

Exercise 2.1

Proof.

(i) (epic \Rightarrow surjective)

For epic arrow $f : A \rightarrow B$ we use the proof by contradiction.

Assuming that f is not surjective, then there exists $b \in B$ such that for all $a \in A$, $f(a) \neq b$.

Let $i, j : B \rightarrow D$ such that $i(x) = j(x)$ for all $x \neq b \in B$ and $i(b) \neq j(b)$. Then for all $a \in A$, $if(a) = jf(a)$ but $i \neq j$. Therefore, f is not epic which is a contradiction.

Therefore f is epic implies f is surjective.

(ii) (surjective \Rightarrow epic)

For surjective mapping $f : A \rightarrow B$. If $if = jf$, then $\forall b \in B. \exists a \in A. i(b) = if(a) = jf(a) = j(b)$. Therefore f is also epic. \square