

# A Robust and Efficient Ensemble of Diversified Evolutionary Computing Algorithms for Accurate Robot Calibration:

## Supplementary File

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This is the supplementary file for this paper. Additional tables and figures regarding the symbol appointment, model structure and experimental results are placed here.

### I. ADDITIONAL TABLES

TABLE S.I. SYMBOL APPOINTMENT.

Symbol	Explanation
$A$	Transformation matrix.
$a, d, \alpha, \theta$	Link length, link offset, link twist angle, joint angle.
$c\theta_i, s\theta_i$	$\cos\theta$ and $\sin\theta$ .
$dA, \delta A$	Holomorphic differential matrix, error matrix.
$dx, dy, dz$	Differential translations.
$\delta x, \delta y, \delta z$	Differential rotations.
$\Delta S$	Position error vector of the end-effector.
$\Delta a, \Delta \alpha, \Delta d, \Delta \theta$	Vector of the D-H parameter deviations
$L, \bar{L}$	Measured length of the cable, nominal length of the cable.
$N$	The total number of samples.
$f$	Objective function.
$S_0, \hat{S}$	Fixed position of the cable encoder on the ground and nominal position.
$f_c$	Perceived magnitude.
$\hat{c}$	Sensory modality coefficient.
$\hat{a}$	Power exponent.
$t$	Current iteration.
$n$	Size of a population.
$\hat{d}$	Dimension of the position vector.
$P_{Mk}$	Population of the $k$ -th expert's output value at $t$ -th iteration.
$k$	Expert algorithm index.
$g^*$	Best position vector.
$r, r_1, r_2$	Random number.
$\bar{p}$	Switching probability.
$cumsum$	Cumulative sum.
$Iter$	Max round of iteration.
$\tilde{r}$	The number of 0 or 1.
$l_{a,i}, l_{b,i}, l_b$	Lower limit of ALO.
$u_{a,i}, u_{b,i}, u_b$	Upper limit of ALO.
$I$	Unit vector.
$R_v$	Radius of random walks of ants.
$A'_{i,j}$	Positions vector of the $j$ -th antlion at the $t$ -th iteration.
$R_{A_i}, R_{E_i}$	Random walks.
$B'_{1,i}, B'_{2,i}, B'_{3,i}, B'_{4,i}, B'_{5,i}, C'_j, Z'_i, O'_i$	Vectors of DA.
$b_1, b_2, b_3, b_4, b_5$	Weights of separation, alignment, cohesion, food, natural enemy and inertia for the dragonfly algorithm.
$L\acute{e}vy(\cdot)$	Lévy flight.
$\tau$	Inertia.
$\bar{\alpha}, \bar{\beta}, \bar{\eta}$	Leader wolves.
$\bar{e}$	An individual in the rest wolves.
$D, \bar{D}, \bar{D}_{best}$	Euclidean distance.
$ \cdot $	Absolute value of a number
$\gamma, \mu$	Coefficient vectors.
$e'$	Convergence factor at $t$ -th iteration.
$R'_i$	Radius.
$\ \cdot\ _2$	$L_2$ norm of a vector.
$\zeta'_i$	Neighbors of $P'_{M4,i-DLH}$
$h, c'$	Constant.
$\kappa$	A parameter of the I-WOA.
$w, w_1, w_2$	Nonlinear variable, maximum of $w$ , minimum of $w$ .
$m$	Distance between the right and left antennas.
$\bar{d}$	Random unit vector.
$\text{sign}(\cdot)$	Sign function.
$\chi_t$	Exploration step.
$v$	Constant.
$Sort\{\cdot\}$	Self-defined function of the non-inferior sort rule.
$\bar{U}_{Mk}$	The set is based on the result of the $Sort\{\cdot\}$ function.
$Intercept\{\cdot\}$	Self-defined function for intercepting the selected agents.
$G_{min}\{\cdot\}$	Self-defined function to select the best agent.
$\Gamma_{Mk}^i$	Contribution factor.
$P^0$	Initial D-H parameters.
$q_i$	Rotation angle.
$\omega$	Dimension of $q_i$ .
$K$	Total number of expert algorithms.

TABLE XI S.II. CALIBRATION ACCURACY OF THE HOES-TEST-BASED MODELS AND THEIR INVOLVED EXPERT ALGORITHMS (M1-8 AND M'1-7) ON D1-3.

