第七章习题参考答案

P107

1. 二项分布的$X \sim B(n,p)$的$\mu = np, \sigma^2 = np(1-p)$,.由矩估计法有

$$\begin{cases} A_1 = \mu \\ B_2 = \sigma^2 \end{cases} \Rightarrow \begin{cases} A_1 = np \\ B_2 = np(1-p) \end{cases} \Rightarrow \begin{cases} \hat{p} = 1 - B_2 / A_1 \\ \hat{n} = A_1^2 / (A_1 - B_2) \end{cases}.$$

2. (1) 矩估计：$A_1 = \mu \Rightarrow \overline{X} = \dfrac{1}{p}$，矩估计值及矩估计量为

$$\hat{p} = \frac{1}{\overline{x}}, \quad \hat{p} = \frac{1}{\overline{X}}.$$

(2) 极大似然估计：

$$f(x_i;p) = p(1-p)^{x_i - 1} \quad (i = 1,2,\cdots,n), \quad x_i \in Z^+$$

$$L(p) = \prod_{i=1}^{n} f(x_i;p) = \prod_{i=1}^{n} p(1-p)^{x_i-1} = p^n(1-p)^{\sum\limits_{i=1}^{n} x_i - n} = p^n(1-p)^{n(\overline{x}-1)},$$

$$\ln L(p) = n\ln p + n(\overline{x}-1)\ln(1-p).$$

令 $\dfrac{1}{L(p)}L'(p) = \dfrac{n}{p} + n(\overline{x}-1)\dfrac{-1}{1-p} = 0$，得$p = \dfrac{1}{\overline{x}}$.

估计值 $\hat{p} = \dfrac{1}{\frac{1}{n}\sum\limits_{i=1}^{n} x_i}$，估计量$\hat{p} = \dfrac{1}{\frac{1}{n}\sum\limits_{i=1}^{n} X_i}$.

3. $E(X) = \int_{-\infty}^{+\infty} xf(x)dx = \int_{\mu}^{+\infty} x \cdot \theta e^{-\theta(x-\mu)}dx = \dfrac{1}{\theta} + \mu.$

$$E(X^2) = \int_{\mu}^{+\infty} x^2 \cdot \theta e^{-\theta(x-\mu)}dx = -[x^2 e^{-\theta(x-\mu)}\Big|_{\mu}^{+\infty} - \int_{\mu}^{+\infty} 2xe^{-\theta(x-\mu)}dx] \quad (\theta > 0)$$

$$= -[-\mu^2 - \frac{2}{\theta}\int_{\mu}^{+\infty} x \cdot \theta e^{-\theta(x-\mu)}dx] = \mu^2 + \frac{2}{\theta}E(X)$$

$$= \mu^2 + \frac{2}{\theta^2} + \frac{2\mu}{\theta}.$$

矩估计： $A_k = \mu_k = E(X^k), (k = 1,2)$

$$\begin{cases} A_1 = E(X) \\ A_2 = E(X^2) \end{cases} \Rightarrow \begin{cases} A_1 = \dfrac{1}{\theta} + \mu \\ B_2 + A_1^2 = \dfrac{2}{\theta^2} + \dfrac{2\mu}{\theta} + \mu^2 \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{1}{\theta} = A_1 - \mu \\ B_2 + A_1^2 = 2(A_1 - \mu)^2 + 2\mu(A_1 - \mu) + \mu^2 \end{cases}$$

$$\Rightarrow\begin{cases}\dfrac{1}{\theta}=A_1-\mu\\ B_2+A_1^2=2(A_1-\mu)^2+2\mu(A_1-\mu)+\mu^2\end{cases}$$

$$\Rightarrow\begin{cases}\dfrac{1}{\theta}=A_1-\mu\\ \mu^2-2\mu A_1+A_1^2-B_2=0\end{cases}\Rightarrow\begin{cases}\dfrac{1}{\theta}=A_1-\mu\\ \mu=A_1\pm\sqrt{B_2}\end{cases}\Rightarrow\begin{cases}\theta=\dfrac{1}{\mp\sqrt{B_2}}\\ \mu=A_1\pm\sqrt{B_2}\end{cases}$$

$\because\ \theta>0,\ \therefore\ \mu=A_1-\sqrt{B_2}$.

