Ion Trap Quantum Computing:
From Theory to Reality

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29 April, 2023

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Outline

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- Why compute with qubits?
- Why use ion qubits?
- How to build a Quantum Computer - Quantum Computer Hardware

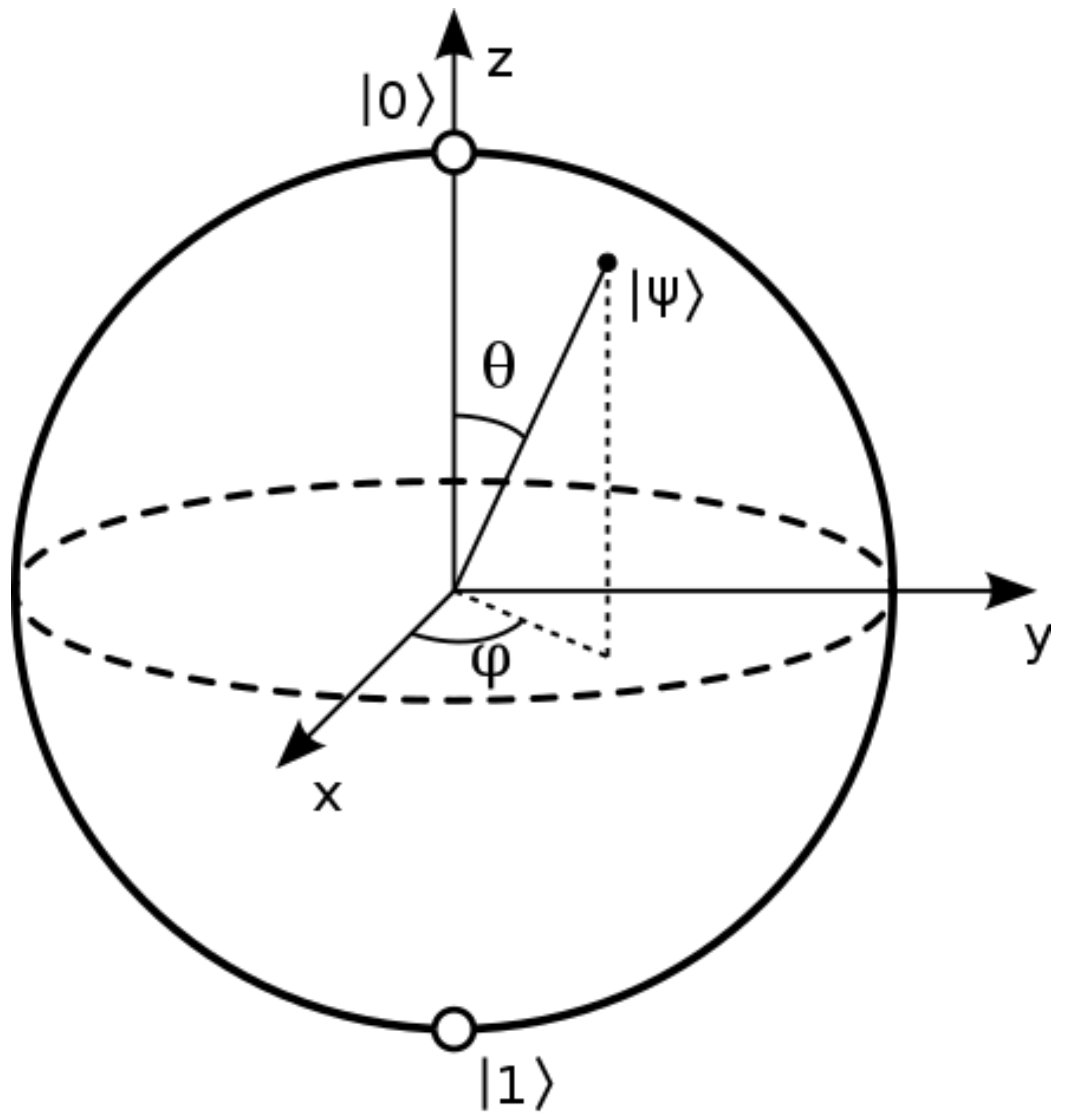
Ions → Trapping Ions → Initialization → Compute With Ions → Readout

- Quantum gates

❗ Mølmer–Sørensen gate

Why compute with qubits ?

Quantum Rules



#1 Superposition

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \qquad |\alpha|^2 + |\beta|^2 = 1$$

$|0\rangle$ or $|1\rangle$

Probability $|\alpha|^2$ $|\beta|^2$

#2 Coherence

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

#3 Entanglement

Why compute with qubits ?

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GOOD NEWS…

quantum parallel processing on 2^N inputs

Example: $N = 3$

$$|\psi\rangle = a_0 |000\rangle + a_1 |001\rangle + a_2 |010\rangle + a_3 |011\rangle + a_4 |100\rangle + a_5 |101\rangle + a_6 |110\rangle + a_7 |111\rangle$$

$N = 300$ qubits: more information than particles in the universe!

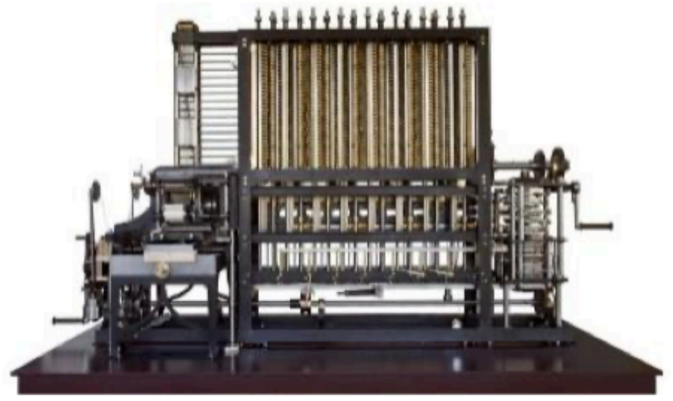
BAD NEWS…

Measurement gives random result

e.g. $|\psi\rangle \Rightarrow |101\rangle$

Classical

32 bits in SRAM store just one number at a time



00000000 00000001 = 1
00000000 00000010 = 2
00000000 00000011 = 3
00000000 00000100 = 4

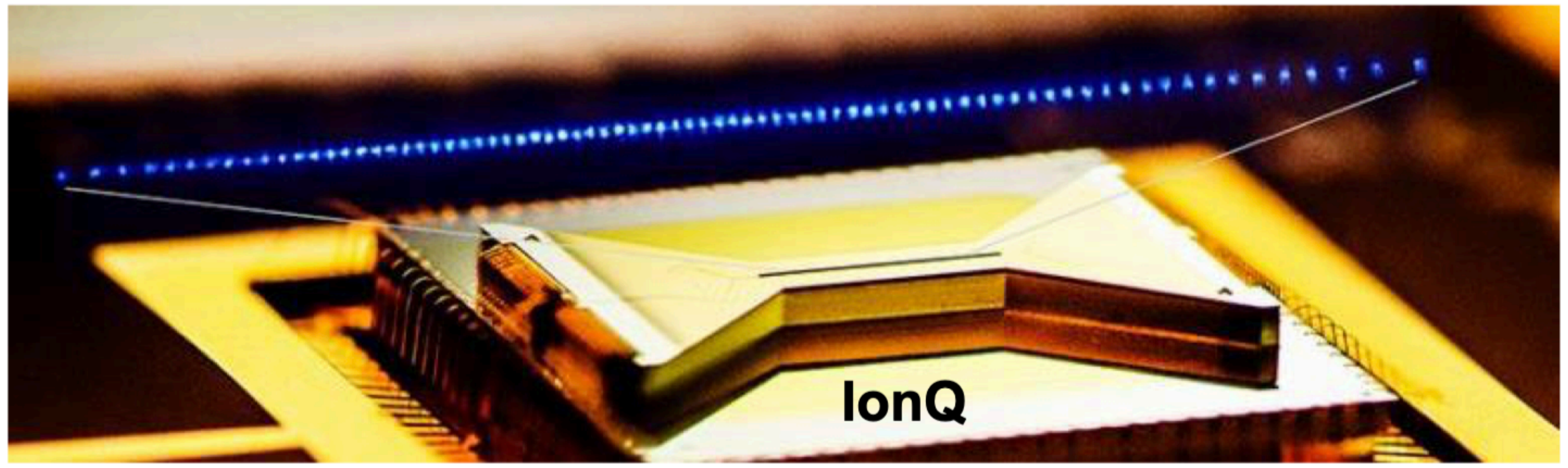
'01' or '10'

Quantum

32 qubits can represent all permutations

'01' **and** '10'

2^{32} > 4 Billion (~Quantum volume)