Algorithm	D1			D2			D3		
	RMSE/mm	Mean/mm	Max/mm	RMSE/mm	Mean/mm	Max/mm	RMSE/mm	Mean/mm	Max/mm
M1	0.791 $\pm 1.1E-2$	0.692 $\pm 1.8E-2$	1.496 $\pm 4.7E-2$	0.689 $\pm 3.1E-2$	0.579 $\pm 2.6E-2$	1.681 $\pm 4.1E-2$	0.642 $\pm 3.1E-2$	0.553 $\pm 3.7E-2$	1.421 $\pm 5.1E-2$
M2	0.767 $\pm 1.1E-2$	0.679 $\pm 9.5E-3$	1.375 $\pm 3.0E-2$	0.632 $\pm 4.6E-2$	0.533 $\pm 3.1E-2$	1.403 $\pm 5.1E-2$	0.553 $\pm 5.1E-2$	0.536 $\pm 4.0E-2$	1.337 $\pm 6.6E-2$
M3	1.060 $\pm 7.4E-2$	0.930 $\pm 7.4E-2$	1.740 $\pm 5.1E-2$	0.661 $\pm 8.3E-2$	0.560 $\pm 7.1E-2$	1.537 $\pm 8.6E-2$	0.675 $\pm 7.3E-2$	0.576 $\pm 6.1E-2$	1.526 $\pm 8.2E-2$
M4	0.673 $\pm 3.3E-2$	0.583 $\pm 4.6E-2$	1.100 $\pm 8.7E-2$	0.525 $\pm 2.1E-2$	0.422 $\pm 8.6E-3$	1.196 $\pm 1.8E-2$	0.525 $\pm 5.5E-2$	0.482 $\pm 3.2E-2$	0.826 $\pm 6.5E-2$
M5	0.652 $\pm 3.5E-2$	0.560 $\pm 6.5E-2$	1.031 $\pm 9.5E-3$	0.461 $\pm 3.3E-2$	0.372 $\pm 2.7E-2$	1.143 $\pm 6.1E-2$	0.505 $\pm 3.7E-2$	0.472 $\pm 2.9E-2$	0.791 $\pm 7.3E-2$
M6	0.905 $\pm 6.7E-3$	0.803 $\pm 6.0E-3$	1.629 $\pm 1.8E-2$	0.702 $\pm 6.1E-2$	0.565 $\pm 5.5E-2$	1.752 $\pm 2.2E-2$	0.635 $\pm 1.1E-2$	0.566 $\pm 8.5E-3$	1.481 $\pm 1.2E-2$
M7	1.012 $\pm 6.0E-2$	0.896 $\pm 2.2E-2$	1.844 $\pm 4.9E-2$	0.591 $\pm 7.3E-2$	0.492 $\pm 3.1E-2$	1.283 $\pm 5.1E-2$	0.993 $\pm 5.3E-2$	0.961 $\pm 2.5E-2$	1.373 $\pm 7.3E-2$
M8	0.867 $\pm 2.6E-2$	0.768 $\pm 1.6E-2$	1.543 $\pm 3.10E-2$	0.633 $\pm 5.1E-2$	0.533 $\pm 2.1E-2$	1.425 $\pm 6.1E-2$	0.622 $\pm 6.1E-2$	0.556 $\pm 3.1E-2$	1.283 $\pm 6.1E-2$
M'1	0.722 $\pm 5.6E-2$	0.621 $\pm 3.2E-2$	1.221 $\pm 1.7E-2$	0.615 $\pm 3.1E-2$	0.523 $\pm 5.1E-2$	1.303 $\pm 3.7E-2$	0.613 $\pm 3.0E-2$	0.546 $\pm 5.3E-2$	1.229 $\pm 6.1E-2$
M'2	0.681 $\pm 2.8E-2$	0.583 $\pm 1.6E-2$	1.166 $\pm 2.8E-2$	0.565 $\pm 4.6E-2$	0.460 $\pm 2.2E-2$	1.261 $\pm 1.1E-2$	0.591 $\pm 2.1E-2$	0.522 $\pm 3.3E-2$	1.052 $\pm 3.6E-2$
M'3	0.632 $\pm 3.7E-2$	0.536 $\pm 5.1E-2$	0.965 $\pm 6.2E-2$	0.512 $\pm 3.2E-2$	0.412 $\pm 3.9E-2$	1.186 $\pm 4.0E-2$	0.521 $\pm 5.3E-2$	0.453 $\pm 4.0E-2$	0.761 $\pm 5.3E-2$
M'4	0.579 $\pm 2.6E-2$	0.486 $\pm 2.6E-2$	0.896 $\pm 3.9E-2$	0.479 $\pm 2.3E-2$	0.383 $\pm 3.2E-2$	1.101 $\pm 5.5E-2$	0.502 $\pm 3.5E-2$	0.430 $\pm 3.6E-2$	0.736 $\pm 4.0E-2$
M'5	<b>0.520</b> $\pm 9.7E-3$	<b>0.431</b> $\pm 1.9E-2$	<b>0.855</b> $\pm 2.0E-2$	<b>0.412</b> $\pm 3.6E-2$	<b>0.331</b> $\pm 3.7E-2$	<b>0.983</b> $\pm 3.5E-2$	<b>0.472</b> $\pm 8.2E-3$	<b>0.433</b> $\pm 7.5E-3$	<b>0.700</b> $\pm 1.1E-2$
M'6	<b>0.520</b> $\pm 1.6E-2$	<b>0.430</b> $\pm 1.9E-2$	<b>0.851</b> $\pm 2.5E-2$	<b>0.411</b> $\pm 3.0E-2$	<b>0.330</b> $\pm 1.9E-2$	<b>0.980</b> $\pm 3.2E-2$	<b>0.472</b> $\pm 2.8E-2$	<b>0.432</b> $\pm 5.2E-2$	<b>0.696</b> $\pm 3.7E-2$
M'7	<b>0.520</b> $\pm 1.3E-2$	<b>0.432</b> $\pm 1.9E-2$	<b>0.857</b> $\pm 1.9E-2$	<b>0.411</b> $\pm 4.1E-2$	<b>0.330</b> $\pm 3.1E-2$	<b>0.982</b> $\pm 4.6E-2$	<b>0.472</b> $\pm 5.8E-2$	<b>0.433</b> $\pm 6.5E-2$	<b>0.701</b> $\pm 6.0E-2$
M'7	<b>0.520</b> $\pm 1.3E-2$	<b>0.432</b> $\pm 2.0E-2$	<b>0.857</b> $\pm 2.0E-2$	<b>0.411</b> $\pm 4.2E-2$	<b>0.330</b> $\pm 3.2E-2$	<b>0.982</b> $\pm 4.8E-2$	<b>0.472</b> $\pm 5.9E-2$	<b>0.433</b> $\pm 6.5E-2$	<b>0.701</b> $\pm 6.1E-2$

TABLE S.III. TOTAL TIME COSTS OF THE HOES-TEST-BASED MODELS AND THEIR INVOLVED EXPERT ALGORITHMS (M1-8 AND M'1-7) ON D1-3.

