Automated Calibration of Experimental Parameters in Trapped Ion Quantum Computing

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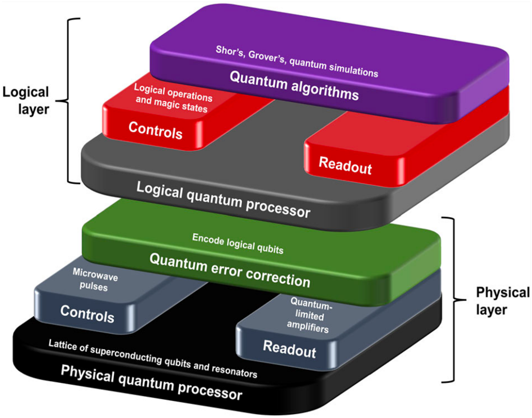
Quantum Computing Architecture

How to build a Quantum Computer?



- **Superconducting** quantum computing
- **Photonic** quantum computing
- **quantum dots** quantum computing
- **Trapped-ion** quantum computing

Comparing the Characteristics of different types of QC[1]

Superconducting QC

Pros

- Fast working
- Build on existing semiconductor industry

Cons

- Collapse easily and must be kept in cold

Photonics QC

Pros

- CMOS compatible photonics waveguide technology

Cons

- Cryogenic single photon sources and detectors

Trapped-Ion QC

Pros

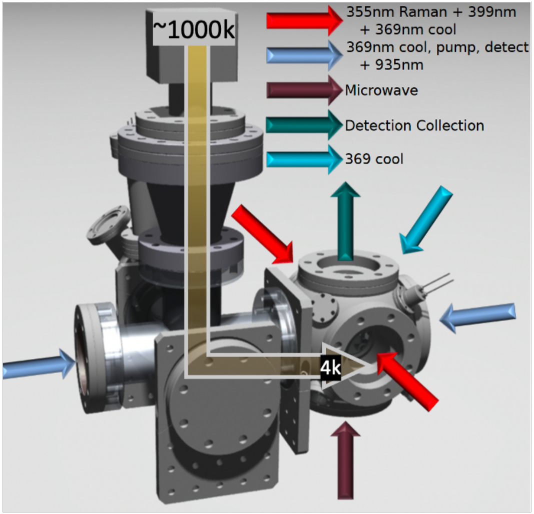
- Very stable $> 1000s$
- Highest achieved gate fidelities (e.g. two-qubit gate 99.9%)
- Fully connected

Cons

- Scalability
- Slow operation

[1] Bruzewicz et al. 2019

Quantum Computing with Trapped Ions



Ions → Trapping ions → Initialization → Controls → Readout

Vibration Modes

Canonical Hamiltonian

$$\hat{H} = \sum_{j=1}^{N} \left(\frac{1}{2m} \hat{p}_j^2 + \frac{m}{2} \omega_m^2 \hat{q}_j^2 \right)$$

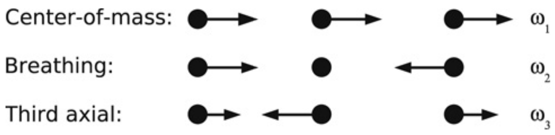
Quantization

$$\hat{q} = \sqrt{\frac{\hbar}{2m\omega}} \left(\hat{a} + \hat{a}^\dagger \right)$$

$$\hat{p} = -i \sqrt{\frac{\hbar m \omega}{2}} \left(\hat{a} + \hat{a}^\dagger \right)$$

Kinetic term

$$\hat{H}_m = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) \sim \hbar\omega \hat{a}^\dagger \hat{a}$$



When N oscillators are coupled, N different canonical modes are obtained, which correspond to **phonons of N different frequencies.**

Two-level Approximation and Raman Process

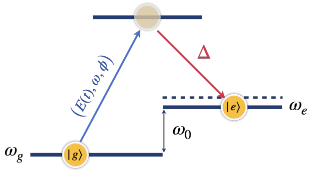


Figure: Internal energy level transition of ions

- Two-level approximation for electronic energy levels

$$\hat{H}_e = \hbar\left(\omega_g |g\rangle\langle g| + \omega_e |e\rangle\langle e|\right) \sim \hbar\frac{\omega_0}{2}\hat{\sigma}_z$$

- Interaction term

$$\hat{H}_{int} = \frac{\hbar}{2}\Omega\left(|e\rangle\langle g| + |g\rangle\langle e|\right)\left[\mathrm{e}^{i(kx-\omega t+\phi)} + \mathrm{e}^{-i(kx-\omega t+\phi)}\right]$$

Why is it necessary to go through a 'bridge'?