矩估计量为 $\hat{\theta}=\dfrac{1}{\sqrt{B_2}}$， $\hat{\mu}=A_1-\sqrt{B_2}$.

(2) 极大似然估计：

$$f(x_i;\theta,\mu)=\theta e^{-\theta(x_i-\mu)}\quad(x_i>\mu,\quad i=1,2,\cdots,n),$$

$$L(\theta,\mu)=\prod_{i=1}^{n}\theta e^{-\theta(x_i-\mu)}=\theta^n e^{-\theta(\sum_{i=1}^{n}x_i-n\mu)}=\theta^n e^{n\theta(\mu-\bar{x})},$$

$$\ln L(\theta,\mu)=n\ln\theta+n\theta(\mu-\bar{x}).$$

$$令\begin{cases}\dfrac{1}{L(\theta,\mu)}L'_\theta(\theta,\mu)=\dfrac{n}{\theta}+n(\mu-\bar{x})=0\\ \dfrac{1}{L(\theta,\mu)}L'_\mu(\theta,\mu)=0+n\theta\cdot 1=0\end{cases}\Rightarrow 方法无效.$$

4. 设总体$X\sim N(\mu,\sigma^2),\mu,\sigma^2$未知. 概率密度$f(x;\mu,\sigma^2)=\dfrac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

似然函数为 $L(\mu,\sigma)=\dfrac{1}{(\sqrt{2\pi}\sigma)^n}e^{-\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}}$,

$$\ln L(\mu,\sigma)=-n\ln\sqrt{2\pi}-n\ln\sigma-\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}$$

$$令\begin{cases}\dfrac{1}{L(\theta)}L'_\mu(\theta)=\sum_{i=1}^{n}\dfrac{x_i-\mu}{\sigma^2}=0\\ \dfrac{1}{L(\theta)}L'_\sigma(\theta)=-\dfrac{n}{\sigma}+\sum_{i=1}^{n}\dfrac{(x_i-\mu)^2}{\sigma^3}=0\end{cases},即\begin{cases}\dfrac{1}{\sigma^2}(\sum_{i=1}^{n}x_i-n\mu)=\dfrac{1}{\sigma^2}(n\bar{x}-n\mu)=0\\ \dfrac{1}{\sigma^3}[-n\sigma^2+\sum_{i=1}^{n}(x_i-\mu)^2]=0\end{cases}$$

得估计值 $\hat{\mu}=\bar{x}$, $.\hat{\sigma}^2=\dfrac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$， 故$\hat{\mu}=997.1$, $.\hat{\sigma}=124.8$

$$P\{X>1300\}=P\{\frac{X-\hat{\mu}}{\hat{\sigma}}>\frac{1300-997.1}{124.8}\}=1-\Phi(2.427)=1-0.9924=0.0076.$$

5. 估计值 $\hat{\mu}=\bar{x}$, $\hat{\sigma}^2=\dfrac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$， 故

$$P\{X<t\}=\Phi(\frac{t-\hat{\mu}}{\hat{\sigma}})=\Phi(\frac{t-\bar{x}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2}}).$$

6. 设 $x_1, x_2, \cdots, x_n$ 是相应样本的观测值，似然函数为

(1) $$L(\theta)=\prod_{i=1}^{n} f(x_i,\theta)=\begin{cases}\theta^n \prod_{i=1}^{n} x_i^{\theta-1} & 0<x_i<1 \\ 0, & 其它\end{cases},$$

当 $0<x_i<1$ 时， $\ln L(\theta)=n\ln\theta+(\theta-1)\sum_{i=1}^{n}\ln x_i$

令 $\frac{1}{L(\theta)}L'(\theta)=\frac{n}{\theta}+\sum_{i=1}^{n}\ln x_i=0$，得

估计值 $\hat{\theta}=-\frac{n}{\sum_{i=1}^{n}\ln x_i}$， 估计量 $\hat{\theta}=-\frac{n}{\sum_{i=1}^{n}\ln X_i}$.

