General Model of Photon-Pair Detection with an Image Sensor

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- Provide a general theoretical framework for **<u>intensity correlation measurements</u>** of two-photon states without approximations made on the source.
- Compare the model to experiments performed with different detection systems:

  (1) An APD-like single-photon counter(SPC) camera, implemented using an EMCCD camera with thresholding.

  (2) A linear **<u>EMCCD</u>** camera with no threshold.

  (3) A standard CCD camera.

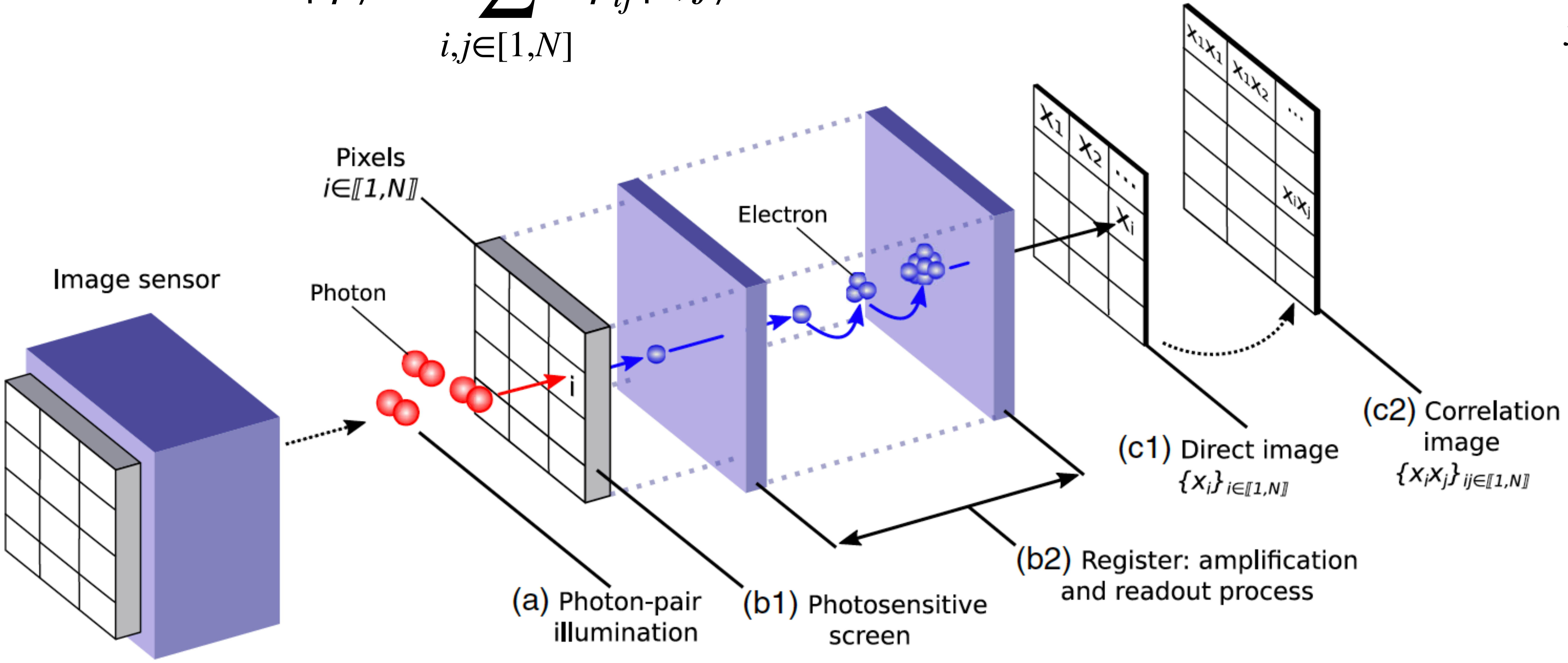
general theoretical framework

- Pixels of the image sensor operate independently
- The input state is a pure two-photon state

$$|\phi\rangle = \sum_{i,j\in[1,N]} \phi_{ij} |i,j\rangle$$

$$\langle x_i x_j \rangle = \sum_{x_i=0}^{\infty} \sum_{x_j=0}^{\infty} x_i x_j P(x_i, x_j)$$

$$\langle x_i \rangle = \sum_{x_i=0}^{\infty} x_i P(x_i)$$



Bayes's theorem: $P\left(B_i \mid A\right) = \dfrac{P\left(B_i\right) P\left(A \mid B_i\right)}{\sum_{j=1}^{n} P\left(B_j\right) P\left(A \mid B_j\right)}$