DiVincenzo Criteria

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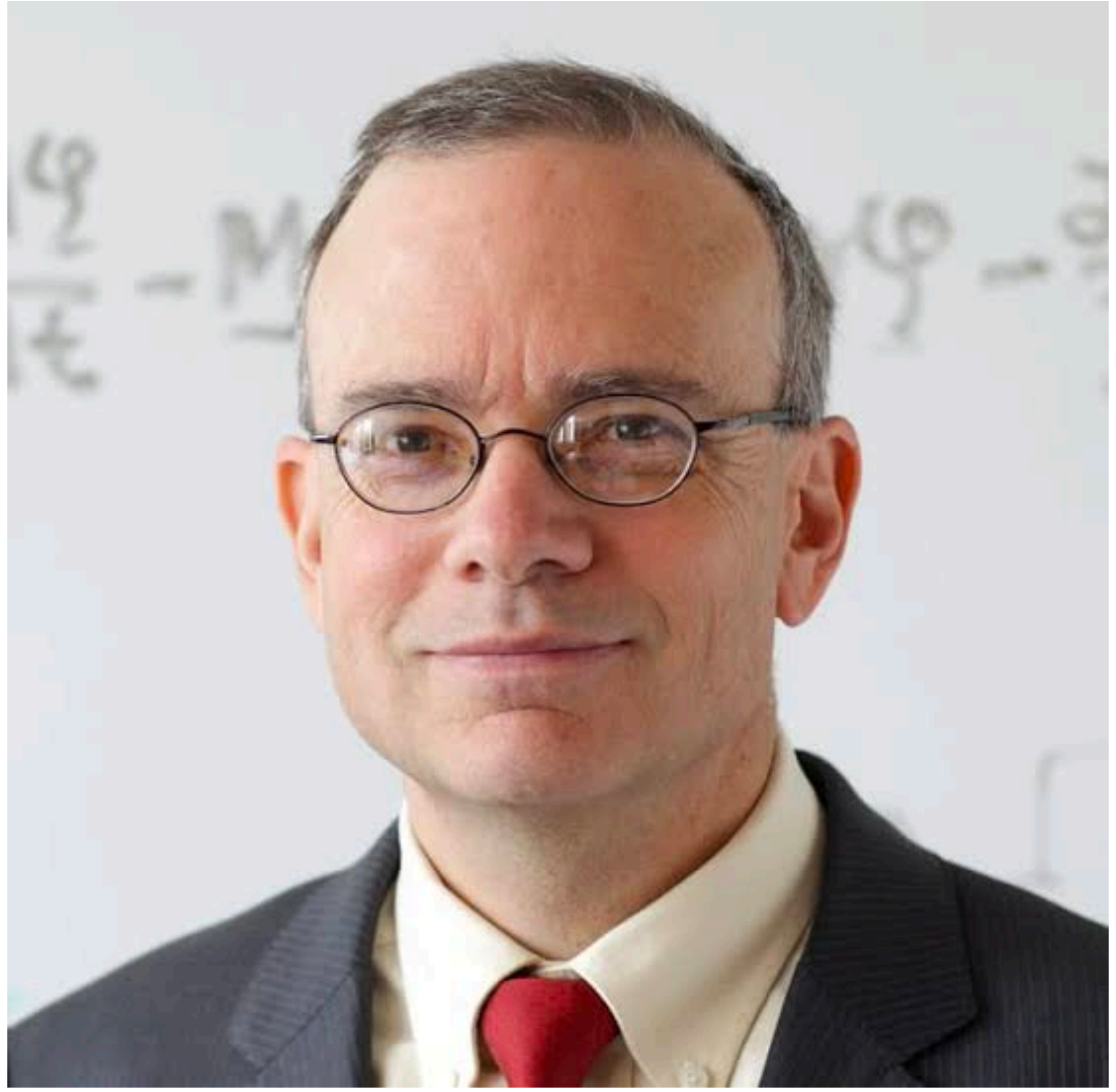

David DiVincenzo
Born 1959 (age 63 - 64)

"Topics in Quantum Computing"

① A **scalable** physical system with well characterized qubits
② The ability to initialize the qubits
③ Long relevant coherence times (vs. gate time)
④ A **"universal"** set of quantum gates
⑤ A qubit-specific measurement capability

DiVincenzo, D.P., 1997. Topics in quantum computers. *Mesoscopic electron transport*, pp.657-677.

Why Use Ion Qubits ?

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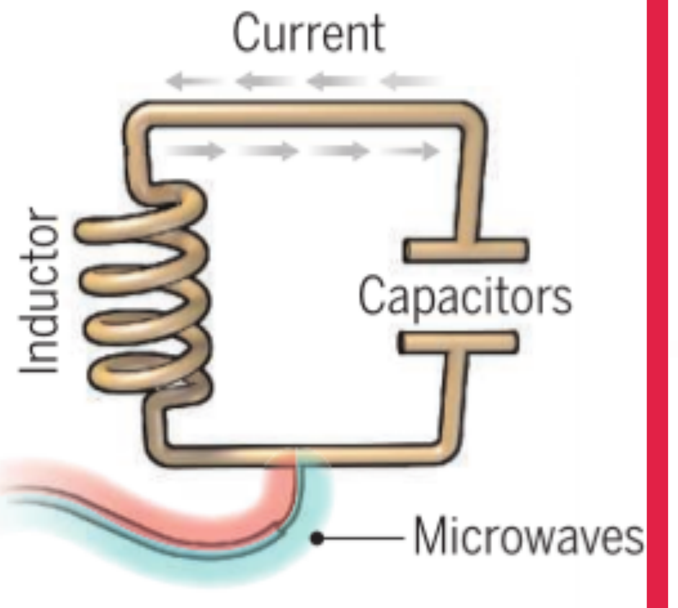
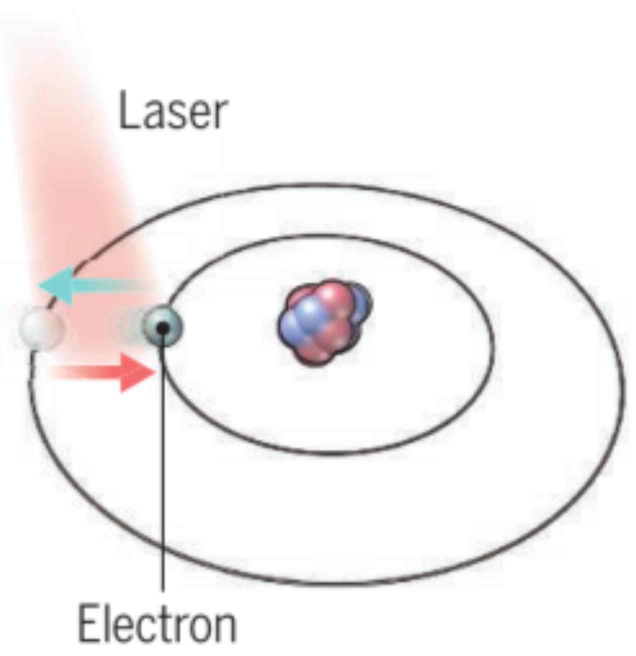
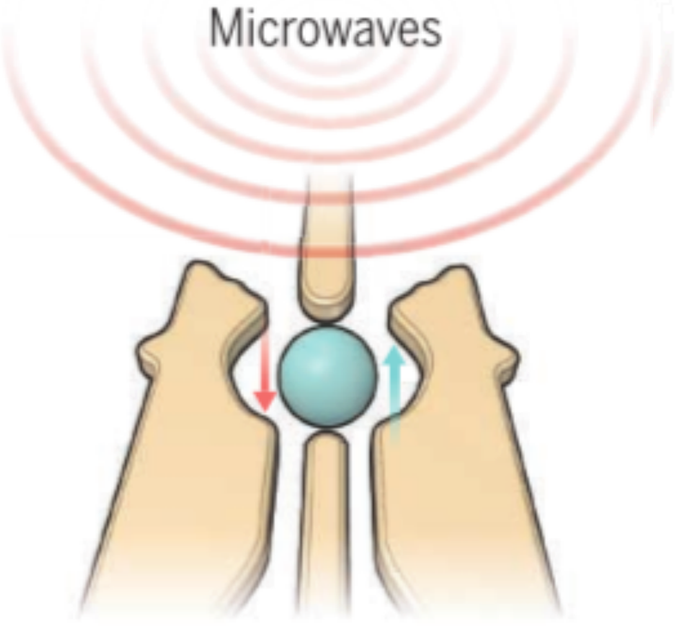
the DiVincenzo criteria

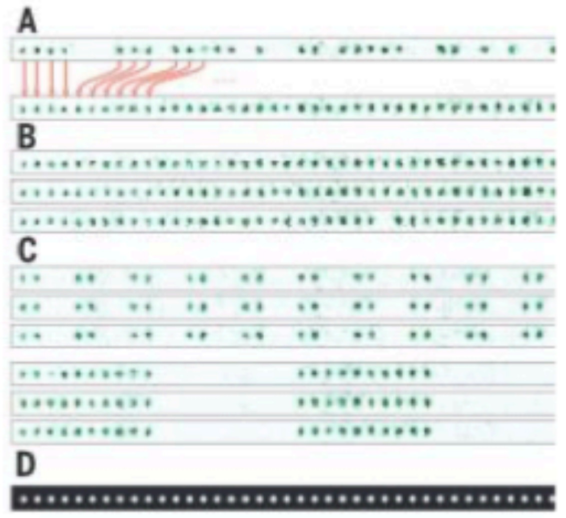
1. A scalable physical system with well characterized qubits
2. The ability to initialize the qubits
3. Long relevant coherence times (vs. gate time)
4. A “universal” set of quantum gates
5. A qubit-specific measurement capability

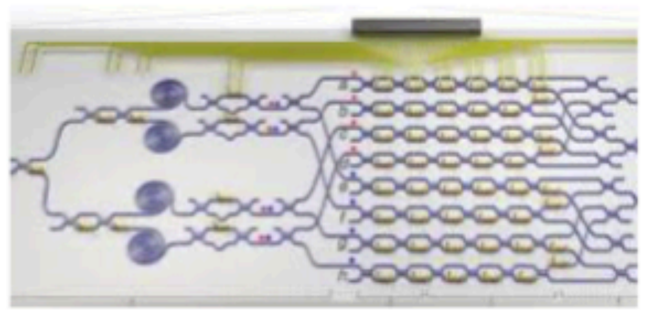






‘New qubits’





	Superconducting loops	Trapped ions	Silicon quantum dots	Neutral atoms	Photonics
	A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into super-position states.	Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.	These “artificial atoms” are made by adding an electron to a small piece of pure silicon. Microwaves control the electron’s quantum state.	Like ions but atoms are trapped with lasers and interact via Rydberg (nearly ionized) excitations	Single photons are guided along integrated pathways and interact via phase shifts during interference
Longevity (seconds)	0.00005	>1000	0.03	1	
Logic success rate	99.4%	99.9%	~99%	97%	97%* (heralded)
Number entangled	9	14	2	20	12
Company support	Google, IBM, Quantum Circuits	ionQ	Intel	Quera , Atom Computing Cold Quanta	Xanadu, PsiQuantum
Pros	Fast working. Build on existing semiconductor industry.	Very stable. Highest achieved gate fidelities.	Stable. Build on existing semiconductor industry.	2D arrays becoming possible, faster gates	CMOS compatible photonics waveguide technology
Cons	Collapse easily and must be kept cold.	Slow operation. Many lasers are needed.	Only a few entangled. Must be kept cold.	Gate fidelity	Cryogenic single photon sources and detectors

DiVincenzo, D.P., 2000. The physical implementation of quantum computation. *Fortschritte der Physik: Progress of Physics*, *48*(9-11), pp.771-783.

