

Homework 3 [Due 9/22]

Problem 1. Prove that for any integer n , $n^2 - n$ is divisible by 2.

Problem 2. Prove that for any integer n , $n^3 - n$ is divisible by 3.

Problem 3. Consider the statement “for any positive integer, $n^4 - n$ is divisible by 4”. Is this a true statement?

1. If you think it is true, then provide a proof.
2. If you think it is not true, provide a number n such that $n^4 - n$ is not divisible by 4. What does the existence of such a number tell you about the original statement (above)?

Problem 4. Prove that for any integer n , $n^5 - n$ is divisible by 5.

Problem 5. Prove that for an integer n , if $3n + 1$ is odd then n is even.

Problem 6. Prove that for an integer n , n and n^3 have the same parity.

Problem 7. Problem 3.47 in the textbook (page 97)

Problem 8. Problem 3.50 in the textbook (page 97)

Problem 9. Problem 3.58 in the textbook (page 98)

Problem 10. Problem 4.10 in the textbook (page 114)

Problem 11. Prove that $n^p \equiv n \pmod{p}$ for $p \in \{2, 3, 5\}$.

Note: Theorems used in your proofs must also be included (with proper references).