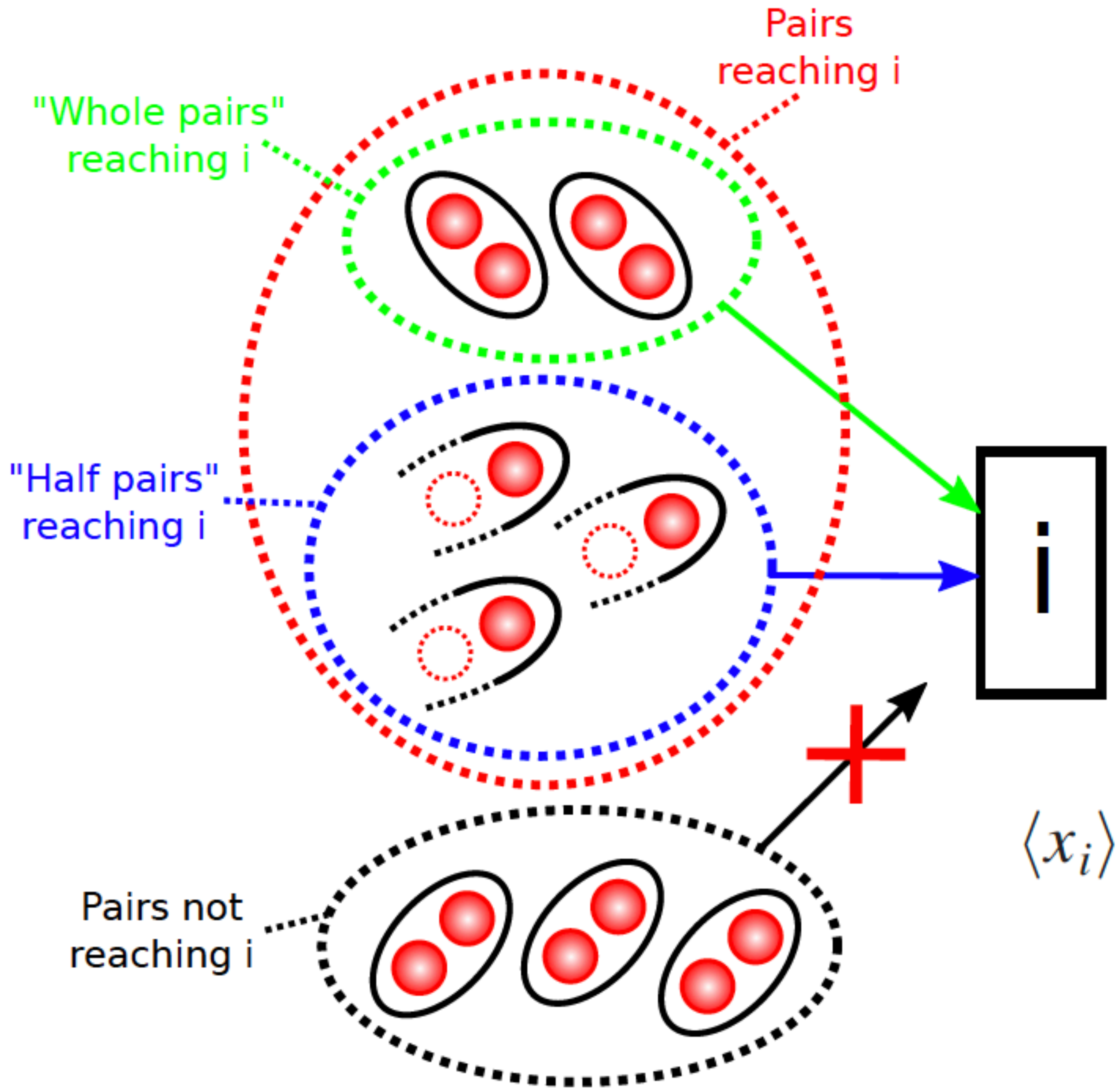
$$\left\langle x_i \right\rangle = \sum_{x_i=0}^{\infty} x_i P(x_i) = \sum_{m=0}^{+\infty} P(m) \sum_{k_i=0}^{2m} I_{k_i} P\left(k_i \mid m\right)$$

$P(x_i)$ is the probability for the sensor to return value x_i at pixel i and $P(x_i, x_j)$ is the joint probability to return x_i at pixel i and x_j at pixel j.

$$I_{k_i} = \sum_{x_i=0}^{\infty} x_i P(x_i \mid k_i)$$

I_{k_i} is the mean of the detector response function at pixel i.

