

proof of 2-approximation guarantee

a neat argument due to [Khuller and Saha, 2009]

- let S^* be the vertices of the optimal subgraph
- let $d(S^*) = \lambda$ be the maximum degree density
- notice that for all $v \in S^*$ we have $\deg_{S^*}(v) \geq \lambda$
- (why?) by optimality of S^*

$$\frac{|e(S^*)|}{|S^*|} \geq \frac{|e(S^*)| - \deg_{S^*}(v)}{|S^*| - 1}$$

and thus

$$\deg_{S^*}(v) \geq \frac{|e(S^*)|}{|S^*|} = d(S^*) = \lambda$$