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Review Lecture

Anti-sound

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The principles by which acoustic and vibrational fields can be mimicked and cancelled by secondary sources are reviewed. Devices for exploiting this aspect of linear wave fields have been discussed and experimented with for some fifty years but it is only in the last decade that active noise control has emerged as a practical possibility. Known applications are reviewed, their performance is summarized and what constraints currently limit system performance are discussed. Developments are described that link the 'anti-sound' problem with that of adaptive beam forming in antenna systems, and both deterministic and statistical criteria for optimal adaptation are discussed. These developments concern multi-degree-of-freedom systems of wide bandwidth, and their application is strictly limited to linear fields. Some nonlinear fields may also be amenable to active control and the paper ends with some speculative discussion of the scope and significance of that area.

Introduction

It is commonly observed that individual conversations remain possible against a noisy background. The ear can tune into a chosen voice at a cocktail party to hear the message that is undistorted by the sometimes louder conversation of neighbours. Signals that retain their identity when superposed on others are the hallmark of linear systems. Incoherent linear signals combine in such a way that the sum of their energies is conserved, but coherent signals interfere in more interesting ways. Destructive interference, fringe patterns (figure 1) and holographic images are all phenomena that depend on the linear superposition of highly coherent parts. The sounds we hear are a mechanical vibration of the air, the ear sensing the pressure fluctuations whose gradients accelerate the material particles. Even very loud noises often involve only minute motions and signal levels. Waves of 1 kHz frequency to which the human ear has high sensitivity, involve particle vibration amplitudes of less than 10^{-4} m and velocity amplitudes smaller than 10^{-4} of the speed at which sound propagates. Higher amplitude waves quickly cause permanent damage to the ear, damage occurring at this frequency whenever the pressure fluctuations exceed 10^{-3} of the mean atmospheric value. Such disturbances are small enough that they can usefully be described by the linearized approximation of the equations of motion, and it is that linearity that permits the various elements of a noise field to be superposed without change of character. Noise is used here

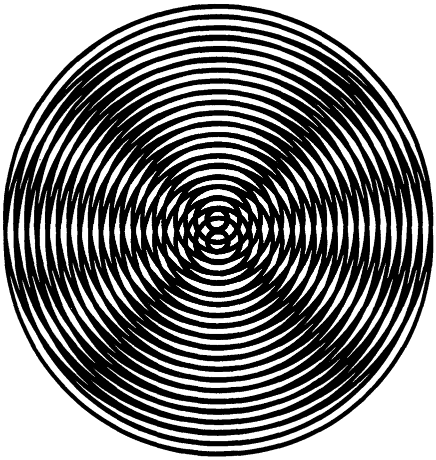

FIGURE 1. The interference pattern between two nearby coherent sources.

as a subjective expression that implies the sound is unwanted; it has no distinguishing physical characteristic.

Two pressure fields arranged to overlap precisely with exactly opposite waveforms would destructively interfere to produce the constant pressure which is the condition of silence. This rather contorted view of silence regards the null field as the superposition of sound and *anti-sound*. This view has emerged over the last half century with the realization that secondary fields can be produced artificially to be superposed in antiphase to suppress primary fields. That process is one of *active noise control* those words being synonymous with anti-sound.

Destructive interference has long been appreciated as the source of great character in acoustic fields. It is the essence of Stokes's explanation (Stokes 1868) of Leslie's experiment in which the audibility of a bell ringing in a partly exhausted bell jar is diminished by the introduction of hydrogen (Leslie 1821). It is also a central theme of Sir James Lighthill's theory of aerodynamic sound production (Lighthill 1952). Aerodynamic sources are naturally arranged in a closely packed quadrupole array in which only a small fraction of the acoustic potential available from any single element of the array escapes the destructive interference. But the term anti-sound is only used for those deliberately created waves that are produced by a controlled source that is superposed on an existing noise field for the purpose of artificially creating a destructive interference. Arthur C. Clarke's *Silence please* (Clarke 1957) in which an opera singer's voice is cancelled by a device which broadcasts its antidote is no longer as fanciful as it once appeared, though Clarke failed to appreciate the energetics of the process: as indeed have some later scientific writers. Craig Thomas's *Sea leopard* anticipates the prospect of a submarine being made immune to SONAR (Thomas 1981); the first steps in that direction might well be the active control of acoustic echoes reported by Guicking *et al.* (1983) in an experimental arrangement with simple geometry.

The scientific development of a subject is inevitably a progression starting from simple experimental arrangements. Paul Lueg's (1936) U.S. patent number **2043416**, a patent that was originally filed in Germany on **27** January **1933**, marks the serious beginning of active noise control. His device was aimed at the suppression of a simple sound wave propagating in one dimension, and modern examples of active controllers still concentrate predominantly on that geometry. Twenty years later Olsen & May (1953) described successful experiments with their 'electronic sound absorber', experiments that mark the beginning of a development that has made possible compact ear defenders in which the ear cavity is actively maintained at the silent condition of constant pressure (Wheeler 1981). It is then a straightforward matter to superpose on this silence signals that are deliberately decoupled from the controller, and that technique allows voiced messages in an environment that would otherwise drown out all possibility of aural communication. The headgear worn by pilots of helicopters, high performance aircraft and powerful surface vehicles are reaching an advanced level of sophistication in this respect.

It is the remarkable developments in modern electronics that has made active control schemes viable outside the laboratory. In particular, the ease with which previously difficult signal conditioning can be achieved with modern digital techniques has revolutionized the subject. Much of the development concerns technique rather than principle and this aspect tends to inhibit communication between the various schools involved in its development. Just as Paul Lueg's first communication was through the publication of a patent, now there are commercial ambitions that act to stifle free and open debate. The interested scientist newly embarking on a study of the subject will be amazed at the style of some recent publications which seem sometimes to be written with a deliberate level of imprecision, the true position and more grandiose interpretations of the words being equally acceptable rationalizations of the printed jargon. Temporal descriptions of a linear control process seem to have been advocated as if they were different from their alternative descriptions in the frequency domain; a large proportion of the literature concerns particular source or receiver arrangements which though interesting at the detailed level of device optimization add very little to the subject in general. It is an unfortunate fact that engineers in this country are among the worst offenders at offering mysterious prospects that encourage the casual reader to expect the early introduction of some transistorized pocket source of silent bliss!

The facts seem to me to be quite different from the simplistic picture visible through the eyes of vocal academic entreprenuers. There exist definitely promising prototype devices that have proved that the subject is poised for commercial use, but the scope of those devices is limited to simple geometrical arrangements at generally low frequencies. This restriction is at once an advantage and an inhibitor of widespread use. Low frequency noise sources are notoriously difficult to silence by conventional passive means and anti-sound techniques offer for them the best prospect of control. But low frequency noises are only very annoying when produced at high power by inevitably bulky and expensive equipment. The production of active suppression for such large scale sources is expensive and for that reason the technology advances are centred on specialist devices that can justify the expense of product development at a scale that is usually beyond the scope of academic activity. This feature also acts to retard the easy dissemination of knowledge for not only are the investors in the technology sometimes anxious to play down the seriousness of a real noise nuisance, they are also anxious to reap what financial reward might accrue from the eventual production of marketable equipment.

The generation and control of linear sound fields

Linear sound fields in a homogenous medium have weak pressure variations, density variations, potential and particle velocity fields all of which conform with the homogenous linear operator,

$$\partial^2/\partial t^2 - c^2\nabla^2 = 0. \tag{1}$$

The source of the waves is largely a point of view. For example, just as the light field reconstituted in a region through a hologram is indistinguishable from that of the real event, so that ambiguity carries over also to sound; perfectly silent source fields are possible. Consider, for example, a source field q of the wave field ϕ and consider that the source field is non-zero only over some bounded source domain. This combination is such that

$$\partial^2\phi/\partial t^2 - c^2\nabla^2\phi = q, \tag{2}$$

and the wave field ϕ is undisturbed if the source field is supplemented by the additional (but null) source field $\partial^2 q/\partial t^2 - c^2\nabla^2 q$. This is because the defining equation

$$\partial^2\phi_1/\partial t^2 - c^2\nabla^2\phi_1 = q + \partial^2 q/\partial t^2 - c^2\nabla^2 q \tag{3}$$

can alternatively be rewritten as

$$\frac{\partial^2\phi}{\partial t^2}(\phi_1 - q) - c^2\nabla^2(\phi_1 - q) = q, \tag{4}$$

a form that makes clear the fact that outside the source region where $q = 0$,

$$\phi_1 - q = \phi. \tag{5}$$

For this reason the definition of the sound source is rather arbitrary and equation (2), which, together with a radiation condition, provides the basis for unambiguously calculating the sound field in terms of q, is just one convenient definition of the source strength. Many other different but equally correct viewpoints could be adopted.

