9/10/2014

Review of tautologies

Logical equivalence

$$\begin{array}{lcll}
\sim(\sim P) & \equiv & P & \text{(double negative)} \\
P \vee Q & \equiv & Q \vee P & \text{(commutativity of } \vee) \\
P \wedge Q & \equiv & Q \wedge P & \text{(commutativity of } \wedge) \\
P \vee (Q \vee R) & \equiv & (P \vee Q) \vee R & \text{(associativity of } \vee) \\
P \wedge (Q \wedge R) & \equiv & (P \wedge Q) \wedge R & \text{(associativity of } \wedge) \\
P \vee (Q \wedge R) & \equiv & (P \vee Q) \wedge (P \vee R) & \text{(distributive "law")} \\
P \wedge (Q \vee R) & \equiv & (P \wedge Q) \vee (P \wedge R) & \text{(distributive "law")} \\
\sim(P \vee Q) & \equiv & (\sim P) \wedge (\sim Q) & \text{(De Morgan's "law")} \\
\sim(P \wedge Q) & \equiv & (\sim P) \vee (\sim Q) & \text{(De Morgan's "law")}
\end{array}$$

Negations

Negation of a negation

$$\sim(\sim P) \equiv P$$

Negation of disjunction and conjection

$$\sim(P \vee Q) \equiv (\sim P) \wedge (\sim Q) \quad , \quad \sim(P \wedge Q) \equiv (\sim P) \vee (\sim Q)$$

Negation of implications: Recall from the truth table for implication that the only case $P \Rightarrow Q$ is false is when P is true but Q is false. Therefore

$$\sim(P \Rightarrow Q) \equiv P \wedge (\sim Q)$$

Negation of biconditionals: Recall that $P \Leftrightarrow Q$ is equivalent to $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$ by definition, so

$$\sim(P \Leftrightarrow Q) \equiv (P \wedge (\sim Q)) \vee (Q \wedge (\sim P))$$

Negations of "for all..."

$$\begin{array}{rcl}
\sim(\forall x \in S, P(x)) & \equiv & \exists x \in S, \sim P(x) \\
\sim(\text{"for all } x \text{ in } S, P(x) \text{ is true"}) & \equiv & \text{"there exists an } x \text{ in } S \text{ such that } P(x) \text{ is false"}
\end{array}$$

Negation of "there exists..."

$$\begin{array}{rcl}
\sim(\exists x \in S, P(x)) & \equiv & \forall x \in S, \sim P(x) \\
\sim(\text{"there exists an } x \text{ in } S \text{ such that } P(x) \text{ is true}) & \equiv & \text{"for all } x \text{ in } S, P(x) \text{ is false"}
\end{array}$$

Example 2 Consider the open sentence

$$P(x, y) \ : \ x < y.$$

Write the following expressions in words and then determine their truth values.

1. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, P(x, y)$.

   In words:

   Its negation in symbols:

Its negation in words:

2. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, P(y, x)$.

   In words:

   Its negation in symbols:

   Its negation in words:

3. $\forall x, y \in \mathbb{R}, P(x, y) \Rightarrow (\exists z \in \mathbb{R}, P(x, z) \wedge P(z, y))$.

   In words:

   Its negation in symbols:

   Its negation in words: