Package "BTNorm"

Hui Lin

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Contents

1 chebite

Description

Compute all the Chebysev-Hermite polnomials up to the degree 'k' for the given array 'xv' of length 'n'.

Usage

$chebite(xv, k)$

Arguments

xv a numeric vector with length n

k the degree of Chebysev-Hermite polnomials

Details

Hermite polynomials are a classical orthogonal polynomial sequence that arise in probability. They are named after Charles Hermite (1864) although they were studied earlier by Laplace (1810) and Chebyshev (1859).There are two different ways of defining Hermite polynomials:"probabilists' Hermite polynomials" and "physicists' Hermite polynomials". Here we used the probabilists' definition. Regier and Hamdan (1971) used Hermite polynomials to derive first and second order moments of Z_1 and Z_2 with single truncation on each normal distributed variable Z_i.

Value

The interative formula $H_{n+1}(x) = x * H_n(x) - n * H_{n-1}(x)$ is used. The output array has dimension $n \times k$.

Reference

1. Laplace, P.S. (1810), Mem. CL. Sci. Math. Phys. Inst. France 58:279-347

2. Regier MH, Hamdan MA (1971) Correlation in a bivariate normal distribution with truncation in both variables. Aust J Stat 13(2):77-82

Example

```
xv<-c(1:5)

k<-5

chebite(xv,k)

     [,1] [,2] [,3] [,4] [,5]

[1,]    1   -1   -4    0   20

[2,]    2    2   -2  -12  -14

[3,]    3    7   12    8  -36

[4,]    4   14   44  120  260

[5,]    5   23  100  408 1540
```

2 cutpoints

Description

Compute the breakpoints of both non-normal and normal bivariates

Usage

$cutpoints(x1, x2, m, p)$

Arguments

x1 : numeric vector of the observations for the first variable

x2 : numeric vector of the observations for the second variable

m : number of breakpoints for piece-wise linear approximation of the first variable

p : number of breakpoints for piece-wise linear approximation of the second variable

Value

A list containing the following components is returned:

- cutpoints1: a data frame of two columns including the breakpoints for x_1 and z_1.
- cutpoints2: a data frame of two columns including the breakpoints for x_2 and z_2.
- m: number of breakpoints for piece-wise linear approximation of the first variable
- p: number of breakpoints for piece-wise linear approximation of the second variable

Example

```
x1<-rgamma(100,1,1)

x2<-rgamma(100,1,2)+x1

cutpoints(x1,x2,30,30)->cut

str(cut)

List of 4

  cutpoints1:'data.frame':  24 obs. of  2 variables:

  .. xcut1: num [1:24] 0.0129 0.1455 0.2782 0.4109 0.5436 ...

  .. zcut1: num [1:24] -2.472 -1.429 -0.969 -0.626 -0.425 ...

  cutpoints2:'data.frame':  26 obs. of  2 variables:

  .. xcut2: num [1:26] 0.11 0.253 0.396 0.538 0.681 ...

  .. zcut2: num [1:26] -2.472 -1.429 -1.302 -1.01 -0.786 ...

  m          : num 30

  p          : num 30
```

3 ecdf2

Description

Compute an empirical cumulative distribution function based on corrected rank-based estimate: $\hat{F}_x(x_i) = \frac{r_i - 0.326}{n + 0.384}$ where r_i is the rank

Usage

$ecdf2(x)$

Arguments

x numeric vector of the observations for ecdf2

Value

The ecdf2 returns a function that calculates empirical cumulative distribution for a given numeric vector **x** based on corrected rank-based estimate: $\hat{F}_x(x_i) = \frac{r_i - 0.326}{n + 0.384}$ where r_i is the rank.

Reference

1. Yu GH, Huang CC(2001) A distribution free plotting position. Stoch Environ Res Risk Assess 15:462-476

Example

```
x<-rnorm(200)
ecdf2(x)->funx
funx(0)
[1] 0.4425999
```

4 flipud

Description

The function flips the rows of a matrix in the up-down direction.

Usage

$flipud(A)$

Arguments

A a matrix with number of rows and columns larger than 1

Value

flipud(A) returns A with rows flipped in the up-down direction.

Example

```
x<-matrix(c(1:9),3,3)
x
     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
[3,]    3    6    9

flipud(x)
     [,1] [,2] [,3]
[1,]    3    6    9
[2,]    2    5    8
[3,]    1    4    7
```

5 invecdf2

Description

Compute the inverse function of empirical cumulative distribution function based on corrected rank-based estimate: $\hat{F}_x(x_i) = \frac{r_i - 0.326}{n + 0.384}$ where r_i is the rank.