(2) 当 $x_i>0$ 时， $L(\theta)=\theta^n\alpha^n\prod_{i=1}^{n}x_i^{\alpha-1}e^{-\theta x_i^{\alpha}}$，

$$\ln L(\theta)=n\ln\theta+n\ln\alpha+\sum_{i=1}^{n}\ln x_i^{\alpha-1}-\theta\sum_{i=1}^{n}x_i^{\alpha}$$

令 $\frac{1}{L(\theta)}L'(\theta)=\frac{n}{\theta}-\sum_{i=1}^{n}x_i^{\alpha}=0$ 得

估计值 $\hat{\theta}=\frac{n}{\sum_{i=1}^{n}x_i^{\alpha}}$， 估计量 $\hat{\theta}=\frac{n}{\sum_{i=1}^{n}X_i^{\alpha}}$.

7. (1) 矩估计：

$A_1=E(X)\Rightarrow \bar{X}=\frac{0+\theta}{2}\Rightarrow$ 矩估计量：$\hat{\theta}=2\bar{X}$

矩估计值：$\hat{\theta}=2\bar{x}=\frac{2}{9}\sum_{i=1}^{9}x_i=\frac{2}{9}\times 11.8\approx 2.62$

(2) 极大似然估计：

设样本观测值 $0<x_i<\theta, (i=1,2,\cdots,9)$

$$L(\theta)=\prod_{i=1}^{9}f(x_i,\theta)=\prod_{i=1}^{9}\frac{1}{\theta}=\frac{1}{\theta^9}$$

$\because \max_{1\le i\le 9}\{x_i\}=x_4=2.2$， $\therefore$ 对任 $\theta\ge x_4$，有

$$L(\theta)=\frac{1}{\theta^9}\le\frac{1}{x_4^9},$$

$\therefore L(\theta)$ 在 $\hat{\theta}=x_4=2.2$ 达到极大值.

8. X表经过路口平均间隔，则 X~$P(\lambda)$.

平均时间间隔即 X 的均值μ

(1) μ的矩估计：

$$A_1 = E(X) \Rightarrow \bar{X} = \mu \Rightarrow \text{矩估计量：} \hat{\mu} = \bar{X}$$

矩估计值：$\hat{\mu} = \bar{x} = \dfrac{24}{6} = 4$

(2) μ的极大似然估计：

设样本观测值$x_i > 0, (i = 1,2,\cdots,9)$,

$$f(x_i;\lambda) = \begin{cases} \lambda e^{-\lambda x_i}, & x_i > 0 \\ 0, x_i \le 0 \end{cases}$$

$$L(\lambda) = \prod_{i=1}^{6} f(x_i;\theta) = \prod_{i=1}^{6} \lambda e^{-\lambda x_i} = \lambda^6 e^{-\lambda \cdot 6\bar{x}}$$

令 $L'(\lambda) = 6\lambda^5 e^{-\lambda \cdot 6\bar{x}} - 6\bar{x}\lambda^6 e^{-\lambda \cdot 9\bar{x}} = 0$,得

$$1 - \bar{x}\lambda = 0 \Rightarrow \hat{\mu} = \frac{1}{\hat{\lambda}} = \bar{x} = \frac{24}{6} = 4$$

9. $\because X \sim B(n, p), \quad \therefore EX = np, DX = np(1-p).$

由题意得

$E(\bar{X} + kS^2) = np^2 \Rightarrow E(\bar{X}) + kE(S^2) = np^2$

$\Rightarrow E(X) + kDX = np^2 \Rightarrow np + knp(1-p) = np^2$

$\Rightarrow k = -1.$

10. 据无偏估计量的定义知：$E(\hat{\theta}) = \theta$. 由

$$E(\hat{\theta}^2) - \theta^2 = E(\hat{\theta}^2) - (E(\hat{\theta}))^2 = D(\hat{\theta}) > 0$$

知 $E(\hat{\theta}^2) \ne \theta^2$，故$\hat{\theta}^2$不是$\theta^2$的无偏估计.