The significance of this observation about source ambiguity in the context of active noise control is that, if two different source distributions can generate the same wave field and one source distribution is under our control, then a simple change in sign makes the primary noise field subject to extinction by the presence of the secondary. Furthermore, even though the source of anti-noise can be of a completely different construction to that of the primary field, the silence is, in principle, achievable everywhere outside the source distribution.

Nor is the position of the source of a wave field well defined. Kempton (1976) has pointed out how the multipole expansion allows the field of a source to be mimicked by that induced by an appropriate series of sources positioned at any desired point. Furthermore, by arranging for that source series to mimic the negative of the primary field, Kempton demonstrated how the widespread annihilation of the sound field would result from the action of a properly constituted anti-sound multipole array. Though this observation has yet to receive material realization it holds open the prospect of anti-sound control of a source field that leaves unimpaired the action of the primary source. But that prospect is a long way off. The expansion he used to represent the field by concentrated singularities fails to converge in the vicinity of the sources and it remains far from clear what practical device might be constructed to approximate the field-mimicking quality of his intriguing proposition.

A less ambitious but more obviously achievable objective characterizes what may be termed the French School of thought. Kirchhoff's theorem provides a formula by which the entire effect of sources contained within a closed boundary can be duplicated by prescribed sources on that boundary; if their opposite is mimicked instead then the combination of primary sources with the surrounding surface distribution of secondary anti-sources constitutes a silent exterior field (figure 2). Alternatively, Kirchhoff's theorem can be used in the form that prescribes

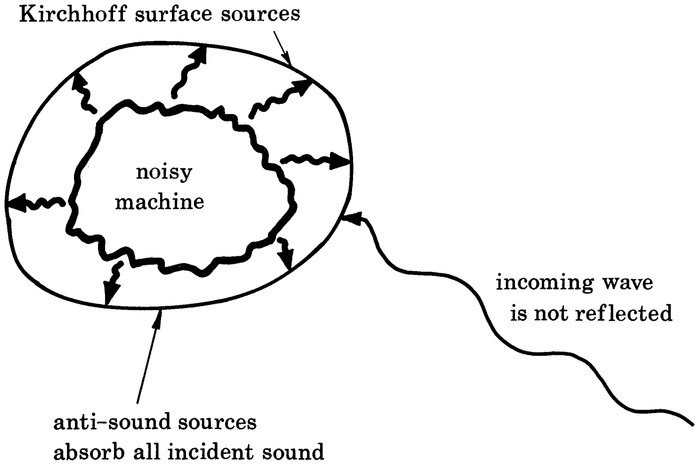

FIGURE 2. Anti-sound sources on a surface surrounding a noisy machine contain its sound and incidentally prevent any echoes of an exterior source.

a source distribution on a closed surface which completely reproduces an interior primary field. Again if the sign of these sources is changed, the surface distribution of negative Kirchhoff sources ensures that the interior is maintained in a state of complete silence. Jessel & Angevine (1980), and Jessel (1972, 1983), in interesting papers with an extensive bibliography, have developed this view theoretically and taken also the first steps towards an experimental demonstration. It is appropriate to dwell on the proposition at some length because it embodies a very wide class of techniques. Their construction of the source field needed to mimic (in anti-phase) a primary field through the application of the Huygens principle is one member of a variety of methods of wave reconstruction, a member that is distinguished by the property that it leaves unchanged the incident wave field outside the region of control; i.e. it is also a means of echo suppression.

At one level the fact that surface conditions around a source can be constrained to inhibit wave propagation beyond the surface is trivial. An impenetrable rigid enclosure does just that as would a surface on which the pressure were artificially maintained at a constant level. Both those would prevent propagation by reflecting the primary field back on itself and would involve no energy production or extraction at the control surface. But there are many other possibilities and some of them are potentially much more attractive.

The hope is that physically connected regions might one day be acoustically separated by a sparse array of secondary sources that constitute an acoustic screen.

The analysis of this situation is extremely complex, and probably not yet attempted in the discrete element approximation of the continuous surface, and it would be wrong to suggest that the technique has moved appreciably from the statement of principle embodied in the theoretical analysis of the continuous case. But that analysis is interesting enough and sufficiently illustrative of current ambitions for the subject that it is worth emphasizing here.

Consider a closed control surface S enveloping a volume V in which there are no acoustic sources and in which it is desired to maintain silence. Any waves inside the volume would have the potential ϕ such that

$$\partial^2\phi/\partial t^2 - c^2\nabla^2\phi = 0. \tag{6}$$

Now suppose that a Green function G is known: a function constrained by the equation

$$\partial^2 G/\partial t^2 - c^2\nabla^2 G = \delta(\boldsymbol{x}-\boldsymbol{y}, t-\tau), \tag{7}$$

for both $\boldsymbol{x}$ and $\boldsymbol{y}$ lying within V, but undetermined to the extent that G can be driven by any source distribution whatsoever exterior to V. These equations can be integrated to express the interior field in terms of conditions on the bounding surface, the field being equal to:

$$\phi(\boldsymbol{x}, t) = -c^2 \int_{\mathrm{S},\,\tau} \left\{ G\frac{\partial\phi}{\partial n} - \phi\frac{\partial G}{\partial n} \right\} \mathrm{dS}\,\mathrm{d}\tau, \tag{8}$$

the integral being over the entire surface, over all past time τ, and n signifying the normal to S in the direction leading into V.

Kirchhoff's theorem results from the selection of G under the constraint that (7) is valid everywhere and is subject to the radiation condition, in which case it is equal to:

$$G_{\mathrm{f}} = \frac{\delta(t-r/c)}{4\pi c^2 r}; \quad r = |\boldsymbol{x}-\boldsymbol{y}|. \tag{9}$$

Active control according to this strategy remains at the speculative stage. It is certainly true that if, for example, ϕ were monitored on S and used to drive transducers to induce a surface distribution $\partial\phi/\partial n$ constrained by the requirement that the surface integral in equation (8) vanish, then there would be silence within

V, and the field exterior to V would remain echo-free. The same would obviously be true if $\partial\phi/\partial n$ was monitored and ϕ driven under the same constraint.

Alternatively, G could be made subject to the requirement that it vanish on S in which case the interior field is

$$\phi(\boldsymbol{x}, t) = c^2 \int_{\mathrm{S},\tau} \phi \frac{\partial G_1}{\partial n} \mathrm{dS}\,\mathrm{d}\tau, \tag{10}$$

$$G_1 = 0 \quad \text{on S.}$$

The condition of maintaining silence within a volume surrounded by sound is then posed as one of monitoring and controlling by secondary surface transducers the vanishing of a surface integral defined in terms of the potential alone, and of course the assumed known Green function G_1. Similarly, by insisting that $\partial G/\partial n$ vanish on S a corresponding surface integral involving only $\partial\phi/\partial n$ is obtained. In both cases the field exterior to V would be supplemented by echoes appropriate, respectively, to soft and hard surface conditions on S.

The suppression of radiation out of a source region into an exterior source-free space is in principle accomplished by the same technique, the volume V now being defined as that *exterior* to a surface S that encloses (at least) all the sources (figure 3). Monitoring of field quantities on S and the excitation of secondary fields with surface mounted transducers controlled to null the integral of (8) again provides a control strategy.

Practical developments based on this technique have inevitably involved approximations of the ideal devices foreseen in the theoretical work. Much of the development has been devoted to the suppression of 'transformer hum' by

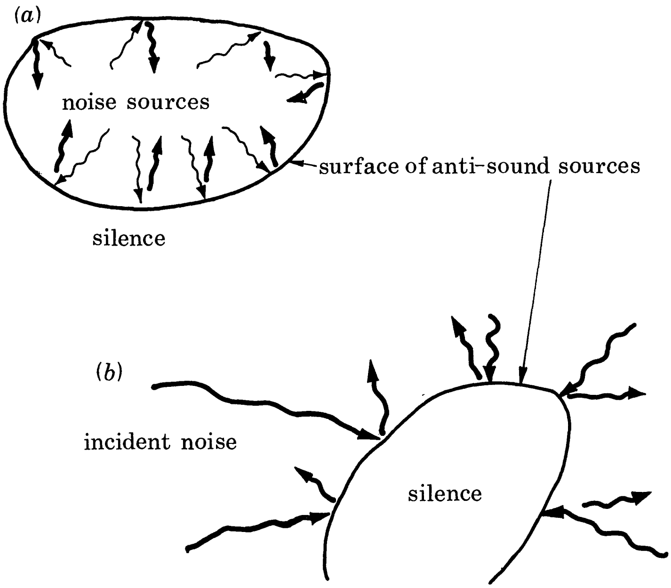

FIGURE 3. An anti-sound source array on a closed surface can either be used to prevent the escape of sound or to maintain a zone of silence in an otherwise noisy area.

appropriately controlled external sources. Ross (1978) and Angevine (1981, 1983) both report successful experiments of this type conducted out of doors on an existing practical noise problem. The laboratory demonstration reported by Mangiante (1977) contains more correlation with the underlying 'Huygens surface' theory but gives less of a feel for the general usefulness of the technique.