1. It is difficult to produce a laser at the GHz level.
2. There may be an electric dipole forbidden transition between the $|g\rangle$ and $|e\rangle$ states.
3. Many factors, like laser intensity, phase, and frequency, can affect the direct transition from $|g\rangle$ to $|e\rangle$, leading to operational inaccuracies.

The Basic Hamiltonian

1. Trans to interaction picture

$$H'_{int} = e^{iH_0 t/\hbar} H_{int} e^{-iH_0 t/\hbar}$$

2. $kx = kx_0 \left(\hat{a} + \hat{a}^\dagger\right) = \eta \left(\hat{a} + \hat{a}^\dagger\right)$
3. Rotating wave approximation
4. Lamb-Dicke approximations $\eta\sqrt{\langle(\hat{a}^\dagger + \hat{a})^2\rangle} \ll 1$

Lamb-Dicke Parameter η

$$\eta = k_x x_0 = k_x \sqrt{\langle 0|\, x^2 \,|0\rangle} = k_x \sqrt{\frac{\hbar}{2m\omega_x}}$$

η quantifies the coupling strength between the internal and motional states of the ion.

The Basic Hamiltonian

$$H \approx \frac{\hbar}{2}\hat{\sigma}_+\left[1 + i\eta\left(\hat{a}^\dagger e^{i\omega_m t} + \hat{a}e^{-i\omega_m t}\right)\right]e^{i(\phi-\mu t)} + \text{h.c.}$$

- Carrier transition $|g\rangle|n\rangle \leftrightarrow |e\rangle|n\rangle$ $(\mu = 0)$

$$H_{\text{car}} = \frac{\hbar}{2}\Omega\left(\hat{\sigma}_+e^{i\phi} + \hat{\sigma}_-e^{-i\phi}\right)$$

- Blue sideband transition $|g\rangle|n\rangle \leftrightarrow |e\rangle|n+1\rangle$ $(\mu = \omega_m + \delta)$

$$H_{\text{bsb}} = i\eta\frac{\hbar}{2}\Omega\left(\hat{\sigma}_+\hat{a}^\dagger e^{i\phi}e^{-i\delta t} + \hat{\sigma}_-\hat{a}e^{-i\phi}e^{i\delta t}\right)$$

- Red sideband transition $|g\rangle|n\rangle \leftrightarrow |e\rangle|n-1\rangle$ $(\mu = -\omega_m - \delta)$

$$H_{\text{rsb}} = i\eta\frac{\hbar}{2}\Omega\left(\hat{\sigma}_+\hat{a}e^{i\phi}e^{i\delta t} - \hat{\sigma}_-\hat{a}^\dagger e^{-i\phi}e^{-i\delta t}\right)$$

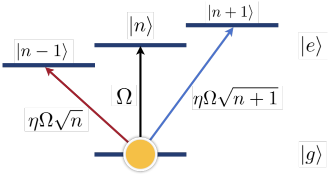


Figure: Transitions

Single-Qubit Gates

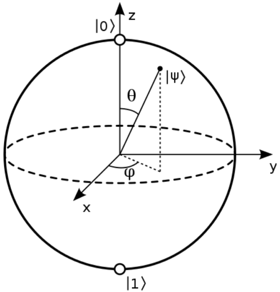


Figure: Bloch sphere

Carrier transition

$$H_{\text{car}} = \frac{\hbar}{2}\Omega\left(\hat{\sigma}_+ e^{i\varphi} + \hat{\sigma}_- e^{-i\varphi}\right)$$

The system can realize a rotation of angle $\theta/2 = \Omega t/2$ around any axis (φ) in the x-y plane under the action of H_{car} after free evolution for a time t.

$$R(\theta,\varphi) = e^{-iH_{\text{car}}t/\hbar} = \begin{pmatrix} \cos\frac{\theta}{2} & ie^{i\varphi}\sin\frac{\theta}{2} \\ ie^{-i\varphi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

- $R_x(\theta) = R(\theta,\varphi=0) = \begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$
- $R_y(\theta) = R(\theta,\varphi=\pi/2) = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$

Two-Qubit Gates

Cirac-Zoller Gate[2]

Red sideband transition

$$H_{\mathrm{rsb}} = i\eta\frac{\hbar}{2}\Omega\left(\hat{\sigma}_{+}\hat{a}e^{i\phi}e^{i\delta t} - \hat{\sigma}_{-}\hat{a}^{\dagger}e^{-i\phi}e^{-i\delta t}\right)$$

1. Apply a π-pulse on ion A
2. Apply a 2π-pulse on ion B
3. Apply a π-pulse on ion A again

$$\begin{array}{ccccccc}
 & (1) & & (2) & & (3) & \\
|gg0\rangle & \rightarrow & |gg0\rangle & \rightarrow & |gg0\rangle & \rightarrow & |gg0\rangle \\
|ge0\rangle & \rightarrow & |ge0\rangle & \rightarrow & |ge0\rangle & \rightarrow & |ge0\rangle \\
|eg0\rangle & \rightarrow & -i\,|gg1\rangle & \rightarrow & i\,|gg1\rangle & \rightarrow & |eg0\rangle \\
|ee0\rangle & \rightarrow & -i\,|ge1\rangle & \rightarrow & -i\,|ge1\rangle & \rightarrow & -\,|ee0\rangle
\end{array}$$

$$|\pm\rangle = (|g\rangle \pm |e\rangle)/\sqrt{2} \quad \Rightarrow \quad \begin{array}{ccc} |g\rangle\,|\pm\rangle & \rightarrow & |g\rangle\,|\pm\rangle \\ |e\rangle\,|\pm\rangle & \rightarrow & |e\rangle\,|\mp\rangle \end{array} \quad \Rightarrow$$

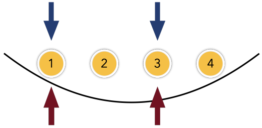
Controlled-Z Gate!

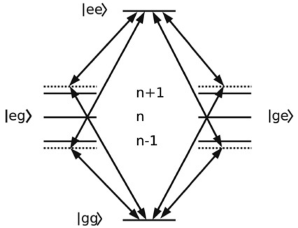
[2] Cirac and Zoller 1995

Two-Qubit Gates

Mølmer-Sørensen Gate

By simultaneously applying a two-frequency laser beam $\omega_0 \pm (\omega_m + \delta)$ to two ions separately, it is possible to drive the red sideband transition and the blue sideband transition of the ions at the same time.





- Red sideband transition

$$H_{rsb} = i\eta\frac{\hbar}{2}\Omega\left(\hat{\sigma}_+\hat{a}e^{i\phi}e^{i\delta t} - \hat{\sigma}_-\hat{a}^\dagger e^{-i\phi}e^{-i\delta t}\right)$$

- Blue sideband transition

$$H_{bsb} = i\eta\frac{\hbar}{2}\Omega\left(\hat{\sigma}_+\hat{a}^\dagger e^{i\phi}e^{-i\delta t} + \hat{\sigma}_-\hat{a}e^{-i\phi}e^{i\delta t}\right)$$

$$H_{MS} = \sum_{j=1}^{2}\left(H_{rsb} + H_{bsb}\right)$$

Two-Qubit Gates

Mølmer-Sørensen Gate

$$U\left(t_g\right) = \exp\left[-i\sum_j^M \hat{\sigma}_x^j \sum_{k=1}^N \left(\alpha_{j,k}\left(t_g\right)\hat{a}_k^\dagger - \alpha_{j,k}^*\left(t_g\right)\hat{a}_k\right) + i\sum_{m<n}^M \chi_{m,n}\left(t_g\right)\hat{\sigma}_x^m \hat{\sigma}_x^n\right]$$

where

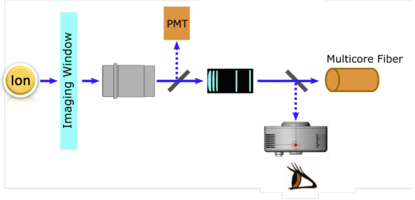
$$\alpha_{j,k}\left(t_g\right) = \int_0^{t_g} dt f\left(\eta_{k,j}, \Omega_j(t), \omega_k\right), \; \chi_{m,n}\left(t_g\right) = \sum_k \int_0^{t_g} dt \int_0^t dt_1 g\left(\eta_{k,j}, \Omega_m(t), \Omega_n\left(t_1\right), \omega_k\right)$$

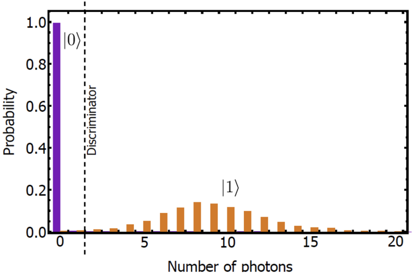
When $\alpha_{j,k}(t_g) \to 0$ and $\chi_{m,n}(t_g) \to \pi/4$, the **maximum entangled state** is obtained:

$$\begin{aligned} |gg\rangle &\rightarrow (|gg\rangle + i\,|ee\rangle)/\sqrt{2} \\ |ee\rangle &\rightarrow (|ee\rangle + i\,|gg\rangle)/\sqrt{2} \\ |ge\rangle &\rightarrow (|ge\rangle - i\,|eg\rangle)/\sqrt{2} \\ |eg\rangle &\rightarrow (|eg\rangle - i\,|ge\rangle)/\sqrt{2} \end{aligned}$$

Readout

Resonance Fluorescence Method





- Fluorescence detected: $|1\rangle$
- Fluorescence NOT detected: $|0\rangle$

- Measurement readout fidelity $> 99.999\%$
- Single measurement time $\sim 150\mu s$

Calibration

Motivation

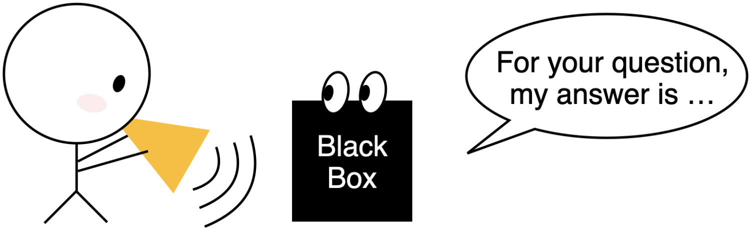
Factors limiting the fidelity (physical layer)

1. Decoherence
2. Preparation error
3. **Imprecise of experimental parameters**
   (e.g. phonon frequency ω_k, Lamb-Dicke parameter η, $\cdots$)

General Calibration Schemes for Ion Trap Systems

1. Insufficient automation
2. Low efficiency
3. Poor accuracy

Calibration



$$H = \sum_{j,k} H_{j,k} = \hbar \sum_{j} \frac{\Omega_j}{2} \hat{\sigma}_j^{+} \mathrm{e}^{i(\phi_j - \mu_j t)} \sum_{k} \exp\left[i\eta_{j,k} \left(\hat{a}_k^{\dagger} \mathrm{e}^{i\omega_k t} + \hat{a}_k \mathrm{e}^{-i\omega_k t} \right) \right] + \text{h.c.}$$

1. Calibrating the phonon frequencies ω_k
2. Calibrating the Lamb-Dicke parameters $\eta_{j,k}$

Calibration of Phonon Frequency

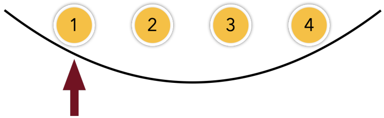
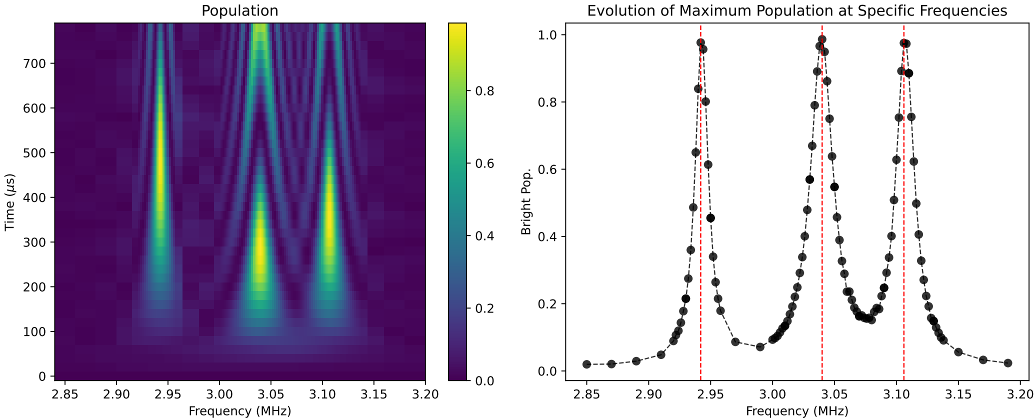


Figure: Calibration of phonon frequencies

Changing the laser frequency and recording the evolution of the ions at different frequencies.

Calibration of Phonon Frequency



Calibration of Phonon Frequency

Example: 3 ions with 3 phonons.

	Phonon frequency	Calibrated Phonon frequency	Error
ω_1	2.9574 MHz	2.9574 MHz	-0.0154 MHz
ω_2	3.0542 MHz	3.040 MHz	-0.0142 MHz
ω_3	3.1222 MHz	3.106 MHz	-0.0162 MHz

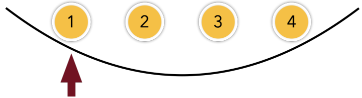
Error Sources? **Stark shift**

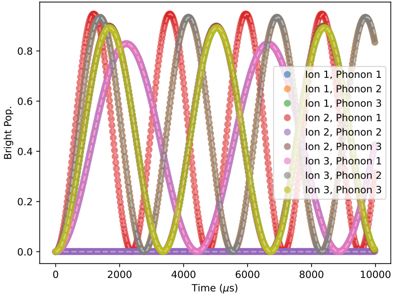
Rabi frequency v.s. **Resonance frequency**

Baidu Research

Calibration of Lamb-Dicke Parameters

Simulation





Calculation

$$P(\Omega, \eta, \Delta, t) = \frac{\eta^2\Omega^2}{\eta^2\Omega^2 + \Delta^2/4} \sin^2\left(\sqrt{\eta^2\Omega^2 + \frac{\Delta^2}{4}}\, t\right)$$

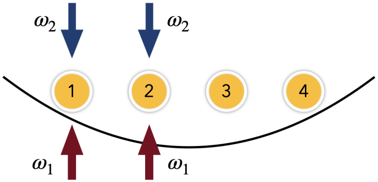
Example: 3 ions with 3 phonons.

	$\eta_{j,k}$	Calibrated $\lvert\eta_{j,k}\rvert$	Error
η_{11}	-0.0457	0.0457	0.00%
η_{12}	0.0776	0.0774	-0.26%
η_{13}	0.0625	0.0625	0.00%
η_{21}	0.0909	0.0908	-0.11%
η_{22}	-2.77×10^{-6}	0.0	-
η_{23}	0.0629	0.0629	0.00%
η_{31}	-0.0457	0.0457	0.00%
η_{32}	-0.0776	0.0774	-0.26%
η_{33}	0.0625	0.0625	0.00%

Calibration of Lamb-Dicke Parameters

Determine the sign of $\eta_{j,k}$

Coupling the ions with different phonon modes



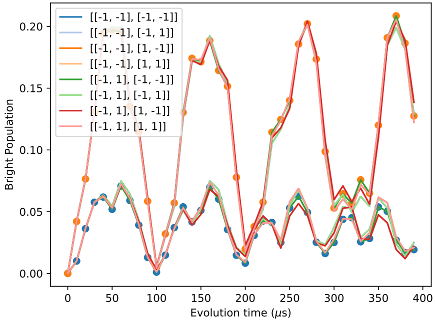


Figure: Example: 2 ions with 2 phonons

Complexity: 2^{N^2-1}

Segmented calibration $\rightarrow$ Complexity: $2N^2$

Summary

Start

↓

Select one of the ions, continuously sweep its frequency, record the changes in the number of bright states during the evolution process, and obtain the phonon frequency. → ω_k

↓

Traverse each ion, set the laser frequency to the phonon frequency, scan separately, record the changes in the number of bright states during the evolution process, and fit out the absolute Lamb-Dicke parameters. → $|\eta_{j,k}|$

↓

Select any two ions, irradiate them simultaneously with two beams of bi-frequency lasers, record the changes in the number of bright states of the two ions during the evolution process, and obtain the sign of the Lamb-Dicke parameters. → $\eta_{j,k}$

↓

End

Thank you!

References I

- Bruzewicz, Colin D. et al. (June 2019). “Trapped-ion quantum computing: Progress and challenges”. en. In: *Applied Physics Reviews* 6.2, p. 021314. ISSN: 1931-9401. DOI: 10.1063/1.5088164.
- Cirac, J. I. and P. Zoller (May 1995). “Quantum Computations with Cold Trapped Ions”. In: *Physical Review Letters* 74.20. Publisher: American Physical Society, pp. 4091–4094. DOI: 10.1103/PhysRevLett.74.4091.