Algorithm	D1		D2		D3	
	Iteration	Time/s	Iteration	Time/s	Iteration	Time/s
M1	57.0 $\pm 1.0$	63.9 $\pm 1.16$	45.3 $\pm 1.6$	50.39 $\pm 1.95$	40.7 $\pm 2.5$	58.3 $\pm 2.83$
M2	28.6 $\pm 1.5$	33.9 $\pm 1.66$	31.7 $\pm 1.5$	37.25 $\pm 0.97$	25.6 $\pm 1.2$	30.4 $\pm 0.83$
M3	16.3 $\pm 1.5$	14.5 $\pm 1.08$	20.6 $\pm 2.5$	18.79 $\pm 1.35$	15.3 $\pm 2.8$	13.5 $\pm 1.58$
M4	44.6 $\pm 4.1$	53.3 $\pm 3.99$	50.9 $\pm 3.6$	60.76 $\pm 3.21$	50.5 $\pm 4.6$	58.6 $\pm 3.97$
M5	73.3 $\pm 6.7$	43.0 $\pm 4.13$	68.3 $\pm 6.7$	40.67 $\pm 4.07$	68.9 $\pm 5.3$	45.67 $\pm 4.62$
M6	35.0 $\pm 4.6$	10.3 $\pm 0.50$	36.7 $\pm 5.3$	12.30 $\pm 0.67$	39.4 $\pm 5.4$	9.62 $\pm 0.68$
M7	30.3 $\pm 1.5$	23.6 $\pm 0.72$	45.6 $\pm 1.9$	25.05 $\pm 0.98$	52.5 $\pm 1.8$	26.9 $\pm 0.76$
M8	90.7 $\pm 3.1$	55.9 $\pm 1.61$	86.9 $\pm 2.9$	53.65 $\pm 0.93$	70.7 $\pm 2.6$	50.5 $\pm 2.69$
M'1	24.3 $\pm 5.1$	60.2 $\pm 0.96$	24.4 $\pm 1.4$	50.8 $\pm 2.62$	20.5 $\pm 1.3$	49.33 $\pm 1.49$
M'2	18.6 $\pm 4.2$	42.3 $\pm 0.96$	16.3 $\pm 3.2$	46.3 $\pm 3.21$	17.3 $\pm 2.7$	53.27 $\pm 2.29$
M'3	12.2 $\pm 3.5$	50.1 $\pm 0.96$	13.2 $\pm 2.0$	52.3 $\pm 2.52$	15.2 $\pm 2.3$	58.23 $\pm 1.23$
M'4	9.3 $\pm 2.1$	40.6 $\pm 0.96$	8.5 $\pm 1.3$	36.2 $\pm 3.22$	12.8 $\pm 1.2$	52.91 $\pm 1.22$
M'5	<b>5.1</b> $\pm 0.2$	<b>22.3</b> $\pm 0.96$	<b>5.1</b> $\pm 2.2$	<b>22.7</b> $\pm 4.42$	<b>7.5</b> $\pm 1.3$	<b>37.65</b> $\pm 1.22$
M'6	5.3 $\pm 1.2$	30.1 $\pm 0.96$	5.2 $\pm 1.0$	27.9 $\pm 1.12$	7.2 $\pm 3.6$	43.25 $\pm 1.35$
M'7	5.2 $\pm 1.8$	36.3 $\pm 0.96$	5.6 $\pm 1.5$	33.1 $\pm 1.80$	7.3 $\pm 0.6$	46.31 $\pm 1.32$
M'7	5.2 $\pm 1.7$	36.3 $\pm 0.93$	5.6 $\pm 1.5$	33.1 $\pm 1.86$	7.3 $\pm 0.6$	46.31 $\pm 1.40$

TABLE S.IV. CALIBRATION ACCURACY OF M9-17 ON D1-3.