How to build a Quantum Computer

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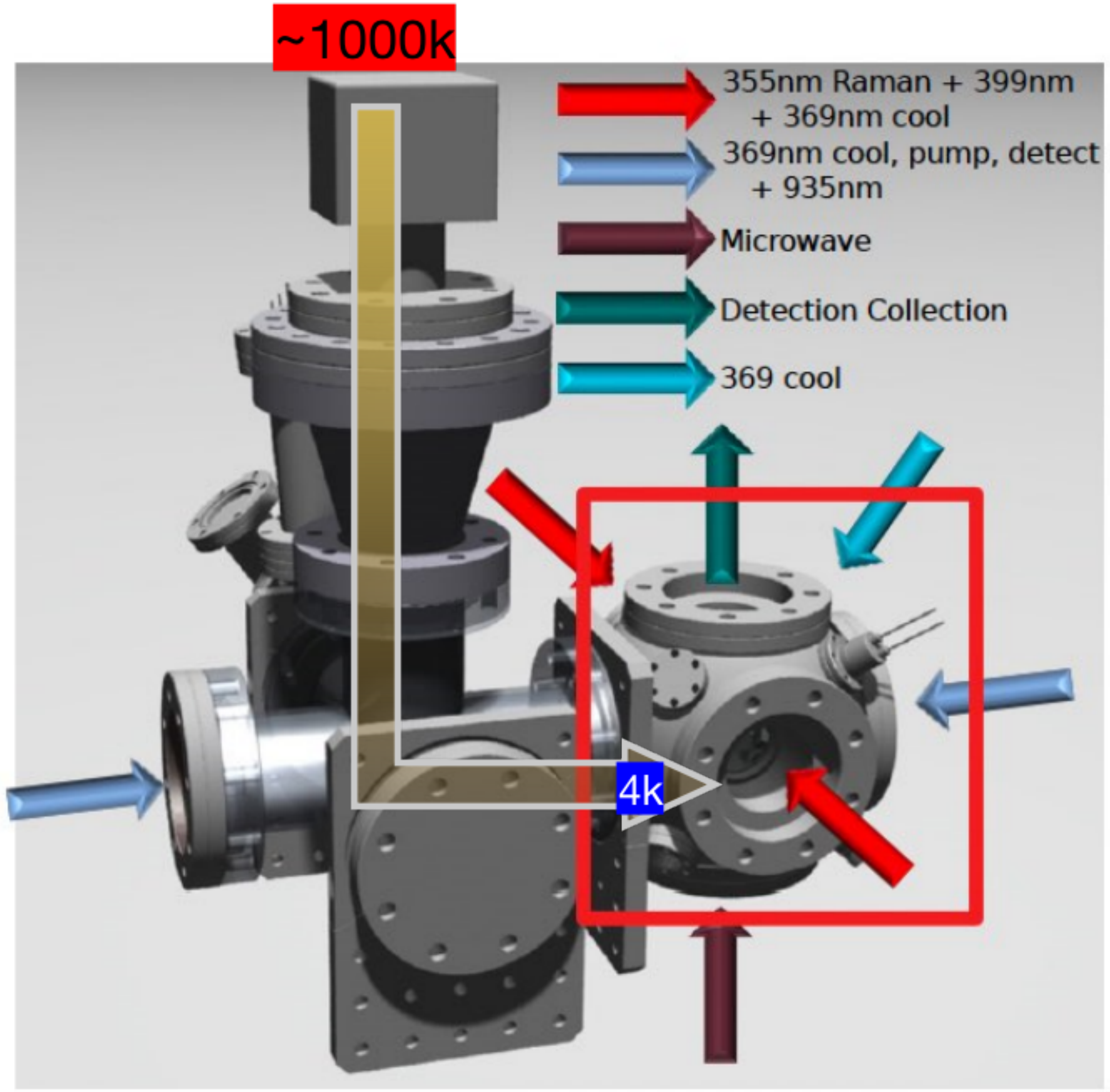
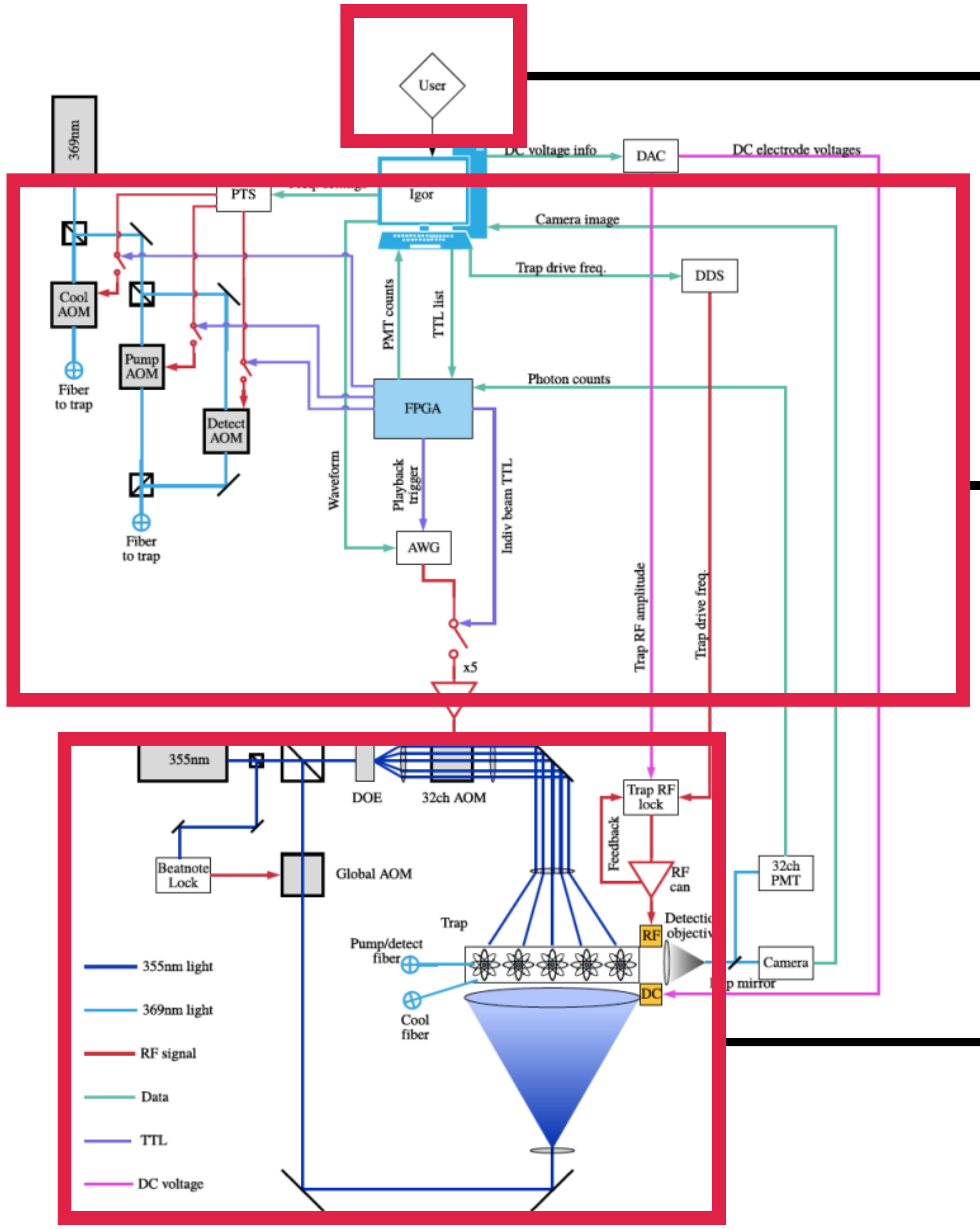


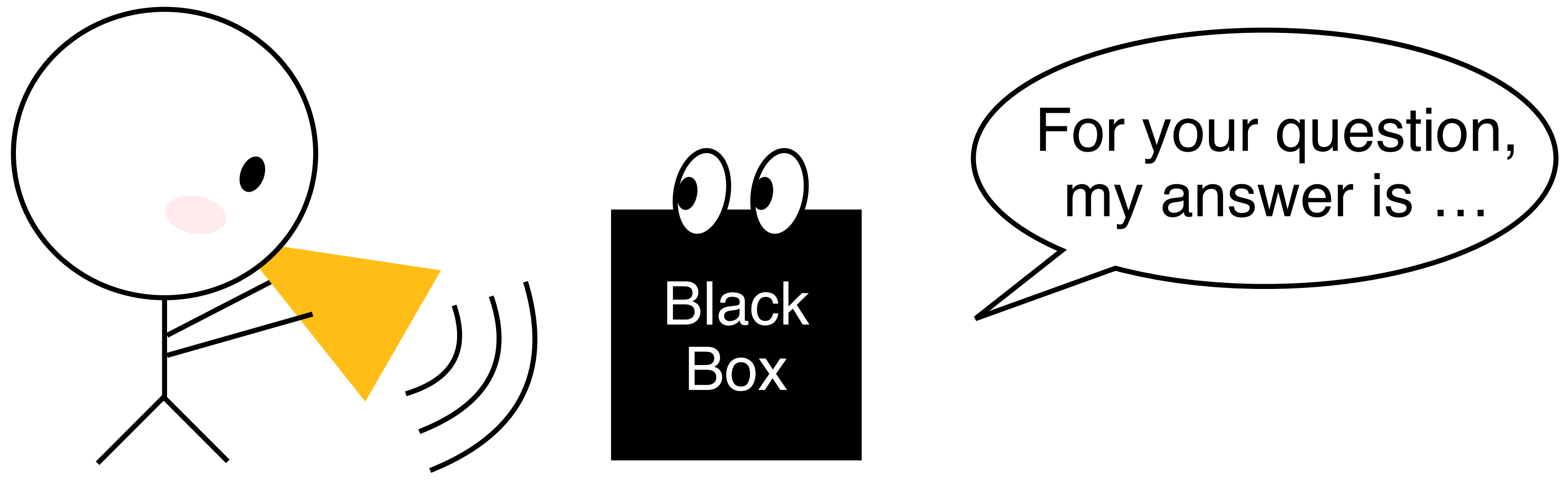


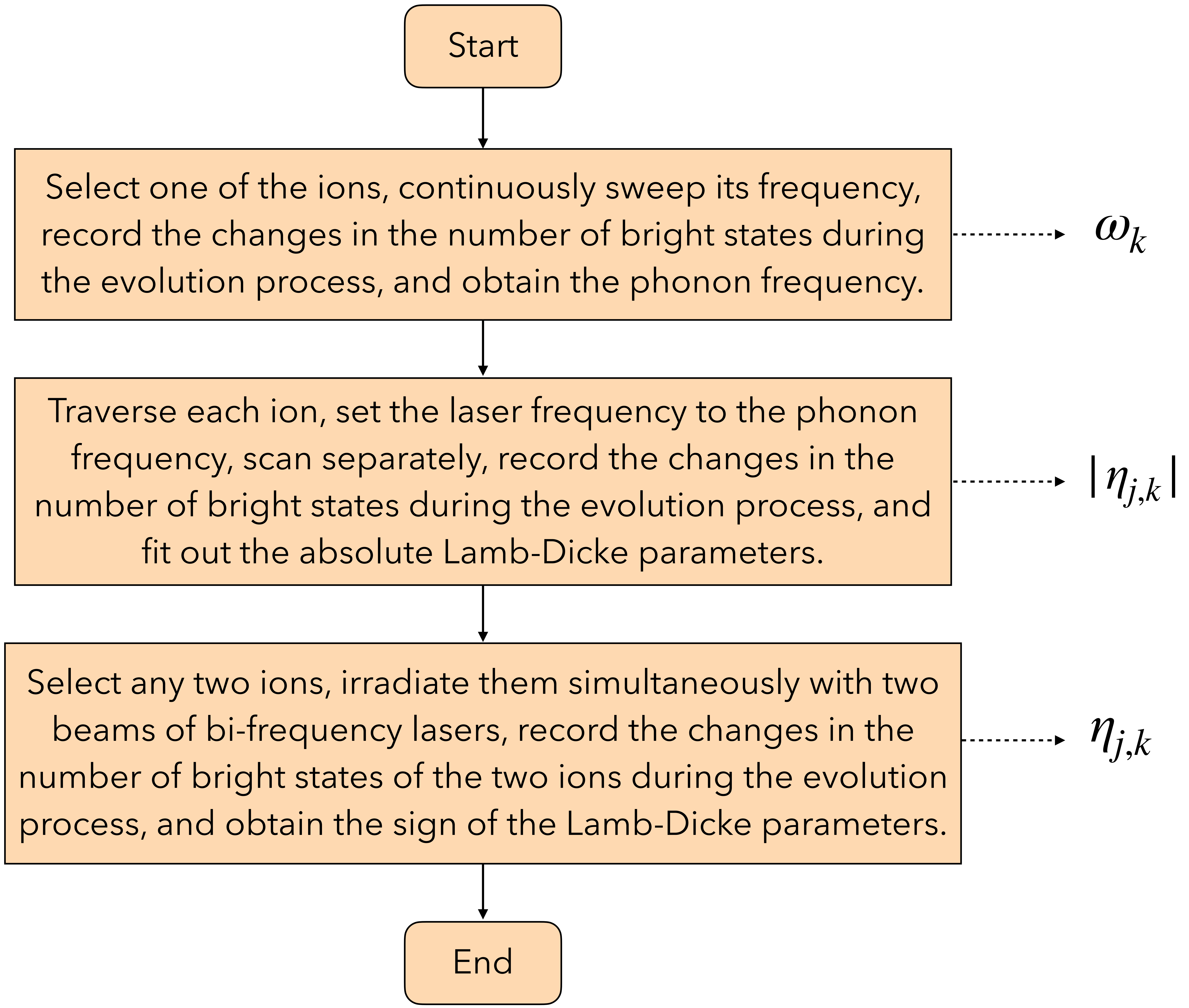
User

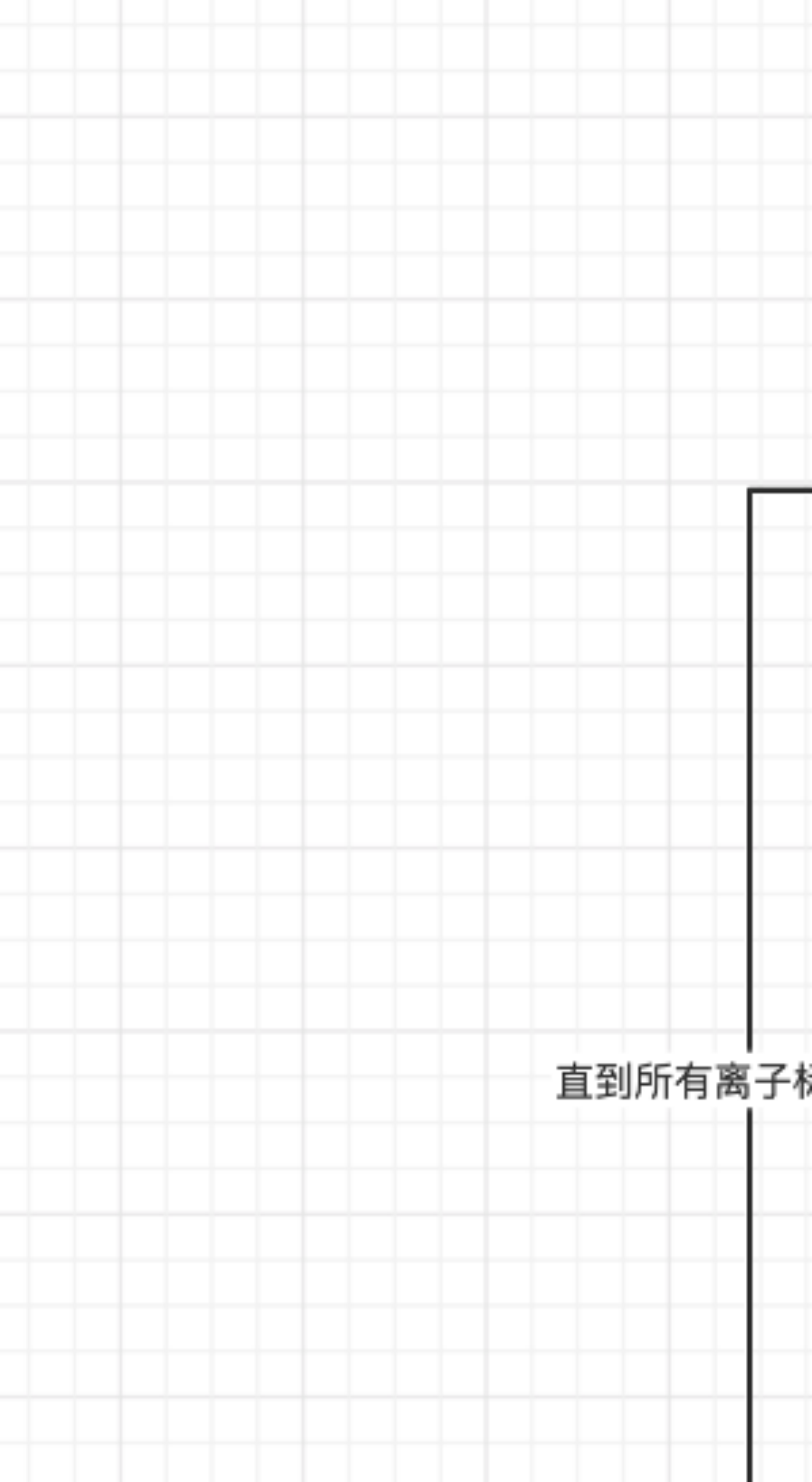
Cooling
Initiation
Measure

Operation

~2ms	30μs	1~300μs	0.5ms
Doppler Cooling	State Initiation	Quantum Operation	State Measurement


Black
Box
For your question,
my answer is …


Start
Select one of the ions, continuously sweep its frequency, record the changes in the number of bright states during the evolution process, and obtain the phonon frequency.
ω_k
Traverse each ion, set the laser frequency to the phonon frequency, scan separately, record the changes in the number of bright states during the evolution process, and fit out the absolute Lamb-Dicke parameters.
$|\eta_{j,k}|$
Select any two ions, irradiate them simultaneously with two beams of bi-frequency lasers, record the changes in the number of bright states of the two ions during the evolution process, and obtain the sign of the Lamb-Dicke parameters.
$\eta_{j,k}$
End


直到所有离子

Laser & Ion

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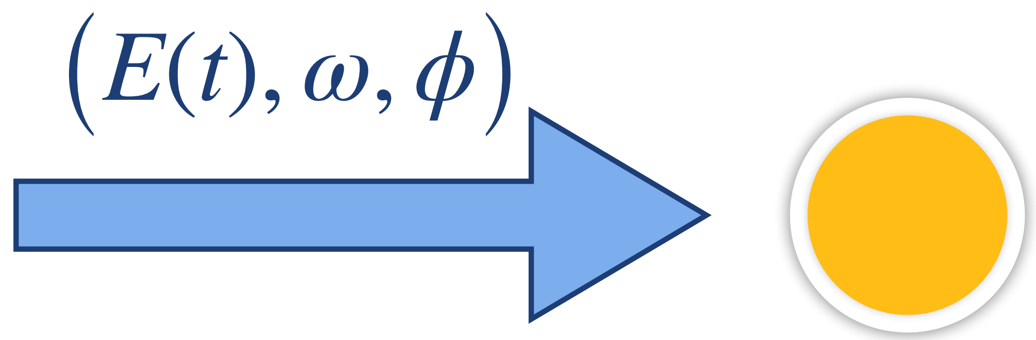
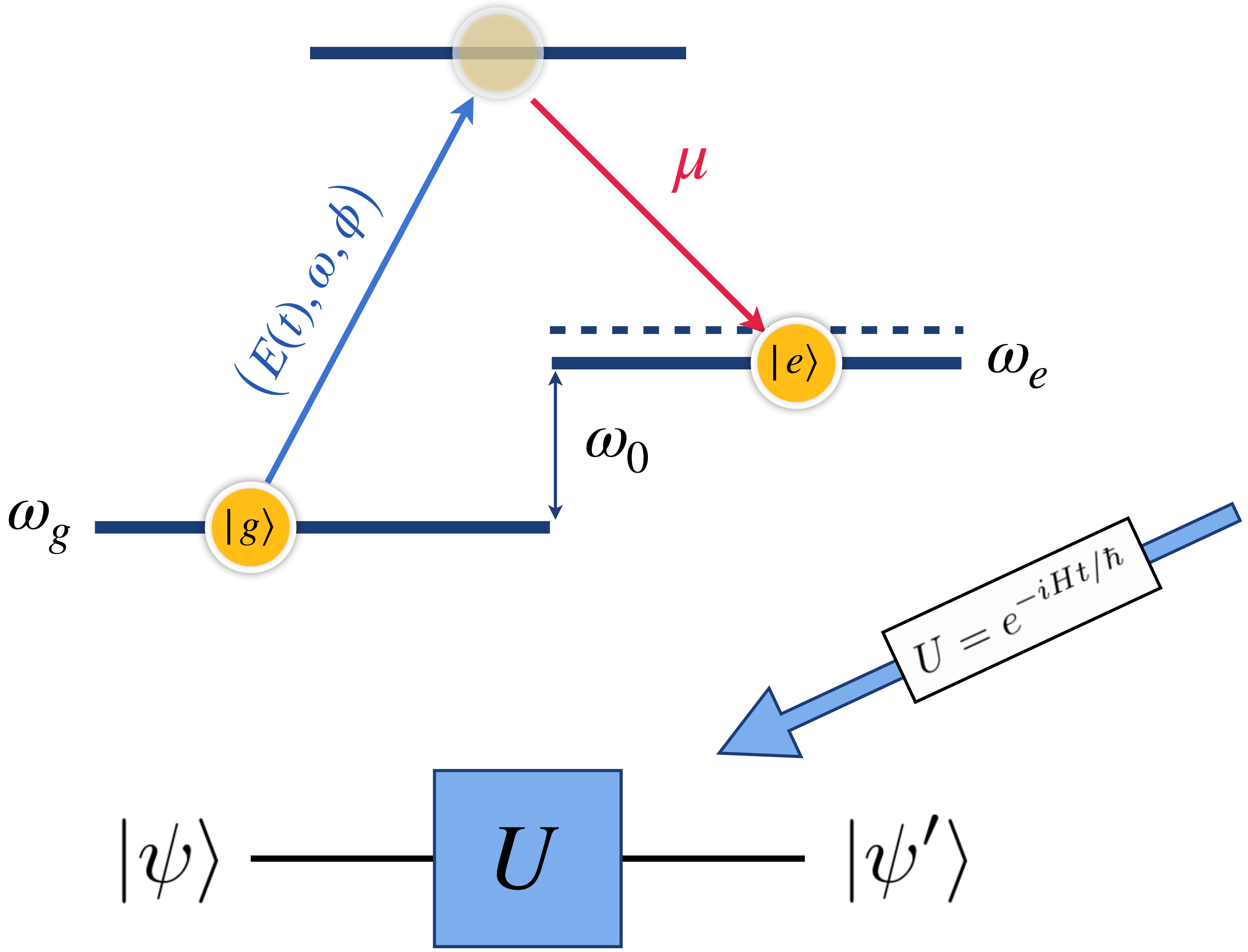


$$H_0 = \hbar\omega_m a^\dagger a + \hbar\frac{\omega_0}{2}$$

$$H_{\text{int}} = \frac{\hbar}{2}\Omega\left(|e\rangle\langle g| + |g\rangle\langle e|\right)\left[e^{i(kx-\omega t+\phi)} + e^{-i(kx-\omega t+\phi)}\right]$$

1. Trans to Interaction picture
2. $kx = kx_0\left(a + a^\dagger\right) = \eta\left(a + a^\dagger\right)$
3. RWA
4. Lamb-Dicke approximations



$$\boxed{H = \frac{\hbar}{2}\Omega\sigma_+\left[1 + i\eta\left(a^\dagger e^{i\omega_m t} + a e^{-i\omega_m t}\right)\right]e^{i(\phi-\Delta t)} + \text{h.c.}}$$

$$\begin{aligned}
&= \frac{\hbar}{2}\Omega\left(\hat\sigma_+ e^{i\phi} + \hat\sigma_- e^{-i\phi}\right)\\
&\quad + i\eta\frac{\hbar}{2}\Omega\left(\hat\sigma_+ a^\dagger e^{i\phi}e^{-i\delta t} - \hat\sigma_- a e^{-i\phi}e^{i\delta t}\right)\\
&\quad + i\eta\frac{\hbar}{2}\Omega\left(\hat\sigma_+ a e^{i\phi}e^{i\delta t} - \hat\sigma_- a^\dagger e^{-i\phi}e^{-i\delta t}\right)
\end{aligned}$$

Carrier transition $\boxed{|n\rangle\,|g\rangle \leftrightarrow |n\rangle\,|e\rangle}$

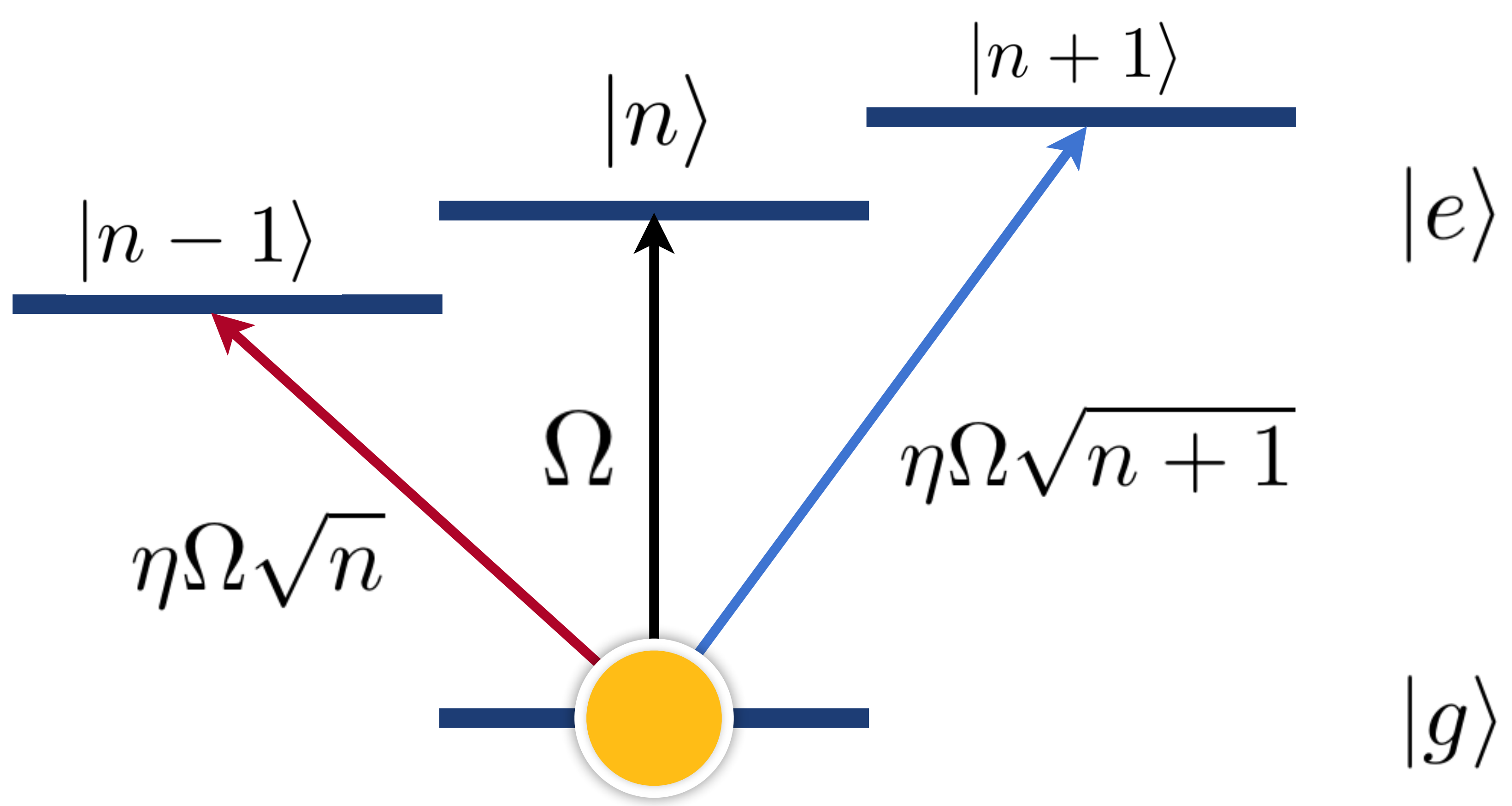
$$H_{\text{car}} = \frac{\hbar}{2}\Omega\left(\hat{\sigma}_+ e^{i\phi} + \hat{\sigma}_- e^{-i\phi}\right)$$

