第六章习题参考答案

P92

1. (4),(5)不是.

2.

1. 设总体 $X \sim N(\mu,\sigma^2)$，$\bar{X}$ 为样本均值，S^2 为样本方差. 则 $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ 服从______，$\frac{\bar{X}-\mu}{S/\sqrt{n}}$ 服从____________.

1. $N(0,1)$, $\quad t(n-1)$ $\quad$ **(P97Th.6.2.3)**

2. 设 $X_1, X_2, \cdots, X_9$ 为总体 $N(0,2^2)$ 的样本，则当 $a=$______，$b=$______，$c=$______时，

$$Y = a(X_1+X_2)^2 + b(X_3+X_4+X_5)^2 + c(X_6+\cdots+X_9)^2$$

服从 χ^2 分布，自由度为____________.

2. 设 $X_1, X_2, \cdots, X_{10}$ 为总体 $N(0,2^2)$ 的样本，则当 $a=$___,$b=$___,$c=$___, (　　) 时，$Y=a(X_1+X_2)^2+b(X_3+X_4+X_5)^2+c(X_6+\cdots+X_9)^2 \sim$________.($\chi^2(3)$)

1. (1)(2)(3)是，(4)(5)不

2. $$\begin{aligned}\sum_{i=1}^{n}(x_i-\bar{x})^2 &= \sum_{i=1}^{n}[(x_i-a)+(a-\bar{x})]^2 \\ &= \sum_{i=1}^{n}(x_i-a)^2 + 2(a-\bar{x})\sum_{i=1}^{n}(x_i-a) + n(a-\bar{x})^2 \\ &= \sum_{i=1}^{n}(x_i-a)^2 + 2(a-\bar{x})(n\bar{x}-na) + n(a-\bar{x})^2 \\ &= \sum_{i=1}^{n}(x_i-a)^2 - 2n(a-\bar{x})^2 + n(a-\bar{x})^2 \\ &= \sum_{i=1}^{n}(x_i-a)^2 - n(a-\bar{x})^2\end{aligned}$$

3. $X \sim f(x) = \begin{cases}\frac{1}{\lambda}e^{-\frac{1}{\lambda}x}, & x>0 \\ 0, & t\le 0\end{cases}$，$(X_1,X_2,\cdots,X_n)$的分布为

$$f(x_1,x_2,\cdots,x_n) = f(x_1)f(x_2)\cdots f(x_n) = \begin{cases}\frac{1}{\lambda^n}e^{-\frac{1}{\lambda}(x_1,x_2,\cdots,x_n)}, & x_1,x_2,\cdots,x_n>0 \\ 0, & \text{others}\end{cases}$$

及 $$F(x_1,x_2,\cdots,x_n) = F(x_1)F(x_2)\cdots F(x_n) = \begin{cases}\prod_{i=1}^{n}(1-e^{-\frac{1}{\lambda}x_i}), & x_1,x_2,\cdots,x_n>0 \\ 0, & \text{others}\end{cases}$$

4. $X_k \sim N(12,4), k=1,2,3,4,5.$

(1) $\bar{X}=\frac{1}{5}\sum_{k=1}^{5} X_k \sim N(12,\frac{4}{5}),$ 由$P97Th.6.2.1$有$Y=\frac{\bar{X}-12}{2/\sqrt{5}} \sim N(0,1).$

$$\therefore \quad P\{|\bar{X}-12|>1\}=P\{\frac{|\bar{X}-12|}{2/\sqrt{5}}>\frac{1}{2/\sqrt{5}}\}=1-P\{-\frac{1}{2/\sqrt{5}}\le Y\le \frac{1}{2/\sqrt{5}}\}$$

$$=1-\left[2\Phi(\frac{\sqrt{5}}{2})-1\right]\approx 2(1-\Phi(1.12))=2(1-0.8686)=0.2628.$$

