ECE521: Assignment 4

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Graphical Models

Graphical models from factorization

Task 1

If we draw a Bayesian network representation of Eq. 1 we get Figure 1.

$$P(a, b, c, d, e, f) = P(a|b)P(b)P(c|a, b)P(d|b)P(e|c)P(f|b, e) \tag{1}$$

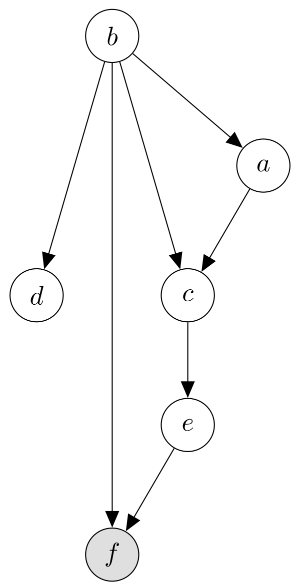

Figure 1: Bayesian Network Representation of Eq. 1.

Task 2

Let us rewrite the joint distribution from Eq. 1 as Eq. 2.

$$P(a,b,c,d,e,f) = f_1(a,b)f_2(b)f_3(c,a,b)f_4(d,b)f_5(e,c)f_6(f,b,e) \tag{2}$$

If we now draw the corresponding factor graph we obtain Figure 2.

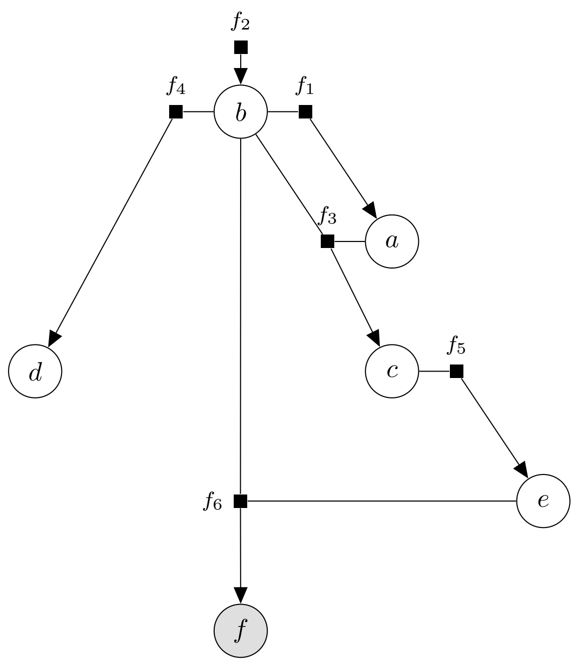

Figure 2: Factor Graph Representation of Eq. 2.

Task 3

Conversion between graphical models

Task 1

Task 2

Task 3

Conditional Independence in Bayesian Networks

Task 1

We can express the joint probability of the Bayesian Network provided using Eq. 3.

$$P(a, b, c, d, e, f) = P(a)P(b|a)P(f|b)P(e)P(c|b, e)P(d|b, c) \tag{3}$$

Task 2

- $\boldsymbol{a} \perp\!\!\!\perp \boldsymbol{c}$: FALSE

  Consider $P(c) = \begin{cases} 1 & P(b) + P(e) > 1 \\ 0 & otherwise \end{cases}$, now let us assume $P(b) = P(a) = \mathcal{N}(0, 1)$. Therefore we have $P(c) = \begin{cases} 1 & P(a) + P(e) > 1 \\ 0 & otherwise \end{cases}$, which clearly shows that **a** is not independent of **c**.

- $\boldsymbol{a} \perp\!\!\!\perp \boldsymbol{c}|\boldsymbol{b}$: TRUE

  If we marginalize the distribution in Task 1 over **a** and **c** we obtain $P(a, c|b) = P(a)P(b|a)P(e)P(c|b, e)$