Soviet interest referred to by Tartakovskii (1974), Mazanikov & Tyutekin (1974) and Fedoryuk (1974) have included a more general case, namely that of inhibiting the radiation from a source region that is both excited from within *and* from outside (figure 4). They have thus concentrated both on inhibiting the direct radiation

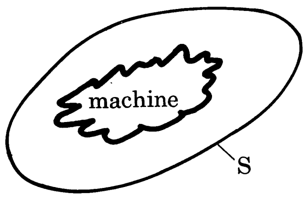

FIGURE 4. A surface of anti-sound sources can prevent both the escape of sound and the generation of echoes.

and the simultaneous prevention of echoes. The fundamental work, that of Malyuzhinets, was published posthumously in 1968. He expressed the total field ϕ outside a region containing the sources to be suppressed as the sum of the field that would be present in the absence of those sources ϕ_i and those scattered by inhomogeneities, the scattered field ϕ_s being expressible as

$$\phi_s(\boldsymbol{x}, t) = -c^2 \int_{S,\tau} \left\{ G_f \frac{\partial \phi}{\partial n} - \phi \frac{\partial G_f}{\partial n} \right\} dS\, d\tau. \tag{11}$$

S is again a surface enclosing the region to be rendered inaudible and n the normal to that surface leading into the volume V containing $\boldsymbol{x}$. G_f is the 'free-space' Green function given by (9) and it is significant that the free field alone contributes nothing to the integral, i.e.

$$\int_{S,\tau} \left\{ G_f \frac{\partial \phi_i}{\partial n} - \phi_i \frac{\partial G_f}{\partial n} \right\} dS\, d\tau = 0. \tag{12}$$

The control strategy for the prevention of noise and of echo suppression is then one of monitoring both ϕ and $\partial\phi/\partial n$ on some convenient closed control surface and driving secondary transducers on that surface under the integral constraint that ϕ_s should vanish. Of course this is easier said than done but it is evident from the extensive Soviet literature in the subject that if there is a principle that renders this goal unachievable that principle has yet to be discovered. Certain discrete frequencies might prove difficult to handle. There seem to be advantages that accrue from the separation of source and receiver surfaces in the active array, and calculations investigating discrete element approximations to continuous control surfaces indicate definitely promising trends at least at low frequencies (see, for example, Konyaev *et al.* 1979).

Though intriguing in concept, this kind of analysis is far removed from any experimental activity reported in the literature and it is probably correct to infer that the impressive recent practical advances in the subject bear little relation to these theoretical observations. At best, realizable surface sources are only sections of the complete surface S and practical ambitions have so far concerned the generation of shadows with simple arrays of microphone and loudspeaker combinations. Mazzanti & Piraux (1983) describe briefly an experiment with a 12 element source array in which attenuations of some 20 dB were measured in the frequency range 250 to 350 Hz; this performance conformed well with their theoretical predictions. Ross (1980) considered the screening properties of similar arrays and showed how their tendency towards instability was likely to pose considerable practical difficulty. In fact Ross demonstrated the requirement for the microphone and loudspeaker array surfaces to be physically separated if such arrays are to be stable, and this conclusion reinforces the earlier work of Zavadskaya *et al.* (1974, 1977) (see figure 5). Both the theory and technology of anti-sound are still in

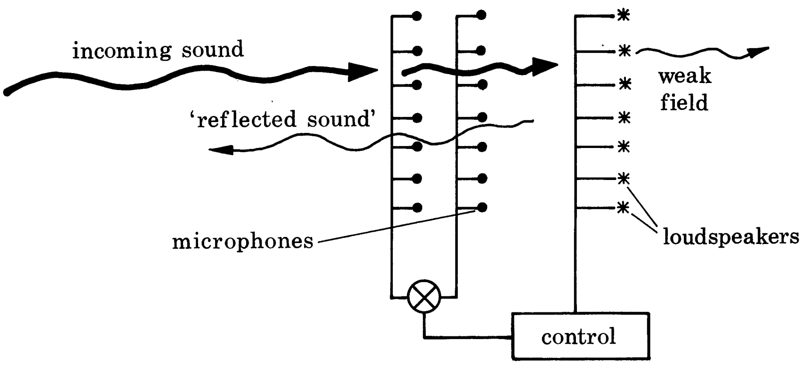

FIGURE 5. A schematic illustration of the separated array of microphone and loudspeaker elements that reflect sound and prevent its passage beyond the active screen.

practice limited either to much simpler geometric arrangements or concerned with techniques that circumvent the need for detailed acoustical calculations by exploitation of the sophisticated data processing and measurement facilities that are now available.

The energetics of anti-sound

Acoustic energy and power are quadratic measures of the sound field and do not therefore add linearly. Precisely how the energy balance is modified when a source of sound competes with an *anti-source* is to be determined after the linear field quantities have been evaluated by superposition. The results are not always in accord with intuition and a whole variety of behaviour can occur in different special cases. Most often it is found that the action of anti-sound is to suppress the power-producing ability of the primary source. But sometimes an anti-source can act to withdraw and consume from a primary source much more sound power than the source could produce in free space or radiate in the presence of the anti-source; the anti-source then acts as a kind of 'sound sucker'. Sometimes also the anti-sound can prevent sound energy escaping to an exterior field only by trapping it in an interior reservoir of ever increasing noise.

Consider for example one of the most popular model problems in active noise control, that of one-dimensional waves in a wave guide. Suppose that a piston at $x = 0$ starts from rest in a duct to maintain at the origin an axial velocity $V_0 H(t) \cos \omega t$, H being the Heaviside function. Suppose also that distance L down the duct is a secondary monopole anti-source controlled to maintain at $x = L^+$ a condition of zero axial velocity for all time. No wave can propagate to $x > L$ with such a control and any wave incident from $x < L$ on to the isotropic anti-source is reflected, in turn to be re-reflected to supplement the wavefield in $L > x > 0$, the entire positive-travelling element of the trapped wave then having a velocity field

$$v_+(x,t) = \sum_{K=0}^{\infty} V_0 H(t - x/c - 2KL/c) \cos \omega(t - x/c - 2KL/c). \tag{13}$$

There are approximately $ct/2L$ elements in this sum which together constitute the positive travelling wave portion of the field whose velocity has amplitude

$$\hat{v} = V_0 |\sin N\omega L/c / \sin \omega L/c|,$$

N being the integer closest in value to $ct/2L$. The 'left travelling' wave has the same amplitude structure in the insonified region.

The wave activity, confined totally to the region between the sources, evidently jumps in amplitude as waves are successively reflected between the two sources (figure 6). Unless the frequency is such that $\omega L/c$ is an integral number times π, the wave amplitude is bounded, changing with each reflection to some value between 0 and $V_0 \operatorname{cosec} \omega L/c$; figure 7.

No energy can be contributed from this totally reflecting control source so that the energy density of the wave field, which changes up and down with successive reflections, but never exceeds a level of $V_0^2 \rho \operatorname{cosec}^2 \omega L/c$ must be maintained by the irregular output of the primary source. Although this maintains a regular vibration, it is conditioned to modulate its power, and sometimes even absorb energy, by the action of the control at $x = L$.

At those resonance frequencies of the insonified guide, when $\omega L/\pi c$ is an integer, then the velocity amplitude of both positive and negative travelling waves grows with each reflection to the every increasing value $(ct/2L) V_0$. The energy in the insonified region then increases in proportion to t^2 as the control source conditions the environment of the primary source in such a way that its power output increases linearly with time. The piston's velocity is then maintained at its prescribed value against the ever increasing back pressure induced by the action of the anti-source whose strength increases with time to exercise the necessary control.

