第五章习题参考答案

1. 记$Z=X+Y$，则

$$P\{|X+Y|\ge 6\}=P\{|(X+Y)-(-2+2)|\ge 6\}=P\{|Z-\mu|\ge 6\}\le \frac{DZ}{6^2}$$

$$=\frac{DX+DY+2\rho_{XY}\sqrt{DX}\cdot\sqrt{DY}}{6^2}=\frac{1+4+2(-0.5)\cdot 1\cdot 2}{36}=\frac{5-2}{36}=\frac{1}{12}.$$

注：书上答案为$\frac{1}{4}$

2. 记事件A发生的概率为p，迟当事件A发生时的频率为$\frac{1}{n}$，

由贝努利大数定律对任意的正数$\varepsilon>0$，有

$$\lim_{n\to\infty}P\{|\frac{1}{n}-p|\le\varepsilon\}=1.$$

即当p很小时，只要n足够大即可使$\frac{1}{n}$很小.

3. $\because\quad E(X_k)=\frac{6-0}{2}=3,\quad D(X_k)=\frac{(6-0)^2}{12}=3,\quad k=1,2,\cdots,100.$

$$\therefore\quad E(Y)=E(\sum_{k=1}^{100}X_k)=\sum_{k=1}^{100}E(X_k)=100\times 3=300,\quad D(Y)=D(\sum_{k=1}^{100}X_k)$$

$$=\sum_{k=1}^{100}D(X_k)=300.$$

$$\therefore\quad P\{260<Y<340\}=P\{260-300<Y-300<340-300\}=P\{|Y-300|<40\}$$

$$>1-\frac{D(Y)}{40^2}=1-\frac{300}{40^2}=1-\frac{3}{16}=\frac{13}{16}.$$

4. (1) $X_i\sim(0-1), P\{X_i=1\}=p,\quad Y=\sum_{i=1}^{2000}X_i\sim B(2000,0.01)$, 德莫佛-拉普拉斯

$$P\{15<Y<25\}=P\{\frac{15-2000\times 0.01}{\sqrt{2000\times 0.01\times 0.99}}<\frac{Y-2000\times 0.01}{\sqrt{2000\times 0.01\times(1-0.01)}}<\frac{25-2000\times 0.01}{\sqrt{2000\times 0.01\times 0.99}}\}$$

$$=P\{\frac{Y-20}{\sqrt{19.8}}<\frac{5}{\sqrt{19.8}}\}-P\{\frac{Y-20}{\sqrt{19.8}}<\frac{-5}{\sqrt{19.8}}\}\approx\Phi(\frac{5}{\sqrt{19.8}})-\Phi(\frac{-5}{\sqrt{19.8}})$$

$$=2\Phi(\frac{5}{\sqrt{19.8}})-1=2\Phi(\frac{5}{4.445})-1\approx 2\Phi(1.12)-1=2\times 0.8686-1=0.7372.$$

(2) $$P\{Y\ge 10\}=1-P\{Y<10\}=1-P\{\frac{Y-20}{\sqrt{19.8}}<\frac{10-20}{\sqrt{19.8}}\}=1-\Phi(\frac{-10}{\sqrt{19.8}})=\Phi(\frac{10}{\sqrt{19.8}})$$

$$=\Phi(2.25)=0.9878.$$

5. 损坏部件数$Y\sim B(100,0.1)$, 系统正常工作的概率

$$P\{\eta<15\}=P\{\frac{Y-100\times 0.1}{\sqrt{100\times 0.1\times 0.9}}<\frac{15-100\times 0.1}{\sqrt{100\times 0.1\times 0.9}}\}=P\{\frac{Y-10}{3}<\frac{15-10}{3}\}=P\{\frac{Y-10}{3}<\frac{5}{3}\}$$

$$\approx\Phi(\frac{5}{3})\approx\Phi(1.67)=0.9525.$$

6. 设需开工机床台数为 Y，保证开工的供电千瓦数为 N，则 $\boldsymbol{Y} \sim \boldsymbol{B}(200, 0.6)$,
以 0.999 开工率的条件得

$$0.999 = \boldsymbol{P}\{\boldsymbol{N} \ge \boldsymbol{Y} \cdot 1\} = 1 - \boldsymbol{P}\{\boldsymbol{Y} < \boldsymbol{N}\} = 1 - \boldsymbol{P}\{\frac{\boldsymbol{Y} - 200 \times 0.7}{\sqrt{200 \times 0.7 \times 0.3}} < \frac{\boldsymbol{N} - 200 \times 0.7}{\sqrt{200 \times 0.7 \times 0.3}}\}$$

$$= 1 - \boldsymbol{P}\{\frac{\boldsymbol{Y} - 140}{\sqrt{42}} < \frac{\boldsymbol{N} - 140}{\sqrt{42}}\} \approx 1 - \Phi(\frac{\boldsymbol{N} - 140}{\sqrt{42}})$$

$$\therefore \quad \Phi(\frac{\boldsymbol{N} - 140}{\sqrt{42}}) = 0.001, \quad \frac{\boldsymbol{N} - 140}{\sqrt{42}} = 3.1, \quad \boldsymbol{N} \approx 160.$$

故供电 142 千瓦即可保证开工率达到 99.9%.