

1.

(1) $\Omega = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 3, 5\}$;

(2) $\Omega = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$, 0 表正面, 1 表反面.

$A = \{(0, 0), (0, 1)\}$, $B = \{(0, 0), (1, 1)\}$, $C = \{(0, 0), (0, 1), (1, 0)\}$;

(3) $\Omega = \{(1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 3, 4), (1, 3, 5), (1, 4, 5), (2, 3, 4), (2, 3, 5), (2, 4, 5), (3, 4, 5)\}$, ——按组合

$A = \{(1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 3, 4), (1, 3, 5), (1, 4, 5)\}$;

(4) $\Omega = \{(1, 1), \dots, (1, 4); (2, 1), \dots, (2, 4); \dots; (4, 1), \dots, (4, 4)\}$,

$A = \{(1, 2), (2, 1), (2, 4), (4, 2)\}$;

(5) 第 1 个球有 3 个盒子可放, 第 2 个球也有 3 个盒子可放, 共 3×3 种:

$\Omega = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$,

$A = \{(1, 1), (1, 2), (1, 3), (2, 1), (3, 1)\}$.

注: 球与盒子加以区别, 共 3 乘 3 种, 第 1 盒至少 1 球的放法有 $3 \times 3 - 2 \times 2$ 种

2. (1) $\overline{AB} = \overline{A} \cup \overline{B} = A \cup B = \{2, 3, 4, 5\}$

(2) $\overline{ABC} = \overline{A} \cup \overline{BC} = \overline{A} \cup BC = \{1, 5, 6, 7, 8, 9, 10\} + \{5\}$
 $= \{1, 5, 6, 7, 8, 9, 10\}$

3. (1) A ; (2) \overline{ABC} ; (3) ABC ; (4) ABC

4.

(1) $(A \cup B) \cap (A \cup \overline{B}) = A \cup (B \cap \overline{B}) = A \cup \emptyset = A$ (第一个等号由对 A 分配的逆运算)

(2) $\underline{(A \cup B)} \cap (B \cup C) = \underline{(B \cup A)} \cap (B \cup C) = B \cup (A \cap C)$ (由对 B 分配的逆运算)

(3) $\underline{(A \cap B)} \cap \underline{(A \cup \overline{B})} \cap \underline{(\overline{A} \cap B)} = \underline{((A \cap B) \cap (\overline{A} \cap B))} \cap (A \cup \overline{B})$
 $= \underline{((A \cap \overline{A}) \cap B)} \cap (A \cup \overline{B}) = \underline{(\emptyset \cap B)} \cap (A \cup \overline{B}) = \underline{\emptyset} \cap (A \cup \overline{B}) = \emptyset.$

5. $P(AB) = P(\overline{AB}) \Rightarrow P(AB) = P(\overline{A \cup B}) \Rightarrow P(AB) = 1 - P(A \cup B)$

$P(AB) = 1 - P(A) - P(B) + P(AB)$

$0 = 1 - P(A) - P(B)$

$P(B) = 1 - p$

6.

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(CA) + P(ABC)$

$= 0.25 + 0.25 + 0.25 - 0 - 0 - 0.125 + P(\emptyset)$

$= 0.625$

7.

(1) Ω 有 C_{10}^3 个基本事件, 当最小号是5时其它两个

取自6~10中, 总数为 C_5^2 , 故

$$p = \frac{C_5^2}{C_{10}^3} = \frac{5!/(2!3!)}{10!/(3!7!)} = \frac{10}{120} = \frac{1}{12}.$$

$$(2) p = \frac{C_4^2}{C_{10}^3} = \frac{1}{20}.$$

$$(3) \text{理解为取自1与9, 2与8, 3与7, 4与6"中间": } p = \frac{4}{C_{10}^3} = \frac{4}{120} = \frac{1}{30}.$$

$$\text{理解为取"之间": } p = \frac{C_4^1 \cdot C_5^1}{C_{10}^3} = \frac{20}{120} = \frac{1}{6}.$$

