

# Package ‘PanelIFE’

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**Title** Panels with Interactive Fixed Effects

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## Description

Provides tools for estimation and inference for a regression coefficient in panel data with interactive fixed effects (i.e., with a factor structure), and either with strong factors or weak factors.

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RcppDE,  
stats

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**LazyData** true

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empirical\_data

*Empirical Dataset (Divorce Rate in the USA from 1956 to 1988)*


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### Description

Data of the divorce rate in the USA from 1956 to 1988, originally from Friedberg (1998, "Did Unilateral Divorce Raise Divorce Rates? Evidence from Panel Data") and Wolfers (2006, "Did Unilateral Divorce Laws Raise Divorce Rates? A Reconciliation and New Results"). Here, this empirical dataset follows Kim and Oka (2014, "Divorce Law Reforms and Divorce Rates in the USA: An Interactive Fixed-Effects Approach") and use their data to construct a balanced panel with  $N = 48$  states and  $T = 33$  years.

### Usage

```
empirical_data
```

### Format

A data.frame with 1,650 rows and 88 variables:

**st** Two-letter state code

**id\_st** Numeric ID for the states

**year** Year of observation

**div\_rate\_rev01** Divorces per 1000 people, 1956-1998

**div\_rate\_rev02** Divorces per 1000 people, 1956-1998

**stpop** State population

**unilateral** Dummy for unilateral law in Friedberg (1998)

**dyn\_uni2** Dynamic treatment effects (years 1-2): dummy for year of unilateral law in Friedberg (1998)

**dyn\_uni3** Dynamic treatment effects (years 3-4): dummy for year of unilateral law in Friedberg (1998)

**dyn\_uni4** Dynamic treatment effects (years 5-6): dummy for year of unilateral law in Friedberg (1998)

**dyn\_uni5** Dynamic treatment effects (years 7-8): dummy for year of unilateral law in Friedberg (1998)

**dyn\_uni6** Dynamic treatment effects (years 9-10): dummy for year of unilateral law in Friedberg (1998)

**dyn\_uni7** Dynamic treatment effects (years 11-12): dummy for year of unilateral law in Friedberg (1998)

**dyn\_uni8** Dynamic treatment effects (years 13-14): dummy for year of unilateral law in Friedberg (1998)

**dyn\_uni9** Dynamic treatment effects (years 15+): dummy for year of unilateral law in Friedberg (1998)

**dyn\_gruber\_2** Dynamic treatment effects (years 1-2): dummy for year of divorce law reform according to Gruber (2004)

**dyn\_gruber\_3** Dynamic treatment effects (years 3-4): dummy for year of divorce law reform according to Gruber (2004)

- dyn\_gruber\_4** Dynamic treatment effects (years 5-6): dummy for year of divorce law reform according to Gruber (2004)
- dyn\_gruber\_5** Dynamic treatment effects (years 7-8): dummy for year of divorce law reform according to Gruber (2004)
- dyn\_gruber\_6** Dynamic treatment effects (years 9-10): dummy for year of divorce law reform according to Gruber (2004)
- dyn\_gruber\_7** Dynamic treatment effects (years 11-12): dummy for year of divorce law reform according to Gruber (2004)
- dyn\_gruber\_8** Dynamic treatment effects (years 13-14): dummy for year of divorce law reform according to Gruber (2004)
- dyn\_gruber\_9** Dynamic treatment effects (years 15+): dummy for year of divorce law reform according to Gruber (2004)
- dyn\_johnson\_2** Dynamic treatment effects (years 1-2): dummy for year of divorce law reform according to Johnson and Mazingo (2000)
- dyn\_johnson\_3** Dynamic treatment effects (years 3-4): dummy for year of divorce law reform according to Johnson and Mazingo (2000)
- dyn\_johnson\_4** Dynamic treatment effects (years 5-6): dummy for year of divorce law reform according to Johnson and Mazingo (2000)
- dyn\_johnson\_5** Dynamic treatment effects (years 7-8): dummy for year of divorce law reform according to Johnson and Mazingo (2000)
- dyn\_johnson\_6** Dynamic treatment effects (years 9-10): dummy for year of divorce law reform according to Johnson and Mazingo (2000)
- dyn\_johnson\_7** Dynamic treatment effects (years 11-12): dummy for year of divorce law reform according to Johnson and Mazingo (2000)
- dyn\_johnson\_8** Dynamic treatment effects (years 13-14): dummy for year of divorce law reform according to Johnson and Mazingo (2000)
- dyn\_johnson\_9** Dynamic treatment effects (years 15+): dummy for year of divorce law reform according to Johnson and Mazingo (2000)
- dyn\_mechoulan\_2** Dynamic treatment effects (years 1-2): dummy for year of divorce law reform according to Mechoulan (2001)
- dyn\_mechoulan\_3** Dynamic treatment effects (years 3-4): dummy for year of divorce law reform according to Mechoulan (2001)
- dyn\_mechoulan\_4** Dynamic treatment effects (years 5-6): dummy for year of divorce law reform according to Mechoulan (2001)
- dyn\_mechoulan\_5** Dynamic treatment effects (years 7-8): dummy for year of divorce law reform according to Mechoulan (2001)
- dyn\_mechoulan\_6** Dynamic treatment effects (years 9-10): dummy for year of divorce law reform according to Mechoulan (2001)
- dyn\_mechoulan\_7** Dynamic treatment effects (years 11-12): dummy for year of divorce law reform according to Mechoulan (2001)
- dyn\_mechoulan\_8** Dynamic treatment effects (years 13-14): dummy for year of divorce law reform according to Mechoulan (2001)
- dyn\_mechoulan\_9** Dynamic treatment effects (years 15+): dummy for year of divorce law reform according to Mechoulan (2001)
- dyn\_ellmanlohr1\_2** Dynamic treatment effects (years 1-2): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition a)

- dyn\_ellmanlohr1\_3** Dynamic treatment effects (years 3-4): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition a)
- dyn\_ellmanlohr1\_4** Dynamic treatment effects (years 5-6): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition a)
- dyn\_ellmanlohr1\_5** Dynamic treatment effects (years 7-8): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition a)
- dyn\_ellmanlohr1\_6** Dynamic treatment effects (years 9-10): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition a)
- dyn\_ellmanlohr1\_7** Dynamic treatment effects (years 11-12): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition a)
- dyn\_ellmanlohr1\_8** Dynamic treatment effects (years 13-14): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition a)
- dyn\_ellmanlohr1\_9** Dynamic treatment effects (years 15+): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition a)
- dyn\_ellmanlohr2\_2** Dynamic treatment effects (years 1-2): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition b)
- dyn\_ellmanlohr2\_3** Dynamic treatment effects (years 3-4): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition b)
- dyn\_ellmanlohr2\_4** Dynamic treatment effects (years 5-6): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition b)
- dyn\_ellmanlohr2\_5** Dynamic treatment effects (years 7-8): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition b)
- dyn\_ellmanlohr2\_6** Dynamic treatment effects (years 9-10): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition b)
- dyn\_ellmanlohr2\_7** Dynamic treatment effects (years 11-12): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition b)
- dyn\_ellmanlohr2\_8** Dynamic treatment effects (years 13-14): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition b)
- dyn\_ellmanlohr2\_9** Dynamic treatment effects (years 15+): dummy for year of divorce law reform according to Ellman and Lohr (1998) (definition b)
- dyn\_brinigbuckley\_2** Dynamic treatment effects (years 1-2): dummy for year of divorce law reform according to Brinig and Buckley (1998)
- dyn\_brinigbuckley\_3** Dynamic treatment effects (years 3-4): dummy for year of divorce law reform according to Brinig and Buckley (1998)
- dyn\_brinigbuckley\_4** Dynamic treatment effects (years 5-6): dummy for year of divorce law reform according to Brinig and Buckley (1998)
- dyn\_brinigbuckley\_5** Dynamic treatment effects (years 7-8): dummy for year of divorce law reform according to Brinig and Buckley (1998)
- dyn\_brinigbuckley\_6** Dynamic treatment effects (years 9-10): dummy for year of divorce law reform according to Brinig and Buckley (1998)
- dyn\_brinigbuckley\_7** Dynamic treatment effects (years 11-12): dummy for year of divorce law reform according to Brinig and Buckley (1998)
- dyn\_brinigbuckley\_8** Dynamic treatment effects (years 13-14): dummy for year of divorce law reform according to Brinig and Buckley (1998)
- dyn\_brinigbuckley\_9** Dynamic treatment effects (years 15+): dummy for year of divorce law reform according to Brinig and Buckley (1998)