Usage

$invecdf2(prob, x)$

Arguments

prob a numeric value of probability, should be in interval [0,1]

x numeric vector of the observations for invcdf2

Value

The invecdf2 returns a value of the inverse function of the empirical cumulative distribution for a given numeric vector **x** and a probability p. The empirical cumulative distribution function is based on corrected rank-based estimate: $\hat{F}_x(x_i) = \frac{r_i - 0.326}{n + 0.384}$ where r_i is the rank. It needs function "ecdf2".

Example

```
x<-rnorm(100)
invecdf2(0.5,x)
[1] 0.0468675
```

6 iterrhox

Description

This function implements the iterative process to find the correct normal correlation coefficient ρ_z given the observations of the non-normal biavariate samples.

Usage

$iterrhox(x1, x2, m, p, rho0, toler)$

Arguments

rho correlation coefficient for z_1 and z_2. The default value is -0.5

toler given tolerance of the iteration. The default value is 0.00001

x1 numeric vector of the observations for the first variable

x2 numeric vector of the observations for the second variable

Details

The ψ function that defines ρ_x from ρ_z is implemented iteratively, starting at a given initial ρ_0. To avoid stack of the algorithm at the bounds -1 and 1 , we reflect and adjust the increment at the bounds to valid correlation coefficient values.

Value

A list containing the following components is returned:

- hist.info: a data frame of four columns recording the iteration history.
  - iterate: iteration number
  - rhox: the correlation of x calculated by $\rho_x = \phi(\rho_z)$ in current value of ρ_z
  - dif: the difference between ρ_x and ρ_z in current iteration
  - rho: the correlation of ρ_z in current iteration
- rho: the estimated value of ρ_z
- toler: given tolerance of the iteration

Example

```
x1<-rgamma(100,1,1)

x2<-rgamma(100,x1[5],1)

iterrhox(x1,x2,20,20)->est

 str(est)

List of 3

 hist.info:'data.frame':  9 obs. of  4 variables:

  .. iterate: num [1:9] 0 1 2 3 4 5 6 7 8

.. rhox   : num [1:9] 0 -0.3668 0.0579 0.1743 0.2021 ...
```

```
.. dif    : num [1:9] 0 0.57671 0.15202 0.03566 0.00787 ...
.. rho    : num [1:9] -0.5 0.0767 0.2287 0.2644 0.2723 ...
 rho      : num 0.274
 toler    : num 1e-05
```

7 linprox

Description

Give the estimated parameters of linear piece-wise approximation.

Usage

$linprox(x1, x2, m, p, rho)$

Arguments

x1 numeric vector of the observations for the first variable

x2 numeric vector of the observations for the second variable

m number of breakpoints for x_1

p number of breakpoints for x_2

rho correlation coefficient of two normal variables Z_1 and Z_2

Value

A list containing the following components is returned:

- c0: parameter vectors of constant term of the linear interpolation at the breakpoints for transforming x_1 to z_1
- c1: parameter vectors of slope term of the linear interpolation at the breakpoints for transforming x_1 to z_1

- d0: parameter vectors of constant term of the linear interpolation at the breakpoints for transforming x_2 to z_2
- d1: parameter vectors of slope term of the linear interpolation at the breakpoints for transforming x_1 to z_1

Example

```
linprox(rnorm(100),rnorm(100),20,20,0.5)->res
str(res)
List of 4
  c0: num [1:19] -0.706 0.341 1.181 -0.259 -0.332 ...
  c1: num [1:19] 0.643 1.135 1.57 0.766 0.717 ...
  d0: num [1:19] -0.525 0.639 -0.855 0.368 -0.336 ...
  d1: num [1:19] 0.715 1.262 0.489 1.345 0.78 ...
```

8 mu

Description

Calculate first moment of univariate truncated standard normal.

Usage

$mu(a1, a2)$

Arguments

a1 left point of the truncated interval

a2 right point of the truncated interval

Value

It will report a number.

9 mu2

Description

Calculate second moment of univariate truncated standard normal.

Usage

$mu2(a1, a2)$

Arguments

a1 left point of the truncated interval

a2 right point of the truncated interval

Value

It will report a number.

