APPENDIX A
DEDUCTION OF THE EQUIVALENT LATENCY INFLUENCE OF THE SPATIAL-TEMPORAL IAQ DATA MEASUREMENT

We collect the massive IAQ data in the student center in Peking University in Jan. 2019 to build the IAQ spatial-temporal model. This data set is a $N \times T$ matrix $\mathcal{U}^0 = [\mu_n^t]_{N\times T}$, the measured IAQ on the N locations through T time slots [**?**].

Temporal deviation: At a certain time slot t, we assume the actual sensing latency (ASL) of a certain location l_n equals τ_n^t ($\tau_n^t \le t$). It implies that, at time slot $t-\tau_n^t$, there is a mobile robot R_m on this location measuring its IAQ value $\mu_m^{t-\tau_n^t}$. During the following τ_n^t time slots, this location l_n keep unmeasured and its IAQ value will gradually deviate from its last measured value. According to the analysis of the collected data set, this temporal deviation approximately follows a normal distribution according to, i.e.,

$$\begin{cases} U_n^t \sim \mathcal{N}(\mu_n^t, \sigma^2(\tau)), & \forall l_n \in \mathcal{L}, t \in [1,T] \\ \mu_n^t \approx \mu_m^{t-\tau}, & \tau = \tau_n^t, \end{cases} \tag{1}$$

where U_n^t is the random variable of IAQ at time slot t on location l_n. μ_n^t is the mean IAQ value of U_n^t, which approximates $\mu_m^{t-\tau}$, the measured value of robot R_m at time slot $t-\tau$. $\sigma^2(\tau)$ is the temporal deviation variance through τ time slots. We utilize $\sigma^2(\tau)$, the temporal change variance to measure the influence of ASL on the uncertainty of the IAQ value. $\sigma^2(\tau)$ is calculated from the collected data set [**?**]:

$$\sigma^2(\tau) = \frac{1}{T-\tau}\sum_{t=1}^{T-\tau}\frac{1}{N}\sum_{l_n\in\mathcal{L}}(\mu_n^{t+\tau}-\mu_n^t)^2. \tag{2}$$

We plot the $\sigma^2(\tau)$ and τ in Fig. 1(a) and fit a curve to find the change of temporal deviation variance as time goes. According to the fitting result in Eqn.(7), $\sigma^2(\tau)$ is the monotonically increasing function of τ, which implies that the longer a location has not been measured, the more uncertain the IAQ value on this location becomes.

$$\sigma^2(\tau) \approx \frac{\tau}{a_1\tau + a_2}, \tag{3}$$

Spatial inference: Considering the spatial inference, the collected data set shows that IAQ difference between the nearby locations also follows a normal distribution,

$$\Delta U_{g'\to g} \sim \mathcal{N}(\mu_{n'\to n}, \sigma^2(d_{n',n})), \tag{4}$$

where $d_{n',n}$ is the Hamilton distance between l_n and $l_{n'}$. $\mu_{n'\to n}$ is defined as the average IAQ difference between location $l_{n'}$ and l_n, and the $\sigma^2(d_{n',n})$ is defined as spatial inference variance. $\sigma^2(d_{n',n})$ can be calculated from collected data set as well:

$$\begin{cases} \mu_{n'\to n} = \frac{1}{T}\sum_{t=1}^{T}(\mu_n^t - \mu_{n'}^t), \\ \sigma^2(d_{n',n}) = \frac{1}{T}\sum_{t=1}^{T}(\mu_{n'}^t + \mu_{n'\to n} - \mu_n^t)^2. \end{cases} \tag{5}$$

We then plot and fit the relation of $\sigma^2(d_{n',n})$ and $d_{n',n}$ in Fig. 1(b). The fitting result in Eqn. shows that the more Hamilton distance between two locations, the high inference variance between them becomes.

$$\sigma^2(d_{n,n'}) \approx a_3(e^{a_4 d_{n,n'}} - 1). \tag{6}$$

To guarantee the accuracy of the spatial inference, we set a threshold $\hat{\sigma}_d^2$, only when $\sigma^2(d_{n',n}) < \hat{\sigma}_d^2$ can the spatial inference between two locations l_n and $l_{n'}$ be adopted. As for each location l_n, we utilize a set $\mathcal{L}_n^d$ to represent all the nearby locations $l_{n'}$ that satisfy $\sigma^2(d_{n',n}) < \hat{\sigma}_d^2$.

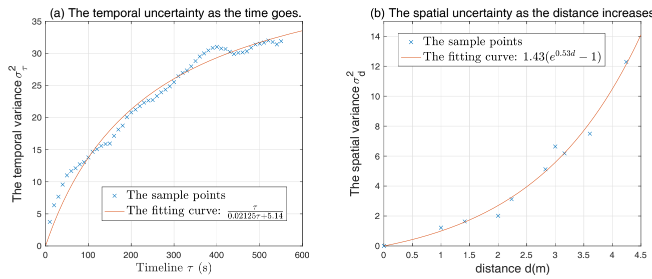

Fig. 1: The fitting of the temporal uncertainty on the time span and the spatial uncertainty on the distance.

The deduction of ESL: To quantify influence of spatial inference, we apply the Kalman Filter to synthesize the result of temporal deviation and spatial inference on location l_n at a certain time slot t:

$$STV(n,t) = 1/\Big(\frac{1}{\sigma^2(\tau_n^t)} + \sum_{l_{n'}\in\mathcal{L}_n^d}\frac{1}{\sigma^2(\tau_{n'}^t)+\sigma^2(d_{n',n})}\Big), \tag{7}$$

where $STV(n,t)$ is the spatial-temporal variance, including the spatial and temporal influence. Obviously, the spatial-temporal variance is lower than the temporal deviation variance, so the spatial inference can reduce the original variance. That is, the effect of spatial inference can improve the reliability of the IAQ value of a certain location and equivalently reduce its sensing latency. Based on the fitting result and the result of Kalman Filter, we can acquire the expression of ESL as follows:

$$ESL(n,t) = 1/\big(\frac{1}{\tau_n^t} + \sum_{l_{n'}\in\mathcal{L}_n^d}\frac{1}{a_2[\sigma^2(d_{n,n'})+\sigma^2(\tau_{n'}^t)]}\big), \tag{8}$$

$$\tau_n^t = ASL(n,t), \qquad \forall l_n \in \mathcal{L}. \tag{9}$$

We can notice that ESL becomes greater in the wake of the increase of ASL and the spatial inference can help to reduce the equivalent sensing latency of each location.