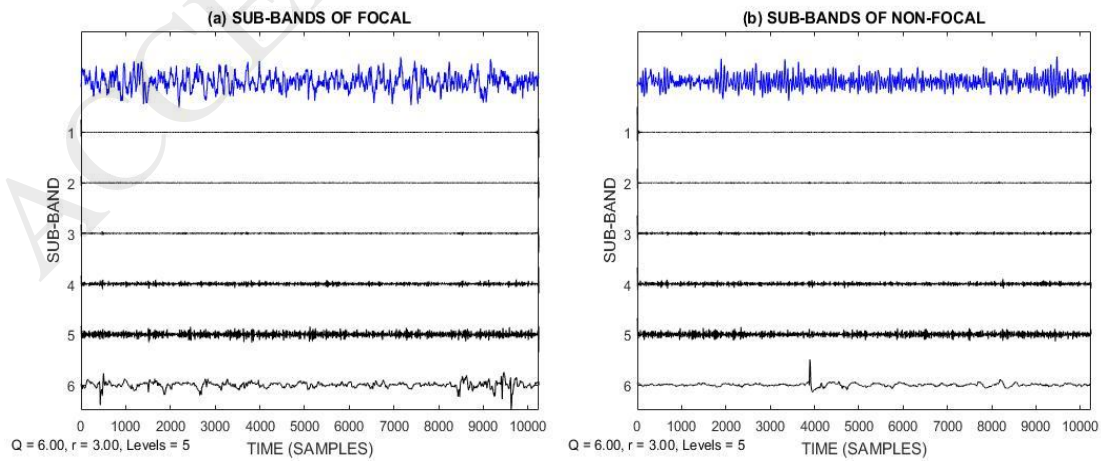
**Fig. 1.** Block diagram of the proposed scheme for the analysis and classification of EEGs.



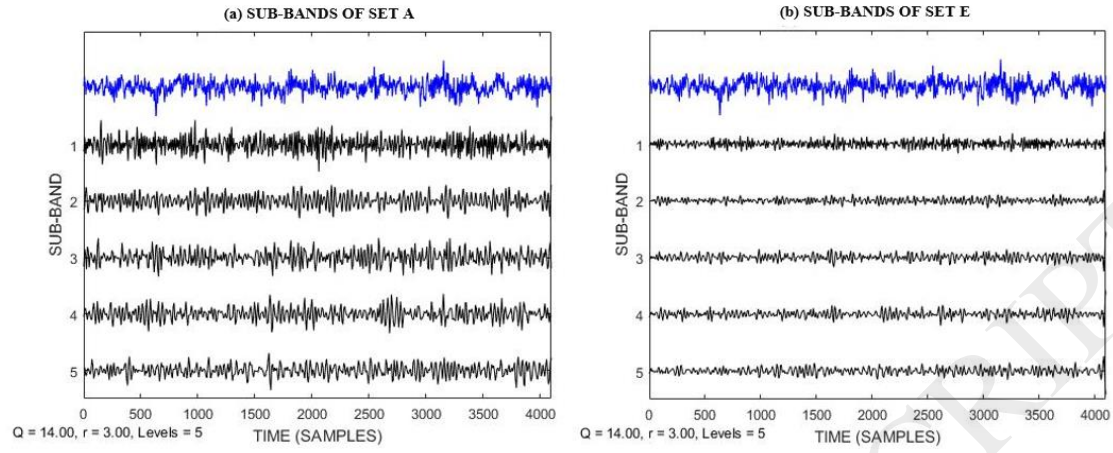
**Fig. 2.** Feature extraction based on Tunable Q-factor wavelet transform and statistical method. Note: SB= Sub-Bands; All-SB<sub>m</sub>= gathering each Sub-Band from all channels in one set; W<sub>n</sub>= the number of windows.