11. 据无偏估计量的定义，有

$$\sigma^2 = E(C\sum_{i=1}^{n-1}(X_{i+1} - X_i)^2) = C\sum_{i=1}^{n-1}\{E(X_{i+1}^2) - 2E(X_{i+1})E(X_i) + E(X_i^2)\}$$

$$= C\sum_{i=1}^{n-1}[(\sigma^2 + \mu^2) - 2\mu^2 + (\sigma^2 + \mu^2)] = C\sum_{i=1}^{n-1} 2\sigma^2 = 2(n-1)C\sigma^2$$

故 $C = \dfrac{1}{2(n-1)}$.

12. (1) 设 $Z \sim N(0,3\sigma^2)$, μ,σ^2 未知. 概率密度 $f(z_i;\sigma^2)=\dfrac{1}{\sqrt{2\pi}\sqrt{3\sigma}}e^{-\frac{z_i^2}{6\sigma^2}}$.

似然函数为 $L(\sigma^2)=\dfrac{1}{(\sqrt{6\pi}\sigma)^n}e^{-\sum\limits_{i=1}^{n}\frac{z_i^2}{6\sigma^2}}=\dfrac{1}{(\sqrt{6\pi})^n}\dfrac{1}{(\sigma^2)^{\frac{n}{2}}}e^{-\frac{1}{6\sigma^2}\sum\limits_{i=1}^{n}z_i^2}$,

$$\ln L(\sigma^2)=\ln\frac{1}{(\sqrt{6\pi})^n}-\frac{n}{2}\ln\sigma^2-\frac{1}{6\sigma^2}\sum_{i=1}^{n}z_i^2$$

令 $\dfrac{1}{L(\sigma^2)}L'(\sigma^2)=-\dfrac{n}{2}\dfrac{1}{\sigma^2}-\dfrac{-1}{6(\sigma^2)^2}\sum\limits_{i=1}^{n}z_i^2=0$, 即 $-n+\dfrac{1}{3\sigma^2}\sum\limits_{i=1}^{n}z_i^2=0$

得估计值 $\hat{\sigma}^2=\dfrac{1}{3n}\sum\limits_{i=1}^{n}z_i^2$, 估计量 $\hat{\sigma}^2=\dfrac{1}{3n}\sum\limits_{i=1}^{n}Z_i^2$.

(2) 又 $E(\hat{\sigma}^2)=\dfrac{1}{3n}\sum\limits_{i=1}^{n}E(Z_i^2)=\dfrac{1}{3n}\sum\limits_{i=1}^{n}[D(Z_i)+(E(Z_i))^2]$

$$=\frac{1}{3n}\sum_{i=1}^{n}(3\sigma^2+0^2)=\sigma^2,$$

故 $\hat{\sigma}^2$ 是 σ^2 的无偏估计量.

14. 由 S_1^2，S_2^2 分别是来自于 $X\sim N(\mu,\sigma^2)$ 的两个独立样本知，

$$E(Y)=E(aS_1^2+bS_2^2)=aE(S_1^2)+bE(S_2^2)=a\sigma^2+b\sigma^2$$
$$=(a+2b)\sigma^2=\sigma^2$$

故 Y 是 σ^2 的无偏估计量.

由 $P100$ 例7.12知，$\dfrac{n_1S_1^2}{\sigma^2}\sim\chi^2(n_1)$, $\dfrac{n_2S_2^2}{\sigma^2}\sim\chi^2(n_2)$ 知，

$$D\left(\frac{n_1S_1^2}{\sigma^2}\right)=2n_1,\quad D\left(\frac{n_2S_2^2}{\sigma^2}\right)=2n_2,$$