Types of pairs falling on pixel i

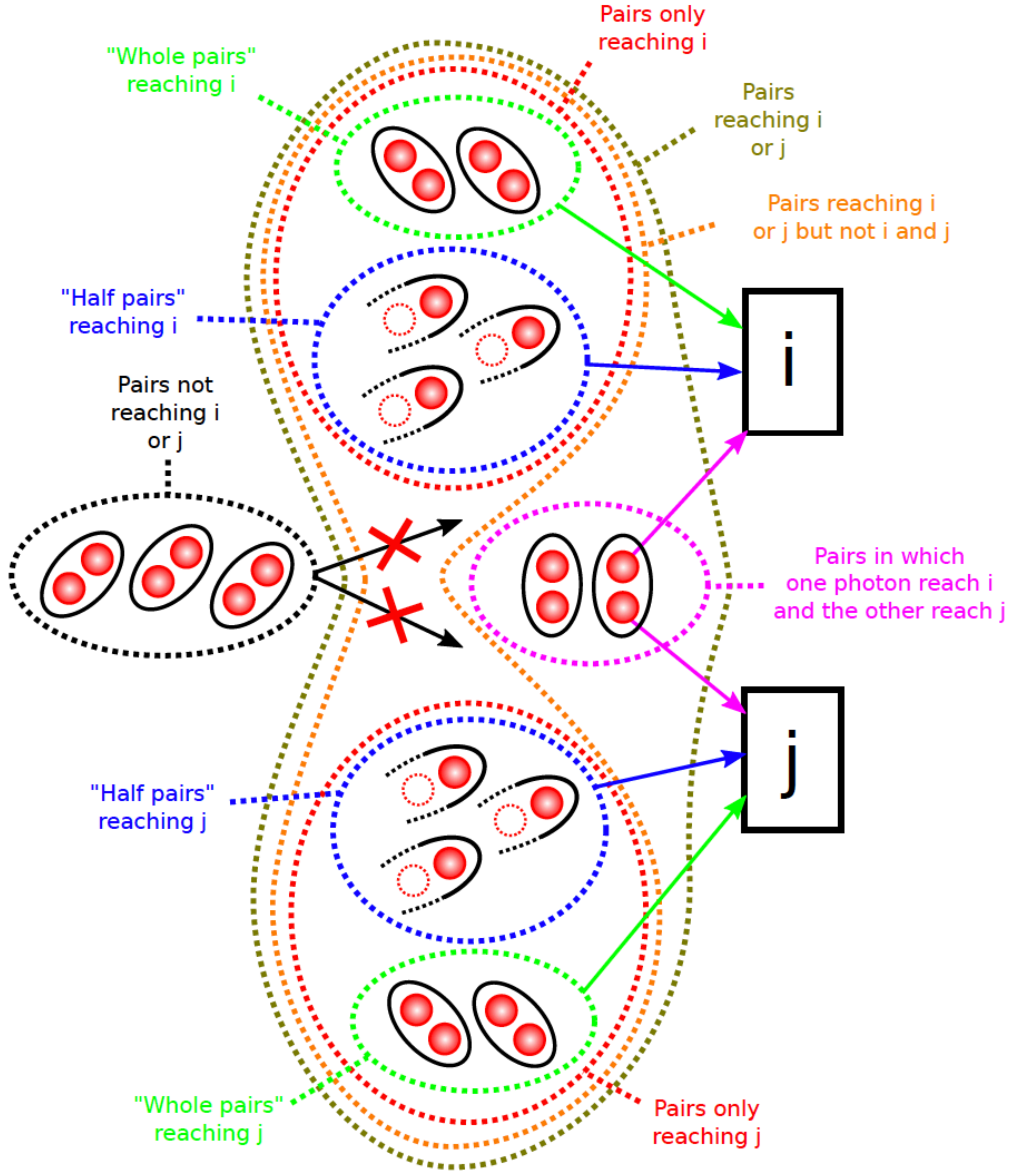


Expression of $P(n_i \mid m)$

$* \Gamma_{ij} = |\phi_{ij}|^2$

- Both photons reach pixel i: $P(2 \mid 1) = \Gamma_{ii}$
- No photons reach pixel i:
  $P(0 \mid 1) = 1 - 2\Gamma_i + \Gamma_{ii}$
- One photon of the pair reaches pixel i and the other does not: $P(1 \mid 1) = 2(\Gamma_i - \Gamma_{ii})$

$$\langle x_i \rangle = \sum_{m=0}^{+\infty} P(m) \sum_{k_i=0}^{2m} I_{k_i} \sum_{q=0}^{\lfloor k_i/2 \rfloor} (\eta^2 \Gamma_{ii})^q (2\eta\Gamma_i - 2\eta^2\Gamma_{ii})^{k_i - 2q} \times (1 - 2\eta\Gamma_i + \eta^2\Gamma_{ii})^{m - k_i + q} \binom{k_i - q}{q} \binom{m}{k_i - q}$$



Subsets of pairs falling on the screen during the exposure time relatively to pixel i and j

$$\langle x_i x_j \rangle = \sum_{m=0}^{+\infty} P(m) \sum_{k_i=0}^{2m} \sum_{k_j=0}^{2m} I_{k_i} I_{k_j} P(k_i, k_j | m).$$

$$\langle x_i x_j \rangle = \sum_{m=0}^{+\infty} P(m) \sum_{k_i=0}^{2m} \sum_{k_j=0}^{2m} I_{k_i} I_{k_j} \sum_{q=0}^{\lfloor (k_i+k_j)/2 \rfloor} \sum_{l=0}^{q} \sum_{p=0}^{q-l} (1 - 2\eta\Gamma_i - 2\eta\Gamma_j + \eta^2\Gamma_{ii} + \eta^2\Gamma_{jj} + 2\eta^2\Gamma_{ij})^{m-(k_i+k_j-q)}$$