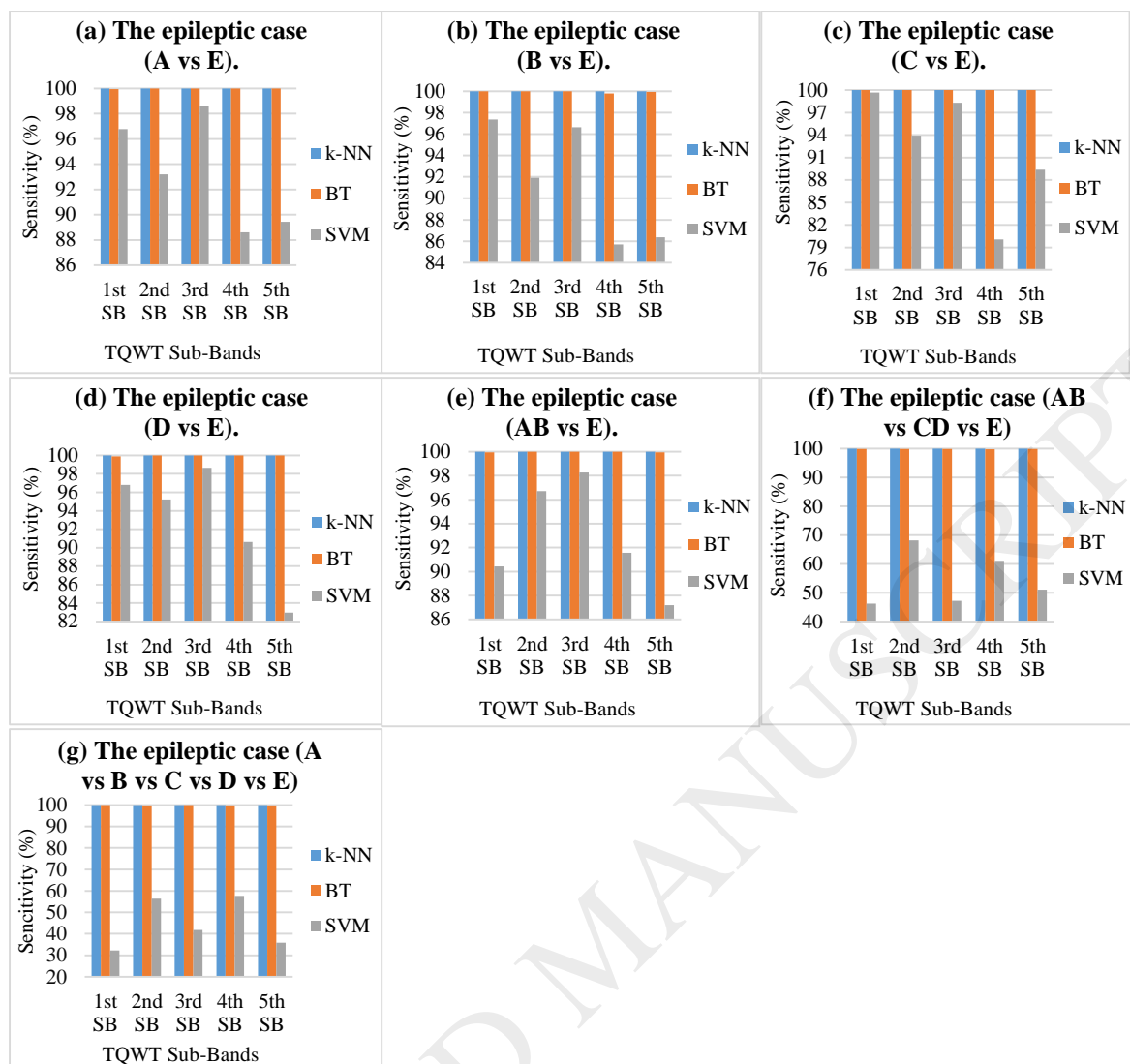
**Fig. 3.** The TWQT decomposes the input EEG signal ( $Sig[m]$ ) into low pass sub-band  $Lsig$  and high pass sub-band  $Hsig$  at  $J^h$  levels.



**Fig. 4.** Examples of Focal ( $F$ ) and Non-focal ( $N$ ) and their five Sub-Bands: (a) **Focal** =  $F$ ; (b) **Non-focal** = ( $N$ ), obtained by using the TQWT.



**Fig. 5.** Examples of five level TQWT decomposition: (a) Healthy persons with eye open (set **A**); (b) Patient with seizure (set **E**).



**Fig. 6.** Performance comparison among the reported classifiers with the proposed feature extraction method for seven cases of Bonn University EEG database in terms of **Sensitivity (SN)**

**Table****Table 1.** Results of ANOVA  $f$ -test of the statistic feature characteristics for each case of Bonn EEG Dataset.

