RESEARCH STATEMENT XIAOYU CHEN

My research interests lie in theoretical computer science, an area pursuing understanding the boundary of computation. One of the most famous problems in this area is the P vs NP problem. It asks whether problems that can be verified[1] in polynomial time can also be solved in polynomial time. It is equivalent to a very philosophical question in mathematics: if a theorem is known to have a short proof, can we actually find that proof by using a computer efficiently?

To investigate the P vs NP problem, it is sufficient to understand the hardest problems in the NP class, which are called NP-complete problems. Independent set decision problem is one of the most famous NP-complete problem. It asks whether a given graph G has an independent set of size k. When I was an undergraduate student, my co-authors and I tried to design a fast algorithm for the weighted version of this problem[2] on graphs with small degree. We obtained an exact algorithm that runs in time $O^*(1.1443^n)$ [1], by exploring the combinatorial properties of independent sets and then doing an exhausted branch-and-bound search over the whole state space.

When I became a Ph.D. student, I was introduced to the area of sampling and counting. I am particularly interested in the Markov chain Monte Carlo (MCMC) methods. It runs a Markov chain in the solution space for sufficiently many steps to produce a random sample, which, ideally, is very close to the target distribution. Such convergence behavior is called *mixing*, and the time required for a "good enough" mixing is called the *mixing time*. Investigating the mixing behavior of Markov chains not only gives fast sampling algorithms but also provides a deep understanding of the structures of the solution spaces.

With this motivation, I am particularly interested in understanding the behavior of MCMC-based samplers for the distributions defined on the solution space of hard problems. Such distributions are also well-known in statistical physics and probability theory, where they are called *graphical models*, also known as *Gibbs distributions*, *Markov random fields*, or *spin systems*. Typically, given a graph $G = (V, E)$, I want to understand the behavior of the MCMC-based samplers on the following distributions:

- The Gibbs distribution μ of the **hardcore model** with fugacity $\lambda > 0$ is supported over independent sets of graph G. The probability of each independent set S is $\mu(S) \propto \lambda^{|S|}$;
- The Gibbs distribution μ for **q-coloring** is defined as the uniform distribution over all the vertex proper q-colorings of the graph G.

The size of the state space is exponentially large in the size of the graph, making sampling from such distributions computationally challenging. The most interesting part of these distributions is that they admit a sharp computational phase transition phenomenon — the computational complexity of sampling changes sharply around certain critical values of the parameters (e.g., λ or q).

Among all the MCMC-based samplers, the *Glauber dynamics* (a.k.a. Gibbs sampling) is of

[1] Given an answer, check if it is correct

[2] Decide if a given graph G has an independent set S whose weight $\sum_{v \in S} w_v$ equals to a given number

special interest. Because it is arguably the most natural and simple Markov chain for the Gibbs distributions. When specified to the hardcore model, it works as follows. It starts from an arbitrary independent set I of the graph. Then, it updates the independent set I by doing the following steps repeatedly:

1. picking a vertex v from the graph uniformly at random;
2. if v has a neighbor u in the independent set I, do nothing;
3. otherwise, add v to I with probability $\frac{\lambda}{1+\lambda}$; and remove v from I with probability $\frac{1}{1+\lambda}$.

My research mainly focuses on understanding the mixing time of the Glauber dynamics on the Gibbs distributions. In particular, My goal is to **understand when MCMC algorithms are fast and to establish optimal bounds on the mixing time in these regions**. This includes two main streams: (1) introducing the **field dynamics** as a new tool to establish sharp mixing time bounds for the Glauber dynamics of anti-ferromagnetic two-spin systems (such as the hardcore model) on general graphs below and at the critical threshold (i.e., the threshold where the computational phase transition happens); (2) generalize such mixing time bounds from two-spin systems to multi-spin systems (such as the proper q-coloring) via a new technique called the **coupling independence** and gain a better understanding of the role of randomness.

Sharp mixing time for critical two-spin systems

The hardcore model has a critical threshold $\lambda = (\Delta-1)^{\Delta-1}/(\Delta-2)^{\Delta}$ for the fugacity λ. When $\lambda > \lambda_c(\Delta)$, Sly showed that the approximate sampling from the hardcore model becomes NP-hard assuming NP $\neq$ RP [Sly10]. Below the critical threshold where $\lambda < \lambda_c(\Delta)$, Weitz gave the first polynomial-time sampler for the hardcore model [Wei06]. This identifies a fascinating computational phase transition at the critical threshold $\lambda_c(\Delta)$.