8. 法1 需按排列求解, 视为连续取7张. 否则, 取单字母与重字母不等可能.

$$p = \frac{P_1 \cdot P_2^1 \cdot P_1 \cdot P_2^1 \cdot P_1 \cdot P_1 \cdot P_1}{P_{11}^7} = \frac{4}{P_{11}^7} = 0.000\ 002405.$$

法2 利用乘法公式计算:

$$\begin{aligned} p &= P(A_a A_b A_t A_l A_i A_y) \\ &= P(A_a)P(A_b | A_a)P(A_t | A_a A_b)P(A_l | A_a A_b A_t)P(A_i | A_a A_b A_t A_l) \\ &\quad \cdot P(A_y | A_a A_b A_t A_l A_i) \\ &= \frac{1}{11} \frac{2}{10} \frac{2}{9} \frac{1}{8} \frac{1}{7} \frac{1}{6} \frac{1}{5} = \frac{4}{P_{11}^7} = 0.000\ 002405. \end{aligned}$$

$$9. (1) \text{不含0和5的情况有 } C_8^3 \text{ 种: } P(A_1) = \frac{C_8^3}{C_{10}^3} = \frac{7}{15} \approx 0.467.$$

(2) 含0但不含5的情况有 C_8^2 种, 含5但不含0的情况有 C_8^2 种,
不含0和5的情况有 C_8^3 种

$$P(A_2) = \frac{C_8^2 + C_8^2}{C_{10}^3} + \frac{C_8^3}{C_{10}^3} = \frac{7}{15} + \frac{7}{15} \approx 0.933.$$

10.

Ω —从10只中取4只共有 C_{10}^4 种取法.

法1 $C_5^1(5 \text{ 双中取1双}) \times C_8^2(\text{剩下的4双中取2只}) - C_5^2(\text{重复的})$

$$p = \frac{C_5^1 \cdot C_8^2 - C_5^2}{C_{10}^4} = \frac{5 \cdot \frac{8!}{2!6!} - \frac{5!}{2!3!}}{\frac{10!}{4!6!}} = \frac{5 \cdot 28 - 10}{210} = \frac{13}{21}.$$

注: 这种算法对恰好取到2双是重复取的。如5双鞋号: 36, 37, 38, 39, 40.

当取5双中36号而另外2只取40号时那双时, 又会出现5只中取到40号而另2只取到36号这一双, 这就重了一次。

法2 $C_5^1(5\text{双中取1双}) \times C_4^1(\text{剩下的4双中取1只}) \times C_6^1(\text{在余下的6只中任取1只})$
 $+ C_5^2(5\text{双中取两双})$

$$p = \frac{C_5^1 \cdot C_4^1 \cdot C_6^1 + C_5^2}{C_{10}^4} = \frac{5 \cdot 4 \cdot 6 + 10}{7 \cdot 3 \cdot 10} = \frac{13}{21}.$$

法3 $C_5^1(5\text{双中取1双}) \times C_4^1(\text{剩下的4双中取1只}) \times C_7^1(\text{在余下的7只中任取1只}) - C_5^2(\text{被重取的})$

$$p = \frac{C_5^1 \cdot C_4^1 \cdot C_7^1 - C_5^2}{C_{10}^4} = \frac{5 \cdot 4 \cdot 7 - 10}{7 \cdot 3 \cdot 10} = \frac{13}{21}.$$

法4 $C_5^1(5\text{双中取1双}) \times C_8^1(\text{剩下的8双中取1只}) \times C_3^1(\text{在不成对的3双中取1只}) + C_5^2$

$$p = \frac{C_5^1 \cdot C_8^1 \cdot C_3^1 + C_5^2}{C_{10}^4} = \frac{5 \cdot 8 \cdot 3 + 10}{7 \cdot 3 \cdot 10} = \frac{13}{21}.$$