Statistical Features	$f$ -test							Result
	{A vs E}	{B vs E}	{C vs E}	{D vs E}	{AB vs E}	{AB vs CD vs E}	{A vs B vs C vs D vs E}	
<i>Min</i>	275.84772	348.95716	274.53135	311.2916	275.84772	268.76247	287.99356	<i>S</i>
<i>Max</i>	311.26012	395.90688	310.12819	352.68545	311.26012	302.2518	323.54456	<i>S</i>
<i>Mean</i>	0.02882	0.02576	0.0239	0.0202	0.02882	0.02536	0.02249	<i>NS</i>
<i>Med</i>	0.3794	0.43788	1.19521	0.25751	0.3794	0.26107	0.0568	<i>NS</i>
<i>Q1</i>	0.19645	0.07705	1.40824	0.33093	0.19645	0.13463	0.06509	<i>NS</i>
<i>Q2</i>	0.3141	0.46503	1.28864	0.28364	0.3141	0.22865	0.09816	<i>NS</i>
<i>Ran</i>	654.6591	826.88195	648.32584	736.28225	654.6591	634.67079	676.89236	<i>S</i>
<i>SD</i>	786.35536	982.60613	799.20694	879.65361	786.35536	775.18764	816.60389	<i>S</i>
<i>SK</i>	0.01688	0.02259	0.00003	0.02199	0.01688	0.01202	0.01589	<i>NS</i>
<i>Var</i>	198.36928	209.44634	198.69604	204.06362	198.36928	198.37071	202.228	<i>S</i>

**Table 2.** Numbers of the training and testing sets used in this study.

Database	Case	Sub-band	Total	Training	Testing
Bonn Database	{A vs E}, {B vs E}, {C vs E}, and {D vs E}	1 <sup>st</sup> and 2 <sup>nd</sup>	4096	3278	818
		3 <sup>rd</sup> , 4 <sup>th</sup> and 5 <sup>th</sup>	2048	1640	408
	{AB vs E}	1 <sup>st</sup> and 2 <sup>nd</sup>	6144	4916	1228
		3 <sup>rd</sup> , 4 <sup>th</sup> and 5 <sup>th</sup>	3072	2459	613
	{AB vs CD vs E} {A vs B vs C vs D vs E}	1 <sup>st</sup> and 2 <sup>nd</sup>	10240	8195	2045
		3 <sup>rd</sup> , 4 <sup>th</sup> and 5 <sup>th</sup>	5120	4100	1020
Born Database	{F vs N}	1 <sup>st</sup> and 2 <sup>nd</sup>	16384	13108	3276
		3 <sup>rd</sup> , 4 <sup>th</sup> and 5 <sup>th</sup>	8192	6554	1638

**Table 3.** Accuracy rates of each sub-band of TQWT based algorithm for different cases of Bonn University database.

No	Case	Classifier	Acc (%)				
			1 <sup>st</sup> SB	2 <sup>nd</sup> SB	3 <sup>rd</sup> SB	4 <sup>th</sup> SB	5 <sup>th</sup> SB
1	A vs E	<i>k</i> -NN	100	100	100	100	100
		BT	99.98	99.98	100	100	100
		SVM	97.6	96.3	96.3	91.1	92.3
2	B vs E	<i>k</i> -NN	100	100	100	100	100
		BT	100	99.98	100	99.97	99.98
		SVM	95.5	94.9	97.6	85.8	89.6
3	C vs E	<i>k</i> -NN	100	100	100	100	100
		BT	100	100	100	100	100
		SVM	99.1	95.8	96.8	85.8	92.3
4	D vs E	<i>k</i> -NN	100	100	100	100	100
		BT	99.97	100	100	100	100

		SVM	97.7	96.4	96.3	91.5	87.2
5	AB vs E	<i>k</i> -NN	100	100	100	100	100
		BT	99.98	100	100	100	99.98
		SVM	92.3	97.4	96.5	92.4	90.2
6	AB vs CD vs E	<i>k</i> -NN	100	100	100	100	100
		BT	99.82	99.77	99.92	99.67	99.76
		SVM	55.02	57.32	50.66	50.22	48.95
7	A vs B vs C vs D vs E	<i>k</i> -NN	100	100	100	100	100
		BT	99.85	99.83	99.82	99.86	99.82
		SVM	45.71	49.63	47.6	49.55	47.12

**Table 4.** Confusion matrix and accuracy for the case of {A vs B vs C vs D vs E} from Bonn University database using *k*-NN

		Expert's Scoring				
		A	B	C	D	E
TQWT	A	204	0	0	0	0
	B	0	204	0	0	0
	C	0	0	204	0	0
	D	0	0	0	204	0
	E	0	0	0	0	204
Accuracy		100%	100%	100%	100%	100%

**Table 5.** Confusion matrix and accuracy for the case of {AB vs CD vs E} from Bonn University database using *k*-NN

		Expert's Scoring		
		AB	CD	E
TQWT	AB	408	0	0
	CD	0	408	0
	E	0	0	204
Accuracy		100%	100%	100%

**Table 6.** Specificity (*SP*) rates from each sub-band by the TQWT based algorithm for different cases of Bonn University database.