Algorithm	D1			D2			D3		
	RMSE/mm	Mean/mm	Max/mm	RMSE/mm	Mean/mm	Max/mm	RMSE/mm	Mean/mm	Max/mm
Before	2.56	2.45	4.51	2.09	2.0	3.36	2.73	2.72	3.09
M9	0.675 $\pm 0.0E-0$	0.589 $\pm 0.0E-0$	1.162 $\pm 0.0E-0$	0.503 $\pm 0.0E-0$	0.416 $\pm 0.0E-0$	1.167 $\pm 0.0E-0$	0.523 $\pm 0.0E-0$	0.504 $\pm 0.0E-0$	0.796 $\pm 0.0E-0$
M10	1.213 $\pm 6.5E-2$	1.110 $\pm 5.6E-2$	2.232 $\pm 7.6E-2$	0.822 $\pm 5.1E-2$	0.728 $\pm 3.1E-2$	1.933 $\pm 7.1E-2$	1.375 $\pm 4.6E-2$	1.347 $\pm 3.5E-2$	2.461 $\pm 5.7E-2$
M11	1.383 $\pm 3.3E-2$	1.285 $\pm 3.8E-2$	2.621 $\pm 6.5E-2$	0.723 $\pm 5.1E-2$	0.635 $\pm 2.1E-2$	1.838 $\pm 6.8E-2$	1.302 $\pm 8.1E-2$	1.273 $\pm 6.1E-2$	2.235 $\pm 8.6E-2$
M12	0.981 $\pm 5.6E-2$	0.880 $\pm 5.0E-2$	1.820 $\pm 5.3E-2$	0.711 $\pm 3.3E-2$	0.578 $\pm 5.1E-2$	1.783 $\pm 3.9E-2$	1.648 $\pm 7.3E-2$	1.438 $\pm 8.3E-2$	1.887 $\pm 6.3E-2$
M13	0.836 $\pm 5.3E-2$	0.738 $\pm 3.8E-2$	1.632 $\pm 6.4E-2$	0.653 $\pm 3.9E-2$	0.552 $\pm 5.1E-2$	1.605 $\pm 6.1E-2$	0.972 $\pm 5.2E-2$	0.936 $\pm 4.0E-2$	1.502 $\pm 3.7E-2$
M14	0.557 $\pm 2.3E-2$	0.462 $\pm 2.1E-2$	0.897 $\pm 2.6E-2$	0.423 $\pm 5.2E-2$	0.348 $\pm 3.7E-2$	1.046 $\pm 5.0E-2$	0.506 $\pm 2.7E-2$	0.465 $\pm 2.1E-2$	0.757 $\pm 3.6E-2$
M15	0.526 $\pm 1.8E-2$	0.435 $\pm 9.2E-3$	0.858 $\pm 1.3E-2$	0.412 $\pm 5.7E-2$	0.325 $\pm 2.1E-2$	0.983 $\pm 1.1E-2$	0.473 $\pm 3.1E-2$	0.431 $\pm 2.6E-2$	0.713 $\pm 1.2E-2$
M16	0.563 $\pm 2.6E-2$	0.462 $\pm 1.5E-2$	0.891 $\pm 2.8E-2$	0.433 $\pm 3.1E-2$	0.353 $\pm 1.6E-2$	1.051 $\pm 2.2E-2$	0.512 $\pm 3.1E-2$	0.465 $\pm 1.1E-2$	0.751 $\pm 1.9E-2$
M17	<b>0.520</b> $\pm 9.7E-3$	<b>0.431</b> $\pm 1.9E-2$	<b>0.855</b> $\pm 2.0E-2$	<b>0.412</b> $\pm 3.6E-2$	<b>0.331</b> $\pm 3.1E-2$	<b>0.983</b> $\pm 3.5E-2$	<b>0.472</b> $\pm 8.2E-3$	<b>0.433</b> $\pm 7.5E-3$	<b>0.700</b> $\pm 1.1E-2$

TABLE S.V. TOTAL TIME COSTS OF M9-17 ON D1-3.

Algorithm	D1		D2		D3	
	Iteration	Time/s	Iteration	Time/s	Iteration	Time/s
M9	58.0 $\pm 0.00$	36.5 $\pm 0.28$	53.0 $\pm 0.00$	34.69 $\pm 0.15$	50.3 $\pm 0.00$	30.9 $\pm 0.12$
M10	13.0 $\pm 0.00$	13.6 $\pm 0.03$	10.0 $\pm 0.00$	10.67 $\pm 0.06$	12.7 $\pm 0.00$	13.5 $\pm 0.16$
M11	15.2 $\pm 1.23$	15.5 $\pm 1.86$	12.6 $\pm 1.50$	13.96 $\pm 1.90$	14.1 $\pm 1.63$	15.5 $\pm 1.57$
M12	11.7 $\pm 1.10$	29.2 $\pm 2.35$	9.5 $\pm 1.60$	19.98 $\pm 3.20$	10.2 $\pm 1.72$	22.3 $\pm 3.56$
M13	62.6 $\pm 3.05$	46.5 $\pm 1.61$	64.2 $\pm 2.95$	49.65 $\pm 0.93$	67.3 $\pm 3.67$	46.5 $\pm 2.69$
M14	5.3 $\pm 0.67$	35.1 $\pm 1.81$	5.5 $\pm 0.97$	40.92 $\pm 3.28$	7.9 $\pm 0.62$	50.2 $\pm 3.67$
M15	5.6 $\pm 0.30$	35.0 $\pm 2.07$	5.3 $\pm 0.46$	39.75 $\pm 2.50$	7.6 $\pm 0.75$	49.6 $\pm 4.60$
M16	5.1 $\pm 0.17$	26.2 $\pm 0.86$	5.2 $\pm 0.22$	30.86 $\pm 1.30$	8.2 $\pm 0.37$	38.7 $\pm 2.53$
M17	5.1 $\pm 0.23$	22.3 $\pm 0.96$	5.1 $\pm 0.33$	22.7 $\pm 1.60$	7.5 $\pm 0.31$	37.65 $\pm 2.20$

TABLE S.VI. THE RESULTS OF THE WILCOXON SIGNED-RANKS TEST ON RMSE/MEAN/MAX OF TABLE XIII.