Blue sideband transition $\boxed{|n\rangle\,|g\rangle \leftrightarrow |n+1\rangle\,|e\rangle}$

$$H_{\text{bsb}} = i\eta\frac{\hbar}{2}\Omega\left(\hat{\sigma}_+ a^\dagger e^{i\phi} e^{-i\delta t} - \hat{\sigma}_- a e^{-i\phi} e^{i\delta t}\right)$$

Red sideband transition $\boxed{|n\rangle\,|g\rangle \leftrightarrow |n-1\rangle\,|e\rangle}$

$$H_{\text{rsb}} = i\eta\frac{\hbar}{2}\Omega\left(\hat{\sigma}_+ a e^{i\phi} e^{i\delta t} - \hat{\sigma}_- a^\dagger e^{-i\phi} e^{-i\delta t}\right)$$



Quantum Gates

Single-qubit gates

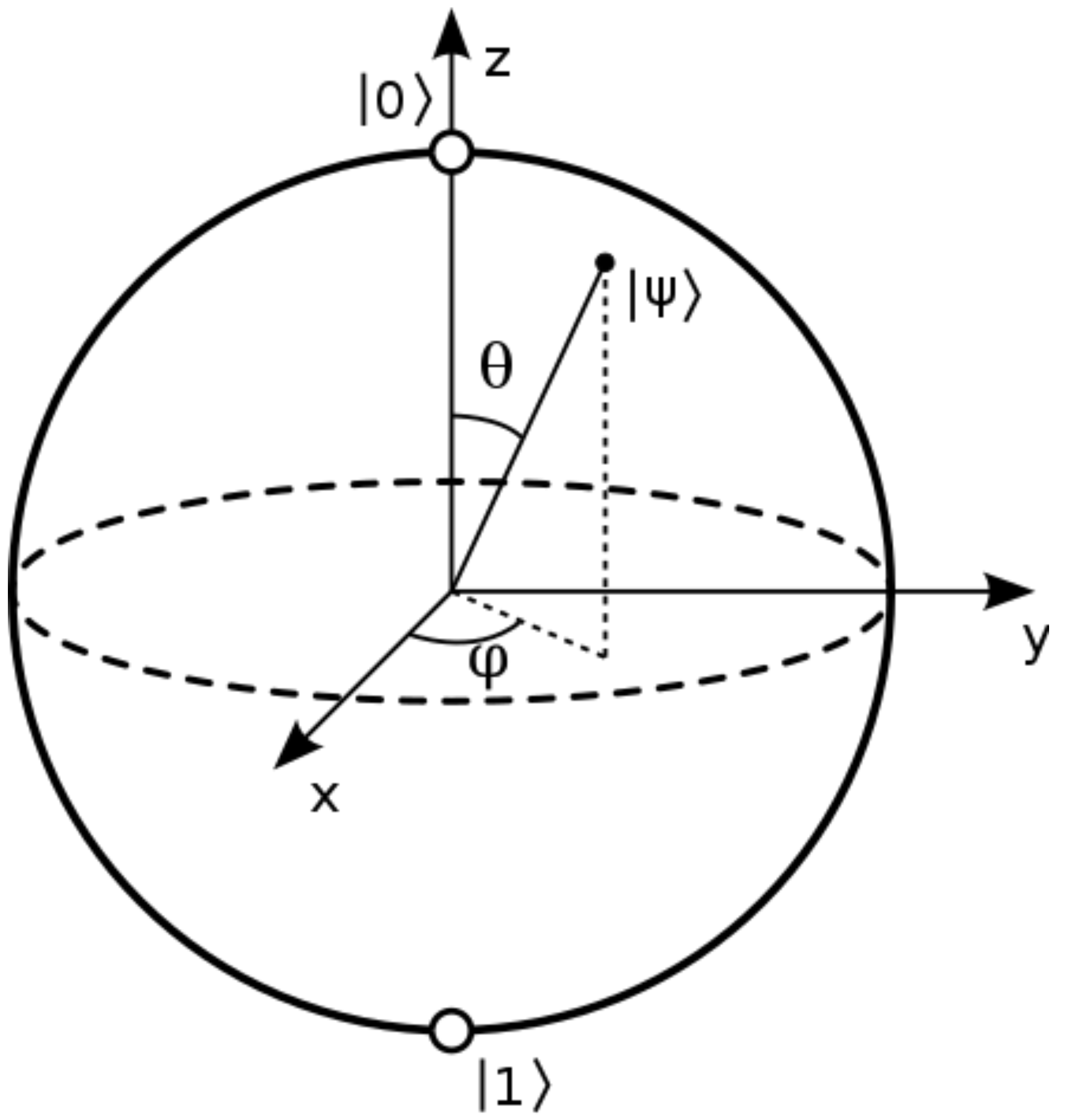
Pauli gate

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Hadamard gate (H)

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle \rightarrow (|0\rangle + |1\rangle)/\sqrt{2}$$
$$|1\rangle \rightarrow (|0\rangle - |1\rangle)/\sqrt{2}$$