The anti-source that determines this power demand of the primary source does no work and needs no power supply; it could be realized by purely passive means, in this case trivially by closing the duct with a rigid diaphragm at $x = L$. But the actively controlled source can effect the same blocking of wave passage while

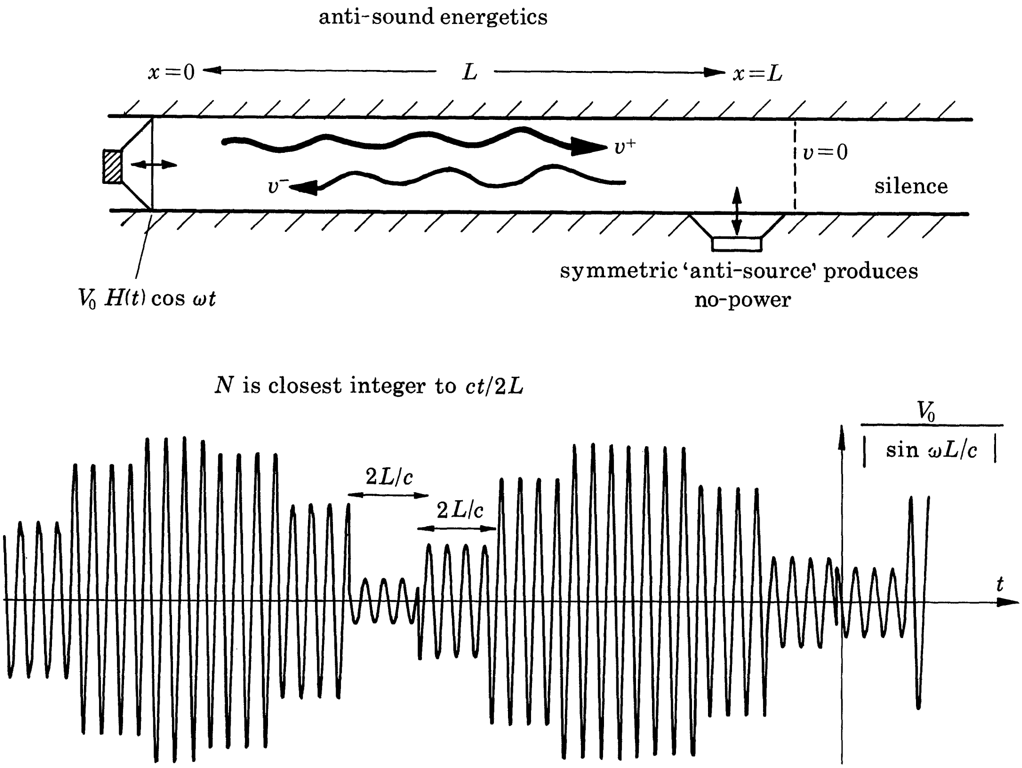

FIGURE 6. Anti-sound energetics.

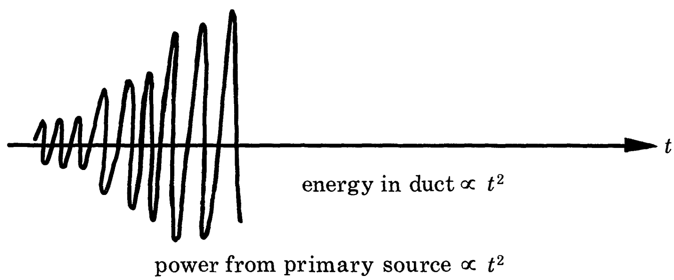

FIGURE 7. Source activity in a resonant duct, when $\omega L/c = n\pi$, causes the signal level to increase without bound.

leaving open a physical connection between the silent and insonified region, a loud-speaker element at the side wall of the duct being a suitable and convenient arrangement for demonstration. The practical device will necessarily have losses that induce bounds on resonance amplitudes but the principles of operation are essentially those of the foregoing idealization. A strong anti-source field can condition primary sources of predetermined strength to give up a large amounts of energy; the sound output of a source depends on the source impedance and this can be controlled by a secondary source.

A secondary elementary example that illustrates interesting energetic properties concerns three-dimensional isotropic sources arranged in a dipole-like array. Consider the superposition of two fields induced by harmonic monopoles. Each monopole in isolation would produce at distance r a pressure field

$$p_{\mathrm{m}} = (p_0/r)\cos\omega(t-r/c), \tag{14}$$

where p_0 is the pressure produced by the source at unit distance at frequency ω and is a convenient measure of source strength. The mean square pressure induced by the isolated source at distance r is

$$\overline{p_{\mathrm{m}}^2} = \tfrac{1}{2}p_0^2/r^2 \tag{15}$$

and its acoustic power output is

$$P_{\mathrm{m}} = 2\pi p_0^2/\rho c, \tag{16}$$

ρ being the mean mass density of the material and c the speed of sound.

Now consider this monopole source to be placed alongside its phase shifted anti-source, which in isolation would also produce an identical mean square field and power output. The two sources are separated by a distance Δ. The total double field is now

$$p_{\mathrm{d}} = (p_0/r)\cos\omega(t-r/c)-(p_0/r')\cos[\phi+\omega(t-r'/c)]. \tag{17}$$

The far field of this double source assembly is the limit as r tends to ∞,

$$\lim_{r\to\infty} p_{\mathrm{d}} \sim (p_0/r)\{\cos\omega(t-r/c)-\cos[\phi+\omega(t-r/c+\Delta/c\cos\theta)]\}, \tag{18}$$

from which it is clear that some ϕ, actually ($\phi = -(\Delta/c)\cos\theta$), could be chosen to guarantee that in any chosen direction this double source combination would guarantee silence. θ is the angle between the line of source separation and the distant observer (figure 1).

The mean square far-field pressure at (r,θ) from this double source arrangement is

$$\overline{p_{\mathrm{d}}^2} = (p_0^2/r^2)\{1-\cos(\phi+[\omega\Delta/c)\cos\theta])\}. \tag{19}$$

The maximum mean square pressure level radiated by the source pair is four times that achievable with one source while the minimum sound achieved in those directions for which $\phi+(\omega\Delta/c)\cos\theta = 2n\pi$, is zero. These are the conditions of constructive and destructive interference.

The sound power output from the double source pair is

$$P_{\mathrm{d}} = 4\pi(p_0^2/\rho c)\left(1-\frac{\sin\omega\Delta/c}{\omega\Delta/c}\cos\phi\right). \tag{20}$$

The maximum power is arranged when $\omega\Delta/c$ is very small and the phase ϕ is set equal to π. This is four times the power output of either one of the sources in isolation (cf. equation (16)) and is the condition when the sound of both sources are in phase. On the other hand, the minimum global sound power radiation is achieved when $\phi = 0$ and $\omega\Delta/c \ll 1$. In the limit this double source array constitutes a dipole of notorious inefficiency as a sound radiator. It is curious to note that when ϕ is chosen to be $\frac{1}{2}\pi$, then the power radiated by the pair of sources to the surrounding field is always simply twice that which would be radiated by either source in isolation. This is curious because, as we shall see, the power output from individual members of the pair can vastly exceed this amount. To show this we consider the structure of the field near the individual sources. The pressure near one is

$$p_{\mathrm{m}} = (p_0/r)\cos\omega(t-r/c), \tag{21}$$

and the radial velocity, determined from the equation of momentum conservation, is

$$v_{\mathrm{r}} = (p_0/r\rho c)\left\{\cos\omega(t-r/c)+\frac{\sin\omega(t-r/c)}{\omega r/c}\right\}. \tag{22}$$

The anti-source nearby, with phase shift ϕ, generates at the position of the primary source a pressure field

$$p_{\mathrm{a}} = -(p_0/\Delta)\cos\{\phi+\omega(t-\Delta/c)\}. \tag{23}$$

The power output of the primary source is the integral of the intensity pv_{r} over a small surface surrounding that source

$$P = \lim_{r\to 0}\overline{(p_{\mathrm{m}}+p_{\mathrm{a}})\,v_{\mathrm{r}}}\,4\pi r^2. \tag{24}$$

$$P = \lim_{r\to 0} p_0\left\{\frac{\cos\omega(t-r/c)}{r}-\frac{\cos[\phi+\omega(t-\Delta/c)]}{\Delta}\right\} \times\frac{p_0}{r\rho c}\left\{\cos\omega(t-r/c)+\frac{\sin\omega(t-r/c)}{\omega r/c}\right\},$$

i.e.

$$P = 2\pi\frac{p_0^2}{\rho c}\left\{1+\frac{\sin(\phi-\omega\Delta/c)}{\omega\Delta/c}\right\} \tag{25}$$

$$= 2\pi\frac{p_0^2}{\rho c}\left\{1-\frac{\sin\omega\Delta/c}{\omega\Delta/c}\cos\phi+\frac{\cos\omega\Delta/c}{\omega\Delta/c}\sin\phi\right\}. \tag{26}$$

Comparing this sound power from an individual source with that produced by the source pair, equation (20), we see that the primary source produces more power than its half of the total field by an amount

$$2\pi\frac{p_0^2}{\rho c}\frac{\cos\omega\Delta/c}{\omega\Delta/c}\sin\phi; \tag{27}$$

this can be very large indeed. This excess production of power is absorbed by the anti-source, its neighbour having a negative relative phase ϕ, so that it produces a power output of

$$2\pi\frac{p_0^2}{\rho c}\left\{1-\frac{\sin\omega\Delta/c}{\omega\Delta/c}\cos\phi-\frac{\cos\omega\Delta/c}{\omega\Delta/c}\sin\phi\right\}; \tag{28}$$

the sum of this power and that of the primary source, equation (26), is that appearing in the surrounding sound field, equation (20).