$\therefore\quad D(S_1^2)=\dfrac{2\sigma^4}{n_1}$，$D(S_2^2)=\dfrac{2\sigma^4}{n_2}$， 利用 S_1^2 与 S_2^2 独立，得

$$\therefore\quad DY=D(aS_1^2+bS_2^2)=a^2D(S_1^2)+b^2D(S_2^2)$$
$$=a^2\frac{2\sigma^4}{n_1}+b^2\frac{2\sigma^4}{n_2}=\frac{2\sigma^4}{n_1n_2}(n_2a^2+n_1b^2)$$
$$=\frac{2\sigma^4}{n_1n_2}[n_2a^2+n_1(1-a)^2]$$

由 $(DY)'_a=\dfrac{2\sigma^4}{n_1n_2}[n_2\cdot 2a+n_1\cdot 2(1-a)(-1)]=0$ 解得

$$a=\frac{n_1}{n_1+n_2},\quad b=\frac{n_2}{n_1+n_2}.$$

即最有效（但与书上答案不同）

分院(系)__________ 班级__________ 分级______ 编组______ §7.1

学号____________ 姓名________________ 第____次

1. 设 0,1,0,1,1 为来自二项分布 $B(1,p)$ 的样本观测值,则 p 的矩估计值为(　　)

(A) $\frac{1}{5}$;　　(B) $\frac{2}{5}$;　　(C) $\frac{3}{5}$;　　(D) $\frac{4}{5}$.

$$\mathbf{1.}A_1=\mu,\quad \frac{1}{5}\sum_{i=1}^{5}x_i=\hat{p},\quad \frac{3}{5}=\hat{p}.$$

2. 设 0,2,2,3,3 为来自均匀分布 $U(0,\theta)$ 的样本观测值,则 θ 的矩估计值为(　　)

(A) 1;　　(B) 2;　　(C) 3;　　(D) 4.

$$\mathbf{2.}\ A_1=\mu\Rightarrow\frac{1}{5}\sum_{i=1}^{5}x_i=\frac{\theta+0}{2}\Rightarrow\hat{\theta}=\frac{2}{5}\cdot 10=4.$$

3. 设 X_1, X_2 为任意总体 X 的容量为 2 的样本，则在下列 $E(X)$ 的无偏估计量中，最有效的估计量是（　　）.

A. $\frac{2}{3}X_1+\frac{1}{3}X_2$　B. $\frac{1}{4}X_1+\frac{3}{4}X_2$　C. $\frac{2}{5}X_1+\frac{3}{5}X_2$　D. $\frac{1}{2}X_1+\frac{1}{2}X_2$

3.D. $\because a^2+b^2\ge 2ab,\quad \therefore \forall a,b:\ a+b=1,$ **have**

$$D(aX_1+bX_2)=a^2D(X_1)+b^2D(X_2)=(a^2+b^2)D(X)$$
$$\ge 2\cdot\frac{1}{2}\cdot\frac{1}{2}D(X)=\frac{D(X)}{2}=D(\bar{X}).$$

补充题. 求 C 使 $\bar{X}^2-CS^2$ 是 μ^2 的无偏估计.

解 $\because\ \mu^2=E(\bar{X}^2-CS^2)=E(\bar{X}^2)-CE(S^2)=D(\bar{X})+\mu^2-C\sigma^2$

$$=\frac{\sigma^2}{n}+\mu^2-C\sigma^2=(\frac{1}{n}-C)\sigma^2+\mu^2$$

$\therefore\ C=\frac{1}{n}.$

8.$(P65)$ $X_i=\begin{cases}1,\text{第}i\text{次取到的是带记号的鱼}\\0,\text{第}i\text{次取到的是不带记号的}\end{cases}$, $i=1,2,\cdots,n,\quad p=\dfrac{r}{N}.$

法1　矩估计法．由P99例1.5解结果知

$$\hat{p}=\bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i=\frac{m}{n},\text{从而由}p=\hat{p}=\frac{m}{n}\text{得}\ \frac{r}{\hat{N}}=\frac{m}{n},\text{即}\ \hat{N}=\frac{r\,n}{m}.$$