My research completes the missing part of this computational phase transition, **by proving a polynomial mixing time for the Glauber dynamics when $\lambda = \lambda_c(\Delta)$ exactly.**

To obtain this result, it is very important to understand the mixing time of the Glauber dynamics in the near-critical regime where $\lambda \leq (1-\delta)\lambda_c(\Delta)$ with the slackness $\delta \in (0,1)$. Indeed, if δ is sufficiently small (e.g., $\delta = 1/n$), then the model is believed to be "similar" to the critical hardcore model.

However, we have no idea whether the Glauber dynamics mixes rapidly when $\delta = 0$ and $\lambda = \lambda_c(\Delta)$ from previous works. In fact, a poor dependency on the slackness δ and the maximum degree Δ appears in all previous algorithm approaches. As a consequence, all previous approaches work only when the slackness δ is bounded away from zero:

- All the previous deterministic counting algorithms, including Weitz's algorithm [Wei06] and Barvinok's polynomial interpolation method [Bar16, PR19], run in time $n^{O((\log \Delta)/\delta)}$ where n is the number of vertices.

- The mixing time of the Glauber dynamics is recently proved to be bounded by $n^{O(1/\delta)}$ and $\Delta^{O(\Delta/\delta)} n \log n$ [ALO20, CLV21] via the novel *spectral independence* technique.
- The hardness result in Sly's work requires λ having a constant gap above $\lambda_c(\Delta)$ [Sly10].

As for the critical hardcore model where $\lambda = \lambda_c(\Delta)$, we barely know anything, either algorithmic results or computational hardness.

My co-authors and I introduced a novel Markov chain sampler called the **field dynamics**. Given a parameter $\theta \in (0,1)$, it is a global Markov chain that produces the next random independent set J from the current independent set I by:

1. drop each vertex in I with probability $1-\theta$, independently;
2. sample J from the hardcore model with fugacity $(1-\theta)\lambda$ conditioning on $J \supseteq I$.

Compared with the Glauber dynamics, the field dynamics enjoys many nice properties. (1) We are still able to use the spectral independence technique to deduce rapid mixing of field dynamics. Moreover, unlike [CLV21], our approach for field dynamics does not depend on the graph structure, and in particular, the mixing time has no dependency on the maximum degree of graphs. (2) The field dynamics, by definition, naturally relates the hardcore model with large fugacity λ to one with small fugacity $(1-\theta)\lambda$. This enables us to boost the mixing properties of Glauber dynamics from small fugacity to large fugacity, including the critical point. (3) The spectral independence approach and the boosting technique allow us to obtain sharp mixing time upper bounds for Glauber dynamics in the form of $\mathrm{poly}(\frac{1}{\delta}) \cdot n \log n$. For the critical hardcore model, in a line of work [2, 3, 4], this gives a $\mathrm{poly}(n)$ mixing time which is the first polynomial-time algorithm at the critical threshold.

The field dynamics is not only helpful for establishing rapid mixing of Glauber dynamics but is an important algorithmic tool itself. Indeed, we are able to generate random samples by implementing the field dynamics efficiently. My co-authors and I also use the field dynamics to (1) refine the uniqueness regime of the bipartite hardcore model ($\#\mathsf{BIS}$) and obtain a near-linear time MCMC-based sampling algorithm in such uniqueness regime [5]; (2) obtain a near-linear time MCMC based sampling algorithm for the ferromagnetic Ising model with external field [6].

Coupling independence for mixing time analysis

Despite the field dynamics serves as a great tool for understanding the critical hardcore model. We do not know how to generalize it to multi-spin systems, so we can also investigate the critical behavior of multi-spin systems, such as the proper q-coloring.

The notion of *spectral independence* (*SI*) plays a central role in analyzing the mixing time of Markov chains such as the Glauber dynamics [ALO20]. In a nutshell, it measures how similar a distribution μ looks to a product distribution by considering the maximum eigenvalue of the covariance matrix of μ. It can also be generalized to general (multi) spin sys-

tems such as q-coloring [CGŠV21, FGYZ22]. However, the eigenvalue nature makes the spectral independence lack intuition. In many applications, spectral independence is often proved by establishing a stronger condition, such as considering the infinity norm of the covariance matrix.

My co-authors and I introduced a new structure-property of high-dimensional distributions, which we called the **coupling independence (CI)** [6, 7]. It measures the stableness of the distribution under small perturbations (i.e., fixing one coordinate with different values) by considering the 1-Wasserstein distance. The coupling independence property is strong enough to immediately imply spectral independence and, meanwhile, is also universal and holds in many spin systems of considerable interest. In fact, many proofs for spectral independence either are obtained by establishing coupling independence [Liu21, BCC$^+$22, CG24, CLMM23][6] or can imply it as a byproduct [CLV20][7].

