16   Refinement

1. (b) and (c) implement (a).

2. (a) - (g) have all the same maximal signature (**value** x : **Int**).

   The theories are as follows:

   (a) **axiom true**

   (b) **axiom** x > 2

   (c) **axiom** x = 2

   (d) **axiom** x ≥ 0

   (e) **axiom** x > 2

   (f) **axiom** x = 2

   (g) **axiom false**

   Considering which theories are consequences of each other we get:

   All implement (a).
   (g) implements all.
   (b), (c), (e) and (f) implement (d).
   (b) and (e) are equivalent and therefore they implement each other.
   (c) and (f) are equivalent and therefore they implement each other.

3. (c) implements (a) and (b).
   (a) implements (b).

4. (a) 'PSTACK2' does statically implement 'PSTACK1'.

   (b) Refinement conditions:

   > **axiom**
   >   ∀ st : Stack • is_empty(st) ≡ st = empty,
   >   ∀ e : E.Elem, st : Stack • top(push(e,st)) ≡ e,
   >   ∀ e : E.Elem, st : Stack • pop(push(e,st)) ≡ st,
   >   ∀ e : E.Elem, st : Stack • empty ≠ push(e,st),
   >   ∀ p : Stack → **Bool** •
   >   p(empty) ∧
   >   (∀ st : Stack, e : E.Elem • p(st) ⇒ p(push(e,st)))
   >   ⇒
   >   (∀ st : Stack • p(st))