Comparison	$R^+$	$R^-$	$p$ -value
M17 vs. M9	45	0	0.002
M17 vs. M10	45	0	0.002
M17 vs. M11	45	0	0.002
M17 vs. M12	45	0	0.002
M17 vs. M13	45	0	0.002
M17 vs. M14	45	0	0.002
M17 vs. M15	45	0	0.002
M17 vs. M16	45	0	0.002

\*The highlighted significance level of acceptance is 0.05.

TABLE S.VII. THE CALIBRATION RESULTS OF THE D-H PARAMETERS DEVIATIONS ON D1-3.

Dataset	Joint $i$	$\Delta\alpha_i/^\circ$	$\Delta a_i/mm$	$\Delta d_i/mm$	$\Delta\theta_i/^\circ$
D1	1	-0.1960	-0.3615	0.6391	0.1698
	2	-0.5361	-2.3644	-0.5903	-0.5987
	3	0.6073	3.7962	-0.1826	0.3161
	4	-0.0618	0.6569	3.8435	0.3668
	5	0.1094	-0.5389	0.1258	0.0396
	6	0.1205	-0.1536	-0.7375	0.9631
D2	1	0.2364	-5.3984	4.3259	-0.2366
	2	0.0035	-3.2165	-2.7589	-0.2698
	3	-0.7653	-0.3962	4.8632	-0.0659
	4	-0.8320	-0.2326	2.3654	-0.0836
	5	-0.0165	-4.2658	1.2369	-0.1265
	6	-0.3658	-0.2354	2.3698	-0.3659
D3	1	0.0562	-1.9678	2.3698	-0.0863
	2	0.0369	-0.0697	-1.5066	0.0532
	3	-0.1698	-2.9634	-0.5436	-0.0221
	4	0.0756	-1.2691	0.5396	0.5132
	5	-0.2689	1.7369	-0.1963	-0.1256
	6	-0.0235	-3.2692	0.2365	-0.3658

## II. ADDITIONAL FIGURES

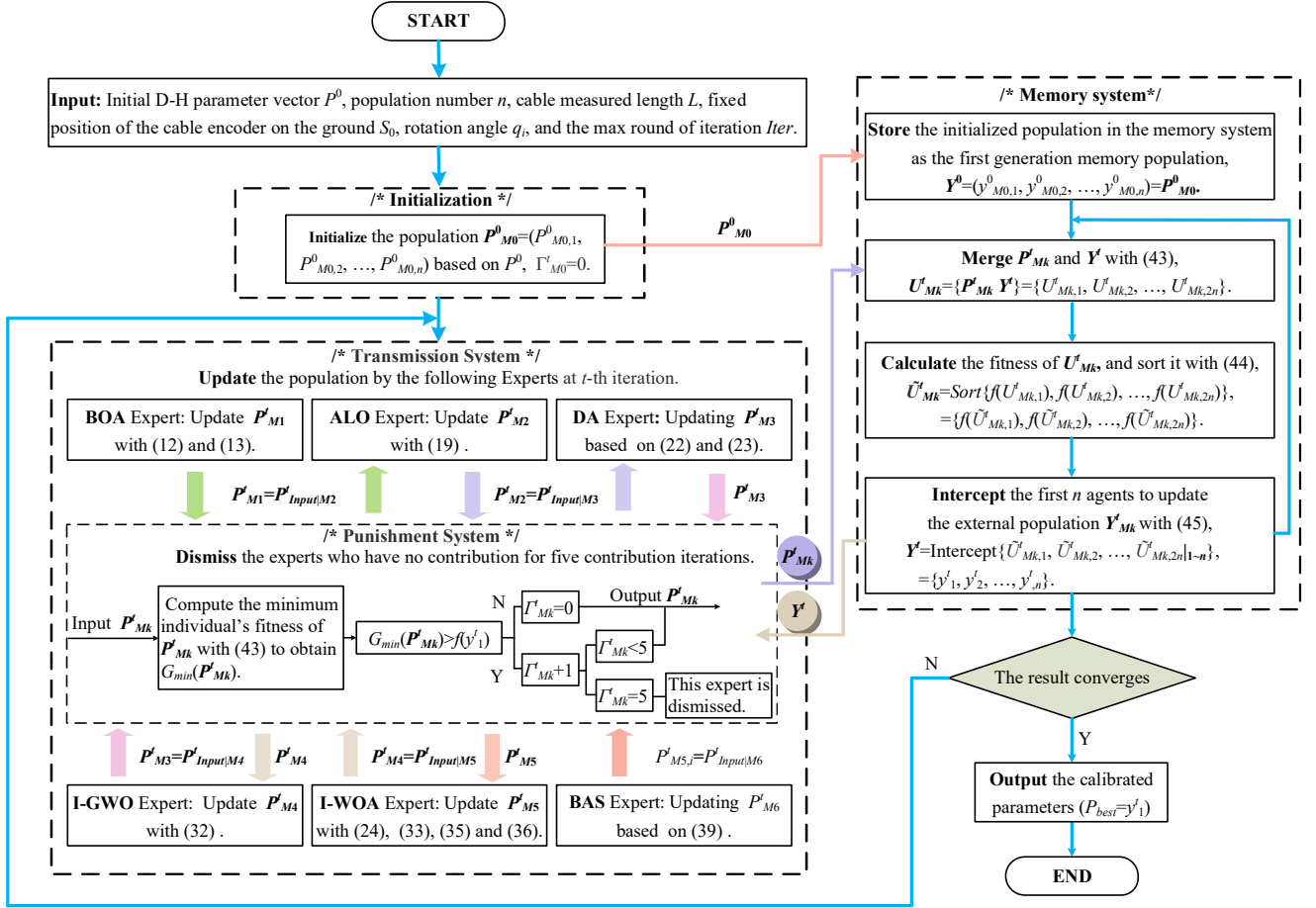


Fig. S.1. The flowchart of the HOEs model.