$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

2-qubit gate: Mølmer-Sørensen Gate

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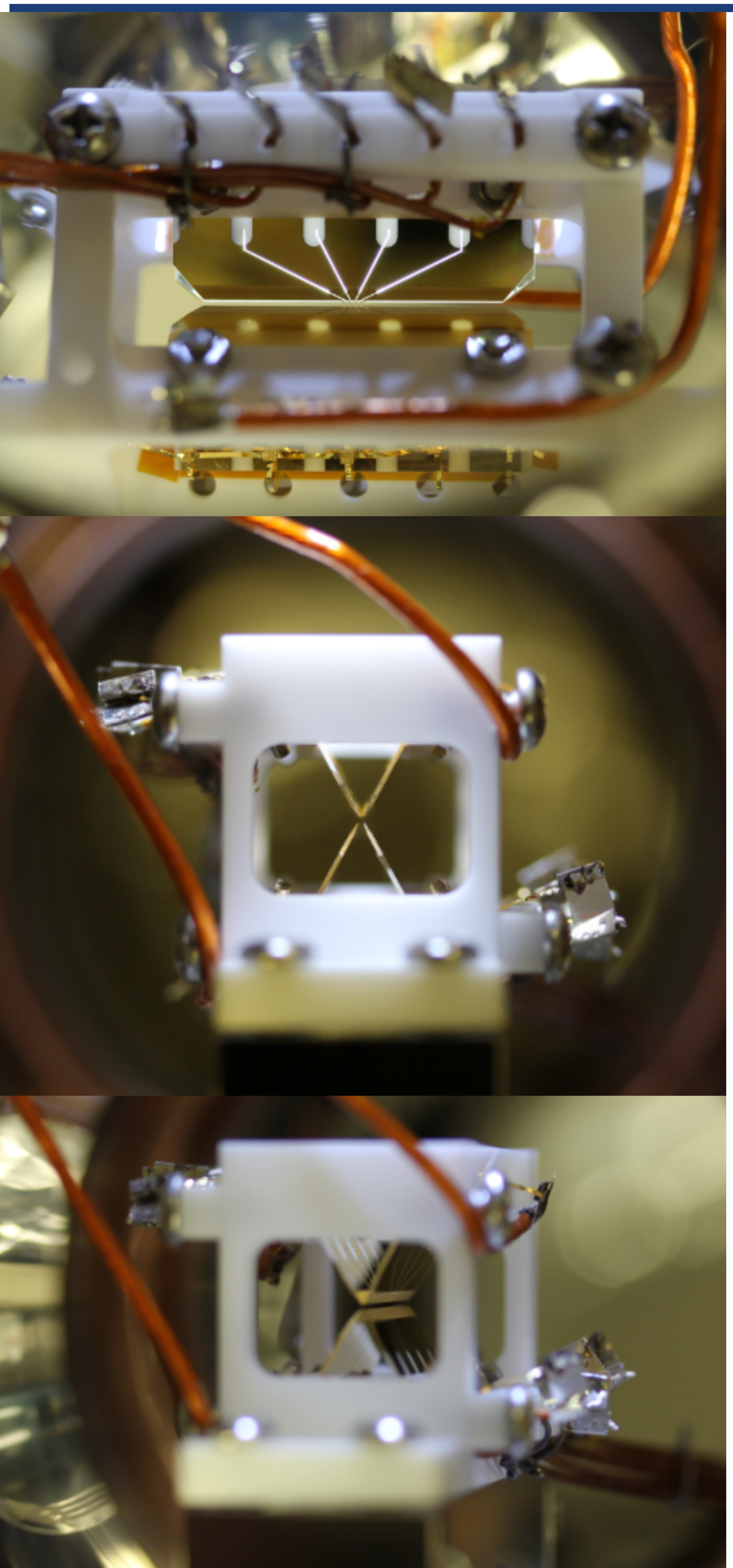
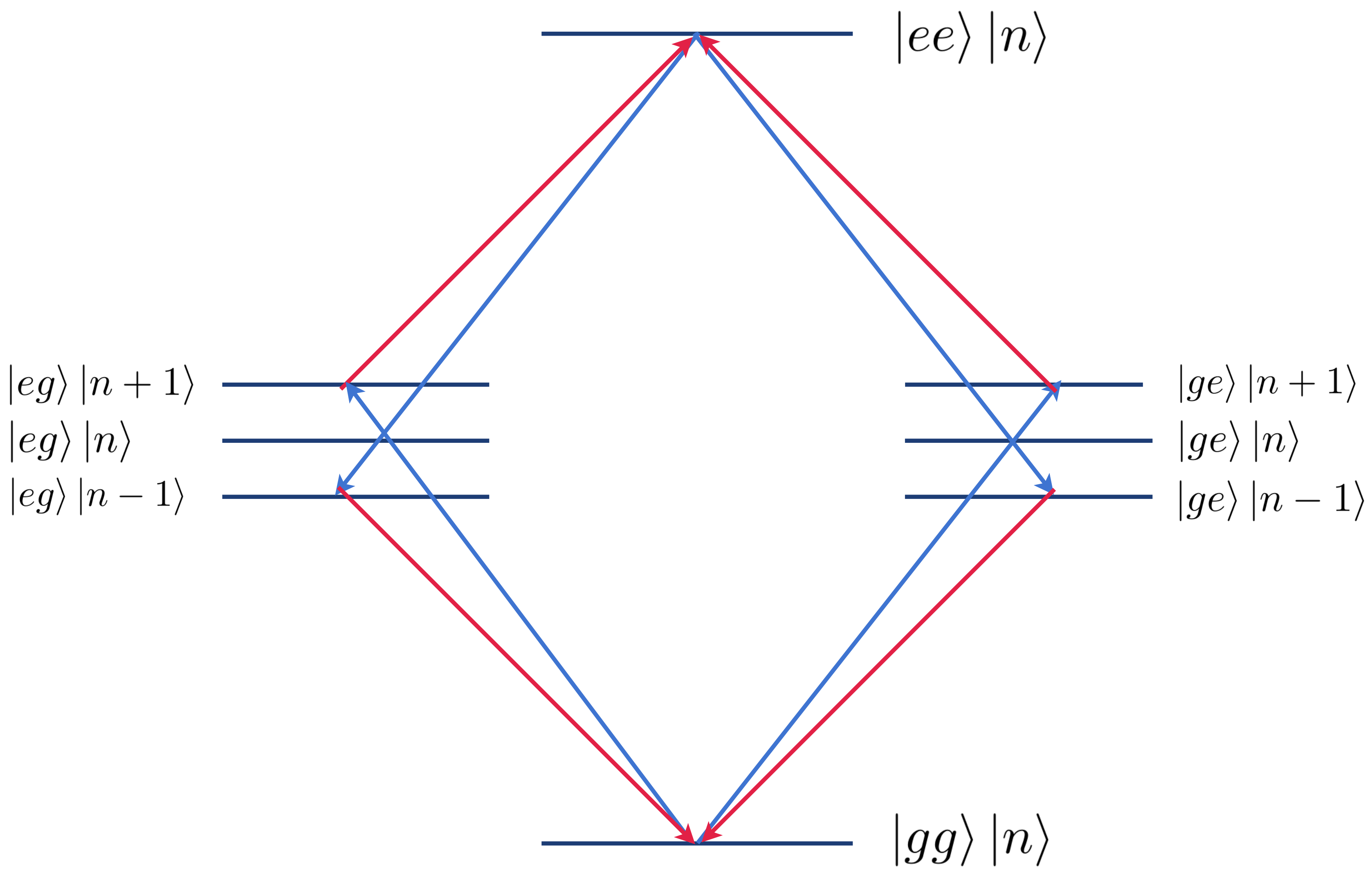
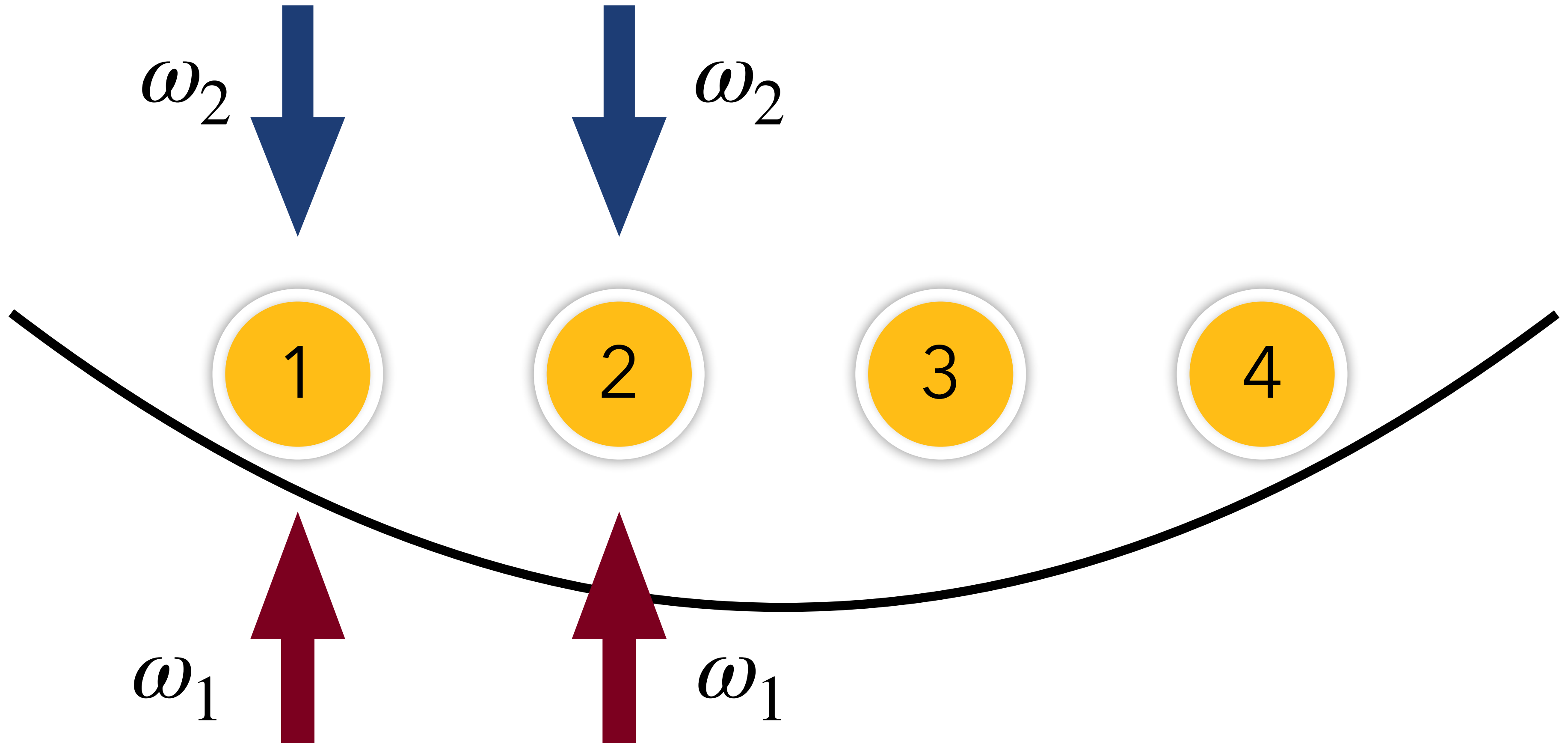