(2) $P\{\max_{1\le i\le 5}(X_i>15\}=1-P\{\max_{1\le i\le 5}(X_i)\le 15\}=1-P\{X_1\le 15,\cdots,X_5\le 15\}$

$$=1-\prod_{1\le i\le n} P\{X_i\le 15\}=1-\Phi^5(\frac{15-12}{2})$$

$$=1-0.9332^5=1-0.70774=0.2923.$$

(3) $P\{\min_{1\le i\le 5}(X_i<10\}=1-P\{\min_{1\le i\le 5}(X_i)\ge 10\}=1-P\{X_1\ge 10,\cdots,X_5\ge 10\}$

$$=1-\prod_{1\le i\le n} P\{X_i\ge 10\}=1-(1-P\{X_i<10\})^5=1-(1-\Phi(-1))^5$$

$$=1-\Phi^5(1)=1-0.8413^5=1-0.4215=0.5785.$$

5. $n_1=10,\quad n_2=15,\quad \mu=20,\quad \sigma^2=3,\ \bar{X}\sim N(\mu,\frac{\sigma^2}{n_1}),\quad \bar{Y}\sim N(\mu,\frac{\sigma^2}{n_2}),$

$$\bar{X}-\bar{Y}\sim N(0,\ \frac{\sigma^2}{n_1}+\frac{\sigma^2}{n_2})=N(0,\frac{1}{2}),$$

$$\therefore \quad \frac{(\bar{X}-\bar{Y})-0}{\sqrt{1/2}}\sim N(0,1),$$

$$P\{|\bar{X}-\bar{Y}|>0.3\}=1-P\{|\bar{X}-\bar{Y}|\le 0.3\}$$

$$=1-P\{-\frac{0.3}{\sqrt{1/2}}\le \frac{\bar{X}-\bar{Y}}{\sqrt{1/2}}\le 0.3\sqrt{2}\}$$

$$=1-(2\Phi(0.3\sqrt{2})-1)=2(1-\Phi(0.42))$$

$$=2(1-0.6628)=0.6744.$$

6. $X_i\sim N(0,0.3^2),\quad \therefore Y_i=\frac{X_i-0}{0.3}\sim N(0,1),$

$$\therefore \sum_{i=1}^{n} Y_i^2=\sum_{i=1}^{n}\left(\frac{X_i-0}{0.3}\right)^2\sim \chi^2(n).$$

$$P\{\sum_{i=1}^{n} X_i^2>1.44\}=P\{\sum_{i=1}^{n}\left(\frac{X_i-0}{0.3}\right)^2>\frac{1.44}{0.09}\}=P\{\chi^2(n)>16\}=0.1\ (\text{查表})$$

7. (1) 22.363, 17.535; (2) 2.4469, 1.3722;

(3) 3.33, $F_{0.90}(28,2)=\frac{1}{F_{0.10}(2,28)}=\frac{1}{2.5}=0.4$

8. $\because T\sim t(n),\therefore T=\frac{X}{\sqrt{Y/n}}$,其中$X\sim N(0,1),Y\sim \chi^2(n),$

$$\therefore T^2=\frac{X^2/1}{Y/n}\sim F(1,n).$$

9. $\sum_{i=1}^{n}(X_i-\mu)^2/\sigma^2=\sum_{i=1}^{n}(\frac{X_i-\mu}{\sigma})^2\sim\chi^2(n).$

10. $U=\frac{(m-1)S_1^2}{\sigma_1^2}\sim\chi^2(m-1),\quad V=\frac{(n-1)S_2^2}{\sigma_2^2}\sim\chi^2(n-1)$，由 F 分布定义有

$$\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}=\frac{U/(m-1)}{V/(n-1)}\sim F(m-1,n-1)$$

11. $E(\bar{X})=\mu, D(\bar{X})=\frac{\sigma^2}{n}, E(S^2)=\sigma^2,$ 故

(1)$X\sim(0-1)$且$P\{X=1\}=p,$ 于是

$$E(\bar{X})=p,\quad D(\bar{X})=\frac{\sigma^2}{n}=\frac{p(1-p)}{n},\quad E(S^2)=\sigma^2=p(1-p)$$

(2)$X\sim P(\lambda)$： $E(\bar{X})=\lambda,\ D(\bar{X})=\frac{\sigma^2}{n}=\frac{\lambda}{n},\ E(S^2)=\sigma^2=\lambda.$