$$\times (\eta^2\Gamma_{jj})^p (2\eta^2\Gamma_{ij})^l (\eta^2\Gamma_{ii})^{q-l-p} (2\eta\Gamma_i - 2\eta^2\Gamma_{ii} - 2\eta^2\Gamma_{ij})^{k_i+l-2(q-p)} (2\eta\Gamma_j - 2\eta^2\Gamma_{jj} - 2\eta^2\Gamma_{ij})^{k_j-2p-l}$$
$$\times \binom{k_j - l - p}{p} \binom{k_i - q + p}{q - l - p} \binom{k_i + k_j - q - l}{k_i - q + p} \binom{k_i + k_j - q}{l} \binom{m}{k_i + k_j - q}.$$

Photon pairs generated through an SPDC process, $P(m)$ can be modeled by a Poisson distribution

$$P(m) = \frac{\bar{m}^m e^{-\bar{m}}}{m!}$$

$$\boxed{\Gamma_{ij} = \frac{1}{2\eta^2 \bar{m}} \ln \left[1 + \frac{\langle c_i c_j \rangle - \langle c_i \rangle \langle c_j \rangle}{\left(1 - \langle c_i \rangle\right)\left(1 - \langle c_j \rangle\right)} \right]}$$

Model of readout process of an EMCCD camera

$$\boxed{I_k = Ak + x_0}$$

The grey value *x* returned by the camera at a given pixel is modeled by a random variable *X* decomposed into [1]

$$X = \alpha \left(X^{sig} + X^{par} + X^{ser} + X^{R}\right)$$

$$\begin{aligned} I_k &= \sum_{x=0}^{+\infty} x P_{ccd}(x|k) \\ &= \alpha \left(gk + \mu + p_{par} g + p_{ser} \frac{g-1}{p_c} \right) \\ &= A\,k + x_0 \end{aligned}$$

[1] E. Lantz, P.-A. Moreau, and F. Devaux, Phys. Rev. A 90, 063811 (2014).

Linking Γ_{ij} to $\langle x_i \rangle$ and $\langle x_i x_j \rangle$ in the case of an EMCCD camera without threshold

$$\boxed{I_k = Ak + x_0}$$

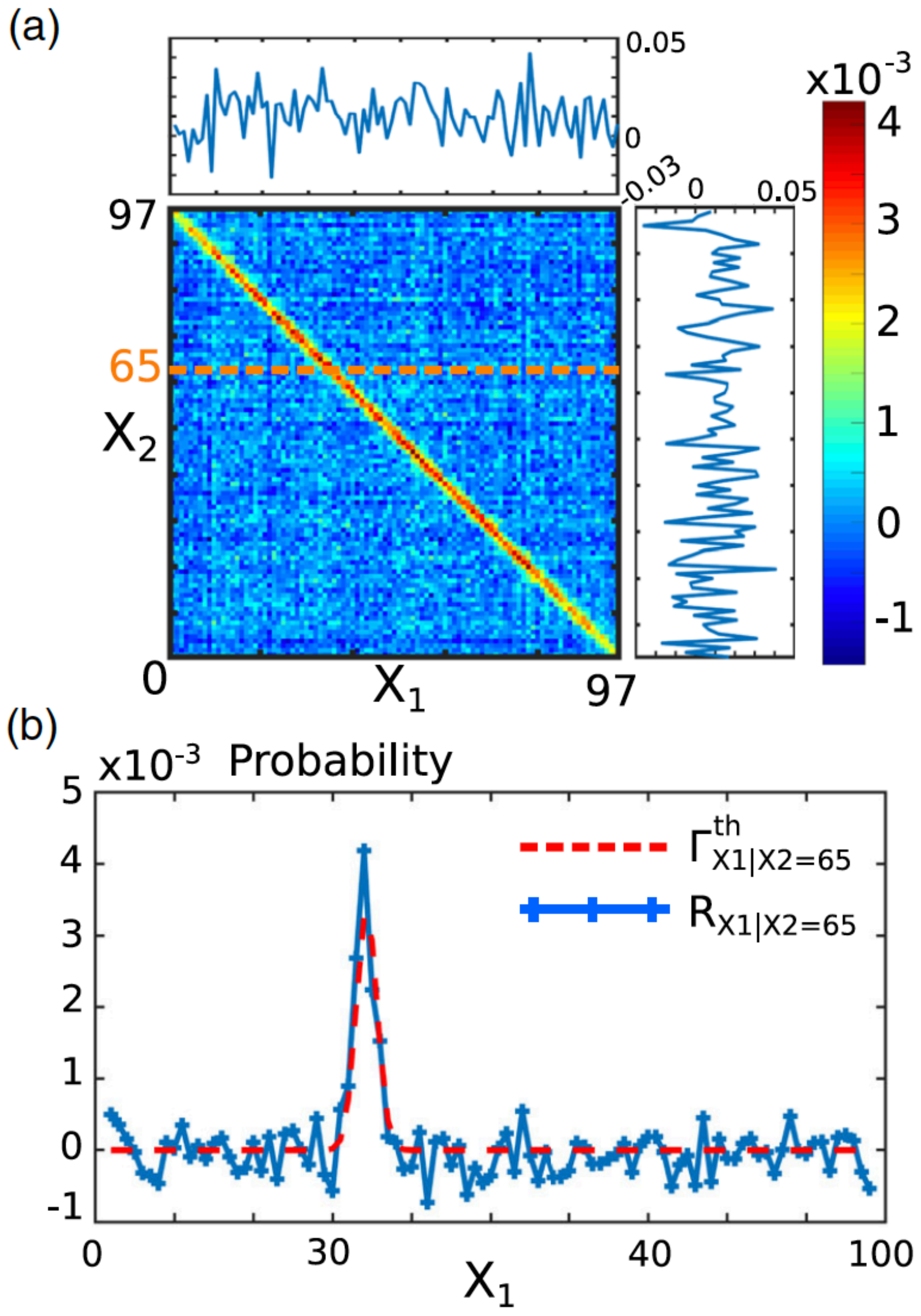
$$\langle x_i \rangle = x_0 + 2A\bar{m}\eta\Gamma_i$$

$$\langle x_i x_j \rangle = x_0^2 + 2Ax_0\bar{m}\eta(\Gamma_i + \Gamma_j) + 4A^2(\bar{m}^2 + \sigma_m^2 - \bar{m})\eta^2\Gamma_i\Gamma_j + 2A^2\bar{m}\eta^2\Gamma_{ij}$$

$$\boxed{\Gamma_{ij} = \frac{1}{2A^2\bar{m}\eta^2}\Big(\langle x_i x_j \rangle - \langle x_i \rangle\langle x_j \rangle\Big)}$$