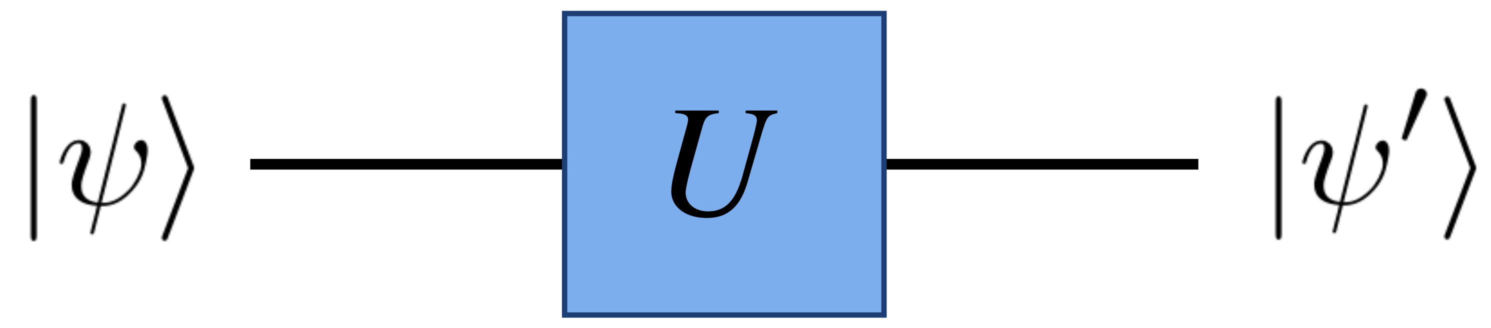


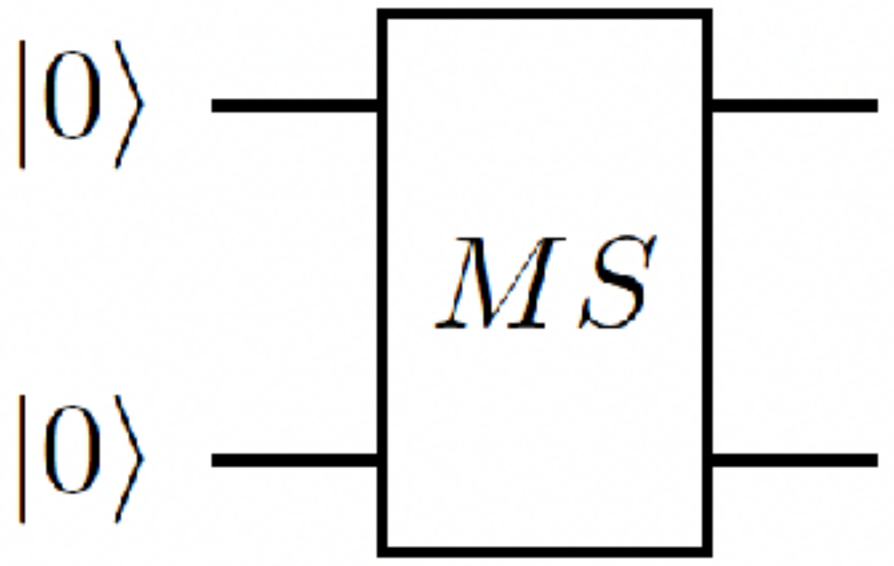
$$\hat{H}_{\mathrm{MS}} = \boxed{\hbar \sum_{j=\{m,n\}} \Omega_j(t)\hat{\sigma}_j} \boxed{\sum_{k=1}^{N} \eta_{j,k} e^{i\mu t}\left(\hat{a}_k e^{-i\omega_k t} + \hat{a}_k^\dagger e^{i\omega_k t}\right)} + \mathrm{h.c.}$$

Ion internal state　　Ion vibration — phonon

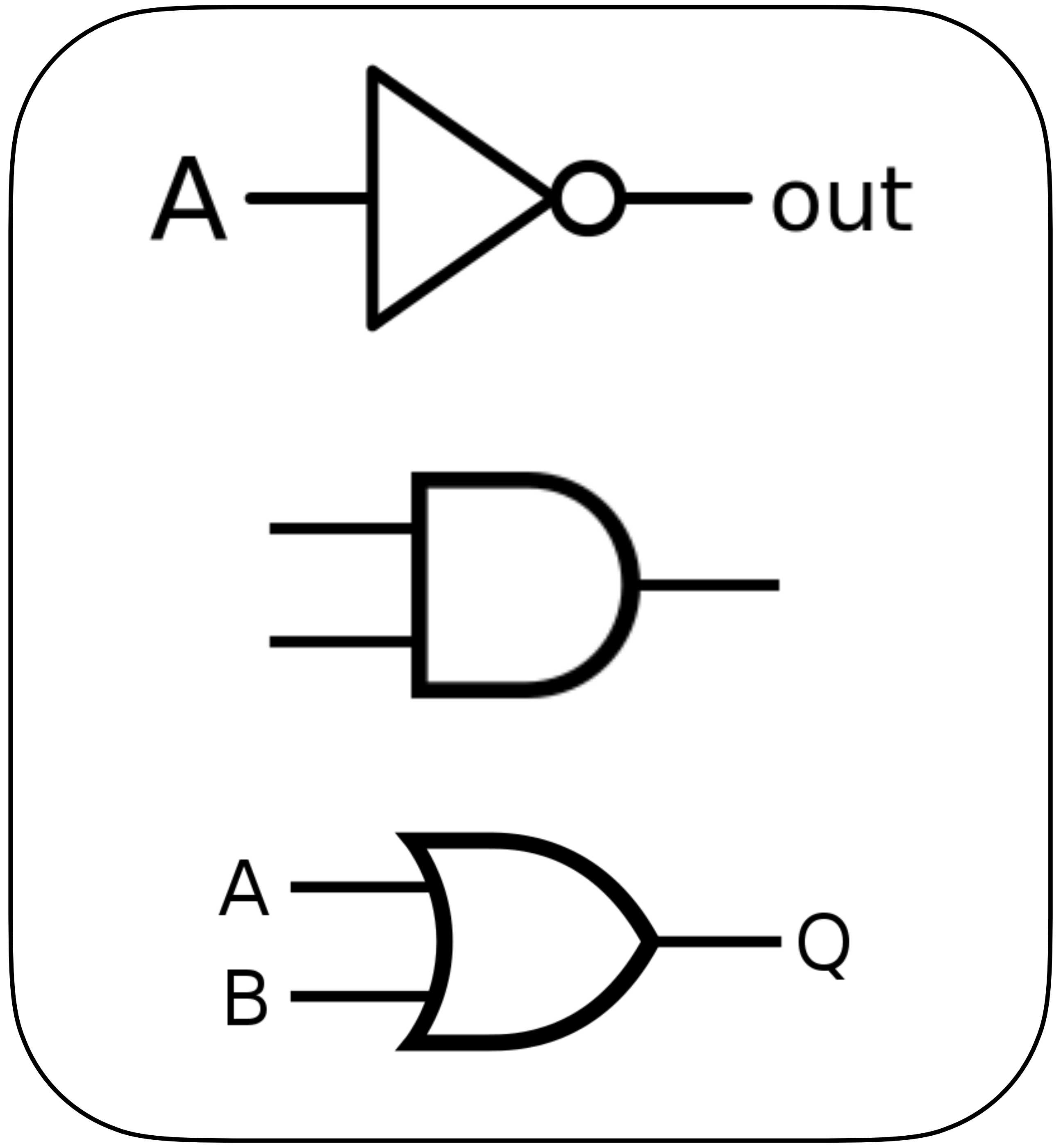
Mølmer-Sørensen Gate

$$MS = U(t_g) = e^{i\frac{\pi}{4}\hat{\sigma}_x^m \otimes \hat{\sigma}_x^n} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & i & 0 \\ 0 & i & 1 & 0 \\ i & 0 & 0 & 1 \end{pmatrix}$$





$$\rightarrow \frac{|00\rangle + i|11\rangle}{\sqrt{2}}$$



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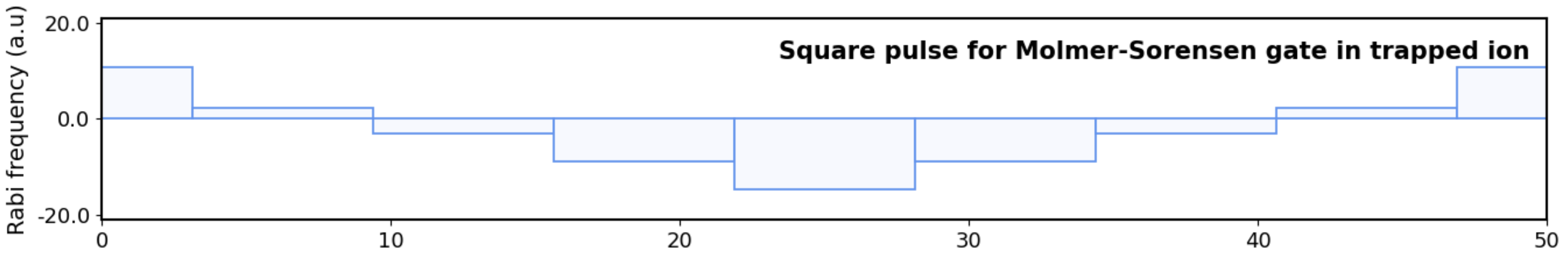
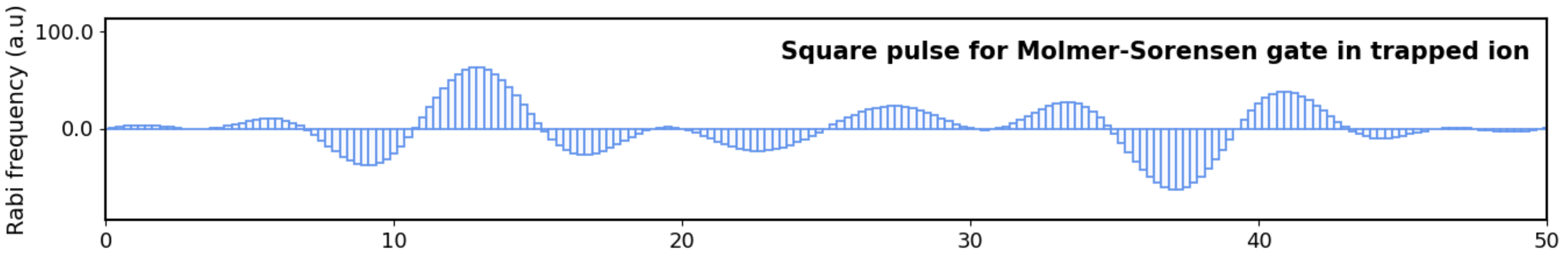
Mølmer-Sørensen Gate

- Square Wave



- Sine Wave



QUANLSE https://quanlse.baidu.com/

Ion Trap Quantum Computing

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Advantages

1. **High fidelity operations**
   2Q gates = 99.8%
   1Q gates > 99.99%
   Readout > 99.99%
   *error from imprecise control (not inherent to ion)
2. **Long coherence time**
   200ms → 600s
   Ratio to gate time ~ 1,000,000
   (Much higher than SCs ~ 1,000)
3. **Shuttling**
4. **Low cross talk with focused lasers**

Challenges

1. **Not in the solid state**
   - Must be loaded into an RF trap
   - They can physically escape
2. **Required MANY laser beams:**
   - Multiple wavelengths
   - Ultra stable and precise
   - Addressed to every qubit
3. **Slow (time scales >> ns)**
   - Gates are slow
   - Shuttling is slow
   - Remote entanglement is very slow

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Thanks for Listening !