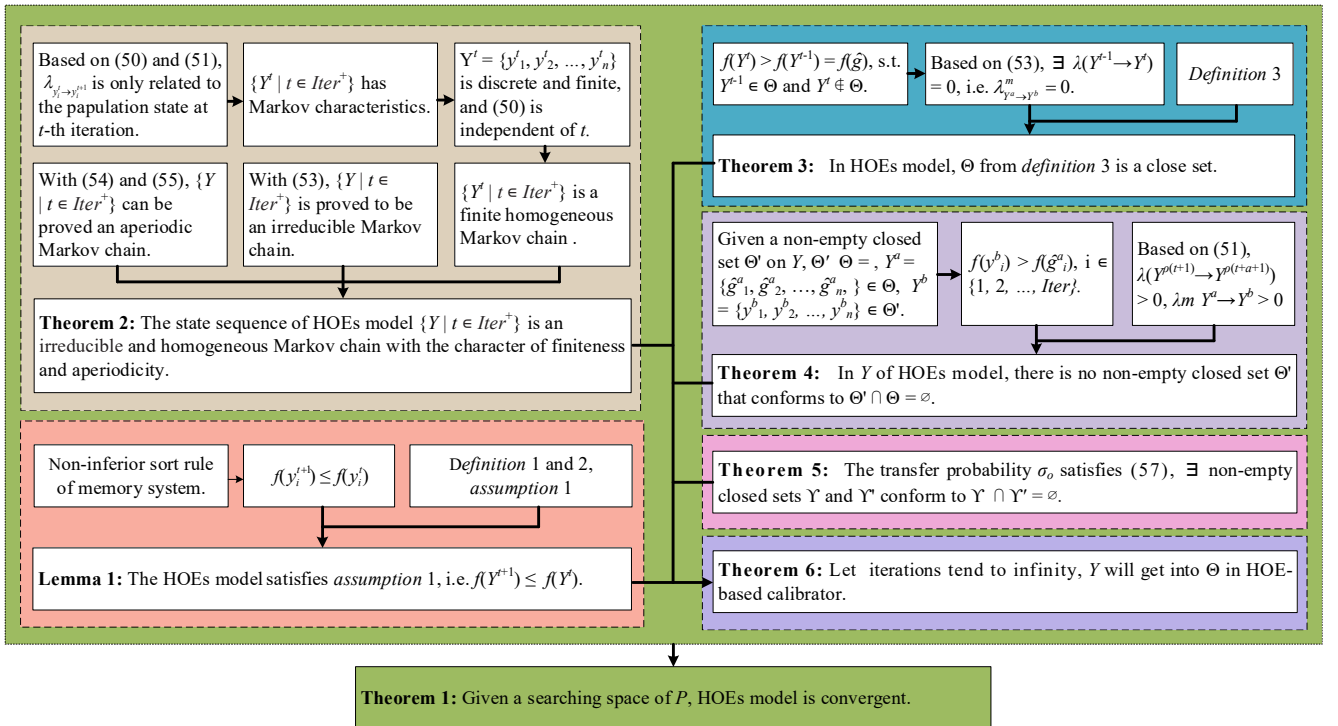


Fig. S.2. Proof sketch of HOEs model's convergence.

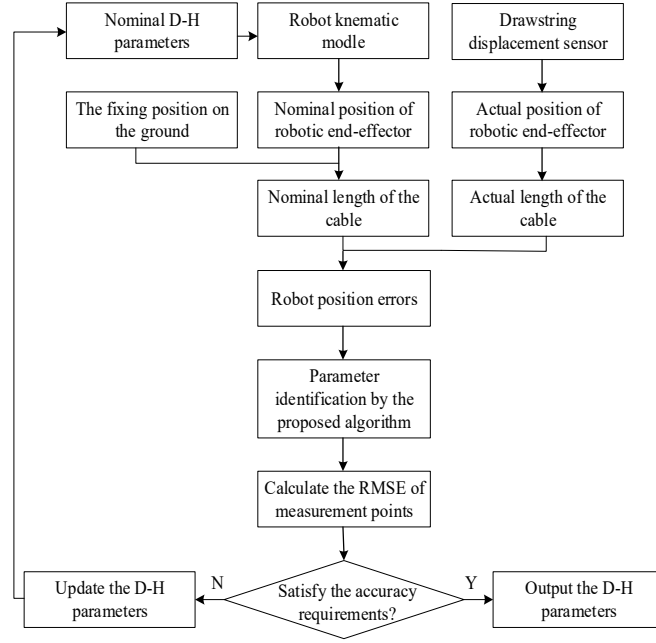


Fig. S.3. The industrial robot calibration process.

