Homework 5 [Due 10/01]

Problem 1. Prove that for integers a, b, p, q with p, q both nonzero, if $a \equiv b \pmod{pq}$ then $a \equiv b \pmod{p}$ and $a \equiv b \pmod{q}$.

Problem 2. For two sets A and B, prove that $A \times B = B \times A$ if and only if $A = B$.

Problem 3. Let A and B be two sets in a universal set U. Prove that $A \subseteq B$ if and only if $\bar{B} \subseteq \bar{A}$.

Bonus problems: (You don't have to do these) you can pick one.

Problem 4. (Bonus) Prove that

$$\bigcap_{x \in \mathbb{R}^+} (-x, x) = \{0\}$$

where $\mathbb{R}^+$ is the set of all positive real numbers.

Problem 5. (Bonus) Consider the statement "For any given $L > 0$, there exists a natural number n such that $x_i > L$ for all $i > n$".

- Rewrite the statement using symbols such as $\forall$ and $\exists$.

- Write down the negation of the statement in words.

- Consider the sequence given by $x_i = \log i$ for $i \in \mathbb{N}$. When applied to this sequence, is the above statement true?

Problem 6. Write a computer program (in any programming language) that construct the power set of a given set. (Print the source code on a separate piece of paper)