法2　用极大似然估计法．视N为待估参数,由

$$f(x_i;N)=p^{x_i}(1-p)^{1-x_i}=(\frac{r}{N})^{x_i}(1-\frac{r}{N})^{1-x_i},\quad i=1,2,\cdots,n.$$

$$\boldsymbol{L(N)=\prod_{i=1}^{n}f(x_i;N)=(\frac{r}{N})^{\sum_{i=1}^{n}x_i}(1-\frac{r}{N})^{n-\sum_{i=1}^{n}x_i},\quad \text{其中},\ \sum_{i=1}^{n}x_i=m.}$$

$$\boldsymbol{\ln L(N)=m(\ln r-\ln N)+(n-m)[\ln(N-r)-\ln N]}$$

$$\text{令}\quad \boldsymbol{\frac{1}{L(N)}L'(N)=-\frac{1}{N}m+(n-m)[\frac{1}{N-r}-\frac{1}{N}]=0,}\quad\text{得}$$

$$\boldsymbol{-\frac{1}{N}m+(n-m)\frac{r}{N(N-r)}=0,\quad -\sum_{i=1}^{n}x_i+(n-m)\frac{r}{N-r}=0}$$

$$\boldsymbol{(n-m)\frac{r}{N-r}=m\quad\Rightarrow\quad r(n-m)=(N-r)m}$$

$$\boldsymbol{\therefore\quad \hat{N}=r+\frac{r(n-m)}{m}=r(1-\frac{n-m}{m})=\frac{r\,n}{m}.\quad(\frac{m}{n}\approx\frac{r}{N})}$$

9. 法1　$f(x_i;\theta)=C_2^{i-1}\theta^{3-x_i}(1-\theta)^{x_i-1},\quad i=1,2,3.$

$$L(\theta)=\prod_{i=1}^{3}f(x_i;\theta)=\prod_{i=1}^{3}C_2^{i-1}\theta^{3-x_i}(1-\theta)^{x_i-1}=\prod_{i=1}^{3}C_2^{i-1}\cdot\theta^{\sum_{i=1}^{3}(3-x_i)}(1-\theta)^{\sum_{i=1}^{3}(x_i-1)}$$

$$\ln L(\theta)=\ln\prod_{i=1}^{3}C_2^{i-1}+\sum_{i=1}^{3}(3-x_i)\cdot\ln\theta+\sum_{i=1}^{3}(x_i-1)\cdot\ln(1-\theta)$$

$$\text{令}\quad\frac{1}{L(\theta)}L'(\theta)=0,\text{即}\sum_{i=1}^{3}(3-x_i)\cdot\frac{1}{\theta}+\sum_{i=1}^{3}(x_i-1)\cdot\frac{-1}{1-\theta}=0,\text{或}\frac{3}{\theta}+\frac{-3}{1-\theta}=0,$$

得估计值 $\hat{\theta}=1/2.$

法 2　矩估计法

$$\boldsymbol{A_1=1\times\theta^2+2\times2\theta(1-\theta)+3\times(1-\theta)^2\Rightarrow A_1=-2\theta+3}$$

$$\boldsymbol{\Rightarrow\hat{\theta}=\frac{3-A_1}{2}=\frac{3-2}{2}=\frac{1}{2}.}$$

习题册 P108

计算器统计功能的使用方法：

(1)按"2ndF",按"stata"(此时2ndF削消失)

(2)输入第一个数字后按M+,接着依次如此

(3)按$\bar{x}$ Σx^2键显示$\bar{x}$值,按"2ndF"后按$\bar{x}$ Σx则显示Σx^2的值

(4)其它"S σ"也如此使用

15. $n=16,\ \sigma=0.1,\quad \bar{x}=2.125,\quad S=0.017127.$

(1) $P\{\left|\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}\right|<Z_{\frac{\alpha}{2}}\}=1-\alpha,\quad P\{\left|\dfrac{\bar{X}-\mu}{0.01/\sqrt{16}}\right|<Z_{0.05}\}=0.90.\quad Z_{0.05}=1.645.$

置信区间： $(\bar{X}\pm\dfrac{\sigma}{\sqrt{n}}Z_{0.05})$, 即$(2.125\pm1.645\times0.01/\sqrt{16})$, 即$(2.125\pm0.004)$或$(2.121, 2.129)$.

