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Review

- Statement
    - "This statement is false".
- Negation
- Disjunction
- Conjunction
- Implication
- Express $P \Rightarrow Q$ in words
- Functional completeness
- NAND

Converse

Given an implication $P \Rightarrow Q$, its **converse** is the implication $Q \Rightarrow P$.

- It is a new statement formed by switching the *hypothesis* and the *conclusion*.
- It is a new statement formed by switching the "if..." and "then..." clause.

Let us fill in the truth table.

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$
T	T		
T	F		
F	T		
F	F		

- Are $P \Rightarrow Q$ and $Q \Rightarrow P$ the same?
- If $P \Rightarrow Q$ is true, then is $Q \Rightarrow P$ automatically true?

Rewrite the following implications in the "if... then..." form, and then construct their converse:

1.

   Implication. Every integer is rational.

   Converse.

2.

   Implication. The continuity of a function at a point is a necessary condition for the differentiability of the function at the same point.

Converse.

3.

Implication. A number is divisible by 6 if it is divisible by both 2 and 3.

Converse.

4.

Implication. When the weather is cold, we sell more snow boots.

Converse.

5.

Implication. The gas price can stablize only if there is no war in the middle east.

Converse.

Biconditionals

For two statements P and Q, the **biconditional** of P and Q is

$$(P \Rightarrow Q) \ \wedge \ (Q \Rightarrow P) .$$

We use the notation

$$P \Leftrightarrow Q .$$

It is true precisely when...

- Both $P \Rightarrow Q$ and $Q \Rightarrow P$ are true
- P and Q have the same truth value

Let us fill in the truth table:

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \Leftrightarrow Q$	$Q \Leftrightarrow P$
T	T				
T	F				
F	T				
F	F				

Notice that in the biconditional $P \Leftrightarrow Q$, the roles of P and Q are symmetric (which is not the case for conditionals).

$P \Leftrightarrow Q$ can be expressed in many different ways:

- P is equivalent to Q
- Q is equivalent to P
- P if and only if Q
- Q if and only if P

- P exactly when Q
- Q exactly when P
- P **iff** Q (use this with care. Actually, don't use it in homework)
- Q **iff** P (use this with care. Actually, don't use it in homework)
- P is a necessary and sufficient condition for Q
- Q is a necessary and sufficient condition for P

Can you think of any other ways of expressing $P \Leftrightarrow Q$?

Are these true?

- Two triangles are congruent *if and only if* their three pairs of sides are equal in length.
- Two lines on the plane intersect at a single point *exactly when* they have different slopes.
- $ax = ay$ if and only $x = y$.
- $x^2 = y^2$ is equivalent to $x = y$.
- 3 is prime if and only if 100 is even.
- 4 is prime if and only if 101 is even.
- 5 is prime if and only if 103 is even.

Compound statements

...a statement composed of some statements and (at least one) logical connectives.

We can also compound statements as functions expressed in terms of logical connectives that takes truth values as input and also output truth values. E.g.

$$f(P_1, P_2, P_3) \;=\; (P_1 \wedge P_2) \vee (\sim P_3)$$

Tautologies

A compound statement involving independent statements $P_1, \ldots, P_n$ is called a **tautology** if it is *true* for all possible combinations of the truth values of $P_1, \ldots, P_n$. Equivalently,

- A logical expression that can never be *false*.
- A logical expression whose truth table containing an entire column of T's.
- A negation of a contradiction.

Let us fill in this truth table:

P	$\sim(P)$	$P \vee (\sim P)$	$P \Leftrightarrow P$
T			
T			
F			
F			

...and this one:

P	Q	$P \Rightarrow Q$	$\sim Q$	$(P \Rightarrow Q) \vee (\sim Q)$
T	T			
T	F			
F	T			
F	F			

...and this one:

P	Q	$P \Rightarrow Q$	$P \wedge (P \Rightarrow Q)$	$(P \wedge (P \Rightarrow Q)) \Rightarrow Q$
T	T			
T	F			
F	T			
F	F			

Can you state the last expression in words? It is commonly known as the **modus ponens** in logic.

How about this more complicated one:

P	Q	$P \Rightarrow Q$	$\sim P$	$\sim Q$	$(\sim Q) \Rightarrow (\sim P)$	$(P \Rightarrow Q) \Leftrightarrow ((\sim Q) \Rightarrow (\sim P))$
T	T					
T	F					
F	T					
F	F					

Can you state the last expression in words? It is commonly known as the **contrapositive**.

Contradictions

A compound statement involving independent statements $P_1, \ldots, P_n$ is called a **contradiction** if it is *false* for all possible combinations of the truth values of $P_1, \ldots, P_n$.

- A logical expression that can never be *true*.
- A logical expression whose truth table containing an entire column of F's.
- A negation of a tautology.

Let us fill in this truth table:

P	$\sim(P)$	$P \wedge (\sim P)$	$P \Rightarrow (\sim P)$
T			
T			
F			
F			

Can you state the last expression in words.