No	Case	Classifier	<i>SP</i> (%)				
			1 <sup>st</sup> SB	2 <sup>nd</sup> SB	3 <sup>rd</sup> SB	4 <sup>th</sup> SB	5 <sup>th</sup> SB
1	A vs E	<i>k</i> -NN	100	100	100	100	100
		BT	100	100	100	100	100
		SVM	98.51	99.84	94.3	93.85	95.68
2	B vs E	<i>k</i> -NN	100	100	100	100	100
		BT	100	99.95	100	100	100
		SVM	93.8	99.47	98.51	85.98	93.46
3	C vs E	<i>k</i> -NN	100	100	100	100	100
		BT	100	100	100	100	100
		SVM	98.55	97.96	95.45	94.32	95.78
4	D vs E	<i>k</i> -NN	100	100	100	100	100
		BT	100	100	100	100	100
		SVM	98.61	97.64	94.22	92.41	92.71
5	AB vs E	<i>k</i> -NN	100	100	100	100	100
		BT	100	100	100	100	100
		SVM	97.41	98.76	93.21	94.59	100
6	AB vs CD vs E	<i>k</i> -NN	100	100	100	100	100
		BT	99.79	99.69	99.9	99.58	99.67
		SVM	65.37	58.17	61.28	54.04	62.82
7	A vs B vs C vs D vs E	<i>k</i> -NN	100	100	100	100	100
		BT	99.9	99.87	99.9	99.95	99.93
		SVM	89.71	72.99	87.6	69.22	79.85

**Table 7.** Accuracy, Sensitivity, and Specificity rates for each sub-band by the TQWT based algorithm for Focal (*F*) vs Non-focal (*N*) with a data full size from the Bonn University database.

Case	Sub-Band	Classifier	Acc (%)	SE (%)	SP (%)
<i>F</i> vs <i>N</i>	1 <sup>st</sup> SB	<i>k</i> -NN	100	100	100
		BT	99.97	99.95	99.99
		SVM	92.47	92.35	92.59
	2 <sup>nd</sup> SB	<i>k</i> -NN	100	100	100
		BT	100	100	100
		SVM	97.0	97.59	96.45
	3 <sup>rd</sup> SB	<i>k</i> -NN	100	100	100
		BT	100	100	100
		SVM	97.35	97.47	97.23
	4 <sup>th</sup> SB	<i>k</i> -NN	100	100	100
		BT	99.99	99.98	100
		SVM	96.57	96.57	95.82
	5 <sup>th</sup> SB	<i>k</i> -NN	100	100	100
		BT	100	100	100
		SVM	95.65	95.79	95.55

<b>Table 8.</b> Comparative report of the proposed method for the epileptic EEG database			
Authors	Technique	Case	Acc (%)
Zhu et al. (2013)	• Sample entropy + Multi-scale K-means	A vs E B vs E AB vs CDE	100 99.0 99.1
Zhu et al. (2014)	• Fast weighted horizontal visibility graph + $k$ -NN	A vs E D vs E ABCD vs E	100 93.0 95.4
Samiee et al. (2015)	• Rational discrete short time Fourier transform+ Multilayer perceptron classifier	A vs E B vs E C vs E D vs E	99.8 99.3 98.5 94.9
Alcin et al. (2016)	• Time-frequency image + Filter vector+ Extreme learning machine	Multi classification	96.4
Patidar et al. (2017)	• TQWT + Kraskov entropy + least square support vector machine	CD vs E	97.75
<b>This study</b>	• <b>Tunable Q-factor WT based technique+ Statistical features +<math>k</math>-NN</b>	A vs E B vs E C vs E D vs E AB vs E AB vs CD vs E Multi classification	100 100 100 100 100 100 100

<b>Table 9.</b> Comparative report of the proposed method for c the Focal and Non-focal epilepsy EEG datasets.			
Authors	Approach	Data size	Acc (%)
Sharma et al. (2015)	• Empirical mode decomposition + least square support vector machine	50	87
Das and Bhuiyan (2016)	• Empirical mode decomposition + discrete wavelet transform + $k$ -NN	3750	89.4
Arunkumar et al. (2017)	• Entropies(approximate entropy, sample entropy, and Reyni's entropy) as features + non-nested generalized exemplars classifier	50	98
Sharma et al. (2017)	• Orthogonal wavelet filter banks + entropies + $t$ -test + least square support vector machine	3750	94.25
Arunkumar et al. (2018)	• Feature entropies(approximate entropy, sample entropy, andfuzzy entropy) + normal inverse Gaussian + non-nested generalized exemplars	50	99
<b>This study</b>	• <b>Tunable Q-factor WT based technique + Statistical features + <math>k</math>-NN</b>	<b>3750</b>	<b>100</b>