When the two sources are close together, and $\phi = \frac{1}{2}\pi$, the sound power radiated to the environment from each source is unaffected by the presence of its neighbour, but energy is transferred from the primary to the anti-source at a rate

$$\frac{2\pi p_0^2}{\rho c} \frac{\cos \omega \Delta / c}{\omega \Delta / c}. \tag{29}$$

This can be very large indeed when the sources are close enough that $\omega \Delta / c \ll 1$. In this case the anti-source is sucking energy out of the primary source in an exceptional way, leaving the surroundings virtually unaffected.

A monopole source placed near its anti-source, which is simply another monopole of exactly opposite field-generating ability, will radiate only a small residual field whenever source and anti-source are separated by a distance small in comparison with the acoustic wavelength. The pressure fluctuation in the vicinity of the source is then suppressed by the action of its neighbour. Practical devices to achieve this kind of control rarely have the simplistic geometrical arrangement of the idealized examples but they all have in common a competition between the primary source's pressure-generating ability and that of the anti-source to suppress that action by the superposition of a carefully controlled secondary field. The controlled suppression of pressure variation in one region is often achievable by the technique suggested by Olson & May (1953), and we shall consider now some aspects of devices based on that technique.

Small zone silencers

Suppose now that at a particular point in space that is normally irradiated by an incident sound wave of perturbation pressure p_i, we wish to exercise control by superposing there an anti-sound field of pressure p_a generated by a nearby loudspeaker. Suppose also that a microphone, or some other sensor, monitors the incident field in advance but does it imperfectly and gives a contaminated estimate $p_i + N$ of the sound that is to be controlled at the microphone M. The noise contaminant N is uncorrelated with the incident field. The control loudspeaker is driven to produce an anti-sound field p_a which is added to the estimate of the incoming sound before amplification by a factor $-A$ before the whole being supplied to the loudspeaker. Figure 8 illustrates this arrangement with supposedly perfect unit gain addition, monitoring, coupling and broadcasting elements, an arrangement that ensures

$$p_a = -A(p_i + p_a + N) \tag{30}$$

$$= \frac{-A}{1+A}(p_i + N). \tag{31}$$

The sound pressure at the microphone M is thereby adjusted from its uncontrolled value p_i to;

$$p_i + p_a = p_i/(1+A) - AN/(1+A). \tag{32}$$

Essentially this scheme was proposed and tested by Olson & May (1953) and shown to be useful. They did not analyse its performance but gave experimental results to indicate that the controlled sound field could be made much less intense

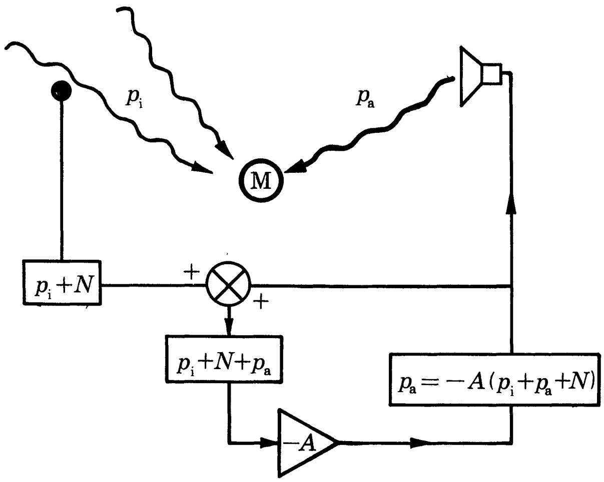

FIGURE 8. Small zone silencers.

than the incident field, a fact that is evident from (32). In the absence of system noise the performance seems limited only by the available system gain A. In practice the imperfections of the system and the well known tendency to instability of a loudspeaker in close proximity to the microphone whose amplified signal it broadcasts, puts definite constraints on performance, but the essentials of a very useful scheme are contained and this example.

The performance is of course limited also by the system noise N, and the optimal controller is one in which the gain A is high enough to overcome the incident field but not so high that it amplifies unreasonably the system noise.

The mean square sound level at the microphone must be minimized by the optimal controller, i.e.

$$\frac{\partial}{\partial A}\overline{(p_a+p_i)^2} = 0; \tag{33}$$

the overbar signifying a mean value;

$$\overline{(p_a+p_i)^2} = [\overline{p_i^2}+A^2\overline{N^2}]/(1+A)^2 \tag{34}$$

has a minimum value

$$\overline{(p_a+p_i)^2_m} = \overline{p_i^2}/(1+\overline{p_i^2}/\overline{N^2}) \tag{35}$$

when the controller gain A is set equal to

$$A_m = \overline{p_i^2}/\overline{N^2}. \tag{36}$$

At this optimal gain setting there is no correlation between the loudspeaker output p_a and the sound pressure at the microphone $(p_a+p_i)_m$,

$$(p_a+p_i)_m = (p_i - N\overline{p_i^2}/\overline{N^2})/(1+\overline{p_i^2}/\overline{N^2}), \tag{37}$$

$$\overline{p_a(p_a+p_i)} = \overline{\frac{-A}{(1+A)}(p_i+N)\left\{\frac{p_i}{1+A}-\frac{AN}{1+A}\right\}} = \frac{-A}{(1+A)^2}\{\overline{p_i^2}-A\overline{N^2}\}, \tag{38}$$

a correlation that vanishes for the optimal controller according to (36). Neither is there any correlation between the contaminated estimate $(p_i + N)$ of the incident field and the optimally controlled field whose value is given in (37),

$$\overline{(p_i + N)\,(p_i - N\overline{p_i^2}/\overline{N^2})/(1 + \overline{p_i^2}/\overline{N^2})} = 0.$$

This observation allows the correlation to be monitored and assessed as a measure of how close the controller is to an optimal condition.

The level of the optimally controlled sound field is in this case determined entirely by the corruption of the microphone estimate of the incident field, the estimate $p_i + N$ having a correlation coefficient of R with the real signal p_i,

$$R = \frac{\overline{(p_i + N)\,p_i}}{\{(\overline{p_i^2} + \overline{N^2})\,\overline{p_i^2}\}^{\frac{1}{2}}} = (1 + \overline{N^2}/\overline{p_i^2})^{-\frac{1}{2}}. \tag{39}$$

The optimal controller sets

$$A_m = R^2/(1 - R^2) = \overline{p_i^2}/\overline{N^2} \tag{40}$$

to produce a minimum sound level of

$$\overline{(p_a + p_i)_m^2} = \overline{p_i^2}(1 - R^2), \tag{41}$$

i.e. an attenuation factor of $(1 - R^2)$ on the local intensity of the sound.

