Homework 1 suggestions for revision [Due 9/15]

Problem 1. Find the maximum and minimum values of the function $f(x) = 2x^3 - 12x^2 + 18x$ on the interval $[0, 3]$. *Explain each step clearly.*

Note. This is a fairly straight calculus problem. Just note that you should perform first or second derivative test to determine which critical points are local maxima and which are local minima. Also, make sure you considered the boundary points.

Problem 2. In decimal notation, if the last digit of a number is even, then this number must be even, and vice versa. *Explain why this technique works.*

Note. The reason behind this technique is likely quite obvious: For numbers having only 1 digit, there is nothing to explain (since that number and its first digit would be the same thing). For a number x having more than one digit, x can be written as $x = 10y + z$ where y is an integer and z is an nonnegative integer less than 10. Hopefully you can see what is the next step here. You will just have to write this down clearly in words.

Problem 3. Is it true that $10^k - 1$ is divisible by 9 for any positive integer k ? Justify your answer.

Note. Again, the answer is probably quite obvious. After all, in decimal notation, $10^k - 1$ is a number having all k digits exactly 9, i.e., $99\cdots9$ which is obviously divisible by 9. While an explanation along this line would be acceptable, I encourage you to consider some alternative approach since you may find them to be useful in Homework 2.

- Take "$11\cdots1$", i.e., $1 + 10 + 10^2 + \cdots + 10^{k-1}$. Multiply it by 9. ??? expand

  $$(10 - 1)(1 + 10 + 10^2 + \cdots + 10^{k-1}),$$

  what do you get?

- Do you know how to expand $(9 + 1)^k$ using the binomial theorem?

- Suppose you already know how to prove the specific case with $k = 3$, i.e., $10^3 - 1$ is divisible by 9, how will that help you with the case with $k = 4$? In general, how does knowing the case of $k = n$, for some fixed n, help you with the case of $k = n + 1$?