- dyn\_nakonezny\_2** Dynamic treatment effects (years 1-2): dummy for year of divorce law reform according to Nakonezny, Shull and Rodgers (1995)
- dyn\_nakonezny\_3** Dynamic treatment effects (years 3-4): dummy for year of divorce law reform according to Nakonezny, Shull and Rodgers (1995)
- dyn\_nakonezny\_4** Dynamic treatment effects (years 5-6): dummy for year of divorce law reform according to Nakonezny, Shull and Rodgers (1995)
- dyn\_nakonezny\_5** Dynamic treatment effects (years 7-8): dummy for year of divorce law reform according to Nakonezny, Shull and Rodgers (1995)
- dyn\_nakonezny\_6** Dynamic treatment effects (years 9-10): dummy for year of divorce law reform according to Nakonezny, Shull and Rodgers (1995)
- dyn\_nakonezny\_7** Dynamic treatment effects (years 11-12): dummy for year of divorce law reform according to Nakonezny, Shull and Rodgers (1995)
- dyn\_nakonezny\_8** Dynamic treatment effects (years 13-14): dummy for year of divorce law reform according to Nakonezny, Shull and Rodgers (1995)
- dyn\_nakonezny\_9** Dynamic treatment effects (years 15+): dummy for year of divorce law reform according to Nakonezny, Shull and Rodgers (1995)
- divx1** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx2** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx3** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx4** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx5** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx6** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx7** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx8** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx9** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx10** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx11** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx12** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx13** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx14** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx15** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx16** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper
- divx17** Friedberg (1998)'s dummies for coding breaks; see appendix of the paper

## Note

**Examples** section provides replication code for Table 5 and 9 in Armstrong, Weidner, and Zeleneev (2024, "Robust Estimation and Inference in Panels with Interactive Fixed Effects").

## References

For the detail of the data and the construction of the balanced panel, see Friedberg (1998, "Did Unilateral Divorce Raise Divorce Rates? Evidence from Panel Data"), Wolfers (2006, "Did Unilateral Divorce Laws Raise Divorce Rates? A Reconciliation and New Results"), and Kim and Oka (2014, "Divorce Law Reforms and Divorce Rates in the USA: An Interactive Fixed-Effects Approach").

## Examples

```
# Following replication requires some time for computation

## Replication of Table 5 in Armstrong, Weidner, and Zeleneev (2024)
## Data cleaning
df <- PanelIFE::empirical_data[ , -1] # Load data without two-letter state code
df <- df[df$id_st != 16 & df$id_st != 33, ] # Drop IN (id == 16) and NM (id == 33)
XX <- df[ , grep("dyn_uni", colnames(df))]
## ===
T <- max(df$year) - min(df$year) + 1
N <- nrow(df) / T
K <- ncol(XX)
## ===
Y <- matrix(df$div_rate_rev02, nrow = N, ncol = T, byrow = TRUE)
X <- array(0, dim = c(K, N, T))
for(k in 1:K) {
  X[k, , ] <- matrix(XX[ , k], nrow = N, ncol = T, byrow = TRUE)
}
## ===
D <- array(0, dim = c(1, N, T)) # Using one regressor only
D[1, , ] <- apply(X, c(2, 3), sum)
X <- D
K <- 1
## Setting estimation parameters
set.seed(1)
beta0 <- rep(0, K)
alpha <- 0.05 # 1 - confidence level
Rmax <- 6 # Maximum number of the factors
lambda_known <- matrix(rep(1, N)) # Standard time effects
f_known <- cbind(rep(1, T), (1:T), (1:T)^2) # Individual effects + time trend + time trend^2
## Least squares estimation of linear panel data model with interactive fixed effects (LS factors)
LS_summary <- setNames(data.frame(matrix(NA, nrow = Rmax, ncol = 4)),
  c("R", "beta", "CI_LB", "CI_UB"))
for(R in 1:Rmax) {
  res <- ls_factor(Y = Y, X = X, R = R,
    lambda_known = lambda_known, f_known = f_known,
    report = "silent", precision_beta = 10^(-8), method = "m1",
    start = beta0, repMIN = 30, repMAX = 300, M1 = 2, M2 = 2)
  LS_summary$R[R] <- R
  LS_summary$beta[R] <- res$beta - res$bcorr2 - res$bcorr3
  LS_summary$CI_LB[R] <- LS_summary$beta[R] - sqrt(diag(res$Vbeta2)) * qnorm(1 - alpha/2)
  LS_summary$CI_UB[R] <- LS_summary$beta[R] + sqrt(diag(res$Vbeta2)) * qnorm(1 - alpha/2)
}
## Robust estimation and inference in panels with interactive fixed effects (honest weak factors)
Debiased_summary <- setNames(data.frame(matrix(NA, nrow = Rmax * 3, ncol = 5)),
  c("R", "Rw", "beta", "CI_LB", "CI_UB"))
for(R in 1:Rmax) {
  res <- honest_weak_factors(Y = Y, X = X, R = R,
    Gamma_LS = NULL, alpha = 0.05, clustered_se = FALSE,
    lambda_known = lambda_known, f_known = f_known,
    itermax = 75, reltol = 10^(-6))
  Debiased_summary$R[(3*R-2):(3*R)] <- R
  Debiased_summary$Rw[(3*R-2):(3*R)] <- c(0, 1, R)
  Debiased_summary$beta[(3*R-2):(3*R)] <- res$beta
  Debiased_summary$CI_LB[(3*R-2):(3*R)] <- res$LB[c(1:2, R+1), 2]
  Debiased_summary$CI_UB[(3*R-2):(3*R)] <- res$UB[c(1:2, R+1), 2]
```

```

# }
# ## Print results
# print(round(LS_summary, 3), row.names = FALSE)
# print(round(Debiased_summary, 3), row.names = FALSE)

# # Replication of Table 10 in Armstrong, Weidner, and Zeleneev (2024)
# ## Data cleaning
# df <- PanelIFE::empirical_data[ , -1] # Load data without two-letter state code
# df <- df[df$id_st != 16 & df$id_st != 33, ] # Drop IN (id == 16) and NM (id == 33)
# XX <- df[ , grep("dyn_uni", colnames(df))]
# # ===
# T <- max(df$year) - min(df$year) + 1
# N <- nrow(df) / T
# K <- ncol(XX)
# # ===
# Y <- matrix(df$div_rate_rev02, nrow = N, ncol = T, byrow = TRUE)
# X <- array(0, dim = c(K, N, T))
# for(k in 1:K) {
#   X[k, , ] <- matrix(XX[ , k], nrow = N, ncol = T, byrow = TRUE)
# }
# # ===
# D <- array(0, dim = c(4, N, T)) # Using four regressors
# D[1, , ] <- apply(X[1:2, , ], c(2, 3), sum)
# D[2, , ] <- apply(X[3:4, , ], c(2, 3), sum)
# D[3, , ] <- apply(X[5:6, , ], c(2, 3), sum)
# D[4, , ] <- apply(X[7:8, , ], c(2, 3), sum)
# X <- D
# K <- 4
# ## Setting estimation parameters
# set.seed(1)
# beta0 <- rep(0, K)
# alpha <- 0.05 # 1 - confidence level
# Rmax <- 6 # Maximum number of the factors
# lambda_known <- matrix(rep(1, N)) # Standard time effects
# f_known <- cbind(rep(1, T), (1:T), (1:T)^2) # Individual effects + time trend + time trend^2
# ## Least squares estimation of linear panel data model with interactive fixed effects (LS factors)
# LS_summary <- setNames(data.frame(matrix(NA, nrow = Rmax * K, ncol = 5)),
#   c("R", "k", "beta", "CI_LB", "CI_UB"))
# for(R in 1:Rmax) {
#   res <- ls_factor(Y = Y, X = X, R = R,
#     lambda_known = lambda_known, f_known = f_known,
#     report = "silent", precision_beta = 10^(-8), method = "m1",
#     start = beta0, repMIN = 30, repMAX = 300, M1 = 2, M2 = 2)
#   LS_summary$k[(K*R-(K-1)):(K*R)] <- 1:K
#   LS_summary$R[(K*R-(K-1)):(K*R)] <- R
#   LS_summary$beta[(K*R-(K-1)):(K*R)] <- as.numeric(res$beta - res$bcrr2 - res$bcrr3)
#   LS_summary$CI_LB[(K*R-(K-1)):(K*R)] <-
#     LS_summary$beta[(K*R-(K-1)):(K*R)] - sqrt(diag(res$Vbeta2)) * qnorm(1 - alpha/2)
#   LS_summary$CI_UB[(K*R-(K-1)):(K*R)] <-
#     LS_summary$beta[(K*R-(K-1)):(K*R)] + sqrt(diag(res$Vbeta2)) * qnorm(1 - alpha/2)
# }
# ## Robust estimation and inference in panels with interactive fixed effects (honest weak factors)
# Debiased_summary <- setNames(data.frame(matrix(NA, nrow = Rmax * K * 3, ncol = 6)),
#   c("R", "k", "Rw", "beta", "CI_LB", "CI_UB"))
# for(R in 1:Rmax) {
#   res <- honest_weak_factors(Y = Y, X = X, R = R,
#     Gamma_LS = NULL, alpha = 0.05, clustered_se = FALSE,

```

```

#                               lambda_known = lambda_known, f_known = f_known,
#                               itermax = 50, reltol = 10^(-4))
#   Debiased_summary$k[(3*K*R-(3*K-1)):(3*K*R)] <- rep(1:K, each = 3)
#   Debiased_summary$R[(3*K*R-(3*K-1)):(3*K*R)] <- R
#   Debiased_summary$Rw[(3*K*R-(3*K-1)):(3*K*R)] <- rep(c(0, 1, R), K)
#   Debiased_summary$beta[(3*K*R-(3*K-1)):(3*K*R)] <- rep(res$beta, each = 3)
#   Debiased_summary$CI_LB[(3*K*R-(3*K-1)):(3*K*R)] <- c(as.matrix(res$LB[c(1:2, R+1), -1]))
#   Debiased_summary$CI_UB[(3*K*R-(3*K-1)):(3*K*R)] <- c(as.matrix(res$UB[c(1:2, R+1), -1]))
# }
# ## Print results
# print(round(LS_summary, 3), row.names = FALSE)
# print(round(Debiased_summary, 3), row.names = FALSE)

```

---

honest_weak_factors	<i>Robust Estimation and Inference in Panels with Interactive Fixed Effects</i>
---------------------	---

---

## Description

This method considers estimation and inference for a regression coefficient in panels with interactive fixed effects (i.e., with a factor structure). As the previously developed estimators and confidence intervals (CIs) might be heavily biased and size-distorted when some of the factors are weak, this method has estimators with improved rates of convergence and bias-aware CIs that are uniformly valid regardless of whether the factors are strong or not.

## Usage

```

honest_weak_factors(
  Y,
  X,
  R,
  Gamma_LS = NULL,
  alpha = 0.05,
  clustered_se = FALSE,
  lambda_known = NA,
  f_known = NA,
  itermax = 75,
  reltol = 10^(-6)
)

```

## Arguments

Y	$N \times T$ matrix of outcomes
X	$K \times N \times T$ tensor of regressors
R	A positive integer, indicates the number of interactive fixed effects in the estimation. Note that this number does not include the known factors and loadings defined below
Gamma_LS	(Optional) A preliminary LS estimate of the matrix of fixed effects; it will be computed if not provided



alpha	(Optional) It determines the $1 - \alpha$ coverage of the constructed confidence interval, where default is set to $\alpha = 0.05$
clustered_se	(Optional) Whether or not performing clustered standard error, where default is set to <code>clustered_se = FALSE</code>
lambda_known	(Optional) $N \times R$ matrix of known factor loadings, e.g., <code>lambda_known = matrix(rep(1, N), nrow = N, ncol = 1)</code> to control standard time dummies. Default is set to <code>lambda_known = NA</code> (or equivalently <code>matrix(NA, nrow = N, ncol = 0)</code> ), i.e., there is no known factor loadings
f_known	(Optional) $T \times R$ matrix of known factor, e.g., <code>f_known = matrix(rep(1, T), nrow = T, ncol = 1)</code> to control standard individual specific fixed effects. Default is set to <code>f_known = NA</code> (or equivalently <code>matrix(NA, nrow = T, ncol = 0)</code> ), i.e., there is no known factor
itermax	(Optional) Maximum iteration allowed while optimizing for the weights $A$ , where default is set to <code>itermax = 50</code> for faster computation
reltol	(Optional) Relative convergence tolerance for the optimization algorithm to stop if it is unable to reduce the value by <code>reltol * (abs(val) + reltol)</code> after <code>0.75 * itermax</code> steps. , where default is set to <code>reltol = 10^(-4)</code> for faster computation

## Details

**Disclaimer:** This function is the implementation of Armstrong, Weidner, and Zelenev (2024, "Robust Estimation and Inference in Panels with Interactive Fixed Effects"). This code is offered with no guarantees. Not all features of this code were properly tested. Please let me know if you find any bugs or encounter any problems while using this code. All feedback is appreciated.

### Linear panel regression model with weak factors:

- We consider a linear panel regression model of the form

$$Y_{it} = X_{it}\beta + \sum_{k=1}^K Z_{k,it}\delta_k + \Gamma_{it} + U_{it}$$

where

- $Y_{it}$ ,  $X_{i,t}$ , and  $Z_{k,it}$  are the observed outcome variable and covariates,
- $\Gamma_{it}$  is the error component that can be correlated with  $X_{it}$  and  $Z_{k,it}$ ,
- $U_{it}$  is the error component modelled as a mean-zero random shock,
- and large panel is considered, where both  $N$  and  $T$  are relatively large.
- Model for  $\Gamma_{it}$  is referred to as a factor model with factor loadings  $\lambda_{ir}$  and factors  $f_{tr}$ , with  $R$  being the number of factors.
- Other than having some strong factors, which requires  $\lambda_{ir}$  and  $f_{tr}$  to have sufficient variation across  $i$  and over  $t$ , model here allows weak factors.
- Proposed method provides the bias-aware confidence intervals that are uniformly valid regardless of whether the factors are strong or not.