12. $\because X_i\sim N(0,2^2)(i=1,2,3,4),$

$\therefore E(X_1-2X_2)=0,\quad E(3X_3+4X_4)=0,$

$D(X_1-2X_2)=D(X_1)+4D(X_2)=20,$

$D(3X_3+4X_4)=9D(X_3)+16D(X_4)=100,$

$\therefore\ \frac{X_1+X_2-0}{\sqrt{20}}\sim N(0,1), \frac{3X_3+4X_4-0}{10}\sim N(0,1),$ 独立

$\therefore(\frac{X_1+X_2-0}{\sqrt{20}})^2+(\frac{3X_3+4X_4-0}{10})^2\sim\chi^2(2).$

$\therefore a=\frac{1}{20},\quad b=\frac{1}{100}.$

13. $\because X_i\sim N(\mu,\sigma^2)(i=1,2,3,4),$

$\therefore E(X_1-X_2)=0,\quad E(X_3-X_4)=0,$

$D(X_1-X_2)=D(X_1)+D(X_2)=2\sigma^2,$

$D(X_3-X_4)=D(X_3)+D(X_4)=2\sigma^2,$

$\therefore\ X_1-X_2\sim N(0,2\sigma^2),\quad X_3-X_4\sim N(0,2\sigma^2),$

$\therefore(\frac{(X_1-X_2)-0}{\sqrt{2}\sigma})^2\sim\chi^2(1),\quad(\frac{(X_3-X_4)-0}{\sqrt{2}\sigma})^2\sim\chi^2(1)$

独立,

$$\therefore(\frac{X_1-X_2}{X_3-X_4})^2=\frac{(\frac{(X_1-X_2)-0}{\sqrt{2}\sigma})^2/1}{(\frac{(X_3-X_4)-0}{\sqrt{2}\sigma})^2/1}\sim\chi^2(2).$$

2. $a=1/8,\quad b=1/12\quad c=1/16.$ **3.**

$$\because Y=8a\left(\frac{\frac{X_1+X_2}{2}-0}{2/\sqrt{2}}\right)^2+12b\left(\frac{\frac{X_3+X_4+X_5}{3}-0}{2/\sqrt{3}}\right)^2$$

$$+16c\left(\frac{\frac{X_6+X_7+X_8+X_9}{4}-0}{2/\sqrt{4}}\right)^2\sim\chi^2(3).$$

3. 设 $\bar{X}$ 和 S^2 分别为正态总体 $N(0,\sigma^2)$ 的样本的样本均值和样本方差，样本容量为 n，则统计量 $\frac{n\bar{X}^2}{S^2}$ 服从____________分布.

3. $F(1,n-1).\quad\because\frac{\bar{X}-0}{\sigma/\sqrt{n}}\sim N(0,1),\quad\therefore U=\frac{\bar{X}^2}{\sigma^2/n}\sim\chi^2(1),$

又 $\because V=\frac{(n-1)S^2}{\sigma^2}\sim\chi^2(n-1),$

$$\therefore\quad\frac{n\bar{X}^2}{S^2}=\frac{\frac{\bar{X}^2}{\sigma^2/n}\Big/1}{\frac{(n-1)S^2}{\sigma^2}\Big/(n-1)}=\frac{U/1}{V/(n-1)}\sim F(1,n-1).)$$

4. 设 $X_1,X_2,\cdots,X_n$ 为正态总体 $N(0,\sigma^2)$ 的样本，μ,σ^2 均未知，则下列样本函数中（　　）是统计量

(A) $\sum_{i=1}^{n}X_i-\mu$；　　(B) $X_i-\bar{X}$；

(C) $\sum_{i=1}^{n}\left(\frac{X_i}{\sigma}\right)^2$；　　(D) $\sum_{i=1}^{n}\left(\frac{X_i-\bar{X}}{\sigma}\right)^2$.