10 mu12

Description

Calculate first order joint moment of X_1 and X_2

Usage

$mu12(rho, zcut1, zcut2, c0, c1, d0, d1)$

Arguments

c0 : parameter vectors of constant term of the linear interpolation at the breakpoints for transforming x_1 to z_1

c1 : parameter vectors of slope term of the linear interpolation at the breakpoints for transforming x_1 to z_1

d0 : parameter vectors of constant term of the linear interpolation at the breakpoints for transforming x_2 to z_2

d1 : parameter vectors of slope term of the linear interpolation at the breakpoints for transforming x_1 to z_1

zcut1 : a vector of breakpoints for z_1

zcut2 : a vector of breakpoints for z_2

rho : correlation coefficient of two normal variables Z_1 and Z_2

Value

It will report a number.

Example

```
# need functions "linprox" and "cutpoints"
x1<-rgamma(100,1,1)
x2<-rgamma(100,1,2)+x1
cutpoints(x1,x2,30,30)->cut
cut$cutpoints1$zcut1->zcut1
cut$cutpoints2$zcut2->zcut2
rho<-0.5
linprox(x1,x2,30,30,rho)->res
res$c0->c0
res$c1->c1
res$d0->d0
res$d1->d1
mu12(rho,zcut1,zcut2,c0,c1,d0,d1)
[1] 1.428648
```

11 pbnorm

Description

The function computes the double truncated standard normal joint probability for binormal (z_1, z_2) on a region $D = (a1, a2) \times (b1, b2)$

Usage

$pbnorm(a1, a2, b1, b2, rho)$

Arguments

a1 left point of the truncated interval for z_1

a2 right point of the truncated interval for z_1

b1 left point of the truncated interval for z_2

b2 right point of the truncated interval for z_2

rho correlation coefficient for z_1 and z_2

Value

It will report a numeric value.

12 pbnormab

Description

The function computes the double truncated standard normal joint probability for binormal (z_1, z_2) on a region $D = (a, \infty) \times (b, \infty)$.

Usage

$pbnormab(a, b, rho)$

Arguments

a left point of the truncated interval for z_1

b left point of the truncated interval for z_2

rho correlation coefficient for z_1 and z_2

Value

It will report a numeric value. Note that it is based on the function "pbnorm"

13 Ptrnormal

Description

Ptrnormal computes the double truncated standard normal joint distribution of two variables $z_1 \& z_2$ ata truncation points given by the vectors **av** for z_1 and **bv** for z_2.

Usage

$Ptrnormal(rho, av, bv)$

Arguments

rho correlation coefficient for z_1 and z_2

av vector of truncation points for $z_1 : -\infty < av[1] < ... < av[n] < \infty$

bv vector of truncation points for $z_2 : -\infty < bv[1] < ... < bv[n] < \infty$

Value

$(n + 1) \times (m + 1)$ matrix of joint pdf. The components of the matrix are ordered as follows:

	$(bv[0], bv[1])$	$(bv[1], bv[2])$	......	$(bv[m], bv[m+1])$
$(av[0], av[1])$	p_{11}	p_{12}	......	$p_{1,m+1}$
$(av[1], av[2])$	p_{21}	p_{22}	......	$p_{2,m+1}$
......	......	......	......	......
$(av[n], av[n+1])$	$p_{n+1,1}$	$p_{n+1,2}$	......	$p_{n+1,m+1}$

where $av[0] = bv[0] = -\infty$ and $av[n+1] = bv[m+1] = \infty$

Example

```
av<-seq(1:5)

bv<-seq(1:5)

rho<-0.5

Ptrnormal(rho,av,bv)

[,1]          [,2]          [,3]          [,4]          [,5]          [,6]

[1,] 9.626532e-09 6.142814e-08 1.238405e-07 7.652139e-08 1.441027e-08 8.247084e-10

[2,] 3.191339e-06 1.095124e-05 1.243541e-05 4.334775e-06 4.574095e-07 1.441027e-08

[3,] 3.101182e-04 5.653872e-04 3.657300e-04 7.258001e-05 4.334775e-06 7.652139e-08

[4,] 9.170596e-03 8.636871e-03 3.214478e-03 3.657300e-04 1.243541e-05 1.238405e-07

[5,] 8.665724e-02 4.003461e-02 8.636871e-03 5.653872e-04 1.095124e-05 6.142814e-08

[6,] 7.452036e-01 8.665724e-02 9.170596e-03 3.101182e-04 3.191339e-06 9.626532e-09
```

14 PEz1tronormal

Description

It computes the mean with respect to z_1 of the truncated standard normal joint distribution of two variables $z_1 \& z_2$ at the truncation points.

