Contents

1 motivation

2 measure of density

2.1 edge density

$d(S) = \frac{2|E(S)|}{|S|}$

2.2 edge ratio

$\delta(S) = \frac{|E(S,S)|}{\binom{|S|}{2}}$

2.3 triangle density

$t(S) = \frac{|T(S)|}{|S|}$

2.4 triangle ratio

$\tau(S) = \frac{|T(S)|}{\left(\begin{array}{c} |S| \\ 3 \end{array} \right)}$

2.5 k-core

every vertex in S is connected to at least k other vertices in S.

2.6 α-quasiclique

$E(S) \geq \alpha \left(\begin{array}{c} |S| \\ 2 \end{array} \right)$

2.7 k-clique

subset of vertices with pairwise distances at most k

- distances defined using intermediaries, outside the set
- not well connected

2.8 k-club

a subgraph of diameter $\leq k$

2.9 k-plex

a subgraph S in which each vertex is connected to at least $|S| - k$ other verteces.

- 1-plex is clique

3 Densest Subgraph

3.1 Undirected Graph

3.1.1 Goldberg's Algorithm

consider: is there a subgraph S with $d(S) \geq c$?

$$\frac{2|E(S,S)|}{|S|} \geq c \tag{1}$$

$$2|E(S,S)| \geq c \tag{2}$$

$$\sum_{u \in S} \deg(u) - |E(S,\bar{S})| \geq c \tag{3}$$

$$\sum_{u \in S} \deg(u) + \sum_{u \in \bar{S}} \deg(u) - \sum_{u \in \bar{S}} \deg(u) - |E(S,\bar{S})| \geq c \tag{4}$$

$$\sum_{u \in \bar{S}} \deg(u) + |E(S,\bar{S})| + c|S| \leq 2|E| \tag{5}$$

transform this constraint to min-cut problem: how to transform

3.1.2 Greedy Algorithm

Algorithm Proof1 Proof2