4. (B)　(因无未知参数)

5. 设 $X\sim N(0,\sigma^2)$，则服从自由度为 $n-1$ 的 t 分布的随机变量是（　　）

(A) $\frac{\sqrt{n}\bar{X}}{S}$；　　(B) $\frac{\sqrt{n-1}\bar{X}}{S}$；

(C) $\frac{\sqrt{n}\bar{X}}{S^2}$；　　(D) $\frac{\sqrt{n-1}\bar{X}}{S^2}$.

$$\textbf{5. } A.\quad\because\frac{\sqrt{n}\bar{X}}{S}=\frac{\frac{\bar{X}-0}{\sigma/\sqrt{n}}}{\sqrt{\frac{(n-1)S^2}{\sigma^2}\Big/(n-1)}}\sim t(n-1).)$$

6. 假设总体 X 的概率密度为 $f(x)$，记为 $X \sim f(x)$，且其期望 μ 与方差 σ^2 都存在，$X_1, X_2, \cdots, X_n (n>1)$ 为其样本，X 为样本均值，则有（　　）

(A) $X \sim f(x)$；　　　　(B) $\min\limits_{1\leqslant i\leqslant n} X_i \sim f(x)$；

(C) $\max\limits_{1\leqslant i\leqslant n} X_i \sim f(x)$；　　　　(D) $(X_1, X_2, \cdots, X_n) \sim \prod\limits_{i=1}^{n} f(x_i)$.

7. 假定 $X \sim N(0,1)$，$X = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$，$S^2 = \frac{1}{n-1}\sum\limits_{i=1}^{n}(X_i - X)^2$，则服从自由度为 $(n-1)$ 的 χ^2 分布的随机变量是______

(A) $\sum\limits_{i=1}^{n} X_i^2$；　　　　(B) S^2；

(C) $(n-1)X^2$；　　　　(D) $(n-1)S^2$.

6. D. $\because$ $X_1, X_2, \cdots, X_n$ 独立，

$$\therefore\ f(x_1, x_2, \cdots, x_n) = f(x_1) f(x_2) \cdots f(x_n).$$

即 $$(X_1, X_2, \cdots, X_n) \sim \prod_{i=1}^{n} f(x_i),$$

(A)不正确. $\because X_1, X_2, \cdots, X_n$ 是样本，故 $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$，

即 $\bar{X} \nsim N(\mu, \sigma^2)$.

(B),(C)均不正确.

7. D. $\because$ 由 $X \sim N(0,1)$ 及 $P93Th2.2(1)$ 知

$$\frac{(n-1)S^2}{1^2} \sim \chi^2(n-1).$$

8. $$\because \bar{X} \sim N(\mu, \frac{\sigma^2}{n}),\quad \therefore X_{n+1} - \bar{X} \sim N(0, \sigma^2 + \frac{\sigma^2}{n}),\quad \therefore Z = \frac{\dfrac{(X_{n+1} - \bar{X}) - 0}{\sqrt{\sigma^2 + \dfrac{\sigma^2}{n}}}}{\sqrt{\dfrac{(n-1)S^2}{\sigma^2}\Big/(n-1)}} \sim t(n-1).$$

$$\therefore f(z) = \frac{\Gamma[(n+1)/2]}{\sqrt{n\pi}\,\Gamma[n/2]}\left(1 + \frac{z^2}{n}\right)^{-\frac{n+1}{2}}, -\infty < z < +\infty.$$

9. $$\because T \sim t(n), \therefore T = \frac{X}{\sqrt{Y/n}}，其中 X \sim N(0,1), Y \sim \chi^2(n),$$

$$\therefore T^2 = \frac{X^2/1}{Y/n} \sim F(1, n).$$

书上习题

1. (1),(2),(3)是.

$$8.\ \bar{x}_k = \frac{1}{k}\sum_{i=1}^{k} x_i = \bar{x}_{k-1} + \frac{1}{k}\sum_{i=1}^{k} x_i - \frac{1}{k-1}\sum_{i=1}^{k-1} x_i = \bar{x}_{k-1} + \frac{1}{k}[\sum_{i=1}^{k} x_i - \frac{k-1+1}{k-1}\sum_{i=1}^{k-1} x_i]$$

$$= \bar{x}_{k-1} + \frac{1}{k}[\sum_{i=1}^{k} x_i - \sum_{i=1}^{k-1} x_i - \frac{1}{k-1}\sum_{i=1}^{k-1} x_i] = \bar{x}_{k-1} + \frac{1}{k}(x_k - \bar{x}_{k-1}).$$