The notorious instability of high gain feedback systems of this kind (instabilities that arise from the inevitable acoustic time delay between the loudspeaker broadcasting p_a and the microphone receiving it) led Ross (1980) to modify the Olson system by subtracting from the amplifier input that part of the measured signal caused by the loudspeaker, whose driving signal is continuously monitored. In that case the loudspeaker induced pressure p_a is simply $-A(p_i + N)$, so that the environment of the microphone is at pressure

$$p_i + p_a = (1 - A)\,p_i - AN. \tag{42}$$

The optimal controller for this system again sets

$$\frac{\partial}{\partial A}\overline{(p_i + p_a)^2} = 0 \tag{43}$$

by choosing
$$A_m = (1 + \overline{N^2}/\overline{p_i^2})^{-1}, \tag{44}$$

at which condition the monitored sound field is

$$(p_a + p_i)_m = \frac{p_i - \overline{p_i^2}/\overline{N^2}\,N}{(1 + \overline{p_i^2}/\overline{N^2})}, \tag{45}$$

just as was the case with the optimally adjusted Olson system, equation (37). Again there is no correlation between the microphone signal and the optimally adjusted loudspeaker output, and the sound in the environment of microphone M is controlled to be less than that with no control by a factor $(1 - R^2)$; R is the normalized correlation between the estimate of p_i available to the controller and p_i itself. Without a coherent estimate of the sound to be controlled no control is possible, and the $(1 - R^2)$ constraint on the magnitude of the attenuation achievable is absolute and general. A correlation coefficient of 90 % between control signal and sound limits the attenuation achievable with a perfect controller to 7 dB; 99 % to 17 dB; 99.9 % to 27 dB and so on. The bounds on achievable anti-sound performance are discussed more fully in the recent paper by Ffowcs Williams *et al.* (1984).

The quality of the estimate of the field to be controlled is crucial. In practical systems the optimal estimate is obtained with an array of sensors, sensor signals being combined and weighted according to the techniques of adaptive beamforming in antenna systems. Roebuck (1982) gives a comprehensive account of these techniques and demonstrates the efficiency with which such weighted arrays can discriminate against corruptive elements and highlight and signal content.

It would be most misleading to imply that the various elements of these zone-silencing arrangements are either easily identified or constructed. In practice some of the connections are acoustical and some electrical, all involve time delays, phase and amplitude changes, and accounting for them is not only intricate but taxes the ingenuity of inventors to overcome inherent tendancies to instability. The main constraint on stability is imposed by the difficulty of getting a good estimate of the sound to be controlled in time that allows the compensating networks to react.

It is for this reason that the basic Olson system is depicted here as driven by a remote estimator of the incident sound field. An estimate can often be determined in advance of the time of control and so make easier the demands on computational speed in the electronic networks. Olson & May's original device had no such provision and neither did their discussion of it involve any mention of noise. Their amplifier was fed directly with the microphone signal at M, and recent attempts to duplicate their experimental results have revealed severe instability problems, some of which are avoided by the technique of measuring in advance an approximate value of what sound signal the controller will be called upon to suppress. Ross (1980) reports the successful construction of a small zone silencer of this type that controlled broad band noise over more than two octaves with a maximum attenuation of some 20 dB. He pointed out the need to compensate for the distorting influence of imperfect components of the system, both mechanical and electrical, and this he achieved with digital filtering techniques. He reports also an unfortunate tendancy for the arrangement to become unstable and howl whenever the acoustical transfer characteristics of the microphone and loudspeaker environment are altered, as they are when a human head is inserted into the quiet zone to hear the benefit of the device!

Small zone silencers of this kind are in practice limited in scope to situations of constant geometry and there are at least two categories of problems for which their application has been demonstrated and proved effective. The first such application is to the cavity between the ear and earpiece in 'earphone' type installations. Applications are reported to be in an advanced stage of development, Wheeler's system having robust broadband sound reducing ability on which can be superposed communication signals or aural entertainment (Wheeler 1981). Jones & Smith (1983) describe a similar device that is specially adapted to the removal of periodic noise in which the seven most dominant spectral lines are suppressed by more than 30 dB. It seems that headgear of this type is ready for commercial use and that it is only a matter of time before it is widespread. Two features help enormously to simplify the task of engineering a successful system of this type. First, the volume to be silenced is very small in comparison with the wavelength of sound to be controlled. Phase changes throughout the volume are therefore negligible and the control system need deal with only one degree of freedom. Second, there is usually a passive 'ear defender' element of the headgear that can be used to decouple partly the interior sound from that outside. This can be monitored to give a slightly advanced measure of the incident sound field, a measure that is only weakly contaminated by the interior control signal.

A related modern practice, in which two estimates of a noise field are combined to highlight the signal content, is illustrated schematically in figure 9.

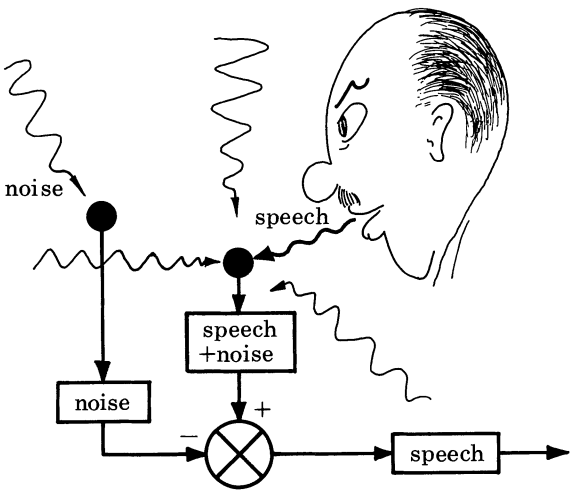

FIGURE 9. A schematic illustration of how multiple microphone estimates of a sound field can be combined to enhance signal levels. The apparatus has been developed by Racal Acoustics and is ready for quantity production.

The second major development is at the opposite end of the dimensional scale and is a practical application of the technique expected by Olson & May (1953) to be suitable for preventing noise escaping from air-conditioning ducts. A small zone silencer was imagined by them to be positioned at the duct exit and the maintaining there of a field of constant pressure makes impossible any transfer of sound out of the duct (cf. figure 10). By suppressing acoustic activity at what is the source region for the exterior field, the benefits of the local silencer are consequently felt everywhere.

The basic skills of selective monitoring of the incident field, the design of stable control algorithms, compact high speed digital computers and data processing equipment had become available in the mid-1970s, partly through the success of research activities at the University of Cambridge; M. S. Swinbanks's contribution is especially outstanding (Swinbank 1973). The British Gas Corporation had a need to investigate all possible means of controlling the noise emission from gas-turbine driven compressor installations, and in 1978 they, with the National Research and Development Corporation, commissioned the construction of a full-scale anti-sound

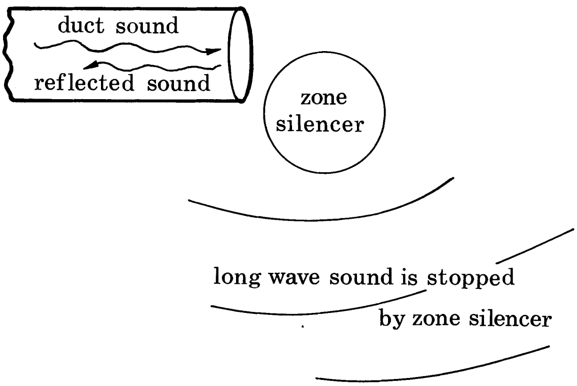

FIGURE 10. A 'small zone silencer' located at the open end of a duct maintains there a condition of silence and prevents the scope of sound from the duct opening.

controller as a test of the technology and as a demonstration to determine the performance and reliability of such a system. The design attenuation of 10 dB of broadband control over an octave was accomplished with equipment that has now run successfully without need for maintainance for nearly three years. One and a half tons of loudspeakers are driven by amplifiers capable of handling 12 kW of power at an average consumption of 1 kW to render inaudible the previously characteristic low frequency rumble of the exhaust flow. Ffowcs Williams (1981) gives some account of this installation though no technical details are disclosed; that information is the property of the N.R.D.C. who are actively seeking to license the technology for routine manufacture.

The schematic outline of the control strategy given above is consistent with the spectral techniques used on such devices. There the signals, depicted in this review as if they were simple functions of time, are handled as complex amplitudes of various spectral elements of the field, and the linear processes performed by the control circuitry are represented by transfer functions which are measured and modified by specially constructed filters. A different style of approach is needed for repetitive sounds and, on the whole, their control by active means is very much more straightforward. Again a single degree of freedom system is useful only if control is needed at a particular point, though control at the location of a compact primary source will effect widespread benefit. Such is the case with the pulsatile exhaust flow of internal combustion engines, and techniques to suppress these pulsations have been brought to an advanced stage by Chaplin (1983). Control strategies and algorithms for monitoring and continually optimizing performance have been developed and used on Diesel engine exhausts and for controlling a variety of other noise and vibration problems. So far they are not in widespread use but the expanding literature on the subject indicates the techniques to be understood widely and available for use. Ross (1982) gives details of effective control algorithms, Crocker (1982) describes several experiments in detail and Smith & Chaplin (1983) indicate that a wide variety of industrial noise and vibration problems can now be tackled in this way. These repetitive problems are simpler in that the characteristics of a signal can be determined from past cycles and compared with that from the controller which can be modified slowly so as to match the basic waveform in anti-phase. In principle, there is no reliance on signal linearity for such empirically formed copies of the primary field to counter the primary signal. But there can be no assumption that combined opposite waveforms will always interfere destructively if the system is not linear, and so there might be intense periodic noise problems that remain immune to these active suppression techniques. Some intense periodic sound fields develop shocks and those are unlikely to be alleviated by any wave superposition device.