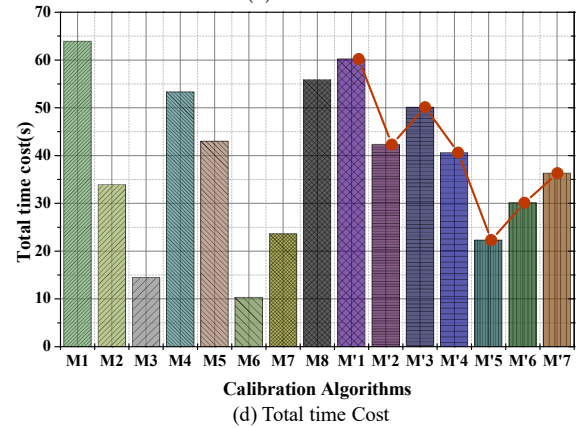
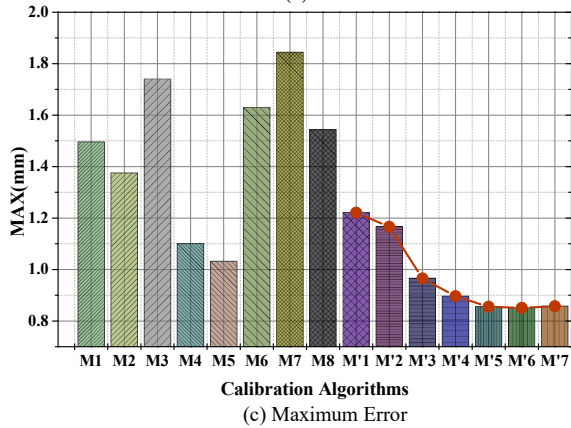
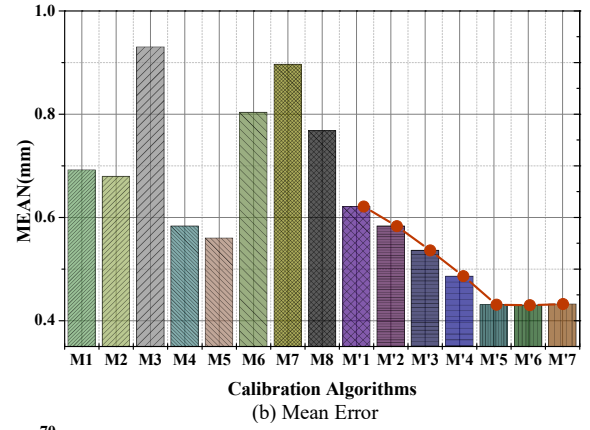
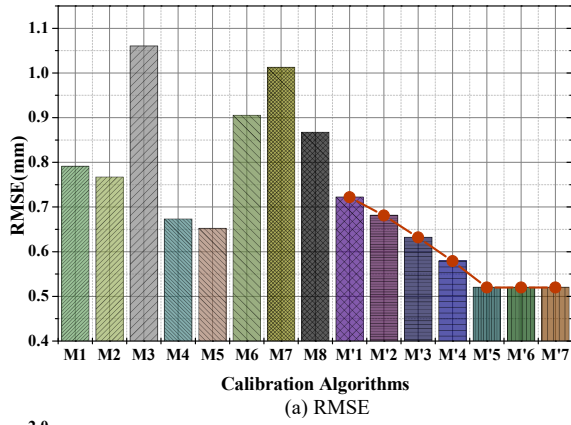


Fig. S.4. Calibration accuracy and total time cost of the HOEs-test-based models and its involved expert algorithms on D1.

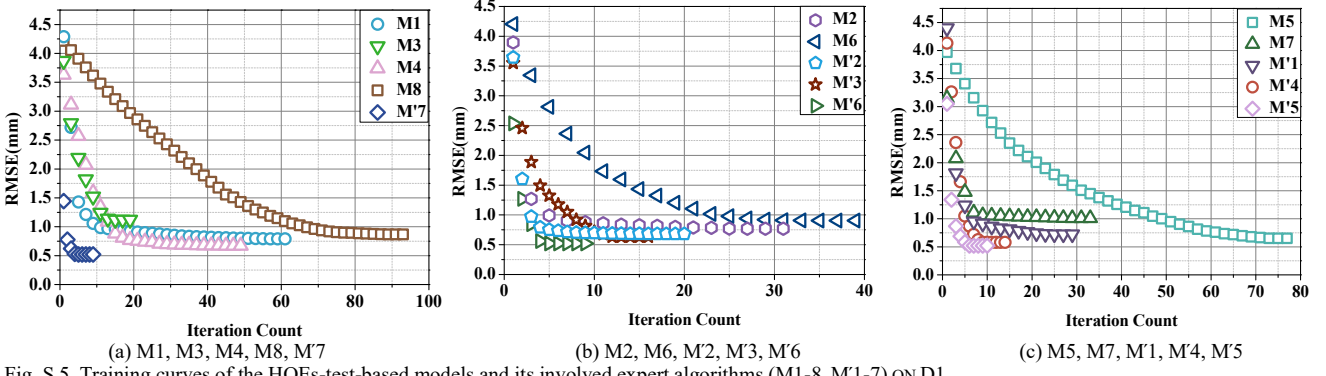


Fig. S.5. Training curves of the HOEs-test-based models and its involved expert algorithms (M1-8, M'1-7) ON D1.

