1.设 $X \sim N(-1,16)$，试计算（1）$P(X < 2.44)$；（2）$P(X > -1.5)$；（3）$P(|X| < 4)$；（4）$P(|X-1| > 1)$.

解：

（1）$P(X < 2.44) = \Phi(\frac{2.44-(-1)}{4}) = \Phi(\frac{3.44}{4}) \doteq 0.8051$

（2）$P(X > -1.5) = 1 - P(X \le -1.5)$

$$= 1 - \Phi(\frac{-1.5+1}{4}) = 1 - \Phi(-\frac{1}{8}) \doteq 0.5498$$

（3）$P(|X| < 4) = \Phi(\frac{4+1}{4}) - \Phi(\frac{-4+1}{4}) = \Phi(\frac{5}{4}) - \Phi(\frac{-3}{4})$

$$= \Phi(\frac{5}{4}) + \Phi(\frac{3}{4}) - 1 \doteq 0.6678$$

（4）$P(|X-1| > 1) = P[(X < 0) \cup (X > 2)] = P(X < 0) + P(X > 2)$

$$= \Phi(\frac{0+1}{4}) + 1 - \Phi(\frac{2+1}{4}) = \Phi(\frac{1}{4}) + 1 - \Phi(\frac{3}{4}) \doteq 0.8253$$

2. 设随机变量 X 和 Y 均服从正态分布，$X \sim N(\mu, 4^2)$，$Y \sim N(\mu, 5^2)$，而 $p_1 = P(X \le \mu - 4)$，$p_2 = P(Y \ge \mu + 5)$，试证明 $p_1 = p_2$.

证明：

$$\because\ p_1 = P(X \le \mu - 4) = \Phi\left(\frac{\mu - 4 - \mu}{4}\right) = \Phi(-1)$$

$$p_2 = P(Y \ge \mu + 5) = 1 - \Phi\left(\frac{\mu + 5 - \mu}{5}\right) = 1 - \Phi(1) = \Phi(-1)$$

$$\therefore\ p_1 = p_2.$$