### Debiasing approach:

- Access the preliminary estimate  $\hat{\Gamma}_{pre}$  along with a bound  $\hat{C}$  on the nuclear norm  $\|\Gamma - \hat{\Gamma}_{pre}\|_*$ .
- Then consider the regression with the augmented outcomes  $\tilde{Y}_{it} := Y_{it} - \hat{\Gamma}_{pre,it}$ .

- Construct the confidence interval with the preliminary estimate  $\hat{\Gamma}_{pre}$  and bound  $\hat{C}$  on the nuclear norm of its estimation error. Such confidence interval is bias-aware, that is, using the bound  $\hat{C}$  will take any remaining bias into account after the previous debias procedure.
- For detailed implementation, please refer to Armstrong, Weidner, and Zeleneev (2024, "Robust Estimation and Inference in Panels with Interactive Fixed Effects").

**Value**

A list of results, where

- beta is the point estimate
- bias is the worst-case bias
- LB are the lower bounds of the 1 - alpha confidence intervals, from number of weak factors being 0 to R
- UB are the upper bounds of the 1 - alpha confidence intervals, from number of weak factors being 0 to R
- A is the matrix of weights
- parameter is the list of input parameters, including Gamma\_LS, alpha, and clustered\_se

**Note**

We assume that all provided input parameters have values and dimensions as described above.

**References**

For a description of the model see Armstrong, Weidner, and Zeleneev (2024, "Robust Estimation and Inference in Panels with Interactive Fixed Effects").

**Examples**

```
dt <- sample_data(N = 100, T = 20, R = 1, kappa = c(0.5))
res <- honest_weak_factors(Y = dt$Y, X = dt$X, R = dt$R,
                           Gamma_LS = NULL, alpha = 0.05)
sum_res <- summary(res)
print(sum_res)
```

---

ls_factor	<i>Least Squares Estimation of Linear Panel Data Models with Interactive Fixed Effects</i>
-----------	--

---

**Description**

This function estimates least squares estimator in a linear panel regression model with factors appearing as interactive fixed effects.

**Usage**

```
ls_factor(
  Y,
  X,
  R,
  lambda_known = NA,
  f_known = NA,
  report = "report",
  precision_beta = 10^(-8),
  method = "m1",
  start,
  repMIN,
  repMAX,
  M1 = 1,
  M2 = 0,
  DoF_adj = FALSE
)
```

**Arguments**

Y	$N \times T$ matrix of outcomes, where we assume a balanced panel, i.e. all elements of Y are known
X	$K \times N \times T$ tensor of regressors, where we assume a balanced panel, i.e. all elements of X are known
R	A positive integer, indicates the number of interactive fixed effects in the estimation; this R does not include the number of known factors and loadings
lambda_known	(Optional) $N \times R \times 1$ matrix of known factor loadings, e.g., <code>lambda_known = matrix(rep(1, N), nrow = N, ncol = 1)</code> to control standard time dummies. Default is set to <code>lambda_known = matrix(NA, nrow = N, ncol = 0)</code> , i.e., there is no known factor loadings
f_known	(Optional) $T \times R \times 2$ matrix of known factor, e.g., <code>f_known = matrix(rep(1, T), nrow = T, ncol = 1)</code> to control standard individual specific fixed effects. Default is set to <code>f_known = matrix(NA, nrow = T, ncol = 0)</code> , i.e., there is no known factor
report	(Optional) Whether or not to report the progress. "silent" has the program running silently; "report" has the program reporting what it is doing
precision_beta	(Optional) Defines stopping criteria for numerical optimization, namely optimization is stopped when difference in beta relative to previous optimization step is smaller than "precision_beta" (uniformly over all $K$ components of beta). Note that the actual precision in beta will typically be lower than precision_beta, depending on the convergence rate of the procedure.
method	(Optional) Optimization method option of choice. Options include "m1" and "m2"
start	(Optional) $K \times 1$ vector, first starting value for numerical optimization
repMIN	(Optional) Minimal number, which is a positive integer, of runs of optimization with different starting point
repMAX	(Optional) Maximal number, which is a positive integer, of runs of optimization (in case numerical optimization doesn't terminate properly, we do multiple runs even for repMIN = 1)