11. 设 $A_i = \text{"第}i\text{次取到黑球"} , i = 1, 2.$

$$(1)\text{法1 } P(A_1 A_2) = \frac{P_5^1 \cdot P_5^1}{7^2} = \frac{25}{49}. \quad \text{法2 } P(A_1 A_2) = P(A_1)P(A_2) = \frac{5}{7} \cdot \frac{5}{7}$$

$$(2)\text{法1 } P(A_1 \bar{A}_2) = \frac{P_5^1 \cdot P_2^1}{7^2} = \frac{10}{49}. \quad \text{法2 } P(A_1 \bar{A}_2) = P(A_1)P(\bar{A}_2) = \frac{5}{7} \cdot \frac{2}{7}$$

$$(3)\text{法1 } P(\bar{A}_2) = \left(\frac{P_7^1 \cdot P_2^1}{7^2}\right) = \frac{2}{7}. \quad \text{法2 } P(\bar{A}_2) = P(A_1)P(\bar{A}_2 | A_1) + P(\bar{A}_1)P(\bar{A}_2 | \bar{A}_1)$$

12. 将线段分成长为 x, y, z 的三段, 则

$$x + y + z = l.$$

欲构成三角形, 当且仅当

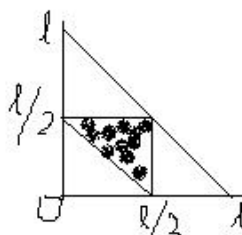
$$0 < x, y < \frac{l}{2}, \quad \frac{l}{2} < x + y < l$$

利用平面坐标系给出上面的关系为

三角形内的点, 故概率为

$$p = \frac{1/2 \cdot l/2 \cdot l/2}{1/2 \cdot l^2} = \frac{1}{4}.$$

注: 解法2在小32开本蔡马的本上 P16~17



13. 需要硬币中心位于与桌边界距离为 r 的正方形内, 故

$$p = \frac{(a-2r)^2}{a^2}.$$

14. 设 A_i 表示 “第一次取到 i 只新球” $i = 0, 1, 2, 3$;

B 表示 “第二次取到 3 只新球”

$$(1) P(B) = P(A_0)P(B|A_0) + P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)$$

$$= \frac{C_3^3}{C_{12}^3} \frac{C_9^3}{C_{12}^3} + \frac{C_9^1 \cdot C_3^2}{C_{12}^3} \frac{C_8^3}{C_{12}^3} + \frac{C_9^2 \cdot C_3^1}{C_{12}^3} \frac{C_7^3}{C_{12}^3} + \frac{C_9^3}{C_{12}^3} \frac{C_6^3}{C_{12}^3} = \frac{7056}{(C_{12}^3)^2}$$

$$\approx 0.145785.$$

$$(2)P(A_3|B)=\frac{P(A_3B)}{P(B)}=\frac{\frac{C_9^3 C_6^3}{C_{12}^3 C_{12}^3}}{\frac{7056}{(C_{12}^3)^2}}=\frac{C_6^3 C_9^3}{7056}=\frac{105}{441}=0.238095.$$

15.

A = "从甲袋取出红球放入乙袋", B = "从乙袋取出红球".

当从乙袋中取球前,其袋内被放入的或是红球或是白球,

这将影响第二次取球,于是有

$$P(B) = P(A)P(B|A) + P(\bar{A})P(B|\bar{A}) = \frac{3}{4} \cdot \frac{5}{7} + \frac{1}{4} \cdot \frac{4}{7} = \frac{19}{28}.$$

16. A_i = "取自第*i*个车间生产的" $i=1,2,3$, B = "取出的次品".

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3) \\ = 0.05 \times 0.25 + 0.04 \times 0.35 + 0.02 \times 0.40 = 0.0345.$$

$$P(A_1|B) = \frac{P(A_1B)}{P(B)} = \frac{0.05 \times 0.25}{0.0345} = \frac{25}{69} \\ = 0.3623 \dots$$

