Problem 2.

```
scheme
  DIGITAL_PICT =
    class
      type
        Colour,
        Colour2 == black | white,
        Colour3 = Intensity,
        Colour4 :: red : Intensity  green : Intensity  blue : Intensity,
        Intensity = {| n : Nat • n ≤ 255 |}

      value
        n : Nat • n ≥ 1,
        m : Nat • m ≥ 1

      type
        Col = {| j : Int • j ≥ 1 ∧ j ≤ n |},
        Row = {| i : Int • i ≥ 1 ∧ i ≤ m |},
        Point = Row × Col

      type Picture = {| b : Point ~m~> Colour • is_total(b) |}

      value
        is_total : (Point ~m~> Colour) → Bool
        is_total(b) ≡ (∀ p : Point • p ∈ dom b)

      value
        colour_of : Point × Picture → Colour
        colour_of(p, b) ≡ b(p),

        one_colour : Picture → Bool
        one_colour(b) ≡ card rng b = 1,

        vertical_stripe : Picture → Bool
        vertical_stripe(b) ≡
          (
            ∃ j : Col, f : Colour •
              (
                ∀ i : Row •
                  colour_of((i, j), b) = f ∧
                  (∀ j′ : Col • (j′ = j − 1 ∨ j′ = j + 1) ⇒ colour_of((i, j′), b) ≠ f)
```

```
        )
    )

  value
    + : Colour × Colour → Colour,
    + : Picture × Picture → Picture

  axiom
    ∀ p : Point, b1, b2 : Picture •
      colour_of(p, b1 + b2) ≡ colour_of(p, b1) + colour_of(p, b2)

  axiom
    ∀ b1, b2 : Picture •
      b1 + b2 = [ p ↦ colour_of(p, b1) + colour_of(p, b2) | p : Point ]
end
```

The last two axioms are alternatives – only one of them needs to be stated.