Contents

1 What is vertex packing problem (VP)

The traditional vertex packing problem dened on an undirected graph identies the largest weighted independent set of nodes, that is, a set of nodes whose induced subgraph contains no edges.

2 The generalized vertex packing problem (GVP-k) that this article cares about

k edges may exist within the subgraph induced by the chosen set of nodes.

3 Some application

A particular context in which such problems arise is in the national airspace planning model

4 Introduction

G = (N, E), weighted c_j, for j = 1, $\cdots$, n.

4.1 Integer Programming Model (for VP)

$$\begin{aligned} \text{Maximize: } & cx \\ \text{Subject to: } & Ax \le e \\ & x \in \{0,1\} \end{aligned}$$

1. A is a p $\times$ n matrix a_{hi} = 1 means vertex i $\in$ edge h
2. e is a all-one vector.
3. $\Rightarrow$ Ax $\le$ e means that each edges 2 end-points should not be in the answer x simutaneously.

4.2 Prefect Graph

chromatic number = maximum clique cardinality (for each G' $\subseteq$ G)

4.3 Integer Programming Model (for GVP-k)

$$\begin{aligned} \text{Maximize: } & cx \\ \text{Subject to: } & \sum_{(i,j)\in E} z_{ij} \le k \\ & z_{ij} \ge x_i + x_j - 1, z_{ij} \ge 0 \\ & x_j \in \{0,1\}, \forall j \in N \end{aligned}$$

note that edge (i, j) is in the answer when $z_{ij} = 1 = x_i x_j$

5 TODO Facets and partial convex hull representations for GVP-k

Proposition 1: Consider a graph G and a subgraph $\hat{G}$ of G. If Dx $\le$ d represents a set of valid inequalities for GVP-k dened on $\hat{G}$, then Dx $\le$ d is valid for GVP-k dened on G.

Proof: Since GVP-k for $\hat{G}$ is a relaxation of GVP-k dened on G, the restrictions that govern a feasible generalized vertex packing solution on $\hat{G}$ are a subset of those valid for G. This completes the proof.

Note: You could use less restrictions on $\hat{G}$ than on G (maybe a subset).