(2) $P\{\left|\dfrac{\bar{X}-\mu}{S/\sqrt{n}}\right|<t_{\frac{\alpha}{2}}(n-1)\}=1-\alpha,\ P\{\left|\dfrac{\bar{X}-\mu}{0.017127/\sqrt{16}}\right|<t_{0.05}(15)\}=0.90.\ t_{0.05}(15)=1.7531.$

置信区间： $(\bar{X}\pm\dfrac{S}{\sqrt{n}}t_{0.05}(15))$, 即$(2.125\pm0.0075)$, 或$(2.1175, 2.1325)$.

16. 由$V\sim N(\mu,\sigma^2)$, μ,σ^2未知. $\dfrac{(n-1)S^2}{\sigma^2}\sim\chi^2(n-1).$

$n=9,\quad \alpha=0.05,\quad S=11,\quad \chi^2_{0.025}(8)=17.535\quad \chi^2_{0.975}(8)=2.180.$

$$P\{\chi^2_{1-\frac{\alpha}{2}}(n-1)<\frac{(n-1)S^2}{\sigma^2}<\chi^2_{\frac{\alpha}{2}}(n-1)\}=1-\alpha,$$

置信区间：$\left(\dfrac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}(n-1)},\dfrac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}\right)$, 即$\left(\dfrac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}(n-1)},\dfrac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}\right)$

$(55.204,\quad 444.037)$.

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3. 设在一群年龄为 4 个月的老鼠中任意抽取雄性、雌性老鼠各 12 只，测得重量(以克计)如下：

雄性	26.0	20.0	18.0	28.5	23.6	20.0	22.5	24.0	24.0	25.0	23.8	24.0
雌性	16.5	17.0	16.0	21.0	23.0	19.5	18.0	18.5	20.0	28.0	19.5	20.5

设雄性、雌性老鼠的重量分别服从 $N(\mu_1,\sigma^2)$, $N(\mu_2,\sigma^2)$ 分布，且两样本相互独立，μ_1, μ_2, σ^2 均为未知，试求 $\mu_1-\mu_2$ 的置信度为 90% 的置信区间.

3. T9. $n_1=n_2=12,\quad \bar{x}=23.2833,\quad S_1^2=2.853^2=8.1396;\ \bar{y}=19.79,\quad S_2^2=3.27^2=10.6929.$

$$Z=\frac{\overline{X}-\overline{Y}-(\mu_1-\mu_2)}{S_W\sqrt{1/n_1+1/n_2}}\sim t(n_1+n_2-2).\quad \sqrt{1/n_1+1/n_2}=\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}=\sqrt{\frac{2}{12}}=\frac{1}{\sqrt{6}}$$

$$S_W=\sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}}=\sqrt{\frac{S_1^2+S_2^2}{2}}=\sqrt{9.416255}=3.07$$

$$P\{|Z|<t_{\frac{\alpha}{2}}(n_1+n_2-2)\}=0.90,\quad P\{|Z|<t_{0.05}(22)\}=0.90,\quad t_{0.05}(22)=1.7171.$$

置信区间为：$((\bar{x}-\bar{y})\pm(1.7171\times3.07\sqrt{1/6}))$,即$(3.4867\pm2.151652)=(1.335,5.6383)$.