Since its introduction, coupling independence has achieved great success in proving spectral independence for general multi-spin systems due to its simple combinatorial nature, including random cluster model [6], colorings [CLMM23], b-matchings [CG24]. It also generalizes previous notions of disagreement percolation for proving Gibbs uniqueness [vdB93] and recursive coupling for showing strong spatial mixing [GMP05].

In my recent work, my co-authors and I have managed to show that **coupling independence implies near-linear running time for MCMC-based samplers** [7]. It also implies near-linear mixing time for Glauber dynamics on monotone systems. This result does not require the underlying graph to have a bounded maximum degree, which is very similar to what we can obtain from the field dynamics on two-spin systems.

Besides the mixing time, the coupling independence also has rich applications. In my recent work, my co-authors and I built **a deterministic approximate counting algorithm for general distribution satisfies coupling independence** [8]. Using this new algorithm, we give a deterministic approximate coupling algorithm for q-coloring when $q/\Delta \geq 1.803$, which matches the latest regime achieved by MCMC-based samplers [CV25]. Previously, a deterministic algorithm is known only when $q/\Delta \geq 2 - \varepsilon$ [BBR24] for some $\varepsilon \approx 0.002$.

Weitz's algorithm for two-spin systems constructs self-avoiding walk trees and then calculates the marginals via the tree-recursion [Wei06]. Our algorithm can be seen as a generalization of Weitz's algorithm, where we generalize the self-avoiding walk tree to the computation tree of some coupling procedure and generalize the tree recursion to an implicit recursion given by the coupling. The coupling independence criterion here provides an exponential decay of the relative error (similar to the strong spatial mixing). We then implement such implicit recursion by using linear programming.

In subsequent work, my co-author and I also find that the **coupling independence criterion has a good compatibility with the canonical path method**. We combine them together to overcome certain worst-case analyses in the original canonical path approach and get optimal mixing time for Glauber dynamics of edge-coloring on trees [9]. It makes the coupling independence an even more valuable tool to investigate.

Future works

New ways to establish spectral independence Currently, there are known examples of distributions where standard approaches for establishing spectral independence fail. Examples include proper q-coloring (SAW tree fails); monomer-dimer model[3] (coupling independence fails) and random cluster model with parameter $1 < q \leq 2$ (log-concavity fails). I want new methods for establishing spectral independence on these distributions.

I am particularly interested in the spectral independence of the monomer-dimer model. The spectral independence of the monomer-dimer on multi-graphs is proven to be $\Omega(\sqrt{\lambda\Delta})$. Meanwhile, it is conjectured that the spectral independence of the monomer-dimer model on the simple graph should be $O(\sqrt{\lambda})$. So, a candidate proof for the $O(\sqrt{\lambda})$ SI should only work on simple graphs and fail on multi-graphs.

For proper q-coloring, I think a good starting point is to find an explicit coupling procedure when $q/\Delta \geq 11/6$. We know coupling independence holds for such regime by an existence proof [Liu21, BCC+22] levering the path coupling for the flip dynamics [CV25]. It would be a great advance if there is an explicit coupling for coupling independence to work with.

Polynomial dependency of SI/CI in mixing time. Polynomial dependency of SI/CI in mixing time (e.g., $\text{poly}(\frac{1}{\delta})n \log n$ mixing time for the Glauber dynamics on hardcore model, where $\text{SI} = O(\frac{1}{\delta})$) plays an important role in establishing polynomial mixing time in the critical threshold. Such dependency also matches the lower bound of the mixing time suggested by the universality result of spectral independence [AJK+24]. However, the current technique of spectral independence only gives an exponential dependency in general. I want to know if there is a general framework for establishing such polynomial dependency of spectral independence/coupling independence in mixing time under some mild assumptions. I think a good point to start this problem is by considering the connection between the spectral independence/coupling independence with the canonical path method, where the dependency of parameters is usually of polynomial magnitude.

A probabilistic local-to-global argument based on coupling independence. The coupling independence criterion implies the mixing of the uniform block dynamics directly via a simple path coupling argument. And eventually, it could imply the near-linear mixing of a censored Glauber dynamics. The whole proof is probabilistic and does not involve functional analysis [7]. I want to know if there is a probabilistic approach for proving the mixing time of the Glauber dynamics assuming coupling independence. Such an approach is likely to exist since we already know the mixing of the censored Glauber dynamics. It would be very interesting since all the currently known probabilistic approaches for proving the mixing time (such as path coupling of Markov chains) only work in a subcritical regime. Moreover, having such an approach will benefit us in the sense that it is more intuitive and direct to work with compared to the entropy approach.

[3] It can be considered as the Glauber dynamics on the line graph

My related works

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