1. STACK2 statically implements STACK1, since the maximal signature of STACK1 is included in the maximal signature of STACK2.

2. Implementation conditions:

$$
\begin{array}{l}
\textbf{axiom} \\
\quad \forall\ \mathrm{st} : \mathrm{Stack} \bullet \mathrm{is\_empty(st)}\ \equiv \mathrm{st} = \mathrm{empty}, \\
\quad \forall\ \mathrm{e} : \mathrm{Elem},\ \mathrm{st} : \mathrm{Stack} \bullet \mathrm{top(push(e,st))} \equiv \mathrm{e}, \\
\quad \forall\ \mathrm{e} : \mathrm{Elem},\ \mathrm{st} : \mathrm{Stack} \bullet \mathrm{pop(push(e,st))} \equiv \mathrm{st}, \\
\quad \forall\ \mathrm{e} : \mathrm{Elem},\ \mathrm{st} : \mathrm{Stack} \bullet \mathrm{empty} \neq \mathrm{push(e,st)}, \\
\quad \forall\ \mathrm{p} : \mathrm{Stack} \rightarrow \textbf{Bool} \bullet \\
\qquad \mathrm{p(empty)} \wedge \\
\qquad (\forall\ \mathrm{st} : \mathrm{Stack},\ \mathrm{e} : \mathrm{Elem} \bullet \mathrm{p(st)} \Rightarrow \mathrm{p(push(e,st))}) \\
\qquad \Rightarrow \\
\qquad (\forall\ \mathrm{st} : \mathrm{Stack} \bullet \mathrm{p(st)})
\end{array}
$$

3. Justifications:

$\llcorner \forall\ \mathrm{st} : \mathrm{Stack} \bullet \mathrm{is\_empty(st)}\ \equiv \mathrm{st} = \mathrm{empty} \lrcorner$
all_assumption_inf:
$\llcorner \mathrm{is\_empty(st)} \equiv \mathrm{st} = \mathrm{empty} \lrcorner$
application_expr_unfold1, value_name_unfold:
$\llcorner \mathrm{st} = \langle\rangle \equiv \mathrm{st} = \langle\rangle \lrcorner$
is_annihilation:
$\llcorner \textbf{true} \lrcorner$
qed

$\llcorner \forall\ \mathrm{e} : \mathrm{Elem},\ \mathrm{st} : \mathrm{Stack} \bullet \mathrm{top(push(e,st))} \equiv \mathrm{e} \lrcorner$
all_assumption_inf:
$\llcorner \mathrm{top(push(e,st))} \equiv \mathrm{e} \lrcorner$
application_expr_unfold1, application_expr_unfold2:
$\llcorner \textbf{hd}(\langle \mathrm{e} \rangle \widehat{\ } \mathrm{st}) \equiv \mathrm{e} \lrcorner$
since
$\llcorner \langle \mathrm{e} \rangle \widehat{\ } \mathrm{st} \neq \langle\rangle \lrcorner$
empty_list_inequality3:
$\llcorner \textbf{true} \lrcorner$
qed
end
hd_concatenation2:
$\llcorner \mathrm{e} \equiv \mathrm{e} \lrcorner$
is_annihilation:
$\llcorner \textbf{true} \lrcorner$
qed

$\llcorner \forall\ \mathrm{e} : \mathrm{Elem},\ \mathrm{st} : \mathrm{Stack} \bullet \mathrm{pop(push(e,st))} \equiv \mathrm{st} \lrcorner$
all_assumption_inf:
$\llcorner \mathrm{pop(push(e,st))} \equiv \mathrm{st} \lrcorner$
application_expr_unfold1, application_expr_unfold2:
$\llcorner \textbf{tl}(\langle \mathrm{e} \rangle \widehat{\ } \mathrm{st}) \equiv \mathrm{st} \lrcorner$
since

$\llcorner \langle e \rangle \frown st \neq \langle\rangle \lrcorner$
empty_list_inequality3:
$\llcorner$**true**$\lrcorner$
qed
end
tl_concatenation2:
$\llcorner st \equiv st \lrcorner$
is_annihilation:
$\llcorner$**true**$\lrcorner$
qed

$\llcorner \forall\ e : Elem,\ st : Stack \bullet empty \neq push(e,st) \lrcorner$
all_assumption_inf:
$\llcorner empty \neq push(e,st) \lrcorner$
application_expr_unfold1, value_name_unfold:
$\llcorner \langle\rangle \neq \langle e \rangle \frown st \lrcorner$
empty_list_inequality2:
$\llcorner$**true**$\lrcorner$
qed