M1	(Optional) A positive integer, bandwidth for bias correction for dynamic bias (bcorr1), M1 is the number of lags of correlation between regressors and errors that is corrected for in dynamic bias correction
M2	(Optional) A non-negative integer, bandwidth for bias correction for time-serial correlation (bcorr3), M2 = 0 only corrects for time-series heteroscedasticity, while M2 > 0 corrects for time-correlation in errors up to lag M2
DoF_adj	(Optional) Whether or not to adjust for degree of freedom, where default is set to DoF_adj = FALSE

## Details

**Disclaimer:** This function is translated and modified from the Matlab function `LS_factor.m` by Martin Weidner, and the documentation details are also mainly from the original function. This code is offered with no guarantees. Not all features of this code were properly tested. Please let me know if you find any bugs or encounter any problems while using this code. All feedback is appreciated.

### Different computational methods:

- Method 1 (recommended default method) iterates the following two steps until convergence:
  - Step 1: forgiven beta compute update for lambda and f as principal components of  $Y - \text{beta} * X$  (same as in method 1)
  - Step 2: forgiven lambda and f run a pooled OLS regression of  $Y - \text{lambda} * t(f)$  on X to update beta
- Method 2 (described in Bai, 2009) iterates the following two steps until convergence:
  - Step 1: forgiven beta compute update for lambda and f as principal components of  $Y - \text{beta} * X$  (same as in method 1)
  - Step 2: forgiven lambda and f run a pooled OLS regression of  $Y - \text{lambda} * t(f)$  on X to update beta

The procedure is repeated multiple times with different starting values.

### Comments:

- Another method would be to use step 1 as in Method 1 & 2, but to replace step 2 with a regression of Y on either  $M_{\text{lambda}} * X$  or  $X * M_f$ , i.e. to only project out either lambda or f in the step 2 regression. Bai (2009) mentions this method and refers to Ahn, Lee, and Schmidt (2001), Kiefer (1980) and Sargan (1964) for this. We have not tested this alternative method, but we suspect that Method 1 performs better in terms of speed of convergence.
- This alternative method and the method proposed by Bai (2009), i.e. "method 2" here, have the property of reducing the LS objective function in each step. This is not true for Method 1 and may be seen as a disadvantage of Method 1. However, we found this to be a nice feature, because we use this property of Method 1 as a stopping rule: if the LS objective function does not improve, then we know we are "far away" from a proper minimum, so we stop the iteration and begin the iteration with another randomly chosen starting value. Note that multiple runs with different starting values are required anyways for all methods (because the LS objective function may have multiple local minimal).
- We recommend method 1, because each iteration step is fast and its rate of convergence in our tests was very good (faster than method 2). However, we have not much explored the relative sensitivity of the different methods towards the choice of starting value. Note that by choosing the quickest method (method 1) one can try out more different starting values of the procedure in the same amount of time. Nevertheless, it may well be that method 2 or the alternative method described above perform better in certain situations.

**Value**

A list of results, where

- beta is the parameter estimate
- exitflag = 1 if iteration algorithm properly converged at optimal beta, and exitflag = -1 if iteration algorithm did not properly converge at optimal beta
- lambda is the estimate for factor loading
- f is the estimate for factors
- Vbeta1, 2, 3 are estimated variance-covariance matrices of beta, assuming
  1. homoscedasticity of errors in both dimensions
  2. heteroscedasticity of errors in both dimensions
  3. allowing for time-serial correlation up to lag M2 (i.e. if M2 == 0, then Vbeta2 == Vbeta3)
- bcorr1, 2, 3 are estimates for the three different bias components (needs to be subtracted from beta to correct for the bias), where
  1. is bias due to pre-determined regressors
  2. is bias due to cross-sectional heteroscedasticity of errors
  3. is bias due to time-serial heteroscedasticity and time-serial correlation of errors
- parameter is the list of input parameters, including Gamma\_LS, alpha, and clustered\_se

**Note**

We assume that all provided input parameters have values and dimensions as described above. The program could be improved by checking that this is indeed the case.

**References**

For a description of the model and the least squares estimator see e.g. Bai (2009, "Panel data models with interactive fixed effects"), or Moon and Weidner (2017, "Dynamic Linear Panel Regression Models with Interactive Fixed Effects"; 2015, "Linear Regression for Panel with Unknown Number of Factors as Interactive Fixed Effects")

**Examples**

```
dt <- sample_data(N = 100, T = 20, R = 1, kappa = c(0.5))
res <- ls_factor(Y = dt$Y, X = dt$X, R = dt$R, report = "silent",
                precision_beta = 10^(-8), method = "m1",
                start = c(0), repMIN = 3, repMAX = 10, M1 = 2, M2 = 2)
sum_res <- summary(res)
print(sum_res)
```

sample\_data

*Generate Sample Data with Interactive Fixed Effects***Description**

This function generates sample data with interactive fixed effects.