Related developments

Any linear field that can be monitored, processed and simulated by secondary sources is, in principle, amenable to control and modification by the techniques of anti-sound. Visual images and radio or radar fields all occur on too short a time scale for the complete control processing of the kind used in 'anti-sound' to be feasible with normal electronic equipment, though active adaptation to mimic and control some simple slowly evolving high frequency radar fields might be quite another matter. The goal set for effective anti-sound systems is that control is established well within the period of one oscillation, and that currently limits the scope of the subject to frequencies lower than about 2 kHz. The one-dimensional controller illustrated in figure 11 has a broad band (200–1600 Hz) sound attenuation that is limited at high frequencies by the computational speed of the 12 bit, 256 co-efficient digital filter.

The vibration of mechanical structures is often linear and occurs in a controllable frequency range, so that is a natural application for these techniques.

The experimental rig demonstrated in this lecture and illustrated in figure 12 is designed to show how periodic vibrations caused by an engine (in this case an electric motor) can be isolated from the structure on which the engine is mounted.

The rig basically consists of a rectangular frame, four electromagnetic vibrators (shakers), a shaft encoder and an electric motor with an eccentric weight mounted on its shaft to induce vibrations. These are detected on the frame by accelerometers, whose signals are monitored by the computer that controls the current of the four shakers. Since the vibrations are periodic a shaft encoder is used to provide a synchronizing signal. This enables the cancelling vibrations, introduced below the motor mounts by the shakers, to be coincident, and therefore interfere destructively, with the primary vibrations. The cancelling vibrations are controlled by an algorithm which continually adapts the shaker currents to minimize the structural vibrations being monitored by the computer.

This technique has been developed for large diesel generators and offers a practical solution in cases where either low frequency structureborne or airborne periodic noise is a problem.

Similar systems in which engine mounts have been stabilized by individual single channel control systems have been studied by Chaplin (1983) in cases where cross-talk between the channels did not lead to system instability. But in the more general case where mounts are highly coupled to each other, a multi-variable controller is called for, an example of which is illustrated here.

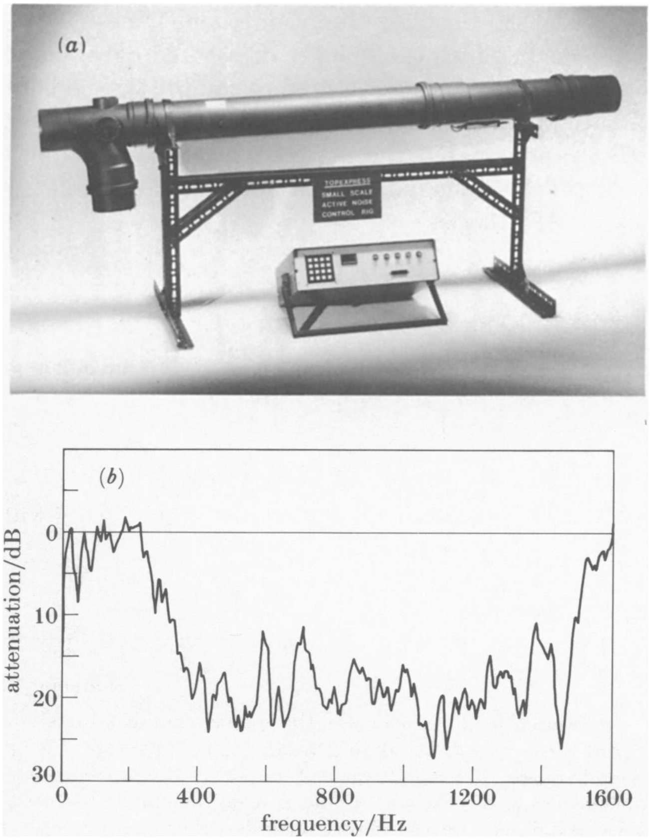

Figure 11. (*a*) A one-dimensional broad band controller for noise in a simple duct. (*b*) Performance of the controller pictured in (*a*).

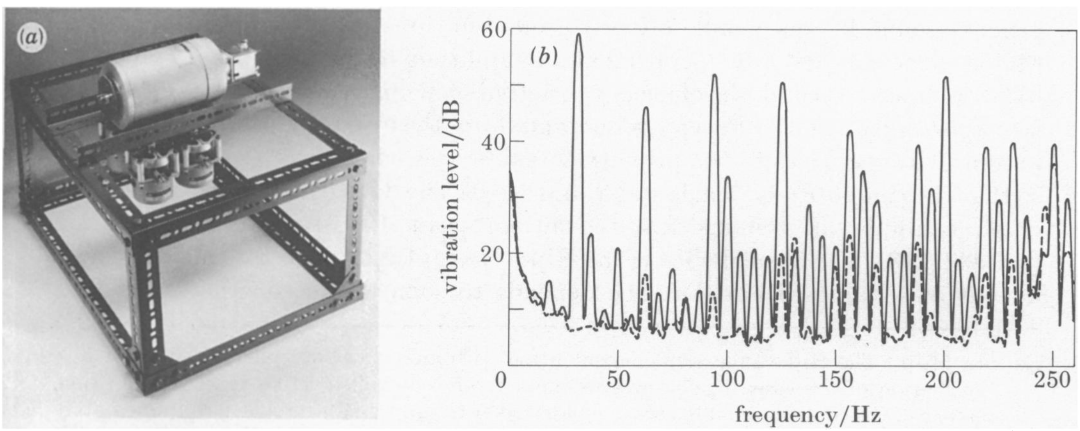

Figure 12. (*a*) A demonstration four-channel controller for isolating machinery noise. (*b*) Performance of the rig pictured in (*a*).

The unsteady burning in a turbulent flame is a process which produces both a noise field and an associated unsteady light emission. Dines (1984) describes an interesting development in which the quickly monitored light emission is used as a signal for the simultaneously produced noise of combustion. A controller adapts this signal to generate at a loudspeaker the antidote of the combustion noise. In this way Dines (1984) attenuated the noise of a turbulent flame over a useful frequency range. Figure 13, which indicates the performance of the device, is taken from his Cambridge Ph.D. thesis.

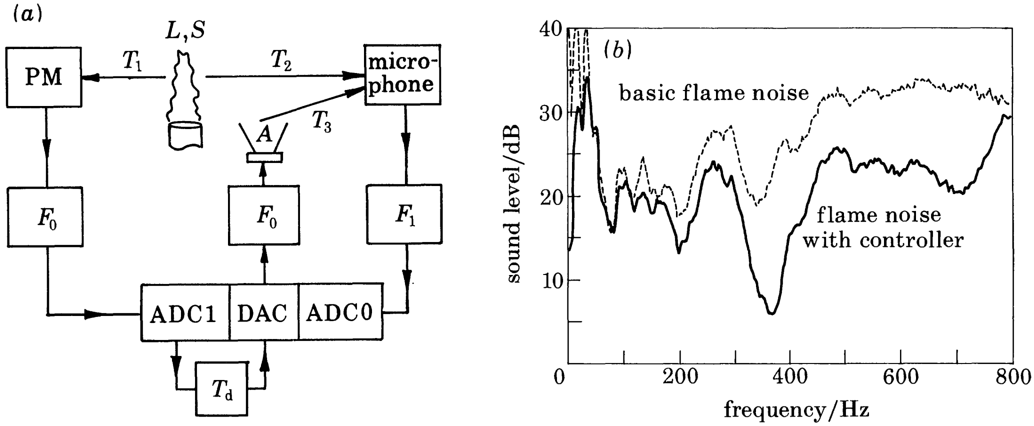

FIGURE 13. (a) A diagrammatic illustration of the experiment conducted by Dines: ADC, analogue to digital converter; DAC, digital to analogue converter; PM, photomultiplier; A is the anti-sound source; F_0 is the transfer function of the anti-aliasing filters; L is the light signal from the flame; S is the sound output from the flame; T_1 and T_2 are the transfer functions between the flame and the photomultiplier, the microphone, respectively; T_3 is the transfer function between the anti-sound loudspeaker and the microphone; T_d is the desired transfer function, $T_d = -ST_2/LF_0^2T_1T_3$. (b) The performance of the controller in the experiment depicted in (a).