Usage

$PEz1tronormal(rho, av, bv)$

Arguments

rho correlation coefficient for z_1 and z_2

av vector of truncation points for $z_1 : -\infty < av[1] < ... < av[n] < \infty$

bv vector of truncation points for $z_2 : -\infty < bv[1] < ... < bv[n] < \infty$

Value

$(n+1) \times (m+1)$ matrix of marginal first moment of z_1 according to the case of joint truncation.

Example

```
av<-seq(1:5)

bv<-seq(1:5)

rho<-0.5

PEz1tronormal(rho,av,bv)

[,1]          [,2]          [,3]          [,4]          [,5]          [,6]

[1,]   6.332613e-09 9.899219e-08 3.108052e-07 2.606828e-07 6.220675e-08 4.340199e-09

[2,]   1.871373e-06 1.711911e-05 3.057859e-05 1.456929e-05 1.958028e-06 7.535490e-08

[3,]   1.517582e-04 8.557272e-04 8.816918e-04 2.410100e-04 1.842373e-05 3.981953e-07

[4,]   3.270318e-03 1.264764e-02 7.606853e-03 1.201578e-03 5.253224e-05 6.418912e-07

[5,]   1.526172e-02 5.674673e-02 2.009475e-02 1.840336e-03 4.602113e-05 3.173500e-07

[6,]  -2.606564e-01 1.177124e-01 2.094493e-02 1.000264e-03 1.334617e-05 4.958792e-08
```

15 PEz2tronormal

Description

It computes the mean with respect to z_2 of the truncated standard normal joint distribution of two variables $z_1 \& z_2$ at the truncation points.

Usage

$PEz2tronormal(rho, av, bv)$

Arguments

rho correlation coefficient for z_1 and z_2

av vector of truncation points for $z_1 : -\infty < av[1] < ... < av[n] < \infty$

bv vector of truncation points for $z_2 : -\infty < bv[1] < ... < bv[n] < \infty$

Value

$(n + 1) \times (m + 1)$ matrix of marginal first moment of z_2 according to the case of joint truncation.

Example

```
av<-seq(1:5)

bv<-seq(1:5)

rho<-0.5

PEz2tronormal(rho,av,bv)

[,1]          [,2]          [,3]          [,4]          [,5]          [,6]

[1,]   4.958792e-08  1.027897e-06  4.122738e-07  1.029523e-07  1.924634e-08 4.340199e-09

[2,]   1.334617e-05  4.602113e-05  5.253224e-05  1.842373e-05  1.958028e-06 6.220675e-08

[3,]   1.000264e-03  1.840336e-03  1.201578e-03  2.410100e-04  1.456929e-05 2.606828e-07

[4,]   2.094493e-02  2.009475e-02  7.606853e-03  8.816918e-04  3.057859e-05 3.108052e-07

[5,]   1.177124e-01  5.674673e-02  1.264764e-02  8.557272e-04  1.711911e-05 9.899219e-08

[6,] -2.606564e-01 -1.257231e-01 -3.389902e-02 -3.071755e-03 -9.738626e-05 6.332613e-09
```

16 PEz1z2trnormal

Description

Compute the joint first moment of the truncated standard normal joint distribution of two variables $z_1 \& z_2$ at the truncation points.

Usage

$PEz1z2trnormal(rho, av, bv)$

Arguments

rho correlation coefficient for z_1 and z_2

av vector of truncation points for $z_1 : -\infty < av[1] < ... < av[n] < \infty$

bv vector of truncation points for $z_2 : -\infty < bv[1] < ... < bv[n] < \infty$

Value

$(n+1) \times (m+1)$ matrix of the joint first moment of the truncated standard normal joint distribution of two variables $z_1 \& z_2$ at the truncation points.

Example

```
av<-seq(1:5)

bv<-seq(1:5)

rho<-0.5

PEz1z2trnomoral(rho,av,bv)

[,1]          [,2]          [,3]          [,4]          [,5]          [,6]

[1,] 1.105341e-07 5.873451e-07 1.644040e-06 1.232241e-06 2.582269e-07 2.266325e-08

[2,] 9.439130e-06 6.967445e-05 1.268401e-04 6.120275e-05 8.300211e-06 3.221917e-07

[3,] 5.680910e-04 2.733875e-03 2.868134e-03 7.926274e-04 6.120275e-05 1.337612e-06

[4,] 7.764350e-03 2.949761e-02 1.798312e-02 2.868134e-03 1.268401e-04 1.571663e-06

[5,] 7.778121e-03 8.250861e-02 2.949761e-02 2.733875e-03 6.967445e-05 4.954605e-07

[6,] 2.713888e-01 8.358639e-03 6.115831e-03 5.398263e-04 1.298974e-05 1.105341e-07
```