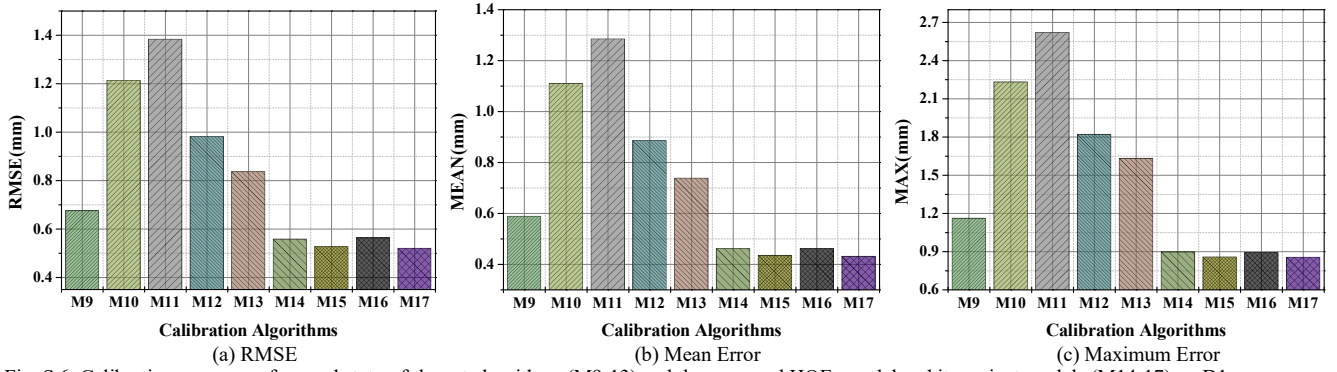


Fig. S.6. Calibration accuracy of several state-of-the-art algorithms (M9-13) and the proposed HOEs model and its variant models (M14-17) on D1.

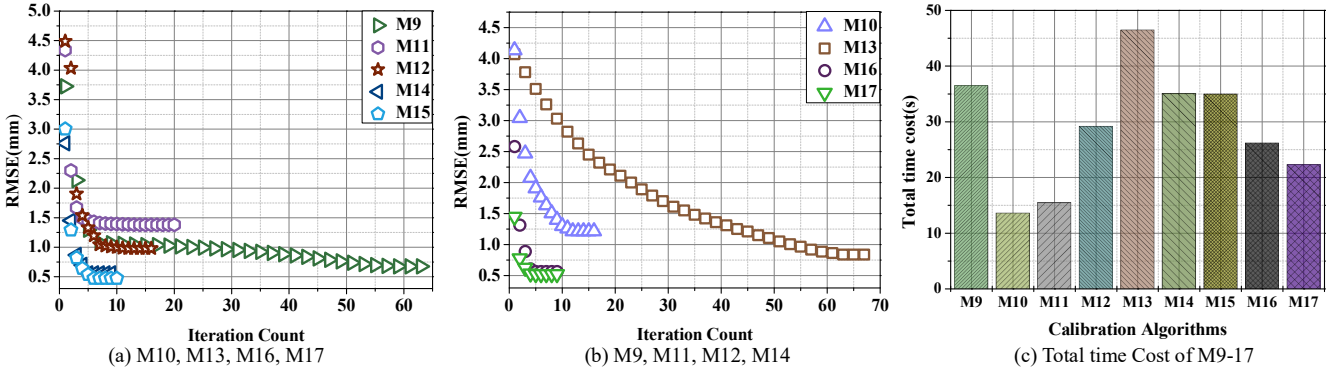


Fig. S.7. Training curves and total time costs of several state-of-the-art algorithms (M9-13) and the proposed HOEs model and its variant models (M14-17) on D1.

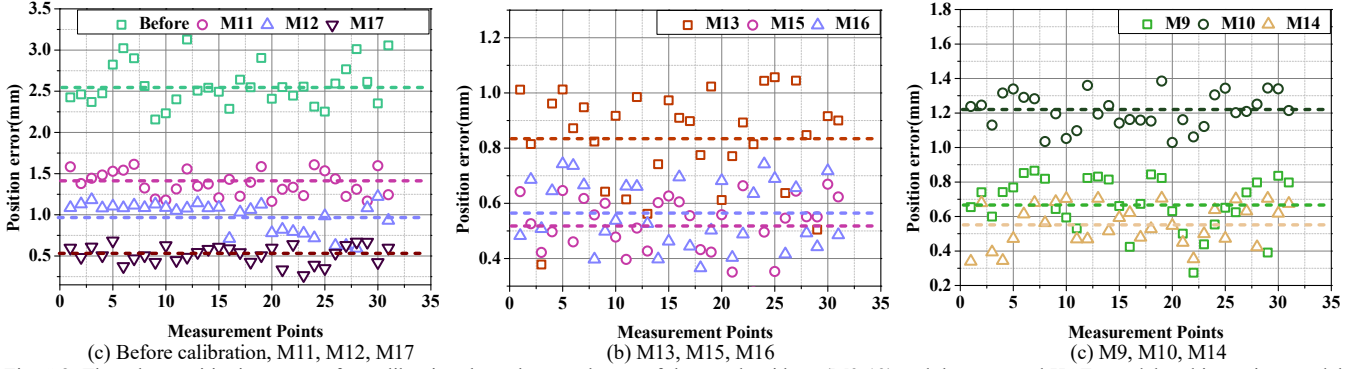


Fig. S.8. The robot positioning errors after calibration through several state-of-the-art algorithms (M9-13) and the proposed HOEs model and its variant models (M14-17) on D1. It shows that the proposed model achieves the lowest calibration error.