A related development in which unsteady combustion is the basic source of the primary sound breaks completely new ground in three respects. First, the system that is silenced is actually operating at an amplitude too large for linear processes to be dominant; second, the efficiency of active silencing is infinite in that the noise is *completely* annulled by the controller; and third, the instability of the combustion process with gave rise to the primary sound field is controlled by the anti-sound so that the composite system is stable and silent. The device illustrated in figure 14 is an example of systems featured in a recent British patent application (no. 8329218). The basic device is the Rijke (1859) tube where the unsteady heat output of a flame within the tube depends to some degree on the sound field

FIGURE 14. (a) The Rijke tube arrangement fitted with an active controller. (b) A schematic illustration of the essential features in the actively controlled Rijke tube. (c) The time history of the control signal fed to the loudspeaker to suppress the oscillation. Immediately following the application of control, the signal is high but quickly falls as the process is stabilized. (d) The time history of the sound field immediately before, during and after the application of control.

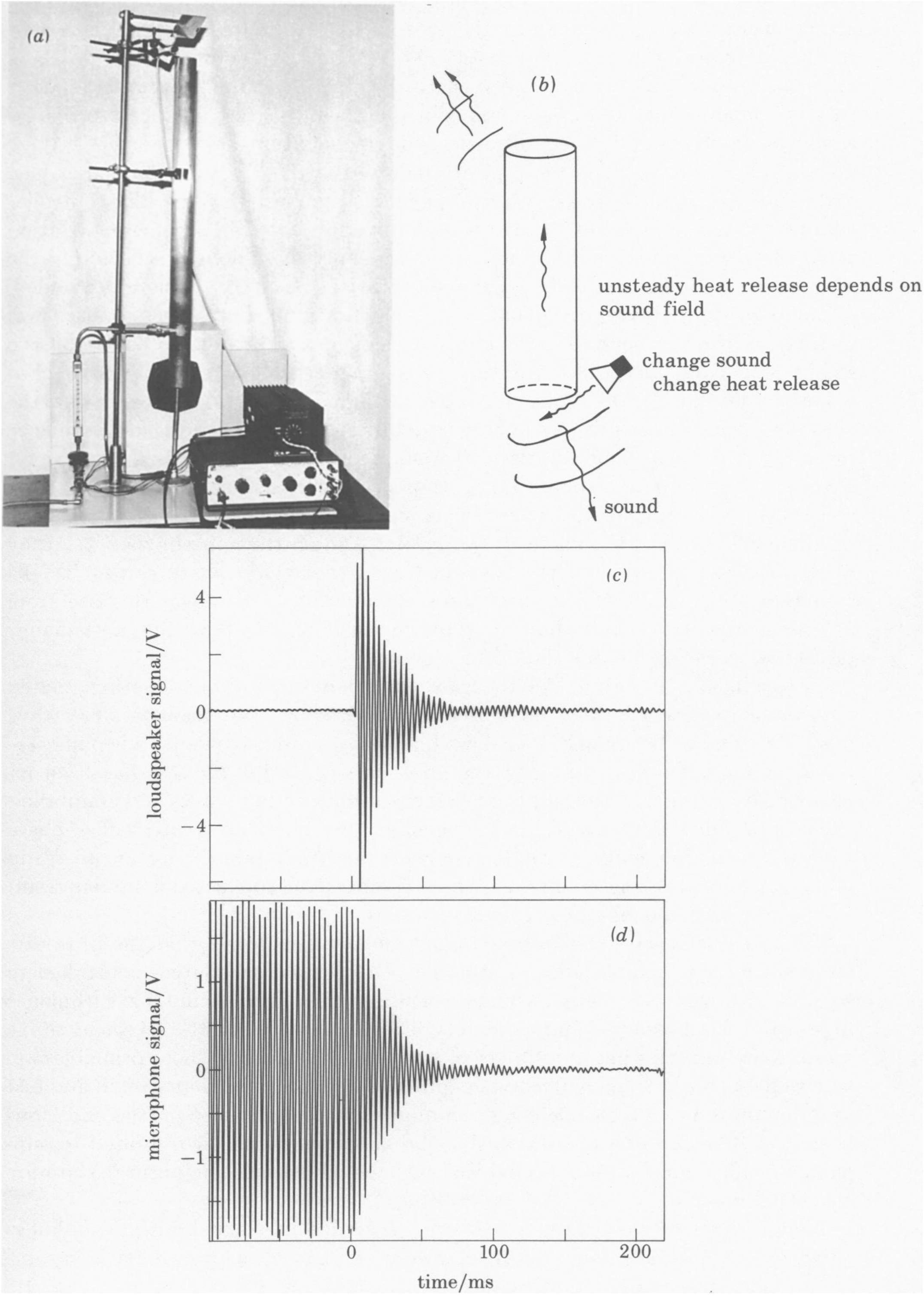

Figure 14. For description see opposite.

produced in the 'organ-pipe' resonator by that flame. When the geometrical arrangement is such as to enhance the heat output at the high pressure phases of the sound wave, then a noisy instability results, the amplitude of which is determined by a nonlinear balance of energy production at the flame and radiation losses through sound. The anti-sound controller is in this case arranged to change significantly the boundary conditions at the open end of the 'organ pipe'. With no control, sound waves in the pipe are reflected at the 'constant pressure' open end to produce an out of phase wave propagating back into the pipe. The controller arranges that the pressure at the open end is maintained in a coherent relation to the emergent sound wave which, in consequence, is no longer reflected in the resonance-producing phase of the natural condition; resonance is thereby avoided.

There are probably many situations in which a fluid flow possesses a natural instability in which sound is a constituent element and in which a control of the sound field would lead to significant changes in the flow characteristics. For example, jet flows are highly unstable at high Reynolds numbers, but the turbulence that results from the flow instability, and the sound that turbulence produces, is known to be sensitive to weak acoustic disturbances (Moore 1977; Bechert & Pfizenmaier 1975). When those acoustic disturbances are carefully controlled in amplitude and phase, then significant control can be obtained on particular features of the unsteady jet motion and on the noise of the jet. Arbey & Ffowcs Williams (1984) have demonstrated that this control can in fact be exercised on parts of the natural jet response that are known to arise from nonlinear effects, so that we have here another instance of interesting 'anti-sound' processes operating outside the linear régime.

In conclusion, I should like to draw attention to the very exciting related developments in which this kind of technique is used on a non-acoustic weak wave in a moving fluid. Boundary layer flows break down into turbulence when linearly unstable Tollmien–Schlichting waves grow exponentially. Success has been reported in experiments that monitor the incipient instability waves and annul their effect by the deliberate creation of a secondary wave that is exactly out of phase with the first. Liepmann *et al.* (1982) controlled the stability by an unsteady boundary-layer heating technique, while Thomas (1983) effected a similar result using vibrating ribbons as the source of waves.

These are still early days in the subject but it seems to me that the prospects are good for the maintenance of stability with secondary sources controlled to counter disturbances which would otherwise grow into sound or turbulence (figure 15). That aspect could have very far reaching effects. It also seems to me appropriate in reviewing the subject of anti-sound, a subject whose principles are now well established and which awaits practical applications that stretch and test its robustness as a viable silencing technique, to speculate also on the enormous benefit that might accrue from anti-sound elements being incorporated to supplement and render stable mechanical systems which in their natural state are chaotic and noisy.

Such control of processes, which are otherwise in some limiting nonlinear balance, by the addition of weak fields whose presence ensures stability is in some sense the ultimate anti-sound device: yet in another it is not there at all. Its

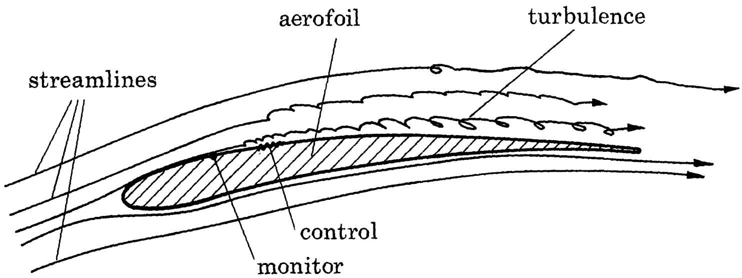

FIGURE 15. An illustration of the arrangement by which surface sensors and exciters might be so arranged as to inhibit the growth of Tollmien–Schlichting waves into downstream turbulence on an aerofoil.

presence has destroyed the primary source which would otherwise be maintained and render silent the conventional 'anti-sound' of its opposing pair!

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