Double-Gaussian function

$$\Gamma_{ij}^{th} = ae^{-\frac{(x_i + x_j)^2}{4\sigma_+^2}}e^{-\frac{(x_i - x_j)^2}{4\sigma_-^2}}$$

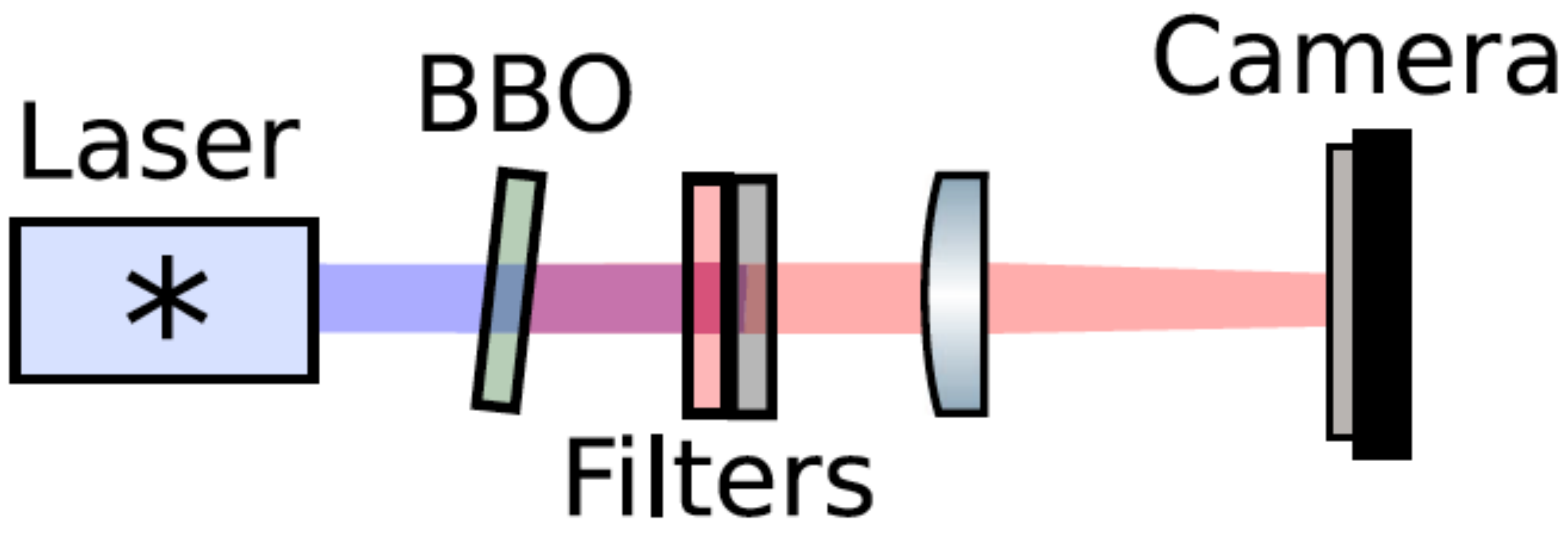


EMCCD camera

$$\boxed{\Gamma_{ij} = \frac{1}{2A^2\bar{m}\eta^2}\left(\langle x_i x_j\rangle - \langle x_i\rangle\langle x_j\rangle\right)}$$

$$R_{ij} = \langle x_i x_j\rangle - \langle x_i\rangle\langle x_j\rangle$$

$$\approx \frac{1}{M}\sum_{l=0}^{M} x_i^{(l)} x_j^{(l)} - \frac{1}{M^2}\sum_{l,l',l\neq l'}^{M} x_i^{(l)} x_j^{(l')}$$



Mixed state

- An attenuated coherent state: unfiltered photons from the pump laser.
- A single-photon term: single-photons created by absorption of one of the two photons of a pair during their propagation through the optical system.

✗ Do not affect the intensity correlation measurements

✓ An extra noise term (non-uniform)

Measurement of a mixed state composed of photon pairs and classical coherent light