4. $T10$. $X\sim N(\mu_1,\sigma^2),\quad Y\sim N(\mu_2,\sigma^2),\quad n_1=4,\quad n_2=6,$

$\bar{x}=91.7,\quad S_1^2=4.23871^2=17.96666246;\quad \bar{y}=94.4667,\quad S_2^2=2.32^2=5.3824.$

$$Z=\frac{\overline{X}-\overline{Y}-(\mu_1-\mu_2)}{S_W\sqrt{1/n_1+1/n_2}}\sim t(n_1+n_2-2).\quad S_W=\sqrt{\frac{3S_1^2+5S_2^2}{8}}=\sqrt{10.101665}=3.1783$$

$$P\{|Z|<t_{\frac{\alpha}{2}}(n_1+n_2-2)\}=0.95,\quad P\{|Z|<t_{0.025}(8)\}=0.95,\quad t_{0.05}(8)=2.3060.$$

置信区间为：$((\bar{x}-\bar{y})\pm3.1783\sqrt{\frac{1}{2.4}}\times2.306))$,即$(-2.76667\pm4.73095)$或$(-7.4976,1.9643)$

5. $T11$. 由lx6-6知 $\dfrac{S_A^2/\sigma_A^2}{S_B^2/\sigma_B^2}\sim F(n_1-1,n_2-1)=F(9,9),$

$$P\{F_{1-\frac{\alpha}{2}}(9,9)<\frac{S_A^2/\sigma_A^2}{S_B^2/\sigma_B^2}<F_{\frac{\alpha}{2}}(9,9)\}=0.95,\ P\{\frac{1}{F_{\frac{\alpha}{2}}(9,9)}\frac{S_A^2}{S_B^2}<\frac{\sigma_A^2}{\sigma_B^2}<\frac{S_A^2}{S_B^2}\frac{1}{F_{1-\frac{\alpha}{2}}(9,9)}\}=0.95$$

置信区间为：$(\frac{1}{4.03}\frac{0.5419}{0.6065},\quad \frac{0.5419}{0.6065}\times4.03)$，即$(0.222,\quad 0.3601)$.--书上错

9. 法2　矩估计法

$$A_1 = 1\times\theta^2 + 2\times 2\theta(1-\theta) + 3\times(1-\theta)^2 \Rightarrow A_1 = -2\theta + 3$$

$$\Rightarrow \hat{\theta} = \frac{3-A_1}{2} = \frac{3-2}{2} = \frac{1}{2}.$$

法3　用极大似然估计法.

$$f(x_i;\theta) = \frac{(-1)^{x_i-1}}{2}(\theta^2)^{(x_i-1)}[(1-\theta)^2]^{(3-x_i)},\quad i = 1,2,3.$$

$$L(\theta) = \prod_{i=1}^{n} f(x_i;\theta) = \frac{(-1)^{\sum_{i=1}^{n}(x_i-1)}}{2^n}\prod_{i=1}^{n}(\theta^2)^{(x_i-1)}[(1-\theta)^2]^{(3-x_i)}$$

$$\ln L(\theta) = \ln\frac{(-1)^{n(\bar{x}-1)}}{2^n} + \sum_{i=1}^{n}[\ln(\theta^2)^{(x_i-1)} + \ln([(1-\theta)^2]^{(3-x_i)}]$$

令 $\frac{1}{L(\theta)}L'(\theta) = 0$,即 $0 + \sum_{i=1}^{n}[\frac{(\theta^2)^{(x_i)}}{(\theta^2)^{(x_i-1)}} + \frac{[(1-\theta)^2]^{(4-x_i)}}{[(1-\theta)^2]^{(3-x_i)}}] = 0.$

如取样本值为3,2,1,　则由矩估计法及极大似然估计法均得 $\hat{\theta} = 1/2.$

又如取样本值为 1,2,1，则有

$$\frac{(\theta^2)'}{\theta^2} + \frac{[(1-\theta)^2]'''}{[(1-\theta)^2]''} + \frac{(\theta^2)''}{(\theta^2)'} + \frac{[(1-\theta)^2]''}{[(1-\theta)^2]'} + \frac{(\theta^2)'''}{(\theta^2)''} + \frac{[(1-\theta)^2]'}{(1-\theta)^2} = 0$$

$$\frac{2\theta}{\theta^2} + 0 + \frac{1}{\theta} - \frac{1}{1-\theta} + 0 - \frac{2}{1-\theta} = 0$$

$$\frac{3}{\theta} - \frac{3}{1-\theta} = 0$$

$$\hat{\theta} = 1/2$$