**Usage**

```
sample_data(N = 100, T = 20, R = 1, kappa = c(0.5))
```

**Arguments**

N	Number of total individuals.
T	Number of total time periods.
R	Number of factors.
kappa	Strength of each factor.

**Details**

The design follows that data generating process of the simulation in Armstrong, Weidner, and Zeileenev (2024, "Robust Estimation and Inference in Panels with Interactive Fixed Effects").

$$Y_{i,t} = X_{i,t}\beta + \sum_{r=1}^R \kappa_r \lambda_{i,r} f_{t,r} + U_{i,t}$$

$$X_{i,t} = \sum_{r=1}^R \lambda_{i,r} f_{t,r} + V_{i,t}$$

where  $\kappa_r$  controls the strength of factor  $f_{t,r}$ , and  $R$  is the number of factors. The factors, loadings, and error terms follows the distributions of:

$$\lambda_i \sim N(0, I_R) \perp f_t \sim N(0, I_R) \perp \begin{pmatrix} U_{i,t} \\ V_{i,t} \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_U^2 & 0 \\ 0 & \sigma_V^2 \end{pmatrix} \right).$$

We consider the setting of  $(\beta, \sigma_U^2, \sigma_V^2) = (0, 1, 1)$ , various of  $N, T$ , and different strength of factors  $\kappa_r \in [0, 1]$ . In this simple simulation data, we only consider  $R \in \{1, 2\}$ .

**Value**

A list of simple simulation data, where

- Y is the  $N \times T$  matrix of outcomes.
- X is the  $K \times N \times T$  tensor of regressors.
- R is the number of total individuals.
- N is the number of total individuals.
- T is the number of total time periods.
- kappa is the vector of strength factors.

---

summary.honest\_weak\_factors

*Summarizing Robust Estimation and Inference in Panels with Interactive Fixed Effects*


---

## Description

summary method for object of class honest\_weak\_factors and returned object is of class summary.honest\_weak\_factors

## Usage

```
## S3 method for class 'honest_weak_factors'
summary(object, ...)

## S3 method for class 'summary.honest_weak_factors'
print(x, digits = 4, labels = TRUE, ...)
```

## Arguments

object	The honest_weak_factors fitted result of class honest_weak_factors.
...	Confidence level alpha can be set for computing confidence intervals. Default is set to 0.05.
x	The summarized result of class summary.ls_factor.
digits	Number of digits to use when printing. Default is set to 4.
labels	Option of whether or not to print labels in the summary. Default is set to TRUE. Only parameters estimates will be printed when it's set to FALSE.

## Details

Summarizing the fitted honest\_weak\_factors object and computing

- Debiased parameter estimate.
- Standard error.
- Worst-case bias of the estimator.
- With and without bias-aware confidence intervals.

## Value

A list of results, where

- beta is the debiased parameter estimate.
- se is the standard errors of the parameters estimator.
- bias is the worst-case bias of the estimator.
- CI is the bias-aware confidence interval.
- CI\_unadj is the confidence interval without bias adjustment.
- A is the choice of weight matrix.

---

summary.ls_factor	<i>Summarizing Least Squares Estimation of Linear Panel Data Models with Interactive Fixed Effects</i>
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---

## Description

summary method for object of class ls\_factor and returned object is of class summary.ls\_factor

## Usage

```
## S3 method for class 'ls_factor'
summary(object, ...)

## S3 method for class 'summary.ls_factor'
print(x, digits = 4, labels = TRUE, ...)
```

## Arguments

object	The ls_factor fitted result of class ls_factor.
...	Confidence level alpha can be set for computing confidence intervals. Default is set to 0.05.
x	The summarized result of class summary.ls_factor.
digits	Number of digits to use when printing. Default is set to 4.
labels	Option of whether or not to print labels in the summary. Default is set to TRUE. Only parameters estimates will be printed when it's set to FALSE.

## Details

Summarizing the fitted ls\_factor object and computing

- Estimates under different bias correction schemes.
- Variance-covariance matrix under different assumptions.
- Confidence intervals and their lengths under different settings.

## Value

A list of results, where

- beta is the table of parameters estimates under different bias correction schemes.
- CI is the table of confidence intervals and their lengths under different settings.
- var\_cov are the variance-covariance matrices under different assumptions.



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