17. 法1 设 A_i = "第*i*只枪击中目标", $i=1,2,\dots,25$. 则

$$P(A_1 + A_2 + \dots + A_{25}) = 1 - P(\overline{A_1 \cup A_2 \cup \dots \cup A_{25}}) \\ = 1 - P(\overline{A_1} \cap \overline{A_2} \cap \dots \cap \overline{A_{25}}) = 1 - P(\overline{A_1})P(\overline{A_2}) \dots P(\overline{A_{25}}) \\ = 1 - 0.992^{25} \approx 0.818$$

法2 按 25 重贝努利概型得

$$P(A_1 + A_2 + \dots + A_{25}) = 1 - P_{25}(0) = 1 - C_{25}^0 p^0 (1-p)^{25-0} = 1 - 1 \cdot 1 \cdot 0.992^{25}$$

18. 法1 设命中率为 p , 且

设 A_i = "第*i*次射击击中目标", $i=1,2,3$. 由射击是独立的, 有

$$0.875 = P(A_1 \cup A_2 \cup A_3) = 1 - P(\overline{A_1 \cup A_2 \cup A_3}) \text{ (德·莫根律)} \\ = 1 - P(\overline{A_1} \overline{A_2} \overline{A_3}) = 1 - P(\overline{A_1})P(\overline{A_2})P(\overline{A_3}) = 1 - (1-p)^3,$$

$$1-p = \sqrt[3]{0.125} = 0.5 \Rightarrow p = 0.5.$$

法2 按 3 重贝努利概型得

$$0.875 = P(A_1 \cup A_2 \cup A_3) = 1 - C_3^0 p^0 (1-p)^{3-0} = 1 - (1-p)^3.$$

A_i = "第*i*个人独立译出密码", $i=1,2,3$.

由于每个人译出是独立的, 所以有

19.

$$\text{法1 } P(A_1 \cup A_2 \cup A_3) = 1 - P(\overline{A_1 \cup A_2 \cup A_3}) \\ = 1 - P(\overline{A_1} \overline{A_2} \overline{A_3}) = 1 - P(\overline{A_1})P(\overline{A_2})P(\overline{A_3}) \\ = 1 - \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} = \frac{3}{5} = 0.6.$$

$$\begin{aligned}
\text{法2 } P(A_1 \cup A_2 \cup A_3) &= P(A_1) + P(A_2) + P(A_3) - P(A_1 A_2) \\
&\quad - P(A_2 A_3) - P(A_3 A_1) + P(A_1 A_2 A_3) \\
&= \frac{1}{5} + \frac{1}{4} + \frac{1}{3} - \frac{1}{5 \cdot 4} - \frac{1}{4 \cdot 3} - \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 4 \cdot 3} \\
&= 0.6.
\end{aligned}$$

20.

A = "甲被击落", B = "乙被击落",

A_i = "甲第 i 次射击击落乙", $i = 1, 2$.

B_1 = "乙第1次射击击落甲".

$$\begin{array}{l}
\left. \begin{array}{l} \text{射中乙概率 } 0.2 \\ \text{射不中乙 } 0.8 \end{array} \right\} \text{甲} \left\{ \begin{array}{l} \text{乙射中甲概率 } 0.3 \\ \text{乙射不中甲 } 0.7 \end{array} \right. \left\{ \begin{array}{l} \text{甲射中乙 } 0.4 \\ \text{甲射不中乙 } 0.6 \end{array} \right.
\end{array}$$

$$\begin{aligned}
(1) P(A) &= P(\overline{A_1} B_1) = P(B_1 | \overline{A_1}) P(\overline{A_1}) = 0.3 \times 0.8 \\
&= 0.24.
\end{aligned}$$

$$\begin{aligned}
(2) P(B) &= P(A_1 \cup \overline{A_1} \overline{B_1} A_2) = P(A_1) + P(\overline{A_1} \overline{B_1} A_2) \\
&= 0.2 + P(A_2 | \overline{A_1} \overline{B_1}) P(\overline{B_1} | \overline{A_1}) P(\overline{A_1}) \\
&= 0.2 + 0.4 \times 0.7 \times 0.8 = 0.424.
\end{aligned}$$

注: 若用全概率公式逐层分析计算则会较麻烦.